

Profiling cognitive biases in primary school children's plane geometry: A dual-process study of prototype and area-perimeter reasoning

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Received 11 July 2026 ▪ Accepted 06 October 2026

Abstract

Early geometric reasoning predicts later mathematical achievement, yet children's errors in this domain persist across instruction and are conventionally labelled "misconceptions." Dual-process theory offers an alternative reading, that many such errors are cognitive biases, in which a fast intuitive response overrides slower analytical reasoning, but this framework has been applied mainly to numerical tasks and to adolescents or adults, leaving elementary geometry largely unexamined. It therefore remains unclear whether children's geometric errors reflect genuine intuitive interference or a simple lack of knowledge. This study profiles cognitive biases in the plane-geometry reasoning of elementary school students, focusing on prototype-based shape recognition and area-perimeter comparison, and uses a congruent-incongruent task design to separate intuitive interference from missing knowledge. Employing a qualitative descriptive design, fourth- and fifth-graders at a high-performing school in South Sulawesi completed task-based clinical interviews combining congruent and incongruent items with think-aloud and probing; episodes (one student × one item) were analyzed thematically for bias forms, sign patterns, and inhibition markers. Prototype bias appeared in 13 of 15 students—most often as orientation-based rejection—and area-perimeter bias in 12, most often as the "larger perimeter, larger area" rule. Crucially, the dominant pattern was accuracy on congruent but not incongruent items, and most such errors were accompanied by self-correction, indicating failed inhibition of available knowledge rather than absent knowledge. The study extends the dual-process framework to spatial-geometric reasoning in early childhood and reframes many geometric "misconceptions" as inhibitable cognitive biases, implying that diagnosis should precede remediation.

Keywords: cognitive bias, dual-process theory, prototype bias, area-perimeter, primary school geometry, inhibitory control

INTRODUCTION

Geometric reasoning is one of the fundamental pillars in early mathematical development, shaping how

individuals make sense of shape, space, and measurement long before formal proofs or deductive arguments are introduced (Mamolo et al., 2015; Seah & Horne, 2019). From the early years of schooling, students

Contribution to the literature

- This study extends the dual-process framework (previously concentrated on numerical reasoning in adolescents and adults) to spatial-geometric reasoning in early childhood, showing that intuitive-interference mechanisms are already operational when inhibitory control is still developing.
- Using a congruent-incongruent task design combined with self-correction tracing, it empirically separates errors caused by a genuine failure to inhibit an available intuitive response (cognitive bias) from those caused by an actual lack of knowledge.
- The findings reframe much of what geometry education calls “misconceptions” as inhibitable cognitive biases, implying that diagnosis must precede remediation because surface-identical errors demand different instructional responses.

are required to recognize and classify two-dimensional shapes, compare their attributes, and measure properties such as length, area, and perimeter (Pollitt et al., 2020; Zambrzycka et al., 2017). Because early geometric competence is associated with future mathematical achievement, the quality of students' reasoning in this domain carries consequences that extend beyond the geometry curriculum itself (Harris, 2023; Libertus et al., 2024; Mulligan, 2015). Therefore, understanding how this reasoning develops is central to early mathematics education.

However, students' geometric reasoning is often disrupted by systematic errors despite being taught with various instructional strategies; in this case, these difficulties reflect students' knowledge that has not yet been deeply formed (Hock et al., 2015; Moss et al., 2015; Seah & Horne, 2025). These errors occur persistently, indicating the presence of bias in students' mental activities. For example, a geometric shape has conceptual properties determined by figural properties rooted in perception, and difficulties arise when the figural-perceptual component dominates the conceptual component (Lehrer et al., 2012). This kind of explanation implies that many geometric errors are not based on a lack of knowledge but are rather the result of mental activities that shape reasoning. Large-scale quantitative evidence supports this pivot, across nearly 10,000 Italian secondary students, geometry items were the most frequently left blank, the lowest-scoring, and the single strongest predictor of overall competition success (Facciaroni et al., 2023). Yet students rated geometry as most similar to classroom material and did not see themselves as weaker in it, citing “using existing knowledge to solve the problem” as their main difficulty.

Dual-process theory provides a powerful lens for reinterpreting these errors as cognitive biases. In this regard, these biases are viewed as the product of fast and intuitive responses that override slower, analytical reasoning (Arvanitaki & Zaranis, 2020; Spelke et al., 2011). According to this view, the intuitive answer appears first and must be inhibited so that a normatively correct response can be produced; bias arises precisely when this inhibition fails. The developmental variant of

this framework places the key mechanism on the child's still-developing inhibitory control, which strengthens from childhood through late adolescence (Deringöl, 2025; Kandaga et al., 2022). This perspective is highly valuable for early geometry because it explains why children who possess conceptual knowledge still fail on tasks that require deep reasoning activities.

One widely documented bias in geometry is prototype-based reasoning, which occurs when children classify figures based on their resemblance to standard mental images, rather than on their defining properties (Haj Yahya & Hershkowitz, 2024; Halas et al., 2019). For instance, most students often fail to respond to a rotated square as a square and instead consider it a “rhombus” (Fujita et al., 2017). Because instructional materials often present shapes in a narrow range of standard orientations and proportions, children form rigid visual prototypes that limit their classification (Kalénine et al., 2013; Verdine et al., 2019). This bias has been theorized through the tension between concept image and concept definition in learners (Bruns et al., 2021) as well as through the dominance of figural properties over conceptual properties (Şahin et al., 2020) and has been proven to persist throughout the elementary school level (Ulusoy, 2023; Ulusoy & Çakıroğlu, 2020).

The second bias relates to the relationship between area and perimeter, namely when students intuitively apply the “larger A, larger B” or “same A, same B” rule to two quantities that are independent of each other (Dooren et al., 2009; Jiang et al., 2019). Students may conclude that a shape with a larger perimeter must have a larger area, or that two shapes with the same perimeter must have the same area, even though both conclusions do not hold true generally (Widjaja & Vale, 2021; Winarti et al., 2012). This pattern reflects a tendency to erroneously extend proportional or linear relationships to inappropriate situations (De Keersmaecker et al., 2025), and has been documented across various grade levels, including the early years of formal education (Im & Jitendra, 2020).

Although such evidence has accumulated, dual-process framework explanations of mathematical biases have thus far been largely concentrated on numerical reasoning and predominantly focused on middle and

high school students (Van Hoof et al., 2020). In line with this, geometry literature tends to describe children's errors as "misconceptions," rather than examining them as cognitive biases within an explicit dual-process framework. For example, recent evidence from senior high school students in Ghana shows that reliance on the visual appearance of a diagram continues to undermine formal geometric reasoning well beyond early childhood, yet this pattern is still interpreted through the lens of a general misconception rather than as a specific failure to inhibit a visually driven response (Davor et al., 2026). Consequently, it remains unclear whether children's geometric errors reflect genuine intuitive interference (e.g., existing knowledge that is uninhibited) or a lack of relevant knowledge; furthermore, the elementary school geometry context, where these biases first take shape, remains relatively understudied.

A related and largely unresolved question is whether students who exhibit bias in one type of geometric task also exhibit bias in other types of tasks. This question of intra-individual consistency bears on whether dual-process biases are best understood as a single, cross-content disposition or as task-specific responses bound to particular problem features, with implications both for how such biases are modeled developmentally and for how instructional interventions might be targeted. Investigating this question, however, first requires reliable evidence on how each bias manifests its own terms—evidence that itself remains limited in the elementary school geometry context. The present study focuses on this necessary first step; the question of cross-task consistency is left for future research.

This study addresses this gap by mapping the profile of two cognitive biases (prototype-based shape recognition and area-perimeter comparison) in the plane geometry reasoning of elementary school students, utilizing a congruent-incongruent task design that distinguishes intuitive interference from a genuine lack of knowledge. We focus on two complementary biases (namely, prototype-based shape recognition and area-perimeter comparison), which were chosen because they represent qualitatively different facets of geometric reasoning. Thus, this study is guided by the following research questions (RQs):

- RQ1:** How are prototype bias and area-perimeter bias manifested in the plane geometry reasoning of elementary school students?
- RQ2:** What student response patterns indicate the presence of these biases?

METHOD

Settings

This study employed a qualitative approach with a descriptive design based on task-based clinical

interviews. This design was chosen because both RQs require an in-depth examination of children's reasoning, rather than merely evaluating right or wrong answers: **RQ1** requires the characterization of the manifested forms of bias, while **RQ2** requires identifying the response patterns that indicate the presence of these biases. This study does not aim to generalize statistically, but rather to characterize the forms and signs of bias in depth.

Participants

The research subjects were fourth- and fifth-grade elementary school students at a high-performing elementary school in South Sulawesi. This grade level was selected because both topics serving as the context for the biases—plane shape classification as well as area and perimeter—have been taught at this level, ensuring that any errors that emerge can be meaningfully distinguished between cognitive bias and a genuine lack of knowledge. The subjects were selected using a purposive sampling technique with the following criteria:

- having received instruction on plane shape classification as well as area and perimeter and
- being willing to participate and able to communicate their reasoning verbally.

The number of subjects was set at 15 students, adhering to the principle of data saturation common in qualitative research; the final number could be adjusted once the variations in the forms of bias ceased to increase. Prior to data collection, permission was obtained from the school, along with informed consent from the parents/guardians and child assent.

Instruments

The study utilized two main instruments. The first instrument is a geometry task with congruent-incongruent items. This task contained items for two types of bias, each designed in two versions that formed the methodological backbone of the research. Prototype bias (shape recognition): Congruent items presented shapes in standard orientations and proportions (e.g., an upright square and an isosceles triangle with a horizontal base), so that matching them with a prototype yielded the correct answer. Shape membership in these items was scored using a mutually exclusive classification of quadrilaterals (e.g., square, rectangle, and rhombus treated as non-overlapping categories) rather than the class-inclusive hierarchy in which a square is treated as a special case of a rhombus. This choice reflects both the instructional convention typically used at this grade level, where shape classes are introduced as mutually exclusive categories before hierarchical class-inclusion relationships are formally taught, and the requirement of the congruent-incongruent design itself, which depends on a single

Table 1. General and item specific in interview

Purpose	Question
Elicit initial justification	Why do you say it's a rhombus?/Why do you think so?
Invite re-examination	Try checking again, are you sure about your answer?
Direct attention to defining properties	Try looking at the angles/the sides
Confirm final answer after re-examination	So, which one is correct?
Prototype bias (shape classification)	Is this a square/triangle/rectangle or not?
Area-perimeter bias (comparison)	Between shape A and shape B, which one do you think has a larger area?
Area-perimeter bias (transformation)	What about its area – Des it change or stay the same?"
Area-perimeter bias (direct calculation)	Try finding the area of this shape (length, width)?

unambiguous target category per item to distinguish typically from atypical responses. Under this scheme, a response of “rhombus” for the rotated square item was scored as a prototype-based rejection rather than as a valid alternative classification; we note this as an explicit methodological choice rather than a neutral geometric fact.

Incongruent items presented legitimate non-prototype members (e.g., a square rotated 45°, an obtuse triangle, and a highly elongated rectangle) or standard-looking non-members, so that prototype matching yielded an incorrect answer. Area-perimeter bias: Congruent items presented comparisons where the intuitive answer (“larger A, larger B” / “same A, same B”) was coincidentally correct. Incongruent items presented comparisons with the same perimeter but different areas (or a larger perimeter but smaller area), so that applying the intuitive rule yielded an incorrect answer. Each type of bias included 3 congruent items and 3 incongruent items of equivalent difficulty. These congruent-incongruent pairs allowed errors to be interpreted as biases rather than merely a lack of knowledge.

The second instrument is a semi-structured interview/probing guideline. This guideline directed the exploration of the children's reasoning for each answer, using a set of general and item-specific probing questions designed to elicit moments of self-correction (Table 1). This probing was crucial as it served as the source of evidence for **RQ2**. Item content for both instruments was informed by bias patterns already documented in the geometry education literature—prototype-based misclassification (Fujita et al., 2017) and the area-perimeter intuitive rules of “larger A, larger B” and “same A, same B” (Jiang et al., 2019; Van Dooren et al., 2009) which were then translated into the congruent-incongruent item pairs and probing prompts described above. The content validity of both instruments was established through expert judgment by 3 mathematics education experts, and they were piloted on 35 non-subject students to ensure item clarity and the appropriateness of the probing questions.

Procedures

Data collection took place over approximately six months, from July to December 2025, encompassing both

instrument validation (expert judgment and piloting with 35 non-subject students) and the main data collection with the 15 subjects. Interviews with the subjects themselves were conducted within this period over approximately two to three months. Data were collected through individual task-based clinical interviews in a quiet room at the school. Each session lasted approximately 20-40 minutes. The children completed the items one by one while voicing their thoughts (think-aloud); the presenting authors then asked probing questions according to the guidelines, particularly on the incongruent items, to trace their reasoning and provide opportunities for self-correction. All sessions were audio- and video-recorded by the recording authors, and the children's worksheets and scribbles were collected as supporting data. The recordings were then transcribed verbatim by the same two authors for analysis.

Data collection was conducted collaboratively by the research team. Four of the six authors, all with a background in mathematics education, were directly involved in conducting the interviews (presenting the geometry tasks, guiding each session, and delivering the probing questions). The remaining two authors were responsible for audio- and video-recording the sessions and subsequently transcribing the interviews verbatim. Inter-coder agreement was independently established by three external researchers not involved in either data collection or authorship.

The classroom teacher was involved in three aspects of data collection, recommending students who met the participation criteria (particularly the ability to communicate their reasoning verbally) scheduling the individual interview sessions around the class timetable, and being present in the room during the interviews. The teacher did not participate in presenting the tasks, guiding the sessions, or delivering probing questions, as these were conducted solely by the interviewing authors.

Analysis

Data were analyzed using thematic analysis (Braun & Clarke, 2006) that was deductive (theory-driven, following the coding scheme below) while simultaneously remaining open to inductively emergent codes if patterns outside the scheme were found. The

Table 2. Coding scheme

Aspect	Code	Brief description
Forms of bias	PB-ORI	Prototype bias: orientation-based rejection (e.g., a rotated square is called a “rhombus”)
	PB-PRO	Prototype bias: rejection based on atypical proportion/size
	PB-INC	Prototype bias: over-inclusion (accepting a non-example because it “looks standard”)
	APB-MM	Area-perimeter bias: “larger A, larger B”
	APB-SS	Area-perimeter bias: “same A, same B”
	APB-CONF	Area-perimeter bias: conflation/substitution of area and perimeter procedures
Signs of bias	SIG-BIAS	Correct on congruent items, incorrect on incongruent items (bias-sign pattern)
	SIG-GAP	Incorrect on both congruent and incongruent items (knowledge-gap pattern)
	SIG-MASTER	Correct on both types of items (mastery)
	INH-SELF	Spontaneous self-correction (knowledge is present, delayed inhibition)
	INH-PROMPT	Correction after probing (knowledge is present, weak inhibition)
	INH-CONFLICT	Conflict detection/doubt even if the final answer is incorrect
	INH-NONE	No conflict & no correction despite being probed
	DX-INTUI	Conclusion: intuitive interference (genuine bias) if SIG-BIAS + one of INH-SELF/INH-PROMPT/INH-CONFLICT
	DX-GAP	Conclusion: lack of knowledge – If SIG-GAP, or INH-NONE without signs of knowledge

Table 3. Frequency of prototype bias forms (N = 15 students; 3 prototype incongruent items)

Form	Code	Number of students	Number of episodes
Orientation-based rejection	PB-ORI	11	18
Proportion-based rejection	PB-PRO	7	9
Prototype over-inclusion	PB-INC	5	6

unit of analysis was an episode, defined as one student × one item (answer + justification + behavior when probed). The analysis follows these steps:

- (1) verbatim transcription,
- (2) coding of manifestation forms (domain A),
- (3) coding of sign patterns and warrant markers (domain B),
- (4) synthesizing each episode into an interpretative tag (DX-INTUI/DX-GAP), and
- (5) compiling a summary matrix per student as the basis for presenting the results.

The resulting thematic coding scheme is shown in **Table 2**.

Trustworthiness

Trustworthiness was maintained through:

- (a) triangulation of evidence sources (answers, worksheets, and reasoning transcripts);
- (b) inter-coder agreement by 3 external coders on a portion of the transcripts, with discrepancies resolved through discussion; and
- (c) an audit trail in the form of coding decision documentation.

Mapping the analysis to the RQs: **RQ1** is answered by domain A, while **RQ2** is answered by domain B (SIG-* patterns, INH-* markers, and DX-* tag).

RESULTS

RQ1: Prototype Bias

Prototype bias in plane shape recognition appeared in most subjects and manifested in three distinct forms: orientation-based rejection, proportion-based rejection, and prototype over-inclusion. Out of 15 subjects, 13 exhibited at least one form of prototype bias on the incongruent items. The frequency of occurrence for each form is summarized in **Table 3**.

The most frequently occurring form was orientation-based rejection (PB-ORI), namely when a student rejected a shape solely because its position differed from the standard orientation. This pattern was most evident in the item featuring a square rotated 45°. Rather than recognizing it as a square, students classified it as a “rhombus.” The following excerpt illustrates this reasoning.

R (researcher): Look at this shape (*pointing to the square rotated 45°*). Do you think this is a square or not?

S07: No, Ma’am. This is a rhombus.

R: Why do you say it’s a rhombus?

S07: Because this tip is at the top, it’s pointy. A square is flat at the bottom, even.

In this excerpt, S07 judged the shape’s identity based on its appearance and position, rather than its defining properties (four equal sides and four right angles).

Table 4. Frequency of area-perimeter bias forms (N = 15 students; 3 area-perimeter incongruent items)

Forms	Code	Number of students	Number of episodes
"Larger A, larger B"	APB-MM	9	14
"Same A, same B"	APB-SS	8	11
Conflation of area-perimeter procedures	APB-CONF	4	5

Similar responses were observed when several students tilted their heads or rotated the test paper so the shape would "stand upright again" before answering—an indication that their recognition relied on standard orientation.

The second form, **proportion-based rejection** (PB-PRO), emerged when students rejected a legitimate member of a class because its proportions were atypical. On the item with a "skewed" obtuse triangle and the item with a highly elongated rectangle, a few students stated that the shapes were not members of their respective classes.

R: Is this a triangle? (*pointing to the tilted obtuse triangle*)

S03: Hmm ... I don't think so. Triangles are neatly pointed, like the one before. This one is tilted; its shape is weird.

R: Is this shape a rectangle or not? (*pointing to the highly elongated rectangle*)

S11: No, it is too long. It's like a line, not a rectangle.

Both excerpts demonstrate that S03 and S11 held specific proportionate images as prerequisites for membership (e.g., a triangle must look "neat" and a rectangle must not be "too long"), thus rejecting shapes that were actually valid.

The third form, **prototype over-inclusion** (PB-INC), is the opposite of the previous two forms: students accepted a non-example because its appearance resembled a prototype. On the item with a parallelogram that at first glance looked "boxy," some students immediately classified it as a square without examining its properties.

R: Do you think this shape is a square? (*pointing to the boxy-looking parallelogram*)

S09: Yes, a square. It's boxy, it has four sides.

R: Try looking at the angles.

S09: (*observing*) Uh ... this one is a bit slanted, not a right angle. But earlier it looked boxy.

S09's episode shows that the initial judgment was based on an overall visual impression ("it looked boxy"), not on an examination of defining properties. Overall, these three forms reveal a common thread: the shape's

identity is judged through its resemblance to a prototype image, rather than through the properties that define it.

RQ1: Area-Perimeter Bias

Bias in reasoning about the relationship between area and perimeter emerged when students treated the two quantities as interdependent, even though they are independent. Out of 15 subjects, 12 exhibited at least one form of area-perimeter bias on the incongruent items. This bias manifested in three forms: application of the "larger A, larger B" rule, application of the "same A, same B" rule, and conflation of area and perimeter procedures. The frequency of occurrence is summarized in **Table 4**.

The most frequently occurring form was the application of the "**larger A, larger B**" rule (APB-MM), namely the belief that a shape with a larger perimeter must have a larger area. This pattern was evident in the item comparing a highly elongated rectangle (large perimeter, small area) with a square (smaller perimeter, larger area).

R: Between shape A and shape B, which one do you think has a larger area? (*A: elongated rectangle; B: square*)

S05: A, Ma'am.

R: Why A?

S05: Because A is longer, its perimeter is larger. So, its area must also be larger.

S05's reasoning directly linked area to perimeter: because one quantity is larger, the other quantity is assumed to automatically follow. Several other students traced the edges of the longer shape with their fingers before concluding it was "larger in area"—an indication that perimeter was used as the basis for judging area.

The second form, application of the "**same A, same B**" rule (APB-SS), emerged when students concluded that two shapes with the same perimeter must have the same area. This pattern was observed in a transformation item, where a square was "changed" into a rectangle with the same perimeter.

R: This square has a perimeter of 16. I changed its shape into this rectangle; its perimeter remains 16. What about its area, does it change or stay the same?

S12: Stays the same, Ma'am.

Table 5. Distribution of sign patterns per bias type (N = 15 students)

Sign pattern	Code	Prototype bias	Area-perimeter bias
Correct congruent, incorrect incongruent	SIG-BIAS	11	9
Incorrect on both	SIG-GAP	2	4
Correct on both	SIG-MASTER	2	2

R: Why are you sure it's the same?

S12: Because the perimeter didn't change, it's still 16. So the area follows and stays the same.

R: These two shapes both have a perimeter of 16. The area?

S04: The same. If the perimeter is the same, then the area is the same too.

Both excerpts show that S12 and S04 equated the conservation of perimeter with the conservation of area, without checking that a change in shape can alter the area even if the perimeter remains constant.

The third form, **procedural conflation** (APB-CONF), emerged when students mixed up the methods for calculating area and perimeter—for example, adding all four sides to answer a question about area.

R: Try finding the area of this rectangle (*length 5, width 3*).

S08: The area is ... 5 plus 3 plus 5 plus 3, so 16.

R: What did you just calculate?

S08: Area ... uh, is that perimeter? But you asked me to find the area earlier (*hesitant*).

S08 demonstrated a boundary between area and perimeter procedures that had not yet been clearly distinguished, such that the perimeter procedure was used to answer an area question. Overall, these three forms reveal a common pattern: area and perimeter were treated as two directly related quantities, rather than as quantities that can vary independently.

RQ2: Distinguishing Bias from Lack of Knowledge

To establish that errors on incongruent items constitute a bias—namely the interference of an intuitive response—and not merely a lack of knowledge, the analysis followed two steps. First, performance on congruent items was compared with incongruent items to determine each student's sign pattern (signature). Second, students' reasoning on incongruent items was examined to find signs that correct knowledge was actually available even though the intuitive response failed to be inhibited, primarily through self-correction episodes.

Distribution of Sign Patterns

The majority of students were correct on congruent items but incorrect on incongruent items—the SIG-BIAS pattern—across both types of bias. The distribution is presented in **Table 5**.

The dominant SIG-BIAS pattern serves as an initial clue that incongruent errors do not stem from a lack of basic knowledge: the same students were able to answer correctly on congruent items that demanded the same concepts. However, this structural clue needs to be reinforced with reasoning evidence, which is presented below.

Self-correction episodes as evidence of an intuitive response that failed to be inhibited

The strongest evidence that incongruent errors are biases comes from episodes where students, after providing an incorrect intuitive answer, demonstrate that the correct knowledge is available. Such episodes appeared in three forms: spontaneous self-correction, correction after probing, and conflict detection without correction.

Spontaneous self-correction (INH-SELF): On the rotated square item, S07 initially provided an incorrect prototype answer, then self-corrected without any probing.

R: Is this shape a square or not? (*square rotated 45°*)

S07: A rhombus ... (*silently observes for a few seconds*) uh, wait, Ma'am. The sides are all equal in length, and the angles are right angles if I straighten it. This is a square, just rotated.

R: So, which one is correct?

S07: A square. Earlier I just looked at its slant.

This excerpt clearly shows that S07 possesses knowledge of the defining properties of a square; the "rhombus" response was an intuitive answer that emerged first but was then overridden by an examination of its properties. This is the hallmark of bias in a dual-process framework: the knowledge exists, but the inhibition of the intuitive response is delayed.

Correction after probing (INH-PROMPT): On the area-perimeter item, S12 stated that the area remained the same because the perimeter remained the same, and only corrected it after being probed to check.

Table 6. Summary per student (excerpt)

Student	Prototype: SIG	Prototype: DX	Area-perimeter: SIG	Area-perimeter: DX
S01	SIG-BIAS	DX-INTUI	SIG-MASTER	-
S03	SIG-BIAS	DX-INTUI	SIG-BIAS	DX-INTUI
S05	SIG-MASTER	-	SIG-BIAS	DX-INTUI
S07	SIG-BIAS	DX-INTUI	SIG-BIAS	DX-INTUI
S08	SIG-BIAS	DX-INTUI	SIG-BIAS	DX-INTUI
S12	SIG-BIAS	DX-INTUI	SIG-BIAS	DX-INTUI
S14	SIG-GAP	DX-GAP	SIG-BIAS	DX-INTUI
S15	SIG-GAP	DX-GAP	SIG-GAP	DX-GAP

R: This square has a perimeter of 16. I change it into this rectangle, its perimeter remains 16. Its area?

S12: The same, Ma'am. The perimeter is the same, so the area is the same.

R: Try counting the unit squares inside each shape.

S12: (*counting*) The square ... 16 squares. The rectangle ... 1, 2, 3 ... only 7. Oh, they are different? That means the areas are not the same, Ma'am.

R: Why did you say they were the same earlier?

S12: I thought it was because the perimeter was the same.

S12 was able to arrive at the correct answer after being directed to count—indicating that the knowledge and skills were available, but the intuitive rule “same perimeter → same area” emerged first and was only inhibited after probing.

Conflict detection without correction (INH-CONFLICT): In some episodes, students exhibited doubt or sensed a conflict, yet their final answer remained incorrect.

R: Which one has a larger area, A or B? (A: *elongated rectangle*; B: *square*)

S05: A, because its perimeter is long ... (*hesitant*) but why does B look fatter, fuller ... (*silent*) ... but still A though, its perimeter is larger.

S05 detected a conflict between the perimeter intuition and the visual impression of area, but was unable to resolve it, thus the intuitive response ultimately won. This episode is still classified as bias because it demonstrates internal conflict, rather than an absence of ideas.

A contrasting case: Lack of knowledge (DX-GAP)

Not all errors are classified as biases. S15 was incorrect on both the congruent and incongruent items

and showed no signs of correct knowledge even after being probed.

R: Is this shape a square or not? (*upright square, congruent item*)

S15: Mmm ... I don't know, Ma'am.

R: Try looking at its sides, are they equal in length or not?

S15: (*hesitant*) I don't know ... they seem different.

Because S15 could not recognize even the prototype shape on the congruent item and was not helped by probing, the error was classified as a lack of knowledge (DX-GAP), rather than a bias. A contrasting case like this confirms that the congruent-incongruent method, when combined with reasoning tracing, is indeed capable of distinguishing between the two phenomena.

Summary of Differentiating Bias from Lack of Knowledge

Table 6 summarizes the sign patterns and interpretative tags of each student for both biases.

Overall, out of the 11 students exhibiting the SIG-BIAS pattern in prototype bias, 10 demonstrated signs of knowledge (DX-INTUI) and only 1 remained uncorrected; regarding the area-perimeter bias, out of the 9 students with the SIG-BIAS pattern, 7 were classified as DX-INTUI. Thus, the majority of errors on the incongruent items constituted biases (the failure to inhibit an intuitive response despite the availability of knowledge) and only a small fraction represented a lack of knowledge. These findings demonstrate that the congruent-incongruent sign pattern, when combined with reasoning evidence, successfully separates genuine bias from a lack of knowledge.

DISCUSSION

This study aims to characterize how prototype bias and area-perimeter bias are manifested in the plane geometry reasoning of elementary school students, while simultaneously determining whether errors on incongruent items reflect genuine cognitive bias or merely a lack of knowledge. The findings show that both

biases emerge in several distinct forms: prototype bias is manifested as orientation-based rejection, proportion-based rejection, and over-inclusion of shapes that merely look standard, whereas area-perimeter bias emerges as the application of the “larger A, larger B” rule, the “same A, same B” rule, and the conflation of area and perimeter procedures. More importantly, the congruent-incongruent sign pattern combined with reasoning tracing indicates that the majority of errors on incongruent items represent genuine biases, students answered correctly on congruent items and, in many cases, were able to self-correct when given the opportunity, while only a small fraction is rooted in a lack of knowledge. These dual findings serve as the foundation for the following discussion, which argues that children’s geometric errors are more accurately understood as a failure to inhibit intuitive responses rather than as a conceptual void.

The most important finding of this study is that the majority of errors on incongruent items are accompanied by evidence of correct knowledge alongside an intuitive response that appears not to have been inhibited, a pattern consistent with an inhibition difficulty rather than a conceptual deficit. Within the dual-process framework (Evans, 2019; Libertus et al., 2024; Spelke et al., 2011), the fast intuitive response emerges first by default, and a normatively correct answer can only be produced if analytical reasoning successfully intervenes and inhibits that initial response; when this inhibition fails or is delayed, the incorrect intuitive answer still emerges even though the correct knowledge is available. The self-correction episodes observed in this study are consistent with this interpretation: when a child initially calls a rotated square a “rhombus” and then, either spontaneously or after mild probing, corrects their answer by referring to the shape’s defining properties, the correction indicates that the relevant conceptual knowledge was available to the child and appears to have been initially overridden by an intuitive response. A similar pattern on the area-perimeter items, where a child corrects the assumption of “the same area” after counting units, points in the same direction. These findings align with developmental models of the dual-process framework that position inhibitory control (which continues to mature throughout childhood) as the key mechanism in overcoming intuitive interference (Clements & Sarama, 2004; Seah & Horne, 2019, 2025). From this perspective, for most students, difficulties on incongruent items are more consistent with a still-developing ability to suppress misleading prepotent responses than with an absence of concepts; however, because inhibitory control was not measured directly in this study, this interpretation is inferred from verbal and behavioral evidence from the interviews.

This distinction between bias and lack of knowledge invites a re-examination of the “misconception” framing that has thus far dominated geometry education. The

majority of literature describes children’s errors—such as rejecting a rotated square or equating perimeter with area—as misconceptions, namely flawed or unformed knowledge structures. This tendency persists even outside the elementary school context: a recent study of senior high school students constructing formal Euclidean proofs found that learners frequently substituted the visual appearance of a diagram for its defining properties (for instance, inferring a midpoint or a specific quadrilateral class from how the figure looked rather than from what was given) and interpreted this pattern as a general misconception rather than as a dual-process phenomenon (Davor et al., 2026). The recurrence of the same figural-over-conceptual pattern across such different ages, tasks, and educational levels suggests that it is not merely a marker of early conceptual immaturity, but a candidate for reinterpretation through the bias framework proposed here.

The findings of this study indicate that, at least in most cases, this label is inaccurate: the “incorrect” child often actually possesses the correct knowledge and merely fails to inhibit the intuitive response that emerges first, so much of what has been traditionally called a misconception is actually better understood as a cognitive bias. This assertion bridges two research traditions that have historically operated relatively separately: the “misconception” tradition in mathematics education, which emphasizes the content of knowledge, and the “cognitive bias” tradition in cognitive psychology within a dual-process framework, which emphasizes the thought process (Surynková et al., 2025; Widjaja & Vale, 2021; Xu et al., 2020). By demonstrating that children’s geometric errors can be empirically separated into bias (knowledge is present, inhibition fails) and a lack of knowledge, this study offers a more nuanced framework than the singular category of “misconception.” It is important to emphasize that this reframing does not claim all misconceptions are biases; the presence of lack of knowledge cases in the data confirms that the two phenomena are distinct and can coexist. Indeed, the ability to separate the two is the conceptual contribution of this research—a shift from asking “what the child does not know” to “why a child who already knows still makes errors.”

The three forms of prototype bias found—orientation-based rejection, proportion-based rejection, and over-inclusion—align with the thesis that children’s geometric difficulties stem from the dominance of figural properties over the conceptual properties of a shape (El Mouhayar, 2018). Orientation- and proportion-based rejections reflect rigid prototype images: when a legitimate shape appears in an atypical orientation or proportion, it no longer matches the child’s standard mental image and is therefore rejected. This pattern reaffirms the long-documented prototype phenomenon (Guerrero-Ortiz et al., 2016) and can be explained as the

dominance of concept image over concept definition (Tall & Vinner, 1981; Vinner, 2013): children judge membership based on resemblance to an evoked image, rather than on defining properties. What is most theoretically intriguing is the form of over-inclusion. While the first two forms represent the error of rejecting legitimate members (over-exclusion), over-inclusion is the opposite—accepting non-members that merely look standard—thus the presence of errors in both directions indicates that prototypes do not function solely as categories that are “too narrow,” but rather as visual benchmarks that can mislead in both directions simultaneously. If children’s errors stemmed solely from overly strict definitions, we would only encounter over-exclusion; the emergence of over-inclusion demonstrates that children’s judgments operate at the level of image (figural), not at the level of defining properties (conceptual). Thus, all three forms point to the same mechanism—shape identity is judged through resemblance to a prototype, rather than through its defining properties—and it is precisely this bidirectional nature that strengthens the interpretation that the core difficulty is figural dominance.

The area-perimeter biases found largely align with the intuitive rules framework (Arvanitaki & Zaranis, 2020; Hock et al., 2015; Lehrer et al., 2012). The first two forms, the application of the “larger A, larger B” and “same A, same B” rules, are classic examples of intuitive rules: relational heuristics that are overgeneralized and applied to two quantities that are actually independent. Both align with the dual-process interpretation, namely a rapid response linking area to perimeter without analytical examination. The third form, procedural conflation, operates through a somewhat different mechanism: rather than an overgeneralized relational heuristic, it reflects a default, unexamined retrieval of the more readily available perimeter procedure when area is requested. This mechanistic difference does not place procedural conflation outside the category of bias: as with the other two forms, the majority of conflation cases in this study were accompanied by self-correction once students were prompted to re-examine their answer, indicating that correct procedural knowledge was available but the more automatic response was not initially inhibited. A small number of conflation cases showed no such correction and were instead classified as a lack of knowledge, confirming that the two phenomena can co-occur within the same error category rather than that the category as a whole is ambiguous. All three forms of area-perimeter bias are therefore best understood as biases, though they are not mechanistically homogeneous—two stems from overgeneralized relational heuristics, while the third stems from competition between two closely associated procedures.

Overall, the findings of this study simultaneously confirm and extend existing understandings of

children’s geometric errors. On the confirmation side, the forms of bias found replicate patterns that have been documented across decades within the elementary school context: the prototype phenomenon (Fujita, 2012) and intuitive rules regarding area and perimeter (Jiang et al., 2019; Van Dooren et al., 2004). Thus, the existence and forms of these biases are not new but rather serve as a foundation that links this study to an established tradition. Its primary contribution, however, lies in extension, not confirmation. The majority of dual-process framework research on mathematical biases has been built upon numerical content and predominantly conducted on adolescents and adults; geometry as a spatial-visual domain is rarely examined through this lens, and the elementary school level even more rarely so. By applying the congruent-incongruent design and inhibition analysis to elementary school geometry, this study extends the dual-process framework simultaneously along two axes: domain (from numerical to spatial-geometric) and age (from adolescents/adults to early childhood). This extension is theoretically significant as it demonstrates that dual-process mechanisms are not unique to numbers but are also evident in spatial-geometric reasoning, and that these mechanisms are already operational and consequential at an age when inhibitory control is still developing, aligning with developmental models of inhibition (Jiang et al., 2019; Lubin et al., 2015); therefore, these findings reinforce the notion that inhibitory control constitutes a relatively cross-domain bottleneck in children’s reasoning.

Practically, the distinction between bias and a lack of knowledge implies that diagnosis should precede remediation, because errors rooted in intuition that fails to be inhibited require a different instructional response from errors rooted in unformed knowledge. Because both types of errors appear identical on the surface, uniform interventions risk missing the mark: building a concept from scratch for a child who already understands it, or merely “reminding” a child who does not yet truly possess the concept. For errors classified as biases, the focus of remediation should be on the process, not the content—namely, activating reflection and inhibitory control so that the intuitive response does not win by default. This can be achieved through explicit warnings that “obvious-looking” answers can be misleading—which has been proven to activate inhibitory mechanisms and improve performance on incongruent items (Hoang, 2020; Jiang et al., 2019)—as well as through the habituation of systematically checking the defining properties of shapes instead of judging by appearance and encouraging students to slow down before answering. Conversely, for errors rooted in a lack of knowledge, remediation must be directed at building the concept itself, because there is no correct response that can be “activated” if the concept does not yet exist. At the curricular level, two preventive

measures can be derived: first, instructional materials should present a diverse range of non-prototypical examples (Clements & Sarama, 2004; Hock et al., 2015; Lehrer et al., 2012; Seah & Horne, 2025); second, area and perimeter should be taught explicitly as two independent quantities, for example through tasks that maintain a constant perimeter while altering the area, so that the intuitive linkage can be addressed directly. It must be emphasized that because this study is descriptive in nature and does not test interventions, these implications are reasoned proposals that still require empirical testing.

CONCLUSION

This study characterizes how prototype bias and area-perimeter bias are manifested in the plane geometry reasoning of elementary school students, while simultaneously examining whether errors on incongruent items reflect cognitive biases or a lack of knowledge. Two key findings emerge. First, both biases appear in distinct forms—prototype bias as orientation- and proportion-based rejection as well as over-inclusion, and area-perimeter bias as the “larger A, larger B” and “same A, same B” rules, along with procedural conflation. Second, by combining congruent-incongruent sign patterns with reasoning tracing, the majority of errors on incongruent items were more consistent with genuine bias—children appeared to possess the correct knowledge but did not inhibit an intuitive response—than with a lack of knowledge, although this interpretation rests on verbal and behavioral evidence from interviews rather than on direct measures of inhibitory control. Thus, the geometric errors of elementary school students, at least in most cases, are more accurately understood as cognitive biases rather than merely a conceptual void.

Several limitations constrain the interpretation of these findings. Subject selection was purposive with a small sample size, so the findings offer transferability to similar contexts rather than statistical generalization to a population. The intra-individual consistency of biases across tasks was deliberately not investigated, thus this study cannot yet state whether bias is a singular disposition or task specific. The primary evidence relies on children’s verbalizations, which at the elementary school age have limitations in fully articulating thought processes. The research also took place within a single curricular and cultural context, without reaction time measures or cognitive processing data for triangulation. Finally, the classification of an error as a bias or a lack of knowledge depends on the quality of probing during the interview, meaning some of the children’s latent knowledge might not have been revealed.

Several directions for future research emerge from these limitations. First, the postponed question of consistency warrants investigation—whether children

who exhibit bias in one type of geometry task also show bias in another type—to test whether bias is a general tendency or content-bound. Second, intervention studies could test whether inhibition-based approaches, such as explicit warnings or habituating property checking, are effective in reducing biases. Third, larger-scale or mixed-methods studies could estimate the prevalence of each form of bias. Fourth, mapping trajectories across grades one through six could illustrate how biases and the ability to inhibit them change over the course of development. Fifth, the addition of inhibitory control tasks or reaction time measures could more objectively strengthen the distinction between bias and knowledge.

Despite these limitations, this study contributes in two ways. Theoretically, it extends the dual-process framework to the geometry domain and elementary school level, which have thus far been relatively underexplored, while also offering a reframing: much of what is commonly termed geometric “misconceptions” may be more accurately understood as cognitive biases—the failure to inhibit intuitive responses despite the availability of knowledge, though confirming this reframing more definitively will require direct measures of inhibitory control that this study’s interview-based design does not provide. Practically, distinguishing bias from a lack of knowledge implies that diagnosis must precede remediation, as the two demand different interventions. By shifting the question from “what the child does not yet know” to “why a child who already knows still makes errors,” this study enriches how we both understand and address children’s difficulties in geometry.

Author contributions: **ZZ:** conceptualization, data curation, formal analysis, investigation, methodology, project administration, writing – original draft; **UM:** conceptualization, methodology, supervision, validation, writing – review & editing; **AR:** methodology, supervision, validation, writing – review & editing; **A:** supervision, validation, writing – review & editing; **MA:** data curation, formal analysis, writing – review & editing; **P:** formal analysis, methodology, writing – review & editing. All authors agreed with the results and conclusions.

Funding: This study was funded by the PNBP (Penerimaan Negara Bukan Pajak) scheme through the Percepatan Guru Besar program of Universitas Negeri Makassar, under contract No. 1827/DST/UN36.12/TU/2026.

Ethical statement: The authors state that the study was approved by the Institute for Research and Community Service, State University of Makassar, in 2026 (approval code/contract no. 1827/DST/UN36.12/TU/2026). Participation was voluntary, informed consent was obtained from all participants, and the confidentiality of participants’ personal data was ensured and used solely for research purposes.

AI statement: The authors state that generative AI tools were used solely to improve the clarity of the English language in the manuscript. AI was not used for content generation or data analysis.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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