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Mathematical argumentation of three-year-old children on a classroom experience based on two STEAM domains

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Abstract

In the last decades, the importance of argumentation as a human competence in general and its close relationship with mathematical comprehension in particular has been highlighted. Thus, in this paper we focus our interest on analyzing the argumentation skills shown by three-year-old children in a STEAM classroom experience. For this purpose, a qualitative study was carried out. For data collection, a STEAM task has been designed in which mathematics and science are worked together from the study of the physical properties of water. Specifically, what color and taste of water has by means of two experiments and, subsequently, photos and video-recordings have been collected during the implementation carried out in a classroom with 20 children of three-year-old. All data collect have subsequently been transcribed and categorized. The results indicate that the children, during the experiments done, mainly use arguments based on mathematical language, numerical figures, or make use of words or spelling, although with considerable imprecision in the type of language used. In addition, the experiments carried out have allowed a high percentage of children to change their previous ideas and have a more comprehensive approach to the concepts introduced, highlighting the importance of working on argumentation in early childhood education through contexts, in this case STEAM, which encourage reasoning and proof.

Keywords: mathematical argumentation, reasoning and proof, STEAM activities, early childhood education

INTRODUCTION

Various research studies suggest the need to analyze in depth the benefits of the STEAM contexts in early childhood education and, in particular, with sustainable topics (Rodrigues-Silva & Alsina, 2023). Previous works such as Berciano et al. (2021), address the analysis of the impact of the inclusion of this type of contexts in the development of mathematical competence. This study proposes, based on an engineering problem adapted to the early childhood education classroom (with four- and five-year-old children), it is shown how it helps to work on different mathematical contents. Furthermore, during the process of finding a solution to the problem posed and assessing its suitability, children perform actions in which mathematical process standards are clearly involved, among others, communication and reasoning and argumentation. This highlights the need to design and include this type of STEAM practices in the classroom in order to develop mathematical competence. Along the same lines, the work of Alsina (2020) establishes a link between engineering and mathematics in which a bridge construction proposal is described, from observation in the context, through design and ending with construction.

Despite these approaches to the problem, there are still few studies that analyze how to design and implement contexts that help to successfully work on mathematical process standards in early childhood education classrooms at younger ages, and therefore, the main objective of this research is to explore and promote the mathematical argumentation skills of three-year-old

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Contribution to the literature

- This research raises the need to analyze the inclusion of teaching practices based on STEM fields in the early ages, which significantly interrelate mathematics and science.
- It analyzes the type of mathematical argumentation that 3-year-old children show in an activity specifically designed from this perspective; highlighting the use of arguments based on mathematical language, numerical figures, or make use of words or spelling.
- The work is a starting point for the design of new activities that generate mathematical argumentation by the youngest students.

children in a childhood education classroom, that is, to explore whether mathematical argumentation emerges in three-year-old students in an early childhood education classroom from tasks designed that involve two STEAM domains, mathematics and science. To this end, after designing and implementing an activity of this type in the classroom, we aim to find out:

(1) the type of initial mathematical argumentation, if any, when the teacher asks them about the properties of water in a contextualized learning situation and

(2) the type of argumentation made by the children, at the end of the classroom experience, after carrying out and experimenting with tasks designed to work on the process of reasoning and proof.

THEORETICAL FRAMEWORK

Argumentation

Current science education emphasizes the development of argumentation skills among students (Marthaliakirana et al., 2022) and has been widely defined among scholars of teaching and learning, especially in relation to mathematics. In general, the proposed definitions are grouped around two broad categories. In the first, it is posited from a social perspective (Kosko et al., 2014; Krummheuer, 2007; Lin, 2018) and understands argumentation as a process of debate; while, in the second, it is posited as a cognitive process responsible for providing evidence as an indicator of assurance to support claims (Osborne et al., 2016). Conjugating both perspectives, argumentation could be considered as the type of discourse by which empirical and theoretical reasons and evidence are presented to support an opinion (Erduran & Jimenez-Aleixandre, 2012), that is defended and validated during the process of debate with others. Argumentation involves not only presenting arguments in favor of a position, but also anticipating and responding to possible objections or counterarguments. It seeks to refute or weaken possible criticisms and strengthen the defended position. It is therefore a crucial skill in the field of critical thinking and effective communication.

Argumentation is not an innate skill, but a competence that must be trained. The teaching and learning of argumentation follow the structure of science education, being based on evidence, justification of claims and effective communication of ideas. Thus, learning argumentation requires practices specific to science that provide the structure, motivation, and modes of communication necessary to support scientific discourse (Erduran et al., 2015).

Argumentation in Mathematics

Mathematical argumentation has gained great interest in the field of mathematics education in recent years. Developing the ability to carry out sound argumentation is essential for generating, establishing and communicating mathematical knowledge (Lin, 2018; Stylianides, 2007), to give meaning (Douek, 2007) and to favor the construction of evidence (Pedemonte, 2007). Likewise, argumentation plays a crucial role in fostering students' learning ability and conceptual understanding to achieve deep learning (Hanna & De Villiers, 2012; Nickel, 2019; Kanellos et al., 2013; Krummheuer, 2007), to reason and challenge, and to achieve a goal (Andriessen et al., 2003).

In mathematics education, it is common to find limitations in the use of argumentation (Tristanti, 2019), which can be explained because throughout the different stages of training, students are usually only asked to solve problems directly without going through a process of argumentation (Tristanti & Nusantara, 2022). In this sense, teachers must create classroom environments that promote the exchange of ideas in order for students to be critical (Noroozi, 2023) and to justify, challenge their mathematical thinking (Andriessen et al., 2003) and develop their reasoning skills (Krummheuer, 2007), increasing their understanding by sharing and defending their ideas with their peers and striving to try to convince each other and arrive to a conclusion (Lin, 2018; Noroozi, 2023).

Thus, in recent years a fundamental role has been given to communication in the classroom. Through it, students reflect, analyze, exchange ideas and opinions and construct meanings by establishing connections, which are implicit in the structure of the argument and are essential for its identification (Rodríguez-Nieto et al., 2023). It also carries out a process of confrontation of hypotheses, which, on the one hand, will allow them to express them in mathematical language.

On the other hand, the fact of having to justify one's own solution to a problem, with the possibility of other classmates disagreeing, enhances a better understanding of the issue by all students as they have to analyze the problem carefully from several points of view in order to reach a final conclusion.

Mathematical Argumentation in Early Childhood Education

Entities that carry out studies for the improvement of teaching practice, such as National Council of Teachers of Mathematics (NCTM) in the United States, suggest that it should be an objective of mathematics education to help students to participate in the production of mathematical arguments, consolidating argumentation as a learning opportunity available from the beginning of schooling (Cervantes-Barraza & Cabañas-Sánchez, 2018).

From the mathematics education, different aspects of children's mathematical argumentation have been studied, due to the importance in the development of mathematical thinking and in the formation of autonomous, critical and reflective citizens from the earliest ages (NCTM, 2003). In this sense, NCTM (2003) proposes a total of five process standards that favor the teaching-learning of mathematics in a meaningful way among which reasoning, and proof and communication stand out. The first, reasoning and proof, has as its objective that students, by means of mathematical justifications, be able to assess whether their arguments are valid or not, and for this purpose, the need to carry out different verifications or proofs of these facts. The second, communication, highlights the need to communicate mathematical ideas through language (verbal or not), that is, to interact with peers and teachers, thus establishing a clear and direct connection between the development of oral or linguistic competence and mathematical competence. The argumentative mathematical strategies used by students or the development of the ability to argue refer to structural, social and logical-semantic aspects (Boero, 2011). These aspects in the early ages are very complex to identify and evaluate, since students express their ideas using short and simple sentences, little elaborated and in an unstructured way; or through non-verbal actions, changing the focus quickly depending on the stimuli (Alsina et al., 2021). For this reason, detailed observation and repetition of tasks are key techniques for argumentative analysis in the early ages. In this line, Cornejo-Morales et al. (2021) present a model for analyzing argumentation in early childhood education, which, from an integrative approach, considers five components in argumentation: argument (what is argued? and why?); interaction (who argues?); functions of argumentation (what is it argued for?); character of the argument (how is it argued?); and Mathematics (what is argued about?).

On the other hand, for the development of mathematical argumentation in childhood, the role of adults is crucial (Nergard, 2023) and for mathematical learning to be meaningful, it must be treated in a holistic way. The contents must be worked on through problem-solving processes and reasoning through classroom activities that encourage the construction of knowledge, and the posing of problems, where varied strategies can be applied and adapted (Berciano et al., 2017). In this teaching and learning context, teachers should plan and manage classroom activities considering mathematical processes such as reasoning and proof (Alsina, 2016), motivating students to reflect on their answers, so that they provide explanations and justifications that allow them to understand the true meaning of mathematics (NCTM, 2003; Salgado et al., 2020). In the early ages, active methodologies such as project-based learning, creating situations of experimentation and play, can favor reasoning and proof more than other activities of repetition and practice (Alsina, 2016). Thus, in this line, it can find works that seek to analyze the impact of the design of the context on the learning process and its reasoning in early childhood education. For example, Berciano et al. (2022) in which, based on a teaching experiment designed to work on concepts associated with spatial notions of three-dimensional objects, in this case, the cylinder, analyze the type of geometric reasoning and apprehension that four- and five-year-old children show in a context that seeks to justify their statements and experiment with manipulative material to test their hypotheses. This work shows that the context designed helps them to move to a higher level of geometric reasoning, which helps them to understand in a more meaningful way the relationship between the characteristics of Euclidean space and the Euclidean plane.

STEAM Educational Approach

Restricting our interest to STEAM practices, we should mention that STEAM (science, technology, engineering, arts, and mathematics) is an educational approach that integrates the disciplines of science, technology, engineering, art and mathematics in an interdisciplinary and project-based approach. This approach seeks to foster active learning, creativity, problem solving, and collaboration among students. It focuses on the connection and practical application of concepts and skills in different areas of knowledge. Instead of teaching disciplines in isolation, it seeks to promote the integration of content and the realization of projects involving elements of science, technology, engineering, art and mathematics (López-Simó et al., 2020). Ocaña et al. (2015), based on Laboy-Rush (2011) and Mastascusa et al. (2011), make explicit as main benefits of STEAM methodology:

(a) transfer of knowledge and skills to real-world problems,

(b) increased motivation to learn, and

(c) improved transfers of acquired knowledge.

In addition, from the perspective of globality offered by STEAM, if scientific-technological areas are incorporated in the design of mathematical proposals, especially engineering and technology, this context will fundamentally allow trial-and-error proposals (Alsina, 2020), allowing students to play an active and meaningful role in the teaching-learning process (Couso, 2017).

Thus, various research studies suggest the need to analyze in depth the benefits of these STEAM contexts in early childhood education and, in particular, with sustainable topics (Rodrigues-Silva & Alsina, 2023). Previous works such as Berciano et al. (2021), address the analysis of the impact of the inclusion of this type of contexts in the development of mathematical competence. In this case, based on an engineering problem adapted to the early childhood education classroom (with four- and five-year-old children), it is shown how it helps not only to work on different mathematical contents, but also that, during the process of finding a solution to the problem posed and assessing its suitability, children perform actions in which mathematical process standards are clearly involved, among others, communication and reasoning and argumentation, thus highlighting the need to design and include this type of STEAM practices in the classroom in order to develop mathematical competence. Along the same lines, the work of Alsina (2020) establishes a link between engineering and mathematics in which a bridge construction proposal is described, from observation in the context, through design and ending with construction.

Despite these approaches to the problem, there are still few studies that analyze how to design and implement contexts that help to successfully work on mathematical process standards in early childhood education classrooms at younger ages, and therefore, as described in the introduction, the main objective of this research is to explore and promote the mathematical argumentation skills of three-year-old children in a childhood education classroom, that is, to explore whether mathematical argumentation emerges in three-year-old students in an early childhood education classroom from tasks designed that involve two STEAM domains, mathematics and science.

METHOD

In order to meet the proposed objectives described in the introduction, qualitative research has been carried out. A STEAM experience on the shape and taste of water has been designed and implemented to serve as a reference to study the argumentation capacity of three-year-old children along the experience. The methodology used for the evaluation of the activities, as well as of the project, has been based on the systematic collection of data information and the subsequent analysis; that is, photos and video-recordings have been collected during the implementation carried out in a classroom with 20 children of three-year-old, which have subsequently been transcribed and categorized. Furthermore, a second assistant teacher carried out a systematic observation with the aim of checking that all the children carried out the experiment according to the indications suggested by the other teacher. In this way, it was verified that the classroom experimentation was in line with the theoretical design, as well as ensuring that all the children had time to think about the question posed in each assembly, which triggered the experimentation, in order to, subsequently, be able to answer what they thought after observing and manipulating the physical models presented. Thus, the second teacher has been the guarantor that each child had the necessary time in the steps of the two experiments.

Participants

Two teachers (one of them assistant) and 20 children of early childhood education from a public school in the province of A Coruña (Spain), participated in the project. The selection of the participants, as well as the school, was intentional.

In relation to the teachers, the principal teacher designed and implemented the tasks carried out in the experiments and the assistant teacher was in charge of systematic observation throughout the process (as described in the previous section).

With regard to the children, it should be noted that they were all three years old at the time of the activity, had been attending school at the center for five months, and for 12 of them it was their first year at school, as only eight had previously attended the zero-three nursery school. Furthermore, none of the children had special needs, all the participants were in an evolutionary stage appropriate to their age, characterized by egocentrism, concrete thinking, where they only think what they perceive and their type of reasoning was transudative, which is defined by establishing analogies from the particular to the particular; their language was developing, it was telegraphic and the use of short sentences of three or four words predominated.

Description of the tasks

The project proposes a STEAM context in which the properties of water are analyzed, focusing on some of its physical characteristics (temperature, taste, weight, shape and color), thus, the project focuses on knowing what water is and what differentiates it from other elements around us. Over a period of two months, five

Table 1. Structure of designed project

Task	Objective	Classroom structure	Materials	Duration
1	Shape. Observe & distinguish different forms & their possibilities	Assembly & individual	Water & containers	5 lessons. 50 min each.
2	Taste of water, discriminate taste of water pending on different elements (minerals)	Assembly & individual	Water from different origins, glasses, solvents, paper, & markers	5 lessons. 50 min each.
3	Weight. Compare, estimate, & discriminate quantities	Assembly & individual	Water, bottles of different shapes & capacities, & scales (digital & analogue	8 lessons. 50 min each.
4	Introduce concept of temperature	Assembly & individual	Water, cups, microwave, thermometer, paper, & markers	8 lessons. 50 min each.
5	Color. Identify colors & their properties	Assembly, small group, & individual	Water & translucent colored glasses	4 lessons. 50 min each.

activities have been carried out, presented as experiments, through which science and mathematics have been linked. Specifically, for each of the attributes, a question was posed to the children to encourage them to answer and/or put forward a hypothesis, in order to subsequently, through experimentation and manipulation, collect data that would help them to validate or not their hypotheses, explain events, carry out verifications, etc., with the aim of acquiring knowledge and skills applicable to other contexts and phenomena. Thus, the characteristics of the activities, together with the teaching objectives, can be consulted in **Table 1**.

Even so, in this paper we only focus on the attributes of form and taste, i.e., that it is a liquid and tasteless element. These two attributes have been selected for the analysis out of the five existing ones, because they are the ones with the greatest argumentative presence.

Experiment 1: Shape of water

The aim of this experiment is for the children to realize that the shape of water is not predetermined, but that it needs a container in order to be handled, and therefore depends on the shape of the container for its subsequent handling. The aim is to break with the preconceived idea that every object has a predetermined shape and to introduce the concept of liquid. This consists of four distinct phases of implementation:

1. In the assembly, the question that triggers the first experiment is presented: What is the shape of water and why?

   At this point, the aim is for the children to make a first approach to their ideas and preconceptions and to come up with their first hypotheses.

   The answers given by the children are then taken up by the teacher as possible hypotheses for the purpose of experimentation in the class group. First of all, she poses questions in an attempt to generate doubts among the pupils, and then invites them to corroborate, to think about how to find out who is right or whether they are all right. The teacher then asks them the question: Where do you see water? So that, through their answers, different objects containing water emerge and they are invited to observe and discriminate what they look like.

   The teacher talks to the pupils, explains that the objects mentioned are many and of different sizes, that not all of them can be brought into the classroom and guided by the teacher, they agree on and select three of them to manipulate and experiment with.
2. The three selected containers with different geometric shapes are presented and filled with water. The aim is to help the children to define and specify an answer to the question posed in the assembly and to see what arguments they come up with. As a first experiment, the children were asked to see what happens when we fill these three containers: a transparent box in the shape of a cube, a cylinder-shaped jar and a balloon in the shape of a face with ears.
3. Individually, children observe and manipulate the containers. They are then asked about the shape of the water and express their beliefs and arguments about it in these containers, which are collected in a double-entry table by one of the teachers. The aim of this phase of the process is that, based on their sense of touch and sight, they reflect and show their critical thinking without external influences.
4. Finally, the question is posed again in the classroom group so that, on the basis of the experimentation carried out, the solution can be specified, defined and concluded and generalized. The teacher writes down the conclusions reached and invites children to compare them with their initial beliefs or ideas.

Experiment 2: Taste of water

The aim of this experiment is to make the children realize that water does not have a taste but is often conditioned by external flavorings. Like the previous experiment, this one is structured in four phases.

1. In an assembly, ask them what water tastes like.

At this point, the aim is for the children to make a first approach to their ideas and preconceptions and for the first answers to emerge.

In view of the answers given, the teacher assumes them as possible hypotheses, in order to create the need for experimentation. To do this, she questions the validity of the answers given through different questions. In this way, she generates doubts among the pupils and invites them to corroborate, to think about how to find out who is right, or if they are all right or if nobody is right. The pupils do not doubt that to prove it, they have to "drink" (try) to which the teacher questions them by asking, "Do you each drink a glass of water from the tap?" The teacher then tells them that as they only have tap water in the classroom, she will bring water from somewhere else, and they will try it. To limit the sample, she asks them some different questions to select three different origins, guided by the teacher.

2. Three bottles and three glasses are presented with water from different sources (tap, smelling mineral spring and bottled). The aim of this experiment is to see the degree of perception of the children about the type of water and its origin and the introduction of concept of tastelessness, an aspect of high complexity at this age.

   At this stage, the pupils are simply shown the three containers. A dialogue is then established in which the children take turns to say which water they usually drink and their preferences. Almost all of them drink from the tap, none of them remember drinking water from the spring (which smelled like the water in the drinking sample).

3. Individually, children taste and sample water from all the bottles. In order to invite them to discriminate the taste of the water (amount of minerals), due to their young age, they are asked to say and record for each of the bottles whether they liked it on a piece of paper (a red "+" if they did not like it, and a green "+" if they liked it). Everyone considered themselves to be participants and all the data were necessary for the experiment to be carried out in a truthful way.

4. Then, in a large group, the results are counted collaboratively and recorded on the blackboard to proceed to interpret the results and the reasons why using a "taste-meter". This way of collecting the results helped children to visualize them, and observing the difference in preferences provoked a dialogue. It was clear to children that there was something that made the distinction. Then, in the classroom group, through dialogue and the teacher's guidance, they agreed on the answers, justifying and arguing, and they specified and defined the results, which the teacher recorded in writing.

Data Collection Techniques & Data Analysis

In order to analyze the type of mathematical argumentation that the children show throughout the STEAM experience, different sources of information were used: photographs, videos and voice recordings of the pupils during the activities, which were transcribed and subsequently analyzed.

For the analysis of the data, bearing in mind the objective of our research, we will center our interest in the aspect "character of the argument" (how it is argued?) of the model defined by Cornejo-Morales et al. (2021). The rest of the aspects are not analyzed, because of the type of the experiment designed, where the other four aspects of the model are determined by this fact. To do this, considering that the mathematical registers and representations of three-year-old children are limited to drawings, few words and reduced mathematical language (Alsina, 2016), we used a rubric defined by Salgado et al. (2020), which consists of the sections shown in **Table 2**.

Thus, this rubric is used to classify the arguments of the children in different phases of each experiment, focusing in three special moments:

(1) at the beginning in the assembly,

(2) during the manipulation-experimentation, and

(3) at the end (i.e., the final argumentation shown at the close of the activity).

RESULTS

To be able to give an answer to the objective of this study, we have differentiated the results depending on the experiment. So, first, we show part of the transcriptions of the assembly; second we detail some

Table 2. Types of argumentation, description,& examples

Type	Description	Example
No argue (NA)	If child's sentence does not express an idea	-
Pictorial argumentation (PA)	That which is made by means of drawings	A generic picture done (a square)
Argumentation with mathematical language (LM)	Verbal argumentation that involves a mathematical term	Terms like "square", "triangle", …
Argumentation with words or graphics (AG)	Oral or written expressions that attempt to express ideas, but without any relation to mathematics	-
Symbolic argumentation (AS)	If, by means of symbols, it establishes some intra- or inter-mathematical relationship	An oral comparison between two geometric elements
Mixed argumentation (AM)	If a mixture of any of the previous ones is used	-

arguments in phase 2, and third, we present all the results throughout the experiment in a table in which, the type of argumentation shown by each child is described:

(1) at the beginning in the assembly,

(2) during the manipulation-experimentation, and

(3) at the final argumentation shown at the close of the activity.

Results of Experiment 1: Shape of Water

First, we show some parts of the transcription of the different phases of the experiment, to describe the type of argumentation done by the children.

In phase 1, the assembly, some of the answers given by the children to the question "what is the shape of water and why?" are, as follows:

> No shape.
>
> Round.
>
> Dots.
>
> ...
>
> Because it rains and then it goes away
>
> Like raindrops.

In this phase of the experiment, the mayor part of the children answers the question, and their answers are classified as a concrete type of argumentation anyway, because of the age of the children, some of them do not answer the question.

With respect to the most common type of argumentation in initial assembly, most children express their ideas and beliefs through words (AG). The other representations are hardly evident, specifically three students use some mathematical term, only one relies on drawings and two do not argue in their response.

Figure 1. Containers used to experiment with shape of water & manipulating containers to determine their shape (left & right, respectively) (Source: Authors' field study)

In view of the different ideas that emerge in the assembly, the teacher challenges them through questions, with the objective to justify the need of the experiment:

> How can it be round and have no shape?
>
> Or dots?
>
> Are you all sure?

In phase 2, containers are selected, and the shape of them is described by children:

> It [the shape] changed [because it filled the balloon], without water it was one balloon and now there are two [ears].
>
> We [have] Mickey mouse (relating it to the balloon).
>
> There is more [water], it is bigger, the round one (to the cylinder).
>
> …

Figure 1 shows the containers.

Finally, **Table 3** collects all the answers related to the shape of water, categorizing the information according to the types of argumentation described in the previous section (no argue [NA], pictorial argumentation [PA], mathematical language [LM], argumentation with

Table 3. Types of children's argumentation about shape of water

Child	IA	Argumentation during experiment (phase 3)						FA
		PA	ML	AW	SA	MA	N	
1	"It has no form" (AG)	-	"Round" "Square"	"Cat shape" "Many forms"	-	"Round + drawing of a circle"	-	"Round" "Water has many forms" (AM)
2	"Small triangles, falling from sky, face downwards" (LM)	-	"Square" "Triangle"	"It has form"	-	-	-	"It is a triangle" (LM)
3	-	-	"Circle"	"Many" "Tooth"	-	-	-	"Water has many forms"
4	"Water is round, like this (doing gest with a finger)" (AM)	-	"Round"		-	-	-	"Round shape" (LM)

Table 3 (Continued). Types of children's argumentation about shape of water

Child	IA	Argumentation during experiment (phase 3)						FA
		PA	ML	AW	SA	MA	N	
5	"Many, I do not know" (AG)	-	"Round" "Square " "Flat"	"Water has no form, it is bottle"	-	--	-	"Round" (LM)
6	"A glass" (AG)	-	"Square" "Round"	"Head" "It only has a shape when it is inside, when it is outside it does not"	-	-	-	"Water has many forms" (AG)
7	-	-	"Square"	"Of all" "Many"	-	-	-	"Square" "Many forms" (AM)
8	"Puddle, there's water there, I like to jump in it" (AG)	-	"Square" "Round in glass"	"Many forms"	-	-	-	"Many forms" (AG)
9	"Dots, mine is painted in this way" (AM)	-	"Round"		-	-	-	"Round" (LM)
10	-	-	-	"Many"	-	-	-	"Water has many forms" (AG)
11	"Round" (LM)	-	"Round"	"Round" 'Water has many forms"	-	-	-	"Water has many forms" (AG)
12	-	-	"Round" "Square"	"Mickey shape" "Water has many forms"	-	-	-	"Water has many forms" (AG)
13	-	-	"Square" "Round" "Flat"	"Many forms"	-	-	-	"Many forms" (AG)
14	"I do not know, because it moves, it has not" (AG)	-	"Flat" "Square" "Round"	"Many forms"	-	-	-	"Water has many forms" (AG)
15	"I do not know" (NA)	-	"Circle" "Square" "Flat"	"It has a shape of a head" "Air balloon" "Shaped in container"	-	-	-	"Water has flat shape" (LM)
16	-	-	"Round" "Triangle"	"It has three forms"	-	-	-	"Round" (LM)
17	-	-	"Round" "Square " "Round"	"Head shape"	-	-	-	"Round shape" (LM)
18	"Many-many" (AG)	-	"Square"	"Many forms"	-	-	-	"Square" (LM)
19	-	-	"Square" "Round"	"It only has a shape when it is in the bottle"	-	-	-	"Many" (AG)
20	"I do not know" (NA)	-		"It does not have"	-	-	-	"It does not have" (AG)

Note. IA: Initial argumentation; PA: Pictorial argumentation; ML: Mathematical language; AW: Argumentation with words; SA: Symbolic argumentation; MA: Mixed argumentation; FA: Final argumentation; N: No argue

words or graphics [AG], symbolic argumentation [AS], mixed argumentation [AM]). Where, if we look at the results of the final assembly, we see that the presence of mathematical language increases considerably, with 50% of children using it to argue their beliefs; the rest of the children use words (GA) to express their ideas.

The results show that, during the experiment, boys and girls argue the shape attribute using mathematical language and words or pictures. Only one case used a mixed argumentation combining words with drawings. It can also be seen that practically all the children use mathematical language to describe the shape of water; specifically, the language used is geometric. In addition, it can be seen how, thanks to the selection of containers with different shapes, the children describe how the shape of the water changes depending on the container used; for example, case 14 describes it as "flat", "square" and "round", depending on the moment of the experiment. But it is also observed that most of the children use an incorrect language to describe the volume, since they resort to terms of flat shapes. Regarding final argumentation, it can be observed that children continue to use mathematical language and/or words or spellings to argue the shape attribute. In this case, the great majority has left mathematical language aside and resorts to words. Also, in this final phase of argumentation, two clear tendencies can be observed: a first one in which some children (40%) persevere in defining the shape of water with certain previous knowledge and beliefs, in spite of the observation and experimentation carried out (subject 18 maintains his argumentation that water is square or 16 that it is round); a second one in which the rest of the children (60%) approaches description of the shape of water from results obtained in observation and experimentation, going on to describe it as an object with an indeterminate shape (e.g., case 14: "It has many forms").

Results of Experiment 2: Taste of Water

First, we show some parts of the transcription of the different phases of the experiment, to describe the type of argumentation done by the children. In phase 1, the assembly, ask them what water tastes like and why. Some of the answers were:

> Nothing
>
> I like
>
> Fridge
>
> …
>
> Because it is not in the mouth
>
> We cannot drink the wine
>
> I like it cold.
>
> ...

In this phase of the experiment, the mayor part of the children answers the question, and their answers are classified as a concrete type of argumentation; anyway, because of the age of the children, some of them do not answer the question, and in this case, the number of children who do not answer is bigger than in the previous experiment, eight who abstained. In fact, in relation to the results regarding to the most common argumentation in the initial assembly, 12 children expressed their beliefs and all of them used words (AG) and in no case was there evidence of the use of mathematical language to justify their beliefs.

In view of the answers given, the teacher assumes them as possible hypotheses, to create the need for experimentation. To do this, she questions the validity of the answers given through questions:

> How nothing?
>
> Is the fridge a flavor, what flavors do you know, does it taste like cake?
>
> Does it always taste the same when you drink water?

To identify where the water comes from, discriminating between drinking water and non-drinking water.

Some of the responses that emerged:

> In the sea
>
> In the river
>
> At the fountain
>
> In the supermarket
>
> ...

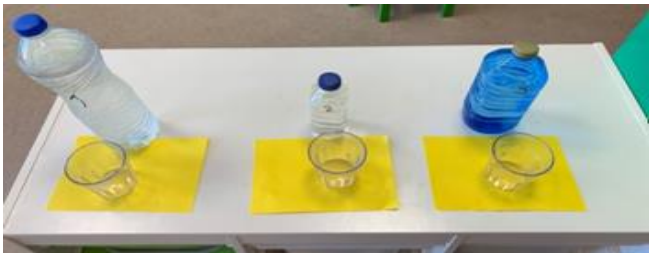

Figure 2. Different types of water (three brands) for taste testing (Source: Authors' field study)

Figure 3. Students tasting water & registering preferences with likes & dislikes (left & right, respectively, phase 3) (Source: Authors' field study)

In phase 2, different bottle with different origins of water are selected. **Figure 2** shows different types of water (three brands) for taste testing. **Figure 3** shows students tasting. **Figure 4** shows responses and count of water taste.

Then, in the classroom group, through dialogue and the teacher's guidance, they agreed on the answers, justifying and arguing, and they specified and defined results, which the teacher recorded in writing (phase 4).

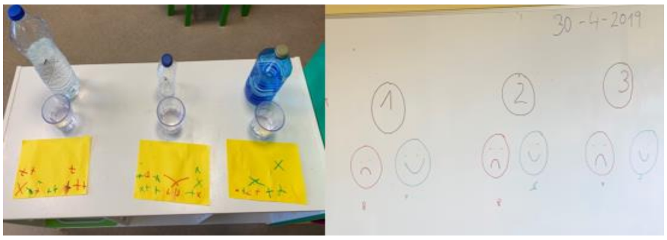

Figure 4. Responses & count of water taste (left & right, respectively) (Source: Authors' field study)

Finally, **Table 4** summarizes the children's responses to the water taste attribute categorized into the types of argumentation described above in the different phases of the experiment (no argue [NA], pictorial argumentation [PA], mathematical language [LM], argumentation with words or graphics [AG], symbolic argumentation [AS], mixed argumentation [AM]).

More concretely, if we examine in details the results of the final assembly, no variation is observed with respect to the type of argumentation, because of all the children again use argumentation with words (AG), but the number of participation increases, with all but one child expressing his or her beliefs through argumentation.

The results show that children have a clear tendency to argue with words to describe the attribute of taste, but, depending on the cases, we see how the boys and girls make a clear distinction in their perception of taste according to the origin of the water.

Table 4. Types of children's argumentation about shape of water

Child	IA	Argumentation during experiment (phase 3)						FA
		PA	ML	AW	SA	MA	N	
1	"Does not taste" (AG)	-	-	"The water tastes" "Tastes a bit this" "Tastes a lot, this more"	-	-	-	-
2	-	-	"1" "2" "3"	The water tastes"	-	-	-	"The water tastes" (AG)
3	"Nothing" (AG)	-	-	"I believe it has no taste"	-	-	-	"The water does not taste; it is all the same" (AG)
4	"I like water" (AG)	-	-	"Water does not taste" "A little bit" "Bad water"	-	-	-	"The water tastes" (AG)
5	"Tastes good" (AG)	-	-	"I believe it has no taste" "Tastes like good" "Almost nothing"	-	-	-	"Does taste" (AG)
6	"Nothing" (AG)	-	-	"This water does not taste" "I believe it has no taste"	-	-	-	"Does not taste" (AG)
7	"Does not taste because it is not eaten" (AS)	-	-	"This water does not taste" "Like an apple"	-	-	-	"Tastes different" (AG)
8	-	-	"Much, one, zero zero zero zero zero zero"	"Yes, it tastes" "It tastes less" "Bad"	-	-	-	"Does taste" (AG)
9	-	-	-	"Yes, does taste" "No, doesn't taste" "Half"	-	-	-	"Water tastes something" (AG)
10	"Like a fridge" (AG)	-	-	"Water does not taste" "Water does not" "I believe it has no taste"	-	-	-	"Water does not taste" (AG)
11	-	-	-	"Water does not taste" "Water tastes" "Tastes a little more"	-	-	-	"Water tastes because it smells" (AS)
12	-	-	-	"Does taste" "Does taste" "A little bit"	-	-	-	"Tastes because smell" (AG)

Table 4 (Continued). Types of children's argumentation about shape of water

Child	IA	Argumentation during experiment (phase 3)						FA
		PA	ML	AW	SA	MA	N	
13	-	-	"More than ten"	"The water tastes" "Tastes a lot"	-	-	-	"Does taste and bad" (AG)
14	"Does not taste" (AG)	-	-	"Does not taste" "The water tastes" 'Almost more taste"	-	-	-	"Does taste" (AG)
15	-	-	-	"Water does not taste" "The water tastes" "This one tastes more"	-	-	-	"The water tastes" (AG)
16	"I like it" (AG)	-	-	"The water tastes" "Doesn't taste" "A little bit"	-	-	-	"The water tastes" (AG)
17	"Tastes nothing" (AG)	-	"5" "5" "6"	"Water does not taste" "Like rotten" "Lore or less"	-	-	-	"The water tastes" (AG)
18	-	-	-	"Water does not taste" "Water tastes" "Tastes a lot"	-	-	-	"The water tastes" (AG)
19	"I like it" (AG)	-	-	"Water tastes like nothing" "The water tastes" "It only tastes when damaged"	-	-	-	"Only has taste when it is damaged" (AG)
20	"I like it cold" (AG)	-	-	"The water tastes" "Tastes less" "Worst"	-	-	-	"Tastes different" (AG)

Note. IA: Initial argumentation; PA: Pictorial argumentation; ML: Mathematical language; AW: Argumentation with words; SA: Symbolic argumentation; MA: Mixed argumentation; FA: Final argumentation; N: No argue

In some cases, in the experimentation phase, comparisons are made that allude to the degree of taste or not (case 11: "Tastes a little more"), and in other cases, it already appears that, depending on the source of the water, no taste is perceived, giving rise to the incipient introduction of the notion of tastelessness (case 6: "Water does not taste"). In the final phase of argumentation, as in the previous experiment, we find two clear tendencies: the first one in which the children do state that water has a taste, although it is important to highlight that they are not able to specify specifically what it tastes like (case 7: "Tastes different"; case 9: "Water tastes something"); and a second, minority in which some children state that water is tasteless (example 10: "The water does not taste").

DISCUSSION

The main objective of this work was to explore and promote the mathematical argumentation skills of three-year-old children in a childhood education classroom. To this end, a STEAM task related to the properties of water was designed and implemented as a context for reasoning and proof.

First, the results show that, as three-year-old pupils, who are at a pre-operational stage of development, are able to draw conclusions from simple everyday problems (Intriago & Murillo, 2022). Furthermore, it has been seen that, related to the "character of the argument" (how it is argued?) of the model defined by Cornejo-Morales et al. (2021), children have been able to construct arguments based on mathematical language, resorting mainly to words or graphs, using and relating their contributions to concepts acquired up to that time. Although it is true that it is common for them to use words or spellings with errors or inaccuracies, because they are young children.

Although there are studies with similar findings in early childhood education (Salgado et al., 2020), few studies have addressed and analyzed the ability and type of mathematical argumentation at such an early age as this one. The results of this study are consistent with those of previous studies (Krummheuer, 2013; Moutsios-Rentzos et al., 2019; Reuter, 2023), which attempt to decipher the way in which children develop mathematical thinking at an early age, showing that the construction of this type of thinking involves both schematic and narrative argumentation.

However, this work has also gone a step further, being able to determine two clear trends in the evolution of children's argumentation at this age: the first, based on preconceived or previous ideas, which have not changed despite the evidence revealed in the two experiments; and the second in which, thanks to experimentation and observation of the different facts, has led to establishing the first connections with the physical concepts that we wanted to work on (the indeterminacy of shape and the insipidity of water). Moreover, between the two experiments, with respect to the type of final argumentation, we also found a difference: in the case of experiment 2, the taste of water, no child gave any argument with mathematical language, while in experiment 1, the shape of the water, this did occur, although in small quantities. The fact that mathematical language does not appear in experiment 2 may be due to the physical properties involved, where the abstract nature of the attribute to be analyzed, insipidity, may have influenced the children's difficulties in being able to specify, describe or explain this quality.

Thus, the work described here has shown that this type of practice should be encouraged much more with infant children, because the experience shows that it favors progress in the development of mathematical and critical thinking and helps to build links between scientific and mathematical reasoning, which is consistent with previous findings such as those of Alsina (2020) and Berciano et al. (2022). In this sense, this type of teaching-learning in the early childhood education classroom (with a holistic approach to mathematics, science, engineering, art and technology) should be included as part of the examples of good practice in the initial and ongoing training of future early childhood teachers, to help them overcome the gap they have between practical and theoretical knowledge of mathematical argumentation, which has a negative impact when designing practices in this framework, revealing serious difficulties in transferring this knowledge to the education of their future students, children aged three to six years (Broeder & Stokmans, 2009; De Gamboa et al., 2010).

It has been saw how simple experiences about the properties of water, adapted to the age of the children, are capable of working with early learners on complex mathematical and physical concepts, as long as a cycle of appropriate experimentation and observation is carried out, together with the teacher's guidance through good questions, which leads the children to want to respond to the problem posed, by means of inquiry. All this leads to the fact that the communication and justification of the conclusions should be established in the evidence obtained, and, therefore, help them to try to argue. On the other hand, it has been found that the most frequently used argumentation, either with graphs, words or a combination of both, is mathematical argumentation, where mainly geometrical concepts emerge in a significative context.

CONCLUSIONS

In this work we have seen how, starting from a STEAM context based on the connections between mathematics and science, we have encouraged children to reflect on their own answers by setting up small experiments that will lead them to interconnect reasoning with the evidence of empirical proof (according to Alsina, 2016). In addition, the atmosphere created in the classroom also influenced the good development of the activities, as well as the attractiveness of the experiments, easily capturing the attention of the children.

On the other hand, it has been shown how three-year-old children are able to elaborate arguments and justify them in evidence derived from the adaptation of experimentation to their young age. From the results obtained, it has been shown that children mainly use arguments based on mathematical language, geometrical argumentation or words or graphs, while other types of argumentations (pictorial and symbolic) are absent. Almost anecdotally, only one student has used a mixed argumentation; combining words with drawings. Thus, along these lines, as stated by Erduran et al. (2015), it has been possible to verify how the use of a practice typical of science has helped and motivated children to establish diverse modes of communication in order to argue their ideas. Thus, children have been protagonists of the experience, and builders of their own knowledge (Couso, 2017). They have had the ability to take advantage of the resources provided by the teachers and benefit from them.

Finally, although this work only presents two experiments out of a total of five that the entire project consists of, children have been able to become familiar with two attributes of water by relating two fields of knowledge: science and mathematics, using inquiry and experimentation. In addition, this theme has made it possible to work on the central axis of the proposal: mathematical argumentation.

About limitations of the study, we would like to comment that, because of the nature of the research, the sample size has been only of 20 children, so, the results cannot be generalize. In this sense, a future line of research would be to extend the field study to a larger sample in order to analyze the results shown here form a quantitative point of view.

Thus, as Berciano et al. (2021) proposed the suitability of the treatment of engineering in the early childhood classroom from a STEAM approach, it cannot be overlooked that this work allows us to conclude that, based on a scientific questioning adapted to the early childhood classroom (with three-year-old children), it has been proven that not only do different mathematical contents emerge, but also that, during the process of justifying the answer, children activate communication and reasoning mechanisms involving different standards of mathematical processes, thus highlighting the need to design and include this type of STEAM contexts in the classroom in order to develop mathematical competence.

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Ethical statement: The authors stated that the study did not require approval from an institutional ethics committee. The study reflects the authors' own research and analysis in a truthful and complete manner. The authors further stated that the highest ethical principles were strictly observed during the planning and conducting of the research. The research project was characterized by its non-invasive nature, ensuring that it does not exert any detrimental effects on the physical or mental well-being of the participants. The dissemination of research findings adheres to the strict ethical guidelines to ensure that the identification of individuals, either directly or indirectly, is not possible. The authors obtained pertinent authorizations from the center and the families to develop and analyze the classroom experience. Written informed consents were obtained from the participants.

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