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Preservice Mathematics Teachers' Experiences about Function and Equation Concepts

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The purpose of this study is to determine the experience of mathematics preservice teachers related to function and equation concepts and the relations between them. Determining preservice mathematics teachers' understanding of function and equation concepts has great importance since it directly affects their future teaching careers. Data were collected by a questionnaire and semi-structured interviews and analyzed using existential- phenomenology. The results show that the experiences of preservice math teachers related to these concepts can be summarized under six different titles, namely, confusion of the definition of function with one-to-one and onto properties, relating it with daily life (exampling), failure to take the functioning conditions into account, the relation between the values which make the function zero and roots of the equation, multiple representations of functions and taking the defining set of functions and equations into account .

Keywords: Function, Equation, Conception, Preservice Mathematics Teachers

INTRODUCTION

Preservice Mathematics Teachers' Experiences about Function and Equation Concepts

The National Council of Teachers of Mathematics (NCTM) (1989) Standards states "Mathematics instruction at the 5-8 level should prepare students for expanded and deeper study in high school through exploration of the interconnections among mathematical ideas" (p. 84). Again, the NCTM (2000) standards also declare; "Instructional programs from pre-kindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and built on one another to produce a coherent whole; …" (p. 354). Also, according to Swadener and Soedjadi (1988), teachers should not neglect the fact that mathematical concepts are related to one another like the rings of a chain's. This situation is particularly important for the concepts of function and equation. They have a prominent place in mathematics curriculum, and function concept plays an important role especially in learning advanced mathematics concepts. Not learning these concepts will cause a breakage in the rings of the above mentioned chain, which makes learning difficult, even impossible. This signifies the importance of learning these concepts at ´meaningful learning` rather than at ´rote learning`.

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A Brief Introduction to Functions

The notion of a function has quitely changed compared to its modern set theoretic definition since the end of seventeenth century and it has been pretty differentiated from the algebraic notation of nineteenth century (Dennis&Confrey, 1995). The word "function"

State of the literature

- The studies indicate that students at every levels have some difficulties in algebraic equations as well as in understanding the concept of function, and in determining the relationships between them.
- Preservice teachers can enrich their teaching as much as their knowledge. In this way, they can increase their students' learning.

Contribution of this paper to the literature

- Preservice mathematics teachers' knowledge about function and equation concepts and the relationships between these two concepts could not go beyond quite defective knowledge based only pure definitions.
- The four categories are oriented towards functions, while two of them are aimed at determining the relationships between function and equation concepts.
- What are the effects of mathematics curriculum, textbooks, and mathematics teacher education programmes on these results?

was firstly introduced by Leibniz originating from the geometry of curves (Kleiner, 1989). Bernoulli, in 1718, introduced the first formal definition of the function as a term as follows:

> *One calls here Function of a variable a quantity composed in any manner whatever of this variable and of constants. (cited in Kleiner, 1989; p.284).*

This ambiguous definition of Bernoulli can be accepted as a start of its transformation into "The multi-faceted concept", which exists today (Jones, 2006; p.3). Afterwards Euler redefined the notion of a function similar to Bernoulli's by adding the phrase "Analytical expression" as following:

> *A function of a variable quantity is an analytical expression composed in any manner from that variable quantity and numbers or constant quantities.(cited Kleiner, 1989; p. 284).*

With the term "analytical expression" added by Euler, the function concept has been transformed from being a geometric concept into being an algebraic one (Jones, 2006). According to this, the concept of function which contains dependent and independent variables was defined by Euler as a procedural concept demonstrating input-output relations and then as a concept representing one to one correspondence between real numbers by Dirichlet (Stallings, 2000; Kieran, 1992) and a century later as a certain subset of Cartesian product by Bourbaki (Kleiner, 1989). He defined functions as follows:

> *Let E and F be two sets, which may or may not be distinct. A relation between a variable element x of E and a variable element y of F is called a functional relation in y if, for all x in E, there exists a unique y in F which is in the given relation with x. (Kleiner, 1989; p. 299).*

Namely, according to him functions are considered as a set of ordered pairs. In 1960's, by "New mathematics" reform frame, it was tried to make definitions of mathematics concepts clearer, making them comprehensible for students. Definition of the concept of function was given as follows: " Let A and B be sets, and let AxB denote the cartesian product of A and B. A subset f of AxB is a function if whenever (x_1, y_1) and (x_2, y_2) are elements of f and $x_1 = x_2$, then $y_1 = y_2$." (Tall, 1992, p. 497). In this way, in contrast to past, the definition was not limited to equations which define relationships between two variables in algebraic expressions (Even, 1988).

Understanding of Function Concept

Function is an essential concept in mathematics and it affects the whole mathematics curriculum (Laughbaum, 2003; Knuth, 2000; Beckmann, Thompson & Senk, 1999; Cooney, 1999; Dossey, 1999; Hitt, 1998; Dreyfus & Eisenberg, 1982). NCTM (1989) standards also stress "one of the central themes of mathematics is the study of patterns and functions." (p.98). However, it seems that students have some problems in understanding of the concept of function (Williams, 1998; Hauge, 1993; Eisenberg, 1991; Gaea, Orit & Kay, 1990). One of the reasons of these problems is that definition of the concept of function has changed in its historical period. As can be seen from the short presentation about the historical development of function concept for three centuries mentioned above, there have been rapid changes in definition of function concept since the time of Leibniz (Jones, 2006). However, modern definition of function presented above did not meet expectations either. As a matter of fact it sometimes caused students not to understand the concept. For, although this modern definition of 1960's has a perfect mathematical base, it does not have a cognitive origin. It is seen the problem students have in comprehending function concept arises from ideas which individuals develop about mathematical concepts rather than the words used in definitions (Tall, 1992). At this point, it seems that there exists differences between a formal concept definition and a concept image and this differentiation is conveyed by Tall and Vinner (1981) as follows:

> *... the term concept image to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes ... The definition of a concept (if it has one) is quite a different matter ... the concept definition to be a*

form of words used to specify that concept. It may be learnt by an individual in a rote fashion or more meaningfully related to a greater or lesser degree to the concept as a whole. (p. 152).

As it can be concluded from above expressions, giving a definition of a concept to students is not enough for them to comprehend it. This is especially valid for the concept of function because it is represented as geometric using graphs, numeric using tables, and symbolic using equations (DeMarois & Tall, 1996). Students' interrelating between these representations and making transitions from one to another is an important indication that shows students understand the function concept (Thompson, 1994). However, multiple representations of functions make understanding difficult (Maharaj, 2008; Thompson, 1994; Thompson & Sfard, 1994). Besides, the concept of function have two dimensions as operational-structural or object-process, Sfard (1991) makes the difference between these two dimensions as follows:

Seeing a mathematical entity as an object means being capable of referring to it was a real things-a static structure, existing somewhere in space and time. It also means being able to recognize the idea "at a glance" and to manipulate it as a whole, without going into details ... In contrast, interpreting a notion as a process implies regarding it as a potential rather than actual entity, which comes into existence upon request in a sequence of actions. (p. 4).

Although these two concepts seem contradictory, "… they are in fact complementary" (Sfard, 1991, p.4). Therefore, these two dimensions should be taken into consideration in order to realize a complete understanding about the concept of function (Schroeder, Schoeffer, Reish & Donovan, 2002). In addition, the fact that functions have many kinds such as polynomial (as constant, linear, quadratic, cubic), trigonometric, reciprocal, continuous and discontinuous functions etc. is one of the factors that make the function concept hard to perceive (Eisenberg, 1991).

The Concept of Equation

A major goal of primary and high education mathematical curricula is to develop algebra and algebraic thinking. According to the NCTM (1989), "Algebra is the language through which most of mathematics is communicated...." (p. 150). It is perceived as the formation and solution of equations using different symbols and expressions (Smith, Eisenmann, Jansen& Star, 2000). The perception of equations and determination of their solution paves the way to understand the advanced mathematical concepts. The previous studies show that students at every levels have difficulties in the solution of algebraic equations (Stacey&MacGregor, 2000; MacGregor&Stacey, 1996; Herscovics& Kieran, 1980). These difficulties stem from failure in simplifying algebraic expressions, difficulties in the transition from arithmetic to algebra (Dooren, Verschaffel&Ongehena, 2003; Van Ameron, 2003), failure to interpret the equations in a correct way (Real, 1996), and difficulties in transition from word problems to algebraic equations (Stacey & MacGregor, 2000; MacGregor & Stacey, 1996; Real, 1996; Herscovics & Kieran, 1980).

Equation and Function Concepts in the Turkish Curriculum

In the Turkish mathematics curriculum, students first meet the concept of equation at 7th class of the primary school. The teaching of the concept of equation starts with the presentation of mathematical expressions followed by propositions and open proposition concepts. Finally the definition of the equation is given: The definition of the "equation": … the mathematical equalities which contain an unknown variable which is correct for certain values are called equations (Özer, 2000; p.91, authors' translation)

In order to learn the concept of equation, the students should acquire the skills of finding the values validating a given proposal, explaining the equation, explaining a first degree equation with examples, writing a first degree equation with one unknown, explaining a second degree equation with examples and writing a second degree equation with two unknowns. Other targets in the teaching of equation concept are enabling students to solve first degree equations with one unknown and a first degree inequality with one unknown, perception of symmetry and coordinates of a point in a plane and being able to plot graphs. The skills required in plotting graphs are drawing the plot of a line passing from (a, y) points with $x = a$ and $y \in R$ and being able to tell that $y = ax + b$ corresponds a line and draw its plot with $(a \neq 0,\ b \neq 0)$ (Ministry National Education Curricula for grades 6-8, 2000).

In the Turkish curriculum, students first meet the concept of function at first class of the high school. Its teaching starts with the presentation of the concept of sets, followed with set of ordered pairs, Cartesian product of two sets and presentation of the concept of relation. Then the definition of function is given. It is followed by teaching multiple representations of functions such as Venn scheme, set of ordered pairs, equations, and graphs.

The definition of the "function": Let A and B are two non empty sets. The relation of f which pairs each element of A to only one element of B is called a function from A to B and shown as $f\colon A \rightarrow B$ or $A \xrightarrow{f} B$. According to this definition in order for a relation f from A to B to be a function;

1. There is at least one $y \in B$ for $\forall x \in A$ whenever $(x, y) \in f$.

2. $y = z$ for $\forall x \in A$ whenever $(x, y) \in f$ and $(x, z) \in f$ (Çetiner, Kavcar & Yıldız, 2003; p. 78, authors' translation).

In order to attain the goal of teaching the concept of function, the students are supposed to acquire the skills of defining a function and showing it with a scheme, defining value and image sets of a function, plotting functions in graphs, defining the equality of two functions, describing one to one, onto and into functions and stating the differences between them, explaining an infinite set, explaining the equality of two sets, and describing equality, constant and zero functions. Another goals related to teaching the function concept are making applications with function and its types (showing whether a given relation is a function or not, representing a given function with a scheme, writing a function given in scheme form into listing rule, plotting its graph, showing its definition and (value) image sets, telling the type of any given function, writing different type of functions etc) … perception of properties of function sets (National Education Ministry, High School Mathematics Program, 1992).

Links between Equations and Functions

Functions are used for determining certain quantities by means of other ones, for representing them, for modeling, for analysis and for interpreting relationships between them. However, symbols play important roles in representation of mathematical situations and expressing generalizations. Symbols are used for studying relationships between quantities (Hull & Seabold, 1995). In the most general meaning, functions can be perceived as dynamics mechanisms (relation) to make transformations (to match). The concept of function has two main conditions. These are: Univalence condition (matching every element of the definition set with only one element of the value set) and arbitrariness condition. When matching elements of the definition set with those of the value set, the function does not have to perform this action based on any algebraic rule. For instance, it may be impossible to represent functions with algebraic formulas when they are demonstrated by Venn-scheme and by listing (expressing by sets) (Breidenbach, Dubinsky, Hawks and Nichols, 1992; Sfard, 1992). However, an equation is a static mathematical structure and they are satisfied by limited or infinite unknown(s) that take their values in this structure (Attorps, 2006). Equation is one of the ways used for posing questions containing functional relationships and for analyzing them (Hull & Seabold, 1995). However, Laughbaum (2003) also states that traditional equation solving approach is only addressed to applications and hence it causes student's motivation on mathematics to decrease. He believes that, by this approach, which is devoted to teach the concept of equation, students can not make connections between real world and the symbol used in the concept of equation. For example, students can not perceive importance of the function concept in realizing that the money they earn in summer holiday is dependent upon their work hours. Therefore, different techniques should be used to make students understand the relationships between functions and equations. This condition is essential for development of the algebraic thinking. For this reason, it is very important to make use of various verbal, pictorial, numerical, concrete, symbolic and graphical representations. In order to model problems into mathematical situations, usage of technological tools such as calculators with graphing capabilities, data collection devices, and computers have also great importance. By this way, notations of equation and of function can be related to each other easier (e.g. $y=x+4$ and $f(x)=x+4$). In this manner, relationships between solutions of equations, zeros of function concerned and x-intercepts of the graph of function can be more easily perceived. Besides, the graphs of functions influenced by the parameters of function can be more easily comprehended and influence of changing parameters of quadratic functions can be interpreted more easily (Hull & Seabold, 1995).

Purpose of the Study

In this study, it has been tried to determine preservice mathematics teachers' experiences about the equation and function concepts in mathematics curriculum, of which students often have difficulties in comprehension and preservice math teachers' abilities in seeing relationships between these concepts have been investigated. For, preservice teachers can enrich their teaching as much as their knowledge and by this way, they can increase their students' learning (Even, 1988). For this purpose, the answer for question below has been sought;

What are the experiences of preservice math teachers about the relations between equation and function concepts?

Method

Research Design

The purpose of this study is to determine the experiences of preservice math teachers about equation and function concepts and the relations between them. For this purpose, qualitative methods in general and existential-phenomenological research in particular were conducted. Because, "qualitative research methods permit the description of phenomena and events in an attempt to understand and explain them … Qualitative methods are used to explore a particular point of view in explaining human behavior" (Krathwohl, 1993, p. 311). The aim of existential-phenomenological research is to investigate the experience of the subjects' being in the world (Giorgi, 1985).The goal of phenomenological research is therefore not to explain the phenomenon under investigation, but rather to describe it and to find meaning in the actual experience" (Hull, 2003, p.82-83) and "questions such as what and how are answered, not why" (Bergman&Norlander, 2005, p.813). Namely, "… phenomenology is primarily oriented towards the immediate phenomena of human experience, such as thinking and feeling …" (Odman, 1988; p.64).

Selecting Participants

Subjects of this study is seventy-one (71) preservice mathematics teachers who were selected by random and studied at Primary Mathematics Education Department in the Education Faculty at Cumhuriyet University in Sivas, a modest city in central Turkey. In this study, the students for interview were selected based on the purposeful sampling strategy. *"The purpose of purposeful sampling is to select information-rich cases whose study will illuminate the questions under study." (Patton, 1990, p.169).* Therefore, nine preservice teachers were selected based on their responses to the questionnaire administered before the semi-structured interviews. Three of the preservice teachers selected were males and six of them were females at the ages between 19 and 22. Four of the selected candidates showed high (one male rest are females), three of them showed medium (one female two males), and others showed low level academic performance. These students gave the researches a strong impression that they are all suitable candidates for interviews capable of freely expressing themselves.

Data Collection

Data gathered from semi-structured interviews and a questionnaire consist of open-ended questions were the basis for the analysis of the present study.

Questionnaire Tasks

The study was used a test consisting of 5 open-ended questions (total 7 questions with sub-items) which was developed by the researchers. It was administered to preservice math teachers for determining conceptions of the relations between equation, function, and polynomial concepts. Exploratory factor analysis showed that the test included three factors and analysis of variance for the entire test was 67.09%; for each factor, analysis of variance ranged as 25.62%, 21.55%, and 19.92% respectively. Factor loading of items in the test also ranged from .564 to .899. It contained items measuring preservice math teachers' conceptions of the relations between equation and polynomial concepts (REP) (factor-1), the relations between both equation and polynomial concepts with function concept (REPF) (factor-2), and the defines of function and polynomial concepts (DFP) (factor-3). Furthermore, Cronbach Alpha Coefficient is also calculated as .82 for reliability of the entire test; for each factor, alpha ranged as .862, .781, and .595 respectively. Based on the reliability and validity analysis, it showed that there were satisfactory factor structure and reliability of the test. Preservice math teachers were asked to answer the test in 60 minutes. In this study only the answers given to the questions related to equation and function concepts and the relations between them were taken into account.

Semi-Structured Interviews

9 in-depth semi-structured interviews with preservice math teachers were conducted and analyzed. The interviews were carried out in Turkish language and they were written on paper based on the participations' request. Later, they were translated into English by the researchers. Each preservice teacher was clearly explained the purpose of interviews before the start. Then each of the participants was asked the questions "What is a function?", "What is an equation?", "Is every function an equation?", and "Is every equation a function?". The questions differed according to the written responses of the questions they gave before. The interviewer employed clinical interview technique using general expressions such as "why?", "explain", and "how?". According to Hunting (1997),"clinical interviews provide the bases for inventions in which explicit strategies, activities, and settings are designed to fit the current state of students' mathematics knowledge" (p. 162). The students were also asked to think aloud when answering the questions .Thinking aloud is a technique employed for clearly displaying the problem solving ability and the cognitive processes of the students (Van Someren, Barnard& Sandberg, 1994). Interviews were conducted in a comfortable and an appropriate atmosphere by the first author. The names of the participants were kept anonymous for reliability purposes using pseudonyms. Each interview lasted around 20-25 minutes. Interviews' transcripts are 24 pages and 2430 words. They were written on paper as average length of three pages and categorized. This categorization process was done by the authors independently. Afterwards, they were compared and adjusted after reaching a consensus by the authors.

Data Analysis

The interviews and the written responses data were analyzed using Giorgi's (1985) existential-phenomenological data analysis. Giorgi's method involves the following four steps: i) Initial reading the text to reach a sense of the whole, ii) Separating the text into meaning units, iii) Transformation of meaning units into a disciplinary language (mathematics), and iv) Synthesis of the structure to describe its essence. In the present study these steps were utilized as detailed below (Bergman & Ncorlander, 2005):

i) Initial reading the text to reach a sense as a whole

The interviews carried out with the participants were read many times. This enable to the researchers to perceive the whole sense of the participants towards the phenomenon investigated. At this stage the researches were not influence by any theory or opinion.

ii) Separating the text into meaning units

After understanding the whole sense of the interviews they were reread and separated to meaning units. The meaning units are not separate pieces but the parts of the whole entity which complete each other. The separation of these meaning units was carried out according to the changes in the meaning rather than a grammatical rule. Then the similarities and the differences of the meaning units in each interview were investigated.

iii) Transformation of meaning units into a disciplinary language (mathematics)

In this step the answers of the participants in a daily language was converted into a mathematical language. For instance the answer of one of the participants as "we think function as a faucet …" was evaluated in Sfard's (1991) operational conception of the mathematical notation. They were then separated into mathematical categories taking the similarities and the differences between the meaning units into account.

iv) Synthesis of the structure to describe its essence

This step involves the synthesis of a general structure of the related phenomenon after the clarification of its current situation.

Trustworthiness of the Study

The research data were obtained by a questionnaire and semi-structured interviews. This enabled us to carry out triangulation by collecting data from different sources. "Triangulation is the application and combination of several research methodologies in the study of the same phenomenon" (Denzin, 1988, p. 511). In order to categorize the data obtained from various sources and determine the common expressions, the written answers and the interview transcripts of the participants were read times and times again. The words used by the participants were exactly written down on the paper without making any changes. Then these written texts were given to the participants for their approvals. That enabled member checking for the reliability of the data. Because according to Creswell (1998) a member check requires "Taking data, analyses, interpretations, and conclusions back to the participants so that they can judge the accuracy and credibility of the account" (p. 203). Peer review or debriefing process was used for the confirmation of the reliability of the research data. This process is an external control tool for the reliability of the research (Lincoln & Guba, 1985). In this study the research data are checked by two independent experts having PhD in pure mathematics and mathematics education.

RESULTS

When the written answers of the preservice teachers were examined, following conclusions were reached:

51 (71.83 %) of the preservice math teachers gave the correct answer to the question "Define the concept of equation and give an example?" while 2 of the them left the question unanswered. 31(43.66%) of the them gave an acceptable answer to the question "describe the concept of function and give an example" while 8 students left the question answered. 47(66.2 %) of the preservice math teachers gave the correct answer to the question "is every equation a function?". However, the answers were generally yes or no. Similarly, the answers of the preservice math teachers to the question "Is every function an equation?" are as follows: Yes, 29 (40.9%), 16 (22.5%). It was observed that only 26 of them (36.6%) answered the question. That is why it is difficult to make a reasonable evaluation by looking at the answers of them. Therefore the preservice teachers were subjected to semi-structured interviews in order to determine their experiences related to these concepts. Totally, six categories emerged from the responses to the questions and semi-structured interviews. Detailed explanations of these categories are shown below:

Category 1. Confusion of the definition of function with one to one and onto properties

It was observed that 44% of the preservice teachers saw one to one and onto conditions as the prerequisites of being a function. For instance Zeynep answered as "… two sets related to each other and pairing them with one to one and onto conditions" while Arda's answer was "…there should be one to one condition in function otherwise an element in the domain set of the function has two images and this contradicts the description of function". Ayla response to this issue was "since every equation cannot be bijective (one-to-one and onto) relation they are not functions". These concept images of preservice teachers are also reported in literature (Vinner, 1983; Dubinsky & Harel, 1992).

Also the preservice teachers who gave these answers are seen to have misunderstanding of relation concept. For instance it would be more appropriate if Ayla had used function concept instead of relation. However, even in this case, there would be a misconception that every function should be one-to-one or onto. Furthermore, it is seen that some preservice teachers are not so much descriptive and they do not take value sets of the function (that is, domain and image sets) into account at all. From the answers of the students given to the questions, one can draw the conclusion that the multivariable function concept where the function has more than one variable or possibility of the expression of any variable in terms of other variables were never considered by some students. An answer of a participant as "… because there is only one unknown in a function and there is one image for each unknown" supports this situation. A function $f: R\times R \rightarrow R$ described by $f(x, y) = x + 3y + 2$ can be given as an example to eliminate this misunderstanding. Also, the expression of "there is one image for each unknown" proves the misunderstanding related to function concept. She should have said "each element in the domain set is paired only one element in the image set". Based upon these answers, we can easily conclude that the preservice teachers are not aware of the fact that a domain set of a function is a Cartesian product.

Category 2. Relating function concept to examples from daily life

It was observed that three (33%) of the participants tried to describe the function concept with the examples from the daily life. For instance Suzan answered the question "would you describe the function?" as "… tea distribution process. Throwing a stone for each passage of sheep" while Arzu said "we went to the village as a visit. We are five people. Nobody should be left out. If these five people stay in different houses it is a one to one function and if they stay in the same house it is a constant function". Mehmet gave an different example in the real life as "we think function as a faucet. When we turn it on the stagnant water starts to flow. In other words the faucet (f) converts x into y". All these answers can be considered within Sfard's (1991) operational conception of the mathematical notation. Suzan stresses one to one and onto properties of the function as a prerequisite saying "nobody should be left out". Her answer of "if they stay in different houses, it`s one to one and if they stay in the same house it is a constant function" indicates the misunderstanding of the definition of function. Of course, the relation of concepts to our everyday lives is of great importance for meaningful learning and erases the questions in the minds of the students related to the use of the function concept. For example; the modeling such as "A child can only have one mother but a mother may have more than one child" is important for drawing of the attention of the students to the lesson during teaching the function concept. A result of a study conducted by Malcolm (2008) on the students from the upper primary and lower secondary years also supports this case. According to this study, it may be easier to build important mathematical concepts by using students' informal and intuitive knowledge concepts of joint variation and function accompanied with modeling life-related. NCTM (1989) approached this issue as "… functions that are constructed as models of real-world problems" (p. 126). However as seen from the answer above if the preservice teachers lack in adequate theoretical background of function concept, there will be problems in relating it to real life situations.

Category 3. Disregarding the conditions of being a function

Of the six preservice teachers, (67%) were observed to make textbook description disregarding the conditions of being a function. The answers of Ayla as "function is the process which carries the elements of non zero set A to a set B", Mehmet as "function … is an equation", and Arda as "making some processes on a variable finding results depending upon them" are the statements. These examples show that the conditions of being a function have not been taken into account. Also the answers such as "function … is an equation" and "upon a function …" shows that the participants have an insufficient knowledge related to multivariable functions. These answers could be considered in "operation", "formula" and "dependence relation" concepts of Vinner and Dreyfus (1989) respectively. These answers could also be evaluated within the context of Sfard's (1991) structural conception of the mathematical notation.

Category 4. The relation between the values which make the function zero and the roots of the equation

This category can be taken into Sfard's (1991) structural conception of the mathematical notation. Four of the preservice teachers (44%) were unable to see the relation between the values which make the function zero and the roots of the equation. The answer of Zeynep as "$x + 2 = 0$ is an equation but not a function. The opposite $y = x + 2$ is a function and an equation" is a good example of this. This answer shows that the student failed to see the fact that the solution of $x + 2 = 0$ is the x-intercept of $y = x + 2$ defined in R (the place where the function is described was not stated by the preservice teacher). This is also documented in literature (Even, 1988). Salih approached the subject as "every equation is a function. But every function is not an equation. The function can take different values but the roots of the equations cannot change". Banu also made a similar approach as "we can find a constant number in the equation. But in function a different value can correspond to a certain number in function". What is emphasized in this answer is different uses of literal symbols as unknown and variable (Philipp, 1992). It is observed that the preservice teachers lack in the understanding of these two concepts. The previous literature also revealed that the students at different levels had lack of understanding related to literal symbols (Dede, 2004; Fujii, 2003; Ursini,&Trigerous, 2001; Graham&Thomas, 2000; Macgregor&Stacey, 1997). Also as understood from Salih's answer above, the student fails to understand the fact that the root/roots of an equation is/are the point/points of the a function intercepting the x-axis. Of course every function may not be represented by an equation. For instance, a function defined from Z^+ to Z^+ matching every integer to the first prime number which is bigger than the integer cannot be represented with an equation. Similarly every equation is not a function. Here Salih might have thought only a single variable equation such as $x^2 - 2x - 3 = 0$ its roots are $x = -1$ and $x = 3$. However $x^2 + y^2 = 1$ is also an equation and there are infinite number of (x, y) pairs which satisfy it. The approach of Banu as "... because zero function is not an equation" shows that she did not have the understanding of "values of functions as solutions to equations" (Even, 1988, p. 305). She gave this answer thinking $f(x) = y = 0$ expression described in *R*. In other words she looked at equal sign as "do something signal" (Herscovics & Kieran, 1980, p. 574) That is why she regards this expression as an answer of something rather than an equation (Attorps, 2003). Related to this topic, Nalan was interviewed. The script is as the following:

I: Is $x + 2 = 0$, an equation in R?
A: Yes it is (she had defined the equation "as the mathematical expressions which include at least one unknown" previously).
I: OK. Is R a function of x?
A: Yes.
I: How? Can you show me the dependent and the independent variable here?
A: x is a dependent variable.
I: Related to what?
A: This expression $f(x) = 2$ is a constant function.
I: How did you find this expression?
A: $x + 2 = 0$ equation has single solution as $x = -2$. x cannot take other values.

Category 5. Multiple representations of functions

This category can also be taken in Sfard's (1991) structural conception of the mathematical notation. When preservice math teachers' written responses are investigated, it is observed that none of them mentioned about the multiple representations of functions. At the end of the interviews four of the participants (44%) were found to be lacking in knowledge related to multiple representations of functions. An except from the interview made with Suzan related to this subject is given below:

…
I: Are there any other representations of functions except the equations?
S: They can be represented with symbols … for instance (she gave the example of set correspondence diagram).
…
I: Are there multiple representations of functions?
S: I've never heard of it.
I: But you wrote an equation a while ago and made a set correspondence. What are these?
S: (Thought a while).Yes I did.
I: OK. Can these x and y values be shown in tables?
S: Yes they can.
I: What sort of conclusion can you draw from all these?
S: We cannot represent functions only with mathematical equations. There are other representations.

Of course, "the equation model refers to the symbolic (algebraic) representation of functions" (Cruz& Armando, 1995, p.6). However, function can be represented by other ways except equations. According to Even (1988) an education which enables the transition between multiple representations of functions geometric, numeric, and symbolic may lead to conceptual understanding. However the interview carried out with Arzu revealed the fact that there may be misunderstandings related to the transition between the multiple representations of functions (e.g. from graphical to set correspondence diagram).

…
I : Is the expression $y = x^2 - 1$ described in R a function

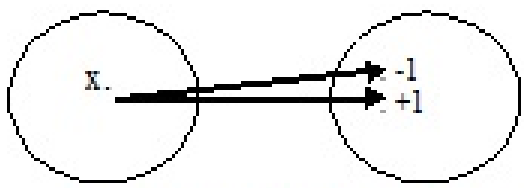

Figure 1. A Student's drawing

?

A: It is not. Because $x = \pm 1$ and has double values (She then drew the following).

It was seen that Arzu and Suzan decided whether a given expression is a function or not by using a set correspondence diagram because, in Turkish secondary mathematics curriculum, set correspondence diagram is given as a prototype in teaching function concept. The answer indicated that there was a confusion between $f(x) = \pm 1$ functions described in R and solving the equation $x^2 - 1 = 0$ (solutions as $x = \pm 1$). It would have been much more appropriate to draw the diagram of the given expression. Dreyfus and Eisenberg (1982) say that multiple representations of functions as tables, arrow diagrams, graphs, formulas or verbal descriptions make it`s understanding difficult. Thompson (1994) stresses that the idea of multiple representations was built without careful thinking and it must be based upon the concept of multiple. He says that graphs, tables and expressions may mean multiple representations for us but it may not be valid for the students and "they will see each as a "topic" to be learned in isolation of the others" (p. 23).

Category 6. Failure to take the defining set of functions and equations into account

It was determined that the preservice teachers failed to take the defining sets of functions and equations into account. Some of the answers related to this subject are as follows:

"$x=y^2$ is an equation but not a function". Although this answer given by three of the participants seems to be correct they didn't state the domain where $x = y^2$ is defined. That is why the importance of the place where a given expression is defined must be emphasized. Also they failed to state which variable is the function of the other. This should also be pointed out. Because although x is a function of y, y is not a function of x. Related to this topic, Arzu was interviewed. The script is as the following:

I: Is expression $x = y^2$ defined in R an equation?

A: Yes it is.

I: Is it also a function? Can you draw its graph? (The student found the values of y for x. But the graph she drew was the graph of $y = x^2$. She also found $y^2 = -1$ and $y^2 = -2$ for $x = -1$ and $x = -2$).

I: Is this graph correct?

A: (… Thinks for a while). No it is not (She then found $y = \pm 2$ and $y = \pm 4$, for $x=4$ and $x=16$ and drew the required graph).

I: Does this graph show a function?

A: If we draw a parallel line to y-axis it cuts the graphical line at two points. Therefore it is not a function. We can conclude from here that every equation is not a function.

Here the preservice teacher tried to explain the fact that a given graph may be decided whether to be a function or not by the interception of the graph at two points by a line drawn parallel to the y-axis. This approach was also supported by earlier studies in Turkey (e.g. Akkoç, 2006). Suzan, on the other hand, gave the right answer by taking the defining set into account. Below are some quotation from her written response and interview:

"It is not. If the defining set of $[x^2/(x-1)] = y$ is R, it cannot be a function".

Suzan's answer is correct. Because the expression $[x^2/(x-1)] = y$ defined in R-{1} is both an equation and a function. But, it is not a function defined in R. Because it is non-defined for $x = 1$. There is a vertical asymptote at $x = 1$. In other words, there is no value of y for $x = 1$. Whole of R can be taken as the image set. Suzan was interviewed related to this topic. The script is as the following:

I: Let $[x^2/(x-1)] = y$ be defined in R. Is this expression an equation?

M: It is not. Because there is no expression for $x = 1$ defined in real numbers.

I: Can x take other values?

M: It can. But (she thought for a while) … In that case … (She thought for a quite a long period and gave no answer).

I: Can the same expression define a function?

M. This expression is an equation. I am correcting what I said before.

I: Why did you change your decision?

M: … (gave no answer).

I: Let me repeat the question. Is this expression (pointing $[x^2/(x-1)] = y$) a function?

M: It is not a function in R.

I: Why?

M: It is not defined in R for $x = 1$.

I: In that case when does this expression define a function?

M: When we subtract 1 from R (…She thought for a while). I mean R/{1} (She wrote the difference symbol wrong).

DISCUSSION

This study involves the application of phenomenological method of analysis developed by Giorgi (1985) to mathematics education. The experiences of mathematics preservice teachers related to function and equation concepts were tried to be determined. To do so, firstly, seventy-one (71) preservice mathematics teachers was asked to answer open-ended questions about function and equation concepts aimed at determining the relationships between them. As a general evaluation, it was seen that most of the preservice mathematics teachers (71.83 %)

defined an equation accurately. However, they had problems in defining a function and only 31 of the 71 (43.66%) gave acceptable answers to the questions. It was seen that situation became worse when they answered the questions towards determining the relationships between function and equation concepts. In this point, we saw that even the students who gave accurate responses made very short statements or some of them could not made any explanations and just responded as true or false. The data gained from the questionnaire was showed that preservice mathematics teachers' knowledge about function and equation concepts were based on only their pure definitions and could not go beyond. In the qualitative part of the study, 9 preservice mathematics teachers at different academic success was interwiewed about their written answers they gave to questionnaire, based on the impressions they had left on researchers. Interview data were collected in six categories. By looking at these six categories from the most general perspective, it can be said that the four categories (categories 1, 2, 3 and 5) are oriented towards functions, while two of them (4. and 6. categories) are aimed at determining the relationships between function and equation concepts. When looked at the literature concerning functions, it is seen students' difficulties may be gathered under those general headlines (Bayazit, 2008): They (a) see a function as a relation that matchs one-to one (Vinner, 1983; Dubinsky & Harel, 1992), (b) could not establish relationships between function and ordered pairs (Dubinsky & Harel, 1992), (c) did not understand the terminology used for teaching of function concept (Gray& Tall, 1994), d) could not detect various function types (polynomial as constant, linear, quadratic, cubic, trigonometric and reciprocal etc.) (Eisenberg, 1991), e) could not comprehend multiple representations of functions and their transitions each to others (Even, 1998, Thompson, 1994; Thompson&Sfard, 1994), and f) did not understand the relationships between functions and algebraic expressions (Breidenbach et al., 1992). Amongst categories gained by this existing research, the 1st and 3rd categories were reported in Vinner's (1983) and Dubinsky and Harel (1992) studies where they tried to determine the concept images of the students related to functions. In categories 2 and 6 where students tried to make definitions without mentioning the value and definition sets can be taken within the context of Vinner and Dreyfus' (1989), "rule" category (p. 360). Among the objectives in Turkish secondary mathematics curriculum (1992) is "definition of the domain, value, and image sets of a function" (p.24), however, it was observed that the preservice teacher lacked in it. It is a clear fact that "no function can be given without some functional concepts (e.g., domain, image)" (Dreyfus&Eisenberg, 1982, p. 361). The case that preservice teachers failed to see the relation between the value/values which makes/make the function zero and the roots of the equation (category 4) was also in good accordance with Even's (1988) data. Fifth category supports findings in the studies carried out by Even (1998), Thompson (1994) and Thompson and Sfard (1994). Furhermore, the categories 1, 3, 4, 5, and 6 can be taken within structural conception of the concept of function (Sfard, 1991). Because, "… they can be grasped simultaneously as an integrated whole" (Sfard, 1991, p.6).

In view of the categories obtained in this study and of the learners' responses, as explained above, it seems also to tie students into giving quite rigidly definition-based responses, rather than moving more flexibly between representations and structural and operational conceptions. Morever, according to the feedback of the preservice mathematics teachers during interviews, it was determined that they usually perceive functions as a mechanism that makes transformation by means of algebraic or arithmetic operations. However, such an approach restricts this concept's function (Dubinsky & Harel, 1992; Breidenbach et al., 1992). The reason for this circumstance may be the fact that mathematics curriculum and textbooks of high schools and those of upper degree has been prepared in accordance with such a mentality mentioned above. In Turkish mathematics curriculum, expectations towards function and equation concepts that students are expected to acquire was given in the introduction part. For instance, in Turkish high school mathematics curriculum, students are expected to define a function and to understand its rich multiple representations. But findings of the present study show that these expectations could not be completely fulfilled (see categories 1, 3, and 5). Also, it is an important problematic issue that there exists no clear statement in Turkish high school mathematics curriculum which indicates relationship between function and equation concepts in Turkish high schools mathematics curriculum. Morever, as we mentioned before, equation and function concepts are presented as two separate and independent concepts in the curriculum. Students are expected to determine the relationship between these two concepts. Therefore, subjects of the present study who had been educated according to such curriculums had difficulties in understanding relations between equation and function concepts (see categories 4 and 6). For this reason, students' knowledge about function and equation concepts and the relationships between these two concepts could not go beyond quite defective knowledge based only pure definitions. The related literature reveals that teachers and preservice teachers have incorrect conceptions and vague about equation and there are a few students who can give a mathematically satisfactory definition of equation (see Attorps & Tossavainen, 2007, 2008, 2009). In Turkish mathematics curriculum expectations towards acquisitions about the function and equation concepts- statements that certainly makes the relationship between these two concepts clear- has been explained clearly. However, the research findings show that these acquisitions have not been gained sufficiently by the preservice mathematics teachers. We consider this case as rather independent from the Turkish settings and as a phenomenon that can also be observed in many other countries.

In recent years the approach of seeing the literal symbols, used for unknowns, as the variables and the relations between the variables has been adapted. It is presumed that this will create necessary impetus to pass from the literal symbols taught as the unknowns to the literal symbols taught as the variables and the relations between them. Moreover this approach is presumed to facilitate the easy learning of functions, the central concept of mathematics, by the students ("Reconceptualising School Algebra", 1997; Wheatley, 1995). This approach will certainly become much more popular with the constructivist learning approach recently adapted in Turkish educational system. The skills expected to be acquired by the primary school students are listed as solving a first degree equation with one unknown, using the equation in solving the problems, examining the linear relationship between two variables with the use of tables and graphs, explaining the change of one variable relative to other, explaining and using the two dimensional Cartesian coordinate system and plotting the graphs of linear equations (National Education Ministry, 2005). The most striking change between the older and the new curriculum is the skill of "examining the linear relationship between two variables with the use of tables and graphs". This enables the students to see the relations between function and equation concepts, transitions between multiple representations of the functions which enrich the concept images of the students. Turkish 7th grade students are taught a solution of an equation of the sort $3x - 7 = 0 \Leftrightarrow x = 7/3$. However, such an expression causes students to develop negative attitudes towards mathematics (Pope, 1994). In Turkish textbooks, the concept of equation is given by means of problem solutions and open sentences that may get true or false values depending upon a number taken from an arbitrary set. However, students have some difficulties in transforming from word problems to equations or from equations to word problems (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007; Dede, 2004; Stephens, 2001; MacGregor & Stacey, 1996; Herscovics & Kieran, 1980). The skills expected to be acquired from the new Turkish high school curriculum related to the concept of function are listed as showing the function in a table, stating the defining, value and image sets of the function, determining the defining and image sets of the functions according to the relations given in graphs, and explaining one to one, onto, into, unit, constant and linear functions. It is also proposed in the new program that function machine should be used in teaching the function concept. In this point, function machines suggested for helping to enrich concept images in students' minds about the function concept by some mathematics educators (MacGowen, DeMories & Tall, 2000; Tall, MacGowen & DeMories, 2000; MacGregor, 1998). Caldwell (1994) also proposes to use TI-81 graphics calculators as a "learning tool" in order to make students comprehend functions and their graphics better. Technological development and its increased usage in classroom atmosphere make it possible to use computer programs such as spreadsheets in teaching these concepts (Rojano, 1988).

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