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Proposal for substantive theory: A connection between mathematics learning and programming for children

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Abstract
In an increasingly digitalized educational context, programming for children emerges as a complementary tool for learning mathematics, facilitating the development of logical thinking and problem-solving skills from an early age. However, in the Colombian school curriculum, coding has not yet been formalized as a pedagogical strategy in mathematics. This study explores the potential of integrating coding into mathematics classrooms through a qualitative analysis based on symbolic interactionism and substantive theories obtained from the literature. These analyses were contrasted with the experiences of children and mathematics teachers who, on their own initiative, incorporated children's coding into their classrooms. The participants were 210 students from fifth, sixth, and seventh grades and six mathematics teachers from four public educational institutions in Colombia. Data was collected through surveys and semi-structured interviews, with the informed consent of the students' guardians and the assent of the teachers; the instruments were validated by experts. The results suggest that the use of electronic devices and the learning of algorithms and sequences contribute to strengthening logical-mathematical and problem-solving skills, which are highly valued in the 21st century. The study concludes that incorporating technological tools into mathematics teaching enriches the understanding of key concepts and offers new possibilities for learning.

Keywords: programming language for children, logical thinking, mathematical problem-solving

INTRODUCTION

In the final decades of the 20th century, programming languages were complex and poorly suited to children's interests, making them difficult to teach at an early age, partly due to the lack of accessible teaching guides (Cerón-Molina, 2022). In fact, programming did not have a significant place in the educational proposals of that time (Resnick et al., 2009). Although programming today arouses great global interest due to the appearance of emerging technologies that require the development of new competencies and skills (Brandsæter & Berge, 2025; Chang et al., 2018; Tiengyoo et al., 2024), curiously, there are still few initiatives that integrate it in the Colombian educational system (Torres & Inga, 2025). This can be explained by the lack of a formal incorporation of programming in school curricula, and the fact that teacher training in this area is not a priority, especially in basic education, where teaching something that is not mastered is a challenge (Valdivieso & Burbano, 2023).

In an increasingly digital and technological world, where students are expected to understand and use tools and techniques related to, for example, Artificial Intelligence (Castro et al., 2024a), programming and computational thinking in general could complement mathematics learning from an early age (García, 2022), promoting the development of logical processes, critical thinking skills, and problem-solving (Abelson et al., 1996). However, teaching programming in mathematics classes is not mandatory in institutions nationwide; its implementation is left to the teacher's discretion. Nevertheless, this situation represents an opportunity to explore the potential of integrating programming into mathematics classrooms (Park & Manley, 2024), to

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Contribution to the literature

- This article demonstrates that programming for children in early childhood education strengthens logical-mathematical and problem-solving skills by facilitating an active understanding of mathematical concepts from an early age.
- It identifies a gap in the Colombian curriculum by highlighting the absence of coding as a formal strategy in mathematics teaching, despite its educational potential.
- It provides empirical findings through grounded theory that show how the voluntary incorporation of programming by teachers and students significantly improves mathematical learning.

design pedagogical and didactic proposals that stimulate the development of mathematical thinking, particularly in children.

In contrast, in recent years there has been a growing international interest in teaching children programming in both online and offline ways. As Karsenti and Burgmann (2017) point out, some countries such as Sweden, England, the United States, and Argentina have begun to integrate programming into their curricula. In the case of Sweden, authors such as Bråting and Kilhamn (2021) describe this case as unique in that it relates mathematics to programming based on algebra at all school levels of the curriculum, especially from childhood. For example, by teaching ways to represent repeated addition by creating a program that performs multiplications using an algebraic representation system with alphanumeric elements, where the use of logical sequences, essential for learning programming, is necessary. In this regard, for Terroba et al. (2021), teaching programming to children offers numerous benefits, especially in problem-solving.

Among these, the ability to break down complex problems into smaller parts and approach each one through an orderly set of steps stands out, which fosters the development of logical thinking, analytical skills, and argumentation. These benefits–along with critical thinking, collaborative work, and the use of technological tools–are essential for living in the 21st century (Monereo & Pozo, 2007). Along these lines, Hu and Wang (2024) assert that unplugged programming improves computational thinking and mathematical creativity in primary education.

Consequently, teachers, especially mathematics teachers, must recognize the importance of pedagogical reflection in today's education, particularly when applying didactic or theoretical approaches that foster creativity and improve students' conceptual understanding through programming (Ye et al., 2024; Zolkower & Bressan, 2012). For Camilloni (2017), for example, it is always possible to teach better, and even more so in a society in which technology is one of the most evolving factors and cannot be left out of school processes or teaching tasks.

The incorporation of robotics education into primary education through the use of block-based programming languages has significantly improved the understanding of computational concepts such as sequences, loops, and conditional statements, thanks to its playful nature (Polo, 2024; Saez-Lopez, 2021). As Castro et al. (2024b) state, robotics, together with programming, constitutes an innovative tool in early childhood education, as it promotes the understanding of the concepts, practices, and perspectives of computational thinking in young students. However, it has also served as a starting point for the development of new methodological strategies that have improved student performance in the mathematics curricular component (Spiess, 2024).

Based on the above, it is appropriate to collect and analyze information from students, teachers, and educational institutions that implement programming in mathematics classes to develop strategies for fostering logical reasoning in children, aligned with the skills needed to meet the challenges of the 21st century.

This article proposes substantive theories (Hernández Sampieri & Mendoza Torres, 2018, pp. 530-531) based on an emergent design that includes a comprehensive documentary review, complemented by the experiences of elementary school students and their mathematics teachers, who, on their own initiative, incorporated the programming in the classroom as a learning strategy for children to develop essential skills in the modern world. The documentary review pursues two purposes: First, to define a priori category on the development of logical thinking in relation to children's coding in the context of 21st century competencies; and second, to contrast the research results with the existing literature.

It is expected that in the future these theories will evolve into a grounded, validated, and implemented theory. Below is a theoretical approach to logical thinking and programming for children, aligned with present-day competencies. For Nieves et al. (2019), logical thinking is so named because it follows the laws of logic, which is why it is developed in the field of mathematics, and its learning is facilitated through representation, abstraction, creativity, and demonstration, which must always be present in this area. Logical thinking is also defined as the ability to analyze, understand, solve problems, and reflect on mathematical situations (Arias & García, 2016).

Regarding programming for children, Nyakundi (2021) points out that the student must learn to code,

Table 1. Competencies for the 21st century

Problem-solving (PS)	Critical thinking (CT)	Collaborative work (CW)	Tools needed to work in the 21st century through technology (TT)
According to Scott (2015), it is "the ability to search, select, evaluate, organize and weigh alternatives and interpret information" (p. 5). He also states that this century draws on multiple domains to find solutions to a given problem. Additionally, authors such as Mason et al. (1989) define a mathematical problem as a situation that requires applying concepts and putting mathematical skills into practice to reach a certain solution that must be reviewed.	It is a process in which knowledge and intelligence are used to find a reasonable position that can be justified (Alanoca, 2016). According to Facione (2007), this type of thinking has a purpose, as it seeks to interpret the meaning of something, prove something, or solve a problem, which can be a collaborative task and therefore does not imply a competitive aspect.	Understood as social construction in which interaction with others leads to teaching and learning processes, where each member contributes to achieving a common goal (Gómez & Álvarez, 2011).	These tools represent resources and programs used to process, manage, and share information through various technological devices such as computers, mobile phones, video game consoles, among others. As Anchundia and Moya (2019) point out, these technologies are the driving force of today's society, and as a result, new professions and jobs emerge, or existing professions undergo changes leading to transformation processes. Therefore, education must stay at the forefront, promoting training in the use of these tools from early childhood and fostering digital literacy.

which involves converting human language into one that the machine can understand in order to execute programs and tasks. Coding, therefore, is closely related to programming, which consists of creating a computer program to perform a specific task. In this context, code is only part of a complete program, because programming requires the use of algorithms, desktop testing, problem modeling, and, according to Briz and Serrano (2018), it is strongly linked to mathematics. The process begins with a formulation of a contextual problem, followed by the structuring of a sequential and logical solution, which is finally translated into machine language for execution (García, 2022). In this sense, a program is defined as a set of instructions that make up an algorithm, where several mathematical concepts of basic education are essential, such as the conceptualization of variables in problem-solving, the use of algebraic language, and the notion of coordinate systems (García, 2022).

There are also the syntactic and semiotic aspects of algebraic concepts, which for Trejos (2018) allow the development of programming as a form of technological expression. In this context, for Duarte et al. (2024) the child applies what he or she has learned in mathematics and thanks to this, it is possible to understand the programming language. According to Cerón-Molina (2023), this language is visual and through blocks it builds logical sequences that form a program; in addition, this approach allows effective communication between humans and machines (Papert, 1987).

As for the 21st century competencies relevant to this research, they are found (**Table 1**).

In this sense, González (2020) proposes that in addition to the aforementioned skills, programming is considered the new literacy of the 21st century, where problem-solving at various levels of abstraction is essential. Childhood is an ideal stage to promote this literacy through new technologies by encouraging computational thinking, which allows students to develop digital skills, which for Caccuri (2018) "(…) is having the ability to search, process, and communicate information, transforming it into knowledge, selecting the most relevant, and making use of different media in a critical, responsible, and safe manner" (p.8). This approach allows for the implementation of inclusive didactics that prepare students to properly understand and use the innovative tools produced in the knowledge society.

In this process of progress, it is essential to have a suitable mediator who, from an educational perspective, fosters logical thinking through programming, facilitating meaning-making processes that enable students to interact with technology, embrace its benefits, and explore its potential, aligned with 21st century competencies.

METHODOLOGY

This research was of the qualitative type, which according to Hernández Sampieri and Mendoza Torres (2018) "focuses on understanding phenomena by exploring them from the perspective of participants in their natural environment and in relation to the context" (p. 390). However, the authors also highlight the importance of key concepts a priori and those that emerge from the data.

In this study, the a priori concepts were derived from an analysis of the existing literature on the topic, with the aim of establishing initial categories. These categories, along with the perceptions of students and their mathematics teachers who incorporated children's programming into their classrooms, were used to define subcategories. These subcategories, in turn, enabled the development of a substantive theory related to the

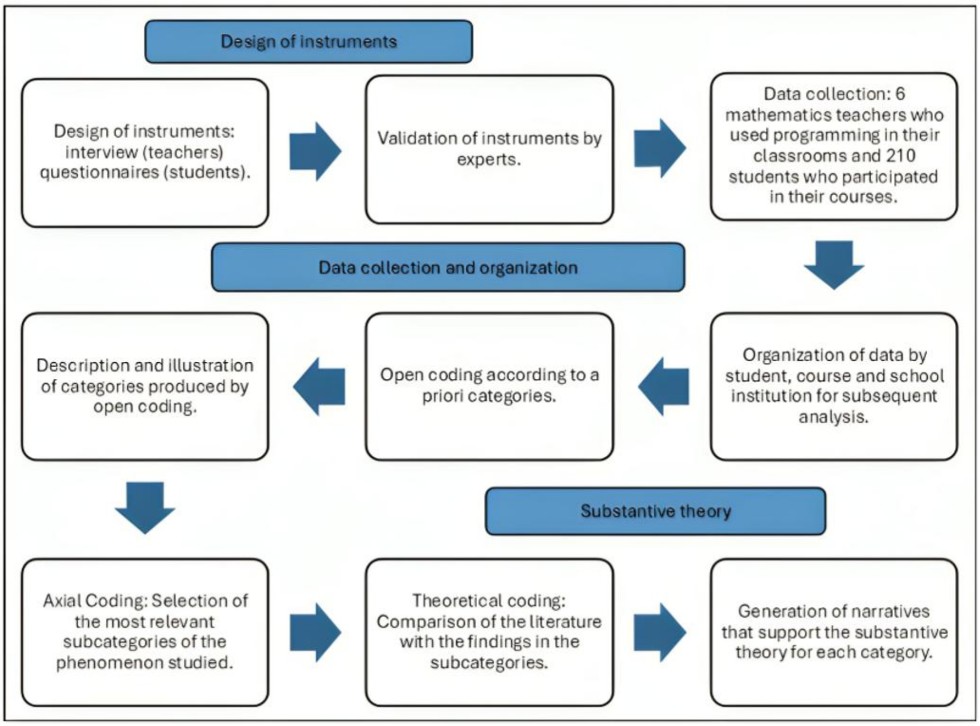

Figure 1. Emergency design based on grounded theory with a hermeneutic interpretation perspective (Source: Authors' own elaboration)

advancement of logical thinking within the framework of current educational competencies.

In this sense, Charmaz (2014) argues that substantive theory must be expressed through narratives that explain the results of data analysis, which constitutes the basis of grounded theory. This methodological design, aimed at generating substantive theory, aligns with qualitative research. It is philosophically grounded in symbolic interactionism, which is derived from data, which is collected systematically, allowing for their subsequent coding and analysis from a hermeneutic perspective. This allows for a greater understanding of the phenomenon studied and provides a guide for action (Strauss & Corbin, 2016; Sudirman et al., 2024). This approach was precisely the one implemented in the present research, allowing the construction of a substantive theory that emerges from the data obtained and rigorously analyzed.

Consequently, the process followed in the design, collection, organization and analysis of data is described (see **Figure 1**), which is explained in detail below.

Context and Participants

This research was carried out in a district educational context, specifically in the Bosa neighborhood of Bogotá, Colombia. The studied population included students from 5 to 11 years old, an age range corresponding to students enrolled in fifth grade of primary basic education, and sixth and seventh grades of secondary basic education in Colombia. In 2022, this locality had 102,490 students from early childhood education to secondary education, distributed among 29 district educational establishments, 10 district schools with contracted administration, and 87 private institutions (Secretaría de Educación de Bogotá, 2022).

The convenience sample for this research was obtained from local public schools, considering aspects such as access, location, and proximity, to ensure similar socioeconomic characteristics among participants. Four educational institutions were selected where, at their own initiative, the mathematics teacher incorporated programming into their classes to foster mathematical thinking, prioritizing students' logical thinking: Colegio Leonardo Posada Pedraza, Colegio Carlo Federici, Colegio Villas del Progreso, and Colegio Carlos Albán Holguín.

In these institutions, two courses were selected from cycle three, which includes the fifth, sixth, and seventh grades, according to the classification of the Mayor's Office of Bogotá and the Ministry of Education (Alcaldía Mayor de Bogotá & Secretaría de Educación, 2009). This cycle is characterized because "learning is guided by inquiry and experimentation, processes anchored in the dynamics of preadolescent development, that begins to master the relationship of proportionality and conservation, and to systematize concrete operations" (p. 11). These aspects are fundamental to the present study; therefore, we sought to guarantee equitable participation of the three grades that make up the cycle.

Table 2 shows the distribution of participants; a total of 210 students and 6 teachers participated.

Table 2. Distribution of information from the selected sample

Institution	System	Grade	Students	Teachers
Colegio Carlo Federici	Public school–Fontibón	Fifth	34	1
Colegio Leonardo Posada Pedraza	Public school–Bosa	Fifth	36	1
		Sixth	36	1
Colegio Carlos Albán Holguín	Public school–Bosa	Sixth	31	1
		Seventh	37	1
Colegio Villas del Progreso	Public school–Bosa	Seventh	36	1
Total			210	6

Note. It is identified that there are more students in seventh grade

Table 3. Definition of a priori categories related to the questionnaire applied to the students

APC	Objectives for developing questions	CC
I. Aspects to consider when selecting content	Identify the main mathematical learning goals that interest the student.	PC
	Identify main mathematical learning goals that may present difficulties for the student.	PC
	Identify some student conceptions regarding the emotions generated by mathematics.	PC
	Describe students' conceptions about the usefulness of mathematics in their environment.	PC & RP
II. Perceptions regarding the use of electronic devices	Identify the electronic devices students interact with in their daily lives.	HT
	Identify the main reasons why students use electronic devices.	HT
	Identify the presence and function of electronic devices during math classes.	HT & RP
	To understand students' perceptions of the relevance of electronic devices in the society in which they live.	HT, TC, & PC
III. Mathematics and programming	Identify the types of software students are familiar with that are related to learning programming.	HT & PC
	Identify whether students have prior knowledge about the use of programming and how they acquired it.	HT & PC
	Describe the mathematical content that students associate with programming.	PC & HT
	Identify whether students relate programming to the mathematical field and how they demonstrate this.	PC & HT
IV. 21st century competencies	To understand students' opinions on learning mathematics through use of programming.	PC & RP
	Establish from the student's point of view why it is important to learn mathematics.	PC & RP
	Establish, from the student's perspective, the skills they consider relevant in relation to the use of electronic devices.	HT
	Identify student's perceptions of 21st century skills that can be developed through programming.	RP, PC, TC, & HT

Note. APC: A priori category & CC: Competency of the 21st century

Design and Collection of Instruments

Two instruments were used to collect data: The first was a survey, which, according to Lanuez and Fernández (2014), is a form of interview conducted using a questionnaire. This technique is a valuable tool for understanding the magnitude of a social phenomenon and its relationship to other phenomena in society. In this study, a questionnaire was designed with four a priori categories that emerged from the literature. These categories served as the basis for formulating sixteen questions addressed to students, which were aligned with specific objectives and 21st century competencies, as presented in **Table 3**. The questionnaire sought to understand students' perceptions of the mathematics class, in which the teacher, on his or her own initiative, incorporated children's programming into the classroom as an educational tool.

The second instrument was the interview, which, according to Hernández Sampieri and Mendoza Torres (2018), is a meeting intended to discuss and exchange information. In this research, semi-structured interviews were designed for mathematics teachers, based on the approach proposed by these two authors, using a guide of topics or questions organized into five a priori categories with a total of twenty questions. Each question is associated with a specific objective associated with 21st century competencies, as shown in **Table 4**. This structure allowed the interviewer flexibility to add questions, clarify concepts, and obtain additional information about the teachers' experience while including coding for children in their mathematics classes.

Once the questionnaire and interview were designed, both instruments were evaluated by four education experts. This process was carried out to ensure the validity and reliability of the instruments in terms of wording, clarity, relevance, and coherence of each question, as well as addressing the research topics with the greatest possible rigor. Once the observations were analyzed, adjustments were made to clarify some key concepts of the research and facilitate the interpretation of each question.

Table 4. Definition of a priori categories related to the interview applied to teachers

APC	Objectives for developing questions	CC
I. Aspects to consider when selecting content	Identify the main topics in the area of mathematics according to the teachers' perceptions.	PC
	Identify the main topics in the area of mathematics in which students have the greatest difficulties.	PC
	Identify the content through which the teacher is integrating the area of mathematics with the field of programming.	PC & HT
	To describe teachers' conceptions about the contribution of the institutional mathematics curriculum to the critical training of third-cycle students.	PC
II. Using devices from the teachers' perspective	To establish the advantages and disadvantages of using electronic devices in cycle three according to the teachers' perspective.	HT & PC
	To identify teachers' perceptions of the impact of electronic device use on mathematics learning.	HT
	Identify the type of software, and its characteristics, commonly used by teachers to teach programming to third-year students.	HT, PC, & RP
	Describe the skills developed through programming according to teachers' perceptions.	HT
III. Didactic and motivational approach	Identify the most relevant didactic aspects in the teaching of mathematics.	RP, TC, & HT
	To identify teachers' perceptions of the role of didactics in the teaching of mathematics.	RP & HT
	Identify aspects related to technology in the teaching of mathematics.	HT, RP, & PC
	Identify the actions teachers take to motivate their students in learning mathematics.	RP, PC, HT, & TC
IV. Educational tools required by the teacher	Describe the characteristics of the main educational tools used in teaching practice in the area of mathematics.	HT & TC
	Identify the main educational tools that teachers use to develop logical thinking.	PC & HT
	To identify teachers' perceptions of access to technological resources to develop their classroom practice.	PC & HT
	To identify mathematics teachers' perceptions of information and communication technology training.	PC & HT
V. Aspects to consider from the 21st century competencies	To identify teachers' perceptions of the development of critical thinking and problem-solving through mathematics learning.	RP, PC, TC, & HT
	Identify the most relevant topics in the area of mathematics to develop 21st century competencies.	RP, PC, & TC
	Identify competencies developed in the area of mathematics during cycle three.	RP, PC, TC, & HT
	Identify teachers' perceptions of the development of thinking-related competencies by using educational tools.	RP, PC, TC, & HT

Note. APC: A priori category & CC: Competency of the 21st century

Data Analysis

Once the instruments were adjusted, they were applied to the selected unit of analysis. Subsequently, the corresponding transcription was completed by the institution and by the participant (student or teacher), using a coding system that allowed for the precise identification of each individual in the research. After transcribing the teachers' and students' perceptions, a licensed version of NVivo software (QSR International, 2020) was used to code the qualitative data following the guidelines of grounded theory (Palop, 2015).

It began with open coding, which, according to Strauss and Corbin (2016), is a process in which key concepts are identified in the data through line-by-line analysis and constant comparison. In this sense, this research analyzed the responses of students and teachers, identifying keywords that were continuously compared until reaching saturation point, at which point the data began to repeat. This process allowed the construction of word clouds for each question in both the questionnaire and the interview, demonstrating the frequency of the terms, an aspect that was useful for conceptualizing and developing hierarchical maps that regrouped the data. Axial coding continued, in which the a priori categories were related to the emerging subcategories of the data, according to the frequency they presented in the hierarchical maps of each question of the questionnaire and the interview, to establish precise and complete explanations with the purpose of regrouping the data that were fractured during open coding and establishing relationships between the categories and their subcategories (Strauss & Corbin, 2016).

This entire process culminated in theoretical coding, where the findings were compared with existing theory to generate a substantive theory. This theory, although local in nature, reflects the reality of third-grade teachers and students, based on their perspectives on the use of programming for children in mathematics learning,

Table 5. Category I and its subcategories obtained from the information provided by the students

A priori category	Sub-categories			
I. Aspects to consider from the selection of content.	Numerical thinking: A fundamental aspect for the development of basic operations and problem-solving in daily life.	Multiplication and division: Among the algorithms that generate the greatest difficulty for third-cycle students.	Learning mathematics generates a variety of emotions, depending on the topics being understood. Positive emotions include joy, surprise, and happiness. In contrast, fear, anger, sadness, and stress arise when problems cannot be solved.	Mathematics and its practical usefulness: A necessity for solving everyday problems and functioning in the context.

Table 6. Category II and its subcategories obtained from the information provided by the students

A priori category	Sub-categories			
II. Perceptions regarding the use of electronic devices	Cell phones have become a daily used device for third-cycle students, along with computers, tablets, and televisions.	Reasons for students' use of electronic devices: Learning, entertainment, fun, and forms of communication.	Explanatory videos: One of the tools used in math classes.	Electronic devices: A medium for learning, play, problem-solving, and personal relationships.

contrasted with information gathered from the literature.

RESULTS

The analysis of the results begins with the a priori categories derived from the literature reviewed, along with their respective subcategories. These emerged from the data to establish the theoretical foundation mentioned in the methodology. The categories are presented in the first column of the tables. In addition, four subcategories were identified for each category, located in column 2 through column 5 of the tables. These subcategories were compared with the literature reviewed, allowing the formulation of nine substantive theories.

Results by Category from the Students' Perspectives

Table 5 shows the initial information obtained from working with the students.

In relation to category I and its subcategories (see **Table 5**), it is essential to consider the topics that generate pleasure and displeasure in students within the logical-mathematical field to foster a more effective motivational process. This is important because it considers the interests, cognitive aspects, and emotions expressed by children, key aspects to enhance their strengths and improve their weaknesses. As Rodríguez (2021) points out, the student is understood as an integral being affected by social, biological, emotional, intellectual, and ethical factors, which is why mathematics education must promote the development of all human dimensions (Szücs & Mammarella, 2020). In this sense, content related to the numerical aspect and its application to solving problems linked to everyday life are attractive to students, who, being part of society, welcome this learning as social products (Gómez & Saldaña, 2019). However, the development of some algorithms is the basis of the difficulty presented by students in the area of mathematics, which is due to limited prior knowledge and "a strict logarithmic structure where each step must be invariably memorized" (Gómez & Saldaña, 2019, p. 7). To mitigate these difficulties, it is important to consider that problem-solving is mediated by electronic devices, commonly used elements that, far from being disqualified, possess great educational value in this century.

Therefore, after comparing the subcategories with previous literature, substantive theory 1 is defined as *Mathematics and emotions: A matter of content learning*.

Regarding category II and its subcategories (see **Table 6**), students consider electronic devices, initially provided by their families, as a means of entertainment, which coincides with what was pointed out by Vélez and Fraile (2019) who mention that from an early age children begin to interact with devices such as cell phones or video games.

Both teachers and students recognize that they have access to computers, tablets, and televisions, which have become tools not only for recreation but also for information gathering and communication among peers. This allows the sharing of aspects of both the school and extracurricular environments, demonstrating that devices are learning tools that have evolved to be easily manipulated since childhood (Peña, 2020). As a result, students are motivated to use them, encouraging teachers to integrate these technological elements into their educational practices, always with proper training,

Table 7. Category III and its subcategories obtained from the information provided by the students

A priori category	Sub-categories			
III. Mathematics and programming	Scratch and MakeCode: The most widely used software programs among third-cycle students for learning to program.	Curiosity about programming stems from computer science class and family environment.	Mathematical topics covered in programming: Problem-solving using basic operations and the use of logic in coordinate systems on the Cartesian plane.	Relationship between mathematics and programming: Mathematical language largely forms the basis of programming language.

Table 8. Category IV and its subcategories obtained from the information provided by the students

A priori category	Sub-categories			
IV. 21st century competencies	Programming makes learning mathematics easier by promoting the development of logical sequences and problem-solving.	Learning mathematics not only facilitates understanding of other subjects, but is also essential for study, work, future development, and technological development.	Computational competencies: Recognize the risks, dangers and benefits associated with the use of technology.	Coding contributes to the development of 21st century competencies, such as problem-solving and learning that focuses on creating and innovating through the use of technology.

because the teaching and learning process is not exclusive to the classroom (Alarcón et al., 2019).

Regarding the availability of these devices in public educational institutions, it may be limited–particularly in mathematics classes, where resources are often restricted to a classroom television and, in many cases, a personal computer owned and provided by the teacher.

Thus, after comparing the subcategories with previous literature, substantive theory 2 is defined as *Mathematics and electronic devices: A relationship motivated by learning and entertainment.*

Regarding category III and its subcategories (see **Table 7**), although programming for children is a little-explored topic in Colombia, some mathematics teachers interested in technology and in offering their students literacy in both mathematics and technology choose to be self-taught and instill this same interest in their students. For this reason, the use of tools such as Scratch and MakeCode, free software programs that can be used to foster the development of logical thinking, stand out. Through these programs, students complement their learning and develop problem-solving skills, which require the use of logic and, at this level of education, mastery of coordinate systems and, in general, the different numerical aspects associated with algorithm development. This coincides with the findings of Moreno León et al. (2021), who observe that students recognize the relationship between mathematics and programming when creating computer codes, where mathematical language becomes a fundamental symbolic system to communicate with machines (Cardona & Rodríguez, 2021).

This relationship is also strengthened in the family setting, because programming language learning is not only acquired at school with teachers; some parents and other family members foster curiosity and interest in programming at home from childhood, according to Pizarro Laborda et al. (2013), which is confirmed by the responses of some students who participated in the study. Consequently, after comparing the subcategories with previous literature, substantive theory 3 is defined as *Mathematics and programming: A combination of languages to learn to communicate with electronic devices from childhood.*

Regarding category IV and its subcategories (see **Table 8**), the evolution of societies has led to the creation of new languages that drive development. In particular, programming languages enhance the learning of areas such as mathematics, which are fundamental to science and technology (Sarmiento, 2020). Therefore, schools must teach literacy in these new languages that emerge from the mathematical field. In this sense, mathematics is essential for the development of a country, and its learning benefits everyday aspects of each individual. This implies that all citizens must have at least a basic command of this field, as well as of technology, to solve problems specific to the 21st century, which include creativity, teamwork, and critical thinking. In this research, students believe that programming facilitates the learning of mathematics by fostering the structure of sequences and patterns in practice, through the creation of programs.

According to Tejera et al. (2020), education in this century demands technological inclusion and, with it, the study of programming languages to solve problems and foster creativity and innovation. These skills, as Valencia and Penaqué (2019) point out, not only represent cognitive capital but also a great social and

Table 9. Category I and its subcategories obtained from the information provided by teachers

A priori category	Sub-categories			
I. Aspects to consider from the selection of content	Numerical thinking and the development of basic operations are fundamental aspects for fostering skills through problem-solving.	Most common difficulty: Problem-solving due to lack of understanding, making conjectures, and performing basic operations.	Topics associated with programming: Numerical thinking, the development of basic operations, algorithms, and the Cartesian plane are highlighted, all related to problem-solving in context.	Context exploration: Developing skills and abilities to understand and argue through logical thinking in problem-solving.

Table 10. Category II and its subcategories obtained from the information provided by teachers

A priori category	Sub-categories			
II. Using devices from a teacher's perspective.	Devices as tools to boost academic processes: Harnessing these elements in students' daily lives to develop mathematical competencies.	The devices facilitate the learning and development of mathematical competencies, such as the ability to create algorithms, design strategies, and recognize patterns.	Unplugged programming: The most widely used option among mathematics teachers due to institutional conditions.	Problem-solving: A relevant skill developed through programming.

cultural asset. Hence, it is important for schools to prepare students to face life in a context that responds to the needs of the current era. Therefore, after comparing the subcategories with previous literature, substantive theory 4 is defined as *21st century skills: A challenge to transform schools from the mathematical perspective.*

Results by Category from the Teachers' Perspectives

Regarding category I and its subcategories (see **Table 9**), teachers' opinions suggest that the priority in the mathematics content of cycle three is numerical thinking, due to its usefulness in students' daily lives, since it is through this that basic operations are developed. However, according to Vargas et al. (2020), basic operations are precisely the aspects in which students have the greatest difficulties, due to a lack of understanding of each mathematical operation, which generates problems when trying to solve exercises. Furthermore, the lack of understanding of problem situations prevents students from following a clear method for solving mathematical problems.

As a result, there is an absence of conjecture and constant review in the process. However, for Skovsmose (2000), the development of mathematical skills is not enough; students must interpret and make decisions based on mathematical knowledge in the face of social and political problems. Mathematics is not only used to make calculations, but is fundamental in the comprehensive education of citizens, allowing them to analyze and understand the environment in which they live (Alvis-Puentes et al., 2019). In this sense, mathematics must facilitate the exploration of the context, since it is the universal language and, in particular, the basis of the computer programming language. Through its symbols, a form of communication is established between humans and machines, which allows solving contextual needs through problem-solving, where logical thinking is developed through sequences and patterns.

Thus, after comparing the subcategories with previous literature, substantive theory 5 is defined as *Mathematics: A possibility of understanding the context through algorithms and problem-solving.*

Regarding category II and its subcategories (see **Table 10**), from the perspective of mathematics teachers in cycle three of basic education, electronic devices are key tools for the development of cognitive processes. However, for Mesa Agudelo (2012), their function goes beyond the academic, because they are elements that fulfill a social role of inclusion by allowing any citizen, regardless of age or economic status, to access educational opportunities and learning communities, where a clear pedagogical intention is highlighted.

Although devices are valuable tools for students, it is the students who provide critical thinking and solve problems using different strategies: Identifying patterns, applying logical thinking, and mathematical concepts (Rentería & Ayala, 2017). However, mathematics teachers have limited access to devices in their schools, therefore, to develop logical thinking through programming, they mostly resort to disconnected activities, while continuing to search for alternatives, amidst difficulties, to include connected programming in the mathematical field. The devices produce in students: "Good disposition, motivation, expectations, a certain degree of curiosity to explore and learn, at the

Table 11. Category III and its subcategories obtained from the information provided by teachers

A priori category	Sub-categories			
III. Didactic and motivational approach	Mathematics didactics: An approach based on the teacher's knowledge of his or her students.	Building environments and resources that facilitate educational processes: The role of didactics.	The role of technology: A resource to motivate learning in face-to-face and virtual environments.	Play: The most effective way to motivate students, whether through physical activities or the use of electronic devices.

Table 12. Category IV and its subcategories obtained from the information provided by teachers

A priori category	Sub-categories			
IV. Educational tools required by the teacher	Electronic devices are educational tools, especially the computer and the television.	Educational tools: The axis of cooperative work.	Mathematics teachers and electronic devices: A relationship with limited access.	The use of ICTs enhances teaching practice by empowering educators and stimulating computational thinking.

same time generating a certain degree of collaboration and support among peers, which was reflected in their desire to manipulate, analyze, and share activities" (Rentería & Ayala, 2017, p. 96). These aspects favor collaborative learning in mathematics.

Thus, after comparing the subcategories with previous literature, substantive theory 6 is defined as *Electronic devices: Teaching tools for learning mathematical skills from childhood.*

Regarding category III and its subcategories (see **Table 11**), the interviewed teachers from cycle three indicate that, in order to transmit knowledge and regulate their practice, they must understand the main characteristics of their students, as well as the context in which they operate. This, in most cases, limits access to the teaching resources they can use to motivate students (Pardo et al., 2020).

According to Hernández Sampieri et al. (2019), the importance of teaching resources lies in the fact that they improve mental calculations, establish problem-solving strategies, and develop a better understanding of mathematics. Among the most relevant resources for teachers are those related to technology, because they consider them a means to motivate students in their learning process, both inside and outside the classroom. In particular, games are of great importance, since through them, skills are developed and acquired knowledge is put into practice, either physically or virtually through challenges designed according to the students' level. Consequently, for Hernández Sampieri et al. (2019), the task of mathematics teachers is to integrate technology into their praxis, thus contributing to the training of students capable of facing a highly technological world, which is currently the case for almost everyone.

Furthermore, from the teachers' perspective, a key strategy is the use of errors as a teaching resource to promote meaningful learning in mathematics. The family can also be used as a teaching strategy to reinforce learning acquired in class through recreational activities related to the topics covered in school. In this way, parents, by actively participating in their children's education, not only guide the topics but also become learning subjects, teaching and learning alongside their children the content covered in their school.

Therefore, after contrasting the subcategories with previous literature, substantive theory 7 is defined as *Technology: A necessity for didactic construction in the mathematical field.*

Regarding category IV and its subcategories (see **Table 12**), it was found that, in the schools studied, technological tools for mathematics teachers are limited mainly to the use of television and computers. This is because electronic devices are intended for computer science, and due to the large number of students, they are generally used by the teachers of that subject.

As a result, access to these tools for mathematics teachers is limited, leaving them dependent on the collaboration of their computer science colleagues. However, according to Vaillant et al. (2020), the most significant problem lies in the techno-pedagogical training necessary to learn how to use tools and platforms with a pedagogical focus in the area of mathematics. For teachers, this enriches their practice, allowing them to develop not only mathematical processes but also computational ones, which helps students connect both fields and feel motivated by strengthening cognitive, affective, and social skills (Maestre & Ávila, 2022). Although limited access to devices can be a factor that hinders teacher's empowerment in digital skills and their use to promote 21st century skills in creative and innovative ways, it is essential to train students with critical thinking skills, capable of solving problems in collaboration with their educational community. In this regard, it is important to highlight that the teachers interviewed have shown motivation to pursue training through courses offered by the District Education Secretariat and the Ministry of

Table 13. Category V and its subcategories obtained from the information provided by teachers

A priori category	Sub-categories			
V. Aspects to consider from the perspective of 21st century competencies	Solving problems in specific contexts through the use of technology: A fundamental competence for the 21st century.	Health, water resources, pollution, basic food baskets, and climate change: Issues relevant to the development of mathematical skills with a critical approach.	Mastery of computational tools and the ability to discern in problem-solving.	The development of skills and abilities related to the way of thinking.

National Education, in addition to being self-taught in this area, due to the limited technological training they received in their educational programs.

Consequently, after comparing the subcategories with previous literature, substantive theory 8 is defined as *Electronic devices: Teaching tools that enrich teaching work.*

With respect to category V and its subcategories (see **Table 13**), the teachers interviewed seek to be at the forefront in the use of technology to train competent students, capable of facing the challenges of the 21st century, who manage to solve problems in the context of a society immersed in technology. Their objective is to instruct not only for the present, but also for the future. In the case of programming, according to Machado and Carrascal (2020), the purpose is not to train children as programmers, but rather to use this strategy to provide students with tools that allow them to develop complex mental schemes. This helps to organize in a structured way the information and knowledge they receive from their environment, and to solve problems that, for the teachers studied, are of greater relevance such as health, water resources, pollution, consumption of public services, basic food basket, and climate change. Which coincides with what Machado and Carrascal (2020) propose, who affirm that training should focus on economic and financial aspects, because they impact daily life and would allow citizens to make sound decisions, mediated by reflection.

From the teachers' perspective, ongoing training is essential to address environmental issues using new technologies in order to be competent in today's classroom. This requires mastery not only of their subject content, but also the use of computational tools and the ability to discern problems specific to their work and those that address the system of professional competencies. For this reason, the mathematics teachers surveyed believe that competencies should focus on the way of thinking, which implies fostering creativity and innovation, problem-solving, and critical thinking. Therefore, after comparing the subcategories with previous literature, substantive theory 9 is defined as *The use of computational tools: A 21st century competency for solving contextual problems through mathematics.*

DISCUSSION

Substantive theories show that students recognize the usefulness of mathematics in daily life, highlighting the importance of numerical thinking and basic operations as key factors in solving problem situations in their immediate environment. However, they perceive certain algorithms as difficult, which generates emotions such as fear, anger, sadness, and stress regarding this discipline. This finding is consistent with Vicente and Barroso (2019), who consider it interesting to delve into the emotional aspect of mathematical learning, as it influences academic performance and can transform learning into a more enriching, practical, and effective experience. Therefore, it is essential to establish educational parameters that take the emotional dimension into account in order to develop mathematical competencies and education from a holistic perspective.

For students, incorporating electronic devices into their learning is motivating because it allows them to access explanatory videos, interact with their peers in extracurricular settings, and engage in academic activities, in addition to serving as a means of entertainment. However, the authors agree with Simó et al. (2020) that the important thing is to know "when, how, and why" to use these digital tools. On the other hand, students consider mathematical language to be essential in programming because numbers are essential for generating coordinate systems and solving problems that require various algorithms. They also recognize that programming requires mathematics, but at the same time strengthens learning in this area through logical thinking. This perception coincides with Briz and Serrano (2018), who affirm that programming can equip students with greater capacity for logical reasoning, structured thinking, and greater imagination.

Being a mathematics teacher today involves a constant search for teaching theories that integrate the use of electronic devices, which students interact with on a daily basis. This also requires self-directed learning, as the Colombian basic education curriculum does not include teaching mathematical processes through literacy in new languages, such as programming. In this way, it seeks that the school respond to the needs of the context and address its problems in a critical way.

However, given that the curriculum must be flexible, teacher initiative is important.

Regarding problem-solving, this is considered the main theme in the relationship between mathematics and programming, because mathematical language constitutes a large part of the development of computer programming in languages such as Scratch and MakeCode, which were used in the development of the research. With them, it is possible to design sequences, establish patterns, develop and verify algorithms, in addition to locating oneself in a given space through the use of a coordinate system. As reaffirmed by Ye et al. (2024), Scratch can function as a creative learning environment and a tool that facilitates the experience of mathematics teachers when doing mathematics. However, mathematics teachers do not always have direct access to electronic devices, so disconnected programming is considered the most viable alternative, given the conditions of the studied population, coinciding with what was stated by Hu and Wang (2024).

It is interesting to note that teacher's commitment involves exploring alternatives that enrich their pedagogical knowledge and promote educational processes that recognize 21st century competencies, as well as the importance of addressing the challenges of this century from childhood. Therefore, studying the development of logical thinking in different contexts through programming for children and developing teaching strategies could contribute to the learning of mathematics in current and future students.

CONCLUSIONS

This research aimed to propose substantive theories through the application of grounded theory, using a methodology appropriate for investigating qualitative data. In this context, the incorporation of technological tools into mathematics teaching enriches the understanding of key concepts and offers new possibilities for learning.

The scope of the research was limited to defining substantive theories. However, it is hoped that by reflecting on these theories, a grounded theory can be proposed in the future. This will encourage mathematics teachers especially in contexts like Colombia, where there is no curriculum that fosters the development of logical thinking through programming for children or the use of various electronic devices for pedagogical purposes, to create innovative strategies that motivate current mathematics learning, enabling students to solve contextual problems using the technological resources with which they interact.

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