# Measures of central tendency in primary education textbooks in Chile 

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#### Abstract

This study analyzed the tasks associated with measures of central tendency proposed to Chilean students from $5^{\text {th }}$ to $8^{\text {th }}$ year of basic education. For this purpose, the activities involving mean, median, and/or mode were identified through the revision of two series of textbooks (16). The units of analysis were (1) type of measure, (2) task, (3) context, (4) support, and (5) cognitive skill. The main findings allowed us to indicate that the texts are aligned with the curricular guidelines, with tasks mostly associated with mean and mode, prioritizing calculation. Additionally, the tasks lack a context that gives meaning to the data. The support in which the data is presented is written and tabular. Finally, regarding the type of cognitive skills with the greatest presence in the textbooks, these are at a basic level.


Keywords: statistics, textbook, basic education, homework, measures of central tendency

## INTRODUCTION

In mathematics, the textbook is considered a fundamental tool in the process of studying and constructing knowledge; the organization of content along with the presentation of activities and pedagogical suggestions can be helpful for understanding the topics (Fernández-Mosquera \& Mejía, 2010; GonzálezAstudillo \& Sierra, 2004). The relevance of this resource has sparked a line of research in mathematics education (Gómez, 2011; Vargas et al., 2020; Wijaya et al., 2015) and in statistics education (Díaz-Levicoy et al., 2020). Authors such as Quispe et al. (2010, p. 112), point out that the analysis of textbooks "is considered an effective and useful way to identify the possible epistemological and methodological origins of students' learning problems".

The textbook allows for the implementation of the official curriculum established by government entities (Herbel, 2007; Occeli \& Valeiras, 2013), thus playing a fundamental role in its successful implementation (Cantoral et al., 2015), often determining the actual or implemented curriculum (Monterrubio \& Ortega, 2011). However, a high dependence on this resource can lead to errors in teaching and learning processes (Barrantes \& Zapata, 2008). In this regard, Ortiz (2002) argues that teachers must constantly monitor the epistemological content presented in textbooks to prevent biased
meanings from being assimilated by students. Given that the choice of textbooks and/or their activities is not a trivial task (Barrantes et al., 2015), their analysis is recommended from the initial training of teachers (Braga \& Belver, 2016) since they will be responsible for implementing the curriculum in the classroom, an activity in which the textbook is fundamental.

It is necessary for every citizen to have the basic knowledge that allows them to read and interpret statistical information (Del Pino \& Estrella, 2012; Jurečková \& Csachová, 2020; McConway, 2016), much of which is summarized through graphs, tables, and measures of central tendency (MCT), the latter being necessary for the construction and comprehension of other relevant statistical concepts (Mayén, 2009). As a result, the contents of statistics, in general, and MCT (mean, median, and mode) in particular, have gained greater prominence both in curricular regulations and in textbooks for different educational stages.

In particular, in the Chilean context, statistics content is present in all courses of basic education ( $1^{\text {st }}$ to $8^{\text {th }}$ grade) and secondary education ( $1^{\text {st }}$ to $4^{\text {th }}$ year) within the content block of statistics and probability (Ministry of Education [MINEDUC], 2012, 2015). Specifically, regarding MCT, these are introduced for the first time in the $5^{\text {th }}$ grade of basic education, specifically the arithmetic mean, through the learning objective

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## Contribution to the literature

- This paper describes the problem-situations related to MCT in textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade of basic education in Chile.
- This work allows identifying the presence or absence of the different situation-problems associated with MCT in mathematics textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade of basic education in Chile, contrasting with what is stated in the current curricular guidelines.
- This study serves as a basis for further research related to statistics topics in textbooks, supported by the onto-semiotic approach.

Table 1. Evaluation indicators of MCT in Chilean curriculum (MINEDUC, 2012, 2015)

| $5^{\text {th }}$ grade | ${ }^{\text {th }}$ grade | $8^{\text {th }}$ grade |
| :---: | :---: | :---: |
| - Explain information provided by mean of a data set. <br> - Determine mean of a dataset. <br> - Provide a context in which mean of a data set is most appropriate measure to communicate a situation. <br> - Compare results of data sets using mean of a data set. <br> - Draw conclusions from information provided by mean of a data set in a specific context. <br> - Solve a problem using data averages. | - Discover that highly dispersed distributions \& homogeneous distributions can have same median. <br> - Show that median remains unchanged even with large variations in extreme values. <br> - Analyze situations \& determine which measure of central tendency is appropriate for making comparisons \& inferences about one or more populations. <br> - Visualize measure of central tendency \& range in corresponding graphs. | - Calculate, describe, \& interpret measures of position (quartiles \& percentiles). <br> - Represent measures of position using box plots. <br> - Recognize when it is appropriate to use any of measures to analyze a sample. <br> - Compare samples from populations using some of measures of tendency. |

"Calculate the average of data and interpret it in its context" (MINEDUC, 2012, p. 249).

In the $7^{\text {th }}$ grade, the study is expanded to include, in addition, the median and the mode, highlighting the need to understand these measures as well as the range (MINEDUC, 2015, p. 109). Specifically, this learning objective is broken down into:
(1) Determining MCT to make inferences about the population.
(2) Determining the appropriate measure of central tendency to address a posed problem.
(3) Using them to compare two populations.
(4) Determining the effect of a data point that is very different from the others.
In the $8^{\text {th }}$ grade, the curriculum guidelines emphasize the understanding of measures of position, quartiles, and percentiles, thus involving the utilization of the median (MINEDUC, 2015, p. 115). The mentioned learning objective considers:
(1) "Identifying the population that is above or below the percentile.
(2) Representing them with diagrams, including the box plot, manually and/or with educational software.
(3) Using them to compare populations".

In Table 1, the assessment indicators established by MINEDUC (2012) are compiled. They highlight the importance of identifying situations, where the use of each measure of central tendency is most appropriate
and interpreting them to solve the proposed problem situation. Note that the curriculum guidelines do not explicitly mention MCT in the $6^{\text {th }}$ grade, focusing mainly on reading and interpreting statistical graphs and introducing the field of probability.

This study analyzes the TCM tasks present in the mathematics textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade of basic education, considering as units of analysis:
(1) type of measurement,
(2) task,
(3) context,
(4) support, and
(5) cognitive skill, with the purpose of updating their characterization in the Chilean context.

## BACKGROUND

## Textbooks \& Measures of Central Tendency

The analysis of mathematics textbooks has become an invaluable resource due to the role they play in the learning process. Authors such as Cockcroft (1985), who categorizes textbooks as invaluable aids, Ortiz (1999), recognizing in this resource a second level in didactic transposition, or Chevallard (1991) who defines textbooks as the legitimized conception of the institutionalization of knowledge to be taught, grant textbooks fundamental relevance in teaching and learning processes. From this recognition of the resource, it is essential to analyze the meaning that these books convey to ensure that they fulfil their purpose by
providing valuable information about the content they collect and their suitability regarding the curriculum guidelines established by different government bodies.

In the field of statistics and probability, we find that a large part of the research has been developed around statistical graphs (Díaz-Levicoy et al., 2020), statistical tables (Amorim \& Silva, 2016; Díaz-Levicoy et al., 2015; García-García et al., 2019; Latorres \& Vásquez, 2022; Pallauta et al., 2021), tables and statistical graphs (Vásquez et al., 2022), and probability (Gómez-Torres et al., 2014). In the case of MCT, there is a limited variety of research (Kus, 2022), and those available have mainly focused on the secondary education stage, leaving aside the analysis of the courses, where the foundation of statistics is laid.

Regarding MCT in primary education textbooks, we highlight the research conducted by Díaz-Levicoy et al. (2020). The authors analyzed the presence of this topic in $4^{\text {th }}, 5^{\text {th }}$, and $6^{\text {th }}$ grades of primary education textbooks published by the Secretaría de Educación Pública [Secretariat of Public Education] as well as in a series of private books in Mexico. Among the results, the predominance of the mean and mode as measures of centralization stand out, along with the task of calculation and representations of data lists and data tables. It is concluded that there is a need to increase the number of activities in textbooks to ensure the achievement of quality learning.

In the Chilean context, the works of Estrella (2008) and García-García et al. (2021) have been found. In the former case, the author analyzes the didactic transposition of MCT in a $7^{\text {th }}$ grade textbook of basic education and in a university-level textbook. Among the results, it is mentioned that the $7^{\text {th }}$ grade textbook lacks precise mathematical language, showing regular writing; it also presents ambiguity in the use of terms and their meanings, where concepts are sometimes confused, one example is the use of "measure" (Spanish: "medida") when referring to the median. Additionally, the contents are presented through definitions, formulas, and activities, where the algorithm associated with each measure of central tendency must be immediately applied. In the same line, García-García et al. (2021) analyze three textbooks, two edited for MINEDUC (2012) and another from a private publisher, at the same educational level, $7^{\text {th }}$ grade. Among the results, the authors highlight the predominance of the mean as a measure of central tendency, the task of calculating a measure of central tendency, the listing of data as a means of presenting the data to be worked on, as well as the modality of individual work.

Kus (2022) conducts a comparative study on how MCT are presented in textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade in Australia and Turkey. Among the findings, it is highlighted that texts primarily focus on mathematical aspects, emphasizing the use of algorithms over the
understanding of meanings of these measures. Also, they incorporate a high number of situations without context, which is more common in Turkish textbooks.

On the other hand, Cobo (2003) analyzes textbooks corresponding to secondary education in Spain, noting that these textbooks address tasks related to the definitions of MCT and their calculation more vigorously. In the specific case of the arithmetic mean, Cobo and Batanero (2004) characterize the components of the associated meaning of this measure, highlighting a high presence of problem fields in which the mean is obtained as an equitable quantity in a distribution and as a representative element of a set of values. VargasDelgado et al. (2021) evaluate the meaning of MCT in four textbooks of secondary basic education in Colombia. The authors, based on the categories of didactic analysis in mathematical education described by Rico (2013), conduct a content analysis (systems of representation and phenomenological elements), cognitive analysis (organization of learning, cognitive limitations, and demands), and instructional analysis (design of the didactic unit, subjects, and resources). The content analysis reveals a low representativeness of the expected meaning elements of MCT for this level, similar results to those previously obtained by Cobo (2003). Likewise, "it is concluded that this can generate cognitive gaps and difficulties in problem-solving skills due to the lack of explanation and depth in MCT" (Cobo, 2003, p. 209).

In relation to the cognitive skills involved in both the teaching and learning processes, Bloom (1956) presents his taxonomy of educational objectives, which organizes cognitive skills hierarchically and cumulatively, assuming increasing complexity. This tool, when applied in the school system, allows for the classification of objectives and educational goals proposed at different levels, to support the development of intellectual skills, mainly those of a higher level. This hierarchy mobilizes in the student a series of intellectual processes and includes six skills in its cognitive dimension: knowledge, understanding, application, analysis, synthesis, and evaluation. Subsequently, Anderson et al. (2001) review and modify Bloom's (1956) original work, incorporating the skill of creating. This new taxonomic proposal stands as a valuable theoretical conceptual framework by presenting a detailed tool for analyzing thinking and cognitive skills.

Regarding mathematical competence or skill, the Organization for Economic Cooperation and Development (OECD, 2003) proposes a theoretical framework in which it is defined as the ability to apply knowledge to solving problems in everyday life situations; a definition similar to that proposed by Anderson et al. (2001), who, when reviewing Bloom's (1956) proposal, specify the levels of their taxonomy of cognitive skills establishing two levels: basic skills

Table 2. Mathematics textbooks, $5^{\text {th }}$ to $8^{\text {th }}$ year

| Code | Title | Authors (year) | Publisher |
| :---: | :---: | :---: | :---: |
| T1 | Matemática $5^{\circ}$ básico: Texto del estudiante [5th grade mathematics: Student text] | Alvarado et al. (2021a) | Santillana |
| T2 | Matemática $5^{\circ}$ básico: Cuaderno de actividades [5th grade mathematics: Activity notebook] | Alvarado et al. (2021b) | Santillana |
| T3 | Matemática $6^{\circ}$ básico: Texto del estudiante [6th grade mathematics: Student text] | Alvarado et al. (2021a) | Santillana |
| T4 | Matemática $6^{\circ}$ básico: Cuaderno de actividades [Mathematics $6^{\text {th }}$ grade: Activity book] | Alvarado et al. (2021a) | Santillana |
| T5 | Matemática $7^{\circ}$ básico: Texto del estudiante [7th grade mathematics: Student text] | Iturra et al. (2019) | S. M. |
| T6 | Matemática $7^{\circ}$ básico: Cuaderno de actividades [7th grade mathematics: Activity book] | Arce (2019) | S. M. |
| T7 | Matemática $8^{\circ}$ básico: Texto del estudiante [84th grade mathematics: Student text] | Torres and Caroca (2019a) | Santillana |
| T8 | Matemática $8^{\circ}$ básico: Cuaderno de actividades [Mathematics $8^{\text {th }}$ grade: Activity book] | Torres and Caroca (2019b) | Santillana |
| T9 | 5 Básico matemática [5 basic mathematics] | Iturra et al. (2016) | S. M. |
| T10 | 5 Básico matemática: Cuaderno de actividades [5 basic mathematics: Activity book] | Banderas et al. (2016) | S. M. |
| T11 | 6 Básico matemática [6 basic mathematics] | Cáceres et al. (2016) | S. M. |
| T12 | 6 Básico matemática: Cuaderno de actividades [6 basic mathematics: Activity book] | León and Rodriguez (2016) | S. M. |
| T13 | 7 Básico matemática [7 basic mathematics] | Bravo et al. (2018) | S. M. |
| T14 | 7 Básico matemática: Cuaderno de actividades [7 basic mathematics: Activity book] | Equipo Editorial (2018a) | S. M. |
| T15 | 8 Básico matemática [8 basic mathematics] | García et al. (2018) | S. M. |
| T16 | 8 Básico matemática: Cuaderno de actividades [8 basic mathematics: Activity book] | Equipo Editorial (2018b) | S. M. |

(remembering, understanding, and applying) and higher-order skills (analyzing, evaluating, and creating).

At the national level, the curricular bases (MINEDUC, 2012, p. 22, 2015, p. 22) define skills as the ability to perform tasks and solve problems with precision and adaptability, highlighting the importance of developing these skills to learn, as they involve knowing how to do, as well as the ability to integrate, transfer, and complement learning in new contexts, being fundamental for the acquisition of new skills and concepts and in the application of knowledge in diverse contexts.

## METHODOLOGY

The research is qualitative in nature (Salgado, 2007), grounded in the interpretive paradigm, based on content analysis (Krippendorff, 2013) of 16 mathematics textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade of basic education in Chile, with half corresponding to textbooks published for MINEDUC (2012) (T1 to T8) and the rest by private publishers (T9 to T16). For each of the courses and series, both the student textbook and its corresponding activity workbook are considered (see Table 2).

A purposive non-probabilistic sampling method was used (Hernández et al., 2010), considering textbooks due to their extensive coverage, use, and prestige in the Chilean school system between the $5^{\text {th }}$ and $8^{\text {th }}$ year of
basic education. The data collection process, based on the steps described by Cobo (2003), has been, as follows:

1. Locate and select the units of the textbooks that present activities on MCT. In our particular case, MCT are located only in the section covering statistical content. Specifically, one chapter from each textbook has been analyzed, totaling 16 chapters.
2. The sections are transformed into units of analysis. Within each section, the tasks addressing the topic of interest have been identified.
3. Based on underlying variables, units of analysis and categories are established.
4. In this study, the following units of analysis and their respective categories have been used:
a. Type of measure of central tendency: Tasks considering three measures are analyzed:
(1) mean,
(2) median, and
(3) mode.
b. Type of task: Following the indications by Díaz-Levicoy et al. (2020), three tasks are considered:
(1) calculate,
(2) find data, and
(3) explain.

Table 3. Frequency (percentage) of types of measures

| MTC | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Media | $48(56.5)$ | $4(20.0)$ | $90(37.2)$ | $14(13.9)$ | $156(34.7)$ |
| Median | $9(10.6)$ | $0(0.0)$ | $57(23.6)$ | $64(63.4)$ | $130(28.9)$ |
| Mode | $28(32.9)$ | $16(80.0)$ | $97(39.3)$ | $23(22.8)$ | $162(36.4)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

c. Context: This study uses different contexts as the third unit of analysis, considering those defined by PISA (OECD, 2003) in its mathematical competency tests:
(1) personal,
(2) social,
(3) professional, and
(4) scientific.
d. Support: Corresponds to the way data are presented in the task statement in the analyzed textbooks. This is to highlight the existence of a variety of representations for communicating data. It is classified according to the criteria of Pino and Blanco (2008), adapted from Chamorro and Vecino (2003). They include:
(1) written text (in numeric and/or logical natural language),
(2) table,
(3) graph,
(4) image.
e. Skills: The cognitive skills considered in each of the analyzed tasks are those declared by Anderson et al. (2001):
(1) remembering,
(2) understanding,
(3) applying,
(4) analyzing,
(5) evaluating, and
(6) creating.

The textbook sections are coded according to the defined units of analysis, considering the following characteristics: series, year group, page number, task number, type of measure of central tendency, type of task, context, support, and cognitive skill. The data is entered into an Excel spreadsheet for summarization and analysis.

## RESULTS

Once the content analysis of the 16 selected texts was conducted, 448 tasks were identified. It's possible to observe that the highest number is concentrated in $7^{\text {th }}$ grade, where there are 242 tasks, while in $6^{\text {th }}$ grade only 20 tasks were found. Regarding the type of MCT, in general terms, it can be observed in Table 3 that these are distributed in a balanced manner, with the median showing the lowest frequency. By year group, the


Figure 1. Example task "calculate" (T9, p. 286)
2. If arithmetic mean of $10,15,12$, and $x$ is 13 , what is value of $x$ ? A. 10
B. 12
C. 13
D. 15

Figure 2. Example task "find data" (T6, p. 115)
Here are Pablo's grades in two subjects:

| Language | Natural sciences |
| :---: | :---: |
| $5.6-5.7-6.0-5.8-6.2-6.2$ | $4.3-5.7-6.3-7.0-5.9-5.9$ |

a. Calculate the range of each subject. What can you say about the obtained values?
b. Calculate Pablo's average in both subjects: -
c. What do those values mean?

Figure 3. Example of the task "explain" (T13, p.273)
highest frequencies are the mean in $5^{\text {th }}$ grade, the mode in $6^{\text {th }}$ and $7^{\text {th }}$, and the median in $8^{\text {th }}$. Furthermore, the alignment of the texts with the curriculum guidelines is observed, as the highest number of tasks coincides with the $7^{\text {th }}$ grade, a grade in which the study of mean, median, and mode is deepened, with 242 tasks.

## Task Types

The task of calculation consists of obtaining a specific value after applying a calculation procedure or algorithm. For example, Figure 1 asks to find the arithmetic mean of the height of $5^{\text {th }}$ grade students and the final grades in mathematics for a certain class. To solve this task, the student must use the algorithm for calculating the mean.

A second category includes tasks that request finding data. This encompasses all those that require the student to find an unknown datum in each series, which may be presented in various formats. The task in Figure 2 is an example of this category, where given the average of a series of four data points (13), the student is asked to find the unknown datum $(x)$ in the presented series.

The last type of task involves explaining. These are tasks that request demonstrating the level of understanding achieved regarding a specific MCT, explaining, whether it be, procedures used, meanings of specific values, situations in which the use of a particular MCT is advisable, advantages and disadvantages of one

Table 4. Frequency (percentage) of task types

| Task | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Calculate | $62(72.9)$ | $16(80.0)$ | $127(52.5)$ | $60(59.4)$ | $265(59.2)$ |
| Find data | $3(3.5)$ | $0(0.0)$ | $2(0.8)$ | $1(1.0)$ | $6(1.3)$ |
| Explain | $20(23.5)$ | $4(20.0)$ | $113(46.7)$ | $40(39.6)$ | $177(39.5)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

Ignacio conducted a survey among his classmates about their favorite pets. Responses were, as follows:


a. Complete table with survey data. Follow example provided below.

\section*{Favorite pet of Ignacio's classmates} | Pet type | Count (quantity) | Preferences (amount) |
| :--- | :--- | :--- |


b. In pairs, answer questions. Explain how you obtained answers.

- How many classmates did Ignacio survey?
- How many classmates prefer rabbits as pets?)
- What is favorite pet among Ignacio's classmates? And least preferred? $\qquad$
Figure 4. Example task with "personal context" (T2, p. 136)
measure over others, etc. For example, Figure 3 presents two sets of data showing Pablo's grades in language and natural sciences. In the third task, students are asked to explain the meaning of the values obtained.

In relation to the type of tasks analyzed, the results are shown in Table 4, indicating that out of the 448 tasks present in the 16 texts, it is possible to observe that the highest frequency corresponds to the "calculate" task (59.2\%). It is important to mention that the "find data" task has a minimal presence in the total number of tasks analyzed ( $1.3 \%$ ). This type of task is particularly interesting because in $5^{\text {th }}$ grade it has its highest frequency with three tasks (3.5\%) and does not appear in $6^{\text {th }}$ grade. This reflects the importance given to mechanical processes over others that demand a greater cognitive challenge.

## Types of Contexts

Within this units of analysis, the personal context refers to situations that consider the student's immediate environment or peers. The task in Figure 4 shows the results of a survey conducted by Ignacio with his class, inquiring about their preferred pets. The results of this survey are presented through an image from which students are asked to construct the corresponding frequency table. In particular, the last task asks the student to identify the pet that has the highest frequency in the collected data, that is, it is associated with finding the mode.

The tasks included in the social context are those that are situated in the local, national, or global community, fulfilling the condition of relating the student to their environment and always highlighting the communal nature of the situation that originates the task. Figure 5


Figure 5. Example task in "social context" (T1, p. 174)

Mrs. Inés produces homemade cookies of various types. Following table shows each type \& quantity sold of each cookie in last week.

| Type | Number of cookies sold | Unit price [\$] |
| :--- | :---: | :---: |
| Raisin | 100 | 100 |
| Chocolate | 400 | 150 |
| Fruit | 280 | 120 |
| Peanut | 320 | 320 |

a. What is mode of sold cookies?

b. How can data above be useful to Mrs. Inés?
c. What is average price of cookies?
d. How can data obtained in previous question be useful to Mrs. Inés?

Figure 6. Example task in "professional context" (T10, p. 137)
is an example of this context in which a statistical graph is presented with the number of traffic accidents that occur during a month in four Chilean communes, all due to the use of mobile phones while driving. This task associated with the mode asks the student to determine, based on the given data, the commune, where the highest number of traffic accidents were detected.

The next context considered in the analyzed mathematics textbooks corresponds to the professional context, understood as one that encompasses all tasks related to the working world, which can range from nonspecialized to highly specialized levels.

Figure 6 shows a task from this category in which the production of cookies sold during the last week by Doña Inés is reported through a statistical table, classified into four types, along with their prices. The task assigned to the student is to find the mode of the type of cookies sold and analyze the importance of knowing this data for the business owner. Additionally, they are asked to calculate the average price of the cookies.

Table shows heights of Chilean volcanoes in southern zone (Austral). Heights are expressed in meters above sea level (m.a.s.l.) \& were rounded to nearest hundred.
a. With data from table, construct a bar graph with all its elements.

| Name | Height |
| :--- | :--- |
| Aguilera | 2,500 |
| Burney | 1,500 |
| Fueguino | 200 |
| Lautaro | 3,600 |
| Rectus | 1,000 |


b. Average height of Andes Mountain range is $4,000 \mathrm{~m}$. What is average of heights of volcanoes
in southern zone (Austral)? What conclusion can you draw regarding both averages?
Figure 7. Example problem situation in "scientific context" (T10, p. 138)
3. Calculate arithmetic mean \& range of following data: $158,160,168$, $156,166,158,160,168,160,168,158,156,164,162,166,164,168,160$, $162,162,158,166,160,168$.


Figure 8. Example problem situation "without context" (T8, p. 111)

Table 5. Frequencies (percentages) of types of contexts

| Context | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Personal | $29(34.1)$ | $14(70.0)$ | $53(21.9)$ | $31(30.7)$ | $127(28.3)$ |
| Social | $20(23.5)$ | $2(10.0)$ | $48(19.8)$ | $12(11.9)$ | $82(18.3)$ |
| Professional | $20(23.5)$ | $2(10.0)$ | $41(16.9)$ | $26(25.7)$ | $89(19.9)$ |
| Scientific | $6(7.1)$ | $2(10.0)$ | $5(2.1)$ | $4(4.0)$ | $17(3.8)$ |
| No context | $10(11.8)$ | $0(0.0)$ | $95(39.3)$ | $28(27.7)$ | $133(29.7)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

A fourth category, called scientific, includes tasks that consider the application of mathematics to nature, or topics associated with science and technology. The task in Figure 7 exemplifies this type of context by showing the height of some Chilean volcanoes in the southern zone in a data table. Students are asked, based on the given data, to create a bar graph and calculate the average height of the southern (Austral) volcanoes. Finally, the average height of the Andes Mountain range $(4,000 \mathrm{~m})$ is provided, and students are asked to compare both values and draw conclusions based on this.

Finally, because of the content analysis, a category called without context is added, which includes tasks, where the given data are not associated with a situation, whether real or fictitious. For example, Figure 8 exemplifies a task in this category since only a set of data is provided, and students are asked to calculate the arithmetic mean.

In summary, Table 5 shows the results of the analysis of the texts according to the contexts expressed in their statements. Considering the total number of tasks, the majority of them lack context ( $29.7 \%$ ), where students are asked to apply algorithms or procedures, with particular emphasis on practicing calculation techniques. On the contrary, the least frequent tasks are those related to the scientific context (3.8\%). When reviewing each grade level, it is observed that in $5^{\text {th }}, 6^{\text {th }}$, and $8^{\text {th }}$ grades, the most frequent tasks are those

Solve problem: Javier \& Daniel know their averages in mathematics, which are $6.3 \& 6.5$, respectively. Is it possible to determine who scored more grades above six? Justify your answer.
Figure 9. Example task with "written text support" (T1, p. 183)

In following scenario, identify which is mode \& answer questions. Manuel is in charge of social activities for class. On next visit to children's home, he collects following data regarding sizes of boys \& girls.

| Size | Number of children |
| :---: | :---: |
| 3 | 12 |
| 4 | 9 |
| 5 | 15 |
| 6 | 11 |
| 7 | 6 |

a. If class wanted to make a donation, what clothing size should they preferably bring?
b. Does this mean that they should not help with clothes of other sizes?

Figure 10. Example task with "data table support" (T13, p. 274)
associated with personal context. Conversely, in $7^{\text {th }}$ grade, which has the highest number of tasks (242), those lacking context prevail (39.3\%).

## Types of Supports

Regarding the types of supports identifiable in textbooks, written support is one in which data is presented through text, which can be in natural, numerical, and/or logical language. For example, Figure 9 shows a task, where information is presented using natural language. In this task, the mathematics averages of two students are provided, and the task is to determine, based on the known averages, which of them obtained a higher number of grades above 6 .

A second support for presenting data in a task is statistical tables, in any of their formats: tally, data, frequencies, and cross-tabulation. Figure 10 shows a task that uses a frequency table as its support, presenting the data collected by Manuel in his class after consulting the sizes of boys and girls in a children's home, with the purpose of planning a future visit to that place. Students are asked to find the mode and based on this data, decide which size should be prioritized for donation.

A new category of support for presenting data or information in a task is one that uses statistical graphs,


Figure 11. Example task with "graphic support" (T13, p. 26)


Figure 12. Example task with "image support" (T9, p. 281)

Table 6. Frequencies (percentages) of support type

| Support | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Written text | $28(32.9)$ | $0(0.0)$ | $127(52.5)$ | $43(42.6)$ | $198(44.2)$ |
| Table | $30(35.3)$ | $7(35.0)$ | $76(31.4)$ | $23(22.8)$ | $136(30.4)$ |
| Graph | $21(24.7)$ | $13(65.0)$ | $35(14.5)$ | $35(34.7)$ | $104(23.2)$ |
| Image | $6(7.1)$ | $0(0.0)$ | $4(1.7)$ | $0(0.0)$ | $10(2.2)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

in any of their formats: bar, line, pie, histogram, box and whisker plot, or other. Figure 11 exemplifies a task that uses a pie chart to communicate distribution of a group of people who, in the last census, reported they considered themselves to belong to indigenous or native peoples, showing the percentages according to the origin tribe. The proposed task, associated with mode, asks the student to identify predominant indigenous tribe in Chile.

Another category analyzed includes tasks that use images as support. Figure 12 shows the faces obtained after rolling a standard die 15 times. Students are asked to complete a frequency table with the visible values and find the mode.

Table 6 reports the results of the analysis of tasks present in the textbooks according to the type of support in which the data are presented. It is observed at a general level that the most used support in textbooks is written text $(44.2 \%)$. On the other hand, the least frequent support is images ( $2.2 \%$ ), present only in $5^{\text {th }}$ ( $7.1 \%$ ) and $7^{\text {th }}$ grade ( $1.7 \%$ ). When analyzed by grade, we observe that the highest frequencies are in $5^{\text {th }}$ grade with

Calculate arithmetic mean of following data sets.
a. 9 - 7-4-12-5-3-20-4-6-30
b. 85-61-35-18-9-84-28-76

Figure 13. Example task of skill "remember" (T9, p. 285)

1. Graph shows time a group of students spends on physical activity. Weekly physical activity hours

a. Which student spends six hours per week on physical activity?
b. Which student spends least time on physical activity?
c. What is average of data?
d. How do you interpret obtained average?

Figure 14. Example task that involves skill "understand" (T3, p. 167)
tabular support ( $35.3 \%$ ), in $6^{\text {th }}$ grade with statistical graphic support ( $65.0 \%$ ), and finally, written support prevails in $7^{\text {th }}(52.5 \%)$ and $8^{\text {th }}$ grade $(42.6 \%)$.

## Cognitive Skill Type

According to Anderson et al. (2001), the skill of remembering is understood as one that demands students to cognitively function at a basic level. It involves receiving, recalling, and reproducing given information, evoking general ideas, methods, processes, models, structures, or orders in general. Its aim is to recall the information stored in memory, not necessarily understanding its meaning or scope.

For example, the task in Figure 13 requires this skill. In the task, students are asked to calculate the arithmetic mean given two sets of data for which it is necessary to recall the algorithm, technique, or procedure to find this value.

Another cognitive skill is understood, which assumes that the student will be able to comprehend the information, construct meanings, generalizing knowledge to new contexts, explaining, or defining a certain knowledge or concept in their own words. This skill is exemplified in Figure 14, which uses a bar graph to report the time a group of students spends doing physical activity.

This task challenges the student to apply their understanding regarding the constituent elements of a simple bar graph and how data is presented in it, to extract the necessary information to calculate the average hours students dedicate to physical activity. They must construct the meaning of this data by calculating this measure of central tendency.

The skill applied is present in all tasks that require students to use the information or knowledge they possess by applying methods, concepts, or theories in


Figure 15. Example task demonstrating skill "apply" (T11, p. 340)
4. Mass of players of a rugby team is represented in following table.

| Mass of players |  |  |
| :---: | :---: | :---: |
| Mass (kg) | $\mathbf{f}$ | $\mathbf{F}$ |
| 76.25 | 7 | 7 |
| 78.75 | $\mathbf{1 0}$ | 17 |
| 81.25 | $\mathbf{1 5}$ | 32 |
| 83.75 | $\mathbf{1 2}$ | 44 |
| 86.25 | 6 | 50 |

a. What is average mass of rugby players?
b. A new player joins team with a mass of 90.75 kg . What will happen to average? What is new value?
c. Coach needs to inform team's characteristics to fitness coach so that they can develop a training plan. Is it convenient to add this data to average to make it representative of group? Why?

Figure 16. Example task demonstrating skill "analyze" (T14, p. 140)

Nicole is offered employment in two different companies for same number of hours. Only data she has, to decide which one to choose, is average salary of both.


Does she have enough information by knowing average of both companies to decide which one is better for her? In pairs, compare \& discuss your answers.
Figure 17. Example problem-solving task demonstrating skill "evaluate" (T5, p. 201)
problem situations that challenge them to discriminate which knowledge to use in their development. Figure 15 exemplifies this skill, based on the data presented in a pie chart showing the results of choosing the best classmate. In this task, the student must select from the information presented in the statistical graph and their knowledge of MCT, the measure that will allow them to identify the best classmate, which is associated with finding the mode.

Another cognitive skill considered is analysis, which includes finding patterns, organizing parts of a whole, recognizing hidden meanings, or identifying components. In summary, it involves decomposing an object of study into its parts and thinking about how these relate to its overall structure.

Figure 16 presents, through a frequency table, the body masses of rugby team players. In particular, task (b) demands the student to determine the effects on the average of incorporating the data of a new player.

Tasks that involve the skill evaluate are those that, based on known data, require comparing and discriminating between ideas, choosing based on reasoned arguments, and verifying the value of the evidence presented.

For example, the task in Figure 17, that presents Nicole's situation, asks to evaluate two job options based on the information of the average hours and salaries offered in both companies. The student, drawing on their understanding of the meaning of averages and their properties, must evaluate and make a judgment about whether it is possible to decide with the available information.

Finally, the skill create, incorporated by Anderson et al. (2001) after reviewing Bloom's (1956) taxonomy, involves using acquired knowledge to construct and develop new ideas, seeking their originality. An example of this skill is shown in Figure 18, where the student is expected to create a problem based on the given information, requiring the application of the knowledge


Figure 18. Example problem-solving task demonstrating skill "create" (T6, p. 114)

Table 7. Frequencies (percentages) of cognitive skill types

| Skill | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Remember | $52(61.2)$ | $1(5.0)$ | $54(22.3)$ | $39(38.6)$ | $146(32.6)$ |
| Understand | $17(20.0)$ | $8(40.0)$ | $27(11.2)$ | $11(10.9)$ | $63(14.1)$ |
| Apply | $2(2.4)$ | $7(35.0)$ | $42(17.4)$ | $4(4.0)$ | $55(12.3)$ |
| Analyze | $12(14.1)$ | $4(20.0)$ | $76(31.4)$ | $33(32.7)$ | $125(27.9)$ |
| Evaluate | $2(2.4)$ | $0(0.0)$ | $38(15.7)$ | $14(13.9)$ | $54(12.1)$ |
| Create | $0(0.0)$ | $0(0.0)$ | $5(2.1)$ | $0(0.0)$ | $5(1.1)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

studied and acquired in the unit for its solution. This task demands from the student both the uniqueness of the problem and the certainty that its solution will require the application of what has been learned.

Regarding the type of cognitive skill considered in the analyzed tasks, Table 7 reports 'remember' as the most frequent ( $32.6 \%$ ), and 'create' as the least frequent (1.1\%). Similarly, it is observed that the skill of creating is only present in $7^{\text {th }}$ grade.

When grouping the tasks according to the levels of required cognitive skills, basic and advanced, it is possible to indicate that, as shown in Table 8, there is a predominance of basic skills ( $58.7 \%$ ) in the textbooks analyzed. When analyzing by grade level, we see a predominance of basic competencies in the first two courses, with $83.5 \%$ in the $5^{\text {th }}$ grade, and then an equal presence in the last two grades.

## CONCLUSIONS

Based on the literature review related to textbook analysis, the findings confirm previous studies (Herbel, 2007; Occeli \& Valeiras, 2013) regarding the fundamental role of textbooks in curriculum implementation, providing teachers and students with organized content. Additionally, according to Cantoral et al. (2015), and considering the observed content distribution, textbooks serve as a vital tool for implementing the official curriculum, playing a fundamental role in its execution.

After analyzing the tasks related to MCT present in the 16 textbooks, we found their presence in all of them, confirming alignment with the current curriculum. The study of this content begins in the $5^{\text {th }}$ grade with the mean, continuing in the $7^{\text {th }}$ grade with all three MCT. Furthermore, it is in the $8^{\text {th }}$ grade, where the median is addressed, a MCT that is associated with positional measures such as quartiles and percentiles. Particularly in the $6^{\text {th }}$ grade, there are few tasks associated with teaching and learning MCT, which can be partially

Table 8. Frequencies (percentages) of cognitive levels

| Skill | $5^{\circ}$ | $6^{\circ}$ | $7^{\circ}$ | $8^{\circ}$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Basic | $71(83.5)$ | $15(75.0)$ | $123(50.8)$ | $54(53.5)$ | $263(58.7)$ |
| Advanced | $14(16.5)$ | $5(25.0)$ | $119(49.2)$ | $47(46.5)$ | $185(41.3)$ |
| Total | $85(100)$ | $20(100)$ | $242(100)$ | $101(100)$ | $448(100)$ |

explained by the absence of learning objectives related to this mathematical concept. Based on the above, we can confirm that the analyzed textbooks are in line with the curriculum guidelines issued by MINEDUC $(2012,2015)$ regarding the coverage and temporal distribution of MCT content.

When analyzing the type of measures present in the tasks of the textbooks, we observe a slight predominance of the mean and mode at a general level. Additionally, it is noteworthy that although the median appears in all courses, it predominates in the $8^{\text {th }}$ year. This aligns with what was reported by Díaz-Levicoy et al. (2020) and García-García et al. (2021). Therefore, it is necessary to include tasks related to mode and mean in the first two years to ensure a balanced treatment for all three MCT. Furthermore, we believe it is necessary to explicitly incorporate the study of MCT in the $6^{\text {th }}$ year to ensure articulation and continuity across these four primary years.

Regarding the type of tasks requested of students, when considering all 16 textbooks together, our results are consistent with those reported by Díaz-Levicoy et al. (2020), García-García et al. (2021) and Cobo (2003) regarding the predominance of tasks involving calculating MCT. This reinforces the intention to prioritize tasks that mainly focus on applying routine procedures over those that are more cognitively challenging. It is interesting to note that, particularly, in all four courses, the task of calculation constitutes over $50.0 \%$ of the activities. Therefore, it is necessary to review these aspects, as prioritizing one type of task over another affects the possibility of working with a greater variety of tasks, promoting diversity in strategies and resolution procedures.

The preceding analysis, in relation to the types of contexts used to present the tasks, where those lacking context predominate, aligning with the reports by Kus (2022) and Vásquez et al. (2022), suggests the need to emphasize the use of varied contexts to demonstrate the applicability of this knowledge. Since statistics require a different way of thinking, recognizing that data are not just numbers, they are values in context (Moore \& Cobb,
2000). It is essential to reduce tasks that emphasize algorithmization and the practice of calculation procedures without contextualizing the data presented in the tasks, as these are obstacles that prevent a deeper understanding of the mathematical object studied. This approach will provide students the opportunity to relate this mathematical knowledge to life-relevant situations, especially those related to scientific and professional contexts, which are less common.

The supports used to present data in the analyzed tasks are mainly written text and statistical tables. This aligns with what was reported by Díaz-Levicoy et al. (2020) and García-García et al. (2021). This finding is concerning, as supports such as statistical graphs are widely used in our society and are of great utility for making connections between mathematics and statistics. Similarly, images, the support with the lowest frequency, should be incorporated more prominently in textbooks, as they allow students to recognize data presented in a much more natural and not always ordered way, as they are presented, for example, in a statistical table. As a result, it is recommended to incorporate various supports with the purpose of providing students the opportunity to demonstrate their competence in translating information from different presentation forms, transforming the textbook into an efficient and effective resource for students' statistical education.

Regarding the cognitive skills aimed to be developed through the tasks involving MCT in textbooks, there is a general predominance of remembering, which aligns with the findings reported by Estrella (2008). Estrella mentions that the contents are presented through definitions, formulas, and activities, where the associated algorithm for each MCT must be immediately applied. This also coincides with Kus (2022), who highlights that the texts prioritize mathematical aspects and the use of algorithms.

From the perspective of Anderson et al. (2001), the analyzed tasks are directly related to basic skills, requiring students to remember and reproduce information, which does not necessarily compel them to understand its meaning. This aspect calls for, first, a review by publishers to incorporate tasks that challenge students and allow for the development of higher cognitive skills, thus fulfilling what is stated in the national curriculum (MINEDUC, 2012, 2015). With this, significant and deep learning is fostered, enabling the formation of citizens capable of demonstrating a statistical culture. Secondly, it challenges teachers to make the necessary adjustments when using the resource to incorporate or expand the cognitive demands of tasks related to MCT.

Furthermore, the analysis of tasks revealed that when cognitive skills are grouped into basic and higher-order categories, they are presented hierarchically and
cumulatively, in accordance with the current curriculum (MINEDUC, 2012, 2015), highlighting basic skills in the first two grades and incorporating higher-order skills in the last two.

Upon completion of this review of tasks related to MCT in Chilean textbooks from $5^{\text {th }}$ to $8^{\text {th }}$ grade, it is possible to conclude that this resource, widely distributed and used in Chilean classrooms, partially meets the requirements demanded by both society and the curriculum. It is necessary to adapt this resource to the needs of $21^{\text {st }}$ century citizens by prioritizing, through well-designed tasks, statistical knowledge (MCT), which, as suggested by MINEDUC (2012, 2015) curriculum guidelines, allows for an understanding of reality. Additionally, students should be able to select strategies for problem-solving, thereby developing critical and autonomous thinking, a purpose to which the analyzed textbooks do not contribute, whether due to the type of tasks prioritized or the skills that the proposed tasks seek to develop. This highlights the nature of mathematics and particularly statistics as creative, multifaceted, and, above all, accessible to students from early levels.

As future projections of this work, it is worth suggesting a deeper study of the cognitive skills targeted by the tasks present in mathematics textbooks. Additionally, conducting comparative studies with textbooks from other countries would be interesting to update existing literature, given the curriculum reforms or adjustments that have been made in this subject. Another challenge worth addressing is the management of the curriculum through the textbook by the teacher in the primary education classroom, for the acquisition of relevant knowledge and the development of higher skills. Likewise, it would be interesting to investigate the levels of understanding achieved by students and teachers about MCT.

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