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Pre-service mathematics teachers' perceptions and knowledge on visual-genetic interpretation of inverse trigonometric functions

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Abstract

This study examines pre-service teachers' perceptions and knowledge regarding the visual-genetic interpretation of inverse trigonometric functions (ITF). The visual-genetic approach provides opportunities for exploring the origin, modification, and graphical and geometrical interpretations of mathematical concepts. Using a convenience sampling technique, a sample of 65 third- and fourth-year prospective mathematics teachers (23 males and 42 females) was selected from the faculty of education in Fergana, Uzbekistan. To collect data on prospective teachers' perceptions and conceptual knowledge related to the visual-genetic interpretation of ITF, a perception questionnaire and an achievement test were employed. The collected data were coded and analyzed using descriptive statistics. The results indicated that prospective teachers perceive the ITF as boring, abstract, and difficult to learn, and that they possess limited conceptual knowledge of the fundamental concepts. As a result, over 50% of the participants were unable to build and recreate the proper mental structures required for gaining a meaningful understanding that would allow them to answer crucial basic inverse trigonometry challenges. To provide high-quality mathematics education for pre-service teachers who would shape our future in the age of artificial Intelligence, teacher educators needed to reconsider their instructional techniques and ensure that perception was well grounded and that understanding and conceptualization were firmly and securely established.

Keywords: inverse trigonometric functions, knowledge, mathematics education, pre-service teachers, perceptions, trigonometry, visual-genetic approach

INTRODUCTION

The concept of inverse functions is important in both secondary and higher mathematics education. International standards have incorporated the concept of inverse functions into their curricula as a topic that students are expected to study (Bergeron, 2015), which highlights the expectation that mathematics teachers provide instruction on inverse functions. Therefore, pre-service teachers should be able to create meanings, provide learning opportunities, and implement instructional practices that develop the concept of inverse functions for their future students. In this way, they can play a significant role in students' conceptual development, which forms much of the foundation of higher mathematics education, particularly in topics related to function families and calculus. One of these inverse function concepts appears in trigonometry. Trigonometry is considered a branch of physical mathematics concerned with understanding concepts and their applications. Its scope includes angles, the measurement of angles, triangles, and the relationships among them (Ahamad et al., 2018; Kamber Hamzić et al., 2025; Kissane & Kemp, 2009; Van Brummelen, 2020). Triangles integrate graphical, geometric, and algebraic reasoning to provide a framework for interpreting trigonometric expressions and graphs (Larson & Hostetler, 2001).

When students and pre-service teachers encounter different conceptual approaches, such as circular and

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Contribution to the literature

- This study enhances the mathematics education literature by systematically analyzing pre-service mathematics teachers' perceptions and conceptual understanding of inverse trigonometric functions (ITF) from a visual-genetic approach. It also emphasizes how pre-service mathematics teachers comprehend the origins, transformations, and conceptual foundations of ITF, in contrast to much previous research that largely concentrates on procedural competency or isolated representations.
- The results offer empirical evidence that unfavorable perceptions—such as regarding ITF as abstract, challenging, and uninteresting—are strongly linked to restricted conceptual comprehension and unstable cognitive frameworks.
- The study elucidates prevalent challenges in reconstructing essential ideas such as inverse connections, domain-range limits, and graphical interpretations, so enhancing the comprehension of why inverse trigonometry continues to pose difficulties in teacher education.

triangular trigonometry (Kamber Hamzić et al., 2025; Mickey & McClelland, 2017), it is not surprising that confusion arises (Bressoud, 2010). This confusion stems from their tendency to perceive ITF as merely algebraic counterparts, which leads to serious misconceptions (Swan, 2019). This, in turn, reveals that the relationships between the geometric and analytic representations of ITF are not being adequately established (Gómez-Chacón et al., 2024).

Efforts to achieve a conceptual knowledge of ITF at high school form the foundation for purposeful mathematics learning in faculties of education at the university. Based on explanatory texts and examples from ten college textbooks actively used, Mesa and Goldstein (2017) identify two different understandings grounded in either a static or a dynamic definition of angles, trigonometric functions, and ITF. Although these textbooks tend to favor trigonometric functions based on a dynamic understanding of angles, the situation changes when it comes to the definition of ITF. It is undeniable that articulating how bridges between these understandings can be constructed (Delice, 2003) may be beneficial for understanding the difficulties that arise when solving problems related to ITF.

In this context, pre-service mathematics teachers hold a position of particular importance. They are not merely individuals who learn mathematical knowledge; rather, they are the key actors who will transmit this knowledge to new generations of students who will shape the future world (Cardoso et al., 2025; Evang, 2025; Woltron, 2024). The visual and genetic (developmental) deficiencies and perceptual limitations that pre-service teachers possess specifically with regard to ITF may lead classroom instruction to take an undesirable direction in the future. In this study, the visual-genetic approach is based on constructing and understanding the formulas that define the fundamental components and arguments of ITF by emphasizing their genetic (origin and development) aspects and by using visual tools (illustrative and graphical), particularly the unit circle, right triangles, vectors, rotations, and reflections. Neglecting this approach in the teaching process may cause pre-service teachers to perceive ITF as difficult and abstract structures. For this reason, examining pre-service teachers' perceptions and levels of knowledge on this topic has emerged as a critical necessity in terms of the quality of mathematics instruction.

Consequently, when individuals have the necessary mental structures for concepts, learning ITF becomes easier. In the absence of such mental structures, it may be nearly impossible to respond effectively to conceptual problem situations or to understand inverse trigonometric concepts and their applications. Understanding ITF serves as a tool for developing the cognitive skills of students and prospective teachers and provides a framework for coordinating concepts (Tallman, 2021). Therefore, ITF within the curriculum create a suitable foundation for exploring, connecting, and relating mathematical ideas, thereby meaningfully integrating different scientific disciplines as well (Ferede et al., 2025).

Perception and Conceptual Understanding in Mathematics

Kurudirek and Berdieva (2024), while advocating for the strengthening of schools to shape a more productive future, Machmud et al. (2025) emphasize the role of education in preparing the developing generation to live at the desired level in the age of artificial intelligence. When individuals know that they will be able to use what they learn, they become more motivated to invest effort in learning. These perceptions of relevance play a motivating role in the learning of mathematics, a subject known for its abstract nature, challenging examinations, and wide-ranging applicability across many areas of society (Vos et al., 2024).

Perception is the process through which individuals record and evaluate information received from their internal or external environment. An important aspect of perception is that the resulting reality may differ from the reality itself (Burn, 2010) and as individuals acquire new information, their perceptions (Anderson et al., 2022) evolve. Canonigo (2025) argued that instructional approaches emphasizing conceptual engagement and multiple representations play a significant role in shaping learners' perceptions and may support improved conceptual understanding in mathematics.

Therefore, perceptions are related not only to our behaviors but also to the responses we give. The way and style in which we think (perceive) determine our actions, which are themselves functions of our perceptions (Zeng et al., 2023). A pre-service teacher's prior experiences with ITF and the meanings constructed with new concepts may lead them to perceive the topic as difficult to understand. Given the importance of ITF and the challenges pre-service teachers face in understanding them, investigating their perceptions of these fundamental concepts they will teach in the future is essential.

Insights into Teaching and Learning Trigonometric Concepts

In this section, studies related to ITF are not only described but also discussed from a comparative and critical perspective. Recent research on learning trigonometry has raised various concerns. Asomah et al. (2023), Dündar (2015), and Ibrahim and Yew (2023), evaluating middle school mathematics pre-service teachers' content knowledge, pedagogical content knowledge, and anticipated practices in trigonometry, found that the participants had an insufficient understanding in areas such as angle measurement in radians, ITF, reciprocal functions, periodicity, and even functions.

Nabie et al. (2018), in a study examining pre-service teachers' perceptions and knowledge of trigonometric concepts, found that participants perceived trigonometry as abstract, difficult, and boring to learn, and had limited conceptual knowledge of fundamental trigonometric concepts. They argue that, to provide high-quality mathematics education, teacher educators need to change their instructional practices and adopt an understanding-focused approach to teaching. Similarly, Orhani (2024) identified core challenges, including graphical representations, real-life applications of trigonometric functions, and the conceptual understanding of angles and ratios, while offering recommendations for improving instructional methods and teaching practices.

Klein (2015), based on conceptual field theory and meaningful learning theory, examined students' knowledge of trigonometry and found that identifying students' prior knowledge and explicitly revealing knowledge in action led to changes in attitudes. While misconceptions and errors are rooted in underlying obstacles, one fundamental barrier is that trigonometry and its related concepts are neither concrete nor intuitive (Dhungana et al., 2023; Gür, 2009). Although students encounter difficulties with angles in degrees, the challenges in understanding trigonometric concepts are more pronounced with angles in radians (Akkoç, 2008). Research addressing students' difficulties with radians (Moore, 2012) suggests explaining their connection to arcs to facilitate understanding. Teaching trigonometric functions by focusing on their relationship to arc lengths can help students overcome conceptual difficulties.

Existing research generally addresses ITF at the level of definitions and rules, without pedagogically examining how these functions are derived from trigonometric functions. In this context, pre-service teachers appear to struggle with constructing ITF within the framework of function-inverse function relationships. Studies that focus on the genetic (developmental) approach are quite limited. This study aims to fill this gap in the literature. It makes an original contribution by examining both pre-service teachers' visual-genetic perceptions of ITF and their conceptual and procedural knowledge.

Visual-genetic interpretation of inverse trigonometric functions

A practical modification of the visual-genetic approach is based on constructing and understanding the formulas that form the fundamental parts and arguments of the functions $y = arcsinx$, $y = arccosx$, $y = arctanx$, and $y = arcctgx$ using their genetic (origin and development) aspects and visual tools (demonstrative-graphical, particularly the unit circle, right triangles, vectors, rotations, and reflections).

Instructional practices indicate that the error rate remains high when solving problems with arguments involving ITF. One main reason for this is often the lack of understanding of the analytical and logical structural essence of the problem. In this context, it is important to visually examine the genetic aspects of expressions with arguments of ITF. A visual-genetic approach allows pre-service teachers to observe the process visually and helps them understand the underlying concept more deeply. Research shows that traditional algorithmic approaches—methods based on memorizing and applying formulas—contribute little to a deep understanding of the essence of the concept.

Practical and scientific studies concerning the methodological characteristics of tests used to diagnose initial knowledge of trigonometry (Kamber Hamzić et al., 2025), trigonometry e-modules (Machmud et al., 2025), systematic errors in solving trigonometry problems (Hanggara et al., 2024; Lorxaypao et al., 2024), the limited contribution of textbook examples to students' functional understanding (Fatmasari & Sutama, 2024; Kamber & Takaci, 2018), types and causes of errors (Ahmad & Khotimah, 2023), analogical thinking in problem solving (Spangenberg, 2021), electronic modules for independent trigonometry learning (Asfyra et al., 2024), integration of trigonometry

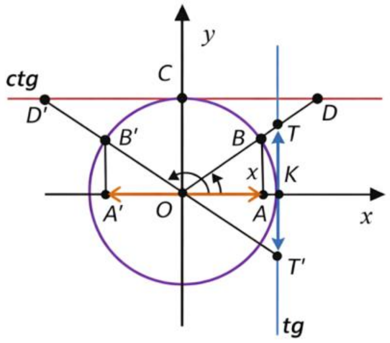

Figure 1. A sample for a visual-genetic interpretation (Source: Authors' own elaboration)

and geometry (Kamber Hamzić et al., 2025), and the use of GeoGebra tools (Yorganci, 2022) provide positive outcomes for innovative approaches to trigonometry instruction. However, the visual-genetic reconstruction of solutions to problems involving ITF requires methodological guides designed to explain the logical-structural aspects of formulas and effectively and easily convey the essence of formulas through genetic-visual methods, systematic grouping, and repetition. For example: $cos(arcsinx) = \sqrt{1-x^2}$, where $x\epsilon[-1,1]$ and $tg(arcsinx) = \frac{x}{\sqrt{1-x^2}}$, where $x\epsilon(-1,1)$.

Deriving similar formulas solely through analytical methods (LaFara, 1954; Tasar et al., 2022) somewhat limits students' and pre-service teachers' mathematical reasoning and proof abilities. Interpreting the development of a concept step by step—from empirical observation to abstract formulation—on a graphical basis (through visualization, comparison, and movement), particularly using visual components (graphs, images, interactive tools, animations, etc.), makes this process intuitive and comprehensible. The visual-genetic approach enables pre-service teachers to understand a mathematical concept sequentially, from empirical experience to abstract formulation, providing conceptual depth through its integrative nature. The visual-genetic method we propose allows for an in-depth examination of the essence of the trigonometric functions $arcsinx$, $arccosx$, $arctanx$, and $arccotx$, helping learners grasp their underlying structure. The content of the visual-genetic method is presented below.

The function $y = arcsinx$ is a multivalued function defined on the interval $x\epsilon[-1,1]$, and the portion corresponding to $y\epsilon\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ is called the principal branch of the $arcsinx$ function (Gaziyev et al., 2006) and is defined as $arcsinx$. The remaining values of the function can be determined through the principal branch using Eq. (1):

$$Arcsinx = arcsinx + 2\pi k, \quad (1)$$

where $k\epsilon Z$.

Let us consider trigonometric expressions with arguments $Arcsinx$ and $arcsinx$ (**Figure 1**).

In the Cartesian coordinate system, we draw the unit circle, the tangent and cotangent lines, and a vector $\overrightarrow{AB}$ parallel to the OY-axis for x ∈ [−1, 1], where $\overrightarrow{AB} = x$.

As a result, we can construct the vectors $\overrightarrow{AB}$ and $\overrightarrow{A'B'}$ on the unit circle; if $x = \pm 1$, these vectors coincide. Therefore, calculations are performed with respect to the OY-axis. $\angle AOB = \alpha$ and $\angle A'OB' = \pi - \alpha$. Then $sin\alpha = x$ and $sin\,(\pi - \alpha) = x$. Considering a full rotation along the unit circle, all angles defined by the points B and B' whose sine equals x are determined by Eq. (2):

$$N = \{\angle AOB + 4lk\} \cup \{\angle A'OB' + 4lk\}, \quad (2)$$

where $k\epsilon Z$ and l represents a quarter of the unit circle. For example, if the angle $\angle AOB$ has a radian measure α and the angle $\angle A'OB'$ has a radian measure α', then the radian measures of the angles in the set N form the set N^* using Eq. (3):

$$N^* = Arcsinx = (\alpha + 2\pi k, \alpha' + 2\pi k), \quad (3)$$

where $k\epsilon Z$.

Since $sin\alpha' = sin(\pi - \alpha) = sin\alpha$ and $sin\alpha = x$, and knowing that $arcsinx = \alpha$, Eq. (3) implies the validity of Eq. (1).

In traditional textbooks, the function $y = arcsinx$ is generally explained using an algebraic approach. This approach spatially illustrates the concept through the unit circle, vectors, angles, and triangles. As a result, pre-service teachers and students can observe the graphical-geometric relationships between the angles corresponding to each value of the argument and the sine values. A geometric correspondence is established between the argument and the functional values. In this context, the intrinsic relationships of the sine, cosine, tangent, and cotangent functions are determined using the OA, AB, and KT vectors (based on triangle similarity). In particular, within this approach, the student understands the symmetric properties of the function in a spatial representation, which naturally aids in comprehending the fundamental laws of trigonometry.

This study is based on the visual-genetic method and aligns with APOS theory, thereby offering a cohesive framework for examining conceptual understanding in trigonometry. APOS theory (Tuktamyshov & Gorskaya, 2024) posits that meaningful mathematics comprehension evolves through a four-stage cognitive framework: action, process, object, and schema (APOS). At the action level, learners execute processes in reaction to external stimuli without internal regulation. At the process level, these activities are internalized and achieve mental coordination. At the object level, learners perceive a mathematical process as a holistic thing that can be examined and modified. At the schema level, many objects and processes are systematically arranged into a cohesive network of relationships.

The visual-genetic approach complements APOS theory by highlighting the inception, evolution, and metamorphosis of mathematical notions using visual representations, including graphs, unit circles, right triangles, vectors, rotations, and reflections. Understanding ITF necessitates both procedural proficiency and the capacity to elucidate their derivation from trigonometric functions using pictorial and geometric reasoning.

Statement of the Problem

Various studies (Dhungana et al., 2023; Gür, 2009; Moore, 2012) have shown that students and pre-service teachers generally experience difficulties in trigonometry. These difficulties arise from several factors, including lack of motivation, the abstract nature of trigonometric concepts, insufficient understanding of fundamental concepts, and the inability to connect ideas within trigonometry. Numerous studies have examined students' and prospective teachers' perceptions and understanding of trigonometry. For example, Klein (2015) investigated the understanding of trigonometry based on conceptual field theory and meaningful learning theory; Akkoç and Akbaş-Gül (2010) focused on radian measurement, Gür (2009) and Dhungana et al. (2023) examined misconceptions and types of errors arising in trigonometry lessons. Tuna (2013) explored future mathematics teachers' conceptual knowledge of degrees and radians, while Fi (2003) investigated subject-matter knowledge, pedagogical content knowledge, and their anticipated applications in the field of trigonometry. Although numerous studies exist in the field of trigonometry, research that specifically examines pre-service teachers' perceptions and conceptual understanding of ITF from a visual-genetic perspective remains limited and has substantial gaps. To address this gap, the topic was examined among pre-service teachers in Uzbekistan. To address this gap, the topic was examined among pre-service teachers in Uzbekistan.

Purpose of the Study

This study investigated the perceptions and knowledge of pre-service teachers at a national university in Uzbekistan regarding the visual-genetic interpretation of ITF. Teacher perceptions are a fundamental aspect of teaching (Fajet et al., 2005). Examining and documenting these perceptions and knowledge can provide insight into classroom practices and highlight what pre-service teachers need to acquire to become competent mathematics educators. The study aimed to contribute to the body of knowledge on prospective teachers' perceptions, knowledge, and understanding of visual-genetic interpretation of ITF. Its findings can guide future researchers and inform policymakers in curriculum design and teacher development initiatives. Since perceptions also influence learning, the results may have a significant impact on how individuals organize and approach mathematical tasks. Therefore, this study aims to evaluate how pre-service teachers in Uzbekistan interpret and learn this issue and how they understand trigonometry.

Research Questions

Since perception plays a central role in the process of training teachers and acquiring knowledge, this study was conducted based on the following research questions (RQs):

RQ1: To what extent are pre-service teachers knowledgeable about ITF?

RQ2: What are pre-service teachers' perceptions regarding the visual-genetic interpretation of ITF?

METHOD

Research Design

This study utilized a descriptive research methodology that included qualitative analysis to investigate the perceptions and conceptual knowledge of ITF among pre-service mathematics teachers. The main goal was not to create a theory but to systematically describe the participants' existing perceptions, levels of knowledge, and the conceptual challenges they faced when learning about the visual-genetic interpretation of ITF in trigonometry. Quantitative data were collected using a Likert-scale perception questionnaire delivered to the participants, providing numerical insights into their self-reported understanding and attitudes and qualitative data were derived from their written replies to open-ended assessment items. The qualitative aspect functioned as a supporting and elucidative element by facilitating a more comprehensive analysis of the misunderstandings influencing participants' conceptual thinking and performance. Consequently, the study should be seen not as a comprehensive mixed-methods design but as a quantitative descriptive study with qualitative descriptive components.

Participants

A convenience sampling technique was applied to select 65 third- and fourth-year mathematics pre-service teachers (23 males and 42 females) from the faculty of education in Fergana, Uzbekistan. The sample size was determined based on practical considerations of accessibility, willingness to participate, and the need for a manageable number of participants for detailed qualitative and quantitative data collection. While the total population of eligible third- and fourth-year mathematics pre-service teachers was larger, this sample provided a reasonable proportion for examining trends in conceptual knowledge and perceptions, allowing for meaningful analysis without compromising depth.

Table 1. Demographic characteristics of the participants (n = 65)

Variable	Category	Frequency (f)	Percentage (%)
Gender	Male	23	35.4
	Female	42	64.6
Grade	3rd	31	47.7
	4th	34	52.3
Type of high school graduated	Public high schools	29	44.6
	Academic lyceum	21	32.3
	Vocational schools	15	23.1
Pre-university academic orientation	Mathematics-science oriented	56	86.2
	Social sciences oriented	9	13.8

The selection of these prospective teachers was also guided by their prior preparation: they had acquired sufficient content knowledge and pedagogical training and were ready to apply what they had learned in practice. Participants had completed the core courses required in basic mathematics, calculus, and trigonometry. This group of prospective teachers was scheduled to begin teaching practice in mathematics, including basic mathematics topics, making them a suitable population for examining both conceptual knowledge and instructionally relevant perceptions related to ITF.

Research Instruments

In this study, two instruments were used for data collection: inverse trigonometry perception questionnaire (ITPQ) and inverse trigonometry assessment test (ITAT). There were twelve questions on the ITPQ, and they were split into two parts. The first segment had four questions that were meant to gather biographical information about each participant (**Table 1**). The second part of the ITPQ had eight five-point Likert-scale questions that were made to find out how pre-service teachers felt about ITF in terms of how hard they were, how abstract they were, how useful they were, and how valuable they were for teaching. The questionnaire items were designed based on existing literature about perception and conceptual understanding in mathematics education and were evaluated by experts for clarity and suitability before distribution.

The ITAT had ten open-ended questions aimed at assessing participants' conceptual, graphical, and visual-genetic comprehension of ITF. The enquiries addressed many facets of comprehension, including: The significance and depiction of ITF, relationships between functions and their inverses, restrictions on domain and range, graphical interpretation and reflection across $y = x$, unit circle analysis, the use of right triangles and certain angles. Participants were instructed to articulate their thinking in writing, facilitating the investigation of both precision and the foundational conceptual frameworks.

Prior to the commencement of the primary data collection, the instruments underwent pilot testing with 17 pre-service mathematics teachers who were excluded from the main study population. The measurement model was examined using structural equation modelling, which confirmed that the instrument had adequate construct validity. The participants were enrolled in an analogous teacher education program and had completed coursework in trigonometric and ITF. Insights garnered from the pilot research were utilized to enhance the clarity of item phrasing, eradicate ambiguities, and guarantee content suitability.

Content validity was determined by expert evaluation. Two mathematics education faculty members from distinct nations, with competence in trigonometry teaching and teacher education, independently assessed the instruments to evaluate the correspondence between the items and the intended conceptual domains. Minor modifications were implemented in accordance with the experts' recommendations.

The internal consistency of the ITPQ was assessed using Cronbach's alpha (α) coefficient, resulting in a reliability coefficient of $\alpha = 0.83$, signifying satisfactory reliability for this research. The ITAT comprises open-ended items aimed at assessing conceptual knowledge instead of yielding a singular scale score; hence, its reliability was evaluated using analytic consistency and coding stability rather than internal consistency metrics.

Additionally, in this research, ITF are represented by the notation $arcsinx$, $arccosx$, $arctanx$, and $arccotx$. The alternative notation $sin^{-1}(x)$ is eschewed to avoid misunderstandings with reciprocal functions.

Data Analysis

In this study, data analysis was conducted using a holistic approach in which quantitative and qualitative dimensions complement each other. Data obtained from the ITPQ and the ITAT were first examined through descriptive statistics, namely frequencies and percentages, to identify general trends in pre-service teachers' perceptions and their levels of achievement across different types of inverse trigonometric tasks, using SPSS. In addition, the written responses to the open-ended items in the ITAT were subjected to a qualitative descriptive analysis process.

Table 2. Pre-service teacher's perceptions of ITF (n = 65) (ITPQ)

Statement	f (%)				
	SD	D	U	A	SA
ITF are a difficult topic in mathematics.	3 (4.6)	11 (16.9)	1 (1.5)	30 (46.2)	20 (30.8)
ITF is a boring topic.	2 (3.1)	7 (10.8)	1 (1.5)	16 (24.6)	39 (60.0)
ITF are very rigid and abstract.	2 (3.1)	4 (6.2)	5 (7.7)	31 (47.7)	23 (35.4)
My academic achievement does not depend on my understanding of ITF.	9 (13.8)	16 (24.6)	3 (4.6)	14 (21.5)	23 (35.4)
ITF should be removed from the curriculum.	16 (24.6)	26 (40.0)	0 (0.0)	14 (21.5)	9 (13.8)
Studying ITF improves my reasoning and analytical skills.	1 (1.5)	4 (6.2)	3 (4.6)	24 (36.9)	33 (50.8)
My perception of ITF depends on their usefulness in my daily activities.	11 (16.9)	12 (18.5)	2 (3.1)	13 (20.0)	27 (41.5)
My perception of ITF has affected my mathematics performance.	8 (12.3)	2 (3.1)	0 (0.0)	26 (40.0)	29 (44.6)

Note. SA: Strongly agree; A: Agree; U: Unsure; D: Disagree; SD: Strongly disagree

Using an analytic coding framework developed in line with the visual-genetic perspective and the relevant literature, participants' responses were evaluated in terms of the conceptual foundations of ITF, graphical and geometrical interpretations, unit circle and right triangle vector relationships, transformation processes, and domain-range restrictions. Responses were classified in a manner that revealed levels of conceptual adequacy and the nature of the difficulties encountered, while document analysis was employed to examine the structure of misconceptions in depth. The reliability of the coding process was supported by a high level of inter-rater agreement, and the resulting findings enabled the interpretation of pre-service mathematics teachers' knowledge levels within a visual-genetic framework. For example, responses that included the phrase $arcsinx = \frac{1}{sinx}$ were coded as wrong due to a notation-based misconception, whereas those that articulated $sin(arcsinx) = x$ for $-1 \leq x \leq 1$ were coded conceptually accurate with domain awareness.

In addition, to ensure theoretical coherence, the items of the ITAT were analyzed by mapping them onto APOS structures and visual-genetic stages. Each item was designed to progress from the action-based level to the schema level. From a visual-genetic perspective, the items were also analyzed according to whether they required understanding of the following aspects: the origin and development of ITF, graphical reflection and transformation, unit circle interpretation, and right-triangle and rotational reasoning. For example: Items asking about the meaning of $arcsin(x)$ primarily targeted understanding at the action level, items involving reflections across $y = x$ and graph construction targeted the process and object levels, items involving compositions such as $sin(arcsin[x])$ or evaluations such as $arcsin(sin[\frac{5\pi}{6}])$ required coordination of inverse relationships, domain restrictions, and representations at the schema level. This alignment allowed the errors and successes of pre-service mathematics teachers to be interpreted not merely as incorrect or correct responses, but also as indicators of incomplete/fragmented mental structures.

FINDINGS

The results of the study were meticulously organized under themes based on the RQs. The questions focused on pre-service teachers' perceptions, knowledge, and visual-genetic understanding of ITF.

Pre-Service Teacher's Perceptions of Inverse Trigonometric Functions

To examine pre-service teachers' knowledge of ITF and their perceptions of the visual-genetic approach, participants were presented with eight statements in a Likert-scale format and were asked to indicate the extent to which they agreed with each statement. The frequency counts and percentages of participants' responses to each statement were calculated and presented as shown in **Table 2** (ITPQ perception results).

In this study, pre-service teachers demonstrated a range of both positive and negative perceptions regarding ITF. Positively, they perceived that the visual-genetic interpretation of ITF enhanced their reasoning and analytical capacities, and that their academic success depended on understanding these concepts. Negatively, as shown in **Table 2** (ITPQ perception results), 83.1% of participants indicated that ITF were very rigid, abstract, and tedious, while 77.0% agreed that they were a difficult topic. The majority of participants (87.7%) believed that ITF enhanced their reasoning and analytical skills. Similarly, 84.6% stated that their perceptions of ITF influenced their performance.

Furthermore, 35.3% of participants agreed that ITF should be removed from the curriculum, while a significant proportion (64.6%) disagreed. This finding demonstrates that, despite the well-documented challenges associated with ITF, their importance within the mathematics curriculum continues to be recognized. Overall, the results suggest that pre-service teachers hold conflicting perceptions of ITF, highlighting the need for instructional approaches that reduce abstraction, enhance conceptual understanding, and emphasize meaningful applications of these functions.

Table 3. Pre-service teachers' responses to ITAT (n = 65)

No	Questions	f (%)	
		Incorrect	Correct
1	What does the expression $arcsin(x)$ mean?	6 (9.2)	59 (90.8)
2a	Is there an alternative expression that is equivalent to $arcsin(x)$?	65 (100)	0 (0.0)
2b	Is the statement $arcsin\ (x) = \frac{1}{sinx}$ true or false? Explain briefly.	65 (100)	0 (0.0)
3a	How can we get the graph of $y = arcsin\ (x)$ using $y = sinx$?	60 (92.3)	5 (7.7)
3b	Sketch $y = sinx$ and $y = arcsin(x)$ on the same coordinate plane.	62 (95.4)	3 (4.6)
4	Briefly explain the range of $y = arcsin(x)$.	59 (90.8)	6 (9.2)
5	Evaluate $sin(arcsinx)$, where $-1 \leq x \leq 1$.	63 (96.9)	2 (3.1)
6	If $arccosx = \frac{\pi}{3}$, what is the value of x?	25 (38.5)	40 (61.5)
7	Find the value of $arcsin\left(\frac{1}{2}\right)$.	61 (93.8)	4 (6.2)
8	Explain the value of $arctan(-1)$ using the unit circle.	63 (96.9)	2 (3.1)
9	Evaluate $arcsin\left(sin\frac{5\pi}{6}\right)$ and explain each step of your reasoning.	61 (93.8)	4 (6.2)
10	Determine whether the statement $sin(arcsinx) = x$ is always true.	60 (92.3)	5 (7.7)

Pre-Service Teachers' Knowledge of Inverse Trigonometric Concepts

A 10-item ITAT was administered to pre-service teachers to evaluate their knowledge of ITF. The responses of the 65 participants were analyzed and presented in **Table 3** (ITAT performance results).

The first question, which investigated participants' basic knowledge of ITF, asked them to explain in an open-ended format the meaning of the most commonly encountered expression, $arcsin(x)$. **Table 3** (identified conceptual difficulties) shows that nearly all participants answered this question correctly with concise and clear responses. Notably, only 9.2% of the participants provided incorrect or unusual answers. This high success rate indicates that most participants were operating at the action level and were familiar with the representation of ITF. However, it also suggests that this success did not extend to deeper levels of understanding and that conceptual development did not go beyond procedural recognition.

Question 2a and question 2b, aimed to test their understanding of the various representations of ITF. **Table 3** (identified conceptual difficulties) indicates that none of the participants (0%) were able to explain that the different basic notations of $arcsin(x)$ convey the same meaning. Furthermore, a detailed analysis of the solutions provided by nearly all candidates indicates that they were influenced by the phrasing of question 2a and fell into a conceptual misunderstanding, incorrectly stating that $arcsin\ (x) = \frac{1}{sinx}$. The participants' insufficient recognition of $arcsin(x)$ and its equivalent expression, along with the widespread misconception that $arcsin(x) = \frac{1}{sinx}$, indicates a failure to transition from the action level to the process level. This situation reflects a lack of internalized coordination of inverse relationships among the participants and, consequently, reveals a fundamental genetic gap in their conceptual development.

Question 3a tested the participants' knowledge, as taught in school mathematics, that the graph of the inverse of a function can be obtained by reflecting the given function across the line $y = x$, without performing lengthy calculations. The results show that participants could not explain how this was done, with 92.3% answering incorrectly. Question 3b examined the drawing of these two functions' graphs on the same coordinate plane. Only a small proportion of participants (7.7%) correctly indicated that the reflection should be taken across the line $y = x$, and an even smaller percentage (4.6%) were able to draw the graphs correctly by overlooking minor, negligible mistakes. These findings indicate that most participants did not construct ITF as objects. The inability to reflect graphs across the line $y = x$ points to weak visual-genetic reasoning and a lack of object-level encapsulation of the function-inverse relationship.

Question 4 examined the participants' ability to read information from a portion of the graph of the function $y = arcsin(x)$. Since most participants were unable to correctly obtain this graph in the previous question, the vast majority (90.8%) also failed to answer this question correctly. Although a very small proportion (9.2%) provided correct answers, these responses lacked any explanation and raised doubts about how the results were obtained. These findings indicate that the participants lacked sufficient development even at the action level and were deficient in fundamental knowledge related to origins, development, and graphical interpretation, thereby also revealing the absence of understanding at the object level.

Question 5 revealed weaknesses in the participants' understanding of composite functions and reflected confusion regarding the constraints associated with ITF. While the expression $sin(arcsinx)$ could be resolved through a very simple operation, many participants attempted various complex procedures, producing inconsistent results. A substantial majority (96.9%) demonstrated a lack of general knowledge about origin, development, and graphical understanding for this question, while only a very small proportion (3.1%) provided satisfactory answers.

Question 6 required participants to demonstrate their knowledge of the relationship between the sides of special right triangles and the acute angles within these triangles, or to use trigonometric tables where the values of trigonometric functions are listed. **Table 3** (identified conceptual difficulties) shows that 61.5% of the participants performed the necessary drawings to support their solutions, while a substantial proportion (38.5%) could not reach correct solutions. These results indicate that the teacher candidates possessed good knowledge in finding ITF ratios using special right triangles. This finding shows that the candidates completed the task by organizing their reasoning at the process level and achieving a coherent schema-level integration. From a visual-genetic perspective, it also indicates that the participants possessed some fundamental developmental and graphical knowledge related to ITF.

Question 7 required knowledge and understanding of the value of $arcsin\left(\frac{1}{2}\right)$ using simple aids similar to those in question 6 (e.g., the sides of special right triangles, relationships between angles, or composite functions), while question 8 required an explanation of the value of $arctan(-1)$ using the unit circle. The results showed that 93.8% of the participants failed to answer question 7 correctly, with only 6.2% providing an accurate explanation. Similarly, only 3.1% were able to correctly perform the drawing, representation, and explanation required in question 8, indicating that very few teacher candidates both knew the concepts and understood them. These findings reflect action-level understanding and object-level reasoning; however, from a visual-genetic perspective, the rare use of the unit circle indicates not only a developmental deficiency but also a fragmented integration of symbolic and visual representations in graphical or functional interpretations.

Question 9, designed with logic similar to question 5 for cross-validation purposes, yielded comparable results: 93.8% of the participants again demonstrated insufficient foundational, developmental, and graphical knowledge. Question 10 examined the participants' ability to apply simple algebraic rules to ITF. This question partially included the cross-validation logic of question 5, but giving the right-hand side of the equality $sin(arcsinx) = x$ revealed that some candidates had partial foundational and developmental knowledge. However, it was evident that this knowledge was insufficient, as 92.3% of participants failed to solve the problem. Tasks 5, 9, and 10 involving compositions such as $sin(arcsinx)$ and $arcsin(sin\theta)$ required the coordination of ITF, domain restrictions, and representations at the schema level. The very low success rates indicate that most participants did not possess a coherent schema integrating the symbolic, graphical, and geometrical aspects of ITF.

Overall, the results indicate that teacher candidates were partially successful on definitional and straightforward computational questions involving ITF, but exhibited serious deficiencies in areas requiring foundational, developmental, and graphical (visual-genetic) knowledge. In particular, the correct response rates for questions requiring graphical and conceptual understanding remained below 10%, suggesting that while participants were superficially familiar with inverse trigonometric representations, more than half lacked sufficient knowledge in these areas. This finding is also consistent with the negative perceptions previously reported by the participants.

Pre-Service Teachers' Conceptual Difficulties in Inverse Trigonometric Functions

Pre-service teachers' conceptual challenges play a significant role in shaping their instructional approaches in the classroom. To identify these conceptual challenges, the responses given by participants to the test items were analyzed. The analysis revealed that teacher candidates generally experienced difficulties with almost all of the fundamental inverse trigonometric concepts they were expected to teach. The impact of these challenges on participants' performance is summarized in **Table 3** (ITAT performance results).

Table 3 (identified conceptual difficulties) also indicates that, except for question 1, more than 50% of the teacher candidates were unable to solve the tested tasks. While participants were largely successful (90.8%) in correctly identifying the meaning of the $arcsin(x)$ notation, this success was largely limited to superficial familiarity with the representation. Their greatest difficulties lay in explaining fundamental concepts related to different representations of ITF, such as $arcsin(x)$, as well as in the misconception that $arcsin(x) = \frac{1}{sinx}$ since none of the participants were able to explain this correctly. These findings reveal a fundamental genetic (origin and developmental) deficiency in understanding the concept of ITF.

Graphical understanding also emerged as a significant area of difficulty. More than 92% of the participants could not explain how the graph of $y = arcsin(x)$ is obtained from $y = sinx$, and 95.4% were unable to accurately draw both graphs on the same coordinate plane.

The second most difficult concept for teacher candidates was calculating expressions such as $sin(arcsinx)$ and $arctan(-1)$. **Table 3** (ITAT performance results) shows that more than 96% of participants were unable to perform these calculations.

The analysis of participants' explanations regarding the fundamental inverse trigonometric expression $arcsin(x)$ and its equivalent representations, as well as the misconception that $arcsin(x) = \frac{1}{sinx}$ together with their approaches to graphing $y = arcsin\,(x)$, revealed that many teacher candidates experienced conceptual difficulties in understanding the relationships among ITF. This highlighted a strong need for visual-genetic approaches.

DISCUSSION

Within the context of this study, the findings indicate that pre-service teachers could benefit from instructional approaches that place greater emphasis on the conceptual understanding of ITF—particularly through graphical and visual-genetic approaches. These instructional strategies, which explicitly address inverse relationships, domain restrictions, and function-inverse transformations, are consistent with prior research on trigonometry learning (Dhungana et al., 2023; Klein, 2015) and may help reduce common misconceptions and insufficiently understood conceptual relationships observed among participants.

These implications should be interpreted as recommendations for similar teacher education programs in which ITF are primarily taught using procedural or algebraic approaches, rather than as a call for broad curricular reform. The results highlight the potential value of integrating visual-genetic reasoning and APOS-based frameworks into existing instructional practices, particularly in courses designed to prepare future mathematics teachers. From this perspective, many participants' responses indicate that understanding often remained at the action or early process level. For example, while participants were able to recall symbolic expressions or compute values, they frequently failed to conceptualize ITF as objects with restricted domains and ranges. This aligns with previous studies reporting that participants tend to interpret inverse trigonometric notation operationally rather than relationally (Tasar et al., 2023; Tuktamyshov & Gorskaya, 2024; Vos et al., 2024).

Participants' problem-solving performance and their capacity to apply trigonometric knowledge effectively support the view articulated by Tuna (2013) that a solid grasp of trigonometric concepts forms the foundation of understanding trigonometry. Solving mathematical problems involves using knowledge to organize mental structures related to origin and extension, graphs, the unit circle, right triangles, vectors, rotation, and inversion. Overall, the results indicate that most pre-service teachers exhibit insufficient competence in the visual-genetic approach (Gaziyev et al., 2006; Kamber Hamzić et al., 2025; Orhani, 2024; Spangenberg, 2021), which limits their ability to actively engage with problems involving inverse trigonometric expressions.

The prevalence of graphical misconceptions further demonstrates limited visual-genetic development. Participants often struggled to interpret inverse functions as reflections across the line $y = x$, suggesting that graphical representations were not well integrated into their conceptual schemas. This finding supports previous research emphasizing that understanding ITF requires coordinated development across multiple representations rather than reliance on algebraic manipulation alone (Fi, 2003; Zeng et al., 2023).

These implications are exploratory in nature and aim to inform reflective practice among teacher educators rather than to prescribe universal instructional change. This perspective aligns with the findings of Nabie et al. (2018), who argue that teacher educators must adapt instructional practices and prioritize understanding-oriented teaching to provide high-quality mathematics education. It also supports the view of Kurudirek and Berdieva (2024) that strengthening schools contributes to empowering future generations, emphasizing that the development of pre-service teachers is a critical component of the educational system.

Limitations

This study has several limitations that should be taken into consideration. First, the sample was taken from only one faculty of education in Fergana, Uzbekistan, hence the results may not be applicable to other institutions or cultures. Second, the educational intervention was conducted over a brief duration, perhaps inadequate for facilitating profound conceptual reorganization of inverse trigonometric knowledge. Third, some of the data is based on how individuals personally understand things. Employing a quasi-experimental and descriptive design may constrain the capacity to formulate robust causal assertions concerning the impacts of the visual-genetic approach. Even with these constraints, the study gives us useful information about how pre-service mathematics instructors think about and understand ITF. It also sets the stage for additional in-depth research in the future.

Recommendations

Considering its limitations, the findings of this study offer several implications for teacher education practices and directions for future research. With similar groups of participants, greater instructional emphasis can be placed on the conceptual and visual-genetic structure of ITF.

Future studies may extend this work through intervention-based designs to examine how targeted instructional approaches grounded in APOS theory and visual-genetic reasoning influence the development of inverse trigonometric understanding over time. Including multiple institutions and larger sample sizes would also enhance the generalizability of the findings.

Further research may investigate the relationship between pre-service teachers' conceptual understanding of ITF and their instructional decision-making processes, particularly regarding how visual representations are selected and used in teaching.

CONCLUSION

This study examined pre-service teachers' perceptions and conceptual understanding of ITF within the context of a single faculty of education in Uzbekistan. The findings indicate that, for this specific group of participants, conceptual understanding of ITF is limited, particularly at the graphical, relational, and visual-genetic levels. Therefore, the study highlights the importance of instructional approaches that emphasize conceptual coherence, visual reasoning, and the coordinated use of multiple representations when teaching ITF to future teachers. It is important to emphasize that these findings are descriptive and context-specific and thus should be interpreted with caution. Nevertheless, they provide valuable insights into common learning obstacles and may contribute to reflective practice in similar teacher education settings.

These findings do not suggest that all pre-service teachers in Uzbekistan or elsewhere have same challenges. Rather, they offer insight into the learning experiences of a particular group and underscore the need for targeted instructional support in comparable teacher education contexts.

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