Eurasia Journal of Mathematics, Science & Technology Education, 2015, **11**(6), 1379-1397

Mathematics Teacher-Candidates' Performance in Solving Problems with Different Representation Styles: The Trigonometry Example

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•*Received 7 January 2015*•*Revised 3 May 2015* •*Accepted 25 May 2015*

Using multiple representations of a problem can reveal the relationship between complex concepts by expressing the same mathematical condition differently and can contribute to the meaningful learning of mathematical concepts. The purpose of this study is to assess the performances of mathematics teacher-candidates on trigonometry problems represented in different formats and to examine the reasons for test failures. This study uses a mixed-method approach and consists of 51 teacher-candidates enrolled in the Department of Mathematics Education at a state university. The data collection tools were a symbolic trigonometry test, a visual trigonometry test, and a verbal trigonometry test. Interviews were conducted with the teacher-candidates to reveal the reasons of their failures and the advantages and disadvantages of each representation style for the trigonometry problems. This study found that the teacher-candidates were most successful on the symbolic test and least successful on the verbal test. The teacher-candidates stated that seeing different representation forms helped them to understand the questions better and produce multiple solutions.

Keywords: mathematics teacher-candidates, representations, teacher education, trigonometry

INTRODUCTION

Trigonometry has attracted research attention due to its historical development and its current importance in mathematics education. Trigonometry is a subject that students have the most difficulty in understanding (Tatar, Okur & Tuna, 2008; Durmuş, 2004), and students do not understand its benefits, historical usage, or application to daily life (Tuna, 2011). Most students who encounter the sine, cosine, tangent, and cotangent concepts for the first time in their high school or university

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doi: 10.12973/eurasia.2015.1396a

ISSN: 1305-8223

education cannot relate these concepts to real-life situations and do not know where they come from. Learning trigonometric concepts can only be achieved when the real-life applications of trigonometry and its importance are shared with the students. From a more optimistic viewpoint, students may consider trigonometry as a component of mathematics, rather than as an independent subject (Adamek, Penkalski & Valentine, 2005). For example, if students see that they can use trigonometry in problems related to real-life situations such as calculating the length of a shadow, while using and understanding the importance of the tangent ratio, they will better understand the subject. Teaching trigonometry theorems and concepts is important for developing students' creative, logical, and analytical thinking skills; trigonometry is the precondition for understanding more advanced concepts—it is the basis of advanced learning in mathematics—and it is necessary for the formation of a mathematical language. One of the most important factors in trigonometry is the problem-solving activity (Markel, 1982; Thompson, 2007).

State of the literature

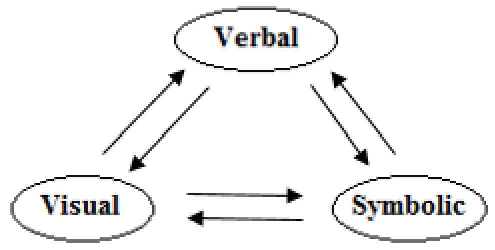
- The studies conducted to date show that presenting a problem in different forms enables students to better understand the problem.
- Using different representation styles is considered important for student learning.
- Few studies have been conducted on teacher-candidates' knowledge regarding representation styles of trigonometric problems.

Contribution of this paper to the literature

- This paper tests teacher-candidates' pedagogical knowledge of the forms of representation in problem solving, because they have active roles in students' learning processes.
- This study will be an important milestone in that it will provide teachers and specialists with information on problem representation.
- It will clarify the purposes of mathematics teaching, and address missing points in problem-solving processes.

The report "Standards and Principles for School Mathematics" by the National Council of Teachers of Mathematics (NCTM) explains the general principles and standards of school mathematics. Problem solving, reasoning, communication, and association are in the process-standards section of this report (NCTM, 2000), which suggests that these process standards in particular should be used in school mathematics. Developing students' problem-solving skills is one of the central purposes of mathematics teaching (Reusser & Stebler, 1997). One of the aims of teaching mathematics is to ensure that students develop the cognitive ability to evaluate everyday situations using problem-solving approaches (Altun, 2005). In problem-solving processes, it is vital to understand mathematical knowledge and the relationships between knowledge structures. Students need to be able to unite concepts and processes and apply them to solving problems. A significant factor in the efficient performance of problem-solving activities is that the problem must be understood (Kieren, 1976; Poon, 2011; Villegas, Castro & Gutierrez, 2009). The type of problem, its structure, and its representation forms are all crucial for understanding the problem (Dündar et al, 2014).

Representation means presenting a mathematical concept or relationship in a certain form (NCTM, 2000). A mathematical concept or relationship can be shown in multiple ways; using different representation styles and understanding the relationship between these representations is necessary for interpretation of mathematical concepts (Lesh, Post & Behr, 1987). The multiple representations express different forms of the same mathematical situation and show the relationship between the complex concepts, thereby contributing to the meaningful understanding of mathematical concepts and relationships. Mathematical educationalists suggest that multiple representations should be used in teaching mathematics (NCTM, 2000). Representations play important roles in understanding problems and in mathematical thinking. Multiple representations develop students' multivariate and flexible thinking skills in problem solving, helping them to understand mathematical concepts. The studies conducted to date indicate that



Verbal representation of the word problem: the word problem as stated, whether written or spoken. *Visual representation:* drawings, diagrams, or graphs, and any kind of related action. *Symbolic representations:* numbers, operations, relational signs, algebraic symbols, and any kind of action referring to these.

Figure 1. Types of Representation Systems (Villegas et al., 2009, p. 287-288)

presenting a problem in different forms helps students to better understand the problem (Villegas et al., 2009). Yan and Lianghuo (2006), Villegas et al. (2009), and Friedlander and Tabach (2001) divided problem representations into three groups: verbal, visual, and symbolic. For conceptual understanding to occur, the different representations must be emphasized (Kieren, 1976). As a student's education level increases, it is important to use representations simultaneously to enable understanding of increasingly abstract mathematical concepts (Baki, 2006). Lesh et al. (1987) state that the ability to understand multiple representations is a good indicator of overall mathematical comprehension. Presenting algebra as only the study of expressions and equations can pose serious obstacles to effective and meaningful learning, as many researchers and teachers know (Kieran, 1992).

Akyüz, Coşkun, and Coşkun (2009) studied the effects of multiple representations on teacher-candidates' problem-solving skills and determined that the teacher-candidates made use of the visual images in their minds while solving problems; if the teacher-candidates did not have a visual image in their minds, they had difficulty solving the problems. Hammill (2010) examined mathematics textbooks in terms of verbal, symbolic, and visual elements and found that textbooks and other teaching materials included all three of these elements; however, Hammill (2010) emphasized the necessity of determining whether or not the necessary interconnections between these representations were made. Kar and İpek (2009) examined the visual representations used in solving verbal problems throughout mathematics history and found that they were an important strategy in understanding and solving a problem, highlighting the need to use visual representations in teaching mathematics and in solving problems. Villegas et al. (2009) claim that students must establish interconnections between various representations in a problem-solving process and concluded that students who could make transitions between representational forms and convert them into different forms were more successful in problem solving; the study found a strong relationship between problem-solving success and the ability to make a transition between visual and symbolic representations. Castro, Morcillo, and Castro (2001) found that while students were trying to solve a verbal problem, they first remembered the visual elements. According to Challenger (2009), students must develop visual techniques in order to comprehensively understand trigonometry. Delice (2004) found that Turkish students performed better on algebraic problems and British students performed better on verbal problems. Weber's (2005) study recommended the use of visual models to teach trigonometry concepts because they enhanced students' understanding. To solve trigonometric equations and systems, using analytical thinking and visual strategies enabled students to think multi-directionally and transfer information; this strategy also ensured the formation of conceptual knowledge and an accurate image of the concept (Doğan & Abdildaeva, 2013). Kardeş, Aydın and Delice (2012) investigated teacher-candidates' ability to transform representation styles to solve problems and found that the teacher-candidates had moderate success. Kardeş, Aydın and Delice (2012) also found that the transition performances of the teacher-candidates were better than their transformation performances among representation styles.

Dikici and İşleyen (2003) state that students who have difficulties in any subject will have trouble achieving general success in the future too. Although trigonometry is an important subject used in daily life, trigonometry studies are not positioned accordingly (Yılmaz et al., 2010). Markel (1982) claims that trigonometry does not receive sufficient consideration in school curricula, although it is widely used. Thompson (2007) agrees that due regard has not been given to trigonometry in the last 25 years. In Turkey, although trigonometry is broadly applied in terms of content, teacher-centered practices make it difficult for students to understand trigonometry (Doğan, 2001). Gürbüz and Birgin (2008) indicate that students' operation processing skills were better developed when using algebraic representations compared to the same skills of students using geometric and number-line representations. Although it is possible to use many representations, excessive use of the algebraic representations makes it difficult for students to understand by conceptualizing (Moseley, 2005). As students' knowledge level increases in mathematics, the subjects become increasingly abstract, so it is important for learning at the primary and secondary school levels to be meaningful. For trigonometry to ensure meaningful learning, teacher-candidates must know how to employ different representations and establish relationships between these representations, and they must be able to make the desired transitions. All of these factors are crucial for teacher-candidates' future mathematical learning and teaching activities.

Teacher-candidates' proficiency is related to the success of the educational system. To train qualified teachers, the necessary first step is determining the teacher-candidates' skills. The problem of teaching as a profession and teacher-candidates' proficiencies is relevant in many countries, and there have been numerous studies on this topic (Lin, Hsieh & Pierson, 2004; Piro, Anderson & Fredrickson, 2015; Villegas, 2007; Wilkerson, 2015). Shulman (1986), Hashweh (2005), Stoessiger and Ernest (1992) argue that there is a relationship between teachers' content knowledge and representation styles. Teachers' proficiencies must include pedagogical, content, and curriculum knowledge (Shulman, 1986). Content knowledge and pedagogical competences affect the learning and teaching environment since teachers have important roles in using different representation styles in the classroom.

How problems are presented is integral in teaching the process of solving problems at all educational levels: it is crucial to show students multiple representational styles (DeBellis & Goldin, 2006; Xenofontos & Andrews, 2014). Failure to solve given problems causes negative cognitive and affective outcomes (Dündar, 2014; Konyalıoğlu, 2003). In this context, it is important to test the pedagogical content knowledge of teachers and teacher-candidates who have active roles in students' learning process. The purpose of this study is to assess the performances of mathematics teacher-candidates on trigonometry tests with problems represented in varying ways and to examine the reasons for test failures. This study will be an important milestone in that it will provide teachers and specialists with information on problem representation, clarify the purposes of mathematics teaching, and address missing points in problem-solving processes. Mathematics teacher-candidates' pedagogical knowledge will develop by increasing their awareness of different representation styles for problems. Teacher-candidates' ability to solve problems presented in diverse formats could provide avenues for further research. Accordingly, this study seeks answers for the following research sub-questions:

1. What are the performance levels of mathematics teacher-candidates on trigonometry tests that use different representation styles?
2. Is there a meaningful relationship between trigonometry problems that have different notations and test takers' scores on the relevant tests?
3. What are the reasons that mathematics teacher-candidates fail tests that have different representation styles?

METHOD

The design of the study

This study uses the mixed-method approach, combining quantitative and qualitative techniques. The quantitative data were collected and analyzed and then the qualitative data were collected from the teacher-candidates to obtain detailed information on the process and support the quantitative findings. This particular mixed-method approach is called explanatory design and was chosen because of the need to support the quantitative data with qualitative data (Creswell & Plano-Clark, 2007). The survey method was used to collect quantitative data, with the purpose of ascertaining and defining the existing situation (Fraenkel & Wallen, 2006). The case study was the qualitative method used to define and understand the details of the situation and to develop possible explanations (McMillian, 2000).

Participants

The purposeful sampling method was used (Creswell & Plano-Clark, 2007) to select a sample of 51 first-year students studying at the Mathematics Education Department at a state university in Turkey. In Turkey, general mathematics lessons are taught in the first level of education at a given institution. General subjects are taught in this course to form a basis for other subjects in the same field (analysis, linear algebra, analytics, geometry, etc.). Trigonometry lessons are taught within the first level of the mathematics curriculum because of its difficulty compared to other subjects in Turkey (Durmuş, 2004; Tatar et al., 2008). The general mathematics lessons are important because they form a basis for other subjects and for the pedagogical content knowledge of teacher-candidates. Therefore, students in the first level of the curriculum were selected for the study.

In the qualitative part of the study, a universe and exemplification assignment was not performed because a generalization will not be made. Instead, interviews were conducted with eight teacher-candidates with lower scores on the tests of different representation styles. In the categorization process of scores (low-medium-high), the formula developed by Alamolhodaei (1996) was used (Figure 2). (Score=Mean Score $\pm\frac{1}{4}$Standard Deviation; $\bar{X}$: Mean score, S: Standard Deviation S_1: Low score, S_2: Medium score, S_3: High score).

The categorization of the teacher-candidates' test scores was determined with the help of the formula in Figure 2. The categorization is high—successful and low—unsuccessful (fail). The criteria for interviews were determined from data obtained at the end of the categorization and used to select the teacher-candidates. The

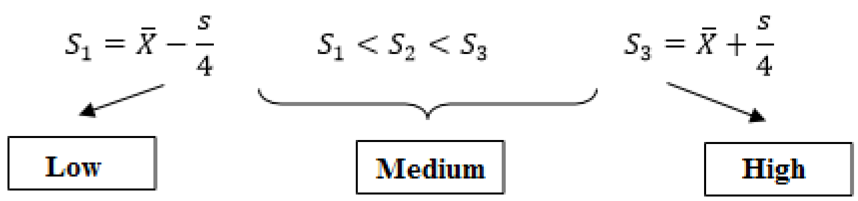


Figure 2. The formula used to determine the scoring categories

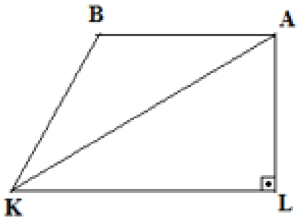
Symbolic Question (STT)	Visual Question (VITT)	Verbal Question (VETT)
When $\tan(\alpha+\beta)=\frac{1}{3}$, $\tan(\beta)=\frac{1}{7}$, what is $\tan(\alpha)$ =?	 When $AL \perp KL, BA//KL, \lvert AL\rvert = 3\ cm,\ \lvert BA\rvert = 12\ cm,\ \lvert KL\rvert = 21\ cm$, what is tan (BKA) =?	A watchman at the watchtower at the K point watches the 12-km-flight of a plane flying parallel to the ground over 3 km from the ground from point A to the point B on the radar. If the orthogonal projection of the point A on the ground is point L and $\lvert KL\rvert$=21 km, what in the tangent of the angle AKB which was scanned by the radar?

Figure 3. VETT, VITT, and STT question sample

following criteria were used for selection: a) unsuccessful in symbolic test, successful in verbal and visual test; b) successful in symbolic and visual test, unsuccessful in verbal test; successful in symbolic and verbal test, unsuccessful in visual test. The purpose was to determine the reasons for the teacher-candidates' failures in different representation forms.

Data collection tools

The verbal trigonometry test (VETT), visual trigonometry test (VITT), and symbolic trigonometry test (STT) were used as the data collection tools to assess the knowledge of the teacher-candidates on trigonometry and to measure their skills at translating the problem situations. Each (Appendix A) of the trigonometry tests had different representations and each had four questions. Each of the trigonometry questions was prepared by using the representations described in the literature (Delice, 2004; Tuna, 2010; Villegas et al., 2009; Yan & Lianghuo, 2006). The questions were prepared according to representation styles based on studies conducted by Villegas et al. (2009), English and Warren (1998), Feifei (2005), Yan and Lianghuo (2006), Friedlander and Tabach (2001). All the questions in the tests had the same numerical solutions but were prepared symbolically in the STT, visually in the VITT, and in the VETT the questions required the trigonometry problem to be applied to a real-life event (See Figure 3).

Since a proper criterion could not be found to calculate the validity coefficients of the tests, specialists' views were obtained. The views of three field specialists who worked in mathematics education and one measurement and evaluation specialist were consulted to ensure the content validity of the tests. The specialists assessed the accuracy of the differentiations between the question representations and the comprehensibility of the verbal questions. The tests were also given to ten teacher-candidates in another university's mathematics education department to ensure the comprehensibility of the questions and the test's structural validity. The final versions of the tests were given to 48 mathematics teacher-candidates at a different university as part of the pilot study, which was conducted in three sessions on different days. Then, the Cronbach Alpha Reliability coefficients were calculated to obtain the reliability of the tests. The Cronbach Alpha results were .75 for the VETT, .72 for the VITT, and .74 for the STT. These results indicated that the tests were reliable (Büyüköztürk, 2007).

Interviews were conducted after evaluating the answers of the teacher-candidates on the VETT, VITT, and STT questions. The teacher-candidates who received low scores (fail) on one test and received high points (successful) on the other two tests were included in the interviews. The interviews lasted 15 minutes for each teacher-candidate and were recorded with a voice recorder. In the interviews, the teacher-candidates viewed their own tests (STT, VETT, and VITT) and were asked to examine their answers. The interviews included questions such as "Could you explain how you found the answer?" and "Why, in your opinion, did you fail this test, although you performed well on the other tests?" and "In your opinion, how are the differing representations of questions beneficial?" The interview questions were clearly understood because the questions were prepared in advance after being given to students at the other university in the pilot study and tested for clarity. However, some additional questions were also asked, such as "Could you explain it again?" Without being manipulative, encouraging verbal expressions (well done, very good, all right, etc.) were used (Clement, 2000). The teacher-candidates were told to "think loudly" while they were answering the questions.

Research process and practice

In the first part of the study, the quantitative data collected from the test takers were analyzed to seek answers for the defined sub-questions. In the second part, the success levels of the participants were categorized according to the statistical analyses. According to these category levels, interviews were performed and qualitative data were collected.

The study was conducted in the fall term of the 2014–2015 academic year because trigonometry is included in general mathematics classes taught during this period. The study used tests that exemplified the three different problem representations. Each test was administered in a separate session on a different day in this order: VETT, VITT, and STT (Figure 4), so that the teacher-candidates had the opportunity to interpret the answers. Two days after the tests were administered, interviews were conducted with the teacher-candidates who earned low scores on one test and high scores on the other two tests.

Data Analysis

Analysis of quantitative data

The answer sheets of the teacher-candidates were coded to ensure the study's reliability (e.g., the fifth teacher-candidate was coded as T5). Table 1 shows the grading key that was prepared by drawing upon the relevant literature (Dündar, 2015; Feifei, 2005; Soylu & Soylu, 2005).

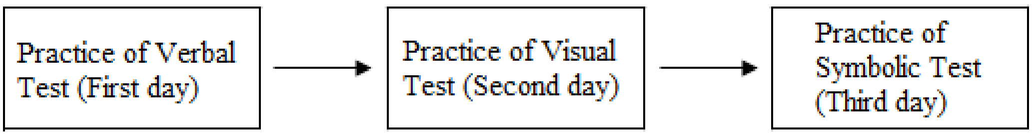


Figure 4. The practice scheme of the tests

Table 1. The grading key for the VETT, STT, and VITT tests

Categories	Description	Score
L0	No answer, irrelevant answer, illegible answer	0
L1	Containing some part of the scientific knowledge Process mistake, missing information, unable to reach the result	1
L2	Obtaining the elements the question asks for via symbolic, visual or verbal approach	2

The three types of tests were graded according to the grading key shown in Table 1. The highest possible grade on the tests was eight points and the lowest was zero points. Three specialists in the relevant field assessed the tests using the grading key. To test the reliability of the grading, agreement among the specialists was examined by calculating the Cohen's Kappa Coefficient, which was .86 in the concordance statistics. According to the Landis and Koch's (1977) classification, a coefficient value between .81 and 1.00 shows that the concordance is perfect. The descriptive statistics values were calculated to determine the performances of the teacher-candidates on the tests of different representation styles. The relationship of between the tests were calculated using the Pearson Correlation Coefficient because of the normal distribution of data (For normality; skewness and kurtosis statistics indicate that all values are acceptable).

Analysis of Qualitative Data

Interviews were conducted with the teacher-candidates who were unsuccessful on one or more of the representation styles to reveal the reasons for their mistakes. Voice recordings were made during the interviews and then transcribed. A specialist from a different field listened to the recording to control the accuracy of the texts (Kvale, 1996). The descriptive analysis technique was used to resolve the data. In this analysis, direct quotations were often used to reflect the viewpoints of the interviewed person (Fraenkel & Wallen, 2006).

RESULTS

General findings on STT, VETT and VITT

The performances of the mathematics teacher-candidates on the three types of tests are presented in Table 2 and categorized as fail (low), medium, and success (high) according to their scores.

Table 2 indicates that the percentage of teacher-candidates in the category of success was higher for the test of visual problems compared to the other tests. There were no teacher-candidates in the medium category for the visual problems test, and there were nearly the same number of teacher-candidates in the medium category for the verbal and symbolic tests. A higher number of the teacher-candidates were in the fail category for the visual and verbal tests. When the test averages were examined, the participants were least successful on the verbal test and most successful on the symbolic test. Figures 5a, 5b and 5c show the distribution of test scores based on the success categories. For example, Figure 5a shows that 21 of 25 teacher-candidates who were successful on the symbolic test were in the success category for the visual test, whereas the rest were in the fail category.

Figure 5a shows that the low-scoring teacher-candidates had low scores on all three tests. The teacher-candidates who were unsuccessful on the symbolic test were also unsuccessful on the visual and verbal tests. Similarly, many of the teacher-

Table 2. Success categories of the teacher-candidates based on their test scores

Test Type	Success category									Test Average & Standard Deviation	
	Fail (low)			Medium			Success (high)				
	f	%	$\overline{X}$	f	%	$\overline{X}$	f	%	$\overline{X}$	$\overline{X}$	ss
Verbal	21	41.2	1.22	15	29.4	4	15	29.4	6.66	3.63	1.01
Visual	23	45.1	2.68	0	0	0	28	54.9	7.28	5.20	1.21
Symbolic	9	17.7	2.88	17	33.3	6	25	49	8	6.43	1.35

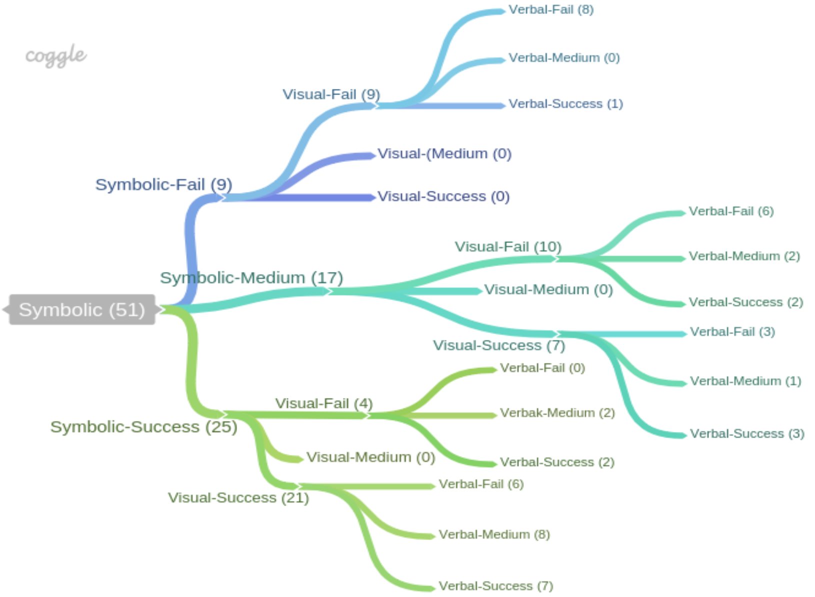


Figure 5a. The distribution of the categories created by comparing the symbolic test scores with the tests of other representation styles *(e.g., Symbolic-Fail (9): there are 9 teacher candidates in this category)*

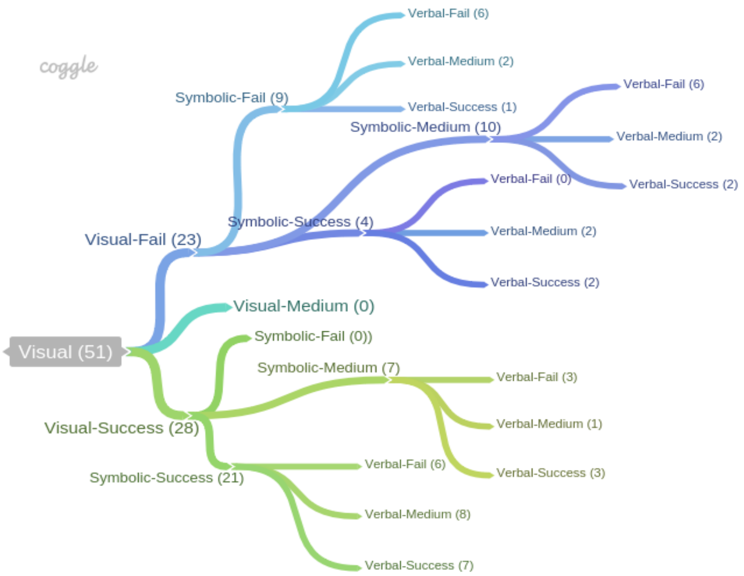


Figure 5b. The distribution of the categories created by comparing the visual test scores with the tests of other representation styles *(e.g., Visual-Fail (23): there are 23 teacher-candidates in this category)*

candidates who were successful on the symbolic test were also successful on the visual test. There were some teacher-candidates, who were successful on the symbolic and visual tests, but of these, some were successful on the verbal test and others were unsuccessful. Figure 5a implies that no precise conclusions can be made about the teacher-candidates in the category of medium. That is, the teacher candidates in the medium category on the symbolic test were in either the success or fail categories for the visual and verbal tests.

Figure 5b shows that success or fail categories exist in the visual test, and more than half of the teacher-candidates were in the success category for the visual test. The teacher-candidates who were unsuccessful on the visual test fell into separate categories on the symbolic test. That is, being unsuccessful on the visual test did not mean being unsuccessful on the symbolic test. The teacher-candidates who were successful on the visual and symbolic tests placed in different categories on the verbal test. There were no teacher-candidates who were successful on the visual test and unsuccessful on the symbolic and verbal tests.

Figure 5c shows that there were some-teacher candidates in all three categories of the verbal test scores. Most of the teacher-candidates were in the fail category. No precise judgments can be made about the success of the teacher-candidates on the symbolic test who were in the fail and medium categories of the verbal test. The teacher-candidates who were successful on the verbal and visual tests did not fail the symbolic test.

The relation between STT, VETT and VITT questions

Table 3 shows the correlation of the scores and the statistical values in order to answer the question "Is there a meaningful relationship between the teacher candidates' scores on the VETT, STT, and VITT?"

Analysis of Table 3 reveals that there was a significant relationship between the tests (Verbal-Visual, Verbal-Symbolic, Visual-Symbolic) when the test scores were taken into account. There is a positive relationship at the medium level between the scores on the VETT and VITT, which express the same numerical situations ($r = .358$, $p < .05$). There is also a positive relationship at the medium level ($r = .360$, $p < .05$) between the scores on the VETT and STT tests. It is found that there is a positive

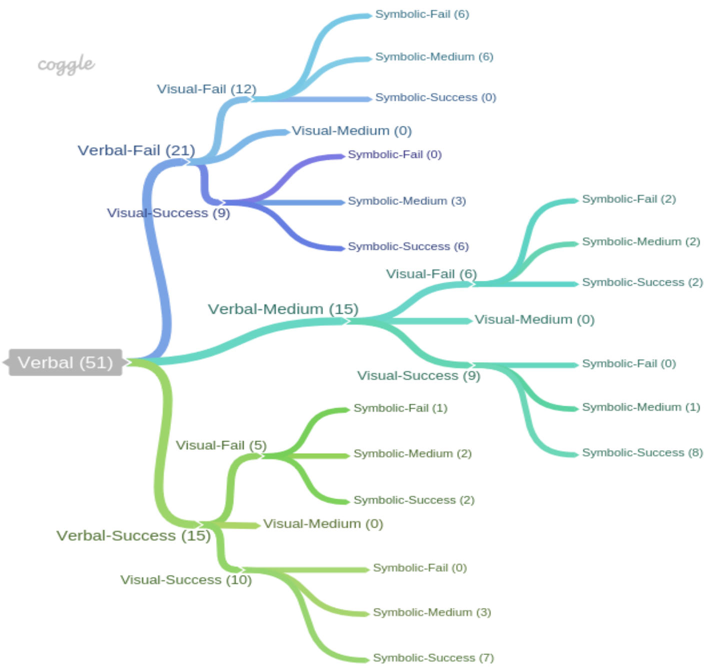


Figure 5c. The distribution of the categories created by the verbal test scores compared to the tests of other representation styles *(e.g., Verbal-Fail (21): there are 21 teacher candidates in this category)*

Table 3. The relationship between the VETT, STT, and VITT test points

		VETT	VITT	STT
VETT	Correlation coefficient	-	.358**	.360**
	Sig. (2-tailed)	-	.010	.009
VITT	Correlation coefficient		-	.707**
	Sig. (2-tailed)		-	.000
STT	Correlation coefficient			-
	Sig. (2-tailed)			-

*** Correlation is significant at the 0.05 level (2-tailed)*

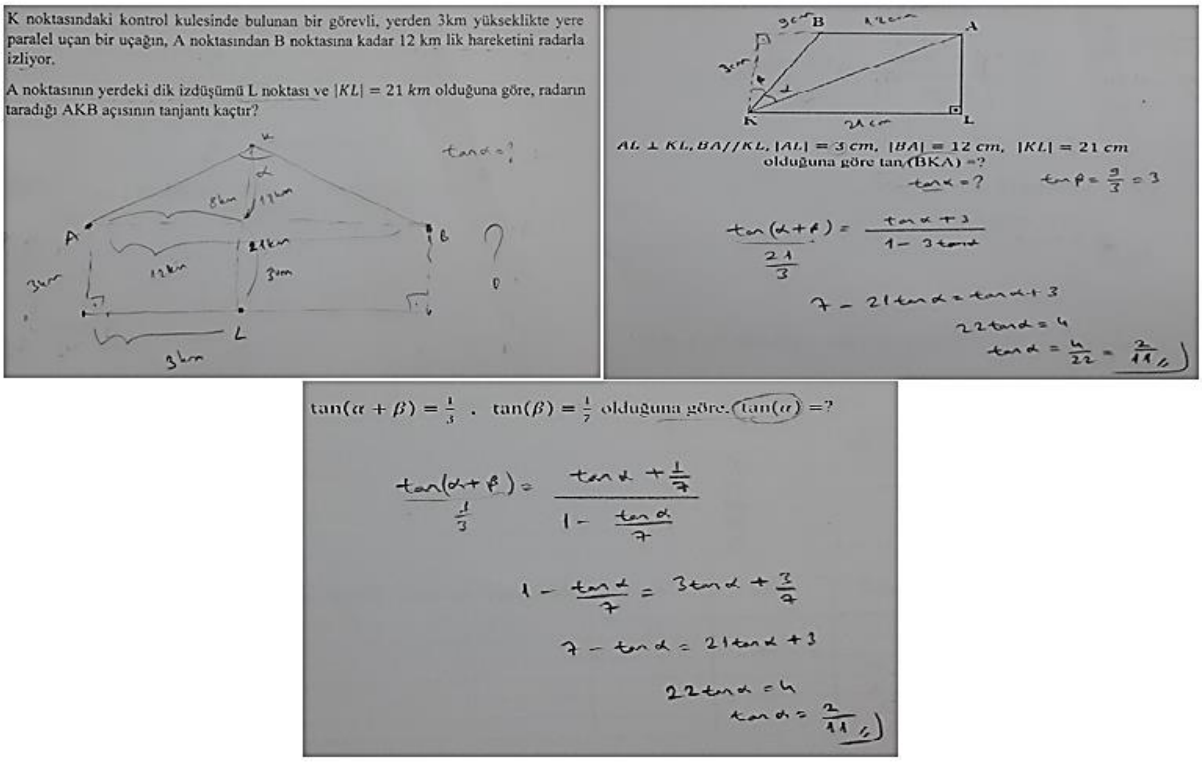


Figure 6a. Teacher-candidate T25's solutions to the problems in different representation forms *(T25: The code given to the teacher candidates)*

relation at high level (r = .707, p<.05) between the points received from the VITT and STT tests. This relationship occurred because the test problems had the same numerical solutions but different representation styles.

The reasons for the failures of the teacher-candidates on the tests with different representation styles

There were six teacher-candidates whose success was high (successful) on the symbolic test, high (successful) on the visual test, but low (fail) on the verbal test. These teacher-candidates and their views on their failures are presented below. Figure 6a shows one of the teacher-candidate's solutions for a problem on all the tests.

The solutions presented by the teacher-candidates who were unsuccessful on the verbal test but successful on the other two tests and their interview data were analyzed. The interviewees stated that they failed this test because the problems were verbal and they could not convert them into visual problems. The interviewees explained that although the questions were the same, the verbal questions were more difficult than the questions presented in other formats; the interviewees interpreted the verbal questions differently and produced divergent visuals or figures.

> T25: I did not place the information in the verbal questions in a correct manner.
>
> T51: I could not convert to the visual representation because the problem was verbal. I failed for this reason.
>
> T46: I failed because I have difficulties in converting the verbal message into a visual form on paper.
>
> T32: The reason I failed on the verbal question is that I could not understand and translate the words I heard into a drawing without mistakes.

> T22: I failed on the verbal questions because while I was converting it into a shape I had difficulty. I interpreted it in a different manner and I drew a figure that was different from the desired one.

Out of the 51 teacher-candidates, 2 candidates were successful on the STT and VETT but were unsuccessful on the VITT. In the interviews with these teacher-candidates, they stated that their failures occurred because they did not see enough question types and they added that they needed geometric knowledge in addition to formulas for these types of questions. The interviewees also stated that they came closer to the solution when they understood the problem and could draw their own shapes. Figure 6b demonstrates the answers of the teacher-candidates on their failures, and one of the solutions provided by a teacher-candidate.

> T10: I cannot use my logic on some questions because trigonometry is not a field that I am fully capable of. I am unsuccessful because I have not solved many problems using different question types. When the question is expressed verbally, I can understand it and convert it into a shape; however, I have difficulty in memorizing the formulas. Since I do not have much talent for memorizing, I cannot memorize all the formulas. This is the reason why I am successful in visual questions.
>
> T7: Not solving enough problems, lack of question types.

The study examined the issue of the benefits or advantages and disadvantages of using different representations for trigonometry questions. The interviewees stated that these representations provided a different perspective and developed solutions in diverse ways, and the participants viewed the questions more carefully. For the verbal questions, the participants became aware of whether their understanding of the problem was correct or not after they read the other questions. The interpretations of the interviewees are as follows:

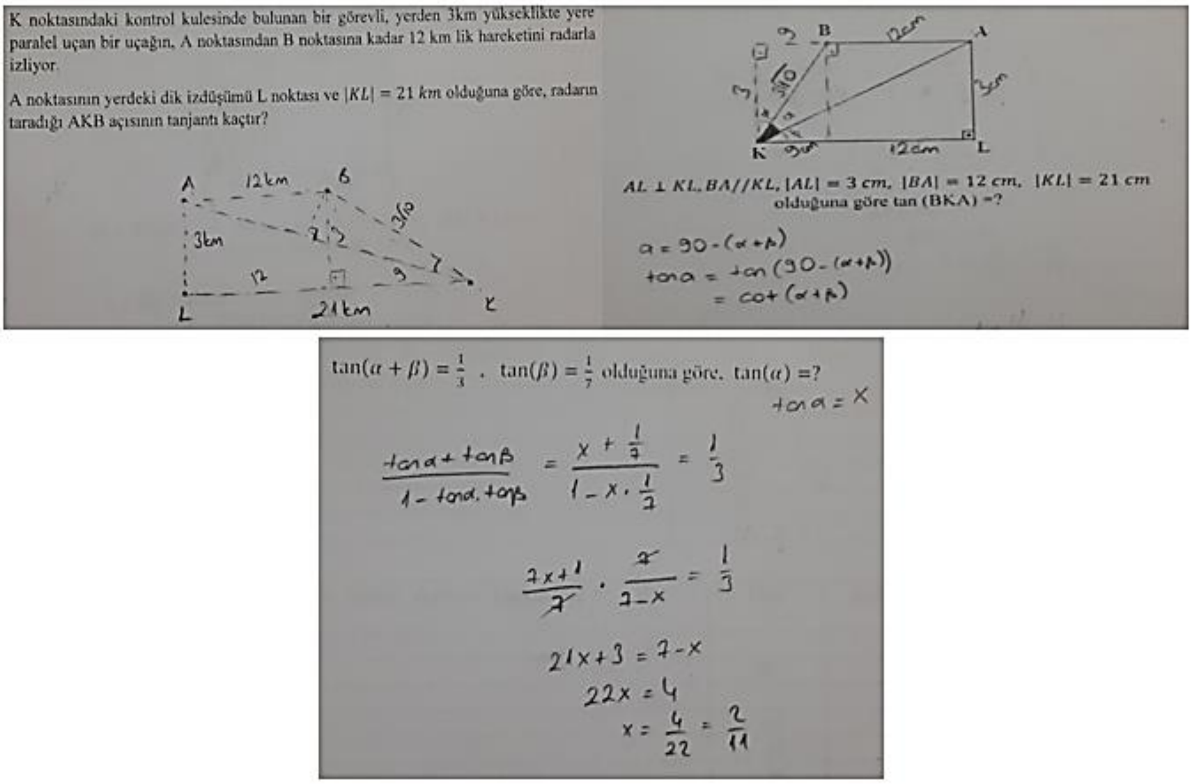


Figure 6b. The solution for a question in different formats, provided by a teacher-candidate who succeeded on the STT and VETT but failed on the VITT *(The teacher-candidate with the code T10)*

> T51: It made me see a question from a different point of view based on the representation. Although they were the same questions, the difficulty level of each was different. The verbal questions are difficult for me.
>
> T46: We can reach the right answer to the question when we perform the necessary mathematical operations in trigonometry questions expressed visually. However, if we explain this question by adding daily life experiences, it is necessary to solve the problem and then do the necessary operations. Therefore, the second style shows that we can provide different solutions to our problem-solving method. It makes us think.
>
> T32: My examining the questions closely prevented me from making operational mistakes. It made me see that I am unsuccessful in verbal questions.
>
> T22: If it is symbolic and visual, it makes me do it in an easy manner because we solve the questions that are directed towards symbols with formulas. If the question is verbal, the possibility of my misunderstanding the desired item and answering the question incorrectly increases.

These responses show that asking questions in different ways provides the teacher-candidates with different insights and makes them aware of the ways they interpret questions:

> T10: We can have different viewpoints and we can see small details by using another method.
>
> T7: It helps me see more question types, which is an advantage. It can make me produce more than one solution.

CONCLUSION AND DISCUSSION

The performances of the teacher-candidates on the tests with different representation styles and the reasons for their failure on these tests were analyzed and suggestions will be made based on the conclusions. The performances of the teacher-candidates differed on the symbolic, verbal, and visual trigonometry tests. The symbolic test had the lowest number of failures, while the visual test had more teacher-candidates in the fail category. On the visual test, no teacher-candidates scored in the medium category, whereas the number of teacher-candidates in that category was nearly the same for the other two tests. In the success category, the visual test had the highest number of teacher-candidates and the verbal test had the least. The test averages indicated that the teacher-candidates' average was highest on the symbolic test and lowest on the verbal test. Baştürk (2010) found that student performances on tests using algebraic representation styles were better than their performances on tests using graphic and verbal representation styles. Trigonometry is difficult for mathematics instructors to teach and students state that trigonometry is very complicated (Yılmaz, Ertem & Güven, 2010). To address these obstacles, preparing the questions on trigonometry tests in different formats could be useful for understanding the questions and increasing students' success in trigonometry, as it forms a foundation for future subjects. In addition, verbal questions ensure that a relationship between daily life and abstract questions is established, which is very important for student learning.

The scores on the tests prepared in different representation styles varied, as did the success levels: the teacher-candidates had greater success on the symbolic test than on the other tests. The verbal test had the lowest success level. When the teacher-candidates encountered verbal problems, they tried transforming them into visual representations and relating them to real-life situations. This result corresponds with Castro et al.'s (2001) study and Sevimli and Delice's (2012) investigation of teacher-candidates' preferences for representation styles. The lower success rates of teacher-candidates on verbal problems compared to the other representation styles could affect their pedagogical content knowledge. Sam, Lourdusamy, and Ghazali (2001) found that students were more successful at solving computational problems than verbal problems. Delice (2004) found that Turkish students were more successful at solving algebraic problems, whereas British students were more successful at solving verbal problems. Although the test questions were the same in terms of their numerical solutions, the representation styles were different, which caused varying success rates. Representation styles might, therefore, have an integral role in solving problems (Leikin & Levav-Waynberg, 2007). Simon and Hayes (1976) claim that student performances can be distinguished based on different representations of some problems and their cognitive processing.

The relationship between the test scores was that there was a positive medium-level relationship between the verbal and visual tests and between the verbal and symbolic tests. There was a positive high-level relationship between the symbolic and visual tests. There were no teacher-candidates who were successful on the verbal and visual tests but unsuccessful on the symbolic test. Since there is a strong connection between the visual and symbolic tests, it is possible that the teacher-candidates could better understand the problem in the verbal test and could convert the problem into a symbolic (numerical) form more easily. Kardeş and colleagues (2012) found that the transition performances of teacher-candidates were better than their transformation performances for the representation styles. In the present study, the teacher-candidates stated that they had difficulties in transforming representation styles. The teacher-candidates were better at transforming the visual representation style into the verbal representation style. This result agrees with the study conducted by Çelik and Sağlam-Arslan (2012).

The teacher candidates were found to be most successful at symbolic, visual, and verbal representation styles respectively when their test performances were examined. Studies have shown that student performances were better on symbolic problems than verbal ones (Castro et al, 2001; Delice, 2004; Moseley, 2005; Sevimli & Delice, 2012). The teacher-candidates who were unsuccessful on the symbolic test were not unsuccessful on the verbal and visual tests, and the teacher-candidates who were successful on the visual test were not unsuccessful on the symbolic test. The teacher candidates who were successful on the verbal and visual tests were not unsuccessful on the symbolic test. However, the teacher-candidates who were successful on the visual and symbolic tests appeared in different categories on the verbal test. The teacher-candidates who were successful on the visual test were not unsuccessful on the symbolic and visual tests. The visual representation style has the role of connecting the symbolic and the verbal representation styles, which corresponds with the finding of Akyüz and colleagues (2009) that teacher-candidates used visual images in their minds while solving problems.

The solutions the teacher-candidates gave to the test questions and the data obtained from the interviews were examined. The teacher-candidates who were unsuccessful on the verbal test but successful on the other two tests connected their failures to the representations of the questions. The interviewees stated that they were successful in symbolic and visual questions but were unsuccessful on the verbal test because they incorrectly imagined the verbal expressions and made mistakes in modelling the forms. The interviewees also stated that they more effectively used their knowledge in visual and symbolic tests; however, in verbal problems they had difficulties in transferring their knowledge. Goldin and Kaput (1996) stated that students had difficulties with problems represented in different styles because the different representations required them to associate concepts with different contexts. This may explain the teacher-candidates' difficulty in solving problems presented verbally in this study. Some of the participants stated that in their previous education they did not encounter such verbal questions that were related to daily life. The teacher-candidates who were successful in verbal and symbolic tests but unsuccessful in visual tests stated that they were successful in verbal tests because they converted the verbal questions into models and shapes that they could understand. In other words, they were successful because they could make sense of the questions themselves. These interviewees also stated that they had difficulties in solving the questions on the visual test because the shapes were already provided, but on the verbal questions, they did their own modelling, which led to their success on the test. According to Kieren (1976), emphasizing different representations in teaching a subject is important. Other studies indicate that different representation styles can help students understand concepts (Lesh et al., 1987), make sense of problems, and develop mathematical thinking (Thompson & Silverman, 2007; Villegas et al., 2009). Normative approaches must be abandoned and different representations must be emphasized; models presenting different representations in learning environments, educational materials, and the numerical axis must be included in the teaching process and students must be given the opportunity to develop methods that work for them. As Jamison (2000) found, students could not understand mathematical concepts that they thought of as abstract when the concepts were conveyed using only verbal expressions or symbols. The importance of using different representation styles was confirmed by the present study.

In this study, the teacher-candidates stated that they became aware of the questions presented in different representations and understood their mistakes better. The teacher-candidates also stated that the different representations changed their perspectives, provided them with multiple possibilities for problem solving, and allowed them to express their cognitive and visualization processes. The different representation styles made it easier to understand the questions; in other words, the efficiency of the representations is obvious. If such important results can be obtained at the teacher-candidate level, positive results can also be obtained at the secondary-school level. The researcher in this study observed the increased awareness of the teacher-candidates, who stated that they supported these kinds of studies, and with the help of continuing education they would learn more about different problem situations. The participants stated that they would perform similar practices in their professional lives and try to better understand students' learning styles. For these reasons, the study findings are important, and similar studies should be conducted on other subjects and at different education levels. For example, future research could examine the transition process between representations and the variability of the representation forms used in solving problems.

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Appendix A

Problems of the Tests with Different Representation Styles *(Tests were applied Turkish Language)*

VETT (Verbal Trigonometry Test)

1- A watchman at the watchtower at the K point watches the 12-km-flight of a plane flying parallel to the ground over 3 km from the ground from point A to the point B on the radar.
If the orthogonal projection of the point A on the ground is point L and |KL|=21 km, what in the tangent of the angle AKB which was scanned by the radar?

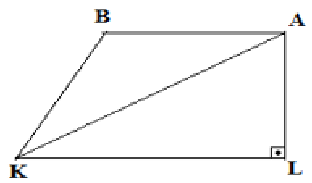
2- The distance from the scene of an accident to an emergency base is 60 km and the distance from the emergency base to the hospital is 140 km. If the measure of the angle formed by the line uniting the emergency base to the scene of accident and the line uniting the scene of accident to the hospital is 120^0, what is the distance from the scene of the accident to the hospital?

3- If the angle between a tree and its shadow whose height is 25 metres, is 53 degrees, then what is the height of the tree? (tan53≅1,32)

4- A pilot who wants to land off Atatürk Airport, does not know the height of the plane from the ground, yet the pilot sees the Bosphorus Bridge whose height is 110 metres from the sea level. The pilot measures the angle of sight of the bridge's top point and abutment at sea level as 15 and 18 degrees, respectively. Find the plane's height and it horizontal distence to the bridge (tan15≅0,268; tan18≅0,325).

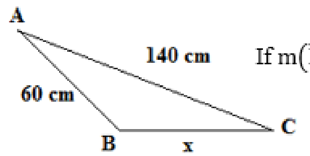
VITT (Visual Trigonometry Test)

5-



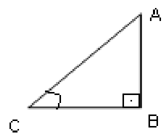
If $AL \perp KL, BA//KL, |AL| = 3\ cm,\ |BA| = 12\ cm,\ |KL| = 21\ cm$ then tan (BKA) =?

6-



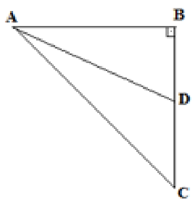
If $m(\widehat{B}) = 120^0$ then |BC| =?

7-



|CB|=25m, s(BCA)=53°,
According to the given information, |AB|=? (tan53≅1,32)

8-



|CD|=110m, s(BAC)=18°, s(BAD)=15°
According to the given information, |AB| and |BC|=? (tan15≅0,268; tan18≅0,325)

STT (Symbolic Trigonometry Test)

9- If $\tan(\alpha+\beta)=\frac{1}{3}$ and $\tan(\beta)=\frac{1}{7}$ then $\tan(\alpha)=?$

10- If $|AB| = 60\ cm, |AC| = 140\ cm\ and\ m(\hat{B}) = 120^0$ then $|BC| =?$

11- tan53=$\frac{a}{b}$ and b=25m, according to the given information, a=? (tan53≅1,32)

12- tan15= $\frac{h-110}{x}$ and tan18= $\frac{h}{x}$ according to the given information, find h and x. (tan15≅0,268;tan18≅0,325)