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Mathematical competencies of prospective teachers when teaching confidence intervals in a microteaching context

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Abstract

Pre-service teacher training in inferential statistics stands out as a priority topic in mathematics education research, with an emphasis on didactic and disciplinary competencies. This qualitative and interpretive study aims to characterize the mathematical knowledge and competence of pre-service teachers when they undertake instructional processes on the confidence interval. The methodology included a cycle composed of an initial video-recorded microteaching intervention, a reflection process, and a re-implementation. For the analysis, we used the didactic-mathematical knowledge and competencies model and the didactic suitability criteria as a theoretical and methodological tool. The results reveal that when teachers reflect on their practice and re-implement their teaching, they significantly improve their mathematical competencies (task-solving, problems linked to the content they teach, and anticipating their students' expected responses). This reflective process proves to be fundamental for developing pedagogical skills in teaching inferential statistics.

Keywords: mathematical competence, didactic suitability criteria, microteaching, confidence intervals, student teachers

INTRODUCTION

Initial teacher training in statistics must be constantly updated to meet the pedagogical demands of the 21st century, which must ensure that educators develop mathematical and didactic competencies that enable them to interpret, analyze, and communicate data effectively in the classroom. Similarly, modern mathematics curricula have integrated the teaching of statistics from the first years of school (Kallivokas, 2024). This with the aim of preparing students to understand, systematize and interpret information that enables them to make logical decisions on global issues stemming from a complex and changing world (Vásquez et al., 2023). Therefore, teaching processes should emphasize statistical thinking, focus on the conceptualization and understanding of specific terms, and promote the interpretation and prediction of future situations through statistical tools (Ben-Zvi & Makar, 2016; Hasim et al., 2024; Schield, 2017).

Similarly, statistics is essential for the interpretation and critical evaluation of information from different contexts. In this context, it has been suggested to promote statistical thinking and statistical estimation skills through the study of concepts such as confidence intervals (CIs), the central limit theorem and hypothesis testing (Chandrakantha, 2014). However, the teaching and learning processes on these topics have raised some difficulties, especially when teachers fail to recognize and interpret the meaning of CI as statistical probability (Choi et al., 2016). According to Han and Jeon (2018), it is only natural that the understanding of CI and hypothesis testing is complex if teachers do not adequately acquire concepts such as representativeness, sampling, sampling variability, and distribution.

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Contribution to the literature

- This study contributes to research in teacher training in inferential statistics, especially confidence intervals, by characterizing mathematical knowledge and skills.
- A methodological framework is provided through a cycle of implementation, reflection, and redesign of a lesson on the concept of CIs in microteaching contexts, representing an innovative empirical approach focused on practice.
- The results show that by reflecting on their own practice, pre-service teachers significantly deepen their disciplinary knowledge and improve their mathematical proficiency. Furthermore, it allows for the development of more effective pedagogical strategies for their future teaching careers.

In this context, the scientific community has emphasized that the teaching and learning process regarding CI has semiotic conflicts, defined as discrepancies between two meanings or interpretations (Godino et al., 2007). Some of these semiotic conflicts are:

(1) the wrong definition of CI (López-Martín et al., 2019; Olivo, 2008),

(2) CI is considered a descriptive statistic, ignoring its inferential nature (Fidler & Cumming, 2005),

(3) limited understanding of the utility of CI (Roldán et al., 2020),

(4) fundamental error of the CI, which always considers the value of the parameter included in the CI (Álvarez et al., 2021; Olivo, 2008; Roldán et al., 2020) and

(5) procedural error in the estimation of the CI (Espinel et al., 2007; Roldán et al., 2020).

According to Choi et al. (2016), teachers' conceptualization is related to what is made explicit in textbooks, i.e., they do not distinguish between CIs and probability intervals by using lowercase and uppercase letters in the sample mean. In addition, they perform teaching processes on CI without situations/problems that activate prior knowledge, such as the concept of population, sample, parameter, standard error, sampling distribution, and critical value (Andrade & Fernández, 2016). The understanding of CI is essential for analyzing estimates and quantitative conclusions from different fields of knowledge (psychology, health sciences, economics, among others) (Aityan, 2022). In Chile, the standards for the teaching profession state that a secondary mathematics teacher must develop knowledge and skills that enable them to calculate CIs, understand their origin and epistemology, interpret their meaning and develop strategies to promote critical analysis, statistical thinking and the interpretation of CI in statistical inference in their students (Ministerio de Educación de Chile [MINEDUC], 2021).

The aim of this article is to characterize the mathematical knowledge and competence of pre-service teachers when they adopt CI teaching processes in a microteaching context. To achieve this goal, we use the model of didactic-mathematical knowledge and competencies (DMKC) (Pino-Fan et al., 2023) and the didactic suitability criteria (DSC) as a theoretical-methodological tool, proposed within the onto-semiotic approach (OSA) to mathematical knowledge and teaching (Godino et al., 2007), especially the epistemic facet (situations/problems, languages, definitions, propositions, procedures, arguments, errors, ambiguities and beliefs) (Breda et al., 2018), which is an essential aspect of mathematical literacy.

The aim of this study is to characterize the mathematical knowledge and competence of pre-service teachers when they carry out teaching processes on the topic of CIs in the context of microteaching. Based on the above, we have posed the following research question in this article: What mathematical competencies are improved by pre-service teachers after participating in a training cycle when they perform teaching processes on the topic of CI?

To answer the research question, the model of DMKC is used (Pino-Fan et al., 2023). The discussion and reflection that are developed are in turn guided by the DSC, especially regarding the epistemic facet (situations/problems, languages, definitions, propositions, procedures, arguments, errors, ambiguities and beliefs) (Breda et al., 2018; Godino, 2011).

THEORETICAL FRAMEWORK

The complexity that trainee teachers experience when tackling professional tasks such as designing or delivering a lesson has been the subject of several studies (Lendínez Muñoz et al., 2024). Internationally, various proposals have attempted to describe the components that make up the DMKC of teachers (Burgos & Godino, 2022; Neubrand, 2018; Parra-Urrea et al., 2022; Pino-Fan et al., 2018; Schoenfeld & Kilpatrick, 2008; Shulman, 1987).

The present study uses the model of DMKC (Pino-Fan et al., 2023). This model represents a theoretical-methodological alternative that enables the analysis and development of knowledge and competencies that are essential for teaching. Competence is understood as "the totality of knowledge and dispositions that enable effective performance in typical professional contexts" (Pino-Fan et al., 2023, p. 5). In the DMKC model, the

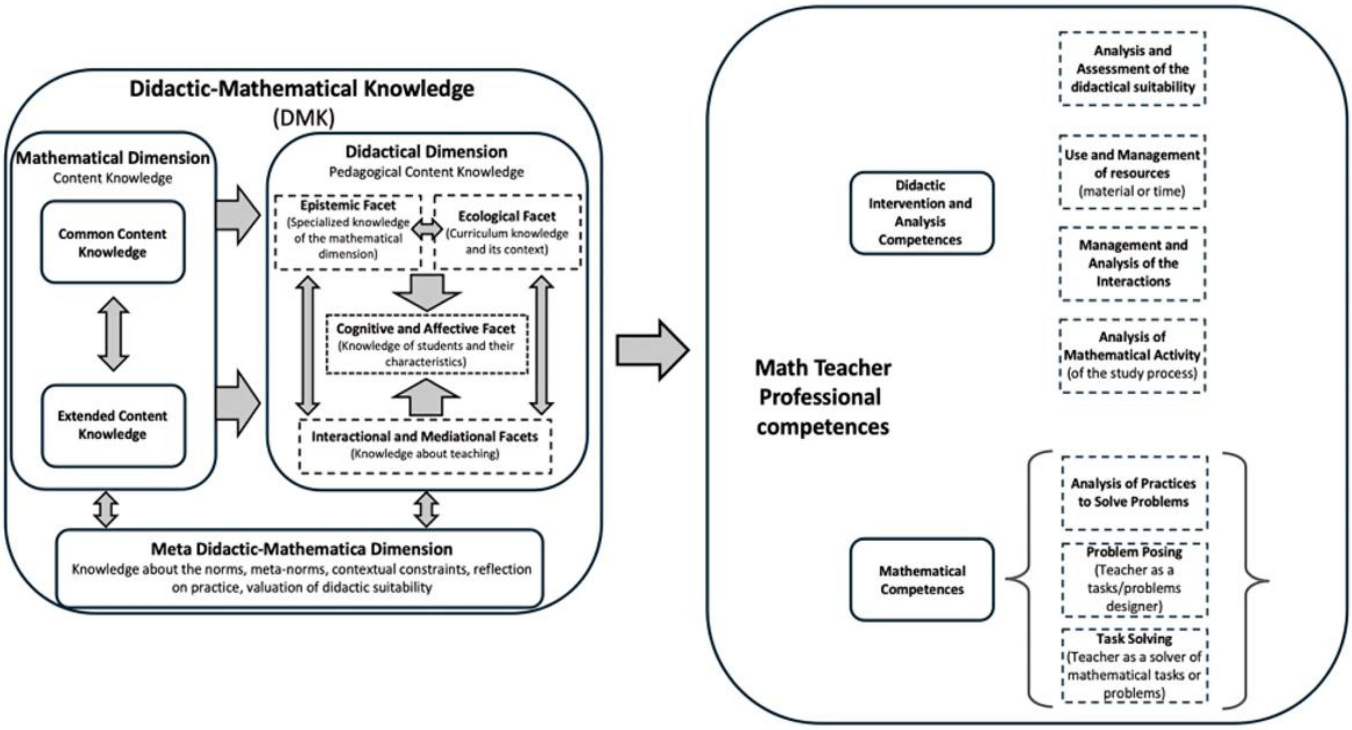

Figure 1. Mathematics teacher's DMKC model (Pino-Fan et al., 2023, p. 1413)

knowledge of the mathematics teacher is divided into three dimensions:

(1) mathematics,

(2) didactics, and

(3) meta-didactics-mathematics (Pino-Fan & Godino, 2015; Pino-Fan et al., 2018).

The professional competencies of the mathematics teacher are divided into

(1) mathematical competence and

(2) didactic analysis and intervention competence.

These have sub-competencies based on international scientific literature (see **Figure 1**). Each sub-competence in turn comprises four performance levels (Pino-Fan et al., 2023). In this study, we analyze the epistemic knowledge and mathematical competence of Chilean teachers-in-training when they adopt CI teaching processes in a microteaching context.

The development of mathematical competence requires knowledge of the mathematical dimension of the DMK and those associated with the epistemic facet of the didactic dimension of the DMK. This competence is divided into three sub-competences:

(1) task solving (solving problems related to a given mathematical object),

(2) problem posing (proposing tasks related to a mathematical object, considering possible conflicts or errors, students' interests and the context), and

(3) analyzing the answers to a posed problem (analyzing the mathematical practice that a student develops when solving a task) (Pino-Fan et al., 2023).

The analysis, discussion and reflection that are developed are guided by the DSCs, which are a methodological theoretical tool proposed by the OSA to mathematical knowledge and teaching (Godino et al., 2007). The concept of didactic suitability, its components and indicators provide guidance for effective classroom management and enable the evaluation of planned or implemented pedagogical actions (Breda & do Rosário Lima, 2016). Furthermore, it is defined as the extent to which mathematics teaching is considered optimal or appropriate, so that there is an adequacy between the personal meanings achieved by students and the intended or implemented institutional meanings (Font et al., 2024).

It is worth mentioning that didactic suitability consists of six criteria: epistemic suitability, cognitive suitability, interactional suitability, mediating suitability, affective suitability, ecological suitability (Godino, 2011; Godino et al., 2019). In order for DSCs to be operational and observable, they are organized into components and indicators (Breda et al., 2018; Godino et al., 2013) that explicitly guide mathematics classroom practices.

METHODOLOGY

This study is qualitative and based on an interpretative paradigm of the non-naturalistic type (Cohen et al., 2018), as the phenomena are studied in simulated microteaching contexts and the situations are interpreted around the meanings that teachers in training assign to the term CI. Microteaching aims to provide initial teacher educators with the opportunity to develop their teaching practice in a controlled context with reduced complexity, the purpose of which is to facilitate the acquisition of pedagogical skills (Donnelly & Fitzmaurice, 2011; Fernández, 2010; Koross, 2016; Mochizuki et al., 2022; Vígh, 2024). This methodology aims to encourage reflection on teaching practice to determine actions for continuous improvement (Allen & Ryan, 1969; Abendroth et al., 2011). Trainee teachers' performance in standardized microteaching environments has been shown to be transferable to real classroom situations (Seidel et al., 2015). In this section, we describe the methodological aspects of our work.

Research Context

This study was conducted at a Chilean college as part of a pedagogical training program in mathematics for college graduates or professionals (continuing studies). The training model of the program aims to consolidate disciplinary knowledge and to train professionals or graduates didactically and pedagogically to work as mathematics teachers in secondary education. The training program is divided into four trimesters, each with 13 teaching weeks and a total duration of 52 weeks. In addition, the curriculum includes two internships (initial and professional internship), which take place in the third and fourth trimester respectively. The first author of this article proposed to develop teaching processes on the concept of CI in micro-teaching environments in the "professional practice" course. The lessons were videotaped and had a maximum duration of 40 minutes.

The Chilean curriculum approaches mathematical learning progressively and with increasing conceptual and procedural complexity. It promotes the conceptualization of mathematical objects and the development of skills such as problem solving, reasoning, discussion and the use of digital tools. The concept of CI is addressed in the common and elective curriculum in the fourth year, i.e., for students aged 17 to 18.

Study Participants

Eleven prospective teachers participated in this study, three of whom played the role of teachers (we call them teachers A, B, and C) and eight of whom played the role of student teachers. The prospective teachers were in the third trimester of the pedagogical training program in mathematics, which has a total duration of four trimesters given its continuation of the study. The teachers in training had previously completed a degree in engineering (industrial engineering, civil engineering and computer engineering) and had experience as mathematics teachers.

Data Collection and Analysis Techniques

In this study, we used the lesson study (LS) phases because it promotes collaboration, identifies responsibilities and favors interaction between teachers (López-Vélez & Galarraga, 2024). In order to achieve the objective of the study, six phases were defined in accordance with the LS cycle, which determine the methodological structure of the research.

1. **Phase 1 – Planning:** three prospective teachers (teachers A, B, and C) planned the teaching of the CI concept, i.e., they designed the lesson together considering the preparation of teaching materials, the identification of resources and the discussion of possible questions and answers. Teachers A, B, and C handed over the detailed planning and lesson plan to the teacher educator, in which the learning objective of the unit, the skills and attitudes to be developed, the aim of the lesson, the moments of the session (beginning-development-conclusion) and the learning resources were made clear.
2. **Phase 2 – Implementation:** In a 40-minute session, teachers A, B, and C carried out a teaching process on CI in a microteaching context, i.e., the participating teachers were given the opportunity to develop teaching practice in a simulated, simplified, controlled and safe environment.
3. **Phase 3 – Analysis:** Immediately after the end of the lesson, the participating teachers discuss with the researchers the relationship between the objectives, the lesson plan and the learning outcomes achieved. In addition, the objectives and implementation are examined to identify development needs and pedagogical issues so that the next lesson can be improved.
4. **Phase 4 – Reflection:** After the lesson, reflection on the intervention is encouraged, i.e., in retrospect, awareness of the key aspects of the intervention is raised. The validity and effectiveness of the intervention is critically analyzed. This is done with the help of the DSCs.
5. **Phase 5 – Redesign:** Based on the formative feedback, analysis and critical reflection, the trainee teachers have planned, designed and implemented a new lesson on CI, thus completing a first LS cycle.
6. **Phase 6 – Final discussion:** The aspects that were improved in the newly designed lesson were analyzed and reflected upon.

The reflection on teaching practice (LS level 4) was based on the DSCs, in particular the components and indicators of the epistemic facet (situations/problems; languages; definitions; procedures; arguments; errors, beliefs and ambiguities) and mathematical competence were considered. The DMKC model (Pino-Fan et al.,

Table 1. General overview of the performance levels of achievement for each sub-competency (Pino-Fan et al., 2023)

Mathematical competences		
Sub-competence	Level	Descriptions of levels of achievement
Task solving	L0	The teacher reproduces both the formulation […]
	L1	The teacher solves problems at the educational level […]
	L2	The teacher solves problems corresponding […]
	L3	The teacher solves problems at the educational grade […]
Problem posing	L0	The teacher reproduces both the statement of […]
	L1	The teacher proposes tasks suited […]
	L2	The teacher proposes tasks corresponding […]
	L3	Additionally, the teacher considers new tasks […]
Analysis of practices to solve problems	L0	The teacher analyzes the students' mathematical […]
	L1	The teacher performs what is indicated L0. Still, […]
	L2	The teacher uses theoretical-methodological tools […]
	L3	At this level, the teacher has appropriated some […]

2023) was used to characterize the mathematical knowledge of teachers A, B, and C, particularly in relation to mathematical competence and its sub-competence:

(1) task solving,

(2) problem proposing, and

(3) analysis of answers to a given task.

This is done with the help of performance levels defined for each of these competencies (see **Table 1**) (Pino-Fan et al., 2023).

The methodology included a cycle composed of an initial video-recorded microteaching intervention, a reflection process, and a re-implementation. For the analysis, we used the DMKC model and the DSC as a theoretical and methodological tool.

With regard to the validity and reliability of the coding in the research interpretation, a triangulation (Creswell & Creswell, 2023) was carried out in the analysis of the study data, using the DSC, the DMKC and the phases of the LS, to characterize the levels of achievement of mathematical competence used by the teachers in the training.

RESULTS AND ANALYSIS

The planning, design, and delivery of lessons on CI within a micro-teaching framework was developed by three Chilean teachers-in-training, whom we refer to as teachers A, B, and C for the purposes of this study.

The trainee teachers plan/design (first phase of the LS cycle) their first lesson on CI by identifying the purposes, learning objectives, skills and attitudes established in the Chilean curriculum on this topic. In this sense, they recognize the intention of providing students with an education that allows them to develop critical and analytical skills. Likewise, they study the course of the curriculum to identify prior knowledge, common frequent mistakes, elements that motivate learning (applications, usefulness, among others) and activities that verify the fulfillment of the proposed objectives. In this way, the teachers-in-training design a 40-minute lesson for fourth grade secondary school students. They consider it appropriate to clarify the terms sampling, frequency table, measures of central tendency, range, probabilities of events, normal distribution and random variables. They also specify the resources to be used, such as Quizziz, GeoGebra and the Mentimeter application. In the second phase of the LS, teachers A, B, and C conduct a lesson on CI in a microteaching context. This lesson is described below.

Description of the First Lesson on Confidence Interval (Implementation)

Teacher A begins the lesson by pointing out that the concept of CI is the last topic they will cover in the unit on probability and statistics. He also points out that the purpose of the unit is to derive information from a sample when the population is "normally distributed".

It then states as a class objective that: *"Students will be able to understand the concept of confidence interval and apply it to estimate parameters in real-world problems"*. Professor A points out that students should have the following prior knowledge: Sampling, frequency table, measures of central tendency, range, probabilities of events, normal distribution, and random variables. To mobilize these concepts, teachers A, B, and C use an online questionnaire with the application *Quizziz*, which contains 11 questions. In this way, teachers A and C give feedback on the activity about the normal distribution and emphasize: *"The graph of the normal distribution is represented by a symmetrical curve, i.e., if you select half of this distribution, a probability of 50% is covered"*.

To summarize, the teachers list the properties of a normal distribution, including: *"(1) it is bell-shaped and asymptotic to the abscissa* ($x = \pm\infty$), *(2) symmetric with respect to the mean* (μ), *with the median* (M_e) *and the mode* (M_o) *coinciding*, and *(3) the inflection points have as values of the abscissa* $\mu \pm \sigma$*"*. The properties described above are illustrated by the teachers A, B, and C in **Figure 2**.

Professor B then begins to explain the concept of CI and presents the graph of a standard normal curve with $N \sim (0, 1)$, stating that: *"It will have two values that are*

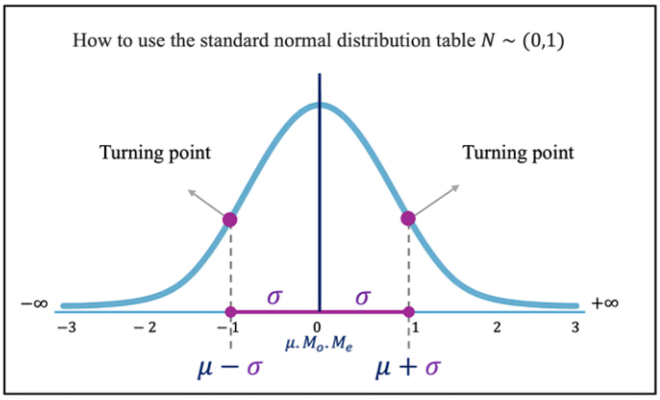

Figure 2. Representation of the normal distribution (illustration used by teachers A, B, and C) (Source: Authors' own elaboration)

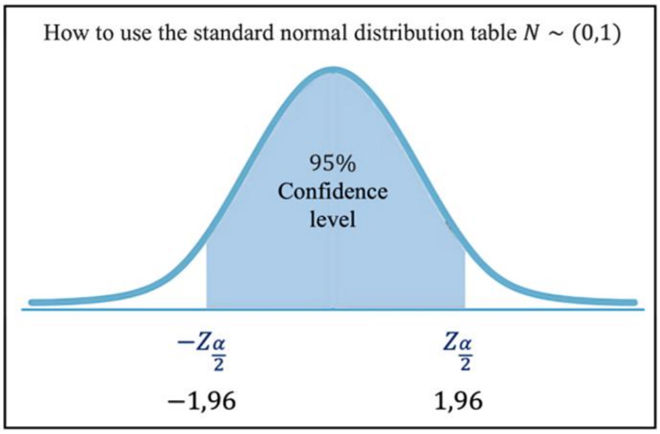

Figure 3. Example presented by Professor B on CI (Source: Authors' own elaboration)

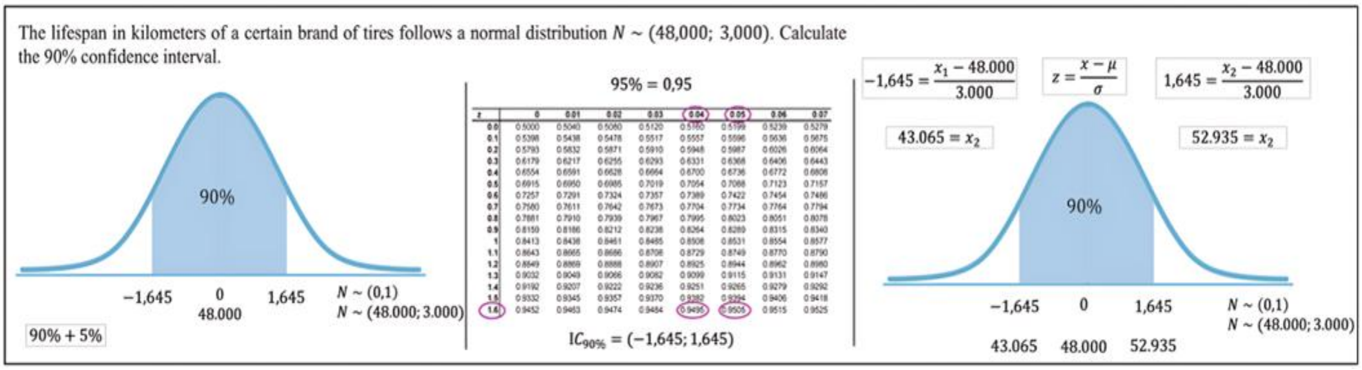

Figure 4. Stages in the resolution of the tire life problem (Source: Authors' own elaboration)

symmetric about the mean, whose value is zero in a standard normal distribution, and are represented by Z and $-Z$ respectively". As an example, Professor B states: *"If I have the value* 10, *the symmetry with respect to the mean is* -10*"*. Specifically, what he presents as the concept of CI is simply the normal curve.

Professor B then gives the following explanation: *"Between these two values ($-Z$ and Z) is a range called the confidence level, which is the probability of being within this interval. What lies outside this confidence level is called the significance level and is denoted by α (alpha)"*. In a situation that requires determining a CI from a given confidence level (0.95), Professor B uses the standardized Z-table to determine the value associated with 0.975 (which corresponds to the confidence level $+\alpha/2$), whose value is 1.96 and whose symmetry is -1.96. This leads to the conclusion that *"the 95% confidence interval is -1.96 and 1.96"* (see **Figure 3**).

In the last part of the lesson, teacher B asks his students to solve the following problem: *"The length of the tires of a certain brand in kilometers corresponds to a normal distribution* $N \sim (48{,}000;\ 3{,}000)$. *Calculate the 90% confidence interval"*. Teacher B starts from the solution of the problem together with the students and concludes that: *"the CI corresponds to an* $CI_{90\%} = (-1.645; 1.645)$ *for a normal distribution, with* $N \sim (0,1)$*"*. Professor B then states: *"the task set informs us that the normal distribution has a mean of 48,000 KM. However, the calculation carried out corresponds to zero mean, and for the 48,000 KM the typing* $Z = \frac{x-\mu}{\sigma}$ *is used, where we substitute the values of Z, μ and σ to obtain the values of x* $(x_1 = 43{,}065\ y\ x_2 = 52{,}935)$*"*. Finally, we decide that: "the tires will last between 43,065 and 52,935 kilometers with 90% certainty" (see **Figure 4**).

At the end of the lesson, teacher B suggests that his students solve a problem (see **Figure 5**). To do this, he suggests using the CI formula to estimate the mean and explains: *"In this kind of situation, it will be necessary to use the following formula* $\left[\mu \pm Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$*"*. Teachers A, B, and C end the lesson with a concluding question: *"What concepts have you learned in today's lesson?"* to gather information about the understanding of the topics covered in the lesson.

Analysis and Reflection of Lesson 1

In the third phase of the LS cycle, the *analysis and reflection* of the teaching practice of teachers A, B, and C is encouraged. In this case, the components of epistemic suitability are discussed. These criteria are known to the prospective teachers.

In the lesson described above, *situations/problems* are used to illustrate the notion of CI through an activity that promotes and graphically illustrates the properties of the

GUIDE 1: CONFIDENCE INTERVAL

The number of televisions in a city is modeled by a normal distribution with mean μ and variance 0.25. A random sample of 100 families is taken from this city, obtaining a mean of 2.75 televisions. For the results of this sample, which of the following intervals corresponds to a 0.95 level confidence interval for μ?

a) $\left[2{,}75 - 1{,}96 \cdot \frac{1}{40}; 2{,}75 + 1{,}96 \cdot \frac{1}{40}\right]$

b) $\left[2{,}75 - 1{,}96 \cdot \frac{1}{200}; 2{,}75 + 1{,}96 \cdot \frac{1}{200}\right]$

c) $\left[-1{,}96 \cdot \frac{1}{40}; 1{,}96 \cdot \frac{1}{40}\right]$

d) $\left[-1{,}95 \cdot \frac{1}{20};\ 1{,}95 \cdot \frac{1}{20}\right]$

e) $\left[2{,}75 - 1{,}96 \cdot \frac{1}{20}; 2{,}75 + 1{,}96 \cdot \frac{1}{20}\right]$

Figure 5. Closing problem posed by teachers A, B, and C (Source: Authors' own elaboration)

normal distribution, in a purely intra-mathematical context (Da Ponte & Chapman, 2010). In a second moment aimed at reinforcing the concept studied and showing the usefulness of the mathematical object, teachers A, B, and C propose the solution of a contextual problem involving estimating the running time of a car tire. This type of problem encourages students to predict future situations, collect data, analyze situations, and make decisions using statistical tools, which promotes meaningful grasp of CI (Watson & English, 2015).

Another *situation/problem* in the lesson relates to an everyday task whose sole purpose is to use the formula $\left[\mu \pm Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$ to determine the appropriate CI. While the algorithmic work is relevant, the importance of interpreting and evaluating the CI to promote statistical literacy should not be neglected (Choi et al., 2016). It is evident from teaching that a teaching approach characterized by the study of standard exercises whose contextualization is not conducive to motivation and interest in the study of CI prevails (Da Ponte & Chapman, 2010).

In terms of *language* use, the prospective teachers use different representations (symbolic, algebraic, graphical, verbal) during the lessons. In terms of *symbolic representations*, teachers A, B, and C use the critical values of the interval $(\mu - \sigma)$, $-Z$, $Z_{\frac{\alpha}{2}}$, and $-Z_{1-\frac{\alpha}{2}}$ interchangeably. This is noted when they present the *situations/problems* that illustrate and/or reinforce the use of the CI. On the other hand, to formalize the conceptualization of the CI, teachers A, B, and C resort to the algebraic representation expressed by the formula $\left[\mu \pm Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$. However, this leads to a conceptual inaccuracy, as the sample mean ($\bar{x}$) is confused with the population mean (μ) (Flider & Cumming, 2005; López-Martín et al., 2019).

In developing the course, GeoGebra is used to illustrate the *graphical representation* of the CI, which is used to discuss the conceptual meaning of the term asymptote, since in **Figure 2** the normal curve intersects the abscissa axis and thus does not meet the definition. Although different representations are used to access the mathematical object, it is necessary to interpret the studied phenomena based on initial premises or conclusions (Choi et al., 2016). This is discussed in detail with the prospective teachers who participated in the study.

As for the *definitions* of teachers A, B, and C, it is concluded that they have only partially mastered the common content knowledge. This is because they commit a conceptual error by confusing the critical values of the confidence level $\left(Z_{\frac{\alpha}{2}} \text{ and } Z_{1-\frac{\alpha}{2}}\right)$ with the CI limits $\left(\bar{x} - Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}};\ \bar{x} + Z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$ (see **Figure 3**). In developing this lesson, teachers do not present the statistically formal definition of CI (Devore, 2008). They also do not emphasize interpretation in the context of the area under the curve of a normal distribution.

As described above, the *procedures* are algorithmic in nature as they encourage the use of formulas that lack meaning (Hasim et al., 2023). This can be seen in the *procedure* presented in the activity of the duration of the mileage of a car tire, when the prospective teachers substitute in the expression $Z = \frac{x-\mu}{\sigma}$ the values of Z, μ, and σ without distinguishing the meaning of each statistic and parameter. In particular, they fail to distinguish the population mean from the sample mean, which prevents them from determining the CI for the population mean.

In the instance of *reflection on action* (Hußner et al., 2023; Schön, 1987), the ambiguity in the meaning of the terms sample and population is discussed with teachers A, B, and C, especially with the statisticians and parameters (López-Martín et al., 2019). In addition, the teacher trainer promotes the critical analysis of the procedure used and strengthens the conceptual aspects associated with it. Similarly, it is noted that the *procedures* used by teachers A, B, and C lack *arguments* to support their development. This is illustrated by the following excerpt.

Teacher trainer: During your lesson, you drew conclusions about a population from a sample, but what do you mean by population and sample?

Teacher C: A population is a set of data that share a common characteristic, while a sample is a subset of the selected population.

Table 2. Description of the level of achievement of the mathematical competence

Mathematical competence–Lesson 1
Mathematical sub-competency 1: Task solving (level of achievement reached: L0)
Observation: Teachers reproduce mathematical tasks and their procedures without considering the relationships to other mathematical objects, e.g. they do not distinguish between the population mean and the sample mean. They also confuse the critical values ($Z_{\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$) with the limits of the CI. Similarly, teachers A, B, and C fail to use a variety of procedures to solve the same task, and these procedures lack justification or reasoning.
Mathematical sub-competency 2: Problem posing (level of achievement reached: L1)
Observation: Teachers propose a task suitable for the educational level, consider the objective set by the curriculum, the prior knowledge (samples, frequency table, measures of central tendency, range, probabilities of events, normal distribution and random variables) required for the study of CI. However, it does not provide for articulation between the concepts, errors or difficulties of students' possible answers, such as (1) inaccurate significance of CI, (2) ignorance of the inferential nature of CI, (3) procedural errors in CI estimation, and others.
Mathematical sub-competency 3: Analysis of practices to solve problems (level of achievement reached: L0)
Observation: Teachers A, B, and C analyze the mathematical practices through questionnaires that they complete using the Quizziz application to provide feedback on their students' productions. However, the teachers do not address specific errors or conflicts that might arise from the solutions presented by their students.

> **Teacher A:** For example, all the students in a school are a population and the students in an educational level would be a sample.
>
> **Teacher trainer:** Your answers and examples are certainly very close to the concept of population and sample. However, it is important to specify all the elements that make up the object under study (parameters, statisticians, type of variables) along with the associated notation. In the lesson, they propose an activity related to the duration of the mileage of a car tire. To do this, they apply the formula $Z = \frac{x-\mu}{\sigma}$. Could you tell me what you mean by the individual parameters of the expression?
>
> **Teacher B:** Sure, the letter Z is the typed value, μ is the average and σ is the standard deviation.
>
> **Teacher trainer:** I agree that Z is a typed value of a normal distribution, but we need to specify that μ and σ correspond to the parameters of the population. Our goal in this problem is to determine a CI for the mean of the population. We must therefore estimate μ from a sample average and must not confuse the critical values of $Z_{\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$ with the limits of a CI.

Regarding the *errors, beliefs and ambiguities* made by teachers A, B, and C in the implementation of the lesson on CI, an error related to the construction of the CI in the graphical representation is detected, since they confuse the critical values ($Z_{\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$) with the limits of the CI (Roldán et al., 2020).

It is important to avoid *ambiguities* and beliefs that lead to misunderstandings about CI in the teaching-learning process. In particular, teachers A, B, and C only specify the numerical values associated with CI, which distracts from the interpretation and inferential nature of the concept (Fidler & Cumming, 2005).

The aspects described above were part of the process of reflection on teaching practice that teachers A, B, and C carried out through questions and interviews related to the eligibility criteria to identify possible improvements. In this case, the instructor and the trainee teachers critically analyzed the validity and effectiveness of the intervention through a cycle that included:

(1) observation of the action,

(2) review of the action,

(3) awareness of the main aspects developed, and

(4) discussion and definition of alternative methods and implementation of an improved proposal (Korthagen, 2001).

After characterizing the epistemic knowledge of teachers, A, B, and C, **Table 2** shows the level of performance in each of the sub-competences of mathematical competence.

The second lesson on CI (redesigned lesson after the analysis and reflection stage) in microteaching context developed again by teachers A, B, and C is described below.

Description of the Second Confidence Interval Lesson (Redesign)

In the newly designed lesson, teachers A, B, and C set the same learning objective as in lesson 1 and then activate prior knowledge by asking two questions: *"What are the characteristics of the normal distribution? And what concepts of statistics do you see in the picture?"*, which promote skills such as discussion, communication and argumentation. This is seen as an area for improvement as teachers A, B, and C use a questionnaire in lesson 1 with questions that have no meaning and are procedural in nature.

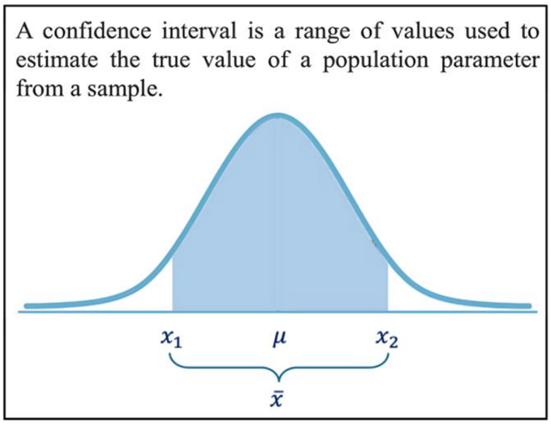

Figure 6. Definition of the CI proposed by professors A, B, and C (Source: Authors' own elaboration)

In the development of the class, unlike in lesson 1, teacher C emphasizes *"that a normal distribution is generally defined by a mean, denoted by μ, and a standard deviation as σ"* and promotes a statistically formal definition for the CI, giving the graphical representation (**Figure 6**), an aspect improved from lesson 1. This is an improved aspect from lesson 1, and the above corresponds to elements discussed in the process of reflecting on practice, encouraging students to correctly interpret the CI to draw conclusions about the situation under study.

Next, the referents pose a *situation/problem* (see **Figure 7**) that aims to reinforce the conceptual aspects described above by emphasizing the elements (sample mean, confidence level, population standard deviation, sample) that form the definition of the CI. It should be noted that the problem posed results from the review of teaching materials by teachers A, B, and C, who also express the intention to incorporate contextualized activities that demonstrate the benefits of CI into the teaching process.

According to the situation/problem described above, teachers A, B, and C define: *"The confidence level is* $(1-\alpha)\cdot 100\%$, *which corresponds to the proportion of cases in which the interval obtained actually contains the parameter, with a significance level of* α*"*. During the feedback and

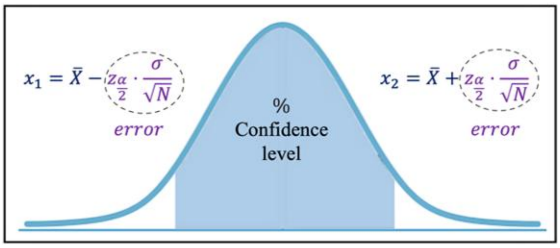

Figure 8. Error associated with CI limits (Source: Authors' own elaboration)

reflection, the distinction between the confidence level and the CI was discussed, the difficulty of which was overcome by teachers A, B, and C in the redesigned lesson. Similarly, in this second lesson, they set the limits of the CI with the expressions $x_1 = \left(\bar{x} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$ and $x_2 = \left(\bar{x} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$ and give an associated error $\left(z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$. The above explanations contribute to a deeper understanding of the concept of the CI.

It should be noted that in the diagram presented by teachers A, B, and C (**Figure 8**), the critical values are expressed as $-z_{\frac{\alpha}{2}}$ and $z_{\frac{\alpha}{2}}$. However, in the algebraic expression, the notation associated with the critical values is represented by $-z_{1-\frac{\alpha}{2}}$ and $z_{1-\frac{\alpha}{2}}$. As we reflect on the relevance of both notations, the importance of strengthening their equivalence is discussed.

Teachers A, B, and C then ask their students the following question: *"Will there be only one confidence interval"*, after stimulating a discussion they point out: *"It depends on the number of samples, because each sample has a different mean, so you get different Cis"*. Similarly, teachers A, B, and C point out that *"when N is very large, the error associated with the CI tends towards zero"*. Therefore, the sample mean is close to the population mean. The above approach shows a better conceptual appropriation of the elements that interact in the CI, unlike in lesson 1 where they could not distinguish the concepts of confidence level, typing and limits of the CI.

Finally, regarding the results of the *situation/problem* (**Figure 7**), teacher C emphasizes the importance of

A pharmacy owner is interested in knowing whether the refrigerator in which the vaccines are stored has maintained an average temperature of 5.5°C over the past few months. If not, he will need to purchase a new one to avoid compromising the vaccines' durability and effectiveness.

Analyze the following example. Then, determine the 95% confidence intervals for the other samples.

Five random samples of different sizes were drawn. It is known, furthermore, that the standard deviation of the refrigerator temperature is 1.2°C:

	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
Sample mean (°C)	6	5.8	4.5	5.4	6.5
sample size (days)	30	45	50	55	60

Figure 7. Situation/problem used by teachers A, B, and C, provided by the text of the student in the middle 3-4 year (Osorio et al., 2019, p. 183)

interpreting the phenomenon associated with the activity and asks the following question to reinforce the CI: *"Is the mean of the problem included in the given CI?"*. In response, the students realize that the mean of the activity (5.5 °C) is not included in the CI (6.19; 6.80), suggesting that the refrigerator does not ensure the shelf life and effectiveness of the vaccines. Thus, unlike lesson 1, where only the algebraic expression is used to calculate the CI, lesson 2 encourages discussion that leads to an appropriate interpretation of the analyzed phenomenon, demonstrating progress in the effectiveness of the teaching process.

Teachers A, B, and C end the lesson by asking five questions with alternatives described as follows: *"(1) What happens to the interval limits as N increases, (2) What happens to the error as the sample size approaches the population size, (3) What is the error associated with confidence intervals, (4) If the population mean is equal to the sample mean, is it necessary to, (a) If the population mean is equal to the sample mean, is it necessary to determine confidence intervals, (b) What happens to the error when the sample size approaches the population size, (c) What happens to the error associated with confidence intervals? (5) For a sample of 100 data, determine a CI to estimate the mean given a* 95% CI ($Z = 1.96$)*, a sample mean of 2 and a population standard deviation of 10"*.

The purpose of this activity is to consolidate and evaluate the knowledge acquired in class. To this end, teachers A, B, and C have collected, reinforced and, if necessary, corrected the answers given by the students. Some of the claims mentioned by the teachers were: *"the amplitude of the CI decreases as the sample becomes larger"*, *"if the sample mean and the population mean of the same phenomenon are known, it is not necessary to estimate a CI"*.

Reflections on Improvements

It can be concluded that the opportunity to practice, analyze and reflect on CI teaching and learning processes in a controlled environment with reduced complexity contributes to the continuous improvement of the professional performance of the teachers participating in the study (see **Table 3**).

Figure 9 summarizes the performance levels achieved by the teachers in training regarding mathematical competence from the first implementation (first lesson) through subsequent reflection to re-implementation (second lesson).

Table 3. Observed improvements in the epistemic facet between the first lesson and the redesigned lesson

Situation/problem
Opportunity for improvement lesson 1 ▪ The concept of CI is illustrated by an activity that promotes and graphically illustrates the properties of the normal distribution in a purely intra-mathematical context. ▪ Tasks that are related to daily life and whose sole purpose is to use formulas to determine the CI. ▪ Neglecting the importance of interpreting and evaluating the CI to promote statistical literacy. The aspects to be improved in relation to the situation/problem are consistent with what was suggested by Choi et al. (2016). *Improved aspect in lesson 2* ▪ Using a situation/problem, conceptual aspects such as sample, population, sample mean, population standard deviation and confidence level are explored in depth, elements that represent the definition of the CI. ▪ Contextual activities that demonstrate the usefulness of the CI are integrated into the teaching process.
Language
Opportunity for improvement lesson 1 ▪ When it comes to the use of language, pre-service teachers use various forms of representation. These include symbolic representations that use the values of the critical interval interchangeably. ▪ Algebraic representations are used to formalize the conceptualization of the CI. However, they lead to a conceptual inaccuracy as they confuse the sample mean ($\bar{x}$) with the population mean (μ). ▪ GeoGebra is used to illustrate the graphical representation of the CI. However, the normal curve they present intercepts the abscissa axis, not satisfying the definition. The complexities described above are consistent with the research developed by López-Martín et al. (2019) and Flider and Cumming (2005). *Improved aspect in lesson 2* ▪ A graphical representation that satisfies the definition of CI is required. ▪ Various representations are encouraged, including symbolic, graphical, algebraic, and verbal representations.
Definitions, propositions, procedures, and arguments
Opportunity for improvement lesson 1 ▪ The definition of Professors A, B, and C commits a conceptual error by confusing the values of the critical confidence level with the CI limits. It also fails to emphasize the interpretation of the area under the curve of a normal distribution. ▪ The methods are algorithmic in nature as they encourage the use of meaningless formulas. They also lack arguments to support their development. This is consistent with the arguments put forward by López-Martín et al. (2019) and Roldán et al. (2020).

Table 3 (Continued). Observed improvements in the epistemic facet between the first lesson and the redesigned lesson

Situation/problem
Improved aspect in lesson 2 ▪ Teachers A, B, and C activate prior knowledge through two specific questions that promote skills such as discussion, communication and argumentation. ▪ They emphasize that a normal distribution is generally defined by a mean denoted by μ and a standard deviation σ. ▪ They promote a formal statistical definition for CI that enables its correct interpretation by allowing conclusions to be drawn about situations and/or phenomena studied. ▪ An understanding of the CI is achieved by defining the confidence level, adjusting the CI bounds and specifying the associated error.
Errors, ambiguities, and beliefs
Opportunity for improvement lesson 1 ▪ Teachers A, B, and C believe it is sufficient to determine the numerical values associated with the IC, thus eliminating the importance of interpretation and the inferential nature of the concept. This is consistent with the arguments made by Fidler and Cumming (2005). *Improved aspect in lesson 2* ▪ Conceptual confusions are corrected, and the relationship between the CI, the significance level, and the probability of containing the population parameter is clarified.

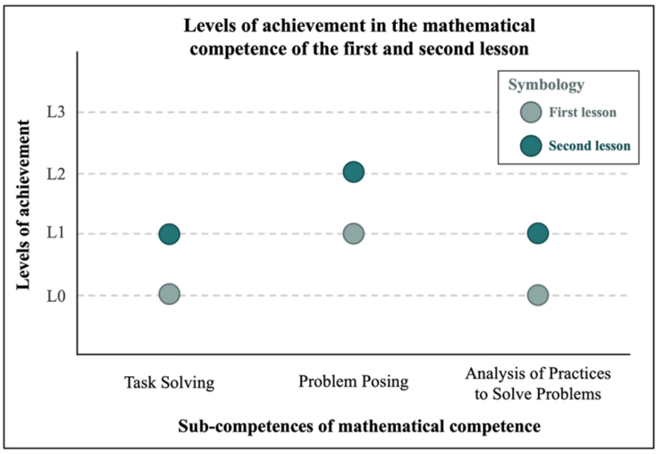

Figure 9. Achievement levels reached in the first and second lessons (Source: Authors' own elaboration)

DISCUSSION AND CONCLUSION

This study reports the results of an investigation that aimed to characterize the mathematical knowledge and competence of pre-service teachers when conducting instructional processes on CI in a microteaching context. Eleven trainee teachers took part in the study. Over the course of an academic semester, they conducted two CI classes in microteaching environments, fostering collaboration and reflection on their teaching practices.

The DMKC model is a tool that emerges from the consensus and findings of mathematics education studies on the knowledge and skills a teacher must possess to conduct effective teaching and learning (Breda et al., 2018; Pino-Fan & Godino, 2015; Pino-Fan et al., 2018, 2023; Schoenfeld & Kilpatrick, 2008; Shulman, 1987). This model was used to infer the epistemic knowledge and mathematical competence of the eleven participating teachers.

During the study, information and recommendations from the scientific literature on the teaching and learning of CI were reviewed. Based on the difficulties encountered in the instructional processes on CI and to promote the learning of this mathematical subject, it was decided to introduce a LS cycle. The systematization of critical and collective reflection on teaching practice is increasingly integrated into innovative experiences in teacher education (Hußner et al., 2023). In this study, we intentionally created the conditions and mechanisms for carrying out the development of collaborative work through the planning/design, implementation, analysis/reflection, and redesign of a lesson on the concept of CI. To guide and orient the reflection, the DSCs were used, specifically the components and descriptors of epistemic suitability that allow the identification of mathematical richness. Based on the above, we found that analyzing a lesson recorded with the DSC in a microteaching context deepens epistemic knowledge and favors mathematical competence. This is confirmed by the fact that when the teaching process is repeated (second lesson), they reach a more advanced level of performance in each of the sub-competencies, thus achieving greater suitability (Pino-Fan et al., 2023).

As for the task solving sub-competency, the prospective teachers were at the performance level (L0) in the first lesson. In the redesigned class, it was observed that the teachers reached a higher level of performance (L1) as they were able to solve problems appropriate to their educational level and their suggested approaches were accompanied by justifications or arguments. In addition, the inclusion of elements specific to the L2 level was observed, such as the use of different representations and the connection between CI and mathematical objects from previous educational levels.

As far as the sub-competence problem posing is concerned, the prospective teachers are used in the first implemented teaching unit at the (L1) level. Later, in the redesigned lesson, progress towards proficiency level (L2) can be seen as they emphasize theoretical aspects and promote the formal statistical definition of CI, achieving a better understanding of the mathematical object in their students. They also correct conceptual errors and clarify the relationship between CI, the significance level and the probability of containing the population parameter.

Regarding the sub-competence of analyzing problem-solving practices, the participating teachers initially position themselves at level (L0). However, after the redesigned lesson 2, they reach level (L1) as they perform experience-based analysis of their students' mathematical practices. These discussions take place in the analysis and reflection phase of LS and enable the clarification of aspects such as the graphical representation that corresponds to the definition of IC.

Finally, in this study, we analyzed mathematical knowledge and skills using achievement levels (Pino-Fan et al., 2023). However, we recommend that future studies focus on analyzing the skills that emerge when teaching in the confidence domain in real classroom contexts. Regarding the latter, other types of variables influence the school environment, allowing us to explore and ensure the impact of practice-based disciplinary pedagogical training, which contributes to professional and holistic development.

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AI statement: The authors stated that no generative AI or AI-based tools were used in any part of the study, including data analysis, writing, or editing.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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