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Pre-service mathematics teachers' mathematical objects and semiotic functions in GeoGebra-based trigonometry tasks: An onto-semiotic analysis

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Abstract

Understanding how pre-service mathematics teachers construct and connect mathematical objects in digital environments remains a persistent challenge in mathematics teacher education. Grounded in the onto-semiotic approach (OSA), this study examined whether pre-service teachers' technically correct GeoGebra constructions of trigonometric functions reflect genuine mathematical understanding, or whether such performance depends on further factors, including how far representations are connected across registers and the quality of mediation the tool affords. Indonesian pre-service mathematics teachers completed GeoGebra-based trigonometric function tasks; data were drawn from written responses, GeoGebra construction files analyzed at the file-structure level, and semi-structured interviews with participants selected for contrasting performance. Results reveal a three-tier proficiency structure, with only a small subset of participants connecting mathematical representations across multiple registers, evidence that technically correct performance does not necessarily indicate procedural mastery. Notably, even the highest-performing participant, once interviewed, revealed an unresolved difficulty translating phase shift between angular and coordinate representations, an obstacle invisible in written work alone. Tool use remained predominantly practical rather than exploratory, suggesting a possible role for instructional guidance in deepening engagement with GeoGebra.

Keywords: onto-semiotic approach, GeoGebra, trigonometric functions, semiotic function, pre-service teachers, mathematical objects

INTRODUCTION

Dynamic mathematics software is now a fixture of pre-service teacher education, and GeoGebra in particular has been studied for how it represents functions and transformations (Hall & Chamblee, 2013; Yildiz & Arpaci, 2024; Zengin, 2019). Hoyles (2018) situates this within a broader shift, arguing that digital tools reshape mathematical practice partly through dynamic graphical representation and partly by offering representational infrastructures that did not previously exist. Specifically for trigonometric functions, GeoGebra has been shown to improve students' ability to associate different representations and interpret graphs (Bedada & Machaba, 2022), and a recent systematic review of pre-service teacher preparation for digital classrooms similarly reports that tools such as GeoGebra can strengthen representational fluency, though integration remains uneven across programs (Wekullo et al., 2026). What is less clear is what pre-service teachers do with

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Contribution to the literature

- This study shows how the onto-semiotic approach can be used to examine pre-service mathematics teachers' mathematical objects and semiotic functions in GeoGebra-based trigonometry tasks.
- It combines written responses, GeoGebra artefacts, and clinical interviews to provide complementary evidence of pre-service teachers' mathematical understanding.
- It identifies difficulties in connecting multiple representations of trigonometric functions, particularly in interpreting phase shift across representational registers, with implications for GeoGebra-supported instruction.

these tools once they are in front of them: how they build mathematical objects, and how, or whether, they connect one representational register to another. Put simply, does working with GeoGebra push a student toward genuine epistemic advancement, or does it mostly support getting the task done without any deeper formalization of the mathematical objects involved?

Trigonometric functions make a good test case. In $f(x) = a \sin(bx + c) + d$, each parameter carries both a procedural rule $T = \frac{2\pi}{|b|}$ and a conceptual object (amplitude, period, phase shift), and making sense of either requires moving between symbolic, graphical, and verbal registers (Duval, 2006). That movement is where students and pre-service teachers tend to struggle: prior work shows the difficulty lies specifically in transforming a mathematical object from one register into another, not in the arithmetic itself (Duval, 2006; Sandoval & Possani, 2016). Within trigonometry specifically, a systematic review of pupils' errors identifies register-related misconceptions as recurring across educational levels (Hamzah et al., 2021), and this extends to pre-service teachers themselves: a recent connection-ability study found first-semester mathematics education students able to link concepts but struggling to relate different representations of trigonometric content (Andriani & Armis, 2025), while an APOS-based analysis of pre-service teachers working with trigonometric limits similarly documented procedural, conceptual, and extrapolation errors tied to representational change (Mangwende & Chirume, 2025). One instance of this broader difficulty, converting between the angle register and the Cartesian coordinate register when interpreting phase shift, is examined closely in this study through clinical interview data.

The onto-semiotic approach (OSA) (Godino et al., 2007, 2019) offers a framework for examining how mathematical objects emerge and connect. It distinguishes six primary mathematical entities and theorizes how they are articulated through semiotic functions (Godino, 2002), which shifts the research question (RQ) away from simply what students do with a tool and toward which mathematical objects they activate, how those objects connect, and at what level of semiotic sophistication; a recent synthesis situates this same framework within cultural historical activity theory, underscoring its continued development as a live research program rather than a fixed model (Godino et al., 2024). Combined with Rabardel's (1995) instrumental genesis theory, and its more recent extension to how learners physically manipulate dynamic geometry objects (Pittalis & Drijvers, 2023), the OSA gives a systematic basis for distinguishing pragmatic from epistemic tool use, a distinction that carries direct instructional consequences.

Three RQs guide the study.

RQ1: Which mathematical objects (OSA entities) emerge in pre-service teachers' problem-solving practices within a GeoGebra-based trigonometry environment?

RQ2: How does digital mediation via GeoGebra function, and to what degree does tool use remain pragmatic versus become epistemic?

RQ3: What semiotic function levels do participants achieve in connecting representational registers, and what learning obstacles impede higher-level semiotic integration? These questions build on each other rather than standing independently.

RQ1 identifies which mathematical objects participants actually activate during the task, a descriptive starting point.

RQ2 then asks about the mechanism behind that activation: whether GeoGebra is functioning as a pragmatic tool for getting the task done or as an epistemic instrument for building knowledge.

RQ3 asks what results from that mediation, namely how far participants manage to connect representational registers, and where that connection breaks down.

Answering all three together, rather than any one in isolation, is what lets the study trace a specific obstacle, the difficulty converting phase shift between registers, back to both the mathematical object involved (**RQ1**) and the mediation process that failed to resolve it (**RQ2**).

This study contributes an integrated design combining written task responses, GeoGebra artefact analysis, and clinical interviews with two purposively selected participants, and examines what each source reveals about pre-service teachers' mathematical activity that the others leave less visible within this dataset. Concretely, this meant working through two linked GeoGebra tasks built around the function, one exploring parameters via sliders and one requiring a manual transformation sketch without software assistance, a design intended to surface exactly where digital and non-digital representational work diverge. Findings are presented and their implications for OSA-informed assessment design are discussed.

THEORETICAL FRAMEWORK

The Onto-Semiotic Approach

The OSA is an anthropological-pragmatic framework built on the claim that mathematical objects are constituted through, rather than prior to, the institutional and personal practices in which they are used (Godino & Batanero, 1994; Godino et al., 2007, 2019). Within this system of practices, the OSA distinguishes six primary mathematical entities that any mathematical practice activates:

(1) language and representations, ostensive objects such as symbols, graphs, and verbal descriptions;

(2) concepts and definitions, the non-ostensive objects that language represents;

(3) propositions and properties, relational statements connecting concepts;

(4) procedures and algorithms, ordered sequences of operations;

(5) arguments and justifications, reasoning that validates propositions; and

(6) problems and situations, the contextual motivators of mathematical activity (Font et al., 2013; Godino et al., 2014).

The interrelations among these six entities are theorized through epistemic configuration, the network of objects and connections that constitutes mathematical content within a given practice situation (Godino et al., 2014).

This entity-based decomposition has proven productive across a range of mathematical objects beyond the present study's focus on trigonometric functions. It has been used to analyze the onto-semiotic complexity of the definite integral (Burgos et al., 2021), diagrammatic reasoning (Giacomone et al., 2023), and algebraic meaning-making across multiple representations (Erbilgin & Gningue, 2023). A recent synthesis situates the same framework within cultural historical activity theory, underscoring its continued development as an active research program rather than a fixed model three decades after its first formulation (Godino et al,. 2024). Together, these applications provide precedent for extending the OSA's entity-based analysis to the representational demands of GeoGebra-based trigonometry tasks in the present study. Central to this application is the OSA construct of semiotic function (Godino, 2002): a correspondence that associates an expression with a mathematical object as content, established according to a rule, code, or criterion within a given practice. Building on this construct and on Duval's (2006) theory of representational registers, the present study operationalizes semiotic function through three analytically distinguishable levels of register connection: level 1, where the function operates within a single register, typically verbal description; level 2, where the function connects two registers, most commonly verbal and graphical; and level 3, where the function connects three or more registers, including the symbolic-algebraic register required for mathematical formalization. This three-level operationalization, adapted specifically for the present analysis, provides the primary lens for **RQ3**.

Instrumental Genesis and Digital Mediation

Rabardel's (1995) theory of instrumental genesis, extending Vérillon and Rabardel's (1995) earlier account of instrumented activity, analyses how users transform tools, physical or digital, into instruments of knowing through the reciprocal transformation of both artefact and user. This transformation proceeds through the development of utilization schemes: stable, generalizable action patterns that users bring to bear across analogous problem situations (Ritella & Hakkarainen, 2021; Shvarts et al., 2021). Rabardel's (1995) broader typology distinguishes several modes of mediation between a subject and the object of an activity, including pragmatic mediation, concerned with action on the object, epistemic mediation, concerned with getting to know the object, and reflexive or heuristic mediation, concerned with the subject's regulation of their own activity (Rabardel & Bourmaud, 2003). The present study centers on the first two of these, since they are most directly diagnostic of whether GeoGebra use in this dataset remains task-oriented or becomes knowledge-oriented, a distinction central to **RQ2**. A student who uses GeoGebra's slider to observe that "the graph gets tighter as b increases" engages primarily in pragmatic mediation; a student who uses the same slider to derive $T = \frac{2\pi}{|b|}$ by tabulating observed period values engages in epistemic mediation. The two are not mutually exclusive, and pragmatic engagement can precede and scaffold epistemic advancement; what matters analytically is whether a given tool interaction terminates in task output or extends to mathematical insight.

This dialectic between technical facility and conceptual work was foundational to early research on computer algebra environments (Artigue, 2002), and has since been extended to dynamic geometry software, where specific tools have been shown to function as instruments of semiotic mediation for the concept of function (Falcade et at., 2007). Dynamic mathematics environments more broadly have been associated with gains in mathematical connection-making (Zengin, 2019), though such gains are not automatic: a recent study of pre-service teachers' continued use of GeoGebra found that technology acceptance and pedagogical content knowledge do not always develop in step with one another (Yildiz & Arpaci, 2024). Most directly relevant to the present analysis, the same instrumental genesis lens has recently been applied to how learners physically manipulate dynamic geometry objects, showing that even embodied actions such as dragging carry epistemic or pragmatic significance depending on how they are coordinated with conceptual goals (Pittalis & Drijvers, 2023). This separation between technical fluency and conceptual integration is not confined to trigonometry or to GeoGebra specifically: a recent systematic review of pre-service teacher preparation for digital classrooms found that tool sophistication and constructivist, knowledge-building use do not automatically co-occur, even where programs actively promote both (Wekullo et al., 2026), consistent with Hoyles' (2018) broader observation that digital tools reshape mathematical practice through multiple, partially independent mechanisms rather than a single uniform effect.

Representational Registers and Conversion

Duval's (1993, 2006) theory of semiotic representation registers distinguishes treatment, transformation within a single register, such as simplifying an algebraic expression or reading a graphical feature without leaving the graphical register, from conversion, transformation between registers, such as moving from a symbolic expression to its graphical form or from a verbal description to a formal definition. Duval (2006) argues that conversion is generally the more cognitively demanding of the two operations, because it requires recognizing that the same mathematical object is being represented despite surface differences between registers; this cross-register recognition is what constitutes semiotic function at level 2 and level 3 in the operationalization. Register conversion difficulties of this kind are not specific to trigonometry. In linear algebra, once students select a working register for representing vectors and planes, they rarely attempt spontaneous transformations into alternative registers, even when doing so would facilitate the task at hand (Sandoval & Possani, 2016). Within trigonometry specifically, a systematic review of pupils' misconceptions identifies register-related errors as recurring across educational levels and instructional contexts (Hamzah et al., 2021); a study explicitly framing trigonometry teaching around representation register changes similarly argues that many persistent difficulties in the topic are best explained as conversion, rather than treatment, failures (Báez Ureña et al., 2023); and an APOS-based analysis of pre-service teachers working with trigonometric limits documented procedural, conceptual, and extrapolation errors that co-occurred with, and were plausibly related to, difficulty moving between representational forms (Mangwende et al., 2025). Complementing this, an experimental study found that GeoGebra use measurably improved students' ability to associate different representations of trigonometric functions, suggesting that dynamic visualization can support, though not guarantee, the register connections that conversion requires (Bedada & Machaba, 2022), while a connection-ability study of first-semester mathematics education students found many already able to link concepts but specifically struggling to relate different representations of trigonometric content (Andriani & Armis, 2025).

The present study examines one specific conversion difficulty that emerged as an obstacle in this cohort: the conversion between the angle register ($\frac{\pi}{4}$ = 45°) and the Cartesian coordinate register (x-position = $\frac{\pi}{8}$) in phase shift interpretation. In the angle register, phase shift is an angular displacement, a rotation measured in degrees or radians; in the Cartesian coordinate register, the same displacement appears as a horizontal translation of the graph, measured in x-units. These are not two notations for the same content but structurally different representational systems, and moving between them requires reconstructing the mathematical object, phase shift, within a new representational logic. As detailed in clinical interview data documented this specific conversion difficulty in both interviewed participants, at different performance levels; confirming how far it extends beyond these two participants would require a larger interview sample, and limitation addressed.

METHODOLOGY

Research Design and Participants

This study adopts a qualitative collective case study design with embedded individual units of analysis (Stake, 1995; Yin, 2018). The case is defined as the cohort of ten pre-service mathematics teacher students engaged in a single GeoGebra-based trigonometry session; individual participants constitute embedded units enabling systematic cross-case comparison. This design suits the RQs for two reasons. First, the phenomena under investigation (learning obstacles, semiotic function levels, and patterns of instrumental mediation) are distributed across participants and are more analytically legible at the group level than within any single case. Second, the embedded unit structure allows movement between levels of analysis: cohort-level patterns in mathematical object emergence can be identified, and individual cases can then be examined in depth where those patterns intensify or break down. The qualitative orientation follows from what the RQs ask. Questions about mathematical object emergence, semiotic connection quality, and learning obstacle anatomy do not reduce to numerical indices; they require reconstructing the internal logic of a

Table 1. Participant demographics, background characteristics, and role in study (N = 10)

Code	Gender	Prior trigonometry	GPA (4.0)	Role in study
S1	F	Completed	3.82	Interview participant (S1)
S2	F	None	3.45	Written + GGB
S3	F	None	3.10	Interview participant (S3)
S4	M	Completed	3.67	Written + GGB
S5	F	None	3.38	Written + GGB
S6	F	None	3.21	Written + GGB
S7	M	Completed	3.55	Written + GGB
S8	F	Completed	3.71	Written + GGB
S9	F	Completed	3.60	Written + GGB
S10	F	None	2.90	Written + GGB

Note. GPA: Grade point average on a 4.0 scale; Prior GeoGebra experience was collected as a single cohort-level self-report item rather than a verified per-participant record, and is therefore not disaggregated by individual code in this table; & S1 and S3 participated in post-session clinical interviews

participant's mathematical activity and identifying the specific point at which a conversion attempt fails. Quantitative performance measures can signal that something goes wrong, but they cannot specify what goes wrong or why. Qualitative analysis of written responses, GeoGebra artefacts, and clinical interview transcripts is therefore the mode of inquiry best suited to answering the questions as formulated in this study.

The study is situated within the OSA tradition of epistemic configuration analysis (Godino et al., 2014), which treats multi-source qualitative triangulation as its primary methodological mode. Written task responses reveal what participants produce; GeoGebra XML artefacts reveal what participants construct and manipulate in the digital environment; clinical interviews reveal how participants reason and where their reasoning stalls. The three sources are not redundant alternatives: each is capable of revealing phenomena less accessible through the other two alone, and collecting all three is a methodological requirement rather than a gesture toward comprehensiveness. Ten pre-service mathematics teacher students (N = 10) enrolled in a university-level mathematics methods course at a state university in Indonesia participated in a 90-minute structured task session on 17 May 2026. Participants completed two trigonometric function tasks. Task 1 required parameter exploration of $f(x) = a \sin(bx + c) + d$ using GeoGebra sliders, with participants manipulating each parameter independently and recording their observations in writing. Task 2 required analysis and interpretation of a specific function, $g(x) = 3\sin(2x - \frac{\pi}{4}) + 1$: participants identified each parameter symbolically, compared the transformed graph against a reference function, produced a manual transformation sketch without GeoGebra assistance, and reflected in writing on how GeoGebra had supported or constrained their understanding. The two-task sequence was designed so that task 1 would engage participants at the level of pragmatic mediation, while task 2 would press them toward epistemic mediation and cross-register semiotic conversion. All participants provided signed informed consent prior to the session; institutional ethical clearance was obtained before data collection began. **Table 1** summarizes participant background characteristics.

Prior GeoGebra experience was collected via a three-point self-report item at the outset of the session (never used it before/used it one to two times before/used it frequently before). Nine of the ten participants reported having used GeoGebra one to two times previously; one participant reported frequent prior use; none reported having never used it. This item was recorded as a single cohort-level self-report rather than a verified, independently corroborated per-participant record, and is reported here at the aggregate level rather than disaggregated by individual code.

Data Sources, Collection, and Analysis Procedures

Three convergent data sources were employed to construct a multi-layered picture of mathematical object emergence and semiotic function. Written task responses: participants completed structured response sheets for both tasks across six sub-items (T1a, T1b, T2a, T2b, T2c, and T2d), scored using an OSA-informed rubric (maximum 23 points). GeoGebra construction artefacts (.ggb files): submitted at session end, analyzed at the XML level for function definitions, slider linkages, command usage, parameter values (indicating authentic exploration vs. default inputs), and file identity (GeoGebra internal ID hash enabling detection of shared files). Clinical interviews: conducted with two purposively selected participants (S1 and S3) on the same session day, each lasting approximately 45 minutes, audio-recorded and transcribed verbatim. A semi-structured protocol comprising 12 items in three sections (B: mathematical objects, **RQ1**; C: digital mediation, **RQ2**; D: semiotic functions, **RQ3**) was used, with the participant's own written responses and GeoGebra file displayed as concrete stimulus material during the interview (**Table 2**).

Written responses were independently scored by two researchers using the OSA rubric; inter-rater reliability on a randomly selected 30% sample (n = 18 sub-task

Table 2. Interview protocol: Item mapping to research questions, OSA entities, and analytical purposes

Section	Item	RQ	OSA entity	Sample probe	Analytical purpose
B	B1	**RQ1**	Language/ representation	When you see $f(x)=a\ sin(bx+c)+d$, what mathematical concepts come to mind first?	Map spontaneous terminological activation and anchor (visual vs. textual)
	B2	**RQ1**	Concepts/ definitions	How do you define amplitude and period? How do they appear on the graph?	Assess conceptual connectivity: does the student link concept to coordinate representation?
	B3	**RQ1**	Procedures/ algorithms	Walk me through your procedure when working with GeoGebra for these tasks.	Identify procedural strategy type: adaptive vs. imitative
C	C4	**RQ2**	Digital mediation	Did GeoGebra give you any new understanding, or did it confirm what you already knew?	Classify mediation type: epistemic (knowledge-building) vs. pragmatic (task-completion)
	C5	**RQ2**	Instrumental genesis	Which GeoGebra features did you use and why?	Assess utilization scheme development and contextual feature selection
	C6	**RQ2**	Representational preference	Which representation was most helpful, formula, graph, or sketch? Why?	Identify dominant representational register and cross-register awareness
	C7	**RQ2**	Instrumental obstacle	Did you encounter any difficulty using GeoGebra? How did you respond?	Identify technical/instrumental obstacles and coping strategies
D	D8	**RQ3**	Semiotic function	What does the "3" in $g(x)=3sin(2x-\frac{\pi}{4})+1$ represent to you? Can you trace that to the graph?	Assess semiotic function: symbol → concept (3-layer vs. 2-layer)
	D9	**RQ3**	Expression-content link	Can you explain the phase shift $\frac{\pi}{4}$? What does it mean and how does it show on the x-axis?	Diagnose register conversion obstacle: angle register → Cartesian coordinate register
	D10	**RQ3**	Representational transformation	What was the hardest step when sketching the transformation manually?	Identify semiotic transfer obstacle: digital-to-manual register migration
	D11	**RQ3**	Arguments/ justification	How do you justify that $g(x)$ is different from $h(x)=sin(x)$?	Classify argument level: deductive-algebraic vs. observational-visual
	D12	**RQ3**	Synthesis	Overall, which concept became clearest through GeoGebra? Why?	Assess metacognitive synthesis and representational self-awareness

Note. Sections B, C, and D correspond to **RQ1**, **RQ2**, and **RQ3**, respectively; The participant's own written responses and GeoGebra file were used as stimulus material throughout; & Interviews were conducted in Bahasa Indonesia and translated to English for analysis

responses) yielded Cohen's κ = 0.87 (strong agreement; Landis & Koch, 1977). GeoGebra artefact analysis proceeded in three stages: XML structure parsing to identify function definitions, parameter linkages, and command usage; file identity analysis to detect shared artefacts via internal GeoGebra file ID matching; and complexity classification into four levels (L1: minimal/non-compliant; L2: functional-basic; L3: functional-exploratory with authentic parameter values; L4: functional-analytic with advanced commands such as intersect and extremum). Interview transcripts were analyzed using Miles et al.'s (2014) framework: first-pass descriptive coding mapped to the interview protocol item and its OSA entity target; second-pass interpretive coding to classify mediation type, semiotic function level, and argument level; and third-pass triangulation against written response and GGB artefact findings. Triangulation across the three data sources followed Denzin's (1978) data source triangulation protocol, with three explicitly defined convergence categories:

(1) CONVERGENT, all three sources support the same analytical claim;

(2) REVISORY, interview data revises the interpretation generated from written and GGB artefact data alone; and

(3) NEW, interview data reveals findings inaccessible to the other two sources.

Selection and Justification of Interview Participants

The selection of two participants for clinical interview reflects a methodological decision grounded in five interconnected principles, detailed below. Criterion 1, the maximum variation principle (Patton, 2002). Total OSA scores ranged from 91% (S1) to 17% (S10). Maximum variation purposive sampling selects cases at extreme poles of a distribution to maximize the

Table 3. Interview participant selection: Sampling strategy, OSA criterion, and analytical justification

Code	Sampling strategy	OSA-specific criterion	What interview data can reveal that written + GGB cannot	Score
S1	Maximum variation: extreme high case (Patton, 2002)	Full activation across all 6 OSA entities; L3 semiotic function; epistemic mediation confirmed from GGB artefact; zero learning obstacles identified in written data	(1) Whether high-level procedural performance reflects genuine analytical procedure or estimation (revealed: phase obtained via visual estimation, not set $bx+c=0$). (2) Quality of semiotic function: does symbol-to-concept connection operate through the general form as a semiotic scaffold? (confirmed). (3) Nature of epistemic mediation: verbal articulation of knowledge-construction process vs task completion.	21/23 (91%)
S3	Maximum variation: extreme low case + negative case analysis (Lincoln & Guba, 1985)	Low activation across 5 of 6 OSA entities; 3 simultaneous learning obstacle types; GGB artefact integrity concerns; L1 semiotic function, densest obstacle cluster in dataset	(1) Distinguish 3 simultaneous obstacles: conceptual misconception vs copying behavior vs GGB instrument confusion, a distinction impossible from written artefacts alone. (2) Verify artefactual evidence of file sharing: does S3 acknowledge non-independent GGB production? (revealed: implicit acknowledgement in B3). (3) Identify whether understanding is at declarative (definitional) or functional-relational level. (4) Assess whether GeoGebra interaction produced any partial insight despite low written scores.	8/23 (35%)

Note. S1 and S3 were selected to maximize theoretical contrast across all three analytical frameworks (OSA entity activation, Rabardel, 1995; mediation typology, Godino, 2002 semiotic function level) & Selection of both from the medial range would have produced theoretically redundant profiles given the written and GGB data already available for those participants

theoretical contrast between successful and obstacle-laden mathematical practice configurations (Godino et al., 2019). The S1-S3 pairing (91% vs. 35%) represents the most productive contrast, placing the two interview subjects at opposite poles of the epistemic configuration spectrum. Selection from the middle of the distribution, for example S4 (74%) or S2 (52%), would have yielded overlapping profiles with limited incremental theoretical yield.

Criterion 2, representational contrast across Rabardel's (1995) mediation typology. Artefact analysis identified S1 as functional-exploratory (level 3, epistemic mediation) and S3 as functional-basic with non-originality indicators (level 2, mediation severely constrained). Verbal corroboration of this contrast is theoretically necessary, since GGB artefact complexity does not by itself distinguish between epistemic and pragmatic mediation. Interview item C4 was designed specifically to elicit each participant's own metacognitive account of whether and how GeoGebra produced new knowledge. Criterion 3, negative case necessity for S3 (Lincoln & Guba, 1985). S3's dataset presents three simultaneous co-occurring learning obstacle types: epistemological (reflection axis misconception), propositional-procedural (period formula errors), and artefactual (identical GGB file ID with S7; verbatim text overlap with S5). Written and artefact data alone would not reliably discriminate between

(a) genuine conceptual misconception,

(b) performance degraded by file copying, or

(c) confusion about the GeoGebra instrument itself, making clinical interview methodologically necessary to avoid misattributing these obstacles to the wrong cause.

Criterion 4, artefact completeness and feasibility of stimulus-based interviewing. The interview protocol required each participant's own written responses and GeoGebra file as concrete stimulus. Both S1 and S3 had complete artefacts. Excluded from interview were S10 (systematic non-completion of written responses, rendering stimulus-based probing unfeasible), S6 (no screenshot evidence, incomplete GGB stimulus), and the medium-tier participants (S2, S4, S5, S7, and S9), whose OSA profiles showed overlapping, medium-activation patterns; the written and GGB data for these profiles showed converging, non-novel patterns that did not warrant further probing. Criterion 5, the same-day interview protocol adopted to avoid retroactive memory contamination. All participants were enrolled students for whom extended participation beyond the session day was ethically constrained, and recall-based retrospection on GeoGebra reasoning is susceptible to post-hoc rationalization. The same-day format, while limiting the number of interviews, is a methodologically defensible approach for capturing genuine in-situ reasoning quality.

Table 3 summarizes these criteria for each interviewed participant and provides the basis on which S6, S9, and S10 in particular were excluded, directly relevant to how each participant's role in **Table 1** was assigned.

RESULTS

Results are presented in five integrated sections. We first address mathematical object configuration (**RQ1**); we then address GeoGebra artefact patterns and digital mediation (**RQ2**); next we present the clinical interview analysis for S1 and S3 across all three RQs; after that we present the formal triangulation synthesis across all

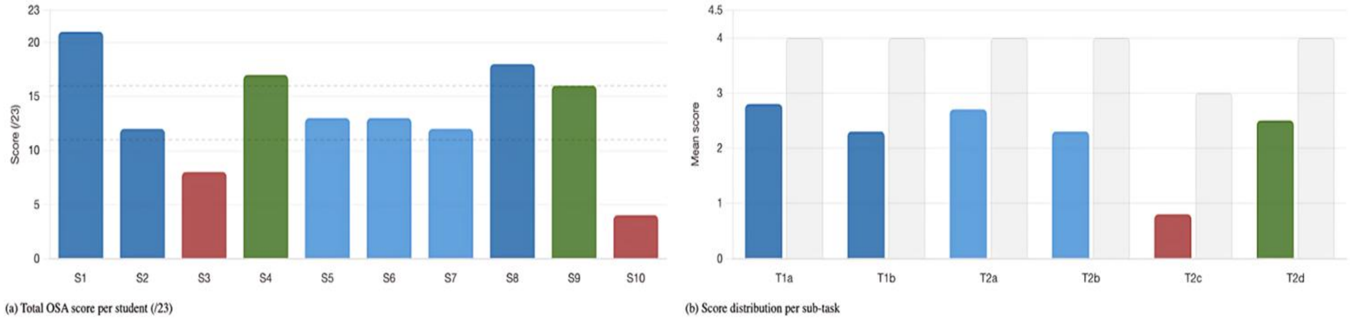

Figure 1. (a) Total OSA scores per participant (/23) (dashed lines demarcate proficiency tiers) & (b) Mean score per sub-task (T2c and T1b show widest variance, identifying semiotic transfer and propositional formalization as primary learning obstacle loci) (created by the authors)

Table 4. Epistemic configuration matrix: OSA entity activation by participant (all three data sources)

OSA entity	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Language/representation	H	M	L	M-H	M	M	M	H	M	L
Concepts/definitions	H	M	L	H	M	M	L	H	H	L
Propositions	H	L	-	H	L	L	L	M-H	M-H	-
Procedures	H	L	L	H	M	H	L	H	H	-
Arguments	H	-	-	H	L	M	-	M-H	M-H	-
Problems/situations	H	M	L	H	M	M-H	L	H	M-H	L
Semiotic function	L3	L1	L1	L2	L1	L1	L1	L3	L2	-
GGB mediation	E	P	P*	EP	P	P	P	EP	P	-

Note. H: High; M-H: Medium-high; M: Medium; L: Low; -: Absent; Semiotic function levels: L1: Verbal only; L2: Verbal + graphical; L3: Verbal + graphical + symbolic; Mediation: E: Epistemic; P: Pragmatic; EP: Mixed; *Pragmatic with non-originality evidence (S3); & GGB mediation classification integrates written, artefact, and interview data

three data sources; and finally we present the learning obstacle taxonomy for the full sample (**RQ3**).

Mathematical Object Configuration: Written Response Analysis (RQ1)

Overall performance and proficiency tiers

Total OSA scores ranged from 21/23 (91%, S1) to 4/23 (17%, S10), with a mean of 13.4/23 (58.3%, SD = 5.2). Three proficiency tiers emerged: high (S1 = 91%, S8 = 78%); medium (S4 = 74%, S9 = 70%, S5 = 57%, S6 = 57%, S2 = 52%, S7 = 52%); and low (S3 = 35%, S10 = 17%). Sub-task T2c (manual transformation sketch; group mean = 0.8/3) was most discriminating, followed by T1b (period formula; mean = 2.3/4) (**Figure 1**).

OSA entity activation matrix

Table 4 presents the comprehensive epistemic configuration matrix across eight analytical dimensions. Two cross-cutting findings emerge. First, the arguments/justifications entity was the weakest dimension across all proficiency tiers; even high performers S1 and S8 showed lower activation on argumentation relative to their procedural performance. This suggests that mathematical justification constitutes a cross-cutting learning obstacle independent of procedural competence. Second, the language/representation dimension exhibited partial formalization: students who correctly used the term "amplitude" (e.g., S4 and S6) did not consistently apply formal terminology across all parameter cases, defaulting to informal language for less familiar cases ($0 < |a| < 1$; negative b). This instrumental linguistic obstacle, access to the sign without full conceptual denotation, is theorized.

Table 4's cell-by-cell profile is necessarily read one entry at a time; the same activation pattern becomes easier to see in aggregate once rendered as a heatmap. **Figure 2** presents this same eight-dimension, ten-participant matrix in that form, and three features stand out immediately that are less visible in tabular format: the near-uniform high activation running across S1 and S8's rows, the largely blank row for S10 reflecting systematic non-completion rather than low performance on completed items, and the concentrated cluster of gaps across five of S3's six core entities rather than a single isolated weak dimension.

Whereas **Figure 2** displays the full cohort across all eight dimensions, **Figure 3** narrows the comparison to three participants representing the proficiency range identified, S1 (high), S4 (medium), and S3 (low), with each profile normalized across the six core OSA entities. Plotting the three tiers together makes the shared dip in the arguments dimension visible as a shape rather than as separate numbers: even S1's otherwise high profile narrows noticeably at arguments, a pattern that is harder to notice when each participant's entity scores are read as isolated table cells.

Figure 2. OSA entity heatmap: 8 dimensions × 10 participants (color intensity = activation level; S1 and S8 show consistent high activation; S10 shows systematic non-completion; S3 shows critical gaps across 5 of 6 core entities) (created by the authors)

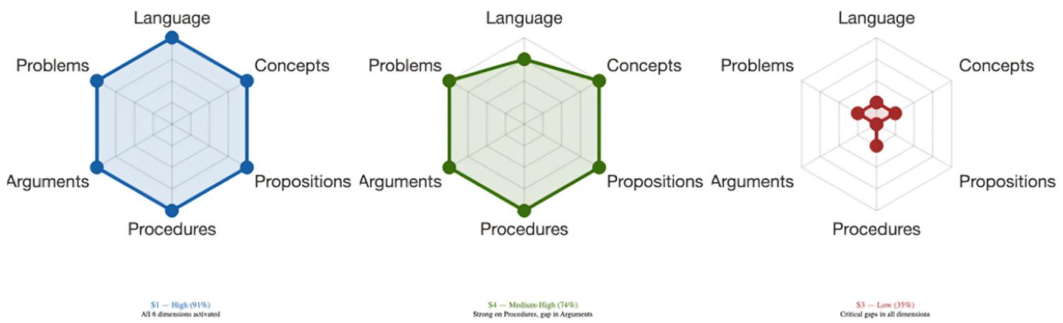

Figure 3. OSA epistemic profiles for three representative participants (S1: high; S4: medium-high; S3: low) across 6 OSA entities, normalized 0-1 (note consistently low arguments dimension across all levels) (created by the authors)

GeoGebra Artefact Analysis and Digital Mediation (RQ2)

Artefact analysis classified GeoGebra files into four complexity levels. S2 (task 2 file) achieved level 4 (functional-analytic), using intersect and extremum commands. S1, S4, S8, and S9 achieved level 3 (functional-exploratory), with authentic non-standard parameter values evidencing genuine exploration (e.g., S8: slider b = -1.74, a = -3). S3, S6, and S7 achieved level 2 (functional-basic); S3 and S6 shared an identical GeoGebra internal file ID (hash 428a56f4 ...), constituting direct artefactual evidence of file duplication. S5 and S10 achieved level 1 (minimal/non-compliant) (**Figure 4**).

Across the sample, GeoGebra mediation was predominantly pragmatic. Eight of ten participants used GeoGebra primarily for visual task completion, observing parameter effects without transitioning to symbolic formalization. Only S1 and S8 demonstrated epistemic mediation from artefact evidence alone (clean exploratory constructions, authentic parameter values, full written formalization). S2 presented an anomaly: the highest GGB complexity (level 4, use of analytic commands) with a medium OSA score (52%) and verbal-only semiotic function. This dissociation, technical sophistication without semiotic integration, is theorized with reference to van Dijke-Droogers et al. (2021) (**Figure 5**).

Clinical Interview Analysis: S1 and S3 (RQ1, RQ2, RQ3)

S1: High proficiency profile, epistemic mediation and residual phase shift obstacle

S1's interview revealed three analytically significant findings that extend and, in one case, revise the interpretation generated from written responses and GGB artefacts alone.

RQ1: Mathematical objects (section B1-section B3). S1's terminological activation was triggered by the graphical register, not the symbolic: "karena grafiknya berbentuk gelombang maka yang terlintas adalah periode dan frekuensi, dimana menentukan panjang dan

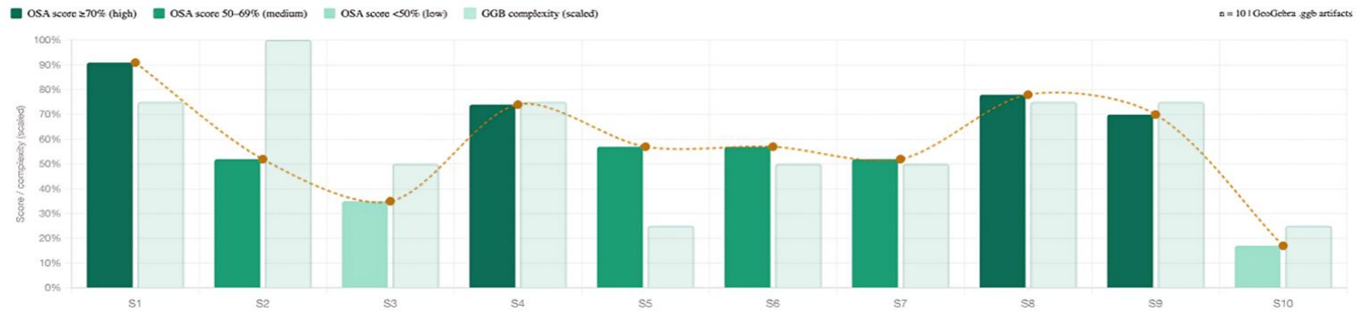

Figure 4. GeoGebra complexity level and OSA score by participant (bar: OSA score (%); tinted bar: GGB complexity [scaled ×25]; dashed amber: OSA trajectory; S2 anomaly: highest GGB complexity [L4] yet medium OSA, technical sophistication does not guarantee semiotic integration) (created by the authors)

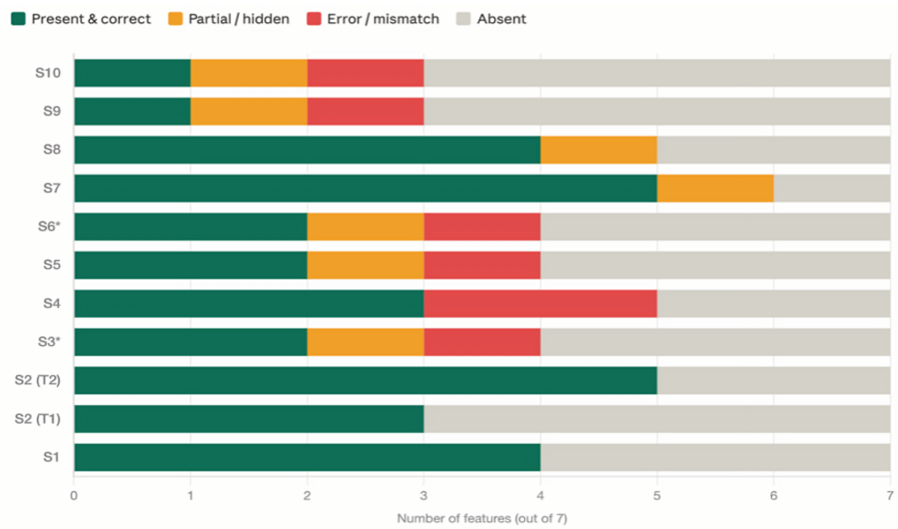

Figure 5. Quadrant scatter plot: digital mediation type (*x*: pragmatic to epistemic) vs. semiotic function level (y: L1-L3) (circle size proportional to GGB complexity; red-bordered: non-original file [S3, S6]; note absence of any participant in Q4 [epistemic, verbal only]) (created by the authors)

juga banyaknya gelombang yang ada" [because the graph is wave-shaped, what comes to mind is period and frequency, determining the length and number of waves]. This visual-functional anchor distinguishes S1's object activation from definitional anchoring observed in lower-performing participants.

S1 also elaborated referential coordinates for amplitude and period ("amplitudo dapat terlihat dengan melihat secara vertikal pada sumbu y, sedangkan periode dapat terlihat secara horizontal pada sumbu x" [amplitude visible vertically on the y-axis, period visible horizontally on the *x*-axis]), an unprompted connection from concept to coordinate space indicating that S1's concepts are linked to propositions (OSA entity 3), not merely named.

> S1: I feel more confident when I already have a concrete visual representation before working on the task.

This visual-first procedural strategy (B3) explains the efficiency of S1's GGB artefact (two relevant functions only, no redundant objects) and confirms that GeoGebra has been instrumentalized as a cognitive scaffold within S1's problem-solving scheme.

RQ2: Digital mediation (section C4-section C7). S1 articulated a clear epistemic mediation account:

> S1: I gained new understanding when changing the values; I was indirectly building an understanding of the importance of values in the general form of trigonometric functions.

The phrase "membangun pemahaman" [building understanding] is a verbal marker of epistemic mediation (Rabardel, 1995): GeoGebra is functioning as a tool for knowledge construction, not merely task execution. S1 further demonstrated a mature utilization scheme through elaborated knowledge of slider directionality (C5) and identified accuracy and speed of GeoGebra graphing as liberating cognitive capacity for analysis (C6), consistent with cognitive load theory (Sweller, 1988). No technical obstacles were reported (C7), confirming successful instrumentation.

RQ3: Semiotic functions (section D8-section D12). S1's semiotic function for the symbol "3" in $g(x)$ operated through three layers: the numeral "3" → positioned within the general form $f(x) = a \sin(bx+c)+d$ as variable a → labelled as the concept "amplitude" (D8). This is the highest-level semiotic function observed in the sample: symbol → semiotic scaffold → named concept.

The most theoretically significant finding from S1's interview is a residual learning obstacle invisible in the written data. S1 scored 4/4 on T2a, including the phase shift. However, D9 revealed that this correct answer was obtained through visual estimation from GeoGebra, not analytical procedure:

> S1: I am still quite confused when faced with phase shift. The meaning of $\frac{\pi}{4}$ is 45 degrees, but I am confused about how to place it on the x-axis.
>
> S1: For variable C as phase shift, I made an estimate because I am still confused about applying the value of C in the Cartesian plane.

This interview-exclusive finding reveals that S1's phase shift answer was correct but obtained via a process different from the expected analytical procedure (set $bx+c=0$, solve for x). The obstacle is not in value comprehension (S1 correctly identifies $\frac{\pi}{4}$ = 45°) but in register conversion: translating a value from the angle register to a position on the Cartesian x-axis. This is precisely the conversion failure theorized by Duval (1993, 2006) as the most cognitively demanding operation in mathematics learning. S1's argument quality was the highest observed (D11): deductive-algebraic, comparing parameter values across $g(x)$ and $h(x)$ to infer graphical consequences via the general form as a propositional bridge.

S3: Low proficiency profile, multiple simultaneous obstacles and pragmatic-perceptual mediation

S3's interview produced four analytically distinct findings that clarify, corroborate, and extend the written and artefact data.

RQ1: Mathematical objects (B1-B3). Unlike S1's visual-functional terminological anchor, S3's activation was triggered by the symbolic-textual register: "I immediately thought of sine function, which I know from the textbook." Concepts were defined correctly as isolated declarative objects (amplitude as graph height, period as cycle length) without any connection to coordinate representations or propositional relationships. In OSA terms, S3's concepts exist at the nominative level: correctly labelled but not propositionally embedded.

S3's procedural description (B3) was characterized by imitative-algorithmic execution without adaptive strategy:

> S3: I followed the steps: opened GeoGebra, determined the function, when entering the formula it might be the same.

The phrase "it might be the same" is an implicit acknowledgement that the formula-entry process was not always independent, directly corroborating the artefactual evidence of file sharing with S7. This verbal-artefactual convergence on non-independent production constitutes the strongest triangulation finding in dataset.

RQ2: Digital mediation (C4-C7). S3 described a global relational insight during slider interaction:

> S3: My thinking is that all of this is interrelated. Both amplitude and period all have different values.

This global relational insight, "all interrelated, all different values," represents a partial pragmatic mediation: GeoGebra successfully conveyed that parameters are distinct and produce different effects. However, this insight was not formalized: no verbal proposition such as $T = \frac{2\pi}{|b|}$ followed from the slider observation. S3 also reported instrumental obstacles (C7) absent in S1: "ketika saya memasukkan rumus dan saya kurang teliti dalam penulisan di buka kurung tutup kurung" [when I enter the formula and I am careless in writing the opening and closing brackets], addressed via wholesale restart rather than targeted error diagnosis, consistent with imitative-algorithmic procedure (B3). Furthermore, S3 referenced the "circles" feature (C5), a GeoGebra feature entirely irrelevant to trigonometric function plotting, revealing an underdeveloped contextual feature-selection map, indicative of instrumentalization in its early stages.

RQ3: Semiotic functions (D8-D12). S3's semiotic function for "3" in $g(x)$ operated at two layers: numeral "3" → multiplication operation → visual effect on graph (points 3 times further or closer to the central axis). This operational-perceptual function lacks the third layer, the named conceptual label "amplitude," distinguishing it from S1's three-layer function and confirming level 1 semiotic operation.

On phase shift (D9): S3 reported still being confused in explaining this. On D12 (synthesis), however, S3 identified phase shift as the concept that became clearer through GeoGebra. This D9-D12 contradiction is analytically productive: S3 possesses perceptual-intuitive understanding of phase shift's graphical effect (graph shifts left or right) but cannot access the formal explanation of the mechanism (why $\frac{\pi}{4}$ in the formula produces $x = \frac{\pi}{8}$ on the graph). This is the same angle-to-Cartesian register conversion obstacle identified in S1, suggesting that the obstacle may not be confined to a single performance tier, although confirming its prevalence across the wider cohort would require interviewing a larger number of participants (see limitations of the study).

S3's justification mode (D11) was purely observational: "saya yakin karena saya melihat

Table 5. Triangulation matrix: Convergence of written response, GGB artefact, and interview data for S1 and S3

Triangulation claim	Written response	GGB artefact	Interview data	Convergence pattern	Methodological implication
S1: Epistemic mediation confirmed	Score 91%; formula $T = \frac{2\pi}{\lvert b \rvert}$ written out; phase procedure $2x - \frac{\pi}{4} = 0$ shown	Efficient construction; two relevant functions only; GGB level 3 (exploratory)	Explicit verbalization: "building new understanding … building an appreciation of the importance of value"	CONVERGENT. All three sources confirm epistemic mediation.	Epistemic mediation is identifiable from convergence across all three sources.
S1: Phase score 4/4 via visual estimation, not analytic procedure	T2a score 4/4 including phase $(\frac{\pi}{8})$; appears fully mastered	No phase-related obstacle indicator visible	Interview D10: "variable C … I was estimating … still confused about applying C on the Cartesian plane"	REVISION. Written response shows a correct answer; interview reveals a different underlying process.	A correct score does not, by itself, indicate mastery of the intended analytic procedure; interview data are indispensable.
S1: Phase shift obstacle, angle-to-Cartesian register conversion	Sketch (T2c) scored 2/3, the only point deducted for S1	No obstacle indicator visible in artefact	Interview D9 explicit: "$\frac{\pi}{4}$ means 45 degrees, but I am confused about entering it on the x-axis"	NEW FINDING. Not accessible from written response or GGB artefact.	This obstacle is detectable only via interview; targeted didactical design is required to address it.
S3: Artefact non-originality confirmed	T2b and T2d responses identical word-for-word with S5	GGB file ID identical to S7; XML title contains another student's ID number	Interview B3: "when entering the formula it could end up the same," an implicit acknowledgement	CONVERGENT (strong). All three sources confirm non-independent production.	Verifying originality requires triangulation; a mandatory Construction Protocol is recommended for future data collection.
S3: Phase shift obstacle present alongside S1	Phase written as "-4" without calculation; score 1/4	Slider c = 1.5, irrelevant to the target $-\frac{\pi}{4}$	Interview D9: "I am still confused about explaining this"	CONVERGENT. All three sources confirm an epistemological obstacle with phase in this participant.	The phase obstacle is present in both the highest- and one of the lowest-scoring interviewed participants; it does not appear to be an individual, isolated deficit within this pair, though confirming its extent beyond these two participants would require a larger interview sample.
S3: Partial pragmatic mediation without formalization	No formula; qualitative description only	GGB level 2 (functionally basic); non-original file; sliders not correctly linked	Interview C4: "my thinking is that all of this is interconnected," a global insight without quantitative elaboration	CONVERGENT. All three sources confirm pragmatic-perceptual mediation without formalization.	Pragmatic mediation does not produce propositional formalization without explicit institutionalization.

Note. GGB: GeoGebra artefact (XML analysis); CONVERGENT: All three sources support the same claim; REVISION: Interview revises the interpretation generated from the written response and GGB artefact; NEW FINDING: Interview reveals a finding inaccessible from the written response or GGB artefact alone; & These convergence categories operationalize Denzin's (1978) triangulation protocol for this dataset

berdasarkan pengamatan grafik" [I am sure because I am looking at graph observations], the lowest argument level in the OSA taxonomy, contrasting with S1's deductive-algebraic arguments. On D12 synthesis, S3 stated the intention to use a visual approach when teaching, reflecting metacognitive awareness of personal learning modality, though without the representational flexibility that would enable differentiated pedagogy.

Triangulation Synthesis: Three Data Sources (All Three RQs)

Table 5 formalizes the triangulation across written responses, GGB artefacts, and interview data for S1 and S3, classifying each claim by convergence pattern.

Table 6. Learning obstacle taxonomy: Categories, multi-source evidence, and instructional implications

Obstacle type	Description & evidence	Subjects affected	OSA dimension	Instructional implication
Epistemological: Reflection axis misconception	S3 writes that $a < 0$ causes reflection over the y-axis (correct: x-axis). Interview confirms a definitional anchor without visual-spatial connection.	S3	Language; concepts	An adidactic situation requiring students to identify the reflection axis via GeoGebra's symmetry point before naming it.
Propositional: Inverted b-period relationship	S6 writes "smaller b, smaller period" (inverting the correct relationship). This is a systematic misconception, not a calculation error.	S6	Propositions	A measurement activity deriving $T = \frac{2\pi}{\lvert b\rvert}$ from direct GeoGebra observation, tabulating values before introducing the formula.
Procedural: Period formula not formalized	8 of 10 students gave only a qualitative description; S3 and S7 equated b directly with period. Interview (S3) confirms b was understood as the period value itself, not as the period-dividing parameter.	S2, S3, S5, S6, S7, S8, S9, S10	Procedures; propositions	An institutionalization sequence: observe, tabulate, derive $T = \frac{2\pi}{\lvert b\rvert}$, then verify symbolically.
Procedural: Phase shift calculation	S3: phase given as -4 with no calculation shown; S7: phase given as -0.4; S5: used the formula $-\frac{c}{d}$ instead of $-\frac{c}{b}$. Interview (S1, D9-D10) showed that even the highest-scoring participant estimated phase visually rather than by solving $bx + c = 0$.	S3, S5, S7 (severe); S1 (residual)	Procedures; semiotic function	Deduce the formula from the condition $bx + c = 0$; repeated notational emphasis distinguishing c from d; an explicit register-conversion activity linking angle values to x-axis position.
Register conversion: Angle to Cartesian	An interview-exclusive finding (S1, D9): the student understands $\frac{\pi}{4}$ as 45° but cannot convert it to the x-position $\frac{\pi}{8}$. This is not a calculation error but a conversion failure between the angle register and the Cartesian coordinate register (Duval, 2006).	Confirmed in S1 &, in related form, in S3; prevalence beyond these two interviewed participants was not established in this study	Semiotic function; concepts	Activities explicitly linking angle values to x-axis positions, for example via a unit circle overlay in GeoGebra, treating phase shift as a coordinate rather than only as an angle.
Semiotic: Digital-to-manual transfer	7 of 10 participants could not produce a correct manual sketch. Interview (S3, D10) identifies the phase-shift step as the hardest part of manual transfer and describes dependence on GeoGebra to perform the transformation. Only S9 produced a fully labelled four-panel sketch, having deliberately set GeoGebra aside first.	S2, S3, S4, S5, S7, S8, S10	Semiotic function	A mandatory representational transition task requiring a manual sketch before returning to GeoGebra, using a four-panel template with labelled axes.
Semiotic: Informal formalization gap	S3 (interview B2) defines amplitude and period correctly but as isolated declarative objects, without connection to the formula or to coordinates. Most medium-tier students used informal language (for example "tighter") without consistently applying formal terminology.	S2, S3, S5, S6, S7, S9	Language; semiotic function	Sentence-starter scaffolds that require formal terminology, with institutionalization of this terminology as a mandatory task component.
Academic integrity: File and text duplication	S3 and S6 share an identical GeoGebra file identifier, confirmed via XML hash. S3 and S5 have verbatim identical written responses for T2b and T2d. Interview (S3, B3): "when entering the formula it could end up the same," an implicit acknowledgement of non-independent production.	S3, S5 (text); S3, S6 (GGB file)	All dimensions	A mandatory individual construction protocol at submission, with personalized parameter sets and simultaneous, independently monitored submission.

Note. Obstacle types follow Brousseau's (1997) epistemological classification & "Register conversion: Angle to Cartesian" is an interview-exclusive finding documented in the two interviewed participants; it cannot be identified from written or artefact data alone, and its prevalence in the wider cohort is not established by this study (see limitations of the study)

Learning Obstacle Taxonomy: Full Sample (RQ3)

Table 6 presents the complete cross-sample learning obstacle taxonomy integrating all three data sources.

Four of the eight obstacle categories in **Table 6**, the phase shift calculation obstacle, the angle-to-Cartesian register conversion obstacle, the digital-to-manual transfer obstacle, and the informal formalization gap, are tagged to the same OSA dimension: Semiotic Function. Each names a different point at which a participant's connection across representational registers broke down, but the table format documents these as discrete qualitative incidents rather than showing how they aggregate across the sample.

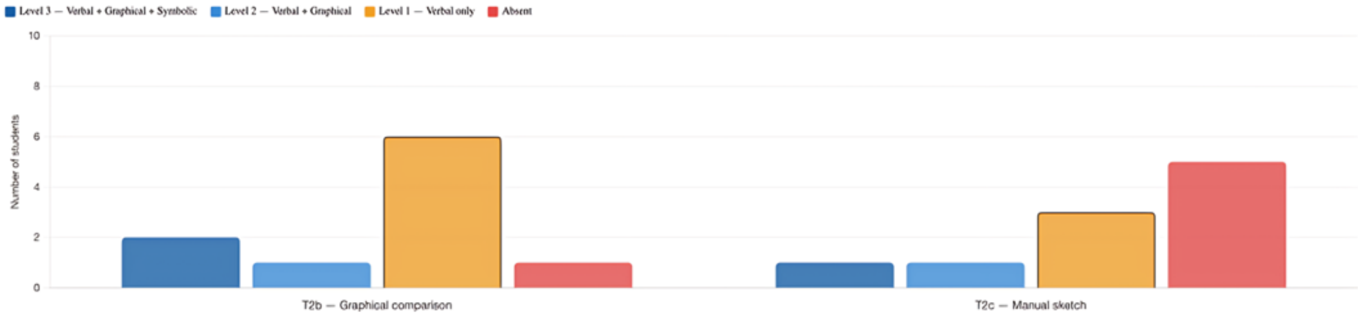

Figure 6. Semiotic function level distribution across T2b and T2c (level 3 [3-register connection] achieved by only 2 students in T2b and 1 in T2c, indicating that GeoGebra mediation alone does not drive multi-register integration) (created by the authors)

Figure 6 supplies that quantitative complement, plotting the distribution of semiotic function levels, L1 (verbal only), L2 (verbal + graphical), and L3 (verbal + graphical + symbolic), across sub-tasks T2b and T2c. The distribution is weighed heavily toward L1 and L2 in both sub-tasks, with level 3 reached by only two participants in T2b and one in T2c, consistent with S1 and S8 being the only participants identified elsewhere in this section (**Table 4**) as achieving full three-register connection. Read alongside **Table 6**, this distribution indicates that the obstacles listed there are not isolated incidents affecting one or two participants each, but symptoms of a cohort-wide ceiling on semiotic function that only a minority of participants moved beyond.

DISCUSSION

Three Mutually Irreplaceable Data Sources: A Methodological Contribution

The triangulation analysis in **Table 5** establishes three categories of finding that differ in their epistemic provenance, that is, in which data sources can access them and under what conditions. The distinction matters because it determines not just what was found, but what kind of evidence each finding rests on.

Convergent findings, epistemic mediation in S1, pragmatic mediation in S3, non-originality of S3's production, are supported by all three sources independently. Each source arrives at the same conclusion through different evidential pathways: written responses show what was produced, GeoGebra artefacts show what was constructed, and interview data show how the participant reasoned. The convergence across pathways increases confidence in the finding but also reveals something important about redundancy: these findings would have been identifiable without interview data, though with lower validity and no mechanism-level explanation. Convergent findings are the most defensible; they are also the least surprising.

The revisory finding, S1's correct phase shift answer obtained via visual estimation rather than analytical procedure, occupies a different epistemic position. The written response alone supports a plausible and positive interpretation: a score of 4/4 on T2a suggests the phase shift procedure was mastered. The GeoGebra artefact provides no counter-evidence. Only the clinical interview, through direct probing of S1's reasoning process, reveals that the correct answer was produced by reading off a GeoGebra graph rather than executing the analytical procedure the task was designed to develop. This is not a marginal correction; it reverses the pedagogically relevant conclusion. A student who estimates correctly from a graph has not demonstrated the same mathematical knowledge as a student who derives the answer analytically, and the difference has direct consequences for instructional decisions about whether further teaching is needed. The revisory finding establishes that written assessment, even when scored accurately, can systematically misrepresent the quality of mathematical practice underlying a correct answer.

The new finding, the angle-to-Cartesian register conversion obstacle underlying phase shift confusion, present in both S1 (score 91%) and S3 (score 35%), is exclusively accessible through clinical interview. Neither written responses nor GeoGebra artefacts contain evidence of this obstacle: S1's written score suggests mastery, and neither artefact displays the internal reasoning that breaks down at the angle-to-Cartesian conversion step. The obstacle only becomes visible when participants are asked to explain their reasoning aloud and are pressed on the specific moment of confusion. Its cross-proficiency presence, shared by the highest and one of the lowest performers in the sample, argues against a purely individual or skill-deficit explanation, although with only two participants interviewed this remains a plausible interpretation rather than a confirmed pattern (see limitations of the study). The obstacle appears to be epistemological in nature: it plausibly inheres in the structure of the mathematical object itself, in the representational discontinuity between how phase shift is encoded in the angle register ($\frac{\pi}{4} = 45°$, a rotation) and how it must be read in the Cartesian coordinate register ($x = \frac{\pi}{8}$, a horizontal displacement). Written-only research designs would have missed this obstacle entirely, producing a false picture of S1's understanding and an incomplete picture of S3's.

Together, these three categories validate the methodological architecture of the study and support a theoretical claim with practical consequences: in OSA-informed research on digital mathematics learning, written response and artefact analysis constitute a necessary but insufficient basis for learning obstacle identification. The insufficiency is not a matter of completeness in the sense of gathering more data of the same kind; it is structural. Written responses and artefacts access the products of mathematical activity, what was written, what was constructed. Clinical interview accesses the process, how the student reasoned, where reasoning broke down, and what the student does and does not understand about what they produced. These are different objects of inquiry, and no accumulation of written or artefact data can substitute for direct access to reasoning process. Clinical interview is not a supplementary data source that improves confidence in findings already accessible elsewhere. It is a theoretically necessary source that accesses phenomena the other two cannot reach.

The Epistemic Configuration Gap: Arguments as a Cross-Cutting Obstacle

The consistent weakness of the arguments/justifications dimension across all proficiency tiers confirms that mathematical argumentation occupies a distinct functional role in pre-service teacher education, one that procedural competence does not automatically generate (Font et al., 2013). Even S1, who produced deductive-algebraic argument quality in the clinical interview, did not write justifications at a comparable level of elaboration in the task response. The gap between S1's oral and written argumentative performance is not a proficiency gap; it is a modality gap. Students who can argue orally may not spontaneously produce written arguments without explicit task design prompts, a distinction that written-only assessment cannot detect and therefore cannot address.

Two mechanisms plausibly account for this suppression. First, written task formats in mathematics instruction conventionally reward procedural output (correct answers, legible working, labelled diagrams) rather than propositional justification. Students who have been socialized into this convention may not recognize written argumentation as expected or valued unless the task explicitly requires it. Second, oral argument benefits from interactional scaffolding: the interviewer's questions, pauses, and follow-up probes create pressure to elaborate and justify. Written tasks provide no such pressure. The implication is not that students lack argumentative capacity, but that the assessment context systematically fails to elicit it.

This finding gives empirical support to Godino et al.'s (2019) prescriptive claim that epistemic configuration design must explicitly target each OSA entity. Argumentation is not an emergent by-product of procedural competence; it requires dedicated task design: prompts that demand justification, not merely solution; rubrics that reward propositional reasoning; and instructional sequences that make the distinction between procedure and argument explicit to students. In pre-service teacher education specifically, this matters doubly: teachers who have not been required to produce written mathematical arguments are unlikely to design tasks that require their own students to do so.

This is consistent with evidence that GeoGebra-mediated argumentation can be strengthened where task design explicitly incorporates collaborative debate and self-reflection stages: when university students worked through GeoGebra tasks structured around teamwork, scientific debate, and self-reflection, the software's mediating role helped them construct, validate, and reach consensus on mathematical arguments (Urhan & Bülbül, 2023). The contrast with the present sample, where no comparable argumentation scaffold was built into the task design, reinforces the conclusion that argumentative quality depends on deliberate task structuring rather than on GeoGebra use in itself.

Pragmatic Mediation Dominance and the Digital-to-Manual Transfer Obstacle

Eight of ten participants maintained pragmatic GeoGebra use, observing slider effects without transitioning to formal propositions. The S2 anomaly is the strongest empirical case in this dataset against the assumption that technical tool sophistication generates conceptual depth: the highest GeoGebra complexity (level 4 analytic commands) paired with a medium-tier OSA score and verbal-only semiotic function. This corroborates van Dijke-Droogers et al.'s (2021) finding that instrumental genesis and conceptual understanding intertwine bidirectionally but non-linearly: technique development does not automatically produce conceptual scheme development.

The failure of most participants to produce manual transformation sketches (7 of 10; T2c mean = 0.8/3) is theorized as register anchoring: the semiotic primacy of the dynamic digital graphical register, reinforced by immediate and responsive slider feedback, produces a cognitive dependency that inhibits voluntary migration to the manual graphical register. This phenomenon extends Duval's (2006) concept of register autonomy to digital learning environments. Register anchoring is distinct from general representational difficulty: it is triggered specifically by the presence and accessibility of a dominant digital register and is therefore an environment-specific obstacle that would not emerge in non-digital instructional contexts.

S9's spontaneous behavior during the task, deliberately abandoning GeoGebra before sketching, then returning to verify, suggests that some pre-service teachers can independently develop strategies to counteract register anchoring. This provides direct empirical grounding for a design principle: task sequences with mandatory non-digital representational phases, explicitly prohibiting GeoGebra use until a manual sketch is produced, can scaffold representational deliberation for those who would not generate it spontaneously.

The Phase Shift Register-Conversion Obstacle: A Novel Cross-Proficiency Finding

The most theoretically significant finding of this study, accessible only through clinical interview, is a specific register-conversion obstacle underlying phase shift misunderstanding. Both S1 (score 91%) and S3 (score 35%) reported the same confusion: understanding $\frac{\pi}{4}$ as an angle value (45°) without being able to convert it to a position on the Cartesian x-axis ($\frac{\pi}{8}$). This is a failure of conversion between the angle register and the Cartesian coordinate register (Duval, 1993), not a calculation error, but a representational interface failure.

The obstacle's presence across proficiency levels, in the highest and among the lowest performers, is suggestive of an epistemological rather than purely individual character, though as noted above this interpretation rests on evidence from two interviewed participants and warrants confirmation in future work with a larger interview sample. S1's correct written answer (4/4 on T2a) was produced via visual estimation from GeoGebra, not via the analytical procedure the task was designed to develop. The task generated a correct answer while failing to generate the intended analytical knowledge, a distinction that standard written assessment cannot make. Only clinical interview can distinguish between an answer that is correct because the procedure was mastered and one that is correct despite the procedure not being mastered. This distinction has direct implications for OSA-informed assessment design: tasks must assess the quality of mathematical practice, not only its outcome.

The instructional implication is specific. An activity that explicitly builds the conversion between the angle register and the Cartesian register for phase shift, for example overlaying a unit circle on the GeoGebra graphical window and asking students to trace the x-position of the angle value, addresses the root cause of the obstacle rather than its symptom.

Limitations of the Study

This study has several limitations that bound the generalizability of its findings and should be considered when interpreting its contribution.

First, the sample size is small (N = 10), and clinical interviews, the data source that produced the study's most theoretically significant findings, were conducted with only two of these ten participants. The maximum variation sampling strategy used to select these two participants is well suited to in-depth exploration of contrasting cases, but it does not support claims of statistical generalizability to a broader population of pre-service mathematics teachers. In particular, the four distinct categories of learning obstacle identified and the phase shift register-conversion obstacle discussed are grounded in evidence from a single cohort at one institution, and their prevalence among pre-service teachers with more diverse academic backgrounds remains an open empirical question. Language describing the register-conversion obstacle as shared across proficiency tiers refers specifically to its presence in both interviewed participants (S1 and S3); it should not be read as a claim that the obstacle is universal across the full ten-participant cohort, let alone across pre-service teachers more broadly.

Second, the study is based on a single 90-minute task session at one institution, which constrains what can be said about how these patterns develop or persist over time, and about whether the same obstacles would appear under different task designs, instructional histories, or institutional contexts.

Third, three participants (S3, S6, and S7) showed artefactual and textual evidence consistent with non-independent production, specifically a shared GeoGebra file identifier between S3 and S6, and near-identical written responses between S3 and S5. These cases were retained in the full-sample statistics reported because the interview data available for S3 corroborated genuine, if incomplete, engagement with the task; however, their inclusion means that some full-sample obstacle frequencies (for example, the count of participants who did not formalize the period relationship) should be interpreted with this caveat in mind rather than as fully independent data points.

Given these limitations, the theoretical claims advanced in this study, particularly the proposed cross-proficiency character of the register-conversion obstacle and the four-category obstacle taxonomy, are best understood as empirically grounded hypotheses generated through in-depth qualitative analysis, rather than as findings already established at the level of the broader pre-service teacher population. Testing the prevalence of these obstacle categories across a larger and more academically diverse sample, ideally through a large-scale quantitative survey informed by the qualitative categories developed here, is a necessary next step and is proposed as a direction for future research.

CONCLUSION

This study investigated pre-service mathematics teachers' construction and connection of mathematical objects in a GeoGebra-based trigonometry environment, drawing on three data sources: written task responses, GeoGebra XML artefact analysis, and clinical interviews with two purposively selected participants. The findings present a complex picture. GeoGebra enabled participants to develop pragmatic familiarity with trigonometric function parameters, but did not, for most participants, promote the propositional formalization, argumentative justification, or multi-register semiotic integration that characterize deep mathematical understanding.

Five substantive contributions to the mathematics education literature emerge from this study. First, this study provides an empirically grounded epistemic configuration map for GeoGebra-based trigonometry instruction, identifying the arguments/justifications dimension as a cross-cutting obstacle independent of proficiency level. Second, the pragmatic-epistemic mediation distinction is operationalized and evidenced across all three data sources, showing that technical GeoGebra sophistication (artefact complexity level 4) does not entail epistemic mediation or semiotic integration. Third, register anchoring is proposed and evidenced as a novel, environment-specific learning obstacle: the inhibition of voluntary register migration caused by the semiotic dominance of the dynamic digital graphical register, distinct from general representational difficulty. Fourth, the phase shift register-conversion obstacle, between the angle register and the Cartesian coordinate register, is identified as a candidate cross-proficiency, epistemologically grounded obstacle that written-and-artefact-only research designs cannot detect, a hypothesis grounded in evidence from two interviewed participants that merits testing at larger scale. Fifth, clinical interview is argued, on the evidence of this study, to be a theoretically necessary data source in OSA-informed research on digital mathematics learning, not merely a supplementary one. Interview alone accessed both a revisory finding, a correct answer obtained via estimation rather than mastered procedure, and a new finding concerning the anatomy of the register-conversion obstacle.

Future research should examine whether epistemically oriented task design produces qualitatively different OSA profiles and semiotic function distributions. Designs of particular interest include tasks featuring mandatory representational phase transitions, explicit institutionalization sequences from GeoGebra observation to symbolic formalization, and dedicated angle-to-Cartesian register conversion activities. Longitudinal designs tracking the co-evolution of GeoGebra mediation quality and epistemic configuration development across a complete teacher education course would further clarify how instrumental genesis and mathematical object construction develop within pre-service teacher learning trajectories.

As outlined before, testing the prevalence of the obstacle categories identified here across a larger and more academically diverse sample of pre-service mathematics teachers, through a large-scale quantitative survey informed by the present qualitative categories, is a necessary direction for future research. In parallel, didactical design research (Suryadi, 2010) offers a promising methodology for iteratively designing and empirically testing the task sequences proposed above, since it explicitly cycles between hypothetical didactic design, implementation analysis, and retrospective analysis in response to observed learning obstacles.

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Ethical statement: This study did not require formal ethics committee approval. It involved a standard educational task session (90 minutes) with pre-service teacher students in a university mathematics methods course. Participation posed no greater than minimal risk: no sensitive personal data were collected, no deceptive procedures were employed, and participants were free to withdraw at any time without consequence. All participants provided written informed consent. Data were anonymized and stored securely. In accordance with applicable national and institutional guidelines for educational research in Indonesia, formal ethics committee review is not mandatory for studies of this nature. The research was conducted in accordance with the ethical principles of the Declaration of Helsinki and the guidelines of the Indonesian National Research and Innovation Agency.

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Data sharing statement: The interview transcripts and raw student response data analyzed contain information that could compromise anonymity of participating students and are thus not publicly available. De-identified data supporting the findings, including OSA scoring rubric, the coded interview transcripts with participant identifiers removed, and GeoGebra artefact analysis records, are available from the corresponding author upon request, subject to institutional approval for secondary use.

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