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# Praxeology review: Comparing Singaporean and Indonesian textbooks in introducing the concept of sets

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**Abstract**

Singapore has successfully implemented mathematics textbooks as a widely referenced part of the curriculum. Nevertheless, not all countries can fit into the offered concept. There are gaps and other interesting factors to investigate, especially regarding the comparison with Indonesia. This study aimed to compare the concepts of Singaporean and Indonesian textbooks on a more specific topic, namely the concept of mathematical sets. The comparative study was conducted on two official handbooks of the two countries, which were reviewed in praxeology (T, τ, θ, and Θ). The research data were limited to the topic of the concept of sets in the two books, ignoring other material as part of the depth of the study. Qualitative research with a hermeneutic phenomenology approach was chosen as an alternative method, and this study is a study at the prospective stage of the entire series of Indonesian versions of didactical design research. The data showed a uniqueness, characterizing the textbooks' quality from each perspective. The similarity and specialization of the textbooks were indicated by the type of task regarding the definition of sets (T1) and the form of the praxis block. There were striking differences in the complexity of task design in influencing the perceptual, memorial, introspective, and a priori. The characteristics of material presentation considered the students' culture and character in each country, making these textbooks worthy of reference. Although studying math topics was quite tricky, a praxeological study of textbooks from both countries provided a new perspective on how task design played an essential role in making material presentation more acceptable to students.

**Keywords:** concept of sets, mathematics, textbook, praxeology review

## INTRODUCTION

Education is an essential need for humans (Acton, 2016). Every person, group, organization, and even a country will not be able to stand without education (Nuraini et al., 2019). Indonesian education is all forms of education held in Indonesia, both structured and unstructured (Gayatri et al., 2022). Education in Indonesia is divided into three main types: formal, non-formal, and informal (Oktapiani et al., 2019). Currently, Indonesia implements a national education system. All levels, pathways, and types of education must implement that system. One of the current educational programs in Indonesia is the 12-year compulsory education (Lewis & Nguyen, 2020), namely six years of elementary school, three years of junior high school, and three years of senior high school. In all levels of formal education in Indonesia, mathematics is one of the subjects that must be studied (Simamora et al., 2018).

Mathematics has become a subject that must be studied at every level of formal education in Indonesia for several reasons, including:

(1) learning mathematics can help us to think more systematically (Yayuk et al., 2020),

(2) learning mathematics makes logical thinking more developed (Kenedi et al., 2019),

(3) mathematics makes us trained in counting (Litkowski et al., 2020),

(4) mathematics trains the ability to conclude deductively (Makowski, 2020), and

(5) mathematics teaches us to be conscientious, careful, and patient.


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**Contribution to the literature**

- This study offers a view of the similarities and differences in mathematics learning opportunities for students from the two countries.
- This study describes a critical evaluation of the didactic designs presented to students.
- This study serves as a basis for conducting further research related to the development of didactic designs, especially on set materials.

However, the quality of mathematics learning in Indonesia is not good enough, even in a low category; the mathematical performance of Indonesian students in global studies such as Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA) was considered very worrying (Fauziyah & Hobri, 2021).

A global study on students' mathematical abilities discovered serious problems in mathematics education in Indonesia (Palinussa et al., 2021). The low achievement of Indonesian students in mathematics tests has become a national crisis. For example, in the 2018 PISA study, the mathematics achievement of 15-year-old students in Indonesia was ranked in the bottom 10 out of 77 countries (Organization for Economic Co-operation and Development [OECD], 2019). The low mathematics achievement of Indonesian students was undoubtedly affected by many factors. The attempts to explore the possible factors contributing to cross-country differences in students' mathematics achievement revealed that curriculum was one of the key factors (Li, 2000). The textbook is considered an important indicator of student learning opportunities, as it can reflect the school curriculum (Erbas et al., 2012).

Textbooks are the physical tools most closely related to teaching and learning. Textbooks are one of the primary learning resources, apart from teachers (Piper et al., 2018). Curriculum content can be conveyed to teachers and students using textbooks to guide students in achieving goals (Remillard et al., 2019). Textbooks have three essential roles in education (Erbas et al., 2012), namely: as a guide to determine which topics to teach, helping the teacher organize topics and materials in sequence, and providing ideas to the teacher about activities for teaching students.

Textbooks are an integral part of teaching mathematics. Mathematics textbooks as supporting materials for teaching and learning mathematics have existed for ages (Fan et al., 2013). In teaching mathematics in schools, textbooks have been used as the main source in many countries (van den Ham & Heinze, 2018). The TIMSS study showed that teachers utilize mathematics textbooks as the main reference to determine teaching strategies (Mullis et al., 2020). Mathematics textbooks affect the topics to be discussed and how they are presented (Alajmi, 2012). Presenting the topic in the text is crucial, as it leads to a different pedagogical approach and opportunities for students to learn. If a topic is not covered in a textbook, it is less likely to be discussed in class (Veldhuis & van den Heuvel-Panhuizen, 2020). Furthermore, the math questions (tasks, problems) in the textbooks used by students largely determine what they learn and how they learn it. This condition confirms that mathematics textbooks provide interpretations of mathematics curricula to teachers and students (Dollah et al., 2019). A good mathematics textbook will also positively impact students' mathematics learning achievement. One way to detect and determine the quality of textbooks used by students and teachers to develop their skills is to compare textbooks across countries.

Researchers believe that studying textbooks used in different countries can provide insights and reveal similarities and differences in mathematics learning opportunities for students worldwide (Haggarty & Pepin, 2002; Kul et al., 2018). Over the past few decades, international comparative studies in mathematics education have resulted in a better understanding of the differences in teaching cultures. International studies related to the comparison of mathematical textbooks between countries have become one of the methods being widely carried out to develop future textbooks, such as the comparison of Japanese and English textbooks using praxeological analysis of geometry materials (Takeuchi & Shinno, 2020), the study on the evidentiary properties taught in geometry in textbooks in France and Japan (Miyakawa, 2017), analysis of methods applied to teach multiplication of fractions in Turkish and American textbooks (Kar et al., 2018), study on the difficulty level of mathematics textbooks in junior high schools in China, Japan, Singapore, South Korea, and the United States (Cao et al., 2017), analysis of algebra problems in Chinese and Indonesian textbooks (Huang et al., 2021), and many other studies.

Indonesia is a large country with ethnic and cultural diversity (Hendriyanto et al., 2021) and different student abilities. Therefore, the Indonesian Government always develops textbooks based on students' needs and abilities adapted to the implemented curriculum (Kristanto et al., 2018). Indonesia and Singapore have their national curriculum, including mathematics. The mathematics curriculum in Indonesia emphasizes a scientific approach to learning that encourages students to observe, question, gather information, associate/reason, and communicate (Anggraena, 2016). Meanwhile, the mathematics curriculum in Singapore is in the form of mathematical problem solving, which depends on five interrelated components: concepts, skills, processes, attitudes, and metacognition (Kaur, 2019).

Sievert et al. (2019) revealed that different textbooks offered different learning opportunities. As a result, different learning achievements would also emerge, as confirmed by the research of Fischer et al. (2015). They found a significant correlation between students' mathematics learning outcomes and the textbooks used. In the present study, researchers compared mathematics textbooks used in Indonesia and Singapore. The selection of Singapore as a comparison for Indonesia was based on the significant difference between the two countries in a PISA study in 2018 concerning students' mathematics achievement. The average math test results of students in Singapore in the PISA study ranked in the top-2, below China (Kaur et al., 2019), while Indonesia was far in the low rank. In addition, Singapore's achievements in international mathematics tests attracted many countries around the world, for example, the use of Singapore textbooks by schools in several districts of the USA (Yang & Sianturi, 2019).

The materials in this study were about sets and focused on introducing the concept of the set. Lipschutz (1986), in the preface to his book, reveals that the theory of sets lies at the foundations of mathematics. Concepts in sets theory, such as functions and relations, appear explicitly in every branch of mathematics. A textbook analysis is important because of its role in teaching and learning, especially in international comparisons. Previous studies had analyzed textbooks from various countries, finding out the advantages and disadvantages of the textbooks, which could eventually become the design of future textbooks. In addition, the results of textbook research potentially provide a broader and more in-depth picture, both of curriculum requirements and in-class practice (Gracin, 2018). Comparative analysis using praxeological theory was carried out at the learning stage. This research was expected to provide an overview of the similarities and differences in textbooks utilized in the two countries, based on praxis block and praxis logos in the theory of praxeology.

Anthropological theory of the didactic (ATD) was employed as a model to describe knowledge from learning activities through a praxeological approach. More specifically, ATD was used to observe human mathematical activity through an epistemological model of mathematical knowledge (Artigue & Bosch, 2014). The first embodiment of ATD theory is called didactical transposition, which is carried out through 3 stages (Artigue & Bosch, 2014). The first stage is creating knowledge initiated by mathematicians. The second stage is making knowledge an essential part of the curriculum to be taught over time. At this stage, the curriculum diverts and even changes scientific knowledge. The third stage is dependent on the

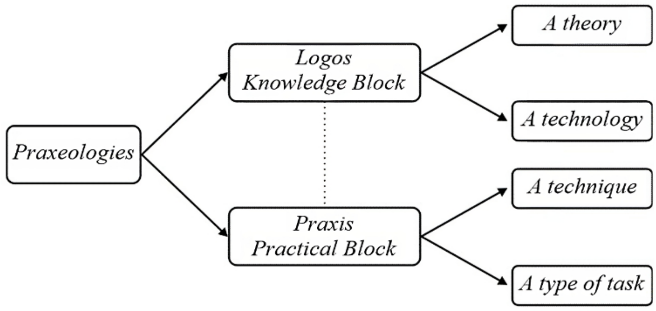



**Figure 1.** Praxeology's model (Putra & Witri, 2017)

previously taught knowledge (classroom learning) and the self-learned knowledge.

Knowledge and its activities have a model of praxeology known by (T, τ, θ, and Θ) (Putra, 2019a) in which T is type of task being completed, τ is technique used in completing the task, θ is the technology being used, which can also be the knowledge to analyze technique or consider discourse, and Θ is theory being referred to justify or examine technology. Task (T) refers to the problems given to students. In classroom learning, tasks can be sourced from certain materials in books. Students need technique (τ) to complete the task (T) (Nazli & Ovez, 2018). Praxeology consists of two main components: praxis and logos (**Figure 1**).

The praxis block consists of two components: task (T) and technique (τ). Meanwhile, the block of logos or knowledge refers to thoughts and how they are justified. The logos block also has two components: technology (θ) and theory (Θ). Technology refers to the justification of the technique (τ) used by the student to complete the task (T). Meanwhile, the material taught acts as a theory (Θ) to explain technology (θ). The four components (T, τ, θ, and Θ) are models to study human knowledge (dos Santos & Farias, 2022).

Commonly, task (T) can be completed using a wide variety of techniques (τ), and technology (θ) can involve various techniques. A series of task types (T) and techniques for completing the task is called punctual organization (Cimatti, 2020). Technology usually justifies various techniques for various types of tasks, becoming a local organization. Because the theory (Θ) is regularly used in diverse technologies, it is referred to as a regional organization (Szałek, 2016). A mathematical organization is a collection of praxeology becoming a part of a particular material domain in mathematics (Winsløw, 2011).

Praxeology is not only used as a model to analyze mathematical knowledge but also didactic knowledge (Putra & Witri, 2017). The task type (T) of didactical praxeology's refers to how the teacher teaches mathematics, such as a way to organize learning situations for students to apply techniques in completing tasks. Didactic techniques also vary. Some teachers may direct instruction of mathematical techniques they know or a context as task-related problems (Putra, 2019b). The blocks of technology and theory of didactical praxeology's for justifying techniques vary based on experience and knowledge. Organization of didactical praxeology's is known as didactical organization.

Textbook analysis of both countries is carried out from a praxiological point of view to answer question in this research. We have not found a comparative study of similar textbooks for Indonesian mathematics textbooks with other countries. We look at the limitations and requirements of knowledge found in textbooks and compare them in different sizes of their praxiological organization. Although if we try to explain many of the praxiological characteristics by focusing on the sub-disciplinary level of the co-determination (from question to domain), This does not mean that this study ignores cultural factors that may have an impact on the textbooks used in each country. The quality of the textbook can be assessed by looking at the questions provided in the textbook, and the study offers such an analysis. This study offers a comparative analysis to describe the characteristics of task design in Indonesian and Singaporean mathematics textbooks and analyze the differences between the two to produce an overview of the offer to correct improper designs. With all this in mind, here is this research question: What are the similarities and differences in knowledge about sets in the content of mathematics textbooks from Singapore and the contents of mathematics textbooks from Indonesia? Overall, research on textbook comparisons has many sides and offers several scopes that range from specific to general, such as how much emphasis textbooks place on what content, method, or cultural component is covered. Although our comparative analysis is content-specific, the theoretical frameworks we use can help conceptualize the various scopes and levels demonstrated by various educational systems. These findings may have contributed to the development of teaching materials.

## METHODOLOGY

### Research Design

Qualitative research with a hermeneutic phenomenology approach was chosen as an alternative method for this study. An approach called hermeneutic phenomenology is based on hermeneutic and phenomenological philosophy. Phenomenology leads to the discovery of the relationship between objectivity and subjectivity, which is present in every moment of human experience (Guillen, 2019), hermeneutics explaining behavior, forms of verbal and non-verbal behavior, culture, and organizational systems and revealing the meaning they contain, but maintaining their singularity (Guillen, 2019). So, phenomenology and hermeneutics complement each other. Friesen et al. (2012) argue that the hermeneutic phenomenological approach is particularly relevant for researchers in the field of education. In general, the phenomena seen in this study are the phenomena that underlie the process of designing learning designs, especially phenomena related to the reflection and evaluation of teaching material design in the set material of two countries (Singapore and Indonesia).

Didactical design research (DDR), developed by Suryadi (2019) is a type of research design used to uncover the phenomenon. DDR is based on two research paradigms: interpretive and critical. The interpretive paradigm aims to investigate real-world phenomena associated with the impact of didactic design on the human ways of thinking. The critical paradigm seeks to make changes by proposing alternative solutions through didactic design hypotheses. In its implementation, DDR contains three stages of analysis which include prospective, meta-pedadidaktic, and retrospective.

This study is an implementation of the prospective stage. At this stage, researchers conduct a study of the phenomena that underlie the process of making hypothetical learning designs which are the findings of the results of didactic transposition analysis on set material. Phenomena found at this stage include:

(1) the transfer of knowledge to 'set material' which includes the analysis of 'set material' as scientific knowledge (scholarly knowledge), 'set material' as knowledge designed in the school curriculum to be taught (knowledge to be taught), and 'set material' as knowledge to be taught (taught knowledge),

(2) didactic situations displayed in textbooks, and

(3) the impact of the transfer of knowledge and the didactic situation displayed on the concept gap, between the student's concept image as a representation of learned knowledge and scholarly knowledge and the learning obstacles he experiences.

### Selection of Compared Textbooks

The books chosen as the data source in this study were the mathematics textbooks used as a reference in almost all schools in both countries, Indonesia and Singapore. Mathematics textbook from Indonesia was the mathematics book (2017 revised edition) for grade VII of junior high school students published by the Ministry of Education and Culture of the Republic of Indonesia (As'ari et al., 2017). It was the primary reference book in schools; around 90% of Indonesian schools (Ulumudin et al., 2017). A diagnostic survey conducted by the Indonesian Ministry of Education and Culture found that many mathematics teachers in Indonesia used the mathematics textbook published by the Indonesian government (Yang & Sianturi, 2017). In addition, the book was available online and could be accessed for free by all parties. Then, a comparative textbook from Singapore entitled New syllabus mathematics (NSM) 4, 7th edition of 2019, was published by Shing Lee Publishers Pte Ltd (Yeo et al., 2019). NSM textbook was the only book approved by the Ministry of Education of Singapore, with the highest market share, up to 80% (Manopo & Rahajeng, 2020).

The sets learning material was the second material in the Indonesian mathematics textbook for the odd semester, with three sub-chapters of discussion, including the concept of sets, the properties of sets, and the operation of sets (As'ari et al., 2017). Meanwhile, in the Singaporean textbook, the sets learning material was presented in chapter one, with five sub-chapters of discussion, namely introduction to set notations, Venn diagrams, universal set and complement of a set, the intersection of two sets, the union of two sets, and combining universal set, the complement of a set, subset, and intersection and union of sets (Yeo et al., 2019). In the present study, the sections were limited to comparing "The concept of sets" in the Indonesian mathematics textbook and "Introduction to set notations" in the Singaporean mathematics textbook. The two sub-chapters covered the same discussion, making them available to be compared in terms of how they were presented.

### Data Collection Procedures

Following the purpose of the study, data are collected by analysis of documents. Document analysis, as a case-based investigative process with a research focus on written materials, notes, or documents, is commonly used in educational studies when a textbook or curriculum is the source of its data. The documents in this study are high school mathematics textbooks available from Singapore and Indonesia, and the task design in these documents is analyzed accompanied by a relevant conceptual framework.

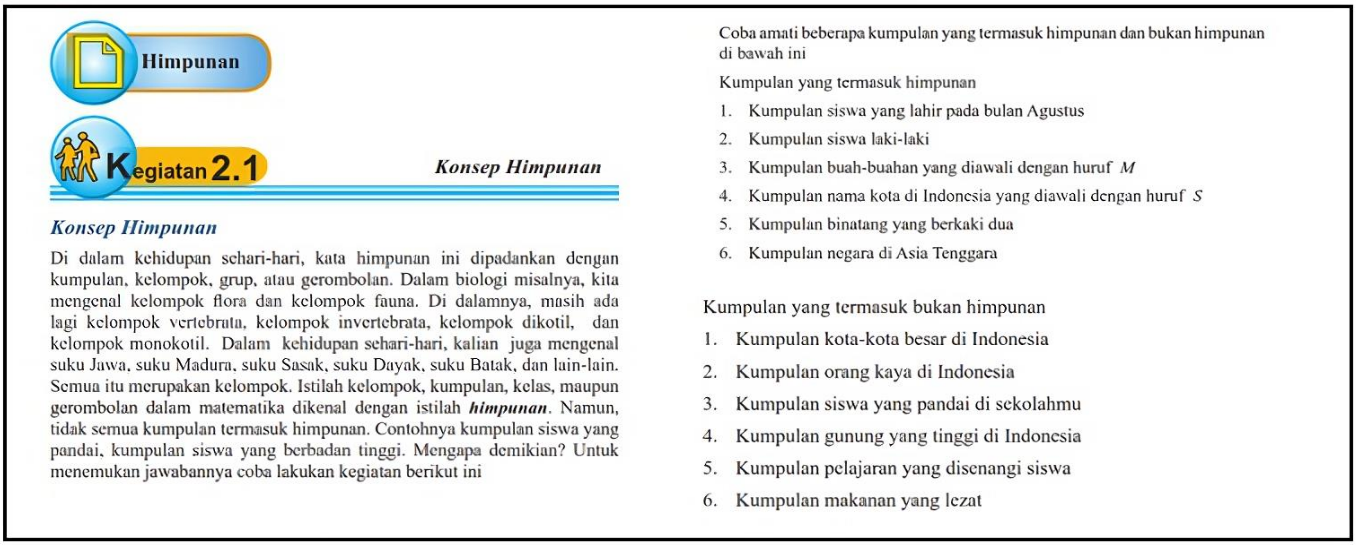
### Data Analysis Methods

Data analysis is searching for meaning (Hatch, 2002). Data analysis in qualitative research takes place in conjunction with other parts of qualitative research development, namely data collection and finding writing (Creswell, 2015). The data analysis procedure in this study will be carried out in three main stages. In the first stage, the task design of arbitrary units in textbooks of both countries is selected. Task design-related topics were inserted in the praxeology table by four researchers. For this, mathematics textbooks of both countries were investigated. In the second stage, each researcher coded the test design independently. In the third stage, the reliability between coders is determined. At this stage, researchers reconverge and identify location inconsistencies in the taxonomic table. These three stages continue cyclically. Then the inconsistent design tasks coded by the researchers were discussed to reach an agreement. During this process, if necessary, an opinion is requested from an expert in measurement and assessment.

## RESULTS AND DISCUSSION

There were two parts of the analysis presented, namely the praxis block and logos block, which were parts of the implementation of the praxeological theory. The initial IB represented the Indonesian mathematics textbook, and SB represented the Singaporean mathematics textbook. Before discussing the differences between the praxis and logos blocks in IB and SB, the researcher first discussed the introduction to textbooks. The visualization of the introduction on IB and SB can be seen in Figure 2 and Figure 3.

The introduction of the two books (IB and SB) was expected to make students understand many equivalent terms for 'set' in everyday life. IB specifically mentioned that the equivalent of 'set' was limited to the synonym of the word *group* (in Indonesian; *kumpulan*, *kelompok*, *grup*,


Himpunan

Kegiatan 2.1 Konsep Himpunan

*Konsep Himpunan*

Di dalam kehidupan sehari-hari, kata himpunan ini dipadankan dengan kumpulan, kelompok, grup, atau gerombolan. Dalam biologi misalnya, kita mengenal kelompok flora dan kelompok fauna. Di dalamnya, masih ada lagi kelompok vertebrata, kelompok invertebrata, kelompok dikotil, dan kelompok monokotil. Dalam kehidupan sehari-hari, kalian juga mengenal suku Jawa, suku Madura, suku Sasak, suku Dayak, suku Batak, dan lain-lain. Semua itu merupakan kelompok. Istilah kelompok, kumpulan, kelas, maupun gerombolan dalam matematika dikenal dengan istilah ***himpunan***. Namun, tidak semua kumpulan termasuk himpunan. Contohnya kumpulan siswa yang pandai, kumpulan siswa yang berbadan tinggi. Mengapa demikian? Untuk menemukan jawabannya coba lakukan kegiatan berikut ini

Coba amati beberapa kumpulan yang termasuk himpunan dan bukan himpunan di bawah ini

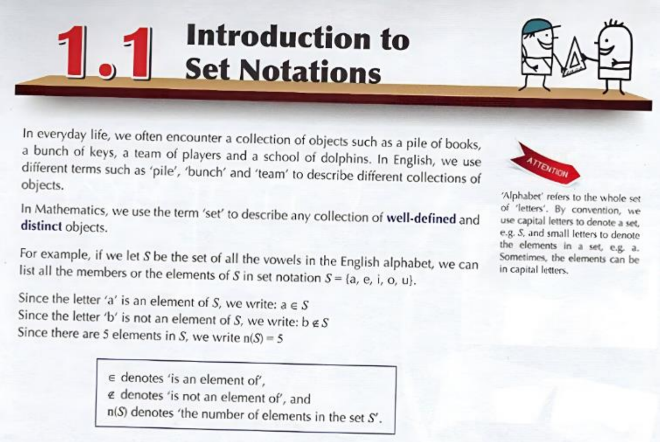
Kumpulan yang termasuk himpunan

1. Kumpulan siswa yang lahir pada bulan Agustus
2. Kumpulan siswa laki-laki
3. Kumpulan buah-buahan yang diawali dengan huruf *M*
4. Kumpulan nama kota di Indonesia yang diawali dengan huruf *S*
5. Kumpulan binatang yang berkaki dua
6. Kumpulan negara di Asia Tenggara

Kumpulan yang termasuk bukan himpunan

1. Kumpulan kota-kota besar di Indonesia
2. Kumpulan orang kaya di Indonesia
3. Kumpulan siswa yang pandai di sekolahmu
4. Kumpulan gunung yang tinggi di Indonesia
5. Kumpulan pelajaran yang disenangi siswa
6. Kumpulan makanan yang lezat

**Figure 2.** Introduction in IB (As'ari et al., 2017)



1.1 Introduction to Set Notations

In everyday life, we often encounter a collection of objects such as a pile of books, a bunch of keys, a team of players and a school of dolphins. In English, we use different terms such as 'pile', 'bunch' and 'team' to describe different collections of objects.

In Mathematics, we use the term 'set' to describe any collection of **well-defined** and **distinct** objects.

For example, if we let $S$ be the set of all the vowels in the English alphabet, we can list all the members or the elements of $S$ in set notation $S = \{a, e, i, o, u\}$.

Since the letter 'a' is an element of $S$, we write: $a \in S$
Since the letter 'b' is not an element of $S$, we write: $b \notin S$
Since there are 5 elements in $S$, we write $n(S) = 5$

$\in$ denotes 'is an element of',
$\notin$ denotes 'is not an element of', and
$n(S)$ denotes 'the number of elements in the set $S$'.

ATTENTION

'Alphabet' refers to the whole set of 'letters'. By convention, we use capital letters to denote a set, e.g. $S$, and small letters to denote the elements in a set, e.g. a. Sometimes, the elements can be in capital letters.

**Figure 3.** Introduction in SB (Yeo et al., 2019)

or *gerombolan*). It triggered the didactical obstacle since there should have been many other equivalent words to refer to the 'set'.

In contrast, SB explained that the equivalent for 'set' followed the object. Therefore, the introduction in SB could be used as an alternative to the didactical obstacle in IB.

The use of examples in IB, 'group of vertebrates, invertebrates, monocots, and dicots', did not correspond to the level of students' understanding and was considered far from their everyday life. In addition, the provided examples did not represent all of the previously mentioned equivalents; the examples merely referred to the word *'kelompok'*. Such conditions triggered epistemological obstacles for students. Meanwhile, the examples in SB involved various equivalent words adapted to the objects and seemed more contextual so that students can imagine and often encounter them in everyday life. In this case, SB could be a reference for resolving the epistemological obstacles in IB. Furthermore, IB gave examples of groups that belong to and do not belong to the sets. Meanwhile, SB justified the meaning of sets in mathematics along with their notations. Overall, the introduction aimed to make students able to define the 'set'.

## Praxis Block Analysis

The type of task (T) in the praxis block was the type of task in the textbook. Each textbook had a different number of T, which were referred to by the initials $T_1, T_2, \dots, T_n$. The praxis block of the learning materials of the concept of set in IB was presented in six types of tasks $(T_1, T_2, \dots, T_6)$, while SB offered the seven types of tasks $(T_1, T_2, \dots, T_7)$ with different presentation characteristics. The six types of tasks in IB were broadly divided into two categories, namely recognizing the definition of the set $(T_1, T_2, T_3, T_4)$ and recognizing the notations of membership of the set ($T_5$, $T_6$). While in SB, the seven types of tasks were categorized into three, namely introducing the notations of membership of the set $(T_1, T_2, T_3, T_4)$, in-depth understanding of the definition of the set $(T_4, T_5)$ and identifying the characteristics of the members of the set $(T_6, T_7)$ (**Table 1**).

The technique (τ) in the praxis block described the method to solve a particular type of T. Each T had at least one type of τ. The type of τ used in this study adopted the findings of Takeuchi and Shinno (2020), consisting of four types of techniques, namely perceptual ($\tau_1$), physical ($\tau_2$), operational ($\tau_3$), and algebraic ($\tau_4$). $\tau_1$ was a T completion using a visual assessment based on the

**Table 1.** Praxis block of IB

| Type of task (T) | Technique (τ) | Description of technique of each textbook |
|---|---|---|
| $T_1$: Why is a collection of cities beginning with the letter $S$ called a set, while a collection of large cities is not? | $\tau_3$ | Performing mental action to formulate the meaning of sets and non-sets based on the previous perceptual process (memorial). |
| $T_2$: What is the difference between a collection called a set and a collection not called a set? | $\tau_3$ | Performing mental action to formulate the meaning of sets and non-sets based on the previous perceptual process (memorial). |
| $T_3$: Write down three examples of collections that belong to a set and three examples of collections that do not belong to a set! Give reasons for each! | $\tau_3$ | Performing mental action to formulate the meaning of sets and non-sets based on the previous perceptual process (memorial). |
| $T_4$: Exchange your answers with your classmates and check the examples of sets and examples of non-sets made by your classmates and discuss if there are any differences! | $\tau_2$ | Reflecting on the results of perceptual and memorial processes and validating findings of the meaning of sets and non-sets through interactions between students (introspective), aiming to draw a valid conclusion about the definition of the set (a priori). |
| $T_5$: After observing the members and non-members, ask questions about the members and non-members of a set!<br>Example: Name the members of the fish! | $\tau_3$ | Performing mental action to formulate the meaning of sets and non-sets based on the previous perceptual process (memorial). |
| $T_6$: Beans are ... of the set of vegetables; beans can be said to be ... of the set of vegetables and are symbolized by ...<br>Catfish is ... of the spices; catfish can be said to be ... of the set of spices and is symbolized by ... | $\tau_3$ | Performing actions in the form of placing the words "member" or "not a member", "element" or "not an element", and the notations "∈" or "∉" to understand the use of membership notation in the set based on the previous perceptual process (memorial). |

**Table 2.** Praxis block of SB

| Type of task (T) | Technique (τ) | Description of technique of each textbook |
|---|---|---|
| $T_1$: $A$ is the set of positive even integers less than 10; "List all the elements of $A$ in set notation" | $\tau_3, \tau_4$ | Performing mental action and mathematical expressions in the form of number identification to conclude the membership of a set through the preceding perceptual and memorial (a priori) processes. |
| $T_2$: $A$ is the set of positive even integers less than 10; "Decide whether each of the following statements is true or false" | $\tau_3$ | Performing mental action to identify the true/false statement about members of a set based on the results of the previous perceptual process (memorial). |
| $T_3$ : $A$ is the set of positive even integers less than 10; "Using the notation $\in$ or $\notin$, describe whether each of the following numbers is an element of, or is not an element of $A$" | $\tau_3$ | Performing mental action to put the notation of members and non-members of a set based on the results of the previous perceptual process (memorial). |
| $T_4$: Given that $B = \{3, 6, 9, 12, 15, \dots, 30\}$, find the value of $n(B)$ | $\tau_3, \tau_4$ | Reflecting on the results of the perceptual and memorial processes to perform an action based on mathematical expression, aiming to formulate the definition of the set and to draw a conclusion about the number of members of a set (a priori). |
| $T_5$: Let $H$ be a collection of all the handsome boys in the class. Is $H$ a set? *Hint: A set is a collection of well-defined objects. Is H well-defined?* | $\tau_2, \tau_3$ | Performing mental action to generate findings from the perceptual and memorial processes about the meaning of sets and non-sets and reflect on them through interaction between students (introspective) to validate the findings, aiming to draw a valid conclusion regarding the definition of the set (a priori). |
| $T_6$: Let $T$ a collection of 2 identical pens. How should we list the elements of $T$? $\{P, P\}, \{P\}$, or $\{P_1, P_2\}$? *Hint: How many elements does T have? A set is a collection of distinct objects. Are the elements in* $\{P, P\}$ *distinct?* | $\tau_2, \tau_3, \tau_4$ | Verifying findings of the membership of a set using contextual content in mathematical notation to draw a valid conclusion (a priori). |
| $T_7$: Let $S$ be the set of letters in the word 'CLEVER'. How should we list the elements? *Hint: Is the letter 'E' distinct?* | $\tau_2, \tau_3$ | Verifying findings of the membership of a set using contextual content in mathematical notation to draw a valid conclusion (a priori). |

appearance of the presented forms; $\tau_2$ was the completion of T performed through physical aids such as rulers, compasses, or others, including performing validation or discussion; $\tau_3$ was a T completion leading student to carry out investigations or discoveries by developing their understanding, and $\tau_4$ was the completion of T employing mathematical expressions.

Overall, the praxis block in IB ranging from $T_1$ to $T_6$ was dominated by $\tau_3$. It indicated that the task design in IB introduced the concept of sets by involving many verifications (observation and development of students' previously acquired knowledge) to form a piece of new knowledge. The acts of observing and verifying knowledge through contextual examples dominated the task design. Thus, there was no opportunity for knowledge construction through an investigation, resulting in a lack of student learning opportunities. Wijaya (2015) argued that the lack of learning opportunities in Indonesian mathematics textbooks potentially caused Indonesian students to struggle to complete their tasks. On the other hand, in Indonesian textbooks, many assignments required students to find solutions without providing any procedures (Hidayah & Forgasz, 2020).

Whereas in SB, knowledge formation was mainly done through observation and identification, as well as the discovery steps. The knowledge formation on the praxis block of SB involved various τ. Overall, $\tau_3$ dominated the process as it was employed in every T, although it collaborated with $\tau_2$ or $\tau_4$ under certain conditions; and even both $\tau_2$ and $\tau_4$ as in $T_6$. Overall, the characteristics of the τ developed constructed a structured and continuous learning trajectory following the referenced formulation (theory). The task design was simple but required a high level of cognitive demand to complete so that the τ used in one T could be of more than one type. It was in line with the findings of Fowler (2015) when comparing Singaporean and US textbooks on the topic of linear functions. Additionally, Sianturi et al. (2021) revealed that Singaporean textbooks required students to master higher levels of cognitive demand, while Indonesian textbooks focused on students' understanding of basic concepts and provided a lower level of cognitive demand (**Table 2**).

$T_1$, $T_2$, and $T_3$ in IB had a similar τ, namely $\tau_3$, to construct a theory (definition of the set), which was later refined by using $T_4$ through $\tau_2$. The expected formulation was $\Theta_1$. However, the task designs $T_1, T_2, T_3$, and $T_4$ only emphasized understanding whether a collection belonged or did not belong to a set. The keyword of $\Theta_1$ was "well-defined". Nevertheless, not all students could achieve the expected formulation using the understanding formed by the $T_1$, $T_2, T_3$, and $T_4$. Due to different levels of intelligence (Guez et al., 2018), some students might not be able to conclude the definition of the set. It suggested that $T_1$ to $T_4$ did not allow students to apply and develop their perceptual, memorial, and introspective abilities in constructing new knowledge in the form of $\Theta_1$. Therefore, overall, the characteristics of the techniques being developed did not form a structured learning trajectory. The justification characteristics of $\tau_3$ in $T_1, T_2$, and $T_3$ did not consider students' diverse knowledge, learning experiences, ways of thinking, and learning potential. In addition, there was no verification of the validity of the new knowledge that students gained as a justified true belief from their findings.

The expectations of $T_5$ and $T_6$ in IB were the students' ability to identify members of a set and understand the use of notation for members and non-members. $\tau_3$ involved in $T_5$ was in the form of questions based on previously acquired knowledge and then proceeded with $\tau_3$ on the $T_6$ by determining "member" or "not a member" and the notation "$\in/\notin$" in the blank space provided, as previously been shown in the related example. Task designs $T_5$ and $T_6$ were relevant to the formulation of expectations, in which $\tau_3$ was used appropriately and interrelated to construct students' understanding of the theory.

$T_1$, $T_2$ and $T_3$ in SB had similar characteristics of τ with one expected formulation that students could understand the meaning of membership of a set and the use of its notation. The context in the task designs $T_1$, $T_2$, and $T_3$ consisted of one mathematical problem, so the solution involved $\tau_4$ as in $T_1$. These three initial task designs were relevant to the expected formulation. The subsequent task designs were related to the introduction of the concept of sets and non-sets. As stated earlier, there was an interrelated correlation between T and τ built by SB in introducing the concept of sets. The context of the problem in $T_4$ was similar to that of the previous problem, namely the mathematical problem. In addition to identifying the members of a set, the problems in $T_1$ to $T_4$ triggered students to understand that the set was a well-defined object. τ developed in $T_4$ was similar to τ developed in $T_1$.

The knowledge obtained independently involving $T_1, T_2, T_3$, and $T_4$ was validated through paired interactions among students in the following task designs ($T_5$, $T_6$, and $T_7$). $T_5$ distinguished the sets and non-sets explicitly. In this case, students had the opportunity to apply and develop their perceptual, memorial, and introspective abilities in constructing new knowledge as a justified true belief. Afterwards, $T_6$ and $T_7$ were constructed to form a broader knowledge concerning the membership of a set. Through the proper content presentation and τ, the gained understanding built could match the expectations, in which students could notice the different or similar members.

## Logos Block Analysis

The components of the logos block included technology (θ) and theory (Θ). θ is a tool or method to justify a τ, while Θ is a conclusion in the form of a theoretical knowledge that serves to generalize the entire process of T, θ, and Θ.

The first three types of T ($T_1$, $T_2$, and $T_3$) in IB served to promote understanding of the meaning of sets and non-sets in a particular collection organized by the $\theta_1$ (a collection of well-defined objects is a set). $\theta_1$ justified the $\tau_3$, as the completion of T was based on mental action to link the perceptual processes and develop the knowledge obtained. $T_4$ in IB was a form of validation to conclude $T_1$, $T_2$, and $T_3$ through $\tau_2$, which was justified by $\theta_1$. Overall, the type of task ($T_1$, $T_2$, $T_3$, and $T_4$) in IB generated a $\Theta_1$ (a set is a collection of well-defined objects). The last two types of T ($T_5$ and $T_6$) in IB were interrelated, but the gained $\Theta$ was different. $T_5$ generated $\Theta_2$ (objects in a set are called members of a set, and other objects are not members of the set). At the same time, $T_6$ resulted in $\Theta_3$ (∈ is a notation for "members/elements of the set" and ∉ is a notation for "not members/not elements of the set"). $T_5$ and $T_6$ in IB were conditioned by $\theta_2$ (use either "words: member/not a member/element/not an element" or "notations: ∈/∉"). $\theta_2$ justified the $\tau_3$ since the completion of T was conducted by connecting the perceptual processes.

In SB, $A = \{2, 4, 6, 8\}$ ($\Theta_4$) was constructed based on $T_1$, $T_2$, and $T_3$, whereas $\Theta_3$ was formed by $T_2$ and $T_3$ using $\theta$ on each of T. $T_1$ in SB was set using $\theta_3$ (2, 4, 6, 8 are positive even integers less than 10). $\theta_3$ justified $\tau_2$ and $\tau_3$ in response to the action of identifying and using mathematical expressions as a basis for completing the T. $\theta_2$ and $\theta_3$ directed the $T_2$ and $T_3$ in SB, and $\tau_3$ in $T_2$ and $T_3$ in SB were justified by $\theta_2$ and $\theta_3$, since the completion of T was carried out by analyzing a statement. $\theta_1$ led the $T_4$ and $T_5$ in SB to understand the definition of sets and non-sets, resulting in a $\Theta_1$. $\tau_3$ and $\tau_4$ were justified by $\theta_1$ due to the identification and the use of mathematical expressions as a basis for solving $T_4$ in SB. $\tau_2$ and $\tau_3$ were justified by $\theta_1$ because of peer validation and identification as a basis for resolving $T_5$ in SB. $T_6$ and $T_7$ in SB were driven by $\theta_4$ (each member of the set has different characteristics compared to the other members). The collaboration of $T_6$ and $T_7$ in SB resulted in $\Theta_5$ (each member of a set has different characteristics). $\tau_2$, $\tau_3$, and $\tau_4$ on $T_6$ in SB were justified by $\theta_4$ due to the validation involving peer interaction and identification employing mathematical expressions as a basis for completing T. $\theta_4$ also justified $\tau_2$ and $\tau_3$ on $T_7$ in SB because there were peer validation and identification as a basis for completing T.

## Comparison of IB and SB

In this segment, researchers discussed the conditions and limitations of the knowledge formation in the

**Table 3.** A regional praxeology of sets concept in IB

| Theme | Number of $T$ | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| Definition of set | $T1$ | $\tau_3$: Operational | $\theta 1$: A collection of well-defined objects is a set | $\Theta 1$: A set is a collection of objects possessing well-defined characteristics |
| | $T2$ | $\tau_2$: Physical<br>$\tau_3$: Operational | | |
| Members of set | $T3$ | $\tau_3$: Operational | $\theta 2$: Each member of the set has different characteristics compared to the others | $\Theta 2$: Objects in a set are called members of the set, and other objects are non-members of the set |
| | $T4$ | $\tau_3$: Operational | $\theta 3$ : Use either "words: member/ not a member/element/not an element" or "notations: ∈/∉" | $\Theta 3$ : ∈ is a notation for "members/ elements of the set" and ∉ is a notation for "non-members/not elements of the set" |

**Table 4.** A regional praxeology of sets concept in SB

| Theme | Number of $T$ | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| Definition of set | $T1$ | $\tau_2$: Physical<br>$\tau_3$: Operational | $\theta 1$ : A collection of well-defined objects is a set | $\Theta 1$: A set is a collection of objects possessing well-defined characteristics |
| Members of set | $T3$ | $\tau_2$: Physical<br>$\tau_3$: Operational<br>$\tau_4$: Algebraic | $\theta 2$: Each member of the set has different characteristics compared to the others | $\Theta 2$: Objects in a set are called members of the set, and other objects are non-members of the set |
| | $T4$ | $\tau_3$: Operational | $\theta 3$: Use either "words: member/not a member/element/not an element" or "notations: ∈/∉";<br>$\theta 4$: 2, 4, 6, 8 are positive even integers less than 10 | $\Theta 3$: ∈ is a notation for "members/elements of the set" and ∉ is a notation for "non-members/not elements of the set" |

textbooks from each country through the analysis of similarities and differences. Previously, researchers had described the T, τ, θ, and Θ of each textbook in praxis block and logos block. The analysis results of the praxis block and logos block could ultimately construct a new type of task (T) to accommodate the entire T in the textbooks of the two countries. T1: defining sets and non-sets; T2: creating examples of sets and non-sets; T3: identifying members and determining the number of members of a set; T4: understanding the membership notations of a set. More specifically, the four new types of tasks could be classified into two themes of discussion: the definition of the set and the members of the set.

**Table 3** and **Table 4** display a similar focus on the elements of praxeology $[T1/\tau_3/\theta 1/\Theta 1]$, the theme of the set's definition $[T3/\tau_3/\theta 2/\Theta 2]$, and the theme of the set's member $[T4/\tau_3/\theta 3/\ \Theta 3]$. The three-point praxeology employed the same techniques on different tasks and theories. One of the techniques used by the three-point praxeology was the operational technique. Meanwhile, the essential differences were identified in $[T2/\tau/\theta 1/\Theta 1]$ in IB, and τ differences in $T1$ and $T2$ in SB in which SB involved more than one technique for this task.

The techniques used by IB in completing the tasks were dominated by operational techniques integrated with contextual problems. The objects' disclosure through the activities familiar to children could provide a sense of closeness (Hong & Choi, 2018). However, the students' understanding was not applied to mathematical problems as in SB. Textbooks providing only a simple context might be good and fundamental but were considered burdensome and not included in moderate or high-level cognitive mastery (Sianturi et al., 2021). It could trigger epistemological obstacles when students deal with mathematical problems requiring higher cognitive abilities (Fuadiah et al., 2019).

For example, students found it challenging to identify the members of the set in the questions such as, "Let $T$ a collection of two identical pens. How should we list the elements of $T$?" and "Let $S$ be the set of letters in the word 'CLEVER'. How should we list the elements?" In this case, the student could not solve problems different from the knowledge they acquired, leading to the breakdown of the process of knowledge formation entirely. The formation of Θ1 through the tasks in SB was more likely to succeed than those in IB. The absence of T2 in SB did not affect the formation of Θ1. Because basically, when students understand that "well-defined" is the key to the definition of the set, mentioning examples would be easy. The task design presented in IB made it difficult for students to understand the concept of "well-defined", even if they engaged in a practical context of everyday life.

In forming Θ2 and Θ3, IB only employed and developed perceptual ability through the duplication of pre-existing displays, so there was no action in constructing knowledge. In the theory of a didactic situation, learning always begins with the action situation (Suryadi, 2019). This situation is considered critical to provide space for students to apply their experiences and knowledge so that the perception of the environment and action in the environment can be adequately realized, and the encapsulation process in the formation of new mental objects can be facilitated. The lack of learning opportunities in IB made it difficult for Indonesian students to complete the tasks that were more difficult or different from the provided examples. Correspondingly, Tumay (2016) states that learning difficulties occur due to errors in delivering the materials, resulting in misconceptions in acquiring knowledge.

The variety of techniques engaged in SB triggered an action situation at the beginning of the introduction of Θ2 and Θ3. For example, in this task, "Given that $B = \{3, 6, 9, 12, 15, \ldots, 30\}$, find the value of n(B)" to determine the number of members of the set, the students first identified the formed composition of the number patterns. It might look simply, but it could trigger students to perform an action (memorial) of knowledge building. The visual elements of the Singaporean textbook were highly emphasized in presenting the topic of discussion, making it richer and easier to understand (Erbas et al., 2012). This pattern has become a part of Singapore's mathematics education curriculum ensuring the mathematical concepts (Ibrahim & Othman, 2010).

## CONCLUSION

Concerning the praxis block, researchers discovered the similarity and specialization of the textbooks of the two countries on the type of task regarding the definition of the set ($T1$). The $T_1$ task type in IB was similar to $T_5$ in SB, aiming to construct new knowledge in the form of $\Theta1$. Nonetheless, the other types of tasks were different in the two textbooks. The dominant operational techniques were used in Indonesian textbooks to complete the given tasks, while such techniques did not stand alone to complete each task in the Singaporean textbook. The task design in the Singaporean textbook was more likely to form a structured learning trajectory than that of the Indonesian textbook. The knowledge building in the Singaporean textbook engaged perceptual, memorial, introspective, and a priori. Meanwhile, in the Indonesian textbook, knowledge formation was only carried out by developing perceptual and memorial. The absence of justification for the conclusion expected by the task design indicated the lack of introspective and a priori development.

This study shows that praxiological analysis helps find and explain knowledge's characteristics (conditions and limitations) in textbooks. In addition, in line with the ATD, our method views textbooks as empirical sources that may demonstrate the knowledge that must be taught during the didactic transposition process. This is different from the methodology of previous textbook research studies. Future research on mathematics textbooks may benefit from this study's theoretical and methodological components. Next, the findings of this study can also help educators, decision-makers, and related parties consider mathematics textbooks from various angles. The advantages and disadvantages of each book can be used as a reference for evaluation actions in studying textbooks in the future.

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