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New insights into the beliefs of secondary pre-service teachers about mathematics

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Abstract

Teachers' beliefs about mathematics can shape how they approach teaching, learning, and assessment in mathematics classrooms. This study examined Australian pre-service teachers' (PSTs') epistemological beliefs about mathematics. A total of 18 PSTs enrolled in Bachelor of Education and Master of Teaching (secondary mathematics) programs participated in the study by answering prompt questions in an open-ended questionnaire. The participants' responses were content analyzed by categorizing the views evident in the responses into four pre-defined mathematical orientations: formalism-related, scheme-related, process-related, and application-related. Findings have shown that the dominant beliefs for the participants reflected process- or application- related orientations to mathematics. These findings are of value to PST educators who seek to explore and expand the beliefs that future teachers hold about the nature and structure of mathematics.

Keywords: beliefs, epistemological beliefs, mathematics teaching and learning, pre-service teachers, secondary mathematics

INTRODUCTION

What teachers of mathematics learn from teacher education or professional development programs, and their classroom practices such as selection of teaching approaches, lesson planning, selection of learning tasks, and decisions on assessment types, can be influenced by their beliefs (An et al., 2006; Hatisaru, 2018; Herppich & Wittwer, 2019; Manderfeld & Siller, 2019; Philipp, 2007). Teacher beliefs have therefore been the subject of many studies in the field of mathematics education (see Safrudiannur et al., 2023). The relevance and importance of mathematics teachers' beliefs are widely considered to be a part of their content knowledge, and associations between content knowledge, beliefs, and practices of mathematics teachers are well acknowledged (e.g., Wilkins, 2008; Yang et al., 2020). Beliefs are often long-lasting (Conner & Gomez, 2019; Pajares, 1992) and can be based on past school experiences (Jao, 2017; Maasepp & Bibis, 2015). For example, as described in Jao (2017), teachers who have been taught mathematics through expository approaches to education (where teachers demonstrate, explain, and describe facts, concepts and knowledge) likely use these approaches in their teaching because they believe that mathematics learning should be directed by the teacher. But teachers who have experienced more explorational approaches to education (e.g., classroom discussions, open-ended problems, alternative assessment methods) seem to use these teaching approaches as they have reflected on the benefits of explorations in mathematics learning.

Teachers can hold beliefs about the nature of mathematics, about the philosophy of mathematics education, or about mathematics teaching and learning (Hatisaru, 2018; Jao, 2017; Viholainen et al., 2014; Woltron, 2024). Sometimes, these are unproductive beliefs such as 'mathematics is unrelated to other subjects', 'mathematics learning should focus on practicing procedures', or 'the role of the student is to memorize information that is presented' (National Council of Teachers of Mathematics [NCTM], 2014). Research has shown that teachers' beliefs can influence their students' beliefs (Beswick, 2007; White et al., 2005), who may develop similar unproductive beliefs about

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Contribution to the literature

- This study examines the beliefs that secondary mathematics PSTs hold about the nature and structure of mathematics.
- The findings of this study are relevant for educators involved in initial teacher education (ITE), who are constructing novice understandings of how to approach the mathematics classroom.
- In ITE for mathematics, the programs can include instruction about the epistemological foundations of mathematics as a discipline; and it is desirable for PSTs to have multiple orientations towards mathematics to be able to inform the most contextually appropriate methods for mathematics teaching and learning in varying situations.

mathematics or mathematics learning (Hatisaru & Murphy, 2019). Teachers' beliefs about students–for example, about the success of socio-economically disadvantaged students in advanced mathematics studies–can influence students' educational outcomes such as their educational aspirations and attainment (see Beswick et al., 2019). Beliefs about mathematics can be changed through teacher education courses (Conner & Gomez, 2019). Thus, to better understand teachers' beliefs and their development, it is crucial to investigate the beliefs of pre-service teachers (PSTs)–who will become the future teachers of mathematics–within teacher education courses in universities or colleges (Beswick, 2006; Jao, 2017).

Research has been conducted about Australian primary school PSTs' beliefs about mathematics or teaching and learning of mathematics (e.g., Ingram et al., 2022; Itter & Meyers, 2017; Maasepp & Bibis, 2015; White et al., 2006), as well as their mathematical content knowledge (see Stephenson, 2018). These research studies have shown that primary PSTs sometimes lack the knowledge needed to teach mathematics effectively (Stephenson, 2018), and that they can hold negative beliefs about mathematics such as dislike, fear or hatred of mathematics (see Itter & Meyers, 2017). Whilst these findings are informative, they are often limited to primary PSTs who are generalist teachers that do not necessarily have mathematics education specialization or are often limited to a dichotomy of positive or negative beliefs, without exploring epistemological beliefs about the discipline. With regard to (potentially) non-specialist practicing secondary mathematics teachers, Beswick et al. (2019) reported that they sometimes hold undesired beliefs about mathematics or its teaching and learning such as 'mathematics learning requires an innate ability; some students can do it, and others cannot' or 'higher-order thinking skills in mathematics can be achieved by only higher achievers'.

Some research has investigated the beliefs of secondary mathematics PSTs who aim to specialize in mathematics education. Findings have shown that these PSTs overall have more productive beliefs about mathematics (Hatisaru, 2018; Hatisaru & Collins, 2023; Lowrie & Jorgensen, 2016) than the primary school PSTs. For example, Lowrie and Jorgensen (2016) reported that a sample of undergraduate students at two Australian universities who were studying to become lower secondary teachers of mathematics had formed quite consistent beliefs that mathematics could be learnt by all, that mathematics was an important subject, and that mathematics had applicability across other discipline subjects. However, little research has been conducted about secondary mathematics PSTs' beliefs about the nature and structure of mathematics or how these beliefs could be developed during initial teacher training courses at university.

In this paper, we examine the beliefs that secondary mathematics PSTs hold about the nature and structure of mathematics. Our findings are grounded in a larger research study where samples of secondary mathematics practicing and PSTs' beliefs about teacher professional knowledge and sources of that knowledge were investigated (Hatisaru, 2023; Hatisaru & Collins, 2023). Our examinations demonstrate the core beliefs about mathematics that inform how the subject is approached in the classroom, proposing ways that beliefs of secondary mathematics PSTs (hereafter PSTs if not otherwise advised) might be expanded during their degree. This investigation is guided by the question: What beliefs about mathematics are manifested in PSTs' responses to the prompt questions concerning mathematics and goals of mathematics education?

CONCEPTUAL BACKGROUND

What Are Beliefs?

There are varying conceptualizations of 'beliefs' in the literature. According to Evans et al. (2013), for example, a belief is a proposition of content accepted as true by the individual who holds the belief. It is useful to distinguish beliefs from related constructs such as 'knowledge' in order to form meaningful questions for inquiry (Törner, 2002). In this paper, beliefs are defined as the cognitively held ideas that bridge a person's knowledge with their actions (Felbrich et al., 2008). Therefore, beliefs are based on what a person knows to be true, and beliefs may influence future actions (Philipp, 2007). A person's beliefs can be challenged, expanded, and disputed when new knowledge is acquired, whether that knowledge is objective and official, or subjective and personal (Berk & Cai, 2019; Furinghetti & Pehkonen, 2002).

Beliefs are generally 'about something' (Törner, 2002), and the beliefs discussed in this paper are about the discipline of mathematics. These beliefs are epistemological and abstract because they are concerned with the nature and structure of mathematical knowledge (Felbrich et al., 2008). Domain-specific beliefs about mathematics are relevant for understanding the affective part of teachers' mathematical work (Furinghetti & Pehkonen, 2002), including how these beliefs might be acquired, expanded, and changed (Leder et al., 2005). Thus, changing beliefs that teachers hold is the first step in changing classroom practice (Ernest, 1989).

Teachers' Beliefs About Mathematics

Literature about the nature and structure of mathematics can be used to characterize beliefs that mathematics teachers hold (Furinghetti & Pehkonen, 2002). These epistemological beliefs also inform teachers' views of the goals of mathematics in terms of why students might benefit from learning it. In this regard, Ernest (1989) was one of the first to suggest that teachers' epistemological beliefs about mathematics are central for understanding their respective beliefs about classroom teaching and learning.

Ernest's (1989) contributions to the field included his original characterization of three varying philosophical views about mathematics: namely, the instrumental view, the Platonist view, and the problem-solving view (detailed in the following sub-section). Ernest's (1989) work laid the foundation for further thinking about how the beliefs teachers hold about the nature and structure of mathematics are related to their beliefs about the goals for the teaching and learning of mathematics (Grigutsch et al., 1998; Viholainen et al., 2014).

Subsequent studies about teachers' epistemological beliefs about mathematics have generated comparable results (e.g., Schoen & LaVenia, 2019; Xie & Cai, 2021). Some of this literature supports two (instead of three) oppositional views between mathematics as a process or a system (Grigutsch et al., 1998). On one side, mathematics is believed to be a static body of knowledge for learners to discover. This perspective corresponds to Ernest's (1989) instrumentalist view, and to some degree the Platonist view. It is outcome-oriented and deductive in nature, starting from existing axioms and using logic to prove new results. On the other side, mathematics is believed to be dynamic, process-oriented, and inductive, wherein learners apply mathematical skills to understand and generalize patterns. This perspective corresponds to Ernest's (1989) problem-solving view. Research shows that expert mathematics teachers are capable of holding multiple views about the structure and nature of mathematics at the same time (Felbrich et al., 2008). Such capability might be useful for teachers to code-switch as necessary when teaching different mathematics content and to students in different contexts. Research suggests that change and expansion of beliefs is possible, because teachers' beliefs about the structure and nature of mathematics develop and change through culture, reflection, and experience (Maasepp & Bibis, 2015; Perry et al., 2006).

Grigutsch et al. (1998) constructed four orientations towards mathematics, and their conceptualization has been applied to research studies to further understand mathematics teachers' beliefs about the nature and structure of mathematics in terms of the teachers' orientation (Felbrich et al., 2008; Viholainen et al., 2014). As described below, and elaborated more in Rott (2020), these orientations can be mapped onto Ernest's (1989) four views:

1. Formalism-related–mathematics as an exact science, deductively learned through logic and abstract thinking. The emphasis is on understanding the structure of mathematics, its theorems, logic, and notation. This and the following orientation relate to Ernest's (1989) Platonist view.
2. Scheme-related–mathematics as a collection of schemas, an application of formulae, terminology, and rules. The emphasis is on developing skills to perform accurate calculations and manipulation of formulae, without necessarily understanding where the formulae or rules come from.
3. Process-related–mathematics as problem solving, a discovery of patterns and structures. The emphasis is on creatively using the structures of mathematics to generate understanding of novel situations. This orientation is equivalent to Ernest's (1989) problem-solving view.
4. Application-related–mathematics as relevant for daily and social life. The emphasis is on applying the rules and formulae of mathematics to real-world situations in order to solve problems. This orientation is equivalent to Ernest's (1989) instrumentalist view.

These four orientations provide another system for understanding the beliefs that teachers hold about mathematics as a subject area. A teacher may favor certain orientations over others, but we argue that it is desirable for teachers to have a holistic view of mathematics that holds all these orientations simultaneously.

Teachers' Beliefs About Mathematics and Their Influence on Teaching

Ernest (1989) not only articulated teachers' epistemological beliefs, but also how these beliefs influence teachers' perspectives of their (and students') roles in the learning of mathematics. For example, an instrumental view of mathematics, in which mathematics is seen as a set of rules and skills, promotes a perspective where the teacher is viewed as an instructor and the student as an active learner or performer (Ernest, 1989). A more Platonist view of mathematics, in which mathematics is seen as an abstract body of static knowledge, supports the teacher as the explainer and student as content learner or understander. Finally, a problem-solving view, in which mathematics is a creative and dynamically changing subject, focuses on the teacher as the facilitator and the student as the explorer. These roles in the teaching and learning of mathematics are appropriate to varying degrees for different educational contexts and purposes (see also Beswick, 2012).

In the line of Ernest's (1989) seminal works, there is a plethora of research studies showing the importance of studying teachers' beliefs because of the ways in which they affect teachers' pedagogical behavior in the classroom. For example, Hashweh (1996) reported the influence of teachers' epistemological beliefs on the way they used strategies in their teaching. These strategies included the use of knowledge representations such as examples or analogies, confronting students' alternative conceptions, or providing opportunities for students to ask questions to reinterpret their experiences, as well as teachers' identification and emphasis of strategies in their students' work. Fennema and Franke (1992) also reported that teachers' epistemological beliefs play an important role in shaping their classroom behavior. The authors illustrate this with a teacher concerned with teaching addition and subtraction to first grade students. Similarly, Stipek et al. (2001) studied 21 teachers teaching addition and comparison of fractions; they found consistent links between the teachers' beliefs and teaching behaviors.

Most recently, Rott (2020) observed and analyzed nine secondary mathematics teachers' lessons that involved problem solving tasks. The teachers holding an instrumentalist view often gave content-related hints and focused on results, with usually one correct way of solving the given problem. The teachers holding a problem-solving view, however, gave process-oriented hints, highlighted students' processes, ideas, errors and strategies rather than majorly considering whether a correct answer was found. These findings agree with the findings reported by Hatisaru (2018) in an earlier study wherein two secondary mathematics teachers' beliefs about teacher professional knowledge, lesson planning and goals of mathematics education, and how these beliefs associated with their mathematics teaching practices, were investigated. Data came from interviews with the teachers (Ali and Fatma, altered names), classroom observations, and written questionnaires. Close associations between each teacher's beliefs and teaching were reported. For example, Ali, who believed that students would need more to develop procedural skills in mathematics learning, mostly focused on procedural aspects of the concept of function (the content area that he was teaching). Fatma, however, believed that the core aim of mathematics education is developing students' conceptual understanding, in addition to procedural skills. She therefore made explicit efforts to support her students' conceptual understanding of functions.

In the Australian context, previous research found that teachers believed in a process-oriented approach to mathematics that did not center on transmission of knowledge, but on the learners' mathematical processes (Perry et al., 2006). For primary PSTs, negative experiences of their own learning of mathematics were common (White et al., 2006), but these perspectives could be shifted during the course of their degree (Maasepp & Bibis, 2015). Australian teachers' beliefs about mathematics teaching are also embedded in the dominant Western cultural beliefs, which value process and risk-taking (Perry et al., 2006). Western cultures like Australia's might thus promote a more process-oriented approach to mathematics when compared with other contexts like Hong Kong, where transmissional, formalist orientations are embedded in a cultural context of beliefs that value obedience, discipline, and rules (Perry et al., 2006). However, no national studies from Australia have investigated PSTs beliefs about the nature and structure of mathematics, despite the assumptions that their beliefs can shape their future classroom practices and influence their future students' relevant beliefs.

THE CURRENT STUDY

This study explored the beliefs about mathematics that were manifested in PSTs' responses to prompt questions concerning mathematics and goals of mathematics education. Informed by qualitative research methods (Wallen & Fraenkel, 2011), data for this study were PSTs' responses to an open-ended questionnaire. While beliefs can also be explored through Likert scale, quantitative surveys (Beswick, 2006; Safrudiannur et al., 2023; Woltron, 2024; Yang et al., 2020) or classroom observations or interviews (e.g., Beswick, 2007), limitations of these surveys are that they can sometimes amplify social desirability bias or/and fail to provide in-depth insights into participants' beliefs (see Woltron, 2024). In this study, responses obtained from PSTs to the prompt questions in an open-ended questionnaire provided us with in-depth information on their beliefs about mathematics (Hatisaru, 2018; Manderfeld & Siller, 2019).

Participants

The study was conducted with PSTs enrolled in the Bachelor of Education (B.Ed.; secondary mathematics education major/minor) and Master of Teaching

(MTeach; secondary mathematics education specialization) degrees at a metropolitan university in Western Australia. Both degrees equip students with the knowledge and skills to teach year 7 to year 12 mathematics. Admission to these degrees requires satisfactory performance in algebra, calculus, and statistics-based mathematics subjects at the end of secondary school. At this university the focus of mathematics training is primarily on scheme- and application- related orientations, with less focus on formalism than at some other universities in Australia and in other countries. In mathematics pedagogy units, students develop pedagogical content knowledge for using the Australian school curriculum, with activities based around number and algebra, statistics and probability, and measurement and geometry, and special coverage of problem solving.

Also reported in Hatisaru and Collins (2023), a total of 32 second-year B.Ed. and first-year MTeach students who undertook the two relevant mathematics pedagogy units offered in semester 2 of 2022 were invited to participate in the research study. Participation in the research was voluntary; the students' informed consent was obtained. Before the research commenced, the students were sent an open-ended questionnaire (detailed next) to complete in their own time, with the completed questionnaires emailed to the unit lecturer. A total of 12 B.Ed. (7 male and 5 female) and 6 MTeach (3 male and 3 female) students responded. All these students had completed some tertiary level mathematics units. At the time of the study, some of them had engaged in teaching practicum in schools, and some worked informally tutoring high school students.

The Instrument

The open-ended questionnaire used in this study was developed in a professional learning initiative wherein the beliefs of secondary mathematics teachers about sources of knowledge for teachers (Hatisaru, 2023) and their content knowledge for teaching algebra (Hatisaru, 2024) were investigated. The questionnaire comprised two main parts–the first part was demographic data that included gender, highest academic credential in mathematics (high school, tertiary), and level of teaching experience; the second part included eight prompt questions. Six of these questions aimed to assess the participants' beliefs about teacher professional knowledge and potential sources of this knowledge (Hatisaru & Collins, 2023); while the other two (the focus of the current study) aimed to inform further understanding of the participants' beliefs about mathematics and goals of school mathematics learning. Prompt #1 was adapted from An et al. (2006), and Prompt #2 came from Sam and Ernest (2000):

> Prompt #1: From your point of view, what are the goals of mathematics education for students?
>
> Prompt #2: Please complete the following sentence. To me, mathematics is …

These two particular prompts have been used in the relevant research literature with practicing mathematics teachers (prompt #1 and prompt #2: An et al., 2006; Hatisaru, 2018; Hatisaru, 2023), as well as with the general public (prompt #2: Sam & Ernest, 2000) and school students (prompt #2: Hatisaru, 2020; Hatisaru & Murphy, 2019), to get insights into their beliefs about mathematics. On this basis, the prompts have a firm foundation in existing literature and established utility in identifying the beliefs of PSTs about the nature and purpose of mathematics. Furthermore, they can provide a comparison with studies about beliefs conducted in other contexts.

Data Analysis

The data for this investigation were the participating PSTs' responses to prompt #1 and prompt #2, and a content analysis method (Shava et al., 2021) was used. This analysis was conducted collaboratively, by the first three authors of this paper, with all coding the data and engaging in discussions to resolve any ambiguities or disagreements to improve trustworthiness and reliability (Hammarberg et al., 2016). As there was little variation between the responses of two cohorts (B.Ed. and MTeach), like the cohorts in Woltron (2024), the data were analyzed for a combined group. In the analysis and reporting, the participants are referred to by using code names such as PST 1, PST 2, PST 3, and so on.

Initial reviewing of the data indicated that the four orientations: formalism-related, scheme-related, process-related, and application-related (Grigutsch et al., 1998) would provide a thorough conceptual framework for the coding of the participants' responses. In the analysis, we considered each participant response (or description) as a unit for coding. Within that unit, we identified the portion of the descriptions (i.e., words, phrases, and/or sentences) referring to the nature of mathematics and connected these descriptions to the four orientations (see **Table 1**).

In a few places, separating the formalism-, scheme- and process-related orientations was difficult, due to vague phrases such as 'increase understanding' (PST 3, PST 17), which could be interpreted as either understanding of concepts (formalism) or procedures (scheme). A phrase like 'understanding mathematical thinking' (PST 6) could be interpreted as either formalism (the structure of knowledge that creates mathematics) or process (developing critical thinking for solving problems). PST 8 wrote that "mathematics is more than just calculation", which could point to formalism (mathematics is an abstract intellectual structure), process (mathematics is a way of problem solving) or application (mathematics explains the world

Table 1. Orientations in the participant responses (n = 18) in prompt #1

Orientation	Example phrases/sentences
Formalism-related	Understand the mathematics concepts; to support advanced mathematics knowledge; learning to work with abstract ideas.
Scheme-related	Aiding scientific literacy; [gain] basic mathematical skills; have mathematical literacy; grow in mathematical capabilities.
Process-related	[Learn] problem solving strategies; develop critical thinking
Application-related	Support for future careers, practical applications of mathematics such as financial literacy, [develop] numeracy to function in society, [see how] mathematics relates to the real world.
Affect/motivation-related	Attitudes towards mathematics; enjoyment; engagement and motivation.
Vague	Understanding mathematical thinking; increase understanding; supporting students on their learning trajectory through mathematical sciences.

Table 2. Orientations in the participant responses (n = 18) to prompt #1 and prompt #2

Orientation	Frequency (prompt #1)	Frequency (prompt #2)	Total
Formalism-related	3	3	6
Scheme-related	5	2	7
Process-related	6	5	11
Application-related	13	9	22
Affect/motivation-related	3	4	7
Vague	4	1	5
No response	-	2	2
Total	34	26	60

around us). We therefore coded these references as "vague" (**Table 1**).

Whilst the four orientations were used as priori codes, we were open to emergent codes such as affect/motivation-related orientations. In some participant responses, there were data reflecting multiple codes. This meant that their words fit with more than one orientation: for example, both application- and scheme-related orientations (as in PST 6's response below). The following examples illustrate the coding; italics are added, and the orientation evident in the response is noted in brackets:

PST 6's response to prompt #1:

> So, they *use of mathematics methods in future life* [application-related]. Understanding "*mathematical thinking*" [vague]. Learning *work with abstract ideas* [formalism-related].

PST 10's response to prompt #2:

> … always challenging me to use the skills I have learned over the years *to solve problems in the most effective and fastest way possible* to make sense of things [process-related].

FINDINGS

When the participants were asked 'what are goals of mathematics education for students?' (prompt #1), they expressed a variety of descriptions related to the conceptual framework from Grigutsch et al. (1998). As shown in **Table 2**, their perspectives of goals of mathematics education for students predominantly reflected application-related orientations of mathematics (13 references), followed by scheme- or process- related orientations (11 references in total).

In the descriptions where formalism-related orientations of mathematics are evident, participants discuss the importance of understanding mathematics concepts, as well as working with abstraction. For example, PST 1 wrote: "[the goal of mathematics learning is] *to understand the mathematics concepts well* & cleaning misconceptions". In the descriptions where scheme-related orientations are evident, the participants often indicated that mathematics learning enables students to gain mathematical skills or numeracy, generally improving their mathematical capabilities and procedural understanding. Examples come from PST 5 and PST 17, respectively:

PST 5's response to prompt #1:

> Depending on the student. All students should have *mathematical literacy*, for some the maths skills to support them in their career, for others advanced maths knowledge.

PST 17's response to prompt #1:

> For students to have a better understanding of *how mathematics works* and how it relates to the real world.

In the data coded as reflecting process-related orientations, the participants often indicated that mathematics learning enables students to learn to solve mathematical problems and develop reasoning, problem solving (PST 14), and critical thinking skills (PST 15). In the data showing application-related orientations, the participants mainly described the goals of mathematics learning including the use or role of mathematics in the real world, in other disciplines, and/or in students' future professions. Typical examples come from PST 2 and PST 16, respectively:

PST 2's response to prompt #1:

> To provide *a good basis for future work and use in everyday life*. Aiding scientific literacy.

PST 16's response to prompt #1:

> To help students leave school with an understanding of *how to apply some level of mathematics in the real world* to assist in day-to-day activities.

In some responses, the scheme-, process- and application- related orientations were so closely connected that it was almost impossible to distinguish them. Here, the process-related learnings are believed by the participants to lead to the development of skill sets (scheme-related learnings) that then support application-related learnings fulfillment in the real world, as in the following response:

PST 12's response to prompt #1:

> I believe the goals of mathematics education is to … *practice and improve mathematics skills and understanding*, and *to use mathematics skills in everyday life*.
>
> Mathematics is used in *building, engineering, home economics, art, sports, ICT, in essential life skills such as managing money and in logical abstract thinking for problem-solving and creating large scale response plans, projections and simulations*. Mathematics is *evident in nature*–in the number of stamens different flowers have, which indicate differences and connections between species. The art of a mathematics teacher is to expose students to *these various aspects of mathematics* and to equip students *to grow in their mathematic capabilities*.

Responses like these suggest an orientation towards the benefits of mathematics learning for life more broadly. Participants with this orientation reflected a belief that mathematics learning enables the development of skills and literacies that are generally beneficial for students' quality of life.

Three PSTs mentioned further affective and motivational outcomes of mathematics learning (in addition to process- or application-related ones), indicating that mathematics education could improve students' affect, such as by "… addressing popular negative attitudes towards mathematics …" (PST 11), "… to engage and motivate students to see the relevance of mathematics" (PST 12) and "to get students to enjoy coming to mathematics class …" (PST 4). The emergence of affective or motivational issues are also part of Ernest's (2015) conceptualization of goals of mathematics education and were also emergent in Viholainen et al. (2014). This study adds to that previous works by finding that for these teachers, the affect may also be in the process-related learning of skills and literacies, as they apply more widely to their future lives–not just in terms of applying mathematical skills, but in terms of applying ways of thinking and being learned through mathematics for increased life quality and enjoyment more broadly.

Within the responses to prompt #2: 'To me, mathematics is …', three participants' responses reflect a formalism-related orientation about mathematics such as: [mathematics is] "the study and use of numbers, pattern and shape … practicing mathematics develops the abstract thinking required to solve problems of logic or pattern" (PST 12), and "the study of formal systems" (PST 8). Two participants did not give a response, and four participant responses expressed attitudes towards mathematics, such as, "[mathematics is] beautiful! And difficult" (PST 2), "everything" (PST 6), "interesting and rewarding" (PST 16), or "often misunderstood/not liked" (PST 4). In the other responses, there were references indicating that mathematics is seen as either a scheme- or process- (7 references in total) or an application- (9 references) related subject (see **Table 2**). **Table 3** presents what beliefs about mathematics are manifested in the PSTs' responses to both prompt questions concerning mathematics and goals of mathematics education.

Out of the 60 references in prompt #1 and prompt #2, in 6 references formalism-related (representing 5 participants), in 7 references scheme-related (representing 6 participants), in 11 references process-related (representing 6 participants), and in 22 application-related (representing 15 participants) orientations were reflected. Affect/motivation-related orientations came up a total of 7 times, representing 6 participants. These findings indicate that many of the participants in this study ($n = 13$) showed more than one orientation towards mathematics, and they viewed the epistemological nature of mathematics from several viewpoints. It was common for the participants to reflect a capacity to move back and forth between the process- and application- related viewpoints, especially.

As seen in **Table 3**, in most of the participant responses–either in their responses to prompt #1 or prompt #2, or to both–more than one orientation was evident. Specifically, only in 5 participant responses was only a single orientation apparent. In the remaining 12 responses, there were references to two ($n = 8$) or three ($n = 4$) orientations. In one participant response, there were references to all five orientations.

Table 3. Orientations apparent in the participant responses (n = 18) by prompt question

Participant	Orientation apparent in prompt #1	Orientation apparent in prompt #2
PST 1	Formalism-related	No response
PST 2	Scheme-related; application-related	Affect/motivation
PST 3	Process-related; vague	Application-related
PST 4	Affect-motivation; vague	Vague; affect/motivation
PST 5	Formalism-related; scheme-related; application-related	Application-related
PST 6	Formalism-related; application-related; vague	Affect/motivation
PST 7	Application-related	Scheme-related
PST 8	Application-related; vague	Formalism-related
PST 9	Application-related	Application-related
PST 10	Process-related; application-related	Process-related
PST 11	Scheme-related; application-related affect-motivation	Application-related
PST 12	Scheme-related; process-related; application-related; affect/motivation	Formalism-related; process-related; application-related
PST 13	Application-related	Vague; application-related
PST 14	Process-related	Process-related; application-related
PST 15	Process-related	No response
PST 16	Application-related	Affect/motivation
PST 17	Scheme-related; application-related	Application-related
PST 18	Process-related; application-related	Process-related; application-related

DISCUSSION

A teacher's capacity to hold multiple orientations in their beliefs about mathematics is a desirable attribute for teachers of mathematics (also see Safrudiannur et al., 2023). Historical emphasis on traditional and formal approaches were largely scheme-related and were criticized as automatizing calculations without emphasizing conceptual understanding and skill development (Jao, 2017; Viholainen et al., 2014). Our findings suggest that this group of PSTs have beliefs that challenge traditional views of mathematics and instead are primarily process-oriented and application-oriented. Previous research in Germany and Australia has also found evidence of a strong dominance of process- and application- oriented viewpoints among PSTs (Felbrich et al., 2008; Perry et al., 2006). The PSTs vantage points reflect a lens through which mathematics is seen as a dynamic discipline, which might lend to belief in constructivist approaches to teaching and learning, as found in Lowrie and Jorgensen (2016).

The findings of the research are relevant for educators involved in initial teacher education (ITE), who are constructing novice understandings of how to approach the mathematics classroom (Beswick, 2006; Jao, 2017). In ITE for mathematics, the programs can include instruction about the epistemological foundations of mathematics as a discipline (Auslander et al., 2019). Students in ITE courses have, and further develop, beliefs about the nature of mathematics that will influence their practices as classroom teachers (Maasepp & Bibis, 2015; Safrudiannur et al., 2023). Beliefs held by teachers shape their approaches and respective actions for leading the mathematics education of their students, bridging knowledge and action (Herppich & Wittwer, 2019; Rott, 2020; Yang et al., 2020). Understanding ITE students' beliefs about the nature and structure of mathematics at the beginning of a teacher education course could be valuable for determining how and what changes are needed (also see Jao, 2017) because attending to their relevant beliefs is as important as providing them with curriculum content, resources and ideas (Beswick, 2007).

In the case of the PSTs in this study, it is desirable for them to expand their beliefs to incorporate other orientations to inform the most contextually appropriate methods for mathematics teaching and learning in varying situations. The ability to code-switch between multiple orientations is vital in a globalized world where there are many diverse student needs (e.g., high-achievers and low-achievers; see Safrudiannur et al., 2023) in a typical Australian classroom (Beswick et al., 2019). Previous research has suggested that PSTs' beliefs about mathematics can be changed through reflection and experience (Conner & Gomez, 2019; Jao, 2017), but further research would be needed to explore effective methods for broadening PSTs' orientations towards mathematics.

We recommend other researchers to explore responses to the survey questions across a range of Australian universities in order to determine which factors reflect a culture of mathematics in Australia more broadly, and which reflect the culture of the local institution. For example, the study university has a strong vocational focus, and hence its mathematics units typically have an application-oriented outlook, with few units focusing on pure mathematics and the formal structures of mathematics. This may explain the lower proportion of formalism-related orientations compared to the study in Finland by Viholainen (2014). There are likely to be differences in outlook between regional universities and metropolitan ones, with students in regional and rural areas being more vocationally motivated than those in urban areas (Beswick et al., 2019; James et al., 2010).

Further research is also needed to understand potential differences in beliefs among groups of teachers, such as undergraduate and postgraduate PSTs, and whether this is related to their different mathematical backgrounds (Safrudiannur et al., 2023). It is typically the case that B.Ed. students do not have a previous STEM-related qualification, while MTeach students are more likely to have mathematics, science, or engineering degrees, and even a PhD in these fields. We would expect that the more mathematical education a PST has, the more orientations towards mathematics they would be able to hold simultaneously, akin to that of professional mathematicians.

Future studies would be of use to disentangle the use of the words 'skills' and 'understanding' as they relate to mathematics learning. Are students valuing the procedural and numerical skills of mathematics, or do they take a more holistic view of mathematics as an abstract and logical way of thinking? Are they interested in understanding where rules come from, or only interested in understanding how to apply them in specific contexts? Answers to these questions will open pathways for how best to challenge, change, and expand the beliefs of PSTs during their degrees.

CONCLUSION

Fundamental to the global mathematics education landscape, mathematics educators and mathematics education associations advocate for teachers of mathematics, as well as for students, to hold productive beliefs about mathematics (e.g., in the USA: NCTM, 2014; in Australia: Australian Association of Mathematics Teachers [AAMT], 2006). Coupled with this advocacy is, in Australia, where this study was conducted, the school mathematics curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2022) emphasizes mathematics as a means for problem solving and encourages conceptual understanding of mathematics (equivalent to process-related or problem-solving orientation). In this qualitative study, when the participants were asked prompt questions on goals of mathematics education for students (prompt #1), and what mathematics means for them (prompt #2), they gave a variety of descriptions where all four orientations of mathematics, as well as affect/motivation-related orientations, were apparent. Whilst all these orientations were apparent in the participants' responses, the findings have shown that a dynamic view of mathematics was predominant. The study by Grigutsch et al. (1998) showed a positive correlation between application- and process- related orientations as well as between formalism- and scheme-related orientations, corresponding to views of mathematics as dynamic and static, respectively. Our participants were just as likely to pair an application-related orientation with a scheme-related orientation as with a process-related orientation. Whilst their orientations towards mathematics were found to be fluent, the findings show that application-related orientations were more dominant among the participants, which may have reflected the culture of the PSTs' university.

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