MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(5), em2834
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/18518

Metacognitive regulation in collaborative math problem-solving among heterogeneous secondary students

Tri Dyah Prastiti [1*], Intan Fathimah Ahmadah [1], Tatag Yuli Eko Siswono [2]

[1] Universitas Terbuka, Surabaya, INDONESIA
[2] University of Surabaya, Surabaya, INDONESIA

Received 24 February 2026 ▪ Accepted 15 April 2026

Abstract
Metacognition is a critical factor in mathematical problem-solving, particularly in collaborative learning contexts. This study aims to explore how secondary students' metacognitive processes (planning, monitoring, and evaluating) emerge during collaborative mathematical problem-solving within heterogeneous ability groups. A qualitative case study was conducted with grade VIII students at an Islamic junior secondary school in Indonesia. Twelve students were purposively selected from a class of thirty-three and organized into four heterogeneous groups based on mathematical ability levels (high, medium, and low). Data were collected through mathematics ability tests, collaborative problem-solving tasks involving non-routine linear equations, classroom observations, video recordings, and semi-structured group interviews. Data analysis followed iterative qualitative procedures, including data reduction, data display, and conclusion drawing, focusing on identifying patterns of metacognitive regulation at both individual and group levels. The findings reveal that planning and monitoring were consistently observed across groups, while evaluative metacognition remained underdeveloped and unevenly distributed.

Keywords: metacognition, collaborative problem-solving, mathematics education, heterogeneous grouping, secondary students

INTRODUCTION

Mathematics education in the 21st century places strong emphasis on students' ability to solve complex and non-routine problems, as this competence is closely linked to higher-order thinking, adaptability, and lifelong learning. Problem-solving in mathematics is not merely a procedural activity but a cognitively demanding process that requires students to make sense of situations, select appropriate strategies, and reflect on outcomes. Within this context, metacognition has been widely recognized as a central construct that supports effective problem-solving by enabling learners to regulate their own thinking processes. Metacognition encompasses students' awareness of how they think and learn, as well as their ability to control these processes through planning, monitoring, and evaluation (Henra et al., 2024; Maras et al., 2017; Wright & Wolff, 2024). Research consistently demonstrates that metacognition plays a significant role in shaping students' mathematical problem-solving performance at the secondary level. Planning allows students to analyze a problem, activate prior knowledge, and select suitable strategies before attempting a solution, which increases efficiency and reduces random trial-and-error approaches (Henra et al., 2024). During problem-solving, monitoring supports students in checking their understanding, detecting errors, and adjusting strategies in real time, thereby strengthening mathematical reasoning and accuracy (Wright & Wolff, 2024). Evaluation, as a reflective process conducted after or during task completion, enables students to judge the effectiveness of their strategies and solutions, fostering deeper conceptual understanding and preparing them for future problem-solving situations (Hidayat et al., 2018; Maras et al., 2017). Together, these components position metacognition as a foundational mechanism underlying successful mathematical learning.

Despite its recognized importance, many secondary students continue to experience difficulties in

✉ tridyahprastiti@ecampus.ut.ac.id **(*Correspondence)** ✉ intanahmadah@gmail.com ✉ tatagsiswono@unesa.ac.id

Contribution to the literature

- This study reconceptualizes metacognition as a socially shared, group-level process in collaborative mathematical problem-solving, using a qualitative, process-oriented case study to provide real-time evidence of student interactions beyond traditional outcome-based assessments.
- The findings directly demonstrate how heterogeneous ability grouping dictates collective regulatory behaviors, specifically revealing a prevalence of shared planning and monitoring but a critical deficit in evaluative metacognition.
- This work provides clear empirical insights into the mechanics of group-level metacognition, offering actionable guidelines for designing instructional environments that promote equitable participation and deeper reflective collaboration.

mathematical problem-solving, particularly when tasks are non-routine and require higher levels of reasoning. These difficulties are often intensified in collaborative learning contexts, where students must not only regulate their own thinking but also coordinate with peers. A lack of metacognitive skills can hinder students' ability to plan collaboratively, monitor shared understanding, and evaluate group strategies effectively. As a result, collaboration may become superficial, with some students dominating the process while others remain passive, limiting the potential benefits of group-based learning (Henra et al., 2024; Wibawa et al., 2025). Recent research continues to emphasize that the efficacy of collaborative learning is contingent upon high-quality metacognitive regulation (Qiao et al., 2024; Wibawa et al., 2025). Specifically, when shared regulation is absent, group dynamics can become fragmented or dominated by a single individual, a phenomenon particularly prevalent when students face non-routine challenges (Henra et al., 2024). This aligns with contemporary science and mathematics education findings suggesting that instructional designs must intentionally promote regulated learning within social contexts to move beyond superficial cooperation (Wada, 2025). In addition to cognitive challenges, affective and social factors further complicate collaborative mathematical problem-solving at the secondary level. Students may experience anxiety, lack confidence in expressing their ideas, or feel uncomfortable negotiating differing perspectives within a group. Such conditions can reduce active participation and weaken group dynamics, ultimately undermining problem-solving effectiveness (Chen et al., 2024). When metacognitive regulation is weak, these affective barriers are less likely to be addressed, as students struggle to reflect on both their own contributions and the collective progress of the group.

A growing body of literature has examined the role of metacognition in mathematics learning and its relationship with problem-solving success. Studies indicate that students who actively engage in metacognitive processes tend to achieve better mathematical outcomes, as these processes enhance self-efficacy, motivation, and persistence when facing challenging tasks (Abdullah et al., 2017; Nováková, 2024). Metacognitive engagement supports learners in becoming more autonomous and confident, enabling them to approach complex problems strategically rather than reactively (Hidayat & Hermandra, 2023; Wibawa et al., 2025). Recent meta-analyses by Hidayat et al. (2025) confirm that metacognitive instruction remains one of the most potent predictors of mathematical achievement, yet its efficacy in collaborative settings is deeply contingent upon how groups negotiate shared goals. These findings underscore the pedagogical value of integrating metacognitive strategies into mathematics instruction. Parallel to this line of research, collaborative problem-solving (CPS) has gained increasing attention as an instructional approach that promotes deeper understanding through social interaction. In mathematics education, CPS is conceptualized as a joint activity in which students work together to analyze problems, share strategies, and construct solutions collectively. Through dialogue and negotiation, students are encouraged to articulate their reasoning, challenge assumptions, and refine their understanding, which can enhance conceptual clarity and critical thinking (Erdoğan & Şengül, 2017; Preiss et al., 2018; Verschaffel et al., 2019). Recent studies further suggest that collaborative environments can stimulate metacognitive reflection, as students are prompted to explain, justify, and reconsider their thinking in response to peers' input (Baumanns & Rott, 2022; Erdoğan & Şengül, 2017).

Beyond immediate learning outcomes, metacognition has been identified as a key competence for effective mathematics learning in the 21st century. By fostering self-awareness and self-regulation, metacognitive abilities enable students to set goals, monitor progress, and evaluate strategies across diverse learning situations. This capacity is increasingly important in an era characterized by complex problems and interconnected knowledge domains (Hidayat & Hermandra, 2023; Salam et al., 2020). Moreover, when metacognitive training is embedded within collaborative contexts, students develop greater flexibility in adapting their thinking, as they learn to reflect not only on individual strategies but also on collective processes (Nováková, 2024; Liu et al., 2022). International frameworks on mathematical problem-solving further highlight the central role of metacognitive regulation. Polya's well-established model emphasizes a structured sequence of understanding the problem, devising a plan, carrying out the plan, and reviewing the solution, with each stage requiring deliberate reflection and control of thinking (García-Moya et al., 2024; Preiss et al., 2018). Contemporary frameworks similarly advocate for integrating metacognition into mathematics curricula, arguing that students' ability to reflect on their thinking is essential for developing advanced problem-solving skills and transferable reasoning competencies (Preiss et al., 2018; Thi-Nga et al., 2024).

Although the existing literature has established a clear link between individual metacognitive awareness and mathematical problem-solving success, a critical gap remains in our understanding of metacognition as a socially shared, group-level process. Most studies to date have focused on individual outcomes or post-hoc self-reports, effectively treating the group interaction as a secondary variable. In particular, there is limited empirical description of how planning, monitoring, and evaluating are distributed among students within heterogeneous collaborative groups at the secondary level. Consequently, there is a lack of empirical evidence regarding how different phases of regulation are distributed among students in heterogeneous ability groups. This oversight is significant because it leaves educators without clear, actionable insights into how specific group configurations (e.g., different mathematic thinking skills) influence the quality of collective reasoning and the equitable distribution of metacognitive roles. Addressing this gap is necessary to move beyond superficial collaboration toward a model of mathematics instruction where group composition is intentionally designed to foster deep, shared reflection. The present study, therefore, employs a qualitative, process-oriented lens to investigate metacognitive regulation. The outcomes of this study are expected to provide empirical foundation needed to optimize collaborative learning in the secondary mathematics classroom. The study is guided by the following research questions:

1. How do planning, monitoring, and evaluating processes emerge during collaborative mathematical problem-solving in heterogeneous secondary-level groups?
2. How are metacognitive roles and participation distributed among students with different mathematical ability levels within these groups?
3. How does group composition influence the quality of shared metacognitive regulation during problem-solving?

The novelty of this research lies in its qualitative, process-oriented focus on metacognition as a socially shared regulatory activity rather than an exclusively individual skill. By analyzing real-time group interactions in authentic classroom settings, this study moves beyond outcome-based evaluations and contributes fine-grained empirical evidence on how heterogeneous group composition shapes collaborative metacognitive dynamics. These insights extend existing theory and provide practical guidance for designing mathematics instruction that fosters balanced participation, shared regulation, and reflective collaboration.

MATERIALS AND METHODS

Research Design

This study employed a qualitative research approach using a case study design to investigate students' metacognitive processes during collaborative mathematical problem-solving. A qualitative case study design was adopted to facilitate an in-depth, holistic exploration of metacognitive regulatory processes as they unfold within the social ecology of a mathematics classroom (Creswell & Miller, 2000; Yin, 2018). Given that metacognition and socially shared regulation of learning are complex, transient, and deeply embedded in social interaction, they cannot be adequately captured through post-hoc self-reports or broad-scale surveys alone. The case study approach allows for the description necessary to trace the trajectory of group thinking and identify the subtle triggers of regulatory behaviors. In this study, the group serves as the primary unit of analysis, enabling a micro-level examination of how varying ability levels influence the collective negotiation of mathematical meaning. Furthermore, a qualitative case study was considered appropriate because the research aimed to obtain an in-depth, contextualized understanding of how planning, monitoring, and evaluating emerge in authentic classroom interactions rather than to test causal relationships or measure outcomes statistically. A figurative representation of research questions and design of our study is presented in **Figure 1**. Qualitative case studies have been widely used in mathematics education research to capture the complexity of learners' cognitive and metacognitive behaviors as they engage with problem-solving tasks (Coughlin & Montague, 2010; Demircioğlu et al., 2010). The case in this study was bounded by context, participants, and activity. The context was a single MTs (Islamic junior secondary school), the participants were grade VIII students working in heterogeneous groups, and the activity focused on CPS of non-routine linear equation tasks. By defining these boundaries, the study was able to examine metacognitive regulation as it naturally occurred within a specific instructional setting, allowing for rich description and analytic depth.

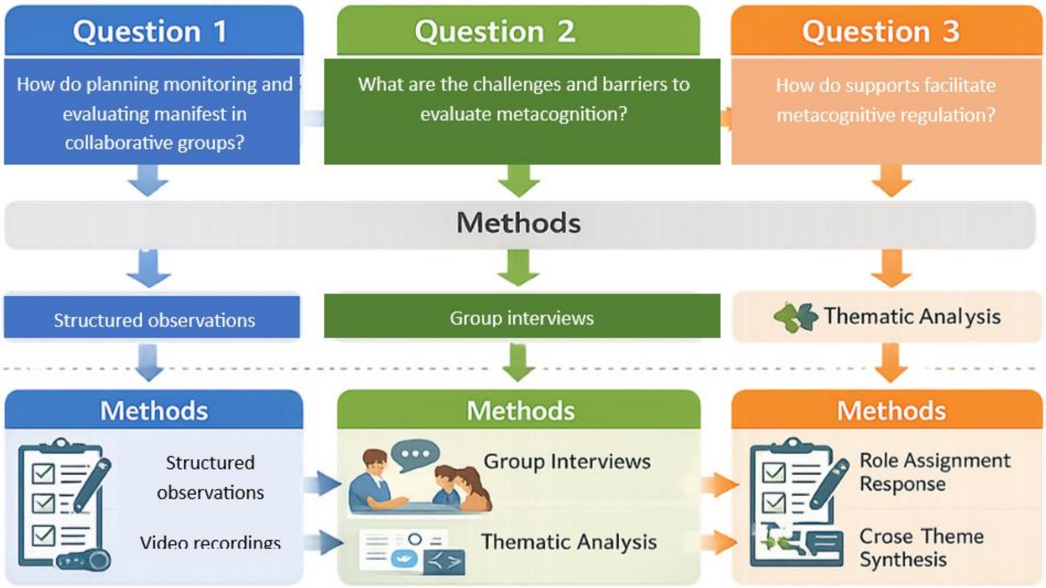

Figure 1. Research questions and study design (Source: Authors' own elaboration)

Research Setting and Participants

The research was conducted at MTsN Gresik, Indonesia, during the 2024/2025 academic year. One grade VIII class (VIII-A) was selected based on teacher recommendations and the heterogeneity of students' mathematical abilities. The class consisted of 33 students who participated in an initial mathematics ability test. Based on the results of this test, 12 students were purposively selected as research subjects. Participants were grouped heterogeneously according to their prior mathematics ability levels: high (labelled as T), medium (labelled as S), and low (labelled as R). Ability levels were determined using score intervals from the mathematics ability test, which consisted of 20 algebra items. Students scoring 80-100 were categorized as high ability, those scoring 65-79 as medium ability, and those scoring 0-64 as low ability. Using stratified purposive sampling, four groups of three students were formed: TSR, TSS (high-medium-medium), TRR (high-low-low), and SSR (medium-medium-low). The four group configurations were selected to provide a representative cross-section of heterogeneous dynamics. Each label (TSR, TSS, TRR, and SSR) represents the distribution of student abilities within a triad. It is important to note that permutations of these distributions (e.g., RSS or SST) were considered functionally equivalent to the selected cases and were thus categorized under the primary distribution labels. This structure allowed for a controlled comparison of how varying ability gaps and the presence or absence of high-achieving students influenced collective planning, monitoring, and evaluation. Ultimately, this grouping structure enabled comparison of metacognitive processes across different heterogeneous configurations.

Data Collection

Data collection was designed to capture both observable interactions and students' reflections on their thinking processes. Four main data sources were used:

(1) the researcher as the primary instrument,

(2) mathematics ability test results,

(3) CPS tasks, and

(4) interviews supported by observation and video recordings.

The mathematics ability test was administered prior to the main study to classify students' initial ability levels and determine group composition. The test consisted of 20 items covering basic algebraic concepts relevant to linear equations. Scores ranged from 0 to 100 and were used solely for grouping purposes rather than for evaluating learning outcomes.

The core data were obtained through CPS tasks. Each group was asked to solve two non-routine linear equation problems designed to require reasoning, strategy selection, and justification. During task implementation, students worked collaboratively without direct teacher intervention. All problem-solving sessions were video recorded to capture verbal communication, gestures, written work, and interaction patterns among group members. In addition to observation and video data, semi-structured group interviews were conducted after the completion of the tasks (**Table 1**). Semi-structured interviews were chosen because they allow researchers to explore participants' metacognitive awareness and regulatory strategies while maintaining flexibility to probe unexpected responses (Kesebir & Oksuz, 2022; Suriyon et al., 2013). Interview questions were guided by indicators of metacognitive knowledge and regulation, focusing on how students planned their approach, monitored

Table 1. Interview guideline indicators

Metacognitive component	Indicator	Description
Metacognitive knowledge	Understanding the problem	Students explain given information, identify what is known and unknown, and restate the problem in their own words.
	Strategy awareness	Students describe the strategies or methods chosen to solve the problem.
	Justification of strategy	Students provide reasons for selecting particular strategies based on prior knowledge or problem characteristics.
Metacognitive regulation	Planning	Students outline solution steps, organize tasks, and allocate roles before or during problem solving.
	Monitoring	Students check their understanding, track progress, identify errors, and respond to difficulties during problem solving.
	Evaluating	Students reflect on the correctness of solutions, assess the effectiveness of strategies used, and consider alternative approaches after completing task.

progress, addressed difficulties, and evaluated their solutions. Although think-aloud protocols are commonly used to capture metacognitive processes, this study prioritized naturalistic interaction during collaboration. Verbalizations produced spontaneously during group work and elaborated during interviews served a similar function by revealing students' metacognitive awareness and control (Sağırlı, 2016).

Data Analysis

Data analysis followed an iterative qualitative procedure involving data reduction, data display, and conclusion drawing. Video recordings, observation notes, students' written solutions, and interview transcripts were first transcribed and organized according to group and task. During data reduction, relevant segments related to metacognitive processes were identified and coded. Coding focused on three main categories of metacognitive regulation: planning, monitoring, and evaluating. Planning included actions such as analyzing the problem, proposing strategies, and allocating roles. Monitoring involved checking progress, identifying errors, and responding to difficulties. Evaluating included reflecting on solutions, assessing strategy effectiveness, and providing feedback to peers. Data display was conducted by organizing coded segments into matrices and narrative descriptions for each group. This allowed comparison of metacognitive patterns across the four heterogeneous group types. Finally, conclusions were drawn by identifying recurring patterns, contrasts among groups, and relationships between group composition and metacognitive regulation.

Data Validity and Ethical Considerations

Triangulation was achieved by combining multiple data sources, including test results, observations, video recordings, interviews, and students' written work. This approach enabled verification of findings from different perspectives and strengthened credibility (Adinda et al., 2023; Kesebir & Oksuz, 2022). Member checking was conducted by discussing preliminary interpretations with participants to confirm the accuracy of representations of their thinking and interactions. In addition, the researcher maintained reflective notes throughout the research process to monitor potential biases and assumptions, enhancing confirmability (Suriyon et al., 2013). These procedures contributed to the dependability and transparency of the study. Ethical considerations were addressed prior to and during the research. Permission to conduct the study was obtained from the school administration. Participants and their guardians were informed about the purpose of the study, and informed consent was secured. Students' identities were anonymized in all records and reports, and all data were used for research purposes, ensuring confidentiality and ethical integrity throughout study.

RESULTS

Research Context and Participant Overview

The study was conducted in a grade VIII mathematics classroom at MTsN Gresik during the 2024/2025 academic year. The school served 1,041 students distributed across 31 classes across grade VII-grade IX. Grade VIII consisted of 10 parallel classes, each comprising approximately 34-36 students. Based on recommendations from mathematics teachers and consideration of ability heterogeneity, class VIII-A was selected as the research site. Class VIII-A consisted of 33 students who completed an initial mathematics ability test. The test scores ranged from 19 to 95, indicating substantial variation in students' prior mathematical knowledge. From this class, 12 students were purposively selected as focal participants to represent different mathematical ability levels. These students were organized into four heterogeneous collaborative groups, each consisting of three students.

Table 2 shows categorization of students' mathematical ability on scores from the mathematics ability test. Of the 30 students who completed the test in class VIII-A, 12 students were purposively selected as research participants to represent high, medium, and low ability levels. This distribution ensured balanced representation across heterogeneous group

Table 2. Distribution of research participants by ability level

Ability level	Score range	Number of students	Number of selected participants	Percentage of selected participants
High	80-100	7	4	33.3
Medium	65-79	13	5	41.7
Low	0-64	10	3	25.0
Total	-	30	12	100

Table 3. Mathematics ability test scores

No	Student initial	Test score	Ability category	Group assignment
1	S1	95	High	TSR
2	S2	90	High	TSS
3	S3	88	High	TRR
4	S4	82	Medium	SSR
5	S5	78	Medium	TSS
6	S6	75	Medium	TSR
7	S7	72	Medium	TSS
8	S8	70	Medium	SSR
9	S9	68	Low	SSR
10	S10	63	Low	TRR
11	S11	58	Low	TRR
12	S12	49	Low	TSR

compositions and supported comparative analysis of collaborative metacognitive processes.

Results of the Mathematics Ability Test

The mathematics ability test consisted of 20 algebra items related to linear equations and prerequisite concepts. The test was administered to 30 students in class VIII-A. Analysis of the results showed that 7 students were categorized as high ability, 13 as medium ability, and 10 as low ability. High-ability students achieved scores within the range of 85 and 100, medium-ability students scored within the range of 65 and 85, and low-ability students scored below 70. Based on these results, 12 students were selected to form four heterogeneous collaborative groups. Group compositions were TSR, TSS, TRR, and SSR. This grouping structure allowed examination of how different combinations of ability levels influenced collaborative interaction and metacognitive regulation. **Table 3** presents the mathematics ability test scores of the twelve selected students from class VIII-A, along with their corresponding ability categories and group assignments. Student identities are anonymized using initials. Data served as basis for forming heterogeneous collaborative groups (TSR, TSS, TRR, and SSR) and were not used for evaluative or grading purposes.

Metacognitive Process Across Different Group

Across all groups, planning and monitoring were the most frequently observed metacognitive activities, while evaluating was consistently underdeveloped. Groups with more balanced ability distributions, particularly TSS and TSR, demonstrated richer interaction and more evenly distributed metacognitive roles. In contrast,

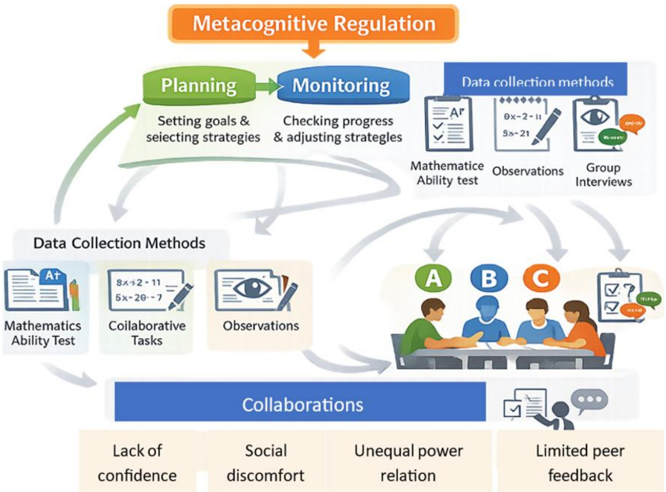

Figure 2. Collaborative interaction patterns of the study (Source: Authors' own elaboration)

groups with greater ability disparity, such as TRR, showed centralized regulation and limited shared evaluation. These findings indicate that group composition influences not only participation patterns but also the quality of metacognitive regulation during collaborative mathematical problem-solving. While heterogeneous grouping can support diverse perspectives and strategic discussion, its benefits depend on balanced participation and opportunities for collective evaluation. Overall, our study also demonstrated that the emergence of metacognitive regulation during CPS was not uniform across the four groups. Rather, it was deeply contingent upon the socio-cognitive dynamics resulting from each group's specific ability composition. While all groups engaged in fundamental regulatory acts, the depth of planning, the frequency of monitoring, and the occurrence of evaluation varied significantly.

Figure 2 illustrate verbal interactions and interaction flow diagrams during the study. In groups with a centralized hierarchy (e.g., TRR), metacognitive acts were often concentrated within a single dominant member, whereas groups with more balanced mid-tier abilities (e.g., TSS) exhibited a more distributed form of socially shared regulation (**Figure 3**). This section provides a comparative analysis of these processes, detailing how different configurations of high (T), medium (S), and low (R) ability students facilitated or constrained the collective transition from cognitive execution to metacognitive reflection.

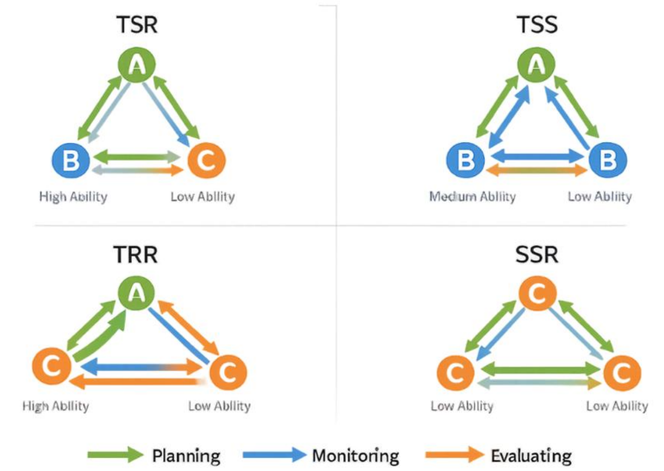

Figure 3. Interaction patterns across different groups (Source: Authors' own elaboration)

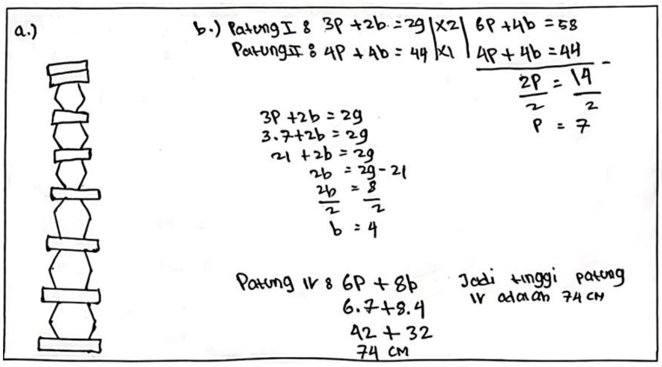

Figure 4. Mathematical problems and students' written solutions of TSR group (Source: Authors' own elaboration)

Metacognitive processes in the TSR group (high-medium-low)

The TSR group demonstrated relatively strong engagement in metacognitive planning and monitoring during CPS. At the beginning of each task, group members discussed the meaning of the problem, identified known and unknown quantities, and proposed potential strategies. This planning phase was characterized by active participation from all members, although the medium-ability student most frequently initiated strategic ideas. During task execution, monitoring behaviors were frequently observed. Group members checked intermediate results, questioned calculation steps, and clarified misunderstandings. The high-ability student often verified solutions and corrected errors, while the low-ability student primarily monitored by asking questions and confirming understanding. Evaluative metacognition, however, was limited. Reflection on the overall effectiveness of strategies or discussion of alternative approaches rarely occurred unless prompted during interviews.

Table 4 shows that planning and monitoring indicators were consistently present across both tasks in the TSR group, while evaluative indicators appeared infrequently and lacked depth, indicating weak development of evaluative metacognition. **Figure 4** demonstrate the given mathematical problems and students' written solutions of TSR group. The diagrams show strong interaction between high- and medium-ability students, with the low-ability student participating mainly through clarification and confirmation.

Metacognitive processes in the TRR group (high-low-low)

In the TRR group, metacognitive regulation was heavily concentrated on the high-ability student.

Table 4. Comparison of metacognitive indicators in the TSR group

Metacognitive component	Indicator	Task 1	Task 2	Description of observed behavior
Planning	Identifying known and unknown information	✓	✓	Group members discussed given data, identified variables, and clarified what was asked in the problem before solving.
	Strategy proposal	✓	✓	Medium-ability student frequently proposed solution strategies, which were discussed and accepted by the group.
	Role allocation	✓	✓	Informal roles emerged (explainer, calculator, checker), though not explicitly assigned.
Monitoring	Checking calculation steps	✓	✓	High-ability student regularly verified calculations and corrected errors during problem solving.
	Clarifying understanding	✓	✓	Low-ability student asked clarification questions to confirm understanding of procedures and results.
	Adjusting strategies	✓	✓	The group modified solution steps when inconsistencies or errors were identified.
Evaluating	Reflecting on solution correctness	△	△	Brief confirmation of final answers occurred, but without in-depth justification or comparison of alternative strategies.
	Discussing alternative strategies	✗	✗	The group did not explicitly explore or compare alternative solution methods after completing the task.
	Peer feedback	△	△	Limited feedback was given among members, usually in response to interviewer prompts rather than spontaneously.

Note. ✓: Observed clearly; △: Observed minimally; & ✗: Not observed

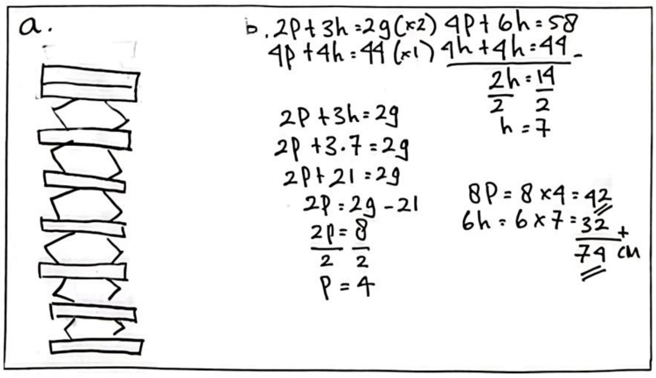

Figure 5. Mathematical problems and students' written solutions of TRR group (Source: Authors' own elaboration)

Planning activities were largely initiated and controlled by this student, who determined solution strategies and delegated tasks. The two low-ability students participated minimally in planning, often accepting proposed strategies without further discussion. Monitoring occurred primarily when low-ability students encountered difficulties, at which point the high-ability student provided explanations or corrections. However, peer monitoring among low-ability students was rare. Evaluative activities were almost absent; the group rarely revisited solutions or reflected on strategy effectiveness once an answer was obtained. This was also reflected in their group written solutions of the given mathematical problems (**Figure 5**), where the solution is only given in mathematical numbers without no clear concluding remarks (as compared to TSR in **Figure 4**) regardless of their correct answer.

Table 5 demonstrates that planning and monitoring were dominated by the high-ability student in the TRR group, while evaluative metacognitive indicators were absent across both tasks, indicating centralized regulation and unequal distribution of metacognitive roles.

Metacognitive processes in the TSS group (high-medium-medium)

The TSS group exhibited the most balanced collaborative metacognitive regulation among all groups. Planning was conducted collectively, with students negotiating strategies and agreeing on solution steps. Each member contributed ideas, and disagreements were resolved through discussion. Monitoring was also shared, as students checked each other's calculations and reasoning. Although evaluative metacognition remained less prominent than planning and monitoring, this group demonstrated more reflective behaviors than others, including brief discussions about alternative strategies and solution accuracy.

In **Figure 6**, we can see that students in the TSS group can correctly answer mathematical problems, gave clear conclusion of their answer, but skipped their most of their mathematical thinking process.

Table 6 indicates that the TSS group demonstrated the most balanced distribution of metacognitive roles, with strong engagement in planning and monitoring across both tasks and emerging, though still limited, evaluative metacognition.

Metacognitive processes in the SSR group (medium-medium-low)

The SSR group experienced notable challenges during CPS. Planning activities were inconsistent, with

Table 5. Comparison of metacognitive indicators in the TRR group

Metacognitive component	Indicator	Task 1	Task 2	Description of observed behavior
Planning	Identifying known and unknown information	✓	✓	The high-ability student identified given information and explained the problem to peers, while low-ability students mainly listened.
	Strategy proposal	✓	✓	Solution strategies were proposed almost exclusively by the high-ability student without substantial negotiation.
	Role allocation	△	△	Roles were implicitly centered on the high-ability student as main problem solver; low-ability students acted as followers.
Monitoring	Checking calculation steps	✓	✓	The high-ability student checked calculations and corrected errors; peer monitoring among low-ability students was rare.
	Clarifying understanding	△	△	Low-ability students asked questions only when they encountered difficulties, not as part of continuous monitoring.
	Adjusting strategies	△	△	Strategy adjustments occurred only after explicit errors were pointed out by the high-ability student.
Evaluating	Reflecting on solution correctness	✗	✗	The group did not engage in reflective discussion about the correctness or efficiency of solutions after task completion.
	Discussing alternative strategies	✗	✗	No alternative strategies were explored or compared once an answer was obtained.
	Peer feedback	✗	✗	Peer feedback was absent; low-ability students did not evaluate or comment on the high-ability student's work.

Note. ✓: Observed clearly; △: Observed minimally; & ✗: Not observed

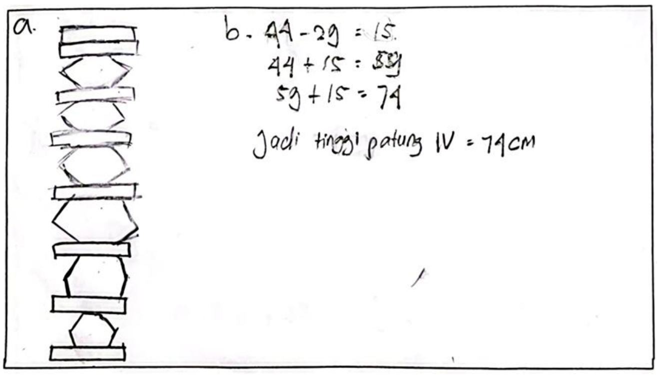

Figure 6. Mathematical problems and students' written solutions of TSS group (Source: Authors' own elaboration)

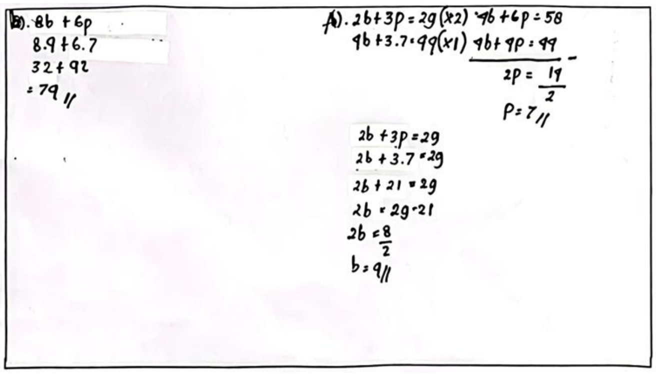

Figure 7. Mathematical problems and students' written solutions of SSR group (Source: Authors' own elaboration)

strategies often proposed without thorough discussion. Monitoring occurred intermittently, typically after errors became evident, rather than as a continuous process. The low-ability student required frequent assistance, and misunderstandings persisted longer than in other groups. Evaluative metacognition was minimal, with little reflection on solutions or strategies after task completion.

Table 7 shows that the SSR group demonstrated partial engagement in planning and monitoring across both tasks, while evaluative metacognitive indicators were not observed, indicating the weakest overall metacognitive performance among all groups.

Figure 7 highlight fragmented interaction and delayed coordination among group members as reflected in their written solution of the given mathematical problems, where no clear mathematical patterns, concluding remarks, and metacognition by illustrating the abstract problem was evident, regardless of their correct answer.

DISCUSSION

Interpretation of Findings in Relation to Theoretical Models

Group composition, particularly heterogeneity in ability, background, and perspectives, has been widely discussed as a key factor influencing the effectiveness of collaborative learning in mathematics classrooms. Research indicates that heterogeneous groups can foster richer discussions, as students bring diverse strategies and viewpoints to problem-solving. Exposure to multiple approaches may encourage flexibility, critical thinking, and deeper conceptual understanding (Cézar et al., 2021; Zhang et al., 2025). Several studies highlight that heterogeneous grouping encourages students to articulate their reasoning more explicitly. When group members possess differing levels of understanding, explanations become necessary, prompting clarification and reflection that can benefit both higher- and lower-ability students (Hurme et al., 2006; Kramarski &

Table 6. Comparison of metacognitive indicators in the TSS group

Metacognitive component	Indicator	Task 1	Task 2	Description of observed behavior
Planning	Identifying known and unknown information	✓	✓	All group members actively discussed problem information, clarified what was known and what needed to be found.
	Strategy proposal	✓	✓	Strategy ideas were proposed by both high- and medium-ability students and negotiated collectively.
	Role allocation	✓	✓	Roles (explainer, calculator, checker) emerged naturally and were rotated during problem solving.
Monitoring	Checking calculation steps	✓	✓	Students checked each other's calculations and reasoning, correcting errors collaboratively.
	Clarifying understanding	✓	✓	Members frequently asked and responded to clarification questions to ensure shared understanding.
	Adjusting strategies	✓	✓	The group revised solution steps when inconsistencies were identified through discussion.
Evaluating	Reflecting on solution correctness	△	△	Short reflective discussions occurred to confirm final answers, but without deep justification.
	Discussing alternative strategies	△	△	Alternative approaches were briefly mentioned but not explored in detail.
	Peer feedback	△	△	Feedback was provided in supportive ways, though it remained limited and non-critical.

Note. ✓: Observed clearly; △: Observed minimally; & ✗: Not observed

Table 7. Comparison of metacognitive indicators in the SSR group

Metacognitive component	Indicator	Task 1	Task 2	Description of observed behavior
Planning	Identifying known and unknown information	△	△	Medium-ability students attempted to identify problem information, but discussion was fragmented and often required clarification from peers.
	Strategy proposal	△	△	Strategies were proposed without thorough discussion, and agreement among members was sometimes unclear.
	Role allocation	✗	✗	No clear role distribution emerged; tasks were undertaken inconsistently by different members.
Monitoring	Checking calculation steps	△	△	Monitoring occurred after errors became evident rather than continuously during problem solving.
	Clarifying understanding	△	△	The low-ability student frequently requested explanations, indicating limited shared understanding.
	Adjusting strategies	△	△	Strategy adjustments were made only after repeated errors or confusion persisted.
Evaluating	Reflecting on solution correctness	✗	✗	The group did not engage in reflective discussion regarding the correctness or efficiency of solutions.
	Discussing alternative strategies	✗	✗	Alternative solution strategies were not discussed after task completion.
	Peer feedback	✗	✗	Peer feedback was absent; students did not evaluate or comment on each other's reasoning.

Note. ✓: Observed clearly; △: Observed minimally; & ✗: Not observed

Mevarech, 2003). In this sense, heterogeneity can act as a catalyst for both cognitive and metacognitive engagement. Nevertheless, the literature also documents significant challenges associated with heterogeneous groups. Differences in ability may lead to communication difficulties, frustration among less proficient students, or dominance by more capable peers. Such dynamics can result in unequal participation and reduced learning opportunities for some group members (Kathayat, 2024; Siagian et al., 2019; Wright & Wolff, 2024). These findings suggest that heterogeneity alone does not guarantee productive collaboration; rather, its effectiveness depends on how interaction and regulation are managed within the group.

There is limited qualitative evidence describing how planning, monitoring, and evaluating are distributed among students within heterogeneous mathematics groups at the secondary level. The interplay between group composition, participation patterns, and shared metacognitive regulation remains insufficiently explored (de Almeida & de Castro, 2023; Wilson & Clarke, 2004). The findings of this study provide strong support for contemporary theories that conceptualize metacognition as both an individual and a socially shared process. Across all groups, metacognitive activities emerged through interaction, discussion, and negotiation among group members rather than solely through individual reflection. This pattern aligns with the concept of socially shared regulation of learning, which emphasizes that regulation can be distributed across members of a group and co-constructed through collaborative engagement (Järvelä et al., 2013). Students' joint discussions at the planning stage and their collective checking of progress during monitoring illustrate how metacognition operates interdependently in collaborative mathematics learning. At the same time, the findings challenge traditional theoretical models that foreground individual metacognitive control as the primary mechanism for successful problem-solving. Although individual ability influenced participation, the quality of CPS was shaped more strongly by interaction patterns and role distribution than by individual competence alone. This observation supports earlier arguments that metacognition in classroom settings cannot be fully understood without accounting for social dynamics and discourse (Goos, 2002). The results thus reinforce calls to extend metacognitive theory beyond individual cognition to incorporate group-level regulatory processes (de Almeida & de Castro, 2023). However, the persistent weakness of evaluative metacognition across all group types of highlights limitations in existing models. While planning and monitoring were frequently observed, evaluation—especially in the form of peer feedback and reflective discussion of strategy effectiveness—remained underdeveloped. This finding is consistent with prior research indicating that evaluation is often the most fragile component of metacognitive regulation in mathematics learning (Hidayat et al., 2018; Muntazhimah et al., 2020).

The remained underdeveloped evaluative metacognition across all ability groups of our study suggests that it is not merely a cognitive lack of skill but is deeply rooted in the socio-emotional climate of the collaborative interaction. While students successfully engaged in shared planning, the act of shared evaluation, which involves questioning, checking, and potentially refuting a peer's contribution, carries a higher social risk. Recent research in suggests that effective evaluation requires psychological safety, which allows students to take the intellectual risk of being wrong or correcting others without fear of embarrassment (Bakhtiar et al., 2017; Mänty et al., 2020). In our study, the lack of spontaneous review may be attributed to a conflict avoidance, where students prioritize social harmony over mathematical rigor. As noted by Huang and Lajoie (2023), groups often engage in superficial consensus to avoid the negative socio-emotional interactions that critical evaluation can trigger. Furthermore, students may view the act of questioning a pee as a breach of social etiquette. This aligns with findings from Wibawa et al. (2025) and Zhou and Colomer (2024), who observed that students in collective cultures often prioritize the maintenance of positive peer relationships over the critical verification of problem-solving steps. Consequently, the evaluation phase is often skipped to minimize interpersonal tension, leading to the "consistently underdeveloped" assessment behaviors observed in this study. Based on our finding, it can be suggested that the current theoretical frameworks may insufficiently capture the social, emotional, and interpersonal demands involved in collective evaluation, underscoring the need for more nuanced models of collaborative metacognition.

Role Distribution, Ability Levels, and Shared Regulation

The results demonstrate that role distribution within groups plays a critical mediating role in shared metacognitive regulation. Groups that exhibited balanced participation, such as the TSS and TSR groups, were more successful in distributing planning and monitoring responsibilities among members. In these groups, students assumed complementary roles—initiating strategies, checking progress, and supporting peers—which facilitated shared regulation and sustained engagement. This finding is consistent with studies suggesting that clearly differentiated yet flexible roles can enhance collaborative learning by leveraging individual strengths while maintaining collective responsibility (Aydan et al., 2025; Vauras et al., 2003). In contrast, groups characterized by poorly distributed roles, particularly the TRR group, showed centralized regulation dominated by a single high-ability student. While this arrangement enabled task completion, it limited opportunities for lower-ability students to engage meaningfully in metacognitive processes. Such dynamics reinforce concerns that without intentional structuring, heterogeneous grouping may reproduce inequities in participation and learning outcomes (de Bruin, 2017). The findings thus extend previous research by demonstrating how ability differences interact with role negotiation to shape shared regulation in collaborative mathematics problem-solving. Importantly, the results indicate that heterogeneity itself is not inherently beneficial or detrimental; rather, its effects depend on how interaction and roles are managed. When roles were negotiated implicitly and remained unbalanced, shared regulation weakened. Conversely, when groups engaged in reciprocal interaction, heterogeneity supported richer discussion and more distributed metacognitive activity. This highlights the importance of pedagogical attention to role design and interaction norms in collaborative learning environments (Chan, 2012; Manathunga & Hernández-Leo 2016).

Educational Implications for Mathematics Instruction

The findings of this study have several implications for mathematics education, which is summarized in **Figure 7**. Specifically, given that evaluative metacognition remained underdeveloped across all groups, there is a clear need for curricula that explicitly integrate metacognitive instruction with mathematical content. Rather than assuming that metacognitive skills will emerge naturally through collaboration, teachers should model and scaffold planning, monitoring, and evaluating strategies within group tasks (Sansone et al., 2023; Sercenia & Prudente, 2023). Teachers should provide prompts for shared reflection during the final third of a problem-solving task (e.g., *"How does this result align with our initial estimate?"* or *"Can we explain this solution in a different way?"*). Such prompts force the group to transition from checking if the steps are correct (monitor) to checking if the logic is sound (evaluation). Explicit prompts that encourage students to articulate plans, check understanding collectively, and reflect on strategy effectiveness may help strengthen underdeveloped evaluative processes. Next, our study highlights that the symmetry of interaction is as important as the diversity of ability. When forming groups, teachers should consider social dynamics alongside test scores. Our study revealed that TSS group showed higher regulatory balance than TRR group, suggesting that a narrower ability gap within a group may sometimes foster more equitable shared regulation than extreme heterogeneity. Moreover, creating structured opportunities for students to evaluate solutions and strategies collaboratively can foster confidence and normalize constructive critique (Husband & Nikfarjam, 2022; Vicario et al., 2024). Such practices must be supported by classroom norms that promote trust, openness, and respect, as students may otherwise hesitate to offer or receive feedback due to social or emotional concerns (Husamah, 2015; Sippel, 2019). Third, the results highlight the importance of intentional role assignment in collaborative learning.

Our findings suggest that, without structure, heterogenous grouping can lead to hierarchical regulation where high-ability students dominate the planning and monitoring phases, often bypassing collective evaluation. To mitigate this, teachers may support shared regulation by assigning or rotating roles related to metacognitive functions. Assigning specific regulatory responsibilities (e.g., strategic planner who is responsible for the roadmap, and critical evaluator who is responsible for challenging the group's assumptions) can distribute the cognitive load and ensure that the 'evaluation' phase is not neglected. In line with this, previous study also demonstrated that clear role expectations can help distribute responsibility more equitably and reduce dominance by particular students, thereby enhancing learning equity (Aydan et al., 2025). While metacognitive regulation has been extensively studied at the individual level, research on group-level metacognition remains comparatively limited. Group-level metacognitive regulation refers to how members collectively plan, monitor, and evaluate their joint problem-solving activities. This includes shared goal setting, mutual monitoring of understanding, and collective reflection on strategies and outcomes. Existing studies acknowledge that collaborative settings can promote metacognitive awareness, yet few investigate how these processes are enacted collectively in real time. Chen et al. (2024) note that limited empirical work has examined how groups engage in shared regulation during collaborative tasks. Similarly, Říčan et al. (2022) argue that although individual metacognitive awareness is well documented, the influence of group dynamics and interaction patterns on shared metacognition remains underexplored. Earlier research suggests that group interaction can support metacognitive development when students are encouraged to question, justify, and evaluate ideas together (Bogart et al., 2017; Hurme et al., 2006).

Furthermore, our observation on underdeveloped evaluative metacognition across all groups, regardless of composition, resonates with recent findings by Zhao and Saleh (2025), who reported that while 100% of students engaged in planning, only a small fraction correctly utilized 'calculation verification' and evaluation in complex tasks. This suggests a systemic evaluation deficit in secondary mathematics, where students prioritize the execution of procedures over the reflection of results. Furthermore, our results regarding the dominance of high-ability students in TRR groups provide a nuanced contrast to the social metacognition framework proposed by Stanton et al. (2021). Their findings argue that group interactions generally stimulate metacognition, our data shows that in heterogeneous mathematics contexts, this stimulation is often asymmetric. Specifically, the 'authority effect' in TRR groups often leads to the suppression of co-regulation, as lower-ability students perceive the high-ability peer's monitoring as definitive, thereby bypassing the collective evaluation phase. Finally, this study extends the work of Sheffler et al. (2022) which identified cognitive and metacognitive planning for learning new skills across the lifespan. While their study focused on individual levels, our research demonstrates how these levels manifest as socially shared regulation of learning of a secondary level students. Our study further revealed that group composition is a primary driver of whether a group reaches 'shared regulation' or remains stuck in 'co-regulation' or individual regulation, such as also demonstrated by Iiskala et al. (2021). Nevertheless, the mechanisms through which group composition shapes these processes are not yet fully understood. In particular, it remains unclear how heterogeneous ability distributions influence who initiates planning, who monitors progress, and who engages in evaluative reflection during CPS.

Limitations and Directions for Future Research

It is important to acknowledge certain limitations inherent in the scope of this study. Primarily, the sample size precludes the statistical generalizability of the findings to the broader population of secondary learners. The research was conducted in a single classroom within one MTs, which limits the generalizability of the findings. However, the objective of this research was not statistical representativeness, but rather analytic generalization. Such approach has been described in detail by Yin (2018). This study contributes a nuanced theoretical understanding of how group composition facilitates or constrains specific phases of shared regulation, such as the underdeveloped evaluative processes observed here. While these findings are context-bound, they offer transferable insights for mathematics educators and researchers seeking to optimize group configurations in CPS environments.

Future research should address these limitations by adopting longitudinal designs to examine how collaborative metacognitive regulation develops over time across different educational contexts (Wada, 2025). Further investigation is also needed into the interplay between individual self-regulation and shared regulation, as focusing on one level alone may obscure important dynamics (Haataja et al., 2021; Maré & Mutezo, 2024). Moreover, future studies should explore the emotional and social dimensions of collaborative metacognition, which remain underrepresented in current research despite their influence on participation and feedback practices (Sobocinski et al., 2020). Finally, the role of technology in supporting shared regulation warrants deeper examination, particularly regarding how digital tools can facilitate communication, reflection, and evaluative dialogue in collaborative mathematics learning (Saab, 2012; Sun, 2025; Volet et al., 2009). Together, these directions point toward the need for more comprehensive and context-sensitive approaches to understanding and supporting collaborative metacognition in mathematics education.

CONCLUSION

This study investigated students' metacognitive processes during collaborative mathematical problem-solving within heterogeneous secondary-level groups. The findings demonstrate that metacognition in collaborative settings is not solely an individual phenomenon but is shaped through interaction, role distribution, and shared regulation among group members. Across all group types, planning and monitoring emerged as dominant metacognitive activities, indicating that students were generally able to discuss strategies and check progress collectively. However, evaluative metacognition—particularly in the form of peer feedback and reflective discussion of strategy effectiveness—was consistently limited. Group composition played a significant role in shaping collaborative metacognitive regulation. Groups with more balanced ability distributions showed richer interaction and more evenly shared metacognitive roles, while groups with greater ability disparities tended to concentrate regulation in a single member. These patterns suggest that heterogeneous grouping alone is insufficient to ensure effective collaborative learning; intentional support for role balance and shared evaluation is essential.

Author contributions: TDP: conceptualization, data curation, formal analysis, investigation, methodology, resources, supervision, writing—original draft, writing—review & editing; **IFA:** formal analysis, investigation, writing—original draft, writing—review & editing; **TYES:** conceptualization, data curation, methodology, resources, writing—review & editing. All authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that ethical clearance was not formally required by the local institutional board as the study involved the assessment of mathematical skills and metacognitive processes through standard, non-invasive pedagogical tasks that do not deviate from the secondary school curriculum. However, the study strictly adhered to the principles of informed consent and data privacy. The authors further stated that written consent was obtained from school authorities and legal guardians, and the identities of all students were anonymized at the point of collection, and no personally identifiable information (PII) was recorded.

AI statement: The authors stated that , during the preparation of this work, the authors used ChatGPT to enhance the clarity of the writing. After using ChatGPT, the authors reviewed and edited the content as needed and took full responsibility for the publication's content.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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