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Mathematics through the 5E Instructional Model and Mathematical Modelling: The Geometrical Objects

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ABSTRACT
The aim of this research is to investigate the effect of education on the mathematical achievement, problem-solving skills and the views of students on the 5E instructional model and the mathematical modelling method for the "Geometric Objects" unit. The students were randomly selected from the 8th grade of a secondary school in Northern Cyprus. One group was the experimental group to which the 5E instructional model applied, and mathematical modelling was applied to the other. As a data collection tool, the "Geometrical Objects Multiple Choice Achievement Test" was applied to the experimental groups. As a result of statistical analysis, it was seen that the teaching provided by the 5E Instructional Model in Experimental group 1 and the Mathematical Modelling Method in the Experimental group 2 increased the academic achievement of the students; however, the mathematical modelling method was more successful in the mathematical achievement and problem-solving skills of the students.

Keywords: geometric objects, 5E instructional model, mathematical modelling, problem-solving skills

INTRODUCTION

Today, every country is questioning its systems to find effective solutions to the problems faced in the field of education, and discussing how to solve these problems with new structures. We can often see that the problems encountered in teaching practice, especially in schools, are derived from traditional methods. However, in recent research, it has been shown that new teaching approaches are more effective than the traditional teaching approaches which have been pushed away, and thus the search for new teaching approaches which are more effective. Unfortunately, it is not possible to ignore it, as many schools today use traditional teaching approaches. Such problems have prompted researchers and educators to develop more efficient and effective teaching practices (Huang and Shimizu, 2016).

The changing living conditions in the world change the type of human being needed. For this reason, people who know themselves and their surroundings well, and how and what they feel about themselves, are required. The way of raising such individuals runs through the understanding of new education, aiming at resolving problems, seeing relationships within them and establishing cause-effect relationships between events (NCTM, 2016).

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State of the literature

- Researchers recommend that mathematics teaching is effective when students actively participate in learning process.
- One of the approaches students participate in the active learning process is the 5E instructional model.
- The mathematical modelling method makes students understand the connection between real life and mathematics better.

Contribution of this paper to the literature

- Teaching provided by the 5E Instructional Model and the Mathematical Modelling Method increased the academic achievement of the students.
- The mathematical modelling method was more successful in the mathematical achievement and problem-solving skills of the students.
- The teaching provided by the 5E Instructional Model and the Mathematical Modelling Method increased the academic achievement of the students.

Mathematics is a science, whose reflection we sometimes see directly in our lives and sometimes use to gain meaning in our lives. Therefore, mathematics, which affects our lives, has great importance in our schools as a lesson. For this reason, it is necessary to carry out mathematics' courses in a way that will give students the ability to solve real-life problems. It may be useful to consider what mathematical concepts are related to events we encounter in our daily lives and to present them as problem situations (Güzel, 2016). Mathematics' education provides individuals with a language that will help them understand the physical world and social interactions. When we think of our changing world, the need to use and understand mathematics in everyday life is constantly increasing and has become important. Therefore, individuals who understand and do mathematics have more choices when they shape their future (MNE, 2009).

As new information is found in all areas of life, it leads to some innovations in mathematics' education and teaching. Since 2004, we have been confronted with a constructivist approach, instead of a behaviourist approach, that is part of these innovations and is reflected in our curriculum. The constructivist approach in the new programme emphasizes learning outcomes and cognitive development, rather than behaviour (Ultay and Calik, 2016). The learning areas at the heart of the programme are built on concepts and relationships. This understanding shapes geometry, which is a natural field of mathematics' education.

This area of mathematics conveys a variety of conceptual associations, enabling the development of the logical and intellectual skills of students (NCTM, 2016). It also contributes to the development of many mental skills, such as mathematical, geometric and spatial thinking. The efficient use of the geometric world surrounding the human being also depends on understanding the concept of this area. We can list all of these as the reasons why geometry discipline has a large place in educational programmes.

Nurseries, which have become important educational institutions in EU countries in recent years, have been selected as targets in the development of early childhood regarding mathematical thoughts, and geometric forms that are included as an area of mathematics learning skills. Students begin to meet with geometric forms during childhood, and then, like in geometry, they understand that geometric shapes are the nature itself. While there are numerous studies in the field of mathematics in the literature, we can easily see that geometry, which is a subdivision of mathematics, is a step behind. The majority of studies are on the teaching of geometric shapes (Thiel, 2010).

Through new approaches, students are expected to establish relations between concept representations, whilst learning geometry. However, this may not always be the case from an educational perspective (Wernet, 2017). Fyfe et al. (2015) suggested that geometric concepts are the main problems behind the most important troubles when teaching mathematics. It is possible to see perspectives, such as misconceptions, conceptual errors, relational disorders and information deficiencies, at almost every level. When these perspectives and the importance of geometry education are taken into consideration, how to overcome the problems that may arise during the process can be understood.

Problem-solving skills, which are amongst the aims of mathematics' lessons in primary education, have an important place. Problem-solving is not only part of the purpose of mathematics' lessons, but also among the common goals for all lessons. For this reason, increasing the success of problem-solving, through the problem and problem-solving structure, is a subject that has been studied by many educators (Toksoy and Akdeniz, 2017). To teach students how to solve problems and to improve their problem-solving skills, the process of problem-solving must be well known. Determining the steps students take during this process will provide data on how they can be helped (Ağaç and Masal, 2012).

While students are evaluating problem-solving skills in mathematics, it is possible to characterize problem-solving, process, concept and process-concept knowledge (Baki and Taliha, 2004). In order for problem-solving skills to be good, we have to characterize both operational and conceptual knowledge. In other words, it interprets the logic of the result of the solution obtained as a result of understanding, using, writing, abbreviating, simplifying symbols and expressions, solving the equation after turning the problem into an equation, associating the connections between them and transforming them into another connection (Gümüş and Umay, 2017).

Problem-solving skills, one of the meta-cognitive characteristics, are not sufficient to reveal the academic achievement of students in mathematics' education, but perceptions within affective traits help to establish this relationship. Thus, dealing with cognitive features as well as affective characteristics will help both to understand the current situation and to predict future behaviour (Usta, 2013).

Cognitive field studies show that learners who are actively involved in the learning process learn better. For this reason, students should be taught the source of information and how to access it, how to evaluate it and how to use it to solve problems. It has been shown in many studies that the 5E instructional model and the mathematical modelling method, which are approaches under the constructivist approach, are effective in gaining these skills (Kaymakci, 2016; MNE, 2013).

Studies in the literature show that the 5E Instructional Model was mostly implemented in the field of science, but today it has become an important model in which the field of mathematics is included and studied. In the majority of the studies, the 5E Model was used extensively in international mathematics education and the effects of students on scientific process skills were investigated (Bybee, 2009). The 5E Model, which started its historical development with the question of "How People Learn", has become an exemplary model of education institutions, especially for science and mathematics education. These are Volusia Country Schools, which have Mathematics Florida Standards.

"Researchers recommend that mathematics teaching is effective when students actively participate in learning process, so mathematics teachers should not use explanatory teaching approaches but should use reconnaissance, manual activities and interactive group works so as to encourage students to learn better. One of the approaches students participate in the active learning process is the 5E instructional model" (Runisah, Herman and Dahlan, 2016).

The stages of the 5E learning cycle model are to engage, explore, explain, elaborate and evaluate. This model was developed by Bybee (2009). This model is called the 5E instructional cycle model as each stage starts with the letter 'E'.

At the beginning of the lesson of the 5E model, the teacher begins with an interesting warm-up activity. The aim here is to encourage students to produce different ideas and ask questions. At the second stage, which is the discovery stage and is student-centred, is an opportunity for students to organize what they have found by developing hypotheses. The next stage contains explanations that the teacher will make and this stage is a guide to the fourth stage, which includes an explanation in depth. The final stage of evaluation is the stage in which both teachers and students evaluate the student's assessment. This stage encourages students to evaluate their own level of understanding and ability. It is possible to combine teaching with computer-assisted education and group work for positive results; this is recommended by many researchers (Eisenkraft, 2003; Sünbül, 2010).

Today, mathematical modelling method is not only used in mathematics education but also in various sciences from health to medicine. When the literature is examined, it is seen that the mathematical modelling is a student-centered method and includes a dynamic process which is carried out in the form of group work (Chapman, 2007; Carreira and Baioa, 2011). According to Bonotto (2007) and Blum (2002), the mathematical modelling method makes students understand the connection between real life and mathematics better. However, studies have shown mathematical modelling is a method that is not known by teachers well (Frejd, 2012).

Mathematical modelling is defined as a periodic cycle in which real-life problems are abstracted, mathematized, solved and evaluated. Modelling problems consist of open-ended questions and these problems include simulation and applied problem-solving (Haines and Crouch, 2007). Therefore, mathematical modelling provides a regular and dynamic method that reduces the gap between mathematics and real life (Ortiz and Dos Santos, 2011).

In much research, we can see that mathematical modelling is an important model in mathematics' education. In addition, the lack of teachers' knowledge about the mathematical modelling method and their lack of experience in new teaching approaches leads them to not use this method in class (Zeytun et al., 2017). However, this model should be part of mathematics' education. It is not possible for mathematics' teachers to develop topics and enrich their activities without knowing this model (Park, 2016).

Although the terms 'model' and 'modelling' look the same, they express different meanings. Whilst the term 'modelling' refers to a process, the term 'model' refers to a product emerging as the result of modelling (Ozturan, 2010). According to Lesh and Fennewald (2010), the model is used to describe a different system of interest in a particular purpose.

The concepts of mathematical models and mathematical modelling are also confusing. Mathematical models are the conceptual tools, which are necessary for individuals to interpret mathematical problems they encounter (Cetinkaya et al., 2016; Kertil, 2008; Sinclair et al., 2016). According to Meyer (1984), mathematical models are mathematical conceptual parts like variables, constants, functions, equalities, inequalities, formulas and graphs. Mathematical models can sometimes be represented by words, symbols, tables, pictures, diagrams or concrete shapes. Mathematical models, for example, are mathematical expressions like functions or equations for such as population growth, supply-demand, the rate of a falling object and the prediction of a baby's life-span (Guzel, 2016; Stewart, 2007). That is, mathematical models are abstract words that represent mathematical terms (Hestenes, 2010). The strong shareable and reusable models used by mathematicians are the most important cognitive objects of mathematics' education (Lesh and Yoon, 2007).

According to Voskoglou (2006), the mathematical modelling process can be performed in five stages in class. First, in order to solve a problem involving mathematical modelling, we always start with the analysis of the problem and follow the model's solution and mathematical steps. If the mathematical relation does not allow the problem to be solved, it is returned to the mathematical stage, in order to change the model and the model is solved. After this solution, the validity of the model is determined and interpreted to check the validity of the model. This interpretation ends by interpreting the results of the real system and the mathematical results.

Mathematical competence, in the information society of the 21st century, is necessary for personal satisfaction, active citizenship, social inclusion and employability. Concerns about low student performance in international research led to the adoption of a benchmark in basic skills across the EU in 2009. This criterion indicates that the proportion of young people aged 15 who lack skills in reading, mathematics and science should be less than 15% by 2020 (Androulla Vasilliou, 2011). We should identify these obstacles and problem areas if we want to eradicate low student performance.

When we follow the national policies monitored, the renewal of the mathematics curriculum, *the promotion of innovative teaching methods and evaluation*, and the development of teacher education, are indispensable. To carry out such a study with 8th graders (aged 15), who are at the last grade of compulsory education, is of utmost

Table 1. Experimental research design

Group	Pre-test	Experimental Process (Implementation)	Post-test
Experimental Group 1	T_1	5E Model	T_1, T_2, T_3
Experimental Group 2	T_1	Mathematical Modelling Method	T_1, T_2, T_3

T1: Geometric Objects Achievement Test (GOAT).

T2: Ministry of Education Problem Solving Skills Assessment Form

T3: Semi-structured Student Interview Form

importance, aiming at this important emphasis and shedding light on the problems in geometry, which is a branch of mathematics. It was tried to discover whether the 5E model or the mathematical modelling method, which are two important teaching approaches belonging to the constructivist approach, is more effective in overcoming these problems. It is thought that it has a key role for many educators, in correcting the existing disruption in mathematics, by trying to find an answer to the question: "Is the 5E model, which put students into active learning processes or the mathematical modelling which is a dynamic process that reduces the gap between mathematics and real-life, an alternative solution to remove low student performance?"

Purpose of the Research

This study was based on the 5E Instructional Model and Mathematical Modelling methods. Without changing the main structure of the model and method, various activities were added to the stages of the 5E model, and materials used in everyday and real-life problems were added to the mathematical modelling method by the researchers. In this study, the 5E Instructional Model was applied to the 1st group from two independent experimental groups, whilst the mathematical modelling method was used in the 2nd group. In order to increase the interaction and competition between the students, groups of five were formed in both experimental groups. All the activities were prepared to a plan in which course notes were taken in order for the geometric objects to be presented more efficiently. An answer was investigated to reply to the question, "How have this lesson and the two different practices that have been taught, affected the academic achievement, problem-solving skills and views of the students?"

METHOD

As a model in the study, the experimental method with a pre-test and post-test control group was used. The experimental design of the study is the experimental method with a pre-test and post-test control group. Tests applied to the students in Experiment group 1 and Experiment group 2 in the study before and after the experiment are shown in **Table 1**.

In the development of GOAT within the scope of validity, five mathematics' teachers (n = 5) were shown the GOAT, in order to decide whether it was appropriate for the students' levels or not. Each correct answer was scored as "1 point" in the scoring of the achievement test in the item analysis process. "0 point" was given to incorrect answers or blank ones. Thus, the total score a student received from the test constituted the number of items he gave correctly.

The preliminary applications of the academic achievement test in the study were carried out on a group of 150 people with similar characteristics who were asked 35 questions from the given questions and the necessary statistical analyses were undertaken by forming a group of 30 for the upper group and a group of 30 for the lower group for each test. As a result of these statistical analyses, 5 questions with discriminative power (r) $r \leq 0.19$ were eliminated and it was decided to use 30 valid and reliable questions in the achievement test.

The KR-20 reliability coefficient obtained for the test reliability was found to be 0.936 by analysing the scores obtained from the item analysis and the test results of the application. At the end of the experimental study, 5 volunteer students from each group formed Experimental group 1 and gave their opinions about the 5E Model and the Experimental group 2 gave their opinions about mathematical modelling. Responses from students were analysed through content analysis, by giving codes to the students.

Table 2. Personal information of the students from experimental groups

	Gender				
	Girl		Boy		
Group	**N**	**%**	**N**	**%**	**Total**
Experiment 1	16	54	14	46	30
Experiment 2	18	60	12	40	30
Total	34	57	26	43	60

Table 3. Geometric Objects Achievement Test Specification Table

Learning Outcomes	Question Number
Student builds the prism, identifies the basic elements and draws the surface angle.	1, 7, 15, 17, 19, 20
Student relates the surface area of the right prisms.	14, 22
Student forms the volume of the right prism.	21
Student identifies and builds a cross section of a plane with a geometric object.	18, 30
Student classifies multiple faces.	6
Student builds pyramids, identifies the basic elements and draws the surface angle.	23, 24
Student forms the volume and surface area of the right pyramid.	10
Student identifies all the elements of the cone and draws the surface angle.	25, 26, 27, 28
Student identifies and builds the basic elements of the sphere.	29
Student forms the surface area and volume relation of the vertical circular cone.	2, 12
Student forms the surface angle of the sphere.	5
Student forms problems and solutions about the angles of geometric objects.	13
Student predicts surface areas of geometric objects by using strategy.	16
Student forms volume of the sphere.	3
Student forms and solves problems about volumes of geometric objects.	4, 8, 11
Student predicts the volume of geometric objects by using strategy.	9

Participants

8th grade students, who study in a state secondary school in Northern Cyprus, constituted this study's sampling. The participants in the research were randomly selected from this sampling group in a secondary school affiliated to the Ministry of Education in the 2016-2017 academic year; 60 students (aged 15), 30 of whom were in the Experimental group 1 while the other 30 were in the Experimental group 2, took part in the study. Students in Experiment 1 and group 2 consisted of 10 weak, 10 moderate and 10 good students. All of these students did not know anything about the GC unit. In **Table 2**, personal information about the gender of the students in the experimental groups is given.

In the experimental study period, the students of the experimental groups in groups of 5 participated in the activities. Consultation was undertaken where necessary. The Experiment 1 group was taught by the 5E instructional model, according to the lesson plans. Prior to the research, appropriate lesson plans were prepared for these models and these prepared plans were checked by the experts and necessary amendments were made. In Experiment 2 group, lesson plans were prepared according to the mathematical modelling method. Prior to the research, the same procedures as for Experiment 1 were applied. The lessons lasted for 7 weeks (7x4 = 28 hours) for both experimental groups.

Data Collection Tools

The Geometrical Objects Achievement Test (GOAT) was used as a data collection tool by the researchers, in order to measure the academic achievement level of the students in the geometric objects unit. **Table 3** shows the GOAT specification table. The achievement test was developed by the researchers to select the most appropriate questions from a range of resource books, even from grade 2 of primary education, after a long-term research.

After the experimental study, the "Ministry of Education Problem Solving Skills Evaluation Form" was used to reveal the effect of the 5E model and the mathematical modelling method on the problem-solving skills of the students. According to this form, whilst evaluating the students' problem-solving skills, the criteria for understanding the problem, using problem-solving strategies, solving the problem, controlling the accuracy of the result, analysing the problem solution, establishing the problem, expanding the problem, trying to solve the problem, confidence in problem-solving and to like problem-solving, were taken into consideration. These criteria were rated 1 = Never, 2 = Rarely, 3 = Sometimes, 4 = Frequently and 5 = Always. The reliability coefficient of the form for this research was found to be Cronbach Alpha = 0.928 after applying it to the 60 students from both groups.

During the development phase of the test, the validity and reliability process was performed and item analysis was carried out. For the validity of the test, content and face validity were applied and for the reliability KR-20 coefficient was applied. In order to ensure the content validity, the achievement test was prepared in such a way that it has a question for each learning outcome in the unit, and the cognitive level of the questions was determined by the researchers and shown on the specification table. The first draft of the achievement test included 35 multiple-choice questions and then a question pool was established with the questions for each learning outcome in the unit; after that, feedback was obtained from the experts in order to ensure face validity. According to Tavsancil (2005), this kind of validity, which is generally evaluated within content validity, is an expert opinion on the measurement of a characteristic of a measuring instrument, and its validity level cannot be determined by numerical values. The preliminary application of the re-established achievement test, in line with the opinions of the experts, was carried out on 150 ninth grade students who were studying at a randomly selected secondary school in Nicosia. Then, in order to determine the differential feature of the questions in the academic achievement test, students were divided into two groups according to their scores, with the most 27% and the least 27%, and the questions whose differential feature was lower than 25, was subtracted from the test. The items whose differential strength is below 20 must be thrown away; the items between .20 and .40 need to be corrected and the items above 40 are very good ones (Tan, 2005). In addition, the difficulty of the choices of the questions in the test was determined and the items below .30 and above .70 were removed from the test. According to Ozguven (1998), the difficulty of the choices of a question is the percentage of respondents correctly choosing the correct choice in the tested group. The choice becomes difficult as the choice strength approaches 0.00, and the choice is interpreted as easy when it approaches 1.00. Choices whose difficulty is around 0.50 are preferred whilst developing a test, and it is important to find out the one who knows or the one who does not know and its reliability must be high. (Tan, 2005). Finally, the questions below .20 in item total score correlation were removed from the test to improve internal consistency. The questions, whose item total score correlations were from .20 to .30, were revised. Buyukozturk (2012) states that, in general, item-total correlations .30 and higher items differentiate individuals well, items that are between .20 and .30 may be discarded if necessary, and items lower than .20 should not be used in the test. The final draft of the test consists of 30 multiple-choice questions. The KR-20 reliability of scale is calculated as .93. Reliability is the degree to which measures are free from error and therefore yield consistent results (Ozguven, 1998). The difficulties of the questions in the test range from .320 to .650. The mean strength of the test was set at .465. The highest score to be taken from the test is 30; the lowest score is 0.

DATA ANALYSIS

The results of the Shapiro-Wilks test ($p > 0.05$), which was used to test whether there was a difference between the GOAT scores of the students in Experiment 1 (5E Instructional Model) and Experiment 2 (Mathematical Modelling Method), showed a normal distribution so independent and paired samples t-test from the parametric tests were applied. Also, independent groups t-test was also used for Statistical Analysis of Problem Solving Skills Evaluation Form. The responses to the semi-structured student interview form, which constitutes the qualitative dimension of the study, were reported by analysing the content. The views of the students were tabulated in line with the codes, which were given to the students.

Büyüköztürk (2017) states that two-way repeated measures ANOVA is a suitable and powerful technique used to analyse if there is an interaction between these two factors on the dependent variable, before and after the experiment. Two-way repeated measures ANOVA design was conducted to analyse the data, in order to compare

Table 4. Independent samples t-test results regarding GOAT pre-test scores of students in Experiment 1 and Experiment 2 groups

Measurement	N	$\bar{X}$	Sd.	df	t	p
Pre-Test (Experiment 1)	30	9.02	2.098	58	0.176	0.861
Post-Test (Experiment 2)	30	9.56	2.236			

Table 5. Independent samples t-test results regarding GOAT post-test scores of students in Experiment 1 and Experiment 2 groups

Measurement	N	$\bar{X}$	Sd.	df	t	p
Post-Test (Experiment 1)	30	51.36	1.651	58	6.045	0.000
Post-Test (Experiment 2)	30	68.13	2.228			

Table 6. Paired samples t-test results regarding GOAT pre-test scores of students in the Experiment 1 group

Measurement	N	$\bar{X}$	Sd.	df	t	p
Pre-Test (Experiment 1)	30	9.02	11.49	29	-14.063	0.000
Post-Test (Experiment 1)	30	51.36	9.04			

the mean difference, to measure whether there was any improvement in the psychological variables and stages of change during intervention in the experimental group.

FINDINGS

In this section, the data obtained by the data collection tools were analysed. Independent samples t-test results for GOAT applied as a pre-test before experimental procedure are shown in **Table 4**.

When **Table 4** is examined, it is seen that the two average ($\bar{X}$) values are very close to each other. It was found that, after checking whether there was a statistically significant difference between these means or not, according to the calculated t-value and significance level measured by t-test for independent samples ($p > 0.05$), there was no significant difference between the groups. The t-test results of the GOAT independent samples applied as post-test in both groups after the experimental procedure are shown in **Table 5**.

When **Table 5** is examined, it is seen that the two averages are quite different from each other. A statistically significant difference was found between the averages and the t-test for the independent samples ($p < 0.01$). There was a significant difference between the groups in favour of the Experimental group 2.

After determining that Experiment group 2 was more successful than the GOAT, according to the post-test scores of Experiment 1 and Experiment 2 groups, it was attempted to reveal the relationship between the pre-test and post-test scores in Experiment 1 and Experiment 2 groups. Results of the pre-test and post-test for GOAT of students in Experiment 1 and Experiment 2 groups are shown in **Table 6** and **Table 7**.

According to **Table 6**, the two mean values are quite different to each other. As a result of paired samples t-test analysis for dependent groups, the calculated t-value and significance level ($p < 0.01$), there was a significant difference between the pre-test and post-test scores of Experiment group 1 in favour of the post-test scores of GOAT.

When **Table 7** is examined, it is seen that the two mean values are quite different to each other. As a result of paired samples t-test analysis for dependent groups, the calculated t-value and significance level ($p < 0.01$), there was a significant difference in favour of the Experiment group 2, in terms of between the pre-test and post-test scores of Experiment group 2 of GOAT.

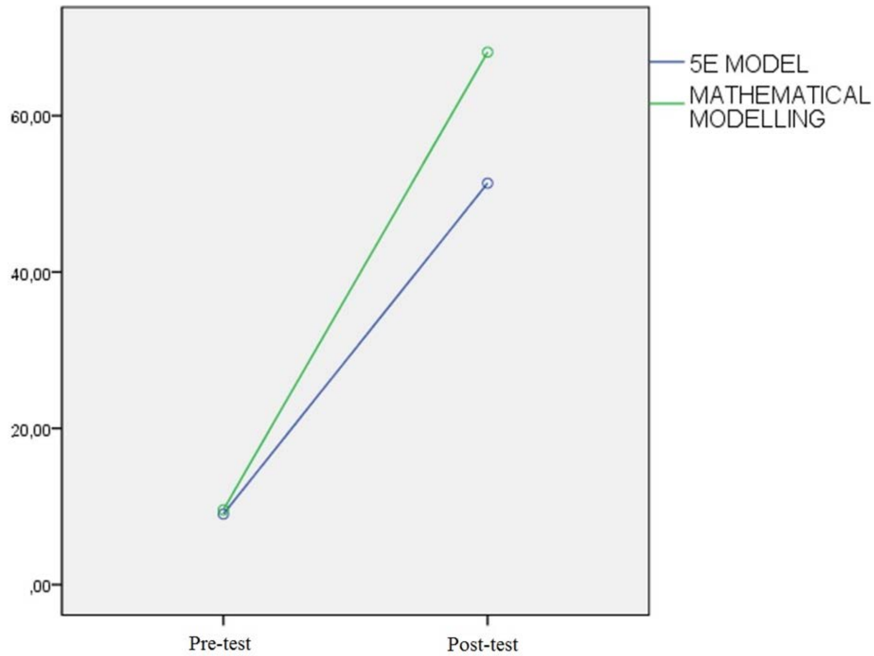

Figure 1. Group intervention and categorical variables based line chart of students

Table 7. Paired samples t-test results regarding GOAT pre-test scores of students in the Experiment group 2

Measurement	N	$\bar{X}$	Sd.	df	t	P
Pre-Test (Experiment 2)	30	9.56	12.24	29	-15.407	0.000
Post-Test (Experiment 2)	30	68.13	12.20			

When the groups are compared, it was seen that the average of GOAT pre-test score of Experiment group 1 was 9.02 and that of Experimental group 2 was 9.56. The post-test scores of Experiment group1 are 51.36 and Experiment group 2 is 68.13. As seen here, although there is no difference in the pre-test scores of the groups, there is a significant difference in the post-test scores in favour of Experiment group 2.

The t-test for the associated samples reveals whether there is a significant difference between the two compared means, but does not give any information about the magnitude of this difference. For this reason, statistical significance, as well as the size of the effect, must be calculated (Can, 2016). Therefore, in this study, we can say that the difference between the groups is moderately close, since the effect size is calculated as Cohen's d = 0.396. The change of geometric objects' achievement pre-test and post-test scores of groups are seen in the line chart (**Figure 1**).

The two categorical variables (pre-test and post-test) were used to test the academic achievement level of the students in the geometric objects unit and group intervention (5E Model and Mathematical Modelling). The two-way repeated measures ANOVA design was conducted to analyse the data, in order to compare the mean difference, to measure whether there was any improvement in the categorical variables during intervention in the experimental Thus, it has been found that geometric objects' achievement scores of the students participating in the two instructional models has a meaningful difference from the pre-test to post-test; the common effects of being in different procedure groups and repeated measures factors are meaningful on geometric objects' achievement scores [$F(1,58)=11.197$, $p<.01$, partial $\eta^2 = .162$]. groups. Regardless of the group, all students' scores were higher on the post-test ($\bar{X}$ = 59.75) than the pre-test ($\bar{X}$ = 9.29). See **Table 8**.

Table 8. Average and standard deviation values of students' geometric objects achievement test results

	Group	$\bar{X}$	Sd.	N
Pre-test	5E Model	9.02	11.49	30
	Mathematical Modelling	9.56	12.24	30
	Total	9.29	11.77	60
Post-test	5E Model	51.36	9.04	30
	Mathematical Modelling	68.13	12.20	30
	Total	59.75	13.59	60

Table 9. Experiment 1 and Experiment 2 Groups Problem Solving Skills Assessment Scores

CRITERIA	5E Model Experimental 1 (N=30)			Mathematical Modelling Experimental 2 (N=30)			Significance Level	
	$\bar{X}$	Sd.	Assessment Results	$\bar{X}$	Sd.	Assessment Results	t	p
1.Understanding problem	3.2	0.836	Sometimes	4	0.000	Frequently	-2.138	$p<0.05$
2.Using problem solving strategies	2.6	0.547	Rarely	3.8	0.447	Frequently	-3.795	$p<0.05$
3.Problem solving	3.6	0.547	Frequently	3.6	0.547	Frequently	0.000	$p>0.05^*$
4.Controlling the solution of the problem	1.6	0.547	Never	2.6	0.547	Rarely	-2.887	$p<0.05$
5.Analysing problem solving	2	0.707	Rarely	2.2	0.447	Rarely	-0.535	$p>0.05^*$
6.Establishing problem	1.8	0.447	Never	2.8	0.447	Sometimes	-3.536	$p<0.05$
7.Expanding the problem	1.6	0.547	Never	3	0.707	Sometimes	-3.500	$p<0.05$
8.Trying to solve the problem	3.8	0.836	Frequently	5	0.000	Always	-3.207	$p<0.05$
9.Confidence in problem solving	3.2	0.707	Sometimes	4	0.447	Frequently	-2.138	$p<0.05$
10.Liking problem solving	3.2	0.707	Sometimes	4	0.447	Frequently	-2.138	$p<0.05$

Independent t-test results for the Problem-Solving Skills Assessment Form in Experiment 1 and Experiment 2 groups are shown in **Table 9**.

According to the results of the independent t-test in **Table 9**, it can be said that the students in the Experimental group 1 had higher problem-solving abilities than the students in the Experimental group 2. We see that the students in the Experimental group 2, who were taught with this mathematical modelling method, "Always" made efforts to solve the problem, whilst understanding the problem, using problem-solving strategies, problem-solving, confidence in problem-solving and liking problem-solving efforts were "Frequently". Also, they rarely used the criteria of checking the correctness and analysing the problem. On the contrary, it was found that the students in Experiment 1 group did not control the result of the problem, did not establish a problem and did not expand the problem, which were shown as 'Never'; both groups analysed the problem "Frequently" and solved the problem "Rarely" ($p> 0.05$) We can see that there is not a significant difference between them.

In **Figure 2** and **Figure 3**, the students' solutions to a problem given to both experimental groups were compared.

Example Problem 1: The radius of the circle given below is 16 cm and the slant height is 20 cm. Find the volume (π = 3).

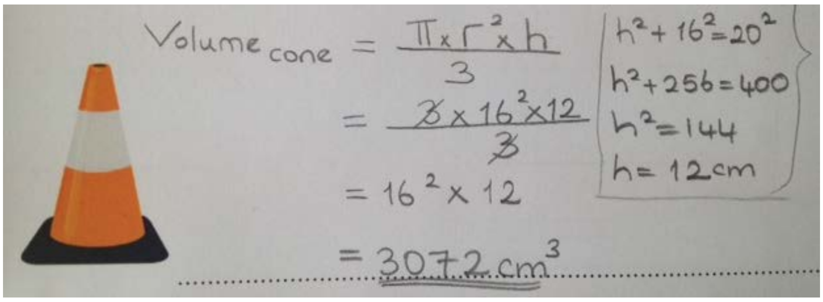

Figure 2. A Solution Evaluating Operationally for the Example Problem (Experiment group 1)

Example Problem 2: The following is an example of a cone used in physical education classes. The radius of this cone is 16 cm and the slant height is 20 cm. Find the volume of this cone (π = 3).

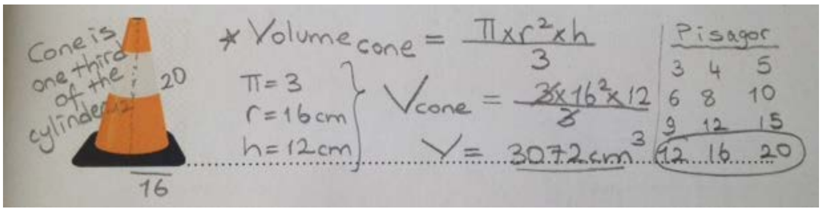

Figure 3. A solution, which was evaluated operationally and conceptually for the example problem (Experiment group 2)

Table 10. Experiment Group 1 Student Views on Model 5E

Views	Student codes (n=5)
I started to like geometry.	Ö1, Ö2, Ö3, Ö4, Ö5
My interest in maths has increased.	Ö1, Ö2, Ö3, Ö4, Ö5
My problem-solving skills have increased.	Ö1, Ö3
My geometry knowledge has increased.	Ö2, Ö3, Ö4
I know the geometric objects.	Ö4, Ö5
I learned how to calculate the surface areas and volumes of geometric objects.	Ö2, Ö5
I have learned to use a compass set.	Ö3

As can be seen in **Figures 2** and **3**, the solution of the Experiment group 1, to which the 5E model was taught, was analytically evaluated for the example problem, and the solution of the Experiment group 2, to which mathematical modelling was taught, was an operational and conceptual solution. According to Akar (2017), students emphasize that by using mathematical modelling, they can discover different mathematical structures and rules in teaching activities, produce more and different solutions, think more flexibly, and make more step by step solutions and relate them. According to this information, the findings obtained in **Figure 2** and **Figure 3** are similar to Akar's (2017).

The views obtained by asking the question from the semi-structured interview questionnaires "What features of the students have developed and what have they acquired?" regarding the 5E model (Experiment group 1) and the mathematical modelling method (Experiment group 2), which was asked of 5 volunteer students, are given in **Table 10** and **Table 11**.

According to **Table 10**, the Experiment group 1 students stated about the 5E model that their interest in mathematics grew and problem-solving skills increased; they started to love geometry, their geometry knowledge increased, they learned how to calculate surface areas and volumes of geometric objects and learned how to use a compass set.

Table 11. Experiment Group 2 Student Views on Mathematical Modelling Method

Views	Student codes (n=5)
This has shed some light on my career choice in future.	Ö6
My geometry knowledge has increased.	Ö7, Ö9
My basic geometry has developed.	Ö8
My visual point of view on the objects has improved.	Ö7, Ö10
I learned how to calculate the surface area and volume of geometric objects.	Ö6, Ö8, Ö9
My problem-solving skills increased.	Ö6, Ö7, Ö8, Ö9, Ö10
I started to understand mathematics better.	Ö7, Ö9, Ö10
Group work has increased our motivation for mathematics.	Ö6, Ö7, Ö8, Ö10

In **Table 11**, according to the views of the students of the Experiment group 2 on the mathematical modelling method, it is seen that their geometry information improved, as their geometry bases were developed; this shed some light on their career choices in future; their visual point of view on the objects and their problem-solving skills improved, as they started to understand mathematics better; they learned how to calculate the surface area and the volume of geometric objects and the group work increased their motivation for mathematics.

Discussion and Conclusion

No significant difference was observed between the results of the GOAT pre-test scores of the students in Experiment 1 and 2 groups. That is, it was determined that the students in the Experiment 1 and the Experiment 2 groups had the same information level about the subject before the experimental study.

There was a significant difference between the results of the GOAT post-test scores of the students in Experiment 1 and Experiment 2 groups. The Experiment 2 group in which the mathematical modelling method was applied was found to be more successful than the students in the Experiment group 1, when the GOAT scores including the "Geometric Objects" of the students in the two groups were observed.

There was a significant difference between the GOAT scores applied before and after the experimental study for the Experiment group 1 which was undertaken according to the 5E Model. As can be seen from the results obtained, this difference was found in favour of the post-test scores. As can be seen in the results, the students in the Experiment group 1 of the 5E model show that they are more successful in the post-test, so this model is an effective teaching method (Biber and Tuna, 2015; Wilder and Shuttleworth, 2005).

A significant difference was observed between the GOAT scores of the pre- test and post- test of the Experiment group 2 students who had their lessons according to mathematical modelling. This difference was found to be in favour of post-test scores and showed similar results with the research of Zeytun et al. (2017). According to this, it is concluded that the mathematical modelling method is more effective than the 5E Instructional Model, although it is seen that the students in the Experiment group 2 in which the mathematical modelling method is applied are more successful in the post-test when the post-test scores are examined.

The experiment using the mathematical modelling method was evaluated and found that the students in group 2 had higher problem-solving skills than the students in Experiment group 1. It was seen that Experiment group 1 students "Never" controlled the result of the problem, "Never" established a problem, and "Never" expanded the problem. However, the Experiment group 2 students "Rarely" checked the result of the problem; they "Sometimes" established and expanded the problem. In addition, whilst the Experiment group 2 students understood the problem "Frequently" and used problem-solving strategies "Frequently", it can be seen that students in Experiment group 1 "Sometimes" understood the problem and "Rarely" used problem-solving strategies. Accordingly, it can clearly be seen that there is a significantly positive correlation between academic achievement and problem-solving skills. This result shows that we reached a similar result to Toksoy and Akdeniz's (2017).

In general, the problem solutions of the students in Experiment group 2 included conceptual and operational solutions, whereas the Experimental group 1 was better at the operational solutions. This result shows that we have the same result as Akar (2017). While the Experimental group 1 tried to solve the questions "Often", group 2 tried to solve the questions "Always".

The result of "Group working has increased our motivation for mathematics", which is one of the views of Experimental group 2 revealed a similar result to Blum's (2002) and the result of "My problem solving skills increased" is similar to Fox's (2006).

When we look at student views in both models, we can see that students increased their problem-solving skills, their knowledge of mathematics and geometry, learned how to calculate the surface area and volume of geometric objects. However, we can say that the mathematical modelling method improved students' visual perceptions of objects and that this shed light on their choice of future professions, positively influencing their spatial reasoning skills, which is an integral element of geometry education. This result is similar to the work of Usta (2013).

In both groups, it was determined that students' views on mathematics and geometry were positive, they became more interested in geometry and their knowledge about geometry increased. This result shows that we have the same opinion as Sakallı (2011).

Recommendations

Teachers or prospective teachers who work in any educational institution can be informed about 5E Instructional Models and Mathematical Modelling Methods by such studies conducted by the academic staff in the universities and supplied with materials especially prepared by lecturers selected from various courses.

Such models can be given for research purposes to senior students studying at the Education Faculties of Universities and examples of courses suitable for these models can be prepared and presented as assignments, projects, etc. However, it should be ensured that these sample lectures are evaluated by knowledgeable people. Through this and similar studies, the use of the constructivist approach and the underlying 5E Instructional Model in teaching mathematics and geometry, and, in particular, recommending mathematical modelling methods in teaching geometry, will enable them to use these methods in their chosen career. It is beneficial to keep the studies published abroad in the libraries of education departments, so as to introduce them to teacher candidates by making translations of foreign publications about these subjects.

Successful implementation of the 5E Model in higher education and schools of the Ministry of Education is only possible by providing an adequate and necessary infrastructure, technical equipment, documents and materials. It would be preferable for these necessary materials to be simple, inexpensive, everyday tools that can be found in daily life, instead of being confusing and expensive tools. In addition, these materials should be provided for all schools as books, Internet sites, CDs, etc., in order to be used by all students. Resources should be provided to all schools. In the absence of such preparations, implementation of this model will be difficult and the desired results will not be achieved.

In the 5E Instructional Model and Mathematical Modelling Approaches applied in this research, intra-group cooperation and a competitive environment between the groups should be established, groups and individual exercise hand-outs should be prepared, students should be given more rights to speak and undertake activities in the classroom environment and assessment should be done by evaluating all these activities.

It is very important to relate the subjects that are undertaken in daily life in mathematics lessons, especially in geometry lessons. If the topics are not supported by vital examples, they are quickly forgotten, draw less attention from the students and so students are not interested. According to the Mathematical Modelling Method, when examples are given in lessons related to daily life and students are asked to give similar examples to these, students get encouraged. The linkage of vital example implementations and the use objects in everyday life will make students attend lessons more willingly and lovingly instead of being afraid of mathematics and geometry.

Whilst evaluating courses that are taught with the 5E Instructional Model and the Mathematical Modelling Method, students should be assessed with all the activities as a whole, in order to test what they have learned during lessons and to enable them to see their own deficiencies. Assessment questions should be addressed in writing and verbally, and these questions should be solved and discussed individually and in groups. Particularly, whilst understanding subjects and solving class study questions, this practice is beneficial.

It should not be overlooked that the preliminary knowledge of pre-instructional students is extremely important in terms of the planning of teaching activities. However, many teachers working in schools do not have enough knowledge about the different methods that can be used to detect unawareness or preliminary information or misconceptions, especially about preparing geometry lesson plans with the Mathematical Modelling Method.

Mathematics, especially geometry lessons, can play a key role in removing low student performance by using the Mathematical Modelling Method about problem-solving strategies, trying to solve problems, liking problem-solving, and having confidence in problem-solving, to improve students' problem-solving skills and structure.

We know that mathematics is one of the most problematic lessons in all the countries in the world. Especially when the mathematics curriculum is intensive, mathematics' teachers need to make the right choices to produce permanent solutions when choosing the teaching model. They can use the mathematical modelling method in the classroom environment to obtain good results in the academic achievement of the students and to improve their perceptions towards mathematics.

In mathematics' education, we can think of the teaching approach used by teachers, the academic achievement of the students and the problem-solving skills as a triangle. These three important elements are crucial points of mathematics. In mathematics' education, especially geometry education, which is a branch of mathematics, in order to eliminate the many deficiencies and problems, the mathematical modelling method can be used by educators as a key to achieve these three important points.

REFERENCES

Ağaç, G., & Masal, E. (2012). The relationship between 8th grade students' opinions about problem solving, beliefs about mathematics, learned hopelessness and academics success. *GAUN JSS*, 216-229.

Akar, Ş. Ş. (2017). *Examining mathematically gifted students' mathematical creativity through the process of model eliciting activities*. (Published PhD thesis). Hacettepe University Education Faculty, Ankara.

Baki, A., & Taliha, K. (2004). Characterizing high school students' algebra knowledge in terms of procedural and conceptual knowledge. *Turkish Education Science Journal 2*(1), 184-210.

Biber, A. Ç., & Tuna, A. (2015). The views of mathematics teachers about the 5E instructional model. *Erzincan University Journal of Education Faculty, 17*(1), 175-196.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education- discussion document. *Educational Studies in Mathematics, 51*(1/2), 149-171.

Buyukozturk, S. (2012). *Deneysel çalışmalar (Experimental designs).* PEGEM-A Publishing. Ankara.

Buyukozturk, S. (2017). *Sosyal bilimler için veri analizi el kitabı istatistik, araştırma deseni spss uygulamaları ve yorum (Data Analysis Handbook for Social Sciences Statistics, Research Design SPSS Practices and Interpretation)*. PEGEM-A Publishing. Ankara.

Bybee, R. W. (2009). The BSCS 5E instructional model and 21st century skills: A commissioned paper prepared for a workshop on exploring the intersection of science education and the development of 21st century skills. *BSCS org Retrieved January*, *26*, 2012.

Can, A. (2016). *SPSS ile bilimsel araştırma sürecinde nicel veri analizi (Quantitative data analysis in the scientific research process with SPSS)*. PEGEM-A Publishing. Ankara.

Carreira, S., & Baioa, A. M. (2011). Students' modelling routes in the context of object manipulation and experimentation in mathematics. In G. Kaiser, W. Blum, R. B. Ferri and G. Stillman (Eds.). *Trends in teaching and learning of mathematical modelling: ICTMA 14*, 211-220, Netherlands: Springer.

Cetinkaya, B., Kertil, M., Erbas, A. K., Korkmaz, H., Alacaci, C., & Cakiroglu, E. (2016). Pre-service teachers' developing conceptions about the nature and pedagogy of mathematical modeling in the context of a mathematical modeling course. *Mathematical Thinking and Learning, 18*(4), 287-314.

Chapman, O. (2007). Mathematical modelling in high school mathematics: teachers' thinking and practice. In W. Blum, P. L. Galbraith, H. W. Henn and M. Niss (Eds.). *Modelling and applications in mathematics education: 14th ICMI Study*, 325-332. New York: Springer.

Eisenkraft, A. (2003). *Expanding the 5E model*. Retrieved May 15, 2017, from http://www.its-about-time.com/htmls/ap/eisenkrafttst.pdf.

Fox, L. J. (2006). *A justification for mathematical modelling experiences in the preparatory classroom.* Paper presented at the *9th Annual Conference of the Mathematics Education Research Group of Australasia*, Canberra, Australia.

Frejd, P. (2012). Teachers' conceptions of mathematical modelling at Swedish Upper Secondary school. *Journal of Mathematical Modelling and Application, 1*(5), 17-40.

Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction, 35*, 104-120.

Galbraith, H. W. H., & Niss, M. (Eds.), *Modelling and applications in mathematics education: 14th ICMI Study*, 161-170. New York: Springer.

Gümüş, F. Ö., & Umay, A. (2017). Effect of problem solving strategies ınstruction on preservice elementary mathematics teachers' conceptual / procedural solution preferences and problem solving performance. *Elementary Education Online, 16*(2), 746-764.

Guzel, E. B. (2016). *Mathematical modeling in mathematics education for researchers, educators and students*. PEGEM-A Publishing. Ankara.

Haines, C., & Crouch, R. (2010). Remarks on a modelling cycle and interpretation of behaviours. In R., Lesh, P. L. Galbraith, C. R. Haines, A. Hurford (Eds.), *Modeling students' mathematical modeling competencies (ICTMA 13)* (145–154), New York: Springer.

Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: an international perspective. *ZDM Mathematics Education, 48*, 393-409.

Kaymakci, Y. D. (2016). Embedding analogical reasoning into 5E learning model: A study of the solar system. *Eurosia Journal of Mathematics, Science and Technology Education, 12*(4), 881-911.

Kertil, M. (2008). *Investigating problem solving ability of pre-service mathematics teachers in modeling process.* (Published master thesis). Marmara University Education Faculty, İstanbul.

Lesh, R., & Yoon, C. (2007). *What is distinctive in (our views about) models & modelling perspectives on mathematics problem solving, learning, and teaching?* In W. Blum, P. L.

Meyer, W. J. (1984). *Concepts of mathematical modeling*. New York: McGraw-Hill.

Ministry of National Education Training and Education Committee Board. (2009). *Primary school mathematics lesson; 6-8. Grades curriculum*. Retrieved May 15, 2017, from http://ttkb.meb.gov.tr/www/ogretim-programlari/icerik/72.

Ministry of National Education. (2013). *Secondary school mathematics (9-12. grades).* Retrieved May 15, 2017, from *http*://ttkb.meb.gov.tr/program2.aspx.

NCTM (National Council of Teachers of Mathematics) (2016). *Executive summary: principles and standards for school mathematics.* Retrieved May 15, 2017, from http://www.nctm.org/uploadedFiles/Math_Standards/12752_exec_pssm.pdf/.

Ortiz, J., & Dos Santos, A. (2011). Mathematical modelling in secondary education: a case study. In G. Kaiser, W. Blum, R. B. Ferri and G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA 14*, 127-135. Netherlands: Springer.

Ozguven, I. E. (1998). *Psikolojik testler (Psychological tests)*. PDREM Publishing, Ankara.

Ozturan Sagirli, M. (2010). *The effects of mathematical modelling method on derivative topic on secondary education students' academic achievements and self-regulation skills.* (Published Ph. D. Thesis). Atatürk University, Graduate School of Education Sciences, Department of Secondary Science and Mathematics Education, Erzurum.

Park, J. H. (2016). How can students generalize the chain rule? The roles of abduction in mathematical modeling. *Eurosia Journal of Mathematics, Science and Technology Education, 12*(9), 2331-2352.

Runisah Hemen, T., & Dahlan, J. A. (2016). The enchancement of students' creative thinking skills in mathematics through the 5E learning cycle with metacognitive technique. *International Journal of Education and Research, 4*(7), 347-360.

Sakallı, A. F. (2011). *The influence of constructivist 5E model about teaching complex numbers subject on students' academic achievement and attitudes.* (Published Master's Thesis). Kahramanmaraş Sütçü İmam University, Mathematics Education Faculty, Kahramanmaraş.

Sinclair, N. et. al. (2016). Recent research on geometry education: an ICME-13 survey team report. *ZDM Mathematics Education 48*, 691-719.

Sünbül, A. M. (2010). *Teaching principles and methods*. Education Publications, Konya.

Tan, Ş. (2005). *SPSS ve Excel uygulamalı temel istatistik-1 (SPSS and Excel Applied Basic Statistics-1).* PEGEM-A Publishing, Ankara.

Tavsancil, E. (2005). *Tutumların ölçülmesi ve spss ile veri analizi (Attitude measurement and data analysis with SPSS).* Nobel Publishing, Ankara.

Thiel, O. (2010). Teachers' attitudes towards mathematics in early childhood education. *European Early Childhood Education Research Journal, 18*(1), 105-115.

Toksoy, S. E., & Akdeniz, A. R. (2017). Determining students' problem solving processes via hint supported problem solving instrument. *Hacettepe University Journal of Education, 32*(1), 185-208.

Ultay, N., & Calik, M. (2016). A comparison of different teaching designs of 'acids and bases' subject. *Eurosia Journal of Mathematics, Science and Technology Education, 12*(1), 57-86.

Usta, N. (2013). *The effectiveness of the problem based learning method on secondary school students' mathematics achievement, mathematics self-efficacy and problem solving skills.* Gazi University, Mathematics Education Department, Ankara. Published PhD Thesis.

Vassiliou, A. (2011). *Mathematics education in Europe. Common challenge and national policies.* Education, Audiovisual and Culture Executive Agency P9 Eurydice. [Electronic Version]. Retrieved May 15, 2017, from http://eacea.ec.europa.eu/education /eurydice.

Voskoglou, M. G. (2006). The use of mathematical modelling as a tool for learning mathematics. *Quaderni di Ricerca in Didattica*, *16*, 53-60.

Wernet, J. L. W. (2017). Classroom ınteractions around problem contexts and task authenticity in middle school mathematics. *Mathematical Thinking and Learning, 19*(2), 69-94.

Wilder, M., & Shuttleworth, P. (2005). Cell inquiry: a 5E learning cycle lesson. *Science Activities*, *41*(4), 37- 43.

Zeytun, A. S., Cetinkaya, B., & Erbas, A. K. (2017). Understanding prospectives teachers' mathematical modeling processes in the context of a mathematical modeling course. *Eurasia Journal of Mathematics, Science & Technology Education, 13*(3), 691-722.

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