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Mathematics teachers' perceptions of student's mathematical writing ability

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Abstract

The research aims to explore how mathematics teachers perceive their students' mathematical writing (MW) skills, with a focus on four key dimensions: students' diverse problem-solving approaches, their proficiency in describing and explaining solutions, their mathematical reasoning and ability to provide proofs, and their capacity to evaluate solution validity. A perception questionnaire specifically designed for teachers was administered to a sample of 580 mathematics educators in Abu Dhabi schools during the academic year 2022-2023. The study utilized one-sample t-test, independent sample t-test, and ANOVA tests to analyze the teachers' perceptions across the four dimensions. The findings reveal positive perceptions among mathematics teachers, reflecting their confidence in their students' capabilities to explore diverse problem-solving approaches, articulate reasoning, provide robust proofs, and critically assess proposed solutions. However, notable differences in teachers' perceptions based on gender, school type, and teaching experience were observed. Female teachers had higher perceptions of students' mathematical reasoning and proof, while male teachers attributed greater significance to students' multiple paths to solutions and their ability to assess solution validity. Additionally, private school teachers demonstrated more optimistic perceptions of students' MW abilities compared to their counterparts in public schools. These findings offer valuable insights into teachers' perspectives on students' MW skills, informing potential enhancements in curricula and instructional practices. Understanding the factors influencing teachers' perceptions is vital to nurture students' MW skills and foster effective mathematical communication and problem-solving abilities. Further research is recommended to explore the underlying reasons for the observed differences and assess the impact of interventions targeting MW improvement in diverse educational settings. Such investigations can enrich mathematics education in Abu Dhabi Emirate schools and beyond, promoting students' proficiency in MW and bolstering their academic achievements. Strengthening MW abilities will empower students to excel in problem-solving and effective communication in mathematics, paving the way for successful academic endeavors.

Keywords: perceptions, mathematics teachers, abilities, mathematical writing

INTRODUCTION

Success in learning mathematics in different classroom settings requires mastery of various skills. The National Council of Teachers of Mathematics (2000) has established fundamental principles and standards to guide the teaching of mathematics, with a particular emphasis on mathematical communication. The National Council of Teachers of Mathematics' (2014) standards have placed greater emphasis on effective teaching practices, one of the key principles highlighted by the National Council of Teachers of Mathematics (2000) is the incorporation of meaningful mathematical dialogue to foster a shared understanding of

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Contribution to the literature

- This study is the first large-scale empirical investigation (N=580) of mathematics teachers' perceptions of students' mathematical writing (MW) skills in the UAE, addressing a major geographical gap in the predominantly Western-focused literature.
- It reveals positive overall perceptions across key MW dimensions (diverse approaches, explanations, reasoning/proof, validity evaluation) while uncovering novel demographic variations by gender, school type, and experience.
- The findings offer region-specific insights to inform UAE curriculum reforms, professional development, and future interventions for enhancing MW equity and mathematical communication.

mathematical concepts through the analysis and comparison of students' methods and arguments. There are numerous teaching practices that support mathematical communication, such as guiding students to express mathematical shapes, phrases, and symbols, describing the steps of the solution, completing texts and mathematical expressions, and paraphrasing (Al-Shahrani & Al-Khuzaim, 2022; Khalil, 2016).

According to Al-Kubaisi and Abdullah (2015), mathematical writing (MW) encompasses the capacity of students to articulate mathematical concepts in written form, describing and solving mathematical problems using the language of mathematics. The integration of MW is crucial in effective mathematics classes, as it plays a significant role in promoting mathematical communication. This integration can occur at two levels: writing without revision and writing with revision, the latter of which may demand more time but ultimately proves advantageous for teachers (Wilcox & Monroe, 2011).

Integrating writing into mathematics teaching and learning has numerous positive effects, including raising the level of mathematical achievement and developing problem-solving skills (Bicer et al., 2013, 2018). Additionally, it encourages the development of metacognitive skills in students, such as constructing meaning and organizing ideas (Kuzle, 2013).

The examination of teachers' beliefs is a critical area of study due to its impact on their ability to effectively present and explain subject matter to their students. A mathematics teacher's belief system is formed based on their perspectives regarding the essence of mathematics, teaching methodologies, and learning strategies , and is influenced by their teaching experiences, colleagues, and interactions with students. These factors contribute to the divergence of teachers' beliefs about mathematics (Al-Tarawneh & Khasawneh, 2018).

Statement Problem

MW holds great importance in facilitating mathematical communication and problem-solving. As math writing classes grow in number, students gain the opportunity to approach problems with creativity and innovation. Moreover, MW aligns with common core practice standards, enabling students to construct convincing arguments, evaluate others' reasoning, explain problem-solving strategies, utilize precise vocabulary, and communicate effectively (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). However, despite the significance of MW in mathematics education, there is a notable gap in understanding mathematics teachers' perceptions of their students' writing abilities in Abu Dhabi Emirate schools. This study aims to bridge this gap by exploring teachers' beliefs and identifying any significant differences in their perceptions based on gender, school type, teacher's qualification, experience, and region. Furthermore, the research seeks to contribute to the field by developing a reliable scale to measure mathematics teachers' beliefs regarding their students' MW abilities, adaptable to various educational settings.

Research Objectives

1. To investigate mathematics teachers' perceptions of their students' abilities in MW.
2. To identify any statistically significant differences in mathematics teachers' beliefs about their students' MW abilities at various levels of significance.

Research Questions

RQ1. What is the level of mathematics teachers' perceptions of their students' MW abilities in Abu Dhabi Emirate schools?

RQ2. Are there any statistically significant differences in mathematics teachers' perceptions of their students' MW abilities in Abu Dhabi Emirate schools based on gender, school type, teacher's qualification, teacher's experience, and the region?

Research Significance

The current study contributes to several aspects:

1. Addressing the subject of MW in detail, according to the latest developments on the subject and based on scientific references.
2. Providing a reliable scale that studies mathematics teachers' beliefs about their

students' writing abilities that can be used in many countries.

3. This study focuses on exploring the beliefs of mathematics teachers, a critical factor that significantly influences the educational process.

Mathematics Writing

There is a limited amount of evidence regarding content-area writing in mathematics, despite the National Council of Teachers of Mathematics (2000) and the National Governors Association Center for Best Practices, & Council of Chief State School Officers (2010) emphasizing the importance of written communication of mathematical reasoning in the curriculum. Teachers often lack familiarity with teaching MW (Cohen et al., 2015).

Mathematical language presents unique challenges, as it includes numbers, characters, words, and symbols to express concepts. Flexibility in language is crucial for good mathematical reading fluency, and students may struggle with mathematical terminology as they engage with mathematics (e.g., Hughes et al., 2016; Powell & Driver, 2015).

Mathematical word problems have a distinctive structure, presenting relevant and irrelevant information before posing a problem for students to solve. Addressing these problems in MW exacerbates the difficulties students face. Four types of MW are identified: exploratory, mathematical creative, argumentative, and informative/explanatory. Informative/explanatory writing is the most commonly used type, frequently seen as open-response word problems in various states (e.g., Pennsylvania) where students must solve a problem and explain their reasoning.

An examination of grade 4 students' MW features revealed that students wrote brief responses, often lacking an introduction or conclusion. They included a significant proportion of mathematics vocabulary terms but encountered challenges in logical organization and describing mathematical relationships between quantities. Students with learning difficulties in mathematics faced further obstacles, as they struggled with introducing and concluding their writing and formulating mathematical ideas coherently.

To address these challenges, empirical findings on MW therapies need evaluation. Prior research has demonstrated the benefits of content writing in various subjects, including science and social studies, in enhancing students' comprehension and content knowledge.

Overall, this study aims to explore mathematics teachers' beliefs regarding their students' MW abilities in Abu Dhabi Emirate schools, considering factors like gender, school type, teacher's qualification, teacher's experience, and region. By investigating teachers' perceptions and identifying potential differences, the study aims to contribute to understanding the importance of MW in the educational process and provide valuable insights for instructional practices and curriculum development.

CONCEPTUAL FRAMEWORK

The study examined four dimensions of mathematics teachers' perceptions of their students' MW abilities in Abu Dhabi Emirate schools, adapted from the framework proposed by Altakhaineh and Alname (2018), which emphasizes key components of mathematical communication. These dimensions are:

1. **Dimension 1:** Students' multiple paths to solutions
2. **Dimension 2:** Students' ability to describe and explain their answers
3. **Dimension 3:** Students' mathematical reasoning and proof
4. **Dimension 4:** Students' ability to consider the validity of a given solution and discuss the validity of two given solutions

These dimensions were selected based on their alignment with established constructs in MW literature. Altakhaineh and Alname (2018) defined MW as encompassing diverse problem-solving approaches, clear explanations, logical reasoning, and critical evaluation of solutions, which informed the structure of our questionnaire. Additionally, Casa et al. (2022) highlighted strategies for fostering multiple solution paths, explanations, reasoning, and solution evaluation, while Al-Nazir and Al-Maliki (2020) emphasized organizing ideas, clarifying relationships, and evaluating solutions as core aspects of written mathematical communication. These studies collectively provided a robust theoretical foundation for the four dimensions. The first dimension, students' multiple paths to solutions, focuses on teachers' perceptions of students' ability to demonstrate understanding, generate innovative ideas, and employ diverse problem-solving approaches in writing. This construct includes five items derived from Altakhaineh and Alname (2018) and Casa et al. (2022), reflecting students' capacity to explore varied strategies. The second dimension assesses teachers' views on students' ability to provide analytically written descriptions of problem-solving steps, drawing on Al-Nazir and Al-Maliki (2020). The third dimension evaluates perceptions of students' proficiency in offering logical evidence and arguments to support mathematical knowledge in writing, aligned with Casa et al. (2022). The fourth dimension examines teachers' perceptions of students' ability to critique solutions and justify their reasoning in writing, informed by Altakhaineh and Alname (2018) and Al-Nazir and Al-Maliki (2020). **Figure 1** illustrates the interconnections

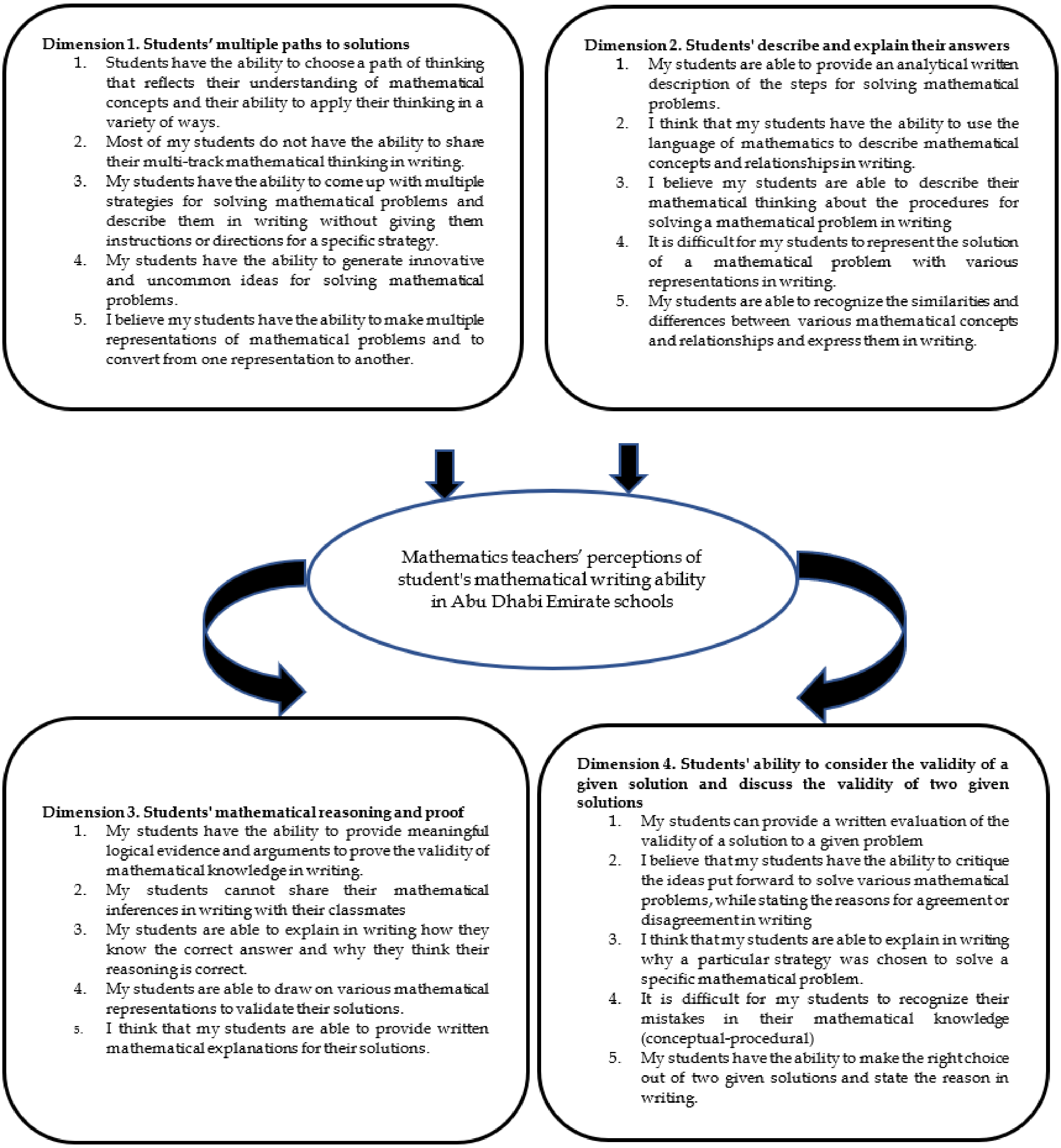

Figure 1. An examination of mathematics teachers' perceptions of students' MW skills in Abu Dhabi Emirate schools (Source: Authors' own elaboration)

between these four dimensions and their corresponding items, which were the primary focus of the study.

The conceptual framework depicted in Figure 1 illustrates the interconnections between the four dimensions investigated in this study. The first dimension focuses on how mathematics teachers perceive MW, encompassing their comprehension and teaching experiences in mathematics. This construct consists of five items derived from existing literature, which directly relate to MW and students' capacity to demonstrate their grasp of mathematical concepts, generate innovative ideas, and employ diverse approaches to problem-solving in writing. The second dimension pertains to mathematics teachers' perceptions of students' ability to describe and explain their solutions in writing, including providing analytical descriptions of the steps taken to solve mathematical problems. The third dimension explores mathematics teachers' perceptions of students' mathematical reasoning and proof, encompassing students' proficiency in offering logical evidence and arguments to support the validity of their mathematical knowledge in writing. The fourth dimension examines students' capacity to evaluate the validity of given solutions and engage in discussions about their validity. This involves critiquing presented ideas to solve mathematical problems, providing written justifications for agreement or disagreement with these solutions, and explaining their reasoning while selecting the correct solution in writing.

LITERATURE REVIEW

The four dimensions of MW used in this study were informed by prior research on mathematical communication. Altakhaineh and Alname (2018) proposed a framework for assessing students' MW, emphasizing diverse problem-solving approaches, clear explanations of solutions, logical reasoning, and critical evaluation of solution validity. Similarly, Casa et al. (2022) outlined strategies for supporting MW, including fostering multiple solution paths, encouraging explanatory writing, promoting reasoning, and evaluating solutions, which align with the four dimensions. Al-Nazir and Al-Maliki (2020) further identified organizing ideas, clarifying relationships, and evaluating solutions as key components of written mathematical communication, providing additional support for the framework. These studies collectively informed the design of the 20-item questionnaire, ensuring its alignment with established constructs in mathematics education.

Craig (2011) emphasizes the significance of explanatory writing in the enhancement of mathematical problem-solving skills. It also helps the math teacher to diagnose students' concepts, beliefs, and understanding of mathematical topics, and learn about students' attitudes (Bicer et al., 2013) It is important for the teacher to be prepared and for the teacher to design well MW activities as they help in enhancing the acquisition of mathematical knowledge (Beavers et al., 2015; Doming, 2022). The use of scaffolding, context, and mother tongue to improve writing and mathematical thinking (Jackaria et al., 2019).

According to Khalil's (2015) study, sixth-grade students demonstrated a low level of written mathematical communication skills. The study recommends including questions that assess written mathematical communication skills in tests and emphasizes the importance of training teachers to improve these skills.

Equally important is the development of classroom mathematical tasks that specifically target MW, alongside the crucial aspect of enabling students to engage with mathematical representations and establish connections among them (Adu- Gyamfi et al., 2010). There are many sports communication skills, and studies have varied in their classification. Khalil (2015) indicated that six written mathematical communication skills are: writing and representing symbols, interpretation, describing graphic representations, summarizing, and forming a question.

Al-Nazir and Al-Maliki (2020) highlighted that written mathematical communication skills encompass three main aspects: organizing and representing mathematical ideas and relationships, clarifying mathematical relationships, and evaluating mathematical solutions and ideas. On the other hand, Casa et al. (2022) presented a comprehensive set of strategies aimed at supporting MW in mathematics classrooms, which include promoting students' exploration of multiple solution paths, encouraging them to describe and explain their answers, fostering the sharing of their mathematical thinking, guiding them to consider the validity of a given solution, and urging them to engage in discussions about the validity of two given solutions.

To design an appropriate classroom environment for learning mathematics, teachers' perspective regarding mathematics teaching and learning must be positive; Therefore, the study of beliefs constitutes an entry point for developing a positive view of learning mathematics, as it is one of the important aspects in building knowledge and effective classroom practices (Al-Ghuwairi, 2020).

According to Phillip (2007), beliefs refer to a collection of ideas and perceptions that an individual holds towards the world and their surroundings. These beliefs encompass emotional aspects, such as inclinations and tendencies, and they play a significant role in influencing an individual's capabilities and guiding their decisions in relation to their surroundings. In a similar vein, my own definition (Cross, 2009) describes beliefs as a combination of conscious and unconscious ideas and perceptions about the world, one's personality, and their position in the world. These beliefs are shaped and developed through membership in collaborative groups.

According to Fives et al. (2014) Mathematics teachers' beliefs about teaching, teaching, and major curricula have a major role in their teaching practices, and explained (Wadmany & Levin, 2006; Scott, 2015) .The decisions made by teachers in the classroom are not solely based on acquired knowledge but also reflect their beliefs. In the context of educational reform, it is crucial to have teachers who are aware of this relationship and hold positive beliefs about mathematics and its learning process. Beliefs play a significant role in influencing a teacher's performance, as higher levels of self-belief are associated with increased efficiency in their teaching practices .

Ibrahim (2016) emphasizes the significance of aligning mathematics teachers' beliefs with modern trends in mathematics education and continuously developing their ideas and beliefs in line with evolving curricula and teaching methods. This is crucial as teachers' beliefs directly influence their approaches to teaching mathematics. Bobis et al. (2016) explored the impact of mathematics teachers' beliefs about learning mathematics on their teaching effectiveness and their awareness of students' understanding. Beswick (2012) found a connection between primary school mathematics teachers' teaching beliefs and their level of professional satisfaction.

Several studies have been dedicated to exploring the beliefs of mathematics teachers. For example, Alotaibi et al. (2021) investigated the teaching practices of mathematics teachers in Saudi Arabia using the PISA 2018 framework and examined the relationship between these practices and the teachers' beliefs towards their students. Saadati et al. (2021) studied the factors associated with the beliefs and practices of mathematics teachers in Chile during the COVID-19 pandemic, revealing high levels of self-efficacy in using technology for personal purposes but only average levels of self-efficacy in integrating technology into teaching practices. Alhunaini et al. (2022) studied the beliefs of mathematics teachers in Oman related to evaluating mathematical thinking, finding a reciprocal relationship between teachers' beliefs and their practices in the classroom environment.

Holm et al. (2020) examined the relationship between primary school mathematics teachers' beliefs about the importance of deepening students' conceptual comprehension and their ability to model and explain mathematical knowledge. Similarly, Bal's (2015) study explored the link between secondary school mathematics teachers' beliefs about the importance of students' problem-solving skills and their teaching practices to support these skills. Studies by Martínez-Sierra et al. (2020) and Barnes et al. (2014) investigated secondary school mathematics teachers' beliefs about assessment in mathematics and its objectives, and how students learn and achieve.

Al-Salouli et al. (2010) examined the relationship between students' comprehension of arithmetic concepts in Saudi Arabia and their teachers' beliefs regarding this comprehension. The results revealed a discrepancy between students' understanding and teachers' beliefs about it, highlighting the importance of teachers' practices aligned with students' comprehension of mathematical concepts.

Sezer (2010) study examined new teachers' pre-service perceptions of sports writing as an educational tool, and the study recommended the importance of training teachers on how to integrate sports writing while teaching sports content, because of its great role in enhancing students' understanding of sports content.

Overall, the importance of mathematics teachers' beliefs lies in their influential role in shaping classroom practices and their potential impact on students' achievements. Ensuring teachers' beliefs are aligned with effective teaching practices is crucial for promoting quality mathematics education.

METHODOLOGY

Research Model and Design

This study employed a quantitative approach to investigate mathematics teachers' perceptions of their students' MW abilities in schools. The choice of a quantitative approach was driven by the need to gather data from many participants and obtain responses with various options through a quantitative tool. This approach allowed systematic data collection and analysis, providing statistical insights into the teachers' perceptions and enabling a comprehensive examination of the research objectives and questions.

Population and Participants

The mathematics teacher perception survey utilized a quantitative survey approach to gather data from schools. The target population comprised full-time mathematics teachers teaching during the academic year 2022-2023. The study collected responses from a total sample of 580 mathematics teachers, consisting of 282 male and 298 female participants who willingly took part in the research.

Regarding the demographic distribution, male respondents accounted for 48.6% of the participants, while females constituted 51.4%. In terms of teacher qualifications, individuals with bachelor's degrees comprised the majority at 83.6%, followed by those with master's degrees at 14.1%, and respondents with PhD at 2.2%. Regarding teacher experience, 9.0% of the participants had less than 5 years of experience, 31.0% had experience between 6 to 10 years, and a substantial 60.0% possessed more than 10 years of experience. In terms of school type, the study had 85.5% of respondents from public schools and 14.5% from private schools. Finally, the regional distribution indicated that 34.4% of respondents were from Abu Dhabi, 55.5% from Al Ain, and 10.1% from Al Dhafrah. **Table 1** shows the distribution of teachers based on their demographic information, including gender, teacher's qualification, teacher's experience, school type, and the region.

Instrument

A 20-item survey was designed to assess mathematics teachers' perceptions of students' MW abilities in Abu Dhabi Emirate schools, as these perceptions may influence students' performance and achievement. The instrument was structured around four dimensions, each assessed by five items, adapted

Table 1. Distribution of teachers on their demographic information, including gender, teacher's qualification, teacher's experience, school type, and the region (n = 580)

Demographic	Frequency (N)	Percentage (%)
Gender		
Male	282	48.6
Female	298	51.4
Teacher's qualification		
Bachelor's degree	485	83.6
Master's degree	82	14.1
PhD	13	2.2
Teacher's experience		
Less than 5 years	52	9.0
Between 6 to 10 years	180	31.0
More than 10 years	348	60.0
School type		
Public	496	85.5
Private	84	14.5
The region		
Abu Dhabi	200	34.4
Alain	320	55.5
Al Dhafrah	60	10.1

from Altakhaineh and Alname (2018), who proposed a framework for MW emphasizing diverse solution paths, explanations, reasoning, and solution evaluation. Additional support came from Casa et al. (2022), who outlined strategies for fostering these skills, and Al-Nazir and Al-Maliki (2020), who highlighted organizing ideas, clarifying relationships, and evaluating solutions. These dimensions are as follows:

1. **Dimension 1: Students' multiple paths to solutions** (e.g., "My students have the ability to come up with multiple strategies for solving mathematical problems and describe them in writing").
2. **Dimension 2: Students' ability to describe and explain their answers** (e.g., "My students are able to provide an analytical written description of the steps for solving mathematical problems").
3. **Dimension 3: Students' mathematical reasoning and proof** (e.g., "My students have the ability to provide meaningful logical evidence and arguments to prove the validity of mathematical knowledge in writing").
4. **Dimension 4: Students' ability to consider the validity of a given solution and discuss the validity of two given solutions** (e.g., "My students can provide a written evaluation of the validity of a solution to a given problem").

Each item used a five-point Likert scale (1 = strongly disagree, 5 = strongly agree) to measure teachers' agreement with statements about students' abilities. The survey also collected demographic information (gender, courses taught, teaching experience, and region) to explore variations in perceptions, aligning with research questions on demographic influences (e.g., Al-Salouli et al., 2010; Beswick, 2012). Item development involved reviewing prior studies (e.g., Alhunaini et al., 2022; Bobis et al., 2016; Holm et al., 2020; Khalil et al., 2023; Saadati et al., 2021; Sezer, 2010) to ensure relevance to MW and teachers' beliefs. The instrument's validity and reliability were verified through expert review, a pilot study, and confirmatory factor analysis (CFA).

Verifying the validity and reliability of the instrument

To ensure the questionnaire's validity and reliability, several steps were taken. Firstly, the questionnaire was reviewed by six faculty members with expertise in mathematics education and eight specialists, supervisors, and mathematics teachers to ensure its relevance and appropriateness. Their feedback led to modifications, including rewording items for clarity and alignment with the four dimensions. Secondly, a pilot study with 16 teachers, not included in the main study, was conducted to test the questionnaire's clarity and effectiveness. Their responses and feedback informed final adjustments to the instrument. The reliability was assessed using Cronbach's alpha, yielding a value of 0.93, indicating high internal consistency. Thirdly, a CFA was performed to examine the factorial construct validity of the 20-item scale, distributed across four dimensions (five items per dimension), as depicted in **Figure 2**. The CFA confirmed that the items strongly loaded onto their respective dimensions, with fit indices (e.g., RMSEA = 0.067, CFI = 0.923) meeting acceptable thresholds. This analysis verified that the instrument accurately measured the intended constructs–students' multiple paths to solutions, ability to describe/explain answers, mathematical reasoning/proof, and evaluation of solution validity–aligned with the study's objectives and theoretical framework (Al-Nazir & Al-Maliki, 2020; Altakhaineh & Alname, 2018; Casa et al., 2022).

Figure 2 displays the high loading of each item in its respective dimension, indicating a strong association. The correlation coefficient results between the four dimensions of the scale show a significant positive relationship. **Table 2** presents the validity indicators of the internal construction of the scale items, confirming the accuracy of the model in predicting the relationships between the items and the data. All indicators meet the study's criteria, ensuring the stability of the model for the item relationships.

Data Collection

The questionnaire was administered to collect data on mathematics teachers' perceptions of students' MW abilities in Abu Dhabi Emirate schools during the 2022-2023 academic year. The 20-item survey, detailed in Section 3.2, was distributed via an online Google Form to 580 full-time mathematics teachers, ensuring convenience and accessibility. Participation was voluntary, with no incentives offered to minimize

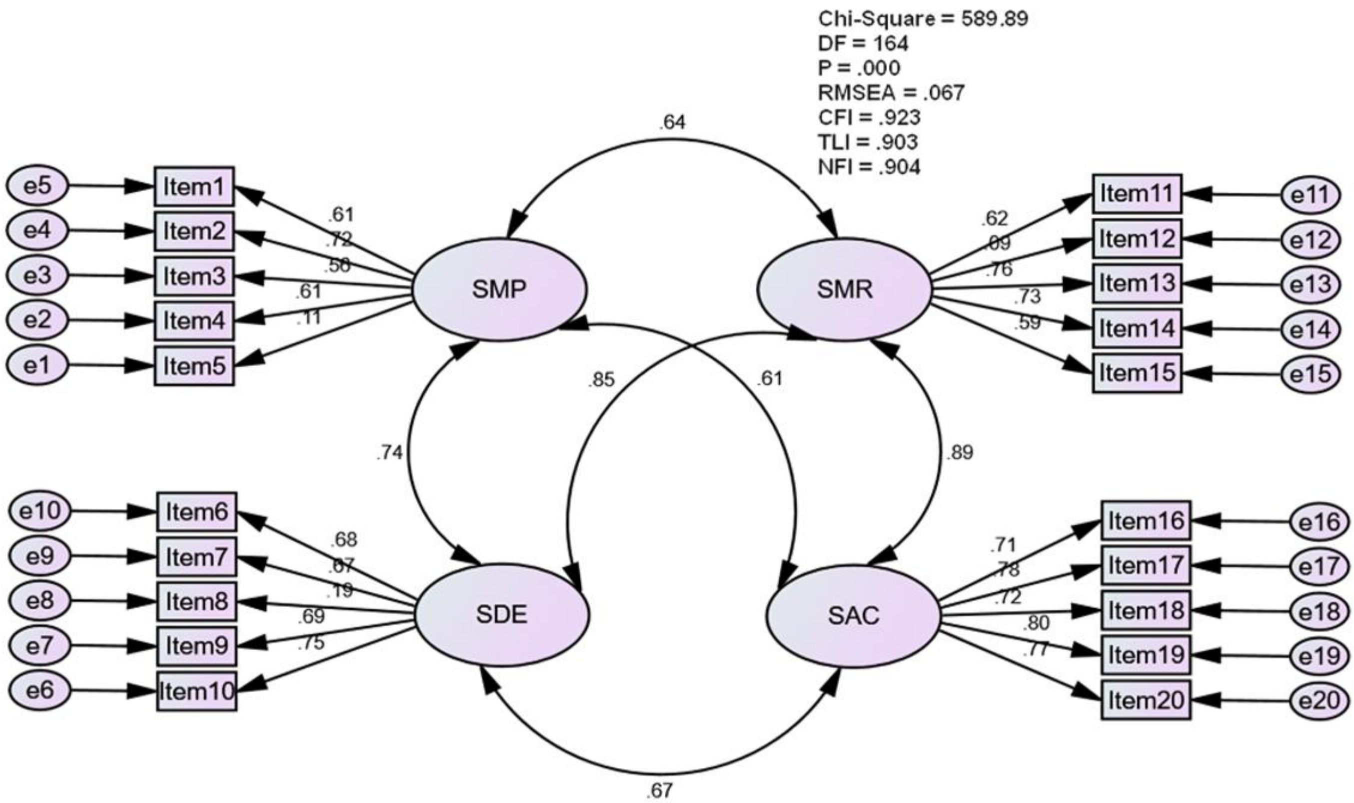

Figure 2. Results of the CFA for the adopted model, indicating the relationship between questionnaire items and their dimensions (Source: Authors' own elaboration)

Table 2. The results of the CFA of the adopted model of the relationship of the instrument items to their dimensions

Name of category	Indicators of the internal construct validity	Level of acceptance	Indexes in the proposed model
Absolute fit	Chi-square	p > 0.05	Significant
	RMSE	RMSE < 0.08	.067
Incremental fit	CFI	CFI > 0.90	.923
	TLI	TLI > 0.90	.903
	NFI	NFI > 0.90	.904
Parsimonious fit	Chi-square/df	Chi-square/df < 5.0	Chi-square/df = 3.6 < 5.0

response bias. The questionnaire collected demographic information, including teachers' gender, years of teaching experience, educational qualifications (bachelor's, master's, and PhD), and the region (Abu Dhabi, Al Ain, and Al Dhafrah) where they taught. This allowed analysis of perception differences by demographics, as outlined in the research questions.

The study proposal and questionnaire were reviewed and approved by the Social Sciences Research Ethics Committee at the United Arab Emirates University, with no major ethical concerns raised. Participants' anonymity was ensured by assigning numbers instead of names, and they were instructed not to share performance details publicly to prevent potential distress. All participants provided informed consent, confirming voluntary participation without coercion.

Data Analysis

The data analysis was structured to address the study's two research objectives:

(1) to determine the level of mathematics teachers' perceptions of students' MW abilities in Abu Dhabi Emirate schools (**RQ1**) and

(2) to identify differences in perceptions based on demographic variables (gender, school type, teaching experience, qualifications) (**RQ**2).

All analyses were conducted using SPSS, with a significance level of 0.05.

For **RQ1**, descriptive statistics (means [Ms] and standard deviations [SDs]) were calculated for the 20-item questionnaire, assessing perceptions across four dimensions: students' multiple paths to solutions, ability to describe and explain answers, mathematical reasoning and proof, and ability to evaluate solution validity. One-sample t-tests were conducted for each dimension's five items and overall dimension scores, using a neutral value of 3 (midpoint of the 5-point Likert scale) as the reference to determine whether perceptions were significantly positive or negative. These results are reported later.

Table 3. The Ms and SDs of the mathematics teachers perception of student's MW in Abu Dhabi Emirate schools

No	Rank	Domain	Frequency (N)	M	SD
1	1	**Dimension 1.** Students' multiple paths to solutions	580	3.66	0.68
2	4	**Dimension 2.** Students' describe and explain their answers	580	3.31	0.74
3	2	**Dimension 3.** Students' mathematical reasoning and proof	580	3.34	0.76
4	3	**Dimension 4.** Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	580	3.32	0.93

For **RQ2**, differences in perceptions were examined based on demographic variables. Independent t-tests were used to compare perceptions by gender (male vs. female) and school type (public vs. private). One-way ANOVA was employed to analyze differences by teaching experience (< 5 years, 6-10 years, and > 10 years) and qualifications (bachelor's, master's, and PhD). For significant ANOVA results, post hoc Tukey honestly significant difference tests were conducted to identify specific group differences. Eta-squared was calculated to determine effect sizes for significant differences, providing insight into the magnitude of demographic influences.

Parametric test assumptions were verified prior to analysis. Normality was assessed using the Shapiro-Wilk test, with $p > 0.05$ for all variables (dimension scores, demographic groups), indicating normal distribution. Boxplots confirmed no outliers in the data. Homogeneity of variances was tested using Levene's test, with $p > 0.05$ for all t-tests and ANOVAs, confirming equal variances. These checks ensured the appropriateness of parametric tests (one-sample t-tests, independent t-tests, and one-way ANOVA).

Results were presented starting with participants' demographic information (gender, qualifications, experience, school type, and region) in **Table 1**, followed by findings addressing **RQ1** and **RQ2**. This structure allowed systematic exploration of perception levels and demographic influences on teachers' views of students' MW abilities.

RESULTS

Mathematics Teachers' Perceptions of Mathematical Writing

The overall perceptions of mathematics teachers in Abu Dhabi Emirate schools were assessed by calculating the Ms for four different domains. These domains include teachers' practices, the variety of approaches students take to arrive at solutions, students' abilities to describe and explain their answers, their mathematical reasoning and ability to provide proofs, as well as their capability to assess the validity of a given solution and discuss the validity of two given solutions (as shown in **Table 3**).

Among the different dimensions that were analyzed to understand the teacher's perceptions, it was found that the dimension related to "students' multiple paths to solutions" had the highest M value of 3.66 and the lowest SD of 0.68. This indicates that respondents in this dimension showed the highest level of perception with the least amount of variability. In simpler terms, the teachers' perception of students' ability to explore multiple ways to reach solutions is highly positive and consistent among the respondents.

On the other hand, the dimension concerning "students' ability to describe and explain their answers" had the lowest M value of 3.31, suggesting that teachers' perceptions in this area were comparatively lower. However, specific variations among the respondents were not as pronounced as in the dimension of considering the validity of solutions.

The dimension regarding "students' ability to consider the validity of a given solution and discuss the validity of two given solutions" had the highest SD of 0.93. This indicates that respondents' perceptions in this area showed high variability or dispersion from the M. In other words, there was a greater diversity of opinions and views among the teachers regarding students' abilities to evaluate and discuss the validity of solutions.

Based on these findings, it can be concluded that dimension 1, which focuses on the teacher's perception of students' multiple paths to solutions, is the most positively regarded dimension, as it has the highest M value and the least SD, indicating strong agreement among the respondents.

Mathematics Teachers' Perceptions of Mathematical Writing on Dimension 1: Students' Multiple Paths to Solutions

In order to investigate potential differences in Teachers' Perceptions of students' MW ability in Abu Dhabi Emirate schools, a one-sample t-test was performed for each item's dimensions, and the results are presented in tables. The distribution of average item scores for each dimension is visually depicted in figures. These analyses aimed to determine whether the respondents' perceptions significantly deviated from a specific reference value or M in each dimension.

A one-sample t-test was performed to assess the teacher's perceptions regarding students' multiple paths to solutions, as shown in **Table 3**. The t-test results indicate that teachers hold positive perceptions towards each of the specific items: item 1 (M = 3.50, SD = 1.11, $p < 0.05$), item 2 (M = 3.47, SD = 1.08, $p < 0.05$), item 3 (M = 3.49, SD = 1.00, $p < 0.05$), item 4 (M = 3.81, SD = 0.94, p

Table 4. One-sample t-test showing mathematics perception of MW on Abu Dhabi Emirate school (n = 580) on dimension 1 items

No	Items	M	SD	t	df	S-2	MD	S
1	Students have the ability to choose a path of thinking that reflects their understanding of mathematical concepts and their ability to apply their thinking in a variety of ways.	3.50	1.11	10.731	579	0.00	.497	SP
2	Most of my students do not have the ability to share their multi-track mathematical thinking in writing.	3.47	1.08	10.547	579	.000	.474	SP
3	My students have the ability to come up with multiple strategies for solving mathematical problems and describe them in writing without giving them instructions or directions for a specific strategy.	3.94	1.00	22.469	579	.000	.944	SP
4	My students have the ability to generate innovative and uncommon ideas for solving mathematical problems.	3.81	0.94	20.594	579	.000	.809	SP
5	I believe my students have the ability to make multiple representations of mathematical problems and to convert from one representation to another.	3.62	1.24	11.895	579	.000	.616	SP
	Dimension 1. Students' multiple paths to solutions	3.66	0.68	23.368	579	.000	.663	SP

Note. S2: Sig. (2-tailed); MD: Mean difference; S: Significant positive/negative perceptions; & SP: Significant positive

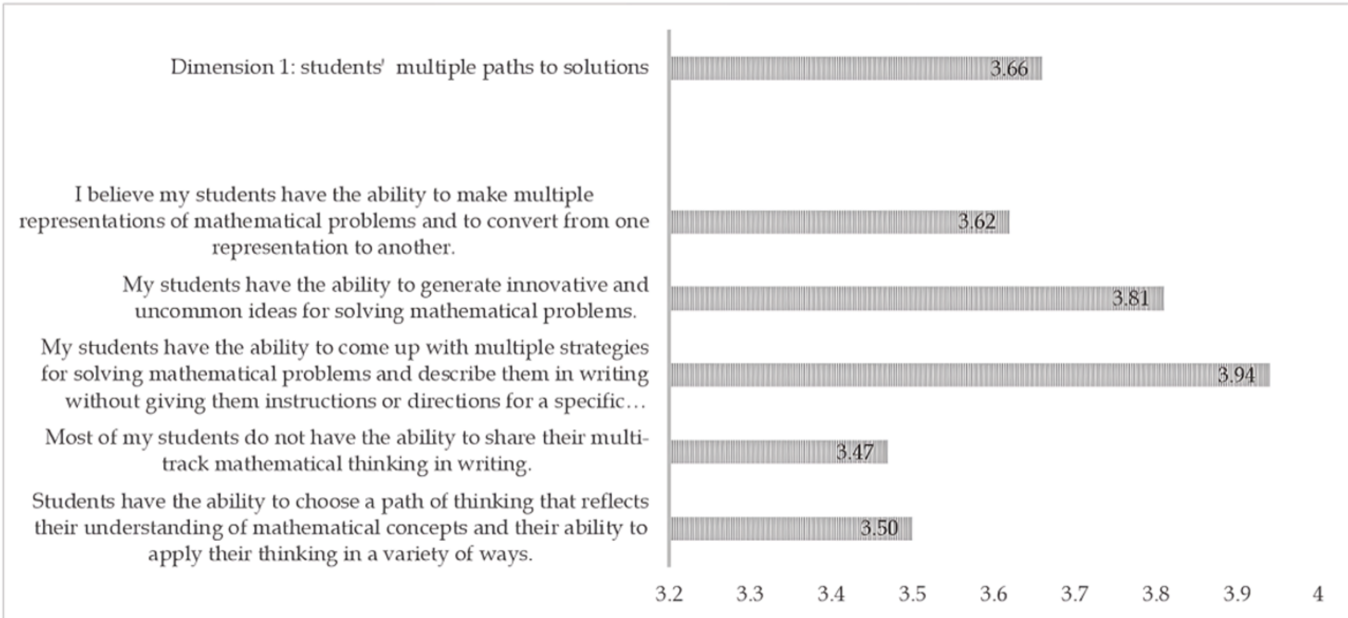

Figure 3. Average scores of the components of dimension 1: students' multiple paths to solutions (Source: Authors' own elaboration)

< 0.05), and item 5 (M = 3.62, SD = 1.124, $p < 0.05$). These results imply that teachers believe students possess the ability to find multiple paths to solutions when working on mathematical problems.

Moreover, the overall teacher's perception towards dimension 1, which relates to students' multiple paths to solutions, is also positive, with an M value of 3.66 and an SD of 0.68 ($p < 0.05$), as presented in **Table 4** and **Figure 3**. This suggests a general consensus among teachers in acknowledging and valuing the students' capacity to approach mathematical challenges through various methods.

In summary, the analysis indicates that teachers in Abu Dhabi Emirate schools hold a favorable view of students' capabilities in exploring multiple paths to arrive at solutions, demonstrating a positive outlook overall in dimension 1.

Mathematics Teachers' Perceptions of Mathematical Writing on Dimension 2: Students' Describe and Explain Their Answers

A one-sample t-test was carried out to assess how teachers perceive dimension 2, which focuses on students' ability to describe and explain their answers. This dimension includes items with responses on a five-point Likert scale, ranging from strongly disagree (coded as 1) to strongly agree (coded as 5). The neutral value of 3 was used as the reference point for the t-test, enabling researchers to examine whether teachers' perceptions significantly differed from this neutral value for each item in dimension 2. The t-test results provided insights into whether teachers generally had positive, neutral, or negative views regarding students' skills in describing and explaining their answers.

Table 5. One-sample t-test showing mathematics perception of MW on Abu Dhabi Emirate school (n = 580) on dimension 2 items

No	Items	M	SD	t	df	S-2	MD	S
1	My students are able to provide an analytical written description of the steps for solving mathematical problems.	3.33	1.160	6.876	578	0.00	.332	SP
2	I think that my students have the ability to use the language of mathematics to describe mathematical concepts and relationships in writing.	3.27	1.054	6.263	579	.000	.274	SP
3	I believe my students are able to describe their mathematical thinking about the procedures for solving a mathematical problem in writing	3.35	1.108	7.530	579	.000	.347	SP
4	It is difficult for my students to represent the solution of a mathematical problem with various representations in writing.	3.32	1.069	7.147	579	.000	.317	SP
5	My students are able to recognize the similarities and differences between various mathematical concepts and relationships and express them in writing.	3.28	1.029	6.577	579	.000	.281	SP
	Dimension 2. Students' describe and explain their answers	3.31	0.74	10.067	579	.000	.3100	SP

Note. S2: Sig. (2-tailed); MD: Mean difference; S: Significant positive/negative perceptions; & SP: Significant positive

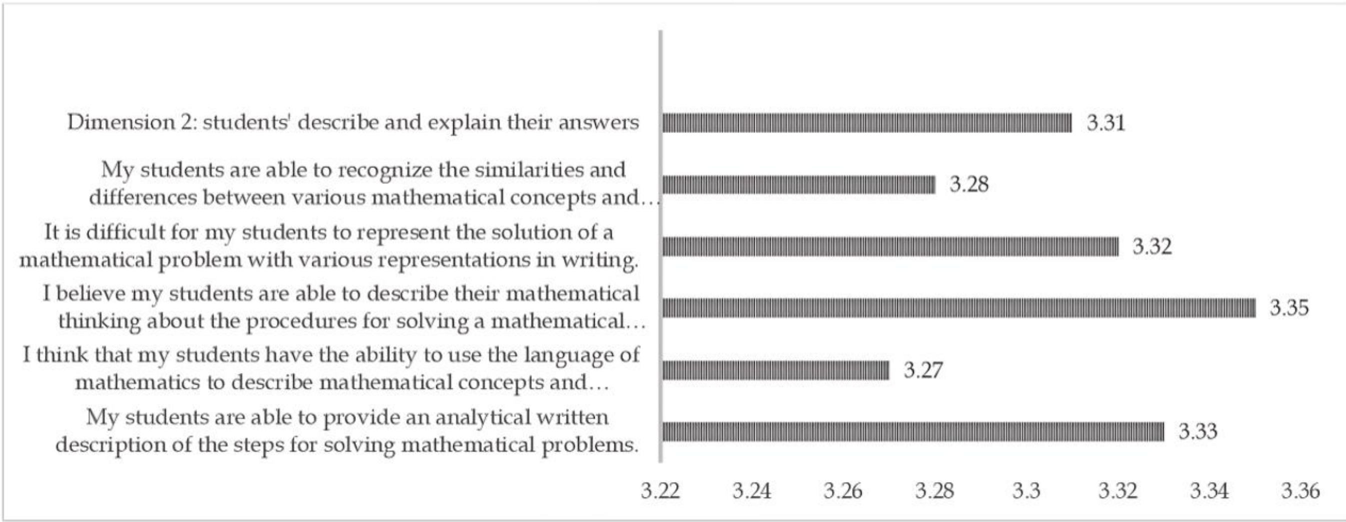

Figure 4. Average scores of the components of dimension 2: students' describe and explain their answers (Source: Authors' own elaboration)

The teacher's perceptions regarding dimension 2, which focuses on students' ability to describe and explain their answers, were examined through a one-sample t-test, as presented in **Table 4**. The results of the t-test revealed that teachers held positive perceptions for each of the specific items: item 1 ($M = 3.33$, $SD = 1.16$, $p < 0.05$), item 2 ($M = 3.27$, $SD = 1.05$, $p < 0.05$), item 3 ($M = 3.35$, $SD = 1.10$, $p < 0.05$), item 4 ($M = 3.32$, $SD = 1.06$, $p < 0.05$), and item 5 ($M = 3.28$, $SD = 1.03$, $p < 0.05$). These findings indicate that teachers believe students have the capability to effectively describe and explain their answers when tackling mathematical problems.

Furthermore, the overall teacher's perception of dimension 2 was positive, with an M value of 3.31 and an SD of 0.74 ($p < 0.05$), as shown in **Table 5** and **Figure 4**. This suggests that teachers, on the whole, hold favorable views regarding students' proficiency in articulating and explaining their solutions to mathematical questions.

In summary, the analysis demonstrates that teachers in Abu Dhabi Emirate schools possess an optimistic perspective on students' abilities to describe and explain their answers, indicating a positive outlook overall in dimension 2.

Mathematics Teachers' Perceptions of Mathematical Writing on Dimension 3: Students' Mathematical Reasoning and Proof

A one-sample t-test was carried out to assess the teacher's perceptions regarding dimension 3, which focuses on students' mathematical reasoning and proof. This dimension includes items with responses on a five-point Likert scale, ranging from strongly disagree (coded as 1) to strongly agree (coded as 5). The neutral value of 3 was used as the reference point for the t-test, allowing researchers to examine whether teachers' perceptions significantly differed from this neutral value for each item in dimension 3. The t-test results provided insights into whether teachers generally had positive, neutral, or negative views concerning students' abilities in mathematical reasoning and providing proofs.

A one-sample t-test was conducted to investigate teachers' perceptions regarding dimension 3, which focuses on students' mathematical reasoning and proof,

Table 6. One-sample t-test showing mathematics perception of MW on Abu Dhabi Emirate school (n = 580) on dimension 3 items

No	Items	M	SD	t	df	S-2	MD	S
1	My students have the ability to provide meaningful logical evidence and arguments to prove the validity of mathematical knowledge in writing.	3.34	1.031	7.892	579	.000	.338	SP
2	My students cannot share their mathematical inferences in writing with their classmates.	3.88	1.041	20.341	579	.000	.879	SP
3	My students are able to explain in writing how they know the correct answer and why they think their reasoning is correct.	3.11	1.179	2.183	579	.029	.107	SP
4	My students are able to draw on various mathematical representations to validate their solutions.	2.83	1.277	-3.254	578	.001	-.173	SP
5	I think that my students are able to provide written mathematical explanations for their solutions.	3.57	1.126	12.130	579	.000	.567	SP
	Dimension 3. Students' mathematical reasoning and proof	3.34	.757	10.941	579	.000	.344	SP

Note. S2: Sig. (2-tailed); MD: Mean difference; S: Significant positive/negative perceptions; & SP: Significant positive

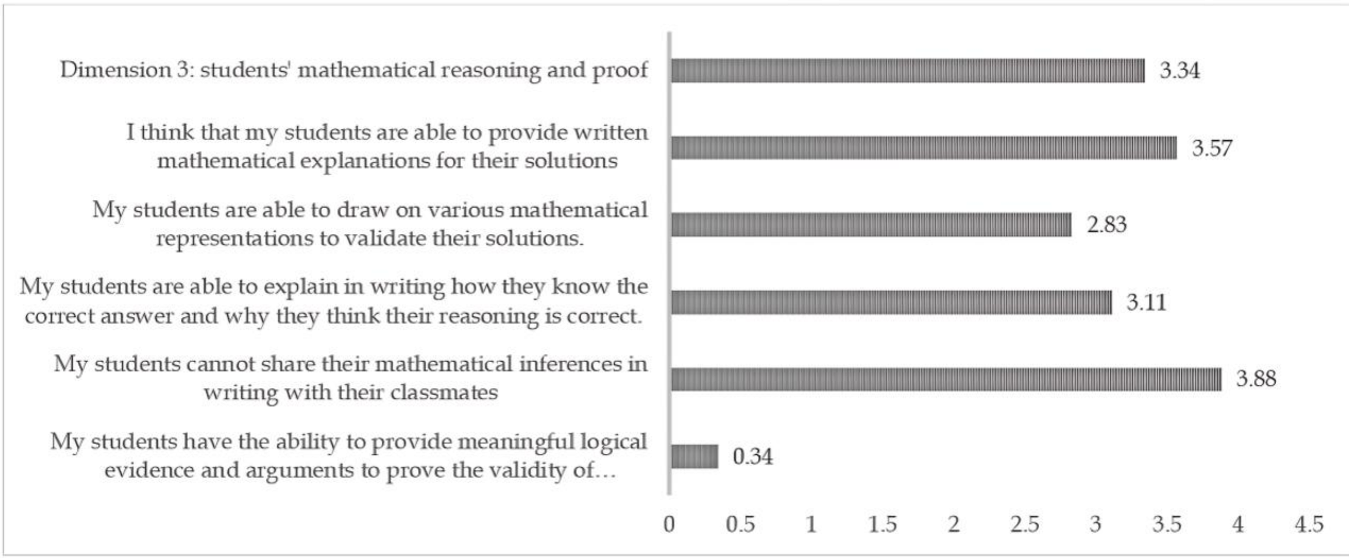

Figure 5. Average scores of the components of dimension 3: students' mathematical reasoning and proof (Source: Authors' own elaboration)

as displayed in **Table 4**. The t-test results provided in **Table 5** indicate that teachers hold positive perceptions towards each specific item: item 1 (M = 0.34, SD = 1.03, p < 0.05), item 2 (M = 3.88, SD = 1.04, p < 0.05), item 3 (M = 3.11, SD = 1.18, p < 0.05), item 4 (M = 2.83, SD = 1.28, p < 0.05), and item 5 (M = 3.57, SD = 1.13, p < 0.05). These findings suggest that teachers believe students possess mathematical reasoning and proof skills.

Furthermore, the overall teacher's perception of dimension 3 was positive, with an M value of 3.34 and an SD of 0.76 (p < 0.05), as shown in **Table 6** and **Figure 5**. This implies that teachers, overall, hold favorable views regarding students' abilities in mathematical reasoning and providing proof.

In summary, the analysis reveals that teachers in Abu Dhabi Emirate schools have a positive perspective on students' mathematical reasoning and proof abilities, indicating a generally optimistic outlook towards dimension 3.

Mathematics Teachers' Perceptions of Mathematical Writing on Dimension 4: Students' Ability to Consider the Validity of a Given Solution and Discuss the Validity of Two Given Solutions

A one-sample t-test was conducted to assess teachers' perceptions regarding dimension 4, which focuses on students' ability to consider the validity of a given solution and discuss the validity of two given solutions. This dimension consists of items with responses on a five-point Likert scale, ranging from strongly disagree (coded as 1) to strongly agree (coded as 5).

The neutral value of 3 was used as the reference point for the t-test, allowing researchers to determine whether teachers' perceptions significantly deviated from this neutral value for each item in dimension 4. The t-test results provided insights into whether teachers generally had positive, neutral, or negative views regarding students' skills in evaluating the validity of a given solution and discussing the validity of two provided solutions

Table 7. One-sample t-test showing mathematics perception of MW on Abu Dhabi Emirate school (n = 580) on dimension 4 items

No	Items	M	SD	t	df	S-2	MD	S
1	My students can provide a written evaluation of the validity of a solution to a given problem.	0.18	1.171	3.689	579	.000	.179	SP
2	I believe that my students have the ability to critique the ideas put forward to solve various mathematical problems, while stating the reasons for agreement or disagreement in writing.	3.28	1.169	5.790	579	.000	.281	SP
3	I think that my students are able to explain in writing why a particular strategy was chosen to solve a specific mathematical problem.	3.53	1.071	12.017	579	.000	.534	SP
4	It is difficult for my students to recognize their mistakes in their mathematical knowledge (conceptual-procedural).	3.28	1.175	5.761	579	.000	.281	SP
5	My students have the ability to make the right choice out of two given solutions and state the reason in writing.	3.30	1.165	6.236	579	.000	.302	SP
	Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	3.31	.929	8.177	579	.000	.315	SP

Note. S2: Sig. (2-tailed); MD: Mean difference; S: Significant positive/negative perceptions; & SP: Significant positive

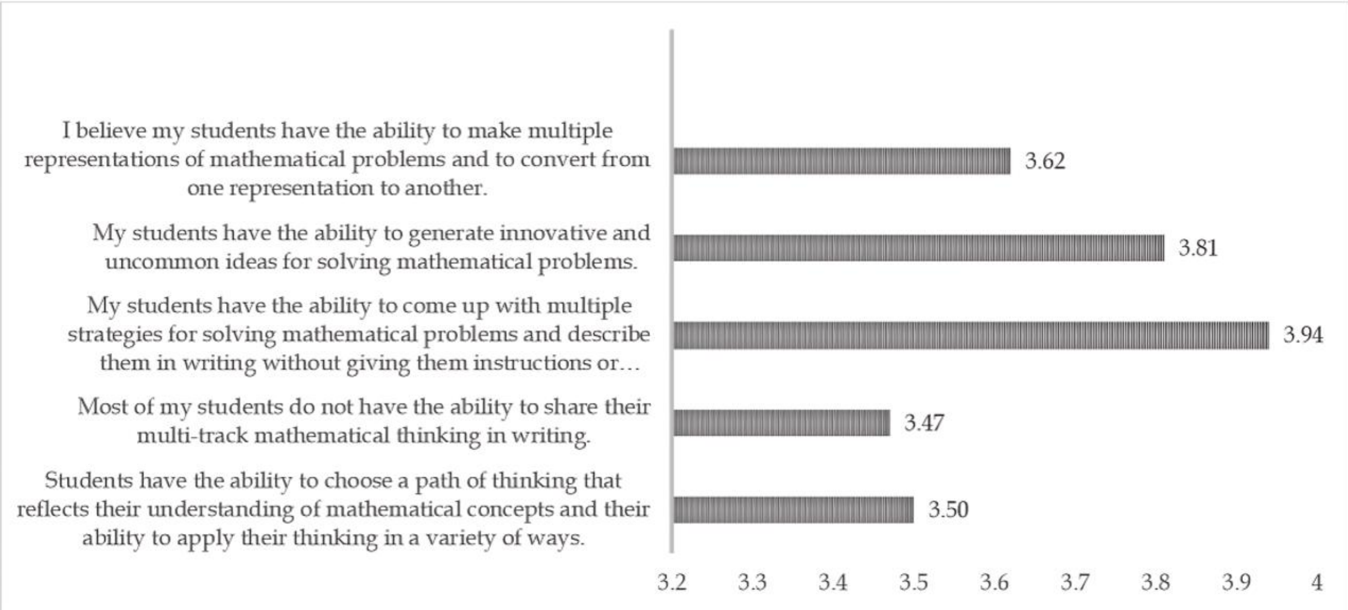

Figure 6. Average scores of the components of dimension 4: students' ability to consider the validity of a given solution and discuss the validity of two given solutions (Source: Authors' own elaboration)

A one-sample t-test was performed to investigate teachers' perceptions regarding dimension 4, which centers on students' ability to consider the validity of a given solution and discuss the validity of two given solutions, as presented in **Table 6**. The t-test results in **Table 6** demonstrate that teachers hold positive perceptions for each specific item: item 1 (M = 0.18, SD = 1.17, $p < 0.05$), item 2 (M = 3.28, SD = 1.17, $p < 0.05$), item 3 (M = 3.53, SD = 1.07, $p < 0.05$), item 4 (M = 3.28, SD = 1.18, $p < 0.05$), and item 5 (M = 3.30, SD = 1.16, $p < 0.05$). These findings suggest that teachers believe students have the capability to consider the validity of a given solution and engage in discussions regarding the validity of two solutions provided.

Furthermore, the overall teacher's perception towards dimension 4 was positive, with an M value of 3.31 and an SD of 0.93 ($p < 0.05$), as shown in **Table 7** and **Figure 6**. This indicates that teachers, on the whole, hold favorable views regarding students' abilities in critically evaluating solution validity and participating in discussions related to given solutions.

In summary, the analysis demonstrates that teachers in Abu Dhabi Emirate schools have a positive perspective on students' capacity to consider the validity of solutions and discuss multiple given solutions, suggesting an optimistic outlook overall in dimension 4.

Differences Between Males and Females With Respect to Mathematics Teachers' Perceptions of Mathematical Writing

To investigate potential differences between males' and females' perceptions of MW in Abu Dhabi Emirate schools, researchers conducted an Independent t-test. The M and SD of each group are presented in **Table 8**.

Table 8. Mathematics teachers' perceptions of student's MW ability in Abu Dhabi Emirate schools on gender

Dimensions	Gender	N	M	SD	F	t	df	S-2	MD
Dimension 1. Mathematical reasoning and proof	Male	282	3.65	.74	10.236	-.61	578	0.001	-.03497
	Female	298	3.68	.62		-.61	548.37		-.03497
Dimension 2. Students' multiple paths to solutions	Male	282	3.35	.80	10.386	1.14	578	0.001	.07026
	Female	298	3.28	.68		1.13	552.47		.07026
Dimension 3. Students' mathematical reasoning and proof	Male	282	3.37	.78	.256	.74	578	0.603	.04661
	Female	298	3.32	.74		.73	571.79		.04661
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	Male	282	3.25	.99	5.869	-1.57	578	0.016	-.12131
	Female	298	3.37	.87		-1.57	559.82		-.12131
Teachers perceptions	Male	282	3.40	.69	8.191	-.18	578	0.004	-.00974
	Female	298	3.41	.59		-.18	555.66		-.00974

Note. S-2: Sig. (2-tailed) & MD: Mean difference

Table 9. Mathematics teachers' perceptions of student's MW ability in Abu Dhabi Emirate schools on school type

Dimensions	ST	N	M	SD	F	t	df	S-2	MD
Dimension 1. Mathematical reasoning and proof	Public	496	3.62	.69	3.482	-3.388	578	0.001	-.27094
	Private	84	3.90	.57		-3.889	128.34		-.27094
Dimension 2. Students' multiple paths to solutions	Public	496	3.21	.71	1.741	-8.150	578	0.063	-.67600
	Private	84	3.89	.65		-8.659	118.87		-.67600
Dimension 3. Students' mathematical reasoning and proof	Public	496	3.27	.75	5.964	-6.048	578	0.188	-.52471
	Private	84	3.79	.66		-6.625	122.34		-.52471
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	Public	496	3.23	.93	12.204	-5.395	578	0.015	-.57767
	Private	84	3.81	.78		-6.095	126.20		-.57767
Teachers perceptions	Public	496	3.33	.63	11.413	-.180	578	0.001	-.51267
	Private	84	3.84	.51		-.180	128.75		-.51267

Note. ST: School type; S-2: Sig. (2-tailed) & MD: Mean difference

The study conducted an independent samples t-test to compare mathematics teachers' perceptions of MW in Abu Dhabi Emirate schools based on gender. The results showed that there was a statistically significant difference between male and female teachers in their overall perception of MW (male: M = 3.40, SD = 0.69; female: M = 3.41, SD = 0.59, and p = 0.004 < 0.05). This suggests that male and female teachers had differing viewpoints when it comes to MW.

However, the study also found that there were no statistically significant differences between male and female teachers in dimension 3, which pertains to students' mathematical reasoning and proof (male: M = 3.37, SD = 0.87; female: M = 3.32, SD = 0.74, and p = 0.613 > 0.05). This indicates that male and female teachers held similar opinions regarding dimension 3.

On the other hand, significant differences were observed between male and female teachers in dimensions 1, 2, and 4. For dimension 1, which involves mathematical reasoning and proof, male and female teachers had different opinions (male: M = 3.65, SD = 0.74; female: M = 3.68, SD = 0.62, and p = 0.001 < 0.05). Similarly, for dimension 2, which relates to students' multiple paths to solutions, male and female teachers exhibited differing viewpoints (male: M = 3.35, SD = 0.80; female: M = 3.28, SD = 0.64, and p = 0.001 < 0.05). Lastly, for dimension 4, concerning students' ability to consider solution validity and discuss given solutions, male and female teachers also demonstrated differing perceptions (male: M = 3.40, SD = 0.87; female: M = 3.41, SD = 0.59, and p = 0.001 < 0.05).

In summary, the study reveals that male and female teachers in Abu Dhabi Emirate schools had varying opinions on the overall perception of MW and dimensions 1, 2, and 4, while their opinions aligned in dimension 3.

Differences Between Public and Private Schools With Respect to Mathematics Teachers' Perceptions of Mathematical Writing

Researchers employed an independent t-test to investigate potential differences between public and private schools' perceptions of MW in Abu Dhabi Emirate schools. The M and SD of each group's perceptions are presented in **Table 9**. This statistical analysis aimed to compare the average perceptions of MW between public and private schools and determine if there are significant distinctions in how the two types of schools view students' MW abilities.

The study conducted an independent samples t-test to compare mathematics teachers' perceptions of students' MW ability in Abu Dhabi Emirate schools based on school type (public and private). The results indicated several significant differences between public and private schools in their perceptions of MW.

Firstly, there was a statistically significant difference in the overall perception of MW between public and private schools (public: M = 3.33, SD = 0.63; private: M =

Table 10. One-way ANOVA test of mathematics teachers' perceptions of students MW ability on Abu Dhabi Emirate schools on teaching experience

		SS	df	MS	F	Sig.
Dimension 1. Mathematical reasoning and proof	Between groups	.741	2	.370	.791	.454
	Within groups	270.025	577	.468		
	Total	270.766	579			
Dimension 2. Students' multiple paths to solutions	Between groups	5.781	2	2.890	5.334	.005
	Within groups	312.681	577	.542		
	Total	318.462	579			
Dimension 3. Students' mathematical reasoning and proof	Between groups	9.591	2	4.795	8.575	.000
	Within groups	322.679	577	.559		
	Total	332.270	579			
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	Between groups	5.103	2	2.552	2.975	.052
	Within groups	494.877	577	.858		
	Total	499.980	579			
Teachers perceptions	Between groups	3.418	2	1.709	4.211	.015
	Within groups	234.198	577	.406		
	Total	237.617	579			

Note. SS: Sum of squares & MS: Mean square

3.84, SD = 0.51, and p = 0.001 < 0.05). This suggests that teachers from private schools hold higher perceptions of students' MW ability compared to those from public schools.

Secondly, there were statistically significant differences in teachers' perceptions of dimension 1, which pertains to mathematical reasoning and proof for MW (public: M = 3.62, SD = 0.69; private: M = 3.90, SD = 0.57, and p = 0.001 < 0.05). Again, teachers from private schools demonstrated higher perceptions in this dimension than teachers from public schools.

Similarly, there were statistically significant differences in teachers' perceptions of dimension 4, which involves students' ability to consider the validity of a given solution and discuss the validity of two given solutions (public: M = 3.23, SD = 0.93; private: M = 3.81, SD = 0.78, and p = 0.015 < 0.05). Teachers from private schools showed higher perceptions in this dimension compared to teachers from public schools.

However, no statistically significant differences were observed between public and private schools in teachers' perceptions of dimension 2, which pertains to students' multiple paths to solutions (public: M = 3.21, SD = 0.71; private: M = 3.89, SD = 0.65, and p = 0.188 > 0.05), and dimension 3, which involves students' mathematical reasoning and proof (public: M = 3.27, SD = 0.75; private: M = 3.79, SD = 0.66, and p = 0.188 > 0.05).

In summary, the study indicates that there are significant differences in teachers' perceptions of students' MW ability between public and private schools, as well as in certain dimensions related to mathematical reasoning and solution validity consideration. However, no significant differences were found in the dimensions of students' multiple paths to solutions and mathematical reasoning and proof between the two types of schools.

Differences in Teachers' Perceptions of Mathematical Writing in Abu Dhabi Emirate Schools Based on Math Teaching Experience

The researchers conducted a one-way ANOVA and post hoc comparisons to investigate potential differences in teachers' perceptions of students' MW ability in Abu Dhabi Emirate schools based on math teachers' experience. The M and SD of each group's perceptions are presented in **Table 10**. This statistical analysis aimed to compare the average perceptions of math teachers with different levels of experience and determine if there are significant distinctions in how they perceive students' MW abilities.

To further explore specific group differences, the post hoc comparisons were conducted and the results are shown in **Table 11**. These post hoc tests help to identify which specific pairs of teacher experience groups have significantly different perceptions of students' MW ability.

The study utilized a one-way ANOVA and post hoc comparisons to examine potential differences in teachers' perceptions of students' MW ability in Abu Dhabi Emirate schools based on teachers' level of math teaching experience. Participants were categorized into three groups: less than 5 years of experience (n = 52), between 6 and 10 years of experience (n = 180), and 16 years and above of experience (n = 348).

The results from the one-way ANOVA indicated statistically significant differences between the groups with different levels of experience for the overall perception scores of math teachers regarding students' MW ability ($F [2, 577] = 4.211$, $p < 0.05$). Post hoc analysis using Tukey's method revealed that the differences in perceptions were significant between the group with 1 to 5 years' experience and both the groups with 5 to 10 years' experience and 10 years' experience and above.

Table 11. Experience multiple comparisons

	(I) Experience	(J) Experience	MD (I-J)	SE	Sig.	95% CI	
						LB	UB
Dimension 1. Mathematical reasoning and proof	1-5 years	5-10 years	.07434	.10770	.769	-.1787	.3274
		More than 10 years	-.00332	.10171	.999	-.2423	.2357
	5-10 years	1-5 years	-.07434	.10770	.769	-.3274	.1787
		More than 10 years	-.07765	.06281	.432	-.2252	.0699
	More than 10 years	1-5 years	.00332	.10171	.999	-.2357	.2423
		5-10 years	.07765	.06281	.432	-.0699	.2252
Dimension 2. Students' multiple paths to solutions	1-5 years	5-10 years	.28487	.11590	.038	.0125	.5572
		More than 10 years	.35522	.10945	.004	.0980	.6124
	5-10 years	1-5 years	-.28487	.11590	.038	-.5572	-.0125
		More than 10 years	.07034	.06759	.551	-.0885	.2292
	More than 10 years	1-5 years	-.35522	.10945	.004	-.6124	-.0980
		5-10 years	-.07034	.06759	.551	-.2292	.0885
Dimension 3. Students' mathematical reasoning and proof	1-5 years	5-10 years	.44718	.11773	.000	.1705	.7238
		More than 10 years	.45155	.11118	.000	.1903	.7128
	5-10 years	1-5 years	-.44718	.11773	.000	-.7238	-.1705
		More than 10 years	.00437	.06866	.998	-.1570	.1657
	More than 10 years	1-5 years	-.45155	.11118	.000	-.7128	-.1903
		5-10 years	-.00437	.06866	.998	-.1657	.1570
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	1-5 years	5-10 years	.16009	.14580	.516	-.1825	.5027
		More than 10 years	.29518	.13769	.082	-.0284	.6187
	5-10 years	1-5 years	-.16009	.14580	.516	-.5027	.1825
		More than 10 years	.13510	.08503	.251	-.0647	.3349
	More than 10 years	1-5 years	-.29518	.13769	.082	-.6187	.0284
		5-10 years	-.13510	.08503	.251	-.3349	.0647
Math teachers perceptions	1-5 years	5-10 years	.24291	.10030	.042	.0072	.4786
		More than 10 years	.27481	.09472	.011	.0522	.4974
	5-10 years	1-5 years	-.24291	.10030	.042	-.4786	-.0072
		More than 10 years	.03189	.05849	.849	-.1055	.1693
	More than 10 years	1-5 years	-.27481	.09472	.011	-.4974	-.0522
		5-10 years	-.03189	.05849	.849	-.1693	.1055

Note. MD: Mean difference; SE: Standard error; CI: Confidence interval; LB: Lower bound; & UB: Upper bound

Regarding dimension 2 (students' describe and explain their answers), the one-way ANOVA revealed statistically significant differences between different experience groups ($F [2, 577] = 5.334, p < 0.05$). Post hoc analysis showed significant changes between the group with 1 to 5 years' experience and both the groups with 5 to 10 years' experience and 10 years and above experience.

Similarly, for dimension 3 (students' mathematical reasoning and proof), the one-way ANOVA results showed statistically significant differences between different experience groups ($F [2, 577] = 8.575, p < 0.05$). Post hoc analysis indicated significant changes between the group with 1 to 5 years' experience and both the groups with 5 to 10 years' experience and 10 years and above experience.

However, no statistically significant differences were observed for dimension 1 (students' multiple paths to solutions) and dimension 4 (students' ability to consider the validity of a given solution and discuss the validity of two given solutions) between the groups with different levels of experience (dimension 1: $F [2, 577] = 0.791, p > 0.05$; dimension 4: $F [2, 577] = 2.975, p > 0.05$).

In summary, the study revealed that teachers' perceptions of students' MW ability, as well as certain dimensions of MW, were influenced by their level of math teaching experience. However, no significant differences were found for other dimensions related to students' multiple paths to solutions and consideration of solution validity.

Differences in Teachers' Perceptions of Mathematical Writing in Abu Dhabi Emirate Schools Based on Math Teachers' Qualifications

To investigate potential differences in mathematics teachers' perceptions of MW in Abu Dhabi Emirate schools based on math teachers' qualifications, the researchers employed a one-way ANOVA test and post hoc comparisons.

The M and SD of each group's perceptions are presented in **Table 12**. This statistical analysis aimed to compare the average perceptions of math teachers with different qualifications and determine if there are significant distinctions in how they perceive MW abilities of students.

Table 12. One-way ANOVA test of mathematics teachers' perceptions of MW ability on Abu Dhabi Emirate schools on math teachers' qualifications

		SS	df	MS	F	Sig.
Dimension 1. Mathematical reasoning and proof	Between groups	.142	2	.071	.152	0.859
	Within groups	270.624	577	.469		
	Total	270.766	579			
Dimension 2. Students' multiple paths to solutions	Between groups	4.192	2	2.096	3.848	0.022
	Within groups	314.270	577	.545		
	Total	318.462	579			
Dimension 3. Students' mathematical reasoning and proof	Between groups	1.936	2	.968	1.691	0.185
	Within groups	330.334	577	.573		
	Total	332.270	579			
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	Between groups	3.194	2	1.597	1.855	0.157
	Within groups	496.786	577	.861		
	Total	499.980	579			
Math teachers perceptions	Between groups	1.743	2	.871	2.132	0.120
	Within groups	235.874	577	.409		
	Total	237.617	579			

Note. SS: Sum of squares & MS: Mean square

Table 13. Qualifications multiple comparisons

Dependent variable	(I) 3. Teacher's academic qualification	(J) 3. Teacher's academic qualification	MD (I-J)	Sig.
Dimension 1. Mathematical reasoning and proof	Bachelor's degree	Master's degree	-.031	.923
		PhD	-.081	.907
	Master's degree	Bachelor's degree	.031	.923
		PhD	-.050	.968
	PhD	Bachelor's degree	.081	.907
		Master's degree	.050	.968
Dimension 2. Students' multiple paths to solutions	Bachelor's degree	Master's degree	-.237	.020
		PhD	-.174	.680
	Master's degree	Bachelor's degree	.237	.020
		PhD	.064	.955
	PhD	Bachelor's degree	.174	.680
		Master's degree	-.063	.955
Dimension 3. Students' mathematical reasoning and proof	Bachelor's degree	Master's degree	-.166	.159
		PhD	-.050	.970
	Master's degree	Bachelor's degree	.166	.159
		PhD	.116	.864
	PhD	Bachelor's degree	.050	.970
		Master's degree	-.116	.864
Dimension 4. Students' ability to consider the validity of a given solution and discuss the validity of two given solutions	Bachelor's degree	Master's degree	-.196	.180
		PhD	.168	.795
	Master's degree	Bachelor's degree	.196	.180
		PhD	.365	.386
	PhD	Bachelor's degree	-.168	.795
		Master's degree	-.365	.386
Math teachers perceptions	Bachelor's degree	Master's degree	-.158	.098
		PhD	-.034	.980
	Master's degree	Bachelor's degree	.157	.098
		PhD	.123	.795
	PhD	Bachelor's degree	.034	.980
		Master's degree	-.123	.795

Note. MD: Mean difference

To delve into specific group differences, the post hoc comparisons were conducted (**Table 13**). These post hoc tests help identify which specific pairs of teacher qualification groups exhibit significantly different perceptions of students' MW ability.

For a comprehensive understanding of the study's findings and more detailed results from the one-way ANOVA test and post hoc comparisons, please refer to **Table 12** and **Table 13**, respectively. **Table 12** and **Table 13** provide valuable insights into how math teachers' qualifications influence their perceptions of students' MW in Abu Dhabi Emirate schools.

A one-way ANOVA test was performed to investigate potential differences in overall teachers' perception of MW and its individual dimensions (dimension 1: students' multiple paths to solutions, dimension 2: students' describe and explain their answers, dimension 3: students' mathematical reasoning and proof, and dimension 4: students' ability to consider the validity of a given solution and discuss the validity of two given solutions) in Abu Dhabi Emirate schools based on math teachers' qualifications. The participants were divided into three groups based on their educational qualifications: bachelor's degree (n = 485), master's degree (n = 82), and PhD (n = 13).

The analysis first assessed data for outliers and normal distribution using boxplot and Shapiro-Wilk tests, respectively. The results showed no outliers and normal distribution for each group ($p > 0.05$). Additionally, the assumption of homogeneous variances was met, as indicated by Levine's test ($p = 0.053$).

For math teachers' overall perception scores, the one-way ANOVA results revealed no statistically significant differences among the groups with different qualifications ($F [2, 577] = 2.132$, $p > 0.05$). The same non-significant pattern was observed for dimension 1 ($F [2, 519] = 1.52$, $p > 0.05$), dimension 3 ($F [2, 577] = 1.691$, $p > 0.05$), and dimension 4 ($F [2, 577] = 1.855$, $p > 0.05$).

However, for dimension 2 (students' describe and explain their answers), the one-way ANOVA results indicated statistically significant differences based on teachers' qualifications ($F [2, 577] = 848$, $p < 0.05$). The post hoc analysis using Tukey's method revealed that dimension 2 scores increased significantly from the bachelor's degree group to the master's degree group ($p = 0.020 < 0.05$), while no other significant differences were observed for the other pairs at a 0.01 level of significance.

In conclusion, the analysis indicates that math teachers' qualifications do not significantly influence their overall perception of MW and its dimensions, except for dimension 2 (students' describe and explain their answers), where a significant difference was observed between teachers with bachelor's and master's degrees.

DISCUSSION

The study demonstrates that mathematics teachers in Abu Dhabi Emirate schools have a positive outlook on their students' MW capabilities across all dimensions. This indicates that teachers recognize the significance of mathematical communication and problem-solving skills in their students. Such optimistic perceptions can influence teaching practices and the development of curricula to further nurture students' MW abilities in the region. Nonetheless, it is crucial to continue exploring effective strategies for supporting and improving students' MW skills to ensure their ongoing academic achievements in mathematics. These findings align with Sezer's (2010) study, which focused on new teachers' perceptions of sports writing as an educational tool. Sezer's (2010) study suggested the importance of providing teacher training on integrating sports writing into sports content instruction to enhance students' comprehension of the subject matter. Indeed, this research aligns with the assertions made in the Al-Maliki (2012), which indicated that the written mathematical communication skills of fifth-grade students were rated as moderate or intermediate.

The study found that male and female mathematics teachers in Abu Dhabi Emirate schools had different perceptions of students' MW abilities in certain dimensions. Specifically, male teachers had higher perceptions in dimension 2 (students' multiple paths to solutions) and dimension 4 (students' ability to consider the validity of given solutions) compared to female teachers. However, there were no significant differences between male and female teachers' perceptions in dimension 3 (students' mathematical reasoning and proof). These findings suggest that gender may influence how teachers perceive certain aspects of students' MW abilities. Further research and exploration of the factors contributing to these differences could provide valuable insights into mathematics education in the region. This outcome corroborates the findings of Al-Surayhi (2022), Abdullah and Jaafar (2017), and Sür and Delice (2016). The likely reasons behind this consistency could be attributed to female teachers' scientific background or their proficiency in MW, as observed in the studies by Rayani and An-Nafish (2019) and Ash-Sharif and Qandil (2020). These studies revealed that mathematics teachers possessed average MW skills. Additionally, Abdullah and Jaafar (2017) identified a positive correlation between teaching performance and MW skills among mathematics teachers. Moreover, this result may be associated with female mathematics teachers' beliefs about their students and their perceptions of MW skills, or possibly due to limited training programs provided to teachers in this particular area.

the study found that public and private schools' mathematics teachers in Abu Dhabi Emirate schools had different perceptions of students' MW abilities in overall perception, dimension 1 (mathematical reasoning and proof), and dimension 4 (ability to consider the validity of solutions and discuss them). Teachers in private schools had higher perceptions in these dimensions compared to teachers in public schools. However, there were no significant differences between public and private school teachers' perceptions in dimension 2 (students' multiple paths to solutions) and dimension 3 (students' mathematical reasoning and proof). These findings suggest that school type may influence how teachers perceive certain aspects of students' MW abilities. Further research and exploration of the factors contributing to these differences could provide valuable insights into mathematics education in both public and private schools in the region.

the study found that there were statistically significant differences in mathematics teachers' perceptions of student's MW ability based on their level of teaching experience in overall perception, dimension 2 (students' describe and explain their answers), and dimension 3 (students' mathematical reasoning and proof). However, no significant differences were observed in perceptions of dimension 1 (students' multiple paths to solutions) and dimension 4 (students' ability to consider the validity of solutions). These findings suggest that the level of math teaching experience may influence how teachers perceive certain aspects of students' MW abilities, particularly in dimensions related to describing, explaining, and providing mathematical reasoning and proof. Further research and exploration of the reasons behind these differences could be beneficial for improving mathematics education in Abu Dhabi Emirate schools. When students engage in MW, the aforementioned difficulties become more pronounced. Casa et al. (2016) identified four types of MW: exploratory, mathematical creative, argumentative, and informative/explanatory. While each type of MW is beneficial to students, informative/explanatory MW is the most commonly used (Hughes et al., 2016; Powell et al., 2017). Many states, like Pennsylvania, incorporate informative/explanatory MW as open-response word problems, requiring students to solve a word problem and provide explanations.

A study examining the MW characteristics of grade 4 students by Hebert and Powell (2016) found that students typically wrote less than one paragraph for each MW task and often omitted introductions or conclusions. Moreover, students included only a fraction of mathematics vocabulary terms in their responses, showing limited use of relevant mathematical language.

Students with learning difficulties in mathematics, as noted by Hughes et al. (2016), face significant challenges when writing mathematically. Like Hebert and Powell's (2016) findings, these students often struggle to include introductory and concluding statements in their MW. Additionally, they find it challenging to organize mathematical ideas coherently and describe mathematical relationships between quantities. The authors also observed various errors in the participants' mathematical reasoning.

Given these findings, there is a pressing need to evaluate empirical evidence for MW interventions. The difficulties faced by students in MW underscore the importance of developing effective instructional strategies and interventions to support their mathematical communication and reasoning skills. Further research in this area is crucial to inform evidence-based approaches that can effectively enhance students' MW abilities.

the study found that there were no statistically significant differences in mathematics teachers' overall perceptions of MW and in their perceptions of dimensions 1, 3, and 4 based on their qualifications. However, there were statistically significant differences in their perceptions of dimension 2 (students' ability to describe and explain their answers). Teachers with a master's degree had higher perceptions in this dimension compared to those with a bachelor's degree. These findings suggest that teachers' qualifications may influence their perceptions of certain aspects of students' MW abilities, particularly in terms of describing and explaining their answers. Further research and exploration of the reasons behind these differences could provide valuable insights for mathematics education in Abu Dhabi Emirate schools. This study is consistent with the scarcity of evidence for content-area writing in mathematics. Despite the emphasis by the National Council of Teachers of Mathematics (2000) and the National Governors Association Center for Best Practices, & Council of Chief State School Officers (2010) on the importance of written communication of mathematical reasoning in the curriculum, there is a lack of peer-reviewed publications on this topic. Moreover, teachers often lack familiarity with how to effectively teach MW in the classroom, as highlighted by Cohen et al. (2015). The study's positive findings regarding mathematics teachers' perceptions of their students' MW abilities provide valuable insights into this underexplored area and contribute to the understanding of the role of MW in the teaching and learning of mathematics. It underscores the importance of further research and teacher training to promote and develop students' MW skills in the field of mathematics education. This result contradicts the findings of Rayani and An-Nafish (2019), who discovered no statistically significant differences related to the effect of years of service on skills. Additionally, it differs from Abdullah and Jaafar (2017), who observed a statistically significant difference between the M scores of mathematics teachers for teaching performance and MW skill, favoring those with five years or more of experience.

The contrasting results may be attributed to several factors. Female teachers with less experience may have recently completed their university studies, and educational programs and courses are continually evolving to align with current trends and advancements. Vocational training programs (in-service) may also focus on supporting new and less experienced female teachers to enhance their expertise and teaching practices in accordance with the curriculum requirements and educational developments.

Furthermore, teachers with less than 10 years of experience may be subject to professional competency tests or professional licensing as prerequisites for working in the educational field and for professional advancement. Additionally, technological advancements and the availability of applications and programs that support professional development could positively impact the practices of less experienced female teachers. These teachers may have a greater interest in technology and find it easier to adapt to it compared to older generations.

This is corroborated by the findings of Khalil et al. (2022), who observed statistically significant differences in teachers' intention to use technology in their teaching, favoring those with less than 10 years of experience. These factors collectively contribute to the variations in the results of studies concerning the influence of years of service on teachers' skills and practices.

CONCLUSIONS

This study provides valuable insights into mathematics teachers' perceptions of students' MW abilities in Abu Dhabi Emirate schools, shedding light on their confidence in students' skills and the influence of demographic factors on these views. Teachers generally perceive students as capable across the four dimensions of MW–exploring multiple paths to solutions, describing and explaining answers, demonstrating reasoning and proof, and evaluating solution validity. This optimism suggests a strong foundation for fostering mathematical communication, which is critical for deepening students' conceptual understanding and problem-solving skills. Notably, teachers expressed the highest confidence in students' ability to explore diverse solution paths, indicating that curricula in Abu Dhabi schools effectively encourage creative approaches to mathematics. However, the relatively lower confidence in students' ability to articulate explanations highlights a potential area for instructional improvement, particularly in developing clear, precise mathematical communication.

Demographic factors, including gender, school type, teaching experience, and qualifications, shape teachers' perceptions in nuanced ways. Variations by gender and school type suggest that female teachers and those in private schools tend to hold more positive views in certain dimensions, possibly due to differences in teaching environments or pedagogical approaches. Similarly, more experienced teachers and those with higher qualifications exhibit stronger perceptions in specific areas, such as explaining answers and reasoning, reflecting the influence of professional expertise. These differences underscore the need for tailored professional development to align perceptions across diverse teacher groups, ensuring consistent instructional support for students' MW. For instance, targeted training for less experienced teachers or those in public schools could enhance their confidence in students' abilities, fostering more equitable educational practices.

The findings contribute to the broader field of mathematics education by highlighting the role of teacher perceptions in shaping instructional strategies and student outcomes. By identifying strengths and gaps in perceptions, this study informs curriculum designers and policymakers about areas where resources and training can enhance MW instruction. Future research could explore how these perceptions translate into classroom practices and their direct impact on student performance, particularly in underrepresented dimensions like explanatory writing. Additionally, addressing the variability in perceptions by school type and experience could guide the development of collaborative professional learning communities to share best practices across Abu Dhabi's diverse educational contexts. Ultimately, this study underscores the importance of nurturing teachers' positive perceptions to empower students' mathematical communication and critical thinking skills.

Implications

The findings of this study have several important implications for mathematics education in Abu Dhabi Emirate schools and beyond:

1. **Curriculum development:** The positive perceptions of teachers regarding students' MW abilities highlight the importance of integrating MW tasks into the curriculum. By providing students with opportunities to express their mathematical reasoning and problem-solving processes through writing, educators can foster deeper understanding and critical thinking skills.
2. **Instructional practices:** Teachers can use their positive perceptions of students' MW abilities to inform their instructional practices. Encouraging and scaffolding students' MW can help them become more confident and proficient in communicating their mathematical ideas effectively.
3. **Teacher professional development:** Understanding the differences in teachers' perceptions based on gender, school type, and teaching experience can inform the design of targeted professional development programs. Providing training on effective strategies for teaching MW can benefit teachers in all educational settings.
4. **Student support:** The study's findings can guide educators in identifying students who may need additional support in developing their MW skills. By addressing individual learning needs, teachers can create a more inclusive learning environment that supports the academic success of all students.

5. **Research in mathematics education:** This study adds to the body of research on MW and its implications for students' learning and achievement. Further research in this area can explore additional factors influencing teachers' perceptions and the long-term impact of interventions aimed at improving students' MW abilities.

Limitations

It is essential to acknowledge the limitations of this study to interpret the findings appropriately:

1. **Sample size:** While the study included a substantial number of mathematics teachers, the sample size may not fully represent the entire population of mathematics educators in Abu Dhabi Emirate schools.
2. **Generalizability:** The findings of this study are specific to Abu Dhabi Emirate schools and may not be directly applicable to other educational settings or regions with different cultural and contextual factors.
3. **Self-report bias:** The data collected through the questionnaire relied on self-reported perceptions of teachers, which may be influenced by personal biases or social desirability.
4. **Cross-sectional design:** The study utilized a cross-sectional design, which limits the ability to establish causal relationships between variables.
5. **Qualitative insights:** While qualitative data were collected, the study's focus was primarily on quantitative analysis. A more in-depth exploration of teachers' perceptions through qualitative methods could provide richer insights into their experiences and beliefs.
6. **Lack of student perspectives:** The study focused on teachers' perceptions of students' MW abilities. Including students' perspectives could offer a more comprehensive understanding of the dynamics of play in the classroom.

Despite these limitations, this study contributes valuable insights into the perceptions of mathematics teachers regarding their students' MW abilities. The implications can guide educational stakeholders in promoting effective mathematical communication and problem-solving skills, thus fostering a deeper understanding of mathematics among students in Abu Dhabi Emirate schools.

Recommendations

Based on the findings and implications of this study, the following recommendations are suggested to enhance students' MW abilities and improve mathematics education in Abu Dhabi Emirate schools:

1. **Integrate MW in the curriculum:** School administrators and curriculum developers should incorporate MW tasks across various grade levels and mathematical topics. This integration should be aligned with learning objectives and can include assignments that require students to explain their problem-solving strategies, provide mathematical reasoning, and present proofs.
2. **Professional development for teachers:** Provide targeted professional development workshops and training sessions for mathematics teachers focused on effective strategies for teaching and assessing MW. Teachers can benefit from learning how to scaffold writing tasks, provide feedback, and foster a supportive environment for students' MW development.
3. **Encourage collaboration and peer review:** Teachers should encourage collaborative activities where students work together to solve problems and engage in peer review of each other's MW. Peer feedback and discussions can provide students with valuable insights and improve their communication skills.
4. **Supportive resources and materials:** Schools should invest in resources and materials that support students' MW development, such as writing prompts, rubrics for assessment, and model solutions that demonstrate effective MW.
5. **Longitudinal studies:** Conduct longitudinal studies to assess the long-term impact of interventions aimed at improving students' MW abilities. Tracking students' progress over time can provide valuable insights into the effectiveness of different instructional approaches.
6. **Student workshops and writing centers:** Establish student workshops and writing centers specifically focused on enhancing MW skills. These workshops can offer students opportunities for guided practice, feedback, and revision of their MW.
7. **Inclusion of diverse perspectives:** Include diverse perspectives in the study of MW, including insights from students, parents, and other stakeholders. Understanding the various perceptions and experiences can lead to a more comprehensive understanding of the factors influencing students' MW abilities.
8. **Research on gender and writing:** Conduct further research to investigate the factors contributing to the observed differences in teachers' perceptions based on gender. Understanding these differences can lead to strategies that promote equitable and inclusive practices in mathematics education.
9. **Cross-cultural studies:** Extend research beyond Abu Dhabi Emirate schools to other regions and

cultural contexts to gain a broader understanding of the factors influencing MW abilities and how they vary across different educational settings.

10. **Ongoing assessment and feedback:** Implement ongoing assessment and feedback mechanisms to monitor students' progress in MW. Regular feedback can help students identify areas for improvement and track their growth over time.

By implementing these recommendations, educators and educational policymakers can create a supportive and enriched learning environment that fosters students' MW abilities, ultimately contributing to their overall success and proficiency in mathematics.

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