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Mathematics performance and the relations to the cognitive reflection, fluid intelligence, conditional reasoning, and logical-mathematical language in the university students

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Abstract

Recent research on university mathematics education has emphasized the need to understand how cognitive factors and specific mathematical skills contribute to students' mathematical performance in advanced mathematical contexts. This paper examines the predictive value of fluid intelligence (Gf), cognitive reflection (CR), mastery of logical-mathematical language, and conditional reasoning on the academic achievement of first-year mathematics undergraduates. Combining theoretical perspectives from cognitive science and mathematics education, this study analyzes the extent to which (individually and jointly) these cognitive and disciplinary variables explain differences in mathematical performance at the university level. Logistic and multiple linear regression models are used to examine the relationships existing between these variables, paying special attention to their role in predicting students' academic outcomes. The findings reveal that both general cognitive abilities and discipline-specific abilities, such as formal language use and conditional reasoning, are significant and independent predictors of academic success in university mathematics. This research contributes to the growing body of knowledge on the factors that shape students' progression and performance in higher mathematics. Moreover, it highlights the complex interplay between cognitive resources and formal mathematical reasoning at the tertiary level.

Keywords: fluid intelligence, cognitive reflection, conditional reasoning, logical-mathematical language, academic achievement, university mathematics education

INTRODUCTION

Research on university mathematics education has found that new students face various difficulties in certain mathematics topics (Gueudet & Thomas 2020; Thomas et al., 2012), as well as in the development of argumentation and formal thinking skills (Balacheff, 2010; Durand-Guerrier, 2020; Durand-Guerrier & Dawkins, 2018; Hanna & de Villiers, 2012; Harel & Sowder, 2007; Stylianou et al., 2015). Therefore, it is necessary to delve further into these issues. As for formal reasoning, study results have been varied. Some have suggested a lack of conceptual or procedural mathematical knowledge, which is necessary for formal reasoning (Leron & Hazzan, 2009; Stylianides, 2007; Stylianou et al., 2015). Others have highlighted the need to examine the cognitive abilities necessary for formal reasoning (Kryjevskaia et al., 2014, Primi et al., 2010). In this regard, an exploration of the interaction between analytical and intuitive thinking is essential.

Similarly, research on factors influencing mathematical performance reveals the dependency on both general cognitive abilities, such as fluid intelligence (Gf) (Cormier et al., 2017), and knowledge and abilities specific to the field of knowledge. Of these general cognitive abilities, fluid intelligence (Gf) strongly predicts math performance during childhood and adolescence (Cormier et al., 2017), as does cognitive reflection (CR) (Dawkins & Norton, 2022; Gómez-Chacón et al., 2014; Morsanyi et al., 2014).

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Contribution to the literature

- Academic achievement and student performance in university mathematics are subject not only to general cognitive variables (cognitive reflection (CR), fluid intelligence (Gf)), but also to specific mathematics abilities (conditional reasoning (C) and logical-mathematical language (L)).
- Predictive statistical modelling techniques allow us to consider the interaction variable between cognitive reflection (CR) and fluid intelligence (Gf). Significance evidence from these techniques suggests relevant interaction effects. This is essential to enriching the understanding of how general cognitive variables and mathematical abilities variables jointly contribute to mathematical reasoning.
- Individual differences regarding the cognitive variables of cognitive reflection (CR) and fluid intelligence (Gf) as variables that qualify individual characteristics and that predict academic performance in university mathematics are explored.

Logical-Mathematical Language and Conditional Reasoning

Language is a key cultural element and plays an essential role in the learning and understanding of mathematics (Ferrari, 2004; Pimm, 1987; Planas et al., 2018; Radford, 2002; Sfard, 2001). In addition to its communicative function, formal mathematical language allows for the creation, objectification, and precise manipulation of concepts, properties, and relationships that are central to mathematical activity (Coppola et al., 2019). In practice, this semiotic dimension of language may be considered as the need for students to be able to translate informal statements (in natural language) into formal language, with the proper use of quantifiers, symbols, and logical structures (Durand-Guerrier, 2008, 2020; Radford, 2002; Sfard, 2001). As Alcock and Simpson (2002) noted, the development of advanced mathematical thinking depends largely on one's mastery of formal language. This language provides individuals with the necessary tools to understand definitions, construct demonstrations, and communicate rigorous reasoning in university contexts, especially during the transition to university period.

Numerous studies have shown that the understanding and mastery of logical-mathematical language are determinants for academic success, especially during the period of transition to university education. Studies such as those by Alcock and Davies (2024), Durand-Guerrier (2015, 2020), Durand-Guerrier and Dawkins (2008), Epp (2003), and Selden and Selden (1995) have suggested that university students have major difficulties in formalizing, negating, and manipulating statements containing quantifiers and logical operators. The interpretation of connectors such as "if… then…" (conditional) and "if and only if" (biconditional) has been found to be especially problematic. These difficulties not only affect students, but they may also impact on teachers (Durand-Guerrier, 2015). Specifically, the negation of statements containing quantifiers and conditionals represents a considerable cognitive and linguistic challenge. This author emphasized that, while negation may be ambiguous or contextual in natural language, in formal mathematical language, it should be precise and follow strict rules of predicate logic.

As for conditional rational thinking, studies such as those carried out by Durand-Guerrier (2003), Epp (2003), and Stylianides (2007) highlight the fact that students tend to interpret conditionals using intuitive models of causality, temporality, or equivalence, instead of relying on classical material logic. This may lead to errors in validation, negation, and the use of counterexamples in mathematical contexts. This phenomenon has also been analyzed from the theory of mental models (Johnson-Laird & Ragni, 2025), which argues that students tend to construct simplified or incomplete representations of the possible cases involved in a conditional statement. This explains the systematic errors made when denying, validating, or seeking counterexamples to universal conditionals.

Studies by Alcock and Inglis (2008), Attridge and Inglis (2013), and García-Madruga et al. (2022) have shown that even advanced students can resort to using alternative schemes (for example, interpreting a conditional as a conjunction or biconditional) or using reduced models that do not consider all the possibilities. Therefore, the effective teaching of logics and formal mathematical language should focus on not only formal training but also the development of metacognition and reflection in one's own reasoning (García-Madruga et al., 2022; Gómez-Chacón et al., 2014).

Fluid Intelligence

At the university level, concept acquisition becomes increasingly abstract. Fluid intelligence (Gf) may be more influential. Research suggests that students having a strong Gf may have a better understanding of complex mathematical theories and may be better at performing advanced mathematical research (Kyllonen & Kell, 2017).

Fluid intelligence (Gf) is understood as the ability to reason abstractly and establish new relationships between multiple mental representations (Cattell, 1987). It tends to be measured through the performance on tests that include deduction and inductive reasoning. This measures the individual's ability to think, solve complex problems using logical reasoning, and make conclusions (Ali & Ara, 2017; Obeidat & Saleh, 2022). Therefore, fluid intelligence is an indicator of academic performance (Passolunghi et al., 2022). This variable is of interest in our study given its influence on abstract and spatial reasoning (Primi et al., 2010). Assuming that Gf is an influential factor in mathematics learning and performance, we shall examine the type of relationship existing between this variable, cognitive reflection, and logical-mathematical language and conditional reasoning in mathematics. In this work, we use a classic measure and Gf index based on visual-spatial reasoning skills: The Raven's Advanced Progressive Matrices Test (Raven et al., 1995).

Studies on adolescents have shown that Gf strongly predicts math performance (Roth et al., 2015; Rubio-Sánchez et al., 2023). The meta-analysis carried out by Peng et al. (2019) found moderate correlations of Gf with mathematics ($r \approx .40$) and indicated that Gf had a stronger relationship with complex skills as opposed to basic ones.

Fluid intelligence has been positively correlated with the degree to which students acquire higher-order thinking skills and achieve better results (Obeidat & Saleh, 2022; Preusse et al., 2011). This indicates that individuals having a high level of fluid intelligence obtain better results on abstract thinking tasks.

Cognitive Reflection

Cognitive reflection (CR), a mental process referring to the ability to override an initial intuitive response and engage in further reflection, has been linked to several cognitive skills and academic achievements, especially in the area of mathematics. Studies have shown that CR is related to certain typical biases and heuristics (Toplak et al., 2011). These authors examined its effect not only on numeric tasks but also on syllogistic and logical reasoning. It has been found that CRT correlates with measures of intelligence and measures of arithmetic calculation (Cokely & Kelley, 2009) or with geometric reasoning (Rubio-Sánchez et al., 2023), explaining significant variances in reasoning and decision-making tasks. Therefore, the meta-analysis on cognitive reflection, intelligence, and cognitive skills carried out by Otero et al. (2022) indicated that cognitive reflection is correlated with several cognitive abilities and skills, with intelligence and numerical skills being responsible for 69% of its variance. Other empirical studies have shown that in high school students, beliefs, cognitive reflection, working memory, and reasoning skills interact to predict students' mathematical achievement (Gómez-Chacón et al., 2014).

Given this background information, this study aims to explore the interrelation between general cognitive variables (cognitive reflection (CR) and fluid intelligence (Gf)) and specific mathematical skills, especially the mathematical ability of students with regard to the use of logical-mathematical language and conditional reasoning. This information will be used to predict how these relations influence mathematics achievement at a university level. The following research questions have been proposed:

RQ1 What interrelationship exists in subjects between general cognitive skills of cognitive reflection (CR) and fluid intelligence (Gf) with the mathematical skills of the use of logical-mathematical language and conditional reasoning?

RQ2 How do the above variables (CR and Gf, the use of logical-mathematical language and conditional reasoning) jointly predict academic achievement in mathematics?

METHODS

Participants and Context

The study group was made up of 66 first-year university undergraduate students in the first year of the mathematics degree. Of these, 62.1% identified themselves as male ($n = 43$) and 37.98% as female ($n = 23$). The average participant age was 17.5 years of age ($SD = 2.75$).

Students from two class groups participated. Both groups shared the same study program, which included the following content: Understanding basic concepts, techniques, and applications of mathematical topics such as mathematical logic, set theory, elementary number theory, discrete mathematics, and complex numbers. The program had the following priority objectives:

1) Understanding mathematical language and differences with natural language,
2) Knowing basic demonstration techniques in mathematics.

Data Collection Instruments and Procedures

Cognitive reflection

The Cognitive Reflection Test (CRT) (Frederick, 2005) has been used, adapted to the Spanish population, to measure the capacity for cognitive reflection in problem solving. This test consists of three problems used in Frederick's initial test and two additional questions proposed in the study of Gómez-Chacón et al. (2014). The CRT assesses an individual's will and aptitude to be reflexive when attempting to find solutions to a verbal mathematical reasoning problem.

A time limit of 20 minutes was established to solve the five problems, which were presented in open answer format. Subsequently, participants were also asked to assess the percentage of their classmates who would give a correct solution to each problem (that is, to qualify the difficulty of the problems). The solutions to each problem and the estimated percentages should be written in data format. Three types of measures were provided for each problem: correct answer, intuitive answer, and a rating of the problem's difficulty. The total score on the CRT was calculated as the number of correct responses.

Fluid intelligence

Raven's Advanced Progressive Matrices (RPMT) (Raven et al., 1995) was used as a test to measure non-verbal abstract reasoning. It is considered a classic measure and an index of fluid intelligence. The RPMT contains sixty visual analogy problems. To solve each test problem, the participant must identify the relevant features of a series of visual abstract figures and shapes, plus an empty box, discover the rules governing the presentation of the various figurative elements, use these rules to determine the missing element in the box, and then choose the correct element from the alternative answers arranged below the matrix. The dependent variable was the number of items solved correctly. Cronbach's alpha was 0.958.

Mathematics problems questionnaire

The main instrument used to assess the logical-mathematical reasoning of university mathematics students consisted of a problem questionnaire made up of two types of problems (P1 and P2). The first type of problem focused on the translation of statements from everyday language to formal mathematical language, the proper use of quantifiers, and the construction of negations corresponding to both natural and formal language. The second type of problem addressed conditional reasoning, asking students to analyze universal propositions with conditional structure and to justify their validity or falsity, by using specific examples and counterexamples or abstract generalizations. Both types of problems are specified below.

Problem statement 1 (P1):

Write the following statements with quantifiers, properly defining the same sets for all of them. Then write their negation, with and without quantifiers:

A: A student enrolled in subjects from all of the courses.

B: At least one student enrolled in each subject.

C: Some students have a scholarship.

Problem Statement 2 (P2):

Let us consider the following sets: A: = {n∈N|n is a multiple of a} and B: = {n∈N|n is a multiple of b}

a) Establish a necessary and sufficient condition so that an element belongs to A and to B.

b) Analyze the validity of the following statement: "∀n∈A and ∀m∈B, the product nm is a multiple of k". Indicate whether the statement is true or false and justify your answer.

Problem 1 (P1) focuses on logical-mathematical language. The student is asked to translate propositions from natural language to formal mathematical language with quantifiers and to adequately define the sets involved. For example, a proposition of the problem is: "A student enrolled in subjects from all of the courses", whereby students must correctly express this proposition with quantifiers, as well as formulate its negation in both mathematical and natural language.

Problem 2 (P2) refers to conditional reasoning through propositions whose structure is universal-conditional, whereby the antecedent consists of the elements' belonging to the respective sets, and the consequent establishes a property of said elements.

Mathematical achievement

According to Roth et al. (2015), school grades are a good measure of academic achievement, since they include information on academic performance over an extensive period and are based on distinct sources. As mathematics performance criteria, this study uses school grades from an introductory subject that aims to establish and lay down the foundations for general university mathematical reasoning during the first year of the mathematics undergraduate degree program (Wood, 2001).

Grades at the end of the academic period are considered in the form of a numerical score on a scale of 0 to 10 points. This assessment is focused on the theoretical and procedural mathematical content of the course syllabus. It is assigned by the mathematics teacher responsible for the subject. Monitoring mechanisms are in place to ensure fair grading, independent of the teacher.

Data Analysis

Variable characterization and categorization

The different instruments require specific data processing methods. For the CR and Gf tests, descriptive analysis was performed initially: calculation of the means, standard deviation, and internal consistency (Cronbach's α) were carried out for each of these, as well as correlation studies.

Table 1. Use of logical-mathematical language (L) variable, categories in problem P1

Category	Definition	Score indicator	Example
L1: Definition of sets	The student clearly defines the sets intervening in the statement's formalization.	It is scored as 1 if the student correctly defines the sets involved; otherwise, it is scored as 0.	"Let A be the enrolled students and C be the courses…"
L2: Affirmative formalization	The student correctly translates the statement from natural language to formal mathematical language using quantifiers.	It is scored as 1 if the translation is correct and respects the logical order; otherwise, it is scored as 0.	"∃a ∈ A: ∀c ∈ C, a is enrolled in c"
L3: Negation in natural language	The student uses natural language to correctly express the negation of the original statement.	It is scored as 1 if the negation maintains the original statement's meaning; otherwise, it is scored as 0.	"No student is enrolled in all of the courses."
L4: Formal negation with quantifiers	The student correctly uses quantifiers when formally expressing the negation of the original statement.	It is scored as 1 if the order and use of the quantifiers is correct; otherwise, it is scored as 0.	"∀a ∈ A, ∃c ∈ C: a is not enrolled in c"

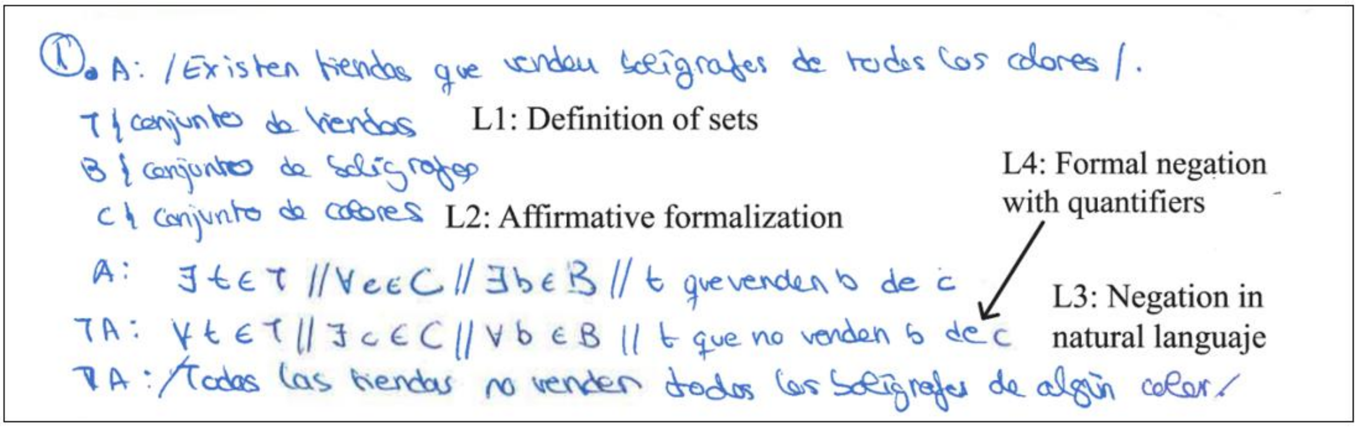

Figure 1. Extract from the P1 resolution protocol with categories, student i22 (Source: Authors' own elaboration)

Data mining techniques, as well as the described variables, cognitive reflection (CR) and fluid intelligence (Gf), and those related to logical-mathematical language and conditional reasoning were used to predict academic achievement in mathematics. In addition, an interaction variable that considered both CR and Gf (called CR_Gf) was used. It is an interaction variable that describes the interdependence between variables and how their combined effects influence the final result. The interaction between these two variables takes place when the effect of the independent variable (CR, Cognitive Reflection) on the dependent variable (Academic Achievement) varies according to the levels of a moderating variable (Gf, Fluid intelligence). In our case, this effect has been recorded using the CR_Gf variable. The results of real phenomena are complex, and they depend not only on the sum of individual effects, but also on the context and the interdependencies between variables.

This variable was defined as the product of both scores, normalized with respect to the maximum possible value on Gf, to be represented as a percentage. The following formula was used:

$$RC_{Gf} = \frac{(RC \cdot Gf)}{60} \quad (1)$$

where 60 represented the maximum possible score on the Raven's matrix test. This transformation has a dual purpose: on the one hand, it reflects an interaction between CR and Gf, a common practice in regression models to study combined effects. On the other hand, it permits expression of interaction on a percentage scale, offering an interpretation that is coherent and comparable with other study variables that are also normalized.

The students' responses to the mathematics problems (P1 and P2) were analyzed and coded using qualitative data processing. Content analysis was used to identify the categories that allow us to define the logical-mathematical language and conditional reasoning variables.

The *use of logical-mathematical language (L)* variable was studied via analysis in *Problem 1* of the following categories: Definition of sets (L1), Affirmative formalization (L2), Formal negation with quantifiers (L3), Negation in natural language (L4), Formal negation with quantifiers (L5) (**Table 1** and **Figure 1**).

The *conditional reasoning (C)* variable is studied by analyzing the following categories in *Problem 2*: Necessary and sufficient condition (C1), Use of counterexample (C2), Generalized counterexample (C3), Use of the example (C4). (**Table 2** and **Figure 2**).

Table 2. Conditional reasoning (C) variable, categories in problem P2

Category	Definition	Score indicator	Example
C1: Necessary and sufficient condition	The student correctly identifies and expresses necessary and sufficient conditions to simultaneously belong to the given sets.	It is scored as 1 if the condition is correctly simplified and expresses a logical equivalence (necessary sufficient); otherwise, it is scored as 0.	"A number is a multiple of 12 and 20 if and only if it is a multiple of 60."
C2: Use of the counterexample	The student correctly uses a numeric counterexample to assess the validity of a conditional statement.	It is scored as 1 if the student correctly identifies at least one specific counterexample that invalidates proposition; otherwise, it is scored as 0.	"The affirmation indicates that there is a multiple of 12 and a multiple of 20 whose product is 70. But the product of any multiple of 12 with any multiple of 20 will always be a multiple of 240; therefore, it can never be exactly 70. Therefore, 70 is specific numeric counterexample demonstrating the falsity of the given statement."
C3: Generalized counterexample	The student offers a generalized or abstract refutation, beyond a specific case, demonstrating the falsity of the conditional proposition.	It is scored as 1 if it provides a correct general formulation to identify all of the possible counterexamples; otherwise, it is scored as 0.	"Given that any common multiple of 12 and 20 is a multiple of 60, any product of these multiples is a multiple of 60, and 70 is not. Therefore, no product of numbers taken from these sets can be 70. This abstract formulation generalizes all possible counterexamples."
C4: Use of the example	The student uses an example or a specific case to argue te validity or falsity of the conditional statement, without reaching a generalization.	It is scored as 1 if the student correctly identifies at least one concrete example relevant to the statement, even if they do not generalize; otherwise, it is scored as 0.	"For example, 12 is a multiple of 12 and 20 is a multiple of 20; the product 12 x 20 = 240, which is a multiple of 60 but not of 70. Therefore, the product cannot be 70."

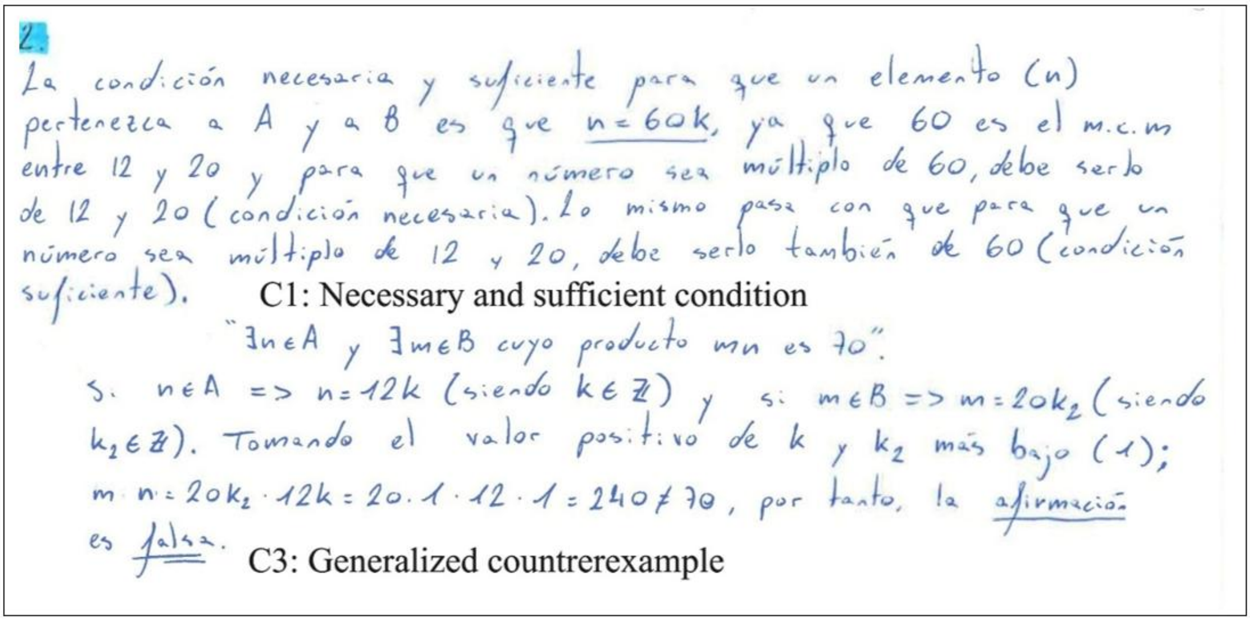

Figure 2. Extract from the P2 resolution protocol with categories, student i2 (Source: Authors' own elaboration)

In short, the coding of the described categories of the questionnaire's mathematical problems into a binary (0/1) format allowed for the transformation of the open-ended responses into quantitative variables. Thus, the logical-mathematical language and conditional reasoning variables are defined as the arithmetic mean of the categories together with the score obtained for the problem.

Preprocessing techniques and regression models for relationship analysis

In this study, two statistical models were employed: logistic regression and multiple linear regression

(Frawley et al., 1992; Nisbet et al., 2017). Applied separately, these models yielded verifiable and specific information about the variables and their relationships. Logistic regression was used to identify whether categorical distinctions were statistically meaningful, while multiple linear regression served as the main tool to analyze how several factors jointly contributed to mathematical performance.

The following sections present the methodological steps: The treatment of missing values, the exploratory use of logistic regression, and the construction of the multiple linear regression model, each selected for its ability to reveal specific relationships in line with the study objectives.

Null value handling

Null value handling is a fundamental part of pre-processing in any data analysis. Statistical and machine learning models, such as multiple linear regression, require complex data sets, and the presence of missing values can lead to technical errors, reducing the reliability of results. Furthermore, bias may be introduced into the estimates if these values are not handled properly.

Various strategies exist to take on this issue. In our case, we have opted for a technique that is adequate according to the sample size: imputation based on nearest neighbors (*k-NN*). This technique consists of estimating the missing values based on the cases that are the most similar to the data set, calculating the missing value as the mean of the closest observations in terms of distance between rows. In this study, the three most similar records have been used for each observation without missing data. This approach offers two major advantages. First, it maintains the internal relationships between the variables, since the imputation is based on real patterns in the dataset. And second, it avoids introducing values that do not reflect the structure of the original data, as may occur with mean imputation. In contexts with little data and high dependence between variables, such as this one, this approach maintains more fidelity to the observed behavior without losing observations.

Logistic regression

Logistic regression is a model used when the dependent variable is categorical, usually binary (e.g., success or failure, 1 or 0). Unlike linear regression, which estimates a continuous value, logistic regression estimates the probability of a specific event taking place.

The model is based on the logistic function or sigmoid function, which transforms a linear combination of independent variables into a value between 0 and 1:

$$P(y = 1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p)}} \qquad (2)$$

Instead of directly interpreting the change in the dependent variable, the coefficients are interpreted as the effect of each variable on the log of the odds ratio (*log-odds*). A positive coefficient indicates that, as the variable increases, the probability of the event also increases (for example, that $y = 1$).

The parameters are estimated using maximum likelihood, and the model's quality is assessed with classification-specific metrics. The *ROC* curve and its associated *AUC* (Area Under the Curve) indicate the model's overall ability to discriminate between the two classes. However, because *AUC-ROC* alone may overlook imbalances between classes, additional threshold-dependent metrics were employed.

Balanced accuracy (*BA*) is defined as the mean of *sensitivity* (true positive rate, *TPR*) and *specificity* (true negative rate, *TNR*):

$$BA = \left(\frac{1}{2}\right) * (TPR + TNR) = \left(\frac{1}{2}\right) * \left(\frac{TP}{(TP + FN)} + \frac{TN}{(TN + FP)}\right) \qquad (3)$$

This metric is particularly useful in the presence of class imbalance, as it ensures that performance on both classes contribute equally to the final score, avoiding dominance by the majority class.

Matthew's correlation coefficient (*MCC*), provides a more comprehensive summary statistic that incorporates all four entries of the confusion matrix (true positives, true negatives, false positives, false negatives):

$$MCC = \frac{(TP * TN - FP * FN)}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}} \qquad (4)$$

The *MCC* can be interpreted as a correlation coefficient between observed and predicted classifications, ranging from –1 (perfect disagreement) through 0 (no better than random) to +1 (perfect agreement). Unlike *accuracy* or even *balanced accuracy*, *MCC* remains reliable when class distributions are uneven or when one class is rare, which is common in many applied binary classification problems.

Taken together, these complementary metrics provide a more reliable assessment of whether the target variable is meaningfully related to the binary predictors. In this study, all accepted variables fell within standard interpretability ranges: *AUC-ROC* between 0.6 and 0.7 (acceptable), *balanced accuracy* between 0.65 and 0.75 (moderate), and *MCC* between 0.3 and 0.5 (moderate). Variables with *MCC* values below 0.2 or *balanced accuracy* below 0.6 were not considered, following a conservative approach to avoid including relationships that might be trivial or due to chance. Since both *balanced accuracy* and *MCC* require the selection of a classification threshold, *Youden's criterion* was applied to determine the optimal cut-off point.

Logistic regression is especially useful when the interest lies in modelling binary decisions or classification processes with clear statistical interpretation. In our case, it was especially useful to decide if the categories (being binary in nature) could characterize the *use of logical-mathematical language* and *conditional reasoning* variables in the model.

The main focus of the analysis was on multiple linear regression. However, in order for the model to be useful and make statistical sense, it was important to first check whether there was a reasonable linear relationship between the independent variables and the dependent variable. When all of the variables are continuous, this verification is quite straightforward: Pearson's linear correlation coefficient can be used to explore potential associations. However, in this case, categorical variables were also being used, so this measure was no longer applicable. As an alternative, a practical strategy was chosen to analyze the relationship between variables when there were scale differences. To do so, a logistic regression model was trained to predict a categorical variable from a numerical variable, and, conversely, a linear regression model was trained to predict a numerical variable from a categorical variable. In the first case, the model's fit was assessed using: the *AUC-ROC*, the *balanced accuracy* and the *MCC*; while in the second, the R^2 coefficient of determination was used. This approach allows us to quantify the extent to which one variable provides information about another, even if they are not on the same measurement scale.

More specifically, in our study, logistic regression analysis was used to identify whether the binary categories detailed in **Tables 1** and **2** were adequate and well-founded to define the *logical-mathematical language* and *conditional reasoning* variables from a statistical point of view. Applying logistic regression, mathematical achievement was used as the independent variable to predict, in each case, the probability of success in each of the binary categories defined in *logical-mathematical language* and *conditional reasoning*. This preliminary analysis served as a sort of informal statistical test, allowing for the filtering out of irrelevant variables and, in turn, the categories (sub-variables) that were to be part of the study, before including them in the multiple regression model. This adds a layer of rigor to the selection process, which is especially important when there is a low volume of data, and the goal is to avoid overfitting or spurious interpretations.

Therefore, the individual variables, use of logical-mathematical language and conditional reasoning, constructed from the students' responses to the mathematical problems and organized according to the previously described categories, will continue to be the model's selected variables.

Multiple linear regression

When the objective of an analysis is to understand how different factors jointly contribute to a quantitative result, one of the most frequently used tools is the *multiple linear regression*. Unlike simple linear regression, which only analyzes the relationship between one independent and one dependent variable, the multiple model permits the simultaneous incorporation of several variables, estimating the individual effect of each one while controlling the influence of the others. This is especially useful in contexts such as that of this study, as it allows us to explain the performance of mathematics based on different individual student abilities. Variables such as mathematics skills (use of logical-mathematical language variable and conditional reasoning variable) and general cognitive skills (interaction variable (CR_Gf)) provide relevant information. However, their relationship with performance can only be suitably understood when considering both of them jointly, to represent a context. Multiple linear regression offers a mathematical framework permitting the quantification of this relationship, its comparison between factors, and an assessment of its statistical consistency.

Formally, multiple linear regression states that the dependent variable y can be expressed as a linear combination of several independent variables and an error term. For a sample of n observations and p variables, the model is written as follows:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i \quad (5)$$

or, in matrix form:

$$y = X\beta + \varepsilon \quad (6)$$

where y is a vector of observed values, X is the design matrix with the independent variables (including a column of ones for the intercept), β is the coefficient vector, and ε is the error vector.

The coefficients are estimated by ordinary least squares, minimizing the sum of squared errors:

$$min\left|\left|y - X\beta\right|\right|^2 \quad (7)$$

with the solution:

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (8)$$

Once the model coefficients have been estimated, their interpretation is straightforward: each β_j represents the expected change in the dependent variable with a unit increase in x_j, keeping the other variables constant. This indicates how much a student's mathematics performance is expected to change when a certain personal characteristic, such as cognitive ability or specific math skill, varies, when all other factors remain unchanged.

To check the internal structure of the model, the variance inflation factor (VIF) has been used. It allows for the quantification of the degree of multicollinearity between the explanatory variables. VIF values greater than 5 suggest that the variable is providing redundant information and may be distorting the results.

Furthermore, to check the reliability of the estimates, confidence intervals were calculated for each coefficient. This allows us to assess whether an effect is statistically

Table 3. Percentages of responses on the cognitive reflection (CR) test and difficulty estimates for each of the five problems (correlations between superficial answers and the difficulty estimate appear between parentheses, **p < 0.01).

Item	Correct answer	Superficial answer	Other incorrect answers	Difficulty estimate
1	97	0	3	93 (n.c.)
2	67	21	11	77 (–0.19)
3	18	54	28	70 (0.39**)
4	79	8	13	74 (–0.13)
5	75	15	10	80 (0.11)
M	67	20	13	78.8

significant or could be due to chance: If the interval includes zero, it is not possible to clearly affirm that the effect exists.

RESULTS

Cognitive Reflection (CR), Fluid intelligence (Gf), and mathematical skills

Table 3 presents the results of the Cognitive Reflection (CR) test in percentages. It also shows the percentage of difficulty estimate for each of the five problems. In this test, the total percentage of correct answers (67%) was greater than the total percentage of superficial answers (20%). However, the variability between the different items was high. In addition, students tended to overestimate the accuracy level of their classmates, with a mean estimation of 78.8%, which was much higher than the real percentage of correct answers. A significant correlation was only observed between this estimate and the frequency of superficial answers ($r = 0.39$, $p = 0.002$). For the other items, no relevant correlations were observed. Overall, the participants' performance was relatively high as compared to the results reported by Frederick (2005), who also studied university students.

Regarding fluid intelligence (Gf), the mean was high (54) with little variability and with a standard deviation of 0.07. As indicated in the methods section, this leads us to explore the option of analyzing both CR and Gf as one variable and exploring its effect on mathematical reasoning and performance.

Table 4 reveals the correlations between the variables. Very significant correlations are observed between high scores on general cognitive skills CR and Gf (CR_Gf) and the mathematical skills variables (logical-mathematical language, conditional reasoning). Significant correlations were also found with academic mathematical achievement (Math achievement).

Descriptive Analysis of the Predictive Model

The results regarding cognitive skills show that the joint consideration of cognitive reflection (CR) and fluid intelligence (Gf) is correlated with academic mathematical performance. It is evident that the separate treatment suggested that the variable CR had a significant correlation with academic performance, whereas the Gf was not significantly correlated. However, given that both of these are supported in scientific literature as factors influencing mathematical performance, their joint effect was examined. By combining them into a new variable (CR_Gf), an improved stability of the model was found, as well as an increased significance of the predictor estimator of mathematical achievement (see **Table 5**, R2 = 0.403). This suggests that those subjects having higher skills in both variables would also display better results on the predictor variable.

Table 4. Means, standard deviations, and Pearson's correlation (p < .05. ** p < .01) between CR_Gf, conditional reasoning, logical-mathematical language, and mathematical achievement

	1	2	3	4
1. CR_Gf	1			
2. Conditional reasoning	.37**	1		
3. Logical-mathematical language	.38**	.69**	1	
4. Math achievement	.39**	.32*	.29*	1
N=66				
M	1.84	1.71	0.88	6.6
SD	0.61	0.73	0.41	1.2
Min–Max	0.00-2.63	0.00-2.25	0.00-1.40	5-10

On the other hand, conditional reasoning had a correlation of 0.32 with academic mathematical achievement, suggesting that this is a relevant predictor, although lower than general cognitive skills. At the same time, the performance on logical-mathematical language displayed a correlation of 0.29 with mathematical achievement, very similar to that of conditional reasoning. Therefore, we conclude that the presented variables had a positive and comparable effect on math achievement.

It is also important to highlight the correlation observed through the logistic regression model used in this study between the correct definition of the sets (L1) on the use of logical-mathematical language and academic mathematical performance. The analysis through AUC-ROC revealed that the capacity to adequately define the sets is a relevant predictor of academic achievement (AUC = 0.63, balanced accuracy = 0.66, MCC = 0.25) highlighting the importance of this specific skill of formal mathematical reasoning.

Multiple Linear Regression Model

Table 5 and **Figure 3** present an integrated summary of the results of the multiple linear regression model developed to predict academic mathematical performance in this study. It includes coefficients,

Table 5. Results of the regression model

Variable / Indicator	Coefficient	p-value	VIF	Lower CI	Upper CI
Gender	0.0237	<0.001	3.15	-0.0357	0.0831
Group	0.0494	<0.001	2.54	-0.0093	0.0923
CR_Gf	0.1963	<0.001	10.78	0.0655	0.3231
Conditional reasoning	0.1165	<0.001	12.26	0.0230	0.2318
Logical-mathematical language	0.1255	<0.001	6.11	0.0177	0.1786
R^2 of the model	0.403				
Adjusted R^2	0.302				
RMSE of the model	0.089				
RMSE using the mean score as the prediction constant	0.1161				
Mean Math achievement (mean score)	0.6611				
Standard deviation Math achievement	0.1199				

*Note***.** CI = confidence interval at 95%; VIF = variance inflation factor; RMSE = root mean square error; R^2 = coefficient of determination

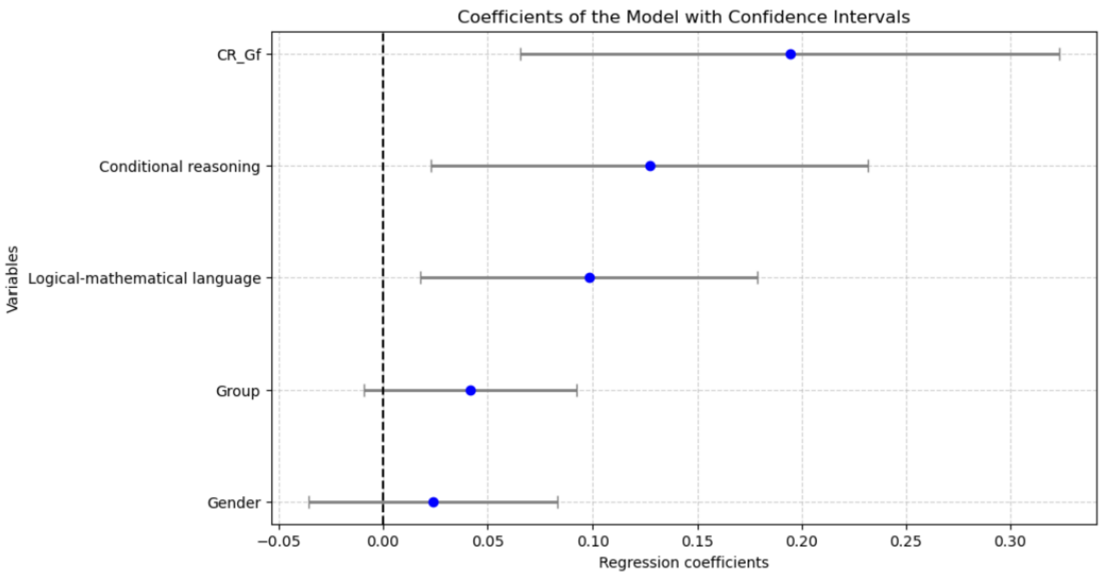

Figure 3. Graph of coefficients with confidence intervals of the variables (Source: Authors' own elaboration)

confidence intervals, and significance levels of each predictor, as well as the main global indicators of the model's fit and dispersion and that of the dependent variable.

The results of the model highlight that the joint consideration of the general skills of cognitive reflection (CR) and fluid intelligence (Gf) is the most relevant predictor of academic Math achievement (coefficient = 0.1943; 95% CI [0.0655, 0.3231]; $p < .001$; VIF = 10.78). This elevated VIF is not necessarily indicative of a misspecification but rather reflects the nature of the constructs: although the subcomponents of the composite variables were not included in the model, their internal correlations still influence the relationship between the aggregated measures and their interaction term. This indirect overlap can inflate VIF values even when the components are not explicitly modeled, pointing to the conceptual and statistical interdependence among the variables. To further confirm that multicollinearity is not a serious concern, we re-estimated the regression models excluding the variables with the highest VIFs one at a time. The resulting coefficients changed only minimally (e.g., CF_Gf from 0.19 to 0.22; Conditional Reasoning from 0.12 to 0.15), and remained statistically significant, indicating that the observed VIF inflation does not substantially affect the robustness of the results.

Regarding the assessment of the role of specific mathematical skills, both the mastery of logical-mathematical language (coefficient = 0.098; 95% CI [0.0177, 0.1786]; $p < .001$; VIF = 12.42) as well as conditional reasoning (coefficient = 0.1165; 95% CI [0.0129, 0.2201]; $p < .001$; VIF = 6.11) are identified as independent and significant predictors of academic Math achievement. These results highlight the importance of the skills of formalization, symbolization, and the handling of conditional structures as necessary components in order to successfully solve problems.

The coefficient of determination, R^2 = 0.403, allows for a global analysis of the model. Although this is a conservative result, it is reasonable given the study characteristics. In this context, with a limited sample size and a focus on ensuring the validity of the model's assumptions, this level of fit is considered adequate.

Likewise, the mean square error (RSME of the model = 0.089) is lower than obtained using only the mean as a predictor (RSME = 0.1161). This indicates that the model has identified relationships between selected variables that contribute to reducing noise in the predictions. This is a positive result for the analysis. The sociodemographic variables of gender and group, although statistically significant, have coefficients of a lower magnitude (0.023 and 0.041, respectively), and their confidence intervals include values approaching zero. This suggests limited practical relevance as compared to the weight of general cognitive and mathematical skills.

In the descriptive analyses of these gender and group class variables, there were no significant differences observed between men (M = 0.669, SD = 0.116) and women (M = 0.658, SD = 0.135), t(61) = 0.30, p = .76. However, significant differences were found between the class groups, with better performance in the MB1 group (M = 0.692, SD = 0.139) as compared to the MB2 group (M = 0.624, SD = 0.076), t(61) = 2.48, p = .016. Although they were methodologically included in the model to control for potential unobserved effects and to reduce bias caused by the omission of variables, they were not statistically significant and did not provide a substantial contribution to the model. Specifically, the coefficient associated with gender was very close to zero, suggesting that it had no appreciable effect on the dependent variable in this context. However, its initial inclusion allowed for verification of the absence of influence, contributing to a more robust estimation of the effect of the other variables.

To summarize, the results of the model integrate and confirm the relative importance of each skill considered in the study objectives, suggesting that the optimization of academic performance in university mathematics requires both the development of general cognitive skills as well as the strengthening of specific disciplinary capabilities.

DISCUSSION

This study aims to explore the interrelationship between general cognitive variables (cognitive reflection (CR) and fluid intelligence (Gf)) and specific mathematics skills, specifically, the mathematical ability of students with respect to the use of logical-mathematical language and conditional reasoning. The objective is to predict how these variables influence mathematics achievement at a university level.

The predictive model explains a sufficient part of the academic math achievement in a university context, revealing that it depends in large part on specific mathematics skills such as logical-mathematical language and conditional reasoning. Overall, the model achieved an R^2 of 0.403 and an adjusted R^2 of 0.302, this indicates that the model explains approximately 40% of the variance in mathematics performance based on the variables included in the analysis, with the adjusted value providing a more conservative estimate that accounts for model complexity and sample size. This is especially relevant in the context of mathematics education.

Among the considered predictors, both general cognitive skills (fluid intelligence and cognitive reflection) as well as specific mathematical skills (logical-mathematical language and conditional reasoning) have a significant influence on performance. The CR variable has a correlation with mathematics performance, confirming previous studies in RC, while that of Gf is not significant for these university students. These results about Gf indicate a variation with respect to the prediction indicated in studies such as those of Cormier et al., (2017). This difference could be explained by the increase in fluid reasoning presented by students who take the mathematics degree, students with good mathematical abilities and with higher fluid reasoning than students of their age.

However, the CR_Gf interaction variable is significant, allowing us to identify and specify the relevant interaction effects. This is necessary to further our understanding of how both general cognitive variables and mathematics skills variables may contribute to mathematical reasoning. It captures the improved synergy between cognitive reflection (deliberative and self-regulating process) and fluid intelligence (ability for abstract reasoning, patterns, and logic).

This supports a hypothesis that already exists in the literature (Gómez-Chacón et al., 2014, Otero et al., 2022; Peng et al., 2019), suggesting that success in mathematics does not depend only on abstract abilities or only on executive control, but rather, on the functional integration of both of these. This suggests that academic mathematical performance is optimized when both cognitive resources are well developed and used in combination with one another.

Recent studies, such as those by Johnson-Laird and Ragni (2025) and Attridge and Inglis (2013), highlight the importance of assessing mathematical reasoning types as well as conditional reasoning, given their impact on learning and mathematics performance. More specifically, the results suggest that logical-mathematical language and conditional reasoning are predictors of academic success, even when the effect of general cognitive variables is regulated. This relevance of language and formal logic confirms observations made in specialized research, which highlights the difficulty faced by students in the translation, symbolization, and negation of propositions, as well as in the proper interpretation and handling of mathematical conditionals (Alcock & Simpson, 2002; Durand-Guerrier, 2015; Epp, 2003). Current literature on mathematics education emphasizes the need to explore how students understand and use formal mathematical language, as well as their ability to reason using conditional structures. This study contributes to filling this gap in the research, offering specific empirical results regarding how these variables affect academic mathematical performance in university contexts, making it especially relevant. Authors such as Durand-Guerrier (2003), Epp (2003), and, more recently, Madruga et al. (2022), have highlighted the fact that these aspects are crucial for the advanced understanding of mathematics and the construction of logical-formal knowledge.

Finally, sociodemographic variables such as class group and gender do not have a significant impact on the model, suggesting that performance depends essentially on the skills and competencies developed by the student, as opposed to external or contextual factors.

CONCLUSIONS

The results of this study confirm the need to consider an integrative perspective in predicting academic Math achievement in a university context. Success in mathematics depends not only on general cognitive skills or executive control but also on the functional integration of both with specific mathematics skills.

It should be noted that the results obtained have clear potential for direct educational intervention in a university context, given that the majority of studies have been conducted with respect to primary and secondary school levels. For example, it highlights the relevance of the mastery of logical-mathematical language, the mastery of strategies to refute or validate sentences, and cognitive reflection as a meta-cognitive skill in which the student may be educated. Likewise, it may be implemented in curricular designs and teaching strategies in initial university mathematics courses (Wood, 2001).

The predictive model developed reveals a robust explanatory capacity, permitting the accurate identification of the main factors associated with academic success during the first years of university for mathematics students. These findings reinforce the need to educate students in cognitive self-regulation, as well as formal logical competence in mathematical reasoning. In this sense, strengthening both skills is key, not only to improving performance but also to facilitating the transition and adaptation of students to university mathematics.

This study has certain limitations that should be considered. First, there is the issue of the specific problem typology, in our case, those linked to the content of a first-year course in the undergraduate mathematics degree. Future studies should rely on a wider variety of problems having distinct mathematical content, contrasting how they correlate with general skills such as CR and Gf.

At a methodological level, although the AUC-ROC, balanced accuracy and MCC values obtained in the logistic regression analyses are modest, their contribution was relevant when considered jointly with the other variables and the study context. These results allow us to delve deeper into case studies on how the definition of Sets and formal negation with quantifiers have a bearing on symbolic formalization.

Finally, as a future line of study, the mathematical skills and strategies considered may be expanded on, including the use of counterexamples to refute conditional statements. According to other authors we emphasized that explicit teaching of cognitive strategies, such as counterexamples and logical argumentation, is key to effective mathematical learning.

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