

Preparing, leading, and reflecting on a mathematics whole-class discussion in a lesson study in initial teacher education

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Abstract

Based on the work in a lesson study carried out during an initial teacher education program, using a comprehensive model for preparing and leading whole-class discussions, we seek to know how a prospective teacher prepared, led and reflected on such discussions. We follow a qualitative approach, and the data was collected through observations, recordings and document collection. The analysis of data is based on the proposed model for whole-class discussions. The results show that the prospective teacher managed to promote a fruitful whole-class dynamic, inviting students to participate, and intervening when necessary and analyzing the students' work with a focus on mathematical ideas. However, she found it difficult to get most students to participate in the whole-class discussion. The work on the lesson study seems to have contributed to the participant to prepare for the discussion in a collaborative environment and to reflect on its strong and weak features.

Keywords: whole-class discussion, lesson study, mathematics teaching, initial teacher education

INTRODUCTION

Whole-class discussions assume great relevance in fostering students' learning, as they enable the exploration of mathematical ideas through the analysis of students' own solutions as well as those of their peers, within the context of tasks assigned by the teacher. The teacher is central in this process, encouraging students to present and justify their ideas, making it essential a thorough preparation of the enactment of a whole-class discussion (Fujii, 2018; Stein et al., 2008). However, leading these discussions has been identified by research as a challenge for teachers and, especially, for prospective teachers (Martins et al., 2023), since it requires a comprehensive knowledge base encompassing content, didactics and understanding of how students learn (Fujii, 2018; Stein et al., 2008). In addition, leading whole-class discussions also requires "solid expectations for what is likely to happen, [and] novices are regularly surprised by what students say and do, and therefore often do not know how to respond to students in the midst of a discussion" (Stein et al., 2008, p. 321). According to Fujii (2018), an effective way to address the challenges of leading whole-class

discussions is through detailed preparation. These plans include anticipating potential strategies for addressing students' difficulties, preparing actions for various stages of the lesson, and carefully considering the teacher's interventions.

Lesson study is recognized as a formative process with the potential to promote the development of prospective teachers, namely on leading whole-class discussions (Martins et al., 2023; Ni Shuilleabhain & Bjuland, 2019). Focused on student learning, lesson study provides opportunities for prospective teachers to prepare lessons in detail, anticipating student work and teacher actions. This teacher education process allows prospective teachers to prepare the lesson, and also to lead it and reflect on the students' work and learning (Bjuland & Mosvold, 2015). Martins et al. (2023) show how reflection on practice is important for the development of prospective teachers' knowledge about leading whole-class discussions. However, further research is needed to explore how lesson study may promote opportunities for prospective teachers to develop their understanding of whole-class discussions, from preparation and enactment to reflection. Although there are several proposals on how to lead a whole-class

Contribution to the literature

- This study contributes to the literature by articulating and empirically applying a practice-based model for leading whole-class discussions within a lesson study context, focusing on a prospective teacher's enactment and learning.
- It offers a fine-grained analysis of the actions used to support, challenge, and inform students, thereby operationalizing constructs that are often treated abstractly in prior research.
- Methodologically, it integrates lesson study with detailed discourse analysis of planning, enactment, and reflection sessions, providing insight into how instructional practices are developed over time. Its findings are particularly relevant for teacher education, as they make evident both the affordances and limitations of lesson study in fostering ambitious discussion-leading practices, especially regarding student participation.

discussion (e.g., Boerst et al., 2011; Cengiz et al., 2011; Kooloos et al., 2023), they often emphasize specific aspects but lack a broader understanding of how to effectively support teachers and prospective teachers in preparing for and leading this activity. This highlights the need for a comprehensive model that can consider all the key elements of a whole-class discussion. To this end, we developed a comprehensive model that encompasses all the fundamental aspects of that part of the lesson. This model was initially used to promote and later to analyze the practices and actions of a prospective teacher in both the preparation for and leading of the discussion. Thus, our research question is: how does a prospective teacher prepare, lead and reflect on a mathematical whole-class discussion within a lesson study?

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

Whole-Class Discussions in Mathematics

The work and learning that students engage in during a mathematics class are significantly influenced by the type of tasks proposed by the teacher, the nature of the communication that takes place, and the roles assumed by both students and teacher. A form of communication particularly valued by research and curriculum reforms is whole-class discussion, a form of group interaction in which participants address a question of common interest, present their ideas, examine different views (Dillon, 1994), and argue with each other to form their answer. As Mosvold (2024) indicates, although there has been extensive and significant research into discussions on teaching and learning mathematics, there is still a need for a "foundation for developing a professional practice that acknowledges the skills and knowing that are involved in leading mathematical discussions" (p. 485).

From a socio-constructivist perspective, Kooloos (2022) states that discourse-based teaching occurs when discourse builds on students' different ways of thinking. He distinguishes between whole-class discourse and whole-class discussion: the former concerns dialogue about students' different ideas, whereas the latter occurs

when students actively engage with their colleagues' mathematical ideas. This type of discussion can be a key moment in an exploratory lesson, organized in three phases:

- (1) presentation and interpretation of the task (collectively),
- (2) carrying out the task (autonomously), and
- (3) presentation and discussion of the solutions and final synthesis (again collectively) (Ponte, 2005).

In an exploratory lesson, students should not have an immediate method for solving the task presented for autonomous work; rather, the task should allow them to use different solution methods and mathematical representations. For Japanese teachers, whose professional development focuses heavily on lesson study (Fujii, 2018), in addition to prior preparation and planning, it is during students' autonomous work that the teacher prepares the whole-class discussion. This moment, called *neriage*, is considered essential in a problem-solving lesson (Takahashi, 2008), as it provides students with opportunities to present and explain their strategies, listen to their colleagues' explanations, and reflect on them.

For a whole-class discussion to be productive, it is important for the teacher to encourage students to justify the ideas and explanations of their solution strategies. It is also significant to include and value incorrect or incomplete contributions that may lead students to formulate alternative justifications, argue, and criticize constructively, resulting in situations of disagreement between them (Shaughnessy et al., 2020; Takahashi, 2008; Wood, 1999). Thus, under the teacher's guidance, a whole-class discussion is a moment when students interpret information, negotiate, and construct knowledge. Students, therefore, need to play a central and active role by presenting and justifying their reasoning, developing mathematical thinking and meaningful learning (Cengiz et al., 2011; Stein et al., 2008; Takahashi, 2008).

However, leading whole-class discussions is a major challenge for teachers and prospective teachers, and they are not commonly used in their usual lessons (Cengiz et

al., 2011; Kooloos et al., 2023; Stein et al., 2008; Takahashi, 2008). A whole-class discussion goes beyond the presentation of students' task-solving strategies, making teacher preparation essential. Therefore, initial teacher education should provide prospective teachers with opportunities to prepare, lead, and reflect on whole-class discussions.

Lesson Study and Whole-Class Discussion

Lesson study is a collaborative and reflective teacher education process based on teachers' practices and focused on student learning (Fujii, 2018). Initially designed for in-service teachers, it has also been disseminated and adapted to initial teacher education (e.g., Martins et al., 2023). In lesson study, prospective teachers have opportunities to plan, lead, and reflect on classroom situations, promoting links between the theory learned in teacher education courses and teaching practice, and developing their knowledge in various fields (Larssen et al., 2018; Ni Shuilleabhain & Bjuland, 2019; Ponte, 2017).

A lesson study is usually organized in five phases:

- (1) problem identification,
- (2) lesson planning,
- (3) conducting and observing the research lesson,
- (4) post-lesson discussion, and
- (5) reflection (Fujii, 2018).

During lesson planning, participants collaboratively select, adapt, and solve challenging mathematical tasks. They then anticipate students' possible strategies, considering different levels of correctness and sophistication, potential difficulties and ways to support students in overcoming them. Based on these anticipated strategies, they discuss how to initiate the whole-class discussion, which strategies to select and sequence, and how to compare them to achieve the lesson aim (Fujii, 2018). They may also anticipate questions or interventions and when to introduce them during the discussion (Orr & Bieda, 2025; Stein et al., 2008).

Participants then teach and observe the research lesson, putting their detailed planning into practice and reflecting on its outcomes. Experiencing in class the strategies that they devised allows prospective teachers to relate theoretical aspects to teaching practice and creates opportunities to reflect on this experience (Leavy & Hourigan, 2016).

The subsequent sessions focus on student learning and reflection on practice. Participants analyze situations from the research lesson using collected data, critically evaluating the results and considering aspects to improve (Bjuland & Mosvold, 2015). This process enables them to review their practice and consider how it can be improved.

Previous studies have shown the potential of lesson study to support prospective teachers' learning about

exploratory mathematics teaching, including the preparation and management of whole-class discussions. Martins et al. (2021), based on a lesson study with two prospective mathematics teachers, showed that collaborative discussions and reflection helped them move from a teacher-centered understanding towards planning exploratory lessons that included autonomous student work and whole-class discussion. In particular, the prospective teachers learned to anticipate students' strategies and difficulties, prepare teacher interventions, and recognize the importance of whole-class discussions. However, leading the discussion remained a major challenge, as they were unable to establish connections among different strategies, manage time, or conduct a final synthesis of the mathematical ideas. Gomes et al. (2022) extended this perspective by examining two lesson studies involving prospective and in-service teachers, showing how planning, enactment, and reflection on tasks enabled participants to develop knowledge about students' strategies and difficulties, representations, and the management of exploratory lessons. Regarding prospective teachers, the study also showed that anticipating students' work and preparing supporting questions helped them to manage students' autonomous work and that reflection on enactment revealed unanticipated strategies. Taken together, these studies demonstrate that lesson study may support prospective teachers in planning exploratory lessons and reflecting on students' work, and they point to the importance and difficulty of leading whole-class discussions. However, neither study focused specifically on whole-class discussion as a distinct object of inquiry, nor did they provide an integrated model that brings together the practices and actions involved in preparing, leading, and reflecting on such discussions. Thus, although previous research has shown that prospective teachers can learn to anticipate strategies, plan interventions, and recognize the role of whole-class discussion through lesson study, it remains unclear how the different practices required for a productive whole-class discussion are articulated across preparation and enactment, and how prospective teachers enact and reflect on these practices. Addressing this gap, the present study uses a comprehensive practice-based model to structure the preparation of a whole-class discussion and to analyze how a prospective teacher prepares, leads, and reflects on it within a lesson study.

Preparing and Leading a Whole-Class Discussion

The quality of a whole-class discussion is strongly influenced by teacher preparation (Fujii, 2018; Takahashi, 2008). Preparation begins by identifying the mathematical content and defining clear learning aims (Boerst et al., 2011), followed by selecting, adapting, or designing tasks that promote students' understanding of key concepts, procedures, and representations. Solving the task in advance enables teachers to anticipate

possible strategies, difficulties, required knowledge, and common misconceptions, drawing on curriculum documents and research (Boerst et al., 2011; Kooloos et al., 2023; Stein et al., 2008). Preparing the discussion is also essential: teachers anticipate how to connect students' strategies to lesson aims and plan questions and interventions that elicit mathematical thinking, introduce productive challenge, and make students' thinking visible (Duarte et al., 2024; Orr & Bieda, 2025; Shaughnessy et al., 2020). Although these elements typically occur in sequence, preparation is iterative and flexible.

In class, the teacher presents and clarifies the task without revealing solution paths or compromising cognitive demand (Duarte et al., 2024). During students' autonomous work, the teacher monitors strategies and reasoning, selects solutions to share, and determines their sequence according to previously defined criteria (Fujii, 2018; Kooloos et al., 2023; Stein et al., 2008). The teacher then orchestrates the discussion in alignment with the lesson aim, continuously adapting interventions to student contributions and unfolding classroom events (Kooloos et al., 2023; Orr & Bieda, 2025).

Research has identified several categories of teacher actions in whole-class discussions. Cengiz et al. (2011) distinguish eliciting actions, which invite students to express their thinking; supporting actions, which interpret, remind, record, or revoice ideas; and extending actions, which invite evaluation, comparison, justification, or counterargument. Similarly, Ponte et al. (2013) identify inviting, supporting/guiding, challenging, and informing/suggesting actions. Kooloos et al. (2023) propose three interrelated categories: teacher moves, involving discursive interactions that support reasoning and connections; teacher noticing, involving interpretation of student thinking and decisions about how to respond; and teacher mathematical thinking, involving anticipation of strategies and connections between student reasoning and lesson aims.

During the discussion, teachers analyze students' work with attention to mathematical ideas and use questioning to clarify reasoning and highlight key ideas (Orr & Bieda, 2025). The diversity of strategies requires careful analysis to establish connections among them and with the learning aims (Stein et al., 2008). Thanheiser and Melhuish (2023) describe mathematically productive routines that make student thinking public, engage students with the reasoning of peers, and position sense-making and justification at the center of instruction. These include conferring to understand thinking, structuring student talk, working with selected and sequenced ideas, using public records of thinking, and orchestrating discussion.

Because whole-class discussions depend on students sharing strategies, teachers should create opportunities

for them to recognize and revise incomplete or incorrect ideas, developing mathematically sound reasoning (Shaughnessy et al., 2021). Incorrect ideas and disagreements can support argumentation, critique, and error analysis, contributing to mathematical reasoning and enabling teachers to identify patterns in student thinking and deepen their understanding of how students learn (Shaughnessy et al., 2020; Stein et al., 2008; Wood, 1999). To sustain engagement, interventions should involve the whole class, with students listening, analyzing, questioning, and responding to one another's reasoning (Boerst et al., 2011; Thanheiser & Melhuish, 2023). Student learning, therefore, depends substantially on how teachers manage the discussion, particularly on whether students are given space to articulate and justify their reasoning or the teacher assumes responsibility for explanations.

The discussion concludes with a final synthesis that systematizes the main mathematical ideas explored and relates them explicitly to the lesson aims. In Japanese lesson study, this moment (*matome*) receives particular emphasis (Fujii, 2018). Its absence can undermine the mathematical work developed during the lesson (Cengiz et al., 2011; Shaughnessy et al., 2021). The synthesis is not a repetition of students' strategies but an articulation of their reasoning with the targeted mathematical concepts and procedures, establishing meaningful connections and linking them to prior knowledge (Stein et al., 2008; Duarte et al., 2024; Ponte, 2005).

Based on this previous work, we developed a model for preparing and leading whole-class discussions (Table 1), identifying key practices that are essential to high-quality discussions but challenging for prospective teachers. By practices, we refer to central professional activities that significantly influence student learning; each practice comprises actions that operationalize it and create opportunities for mathematical learning (Ponte et al., 2013).

The model distinguishes preparation and enactment. Preparation incorporates established elements from the literature (Boerst et al., 2011; Cengiz et al., 2011; Fujii, 2018; Kooloos et al., 2023; Stein et al., 2008): defining aims, selecting and solving tasks, anticipating strategies, and planning connections. Leading the discussion involves three simultaneous practices identified in prior research (Boerst et al., 2011; Kooloos et al., 2023; Ponte et al., 2013; Stein et al., 2008; Wood, 1999): initiating discussion and intervening according to aims and context; analyzing students' work with attention to mathematical ideas; and promoting broad student participation. In the final synthesis, in addition to establishing connections (Stein et al., 2008), the model includes explicitly summarizing and highlighting the main mathematical ideas and procedures addressed (Ponte, 2005).

Table 1. Model of practices and actions to consider when preparing and leading the whole-class discussion and final synthesis

To prepare the whole-class discussion		To lead the whole-class discussion	To conclude the whole-class discussion with a final synthesis
Before class	During class		
Initial preparation	Present the task	Initiate the whole-class discussion and intervene according to the situation and the aim	Conclude, with the collaboration of the students
- Identify the mathematical content	- Challenge the students and clarifying aspects of the statement, without suggesting or indicating the answer	- Invite a student or students to present the first solution you have selected and invite new students to present other solutions	- Summarize and highlight the main mathematical ideas or procedures explored
- Define the learning aims considering the curriculum documents	Monitor the students' work	- Supporting/ guiding, including encouraging, redirecting and keeping the discussion on track	- Show connections with other mathematical ideas already known to the students
- Consult articles on students' common difficulties with the topic	- Observe their solving of the task and their reasoning	- Challenging and urging students to clarify and justify their statements	
Choose the task	Select students' answers	- Informing/ suggesting, including highlighting important mathematical ideas	
- Select, adapt or create the task	- To share in whole-class discussion	Analyze students' work with a focus on mathematical ideas	
Solve the task	Sequence	- Making connections between ideas, solutions, and strategies used by students	
- Anticipate students' solving strategies	- The selected answers	- Asking questions to support students when they express their reasoning	
- Anticipate students' difficulties in interpreting and solving the task		- Emphasizing important mathematical ideas	
Prepare to carry out the task in class		- Reformulate incomplete or erroneous ideas	
- Anticipate how to establish connections between the different strategies and the lesson aim		Promote student participation	
- Prepare their interventions		- Encourage students to participate in the discussion, involving the whole class	
		- Encourage students to listen, analyze and respond to their colleagues' reasoning	
		- Asking another student to explain a colleague's answer	
		- Promote situations of disagreement.	

RESEARCH METHODOLOGY

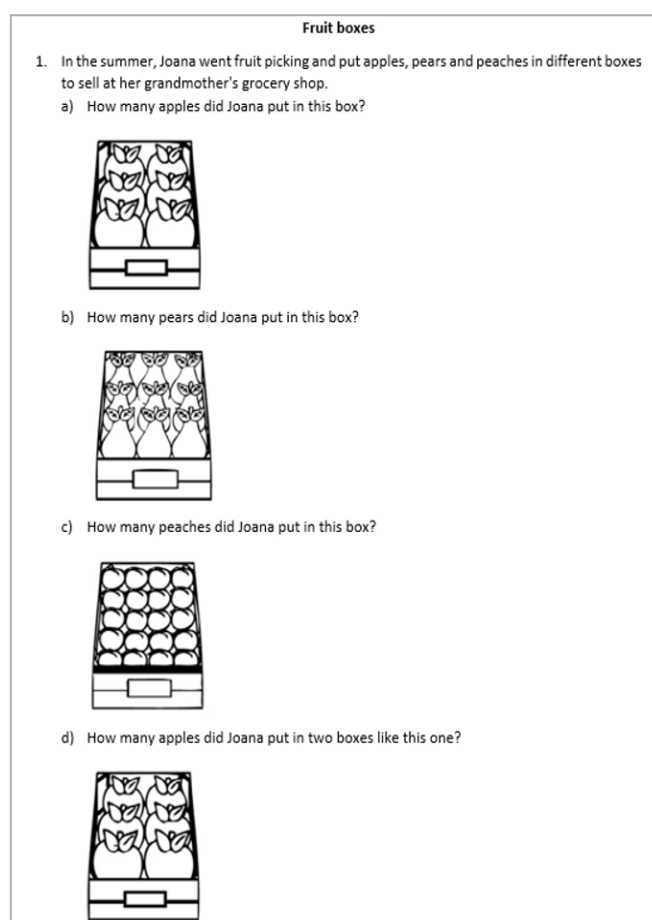
This research follows the qualitative approach within the interpretive paradigm (Bogdan & Biklen, 2007) and addresses a lesson study carried out at a higher education institution in Portugal in the initial teacher education of primary teachers. The lesson study was carried out in the 2023/2024 academic year and involved four prospective teachers who were attending the first semester of the first year of their master's degree program in teaching. The prospective teachers had several courses on mathematics (32 ECTS). Prior to the lesson study, the prospective teachers completed courses in sociology and psychology of education, didactics of mathematics, curriculum subjects and school organization. Given that they were at the beginning of their master's degree program, they had very limited knowledge, if any, of whole-class discussion, as that is not studied in depth during their undergraduate studies.

All of them observed several lessons taught by experienced teachers before the lesson study so they were familiar with the students' dynamics in the classroom. There was a total of 13 sessions, organized as shown in **Table 2**, held weekly, between October and December 2023, and lasting approximately two hours each.

In this study, we distinguish between the discussions that took place among the prospective teachers and the facilitator during the lesson study and the mathematical whole-class discussion that Júlia led with the school students during the research lesson. We use "lesson-study discussions" to refer to the former, including planning and post-lesson reflection, and "whole-class discussion" to refer to the classroom interaction in which Júlia and the school students collectively examined and discussed mathematical ideas.

Table 2. Relation between the stages of the lesson study and the session number

Stages of the lesson study	Sessions (Sn)	Main activities	Duration	Data collected
Goal setting	S1-S2	Curriculum analysis; definition of learning aim; task discussion	4 h	Audio recordings; documents
Lesson planning	S3-S6	Task solving; anticipation; selection; sequencing; questioning	8 h	Audio recordings; lesson plans
Research lesson 1	S7	Teaching and observation	-	Video; observation notes; student work
Post-lesson 1 discussion	S8	Analysis of lesson	2 h	Audio; notes
Research lesson 2	S9	-	-	-
Post-lesson 2 discussion	S10	-	-	-
Reflection	S11	Reflection on practice	2 h	Audio recordings; written reflection
Preparing to disseminate results	S12-S13	-	-	-

**Figure 1.** Task proposed to the students in the research lesson (adapted from Rocha & Menino, 2009)

The lesson study was an integral part of the practicum program, which was carried out in a grade 2 class with students that were between the age of six and seven. The prospective teachers were supervised and guided by a teacher from their higher education institution (first author), who took on the role of facilitator. The supervisor organized the lesson study preparation sessions, guiding the prospective teachers through the various tasks to carry out, such as defining the learning aim for the lesson and choosing the task. During the research lesson, the supervisor also took on the role of observer, video recording the lesson and

Table 3. Table used in the final synthesis of the research lesson

Quantity of fruit	Addition	Multiplication
6 apples	$2 + 2 + 2$	3×2
	$3 + 3$	2×3
9 pears		
20 peaches		
12 apples		
Class suggestion		

making written records of relevant moments for the post-lesson discussion. In the final discussion and reflection sessions, she shared her written records of the lesson observation and selected video excerpts to be analyzed by the group. Throughout all the phases of the lesson study, the supervisor promoted moments of reflection through open questions and ensured that the participants were actively involved in doing the tasks. Additionally, the prospective teachers were supported by a cooperating teacher from the school where they were doing their practicum.

After analyzing the timetable and medium-term planning provided by the prospective teacher's practicum school, the participants decided that the research lesson would focus on the theme of numbers and the introduction of the topic of multiplication. The lesson's learning aim is to introduce the concept of multiplication based on the students' previous knowledge of addition. This would be achieved by solving problems with rectangular arrangements that fostered an understanding of the relationship between multiplication and addition. Three articles from professional literature were analyzed in the lesson preparation sessions, promoting the discussion of key aspects in preparing an exploratory lesson and, in particular, the whole-class discussion. This work began by paying attention to the selection of the task (Figure 1).

To promote the final synthesis, the prospective teachers decided to explore with the students a table (Table 3), which systematizes some of the strategies that may be used to solve the task and includes a space to fill in with the students' suggestions.

Among the four prospective teachers, only two Júlia and Alda (pseudonyms) took on the role of leading research lessons. In addition to the supervisor, the prospective teacher who did not conduct the lesson took on the role of observer for her colleague. In this article, we make a case study of Júlia, analyzing how she prepared, conducted and reflected on the research lesson. This prospective teacher had previously conducted exploratory lessons in student teaching situations, valuing their contribution to student learning. She was selected for this study because her participation throughout the lesson study showed a remarkable self-reflection on her own practice.

The data was collected through observation of the lesson planning sessions, the research lesson, and the post-lesson discussion and reflection sessions and through the collection of documents produced, including the reflection written by Júlia. The lesson study sessions were audio recorded and relevant excerpts for this study were transcribed. The data was analyzed using a deductive method (Amado, 2013), considering the model presented in **Table 1**, which organizes the teacher practices to consider when preparing and leading a whole-class discussion and the actions that make up these practices.

In our analysis, we carried out a content analysis of the data, starting by making notes and initial analytical observations in order to have a clearer view of the data and what it suggests. Then, we selected transcribed episodes that highlight relevant situations regarding the presence of practices and actions shown in **Table 1**. Each extract was then coded using these practices and actions. To ensure trustworthiness in the interpretations, the three authors discussed the coding of data, in all cases reaching a consensus. This approach allowed us to identify substantial incidents from the preparation, leading and reflection stages of the research lesson conducted by Júlia. Through this analysis we sought to know how the prospective teacher prepared and led the whole-class discussion during her participation in the lesson study.

We followed the usual ethical procedures in qualitative research in education, informing the participants in a meeting prior to the beginning of the study of the aims and methods of the research, guaranteeing that they would remain anonymous and that the results would only be used for research and teacher education purposes. The research was approved by the Ethical Committee of Instituto de Educação da Universidade de Lisboa by decision of 28.02.2022.

RESULTS

In this section we present situations of the lesson study work and from Júlia's reflection that we relate to specific features of the model for preparing and leading whole-class discussions.

Preparation of the Whole-Class Discussion

Initial preparation

After analyzing the planning provided by the prospective teachers' practicum school, the participants decided that the research lesson would focus on the theme of numbers and the topic would be the introduction of the notion of multiplication:

Supervisor: Please open the curriculum document. We just need to define the learning aim for this lesson.

Alda: So, the theme will be numbers. The topic is multiplication.

Supervisor: What about the subtopic?

Júlia: Meanings and uses of multiplication.

Supervisor: And what does the learning aims column tell us?

Júlia: It should be the first one that says, "interpret and model situations within the additive sense and solve associated problems".

Supervisor: Let's just adapt it.

Júlia: It would be better to "solve problems that mobilize an understanding of the additive sense of multiplication".

Alda: And if we simplify, as "introduce the concept of multiplication based in the students' previous knowledge of addition"?

Supervisor: Sounds good, we can also add "by solving problems with rectangular arrangements". (S1)

Since the prospective teachers had limited experience in defining learning aims for the students, this lesson-study discussion was directed by the supervisor. She sought the prospective teachers' contributions in order to arrive at a statement that they could interpret and use to begin the planning of the lesson. Júlia, Alda and also the supervisor provided suggestions for the final formulation.

Choose the task

The selection of the task began with the participants discussing the different types of mathematical task, depending on characteristics such as the degree of challenge and the degree of openness:

Supervisor: What kind of task would you choose for this exploratory lesson?

Júlia: Problems or exploratory tasks. Or investigations.

Supervisor: Why?

Júlia: Because of the degree of challenge and the degree of openness.

Alda: An exercise has little challenge, it does not motivate or stimulate students, and it only has one answer ... I agree with Júlia about choosing a problem or an exploratory task. (S1)

Drawing on what they had learned from reading articles during the preparation phase of this lesson study, the prospective teachers considered that preparing an exploratory lesson, tasks such as problems or explorations would be more suitable to promote student learning.

In the next session, the prospective teachers shared some tasks they had selected with these characteristics. They analyzed several tasks and, prompted by the supervisor to select a task that could promote an interesting whole-class discussion, they opted for one that they considered to be "more interesting, because there are several ways to solve it, it's not so straightforward" (Júlia, S2).

Finally, the participants analyzed possible adaptations to make in the statement of the task. They began by thinking of a context that would be meaningful to the students, opting to "tell them that someone went to pick fruit in the summer and that the fruit is organized in boxes in a certain way" (Júlia, S2).

They also decided to organize the statement into parts in order to guide the students' thinking by increasing degree of complexity and to present illustrative figures in each part to make the statement easier to understand.

Solve the task

After selecting and adapting the task to present to the students in the research lesson, the participants anticipated possible strategies that the students could use. As such, they solved the task in several ways, with different levels of correctness and sophistication:

Júlia: I, sincerely, think that on (b) they [students] may use ...

Alda: Counting one by one.

Júlia: [Counting] by rows and by columns: $3 + 3 + 3$ and $3 + 3 + 3$. Here it is the same. To say 3 columns of 3 or 3 rows of 3 or 3 horizontally, and 3 vertically ... And whereas in the previous question the factors changed, replacing their position, but the result was the same, here [question (b)], no. The factors are equal, it does not

matter to replace position. In question (c), in addition to these, there are two products: 4×5 or 5×4 and the position of the factors matters again. (S3)

The prospective teachers carefully considered the possible solution strategies that the students might come up with, beginning by using the simplest strategy (addition). Drawing on previous knowledge about the properties of multiplication, they discussed the fact that, in (b), the order of the factors does not influence the product. When discussing possible student answers in (c), they analyzed the fact that the position of the factors mattered again, taking on different meanings (rows or columns).

Prepare to carry out the task in class

Given the variety of possible strategies for solving the task, the participants discussed how, during the monitoring of students' work, they could select and sequence the students' answers for the whole-class discussion, namely considering wrong or incomplete strategies. They assumed that these "should be presented first, if we want disagreements to arise" (Júlia, S4) and lead the students to explain and justify their solutions. In addition, the prospective teachers also chose to sequence the students' answers according to an increasing degree of complexity and sophistication:

Alda: Counting one by one is simpler, I think it makes sense to begin with that one.

Júlia: And it must be the most used, at least in this [first] question. Multiplication should be the most difficult for the students and it is also where we want to get, so it makes sense to be the last one. And if we want them to connect addition and multiplication, I think that is the order. From additions to multiplications.

Supervisor: And in (b)?

Júlia: I would do the same. However, I would not select counting one by one, because that was talked before ... The aim of the lesson is not counting one by one ... Therefore, I would not select it. (S4)

Júlia also suggested giving priority to strategies that made it possible to explore the aim defined for the lesson, choosing not to insist in less powerful strategies that were already discussed. Thus, she shows the aim of the lesson and how to promote it and, with confidence, she chooses to filter out the students' strategies that are not important for this lesson.

Depending on the different solution strategies that the participants anticipated, they also prepared questions and other teacher interventions during the

whole-class discussion to promote student learning. Júlia considered it important to refrain from frequent interventions, allowing students to take a central role in explaining their reasoning. She also felt it was beneficial to not validate or prompt answers, and often challenge other students to participate in the whole-class discussion:

Supervisor: And these interventions [support and challenge] are directed towards a group, a student, the whole class?

Júlia: To the groups, I think. The class must be provoked to always participate. Moreover, they must listen with attention to what a group says to be able to criticize. To say: "Are there other ways to count, group x? That group counted in another way. Explain how you thought" or, for question (b), "the group A added 3 with 3 and group B did 2 times 3. However, the result is 6 in the two solutions ... Is this right? Why? Group C, explain why it yields the same result". (S4)

These and other interventions to be carried out during the whole-class discussion were included in the planning of the research lesson. Júlia took into consideration encouraging students to participate frequently in the whole-class discussion, and to listen to their colleagues' statements, validating or criticizing these with well-formulated arguments. Paying attention to the suggestions from the articles that she read in this lesson study (Table 1), she suggested some questions, which were included in the lesson plan, to promote interactions among students from different groups, responding to each other's mathematical thinking or rephrasing explanations.

Present the task

Júlia began the lesson by organizing the students into groups and distributing the task statement to them. She then assigned a number to each group of students to identify them and read the task statement aloud, drawing the students' attention to the accompanying illustration. The prospective teacher said to the students, "I'm going to give you another challenge! What do I want you to do? Find out how many apples there are in the box. Write down on the sheet how you thought about it and how you arrived at the answer. Are there any questions?" (S7). So, Júlia presented the task to the students, encouraging their interest and enthusiasm for solving it, and asked if they had any questions about the instructions, to which the students replied that they did not.

Monitor the students' work

During the students' autonomous work, Júlia circulated around the classroom, observing their written

records, strategies, and representations. The prospective teacher asked open questions such as "How did you think about this?" (S8) to various groups in order to understand their reasoning in solving the task. She asked other more guiding questions such as "And how did you count the apples?" to help the students communicate their reasoning clearly. In addition, she intervened to validate the students' answers by confirming the results they had reached ("Yes, the result is 6. Exactly!" (S8)).

Select and sequence students' answers

Júlia selected the students' strategies that included adding apples, either by rows or columns, and those that included multiplication and decided not to include counting one by one. She sequenced the strategies according to the criteria already defined in the previous sessions: first the wrong answers and second the correct ones in an increasing degree of complexity and sophistication (from addition to multiplication).

Leading the Whole-Class Discussion

Initiate the whole-class discussion and intervene according to the situation and the aim

In the research lesson, Júlia began the whole-class discussion with an invitation to the group that she selected to be the first to present a solving strategy: "the group of Carolina, how did you do it? Explain to your colleagues" (S7). This action led the students to share their solution to question (a) and explain it. Júlia contrasted it with the solution of another group:

Júlia: Ricardo did $2 + 2 + 2$. Is there another way to say this?

Ricardo: 2×2 .

Júlia: 2×2 ? So, let us pick here in this operation that Ricardo told us.

Hugo: 2×2 is 4!

Júlia: 2×2 is 4. Very well. However, you know how we call the operation that has this symbol [point towards $\times$]?

Ricardo: Are "times"!

Júlia: Is "times". And how do we call the operation?

Fabiana: Is "x".

Júlia: But it has a name ... the "x" is called "times", not "x". But what is the name of the operation? This here is an addition [points towards +], and this one uses "times", how do we call it? You do not know? Say, Hugo?

Hugo: Mult ...?!

Júlia: Multiplication! When we use “times”, we are doing a multiplication. Let us go back to your responses. Hugo, it is your turn, say it.

Hugo: It can be 2×3 .

Júlia: Very well. So, it is 2×3 and not 2×2 . What does it yield?

Hugo: 6.

Júlia: 6. But look at one thing. Do we have two times three or three times two?

Students: Three times two!

Júlia: Then, the correct operation is 3×2 . (S7)

Júlia began by inviting Ricardo to elaborate on his response. Then, she challenged the students to name the operation that was symbolically written by Ricardo. Drawing on the incomplete response of Hugo, she introduced the word “multiplication”. In the sequence, she guided the students to understand the different role of factors in multiplication.

Analyze students' work with a focus on mathematical ideas

This is also visible in the extract above. In the dialogue, through Júlia's prompting, Ricardo made a connection with multiplication, even if his initial response was incorrect. Hugo said that the product of 2×2 is 4 and mentioned the name of the operation being discussed. These two students showed that they already knew multiplication, from everyday life or from private mathematics lessons. As this was the first moment of the lesson in which the operation appeared, Júlia chose not to immediately highlight Ricardo's error, but took advantage of his intervention to explore multiplication with the students. Then, she returned to the answer 2×2 , with intervention actions, challenging the students to validate this response, with Hugo correcting the operation indicated by Ricardo.

In this part of the whole-class discussion, Júlia also interrupted students' interventions to support and guide them. She encouraged students to clarify their explanations and aimed to sustain their engagement and participation in the discussion. Additionally, she employed informing and suggesting actions to highlight key mathematical concepts, introduce relevant information, validate students' responses, and analyze their work. For instance, she promoted connections between the students' answers of 2×3 and 3×2 .

Promote student participation

Júlia was able to do this all though the whole-class discussion. For example, in the discussion of question (c), Júlia promoted this participation as she handled a situation of disagreement in which Francisco, when explaining his reasoning to his colleagues, making a question about the position of the factors:

Francisco: First, we counted, 1, 2, 3, 4, 5 [a column]. After 1, 2, 3, 4 [a row]. And we understood that it could be 5×4 , I mean 4×5 .

Júlia: And why it could not be 5×4 ?

Igor: It would yield I don't know.

Júlia: Francisco said that you thought that way [by columns] 4 times, and in this way [by rows] 5 times. And it could not be also 5×4 ?

Igor: It would not be. It would yield another number. I need to do the computation.

Júlia: Children, what do you think?

Maria: Would it be another thing.

Júlia: They thought 5, 5, 5, 5, four times [columns]. And would not yield 5×4 ?

Rui: It would be 20. So, that would be another thing.

Júlia: It would be? How that would be?

Rui: I think it is in both ways.

Júlia: Why?

Rui: I don't know.

Júlia: Let's all discuss. Ricardo, say?

Ricardo: Because there are 5 rows with 4 up front [rows] and 4 rows with 5 downwards [columns].

Júlia: Do you want to come here [to the board] explain?

Ricardo: Here [points the board] there is $4 + 4 + 4 + 4 + 4$ [points to the columns], equal to 20. [Points to the rows] $5 + 5 + 5 + 5$ equals 20.

Júlia: So, we can do 5×4 if we think of Francisco and we can do 4×5 if we think as Ricardo. How is it? Explain once more.

Ricardo: $4 + 4 + 4 + 4 + 4$ [points, at the board, to the columns downwards] and $5 + 5 + 5 + 5$ [points at the board, left to right]. (S7)

This dialogue shows again how Júlia analyzed the student's work with a focus on mathematical ideas. When faced with Francisco's difficulty, Júlia sought to connect the mathematical aspects of his strategy with the lesson aim. She also strived to promote the participation of students, intervening by challenging them and insisting that they explain why "it could not be 5×4 ". When she realized that the students were not as engaged and participative in the whole-class discussion, she intervened and encouraged them to share their opinions on their peers' contributions. Júlia's actions, which often took the form of challenges, involved analyzing students' work and making connections between their strategies and the lesson's aim. She presented mathematical questions and situations that required students to justify their explanations. Her approach demonstrated a careful analysis of students' reasoning, as she responded explicitly and highlighted key information through informing and suggesting actions. When confusion arose due to certain students struggling to understand that the position of the factors does not alter the product, the issue was resolved by Ricardo's explanation. Throughout the discussion, although Júlia made multiple interventions, she allowed students to justify their reasoning and work towards a consensus.

Concluding With a Synthesis

Conclude, with the collaboration of the students

The whole-class discussion ended with a final synthesis, promoted through the exploration of the previously prepared board organization, which also led to the lesson-study discussion of some of the students' ideas:

Júlia: This first part is correct? $2 + 2 + 2$ is equal to 3×2 ?

Some students: No!

Júlia: No? Why not? What is wrong here?

João: It is the same.

Júlia: Is it the same? You all agree?

Students: Yes.

Júlia: Why so? I want to know why you agree that $2 + 2 + 2$ is equal to 3×2 . Why is that correct?

Tomás: Because it is 3 times 2.

Júlia: We have 3×2 . We are adding 3 times 2. So, now $3 + 3$ is equal to what?

Tomás: To 2×3 .

Júlia: Mariana?

Mariana: 6.

Júlia: Yes, is 6. However, I do not want to know the result. I want to know $3 + 3$ is equal to what in a multiplication?

Sofia: 6.

Júlia: Sofia, I just said to Mariana that I don't want to know the result. Carolina, how is $3 + 3$?

Carolina: It is 2×3 .

Júlia: All agree?

Students: Yes.

Júlia: Because we are adding three twice. So, now, $3 + 3 + 3$. How is it in multiplication?

Francisco: 3×3 . (S7)

In this segment of the final synthesis, Júlia encouraged the involvement of the students, who explained their reasoning and expressed themselves in relation to the explanation of their colleagues, validating the answers presented. In this way, the final synthesis was promoted through the collaboration of the students in filling out the table where they summarized and highlighted the main ideas explored in the lesson and evidenced connections between the addition of equal numbers and multiplication, namely between the students' answers presented in this episode: $2 + 2 + 2$ and 3×2 .

Reflection About the Whole-Class Discussion

Select and sequence strategies

Reflecting on the lesson she conducted, Júlia said she was "very happy! I really enjoyed the lesson; I want to continue doing exploratory lessons more often" (S8). She also indicated that she was "surprised by the students, they were very predisposed to the task, maybe because of the way I introduced it, by telling them, 'I'm going to give you another challenge!' They were enthusiastic" (S8). Reviewing episodes of the lesson, the prospective teacher explained that she had "no difficulty in selecting and sequencing [the students' strategies to take to the whole-class discussion], since I always circulated through the different groups and checked which were the most relevant for the discussion" (written reflection). She also explained that she was confident in her sequencing, which she associated with the detailed preparation of the lesson made in the lesson study sessions: "the activity of anticipating what may arise, the strategies and difficulties, was very important so that I did not feel lost in the lesson. I felt safer and more supported with all this preparation" (S8).

Interventions

In contrast, Júlia named some aspects in which she had difficulties and that influenced the whole-class discussion, namely some of her interventions:

Júlia: I asked horrible questions!

Supervisor: For example?

Júlia: I said things such as “what do I put here?”, “what do I do here?”, “ $3 + 3$ is what (in multiplication)?”

Supervisor: And many times the students responded to the result of addition, wasn't it?

Júlia: Yes, because the way I put the questions, they could only tell the result. After the lesson, when I realized the kind of questions I had asked, I really thought “what did I expect the students to answer me?” Because, really, they could only give me the result, and it wasn't what I wanted. I wanted them to turn addition into multiplication or vice versa. Moreover, maybe I shouldn't have asked so many times, “Do you agree? Does everyone agree?” because no one would answer me, or some students would say “yes”, just so I could move forward. Moreover, we had seen here [in the lesson study] that video [of an exploratory lesson] in which the teacher asked these open-ended questions, and I saw it and said that it didn't work well, because the students didn't answer. And I went on to repeat the same! (S11)

Reflecting on her interventions, in addition to understanding that she sometimes validated the students' answers, Júlia demonstrated that she was afraid that the way she formulated some questions to the students during the whole-class discussion and final synthesis did not challenge them to develop their own explanations and understand the mathematical topic addressed on in that lesson.

Analysis of students' work

Reflecting on the way she analyzed the students' work, Júlia indicated that another aspect that she would reformulate in a possible next lesson would be “the fact that she did not mention that multiplication is a successive addition of equal numbers” (written reflection) and “if I repeated the lesson, I would reinforce this idea a lot. And I think that the biggest difficulty for the students was to go from multiplication to addition or vice versa and I think it was my fault for not having clarified that” (written reflection). For her, this difficulty of the students became clearer in the final part of the whole-class discussion, which is why she chose to “use less time in solving and discussing the task and more time in exploring the synthesis” (S8). She justified:

Júlia: Because in the synthesis I had more perception of who was understanding and who wasn't. And it was also in the synthesis, because they chose numbers like 21, that I realized the lack that made me explain that the addition has to be of equal numbers. Because then they filled the board with “ $10 + 10 + 1$ ”. Moreover, since there were several number suggestions, I took a lot of time to explore each one. And the synthesis was where I had the most difficulty, without any doubt.

Supervisor: Why?

Júlia: Because I wanted the content to be so clearly explicit and understood by the students... I hope I haven't confused them more! I don't think it happened, but for me it was the hardest part. I want so much that the students are all right with the content ...

Supervisor: But how do you think you could confuse them more?

Júlia: Because of my questions, which should be clearer, less direct and more directed to a student or group. (S11)

The importance of the final synthesis was also evident for her, indicating that it allows students to have a clear view of the main ideas and mathematical procedures explored. Reflecting on the final synthesis, Júlia had the opportunity to re-analyze the influence that her questioning had during this moment of the lesson, realizing that in several instances the questions posed to the students were unclear, closed-ended and addressed to the whole class, which did not promote students' involvement and participation as she intended.

Promote student participation

Júlia said she had difficulties in promoting the participation of most students during the whole-class discussion. Although she managed to get the students to actively participate in the discussion of the task, “always talk and be really involved... It seemed like it wasn't enough. That is, I expected that when a student was speaking, others would respond to that student. And no. [The whole-class discussion] was only between me and this student” (S11). The prospective teacher considered that she was able to promote the participation of the students that she spoke to directly to share their solution and reasoning strategies, but she did not engage, in these moments, the whole class.

DISCUSSION

In this lesson study, Júlia and the other participants carefully planned the whole-class discussion, beginning with the selection and adaptation of the mathematical task for the research lesson, based on previously defined

content and learning aims, as suggested by Kooloos et al. (2023). They solved the task using multiple strategies, considering correctness and sophistication, which enabled them to anticipate possible student responses and plan their selection and sequencing – an essential aspect of preparation highlighted by Stein et al. (2008) and Kooloos et al. (2023). They also discussed how to foster students' active participation, encouraging them to share answers and explain reasoning (Stein et al., 2008). In line with Orr and Bieda (2025) and Duarte et al. (2024), interventions were designed to introduce challenge and elicit mathematical thinking.

The lesson began with Júlia presenting the task and ensuring students understood it. During autonomous work, she monitored strategies and reasoning, while selecting and sequencing responses for the whole-class discussion according to prior criteria (Stein et al., 2008).

Júlia initiated the whole-class discussion by inviting a selected group to present their strategy, promoting initial involvement and encouraging others to comment (Cengiz et al., 2011; Kooloos et al., 2023; Author et al., 2013; Thanheiser & Melhuish, 2023). Her interventions included:

- (1) supporting/guiding students by revoicing explanations or prompting clarification,
- (2) challenging students through mathematical questions and requests for justification, and
- (3) informing/suggesting ideas to highlight key mathematical aspects (Cengiz et al., 2011; Kooloos et al., 2023; Orr & Bieda, 2025; Ponte et al., 2013; Thanheiser & Melhuish, 2023).

Based on her analysis of students' work, Júlia promoted connections among strategies, particularly between repeated addition and multiplication, and between solutions differing in factor order (Cengiz et al., 2011; Kooloos et al., 2023; Stein et al., 2008). However, she did not consistently ask questions to support students in articulating their reasoning (Thanheiser & Melhuish, 2023), nor did she systematically reformulate incomplete or incorrect ideas, an action emphasized by Duarte et al. (2024).

Her main difficulty was engaging the entire class. Contrary to Boerst et al. (2011), the whole-class discussion centered largely on presenting groups. Although disagreement emerged regarding factor order and product, she encouraged students to justify their reasoning (Shaughnessy et al., 2020; Stein et al., 2008; Thanheiser & Melhuish, 2023; Wood, 1999), yet without broad participation. Interactions across groups were limited, despite their importance (Boerst et al., 2011; Stein et al., 2008). Only occasionally did she prompt students to analyze peers' reasoning or explain classmates' answers (Duarte et al., 2024; Thanheiser & Melhuish, 2023).

Our findings confirm and extend those of Martins et al. (2021) and Gomes et al. (2022). Similar to Martins et

al. (2021), Júlia's participation in lesson study supported the preparation of an exploratory lesson by encouraging attention to students' possible strategies and difficulties, teacher interventions, and the role of whole-class discussion. Moreover, as in that study, leading the discussion proved to be more challenging than planning it: although Júlia had carefully anticipated students' work and discussed possible interventions, she experienced difficulties in promoting broad participation and in using sufficiently challenging questions to sustain students' engagement. This convergence suggests that the difficulties identified by Martins et al. (2021) are not simply a consequence of limited awareness of exploratory teaching but may persist when prospective teachers have to coordinate multiple demands during the enactment of a discussion. Our findings are also consistent with Gomes et al. (2022), particularly in showing that anticipation and collaborative preparation supported prospective teachers' attention to students' strategies and difficulties and that reflection on enactment provided opportunities to identify aspects not fully anticipated. However, the present study goes further by examining these processes specifically through a comprehensive model of whole-class discussion practices. This makes visible a distinction that is less explicit in previous studies: knowing and planning for particular whole-class discussion practices does not necessarily translate into enacting them with sufficient quality. Júlia enacted the practices identified in the model, including initiating and intervening, analyzing students' work with a focus on mathematical ideas, and inviting participation, but the quality of these actions varied, particularly when she had to respond to students' contributions in the moment. Therefore, the findings suggest that lesson study can provide a strong environment for anticipating and reflecting on whole-class discussion, while the enactment of ambitious discussion practices requires further opportunities to analyze and rehearse how different practices are coordinated during real-time classroom interactions.

To conclude, Júlia led a collaborative final synthesis summarizing key ideas and highlighting the relationship between repeated addition and multiplication (Ponte, 2005). However, she did not connect these ideas to other previously learned mathematical concepts (Duarte et al., 2024).

In reflection sessions, Júlia recognized that her interventions sometimes limited participation, as some questions lacked clarity and sufficient challenge and were occasionally accompanied by implicit validation (Stein et al., 2008). She acknowledged that the reduced cognitive demand of her questioning contributed to lower engagement, reinforcing Orr and Bieda's (2025) argument for careful preparation of questioning.

She also identified students' difficulty in understanding multiplication as repeated addition of

equal numbers. Finally, she acknowledged her challenge in involving the whole class (Boerst et al., 2011), attributing it partly to unclear interventions and insufficient emphasis on key ideas.

Overall, Júlia enacted all practices and most actions of our model. The main weakness concerns promoting student participation. She critically reflected on this limitation and on the need for more challenging questions. During the lesson, she realized she had not explicitly emphasized multiplication as successive addition of equal numbers, an aspect that should have been more carefully addressed during planning.

CONCLUSION

The preparation of a whole-class discussion has been extensively studied, with a broad consensus supporting the model proposed by Stein et al. (2008). However, there remains a need for further clarification regarding the initial phase of lesson preparation, specifically the selection and solving of tasks. This study addresses this gap through our model (Table 1), which serves as a tool for both preparing and analyzing whole-class discussions. Regarding the leading of the lesson, our model intertwines three practices, initiating the discussion and intervening based on the situation and the aim, analyzing the students' work with a focus on mathematical ideas, and promoting student participation. Several of these aspects are mentioned in previous studies, with emphasis on one dimension or another, and often with different terminologies (such as anticipating strategies and monitoring students' work) (e.g., Fujii, 2018; Kooloos et al., 2023; Orr & Bieda, 2025; Stein et al., 2018; Wood, 1999). The model we present also emphasizes the importance of concluding the whole-class discussion through a final synthesis. This conclusion should involve students and focus on summarizing essential ideas and connecting them to previously known mathematical concepts.

Júlia's case illustrates the great potential of using this model during the lesson planning phase. The model encourages prospective teachers to anticipate students' strategies and difficulties and plan the discussion to encourage active student participation in knowledge construction. Furthermore, the lesson study structure allows the participants to prepare for the discussion, to lead it, and to reflect on the discussion led in a collaborative environment and deep way.

The model can act as a catalyst for dialogue about the intended practice of prospective teachers and support decision-making during the whole-class discussion. It brings together the main aspects to be considered when preparing and leading the discussion and final synthesis. In our view, they are necessary conditions. They will be sufficient conditions if performed with high quality. This study demonstrates that a prospective teacher with some prior experience in leading whole-class discussions

achieved a dynamic discussion environment within the framework of an exploratory approach. She effectively invited student participation, made situational interventions, and focused on mathematical ideas. However, as she reflected, she did not always make the most appropriate interventions and, above all, she exhibited difficulty in promoting a broad participation from the majority of students. This indicates a need for further attention in future lesson studies to strategies for promoting broader student involvement. How to achieve this is a question for future research. Despite this, the lesson study and the model presented appear to have been valuable tools that supported the prospective teacher's learning process in preparing, leading, and reflecting on whole-class discussion.

The value of this study lies in showing how lesson study may provide a practice-based context in initial teacher education for prospective teachers to anticipate, enact, and reflect on the complex practices involved in whole-class discussions, while the proposed model offers a framework for making visible the gap between planning these practices and enacting them effectively.

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REFERENCES

- Amado, J. (2013). *Manual de investigação qualitativa em educação* [Handbook of qualitative research in education]. Imprensa da Universidade de Coimbra. <https://doi.org/10.14195/978-989-26-0879-2>
- Bjuland, R., & Mosvold, R. (2015). Lesson study in teacher education: Learning from a challenging case. *Teaching and Teacher Education*, 52, 83-90. <https://doi.org/10.1016/j.tate.2015.09.005>

- Boerst, T., Sleep, L., Ball, D., & Bass, H.P. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, 113(12), 2844-2877. <https://doi.org/10.1177/016146811111301207>
- Bogdan, R. C., & Biklen, S. K. (2007). *Quality research for education: An introduction to theory and methods* (5th ed.). Pearson.
- Cengiz, N., Kline, K., & Grant, T. J. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education*, 14, 355-374. <https://doi.org/10.1007/s10857-011-9179-7>
- Dillon, J. T. (1994). *Using discussion in classrooms*. Open University Press.
- Duarte, N., Faria, F., & Ponte, J. P. (2024). Preparar e conduzir a discussão coletiva em matemática [Prepare and lead collective discussion in mathematics]. *Educação e Matemática*, 171, 11-14.
- Fujii, T. (2018). Lesson study and teaching mathematics through problem solving: The two wheels of a cart. In M. Quaresma, C. Winsløw, S. Clivaz, J. P. da Ponte, A. Ni Shuilleabháin & A. Takahashi (Eds.), *Mathematics lesson study around the world. ICME-13 monographs* (1-21), Springer. https://doi.org/10.1007/978-3-319-75696-7_1
- Gomes, P., Martins, M., Quaresma, M., Mata-Pereira, J., & da Ponte, J. P. (2022). Task design and enactment: Developing in-service and prospective teachers' didactical knowledge in lesson study. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(7), Article em2131. <https://doi.org/10.29333/ejmste/12172>
- Kooloos, C. (2022). *Eye on variety: The teacher's work in developing mathematical whole-class discussions* [PhD thesis, Radboud Universiteit Nijmegen].
- Kooloos, C., Oolbekkink-Marchand, H., Kaenders, R., & Heckman, G. (2023). Developing mathematical whole-class discussions: An exploratory study of teachers' learning paths. *Teaching and Teacher Education*, 134, Article 104257. <https://doi.org/10.1016/j.tate.2023.104257>
- Larssen, D., Cajkler, W., Mosvold, R., Bjuland, R., Helgevold, N., Fauskanger, J., Wood, P., Baldry, F., Jakobsen, A., Bugge, H., Næsheim-Bjørkvik, G., & Norton, J. (2018). A literature review of lesson study in initial teacher education: Perspectives about learning and observation. *International Journal for Lesson & Learning Studies*, 7(1), 8-22. <https://doi.org/10.1108/IJLLS-06-2017-0030>
- Leavy, A., & Hourigan, M. (2016). Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education*, 57, 161-175. <https://doi.org/10.1016/j.tate.2016.04.002>
- Martins, M., da Ponte, J. P., & Mata-Pereira, J. (2023). Preparing, leading, and reflecting on whole-class discussions: How prospective mathematics teachers develop their knowledge during lesson study. *European Journal of Science and Mathematics Education*, 11(1), 33-48. <https://doi.org/10.30935/scimath/12432>
- Mosvold, R. (2024). Research on discussion in mathematics teaching: A review of literature from 2000 to 2020. In J. Wang (Ed.), *Proceedings of the 14th International Congress on Mathematical Education: Volume II: Invited Lectures* (pp. 473-488). https://doi.org/10.1142/9789811287183_0032
- Ni Shuilleabhain, A., & Bjuland, R. (2019). Incorporating lesson study in ITE: Organisational structures to support student teacher learning. *Journal of Education for Teaching*, 45(4), 434-445. <https://doi.org/10.1080/02607476.2019.1639262>
- Orr, S., & Bieda, K. N. (2025). Learning to elicit student thinking: The role of planning to support academically rigorous questioning sequences during instruction. *Journal of Mathematics Teacher Education*, 28, 523-544. <https://doi.org/10.1007/s10857-023-09603-5>
- Ponte, J. P. (2005). Gestão curricular em matemática [Curriculum management in mathematics]. In Grupo de Trabalho sobre Investigação em Educação Matemática (Ed.), *O professor e o desenvolvimento curricular* (pp. 11-34). APM.
- Ponte, J. P. (2017). Lesson studies in initial mathematics teacher education. *International Journal for Lesson and Learning Studies*, 6(2), 169-181. <https://doi.org/10.1108/IJLLS-08-2016-0021>
- Ponte, J. P., Mata-Pereira, J., & Quaresma, M. (2013). Ações do professor na condução de discussões matemáticas [Teacher actions in facilitating mathematical discussions]. *Quadrante*, 22(2), 55-81.
- Rocha, I., & Menino, H. (2009). Desenvolvimento do sentido do número na multiplicação. Um estudo de caso com crianças de 7/8 anos [Developing number sense in multiplication: A case study with 7/8 year old]. *Revista Latinoamericana de Investigación em Matemática Educativa*, 12(1), 103-134.
- Shaughnessy, M., DeFino, R., Pfaf, E., & Blunk, M. (2021). I think I made a mistake: How do prospective teachers elicit the thinking of a student who has made a mistake? *Journal of Mathematics Teacher Education*, 24, 335-359. <https://doi.org/10.1007/s10857-020-09461-5>
- Stein, M. K., Engle, R. A., Smith, M., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10, 313-340. <https://doi.org/10.1080/10986060802229675>

- Takahashi, A. (2008). Beyond show and tell: Neriage for teaching through problem-solving ideas from Japanese problem-solving approaches for teaching mathematics. In *Proceedings of the 11th International Congress on Mathematics Education (Section TSG 19: Research and Development in Problem Solving in Mathematics Education)*.
- Thanheiser, E., & Melhuish, K. (2023). Teaching routines and student-centered mathematics instruction: The essential role of conferring to understand student thinking and reasoning. *Journal of Mathematical Behavior*, 70, Article 101032. <https://doi.org/10.1016/j.jmathb.2023.101032>
- Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education*, 30(2), 171-191. <https://doi.org/10.2307/749609>

<https://www.ejmste.com>