

Python-based simulations of the probabilistic behavior of random events for secondary school students

Supot Seebut^{1*} , Patcharee Wongsason¹ , Dojin Kim² , Thanin Putjuso³ , Chawalit Boonpok⁴ 

¹ Department of Mathematics Statistics and Computers, Ubon Ratchathani University, Ubon Ratchathani, THAILAND

² Department of Mathematics, Dongguk University, Seoul, SOUTH KOREA

³ School of General Science, Rajamangala University of Technology Rattanakosin, Prachaub Khiri Khan, THAILAND

⁴ Department of Mathematics, Mahasarakham University, Mahasarakham, THAILAND

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Abstract

Simulation modeling is an effective tool for solving problems that cannot be explained analytically or when data cannot be collected. This is done by simulating the observed behavior of a problem under study using a computer program. In math education, this can develop knowledge and fundamental competencies of simulation modeling at a higher level and foster its applications in everyday life. This study created learning activities for secondary school students to simulate the probabilistic behavior of random events using Python. 28 grade 12 students took part in these activities using appropriate scaffolding strategies and a powerful mathematical tool, Python. After completing the activities, student competency in simulating the probabilistic behavior of random events with Python was evaluated using rubrics and the factors of student enjoyment, perceived value, interest, and self-efficacy were determined through a Likert-scale questionnaire. The assessment results showed that the activities had a positive effect on student competencies and emotions. The outcomes of the study can serve guidelines for teachers who are interested in expanding the results for further student development.

Keywords: mathematical modeling, simulating probabilistic behavior, simulation, simulation modeling

INTRODUCTION

Mathematical model is the application of mathematics to explain observable phenomena. If a mathematical model accurately describes or represents the phenomenon, it can be extraordinarily useful in any field. Consequently, research is being conducted to investigate the learning management process in mathematical modeling. This endeavor began with research on mathematical modeling in elementary (English, 2012; Kazak et al., 2018; Patel & Pfannkuch, 2018; Shahbari & Peled, 2017) and secondary schools (Balakrishnan et al., 2010; Krutikhina et al., 2018; Stillman, 2010). The objective is to develop the learners' ability to create mathematical models that solve everyday problems (Blum & Leiß, 2006; Galbraith et al., 2020; Hartmann et al., 2021; Schukajlow et al., 2015b; Wake, 2015), as well as developing student competency

in creating mathematical models of various types. Such models could be deterministic (Fariyah, 2019; Ortega & Puig, 2017; Yanagimoto & Yoshimura, 2013), probabilistic (Frejd & Årlebäck, 2017; Greefrath & Siller, 2017; Kazak, 2010; Kotelawala, 2011), or dynamic (Blomhøj, 2020; Kaiser et al., 2011; Leung, 2013; Rodríguez, 2015). This leads to proficiency in applying specific solutions that correspond to the type of model in question and provide a foundation for studying mathematical models at a higher level.

Simulation modeling is another fascinating modeling technique. It is crucial in real-world situations where the behavior of the problem cannot be explained analytically or where data cannot be collected directly. This is because analyzing such real-world scenarios is difficult, expensive, and time-consuming. Using the appropriate computer technique, modelers create simulations that

Contribution to the literature

- A simulation model is an important specific mathematical model for dealing with contextual problems when the problem cannot be explained analytically or data cannot be directly collected .
- The development of subject-specific competencies in simulating the probabilistic behavior of random events with Python is one approach to introducing students to simulation modeling. This serves as basic knowledge and competency to learn at a higher level and to apply these skills in everyday life.
- Student competency in simulating the probabilistic behavior of random events with Python was enhanced through the development of specific learning activities.

mimic the behavior observed in actual problems. The outcomes of the simulation study should then be used to address the appropriate issues (Kin & Chan, 2011; Maria, 1997; Tobrawa et al., 2022; Volosencu & Ryoo, 2022; Zárate Ceballos et al., 2021). Previously, simulations had to be done manually, which was inefficient. However, currently there are a variety of computer programs available for simulation. This simplifies model development. Therefore, computer simulation for behavioral modeling is widely used (Albright & Fox, 2019; Fox, 2013; Gordon & Guilfoos, 2017).

It was discovered, however, that there were not many school-level learning management systems that prepared students with a foundational competency in simulation modeling. Students may participate in simulations of the probabilistic behavior of random events, if deemed appropriate. It is highly likely that such simulations will serve as a starting point for developing fundamental competency in simulation modeling (Giordano et al., 2013). This is because simulating the probabilistic behavior of random events is an important foundation for simulation modeling. It is linked to the probability content that students have already studied (Giordano et al., 2013).

Another factor to consider when deciding to simulate the probability behavior of random events is the computer program that will be used as a simulation tool. Learning management research seeks to use a free program to make it easier for those interested in research findings to apply them in the classroom. Python is a powerful mathematical tool that can be used in simulations (Gordon & Guilfoos, 2017; Liu, 2020; Tendeloo et al., 2019). This program is unique in that it is free and has many useful features. It is convenient to use without the need to install programs on the computer, while the code and results are automatically saved in cloud storage. Since Python can run online programs on a variety of devices, including smartphones, tablets, portable computers, and personal computers, its use in learning activities based on simulating the probability behavior of random events improves learning management.

Moreover, the inclusion of Python in this learning environment has alleviated some of the common criticisms of using the program, which is that lesson designers often create automated instructions for

learners to follow to get results, which affect the learning efficacy of the learners relatively little. Although it is convenient to use and has many functions to choose from, the process of using it is not an automatic button press, but users will have to start by analyzing problems that require the use of Python in ways that help find answers. It converts problems into mathematical and logical forms. Use the mathematical and logical models created to write an algorithm, such as pseudo-code, are necessary before implementing the actual program code. Running the output that will bring back the original problem description. This, in addition to promoting on-demand competence, is also an important motivation for developing the initiative to adopt mathematics in a STEM-oriented manner, enhancing mathematical and digital literacy and fostering computational competence.

This study developed learning activities for secondary school students that simulate the probabilistic behavior of random events to provide them with the knowledge and basic competency required for more advanced studies in simulation modeling and applications to real-world problems. Following completion of an activity, student competency in simulating the probabilistic behavior of random events was evaluated using a rubric whose scores could be analyzed and interpreted both quantitatively and qualitatively. Furthermore, a Likert-scale questionnaire was used to evaluate student enjoyment, values, interests, and self-efficacy. The evaluation results will be used to improve the activities, making them more beneficial and applicable to those who want to apply learner development in other contexts.

SIMULATING THE PROBABILISTIC BEHAVIOR OF RANDOM EVENTS USING PYTHON

Monte Carlo simulation is a popular and well-known technique that generates results based on the probability of an event. The probability of each event is proportional to its likelihood. Simulations are run using a random sampling method, but theoretical sampling based on random numbers is used instead of sampling actual data. In this randomization, the data distribution is sampled to match or closely resemble that of the actual data in the problem scenario. Random events that

Table 1. Activities for learning about simulation of probabilistic behavior of random events. Each activity yields a number of occurrences and a probabilistic proportion of each outcome of the simulation

Simulation activity	Description	Type of activity
 Tossing dice	Create a simulation of the probabilistic behavior of tossing a die using Python.	Demonstration by a teacher
 Flipping two coins	Create a simulation of the probabilistic behavior of flipping two coins with Python.	Demonstration by a teacher
 Tossing a coin & a die	Create a simulation of the probabilistic behavior of tossing a coin and a die with Python.	Teachers led students to practice.
 Determining a random event	Create a simulation of the probabilistic behavior of random events as determined by each group of students with Python.	Group activity
 Picking a card	Create a simulation of the probabilistic behavior of randomly picking a card numbered 0-9 with Python.	Individual activity
 Flipping three coins	Create a simulation of the probabilistic behavior of flipping three coins with Python.	Evaluation of competency in simulating probabilistic behavior of random events with Python

students have already studied are highlighted for the problem situations in this activity.

The activities listed in **Table 1** were designed for students based on a previously presented academic concept of simulating the probabilistic behavior of random events (Albright & Fox, 2019; Fox, 2013; Giordano et al., 2013; Gordon & Guilfoos, 2017).

It is necessary to describe how to simulate the probabilistic behavior of random events. A simulation of simultaneously flipping two coins can be done according to the steps listed below. The sample space for random occurrences can be described as

$$S = \{HH, HT, TH, TT\}.$$

A sample space is often required that has two possible random outcomes with equal probability. This is referred to as a “fair coin” random event that simulates a coin toss. The function, $f(x)$ below, is similar. However, it allows for four outcomes, each with a probability of 0.25. Consider the set S to create a simulated function as

```

Simulation algorithm of flipping two coins
Input  Number of simulations
Output The number and probability of each sample point
Start
Step 1 Set counterHH = 0, counterHT = 0, counterTH = 0 counterTT = 0
Step 2 For i = 1,2, ..., n do step 3 and 4
    Step 3 Generate a random number corresponding to 0 ≤ xi ≤ 1
    Step 4 If xi is in these ranges and add the appropriate counter value
        0 ≤ xi ≤ 1/4    counterHH = counterHH + 1
        1/4 ≤ xi ≤ 1/2  counterHT = counterHT + 1
        1/2 ≤ xi ≤ 3/4  counterTH = counterTH + 1
        3/4 ≤ xi ≤ 1    counterTT = counterTT + 1
Step 5 Compute
    P({HH}) = counterHH/n    P({TH}) = counterTH/n
    P({HT}) = counterHT/n    P({TT}) = counterTT/n
Step 6 Displays the number and probabilities of each sample point.
STOP
    
```

Figure 1. An algorithm for simulating a simultaneous flip of two coins

$$f(x) = \begin{cases} HH, 0 \leq x < 1/4 \\ HT, 1/4 \leq x < 1/2 \\ TH, 1/2 \leq x < 3/4 \\ TT, 3/4 \leq x \leq 1 \end{cases}$$

Then apply the simulation function to write the simulation algorithm as follows in **Figure 1**.

```

from random import seed
from random import random
x=float
y=str
q=[]
d=[]
countHH=0
countHT=0
countTH=0
countTT=0
print ('Simulating two fair coins flipping')
print ('Please specify how many times you want.')
```

`n=int (input ())`

```

for i in range (1, n+1):
    x=float (random ())
    if 0<=x<1/4:
        y="HH"
        countHH=countHH+1
    elif 1/4<=x<2/4:
        y="HT"
        countHT=countHT+1
    elif 2/4<=x<3/4:
        y="TH"
        countTH=countTH+1
    else:
        y="TT"
        countTT=countTT+1
    q.append(y)
    d.append(i)
print ('No.' ' Outcomes')
```

`for i in range (0, n):`

```

    print (d[i], '\t', q[i])
print ('Number of HH =', countHH)
print ('P({HH})=', round(countHH/n, 5))
print ('Number of HT=', countHT)
print ('P({HT})=', round(countHT/n, 5))
print ('Number of TH=', countTH)
print ('P({TH})=', round(countTH/n, 5))
print ('Number of TT=', countTT)
print ('P({TT})=', round(countTT/n, 5))
```

Figure 2. Python code for a computer simulation of flipping two coins

During the simulation, Python commands can be used for simulation as shown in **Figure 2**.

After executing the program, the following simulation results in **Figure 3** for a two-coin flip can be obtained.

From this example, the sub-competencies in simulating the probabilistic behavior of random events with Python are analyzed and summarized in **Figure 4**.

SCAFFOLDING

Since students are unfamiliar with the learning activities that involve simulating the probabilistic behavior of random events, the instructor must incorporate a support process into the learning management activities to increase student abilities to achieve the set goals. Scaffolding is an effective learning management technique when students must learn something new or difficult. We can help our students help one another by demonstrating, acting out, or using prompting questions to promote learning. A rescue goal is set to help learners who are unable to work alone at first, until they are able to finish a task independently.

```

Simulating two fair coins being flipped
Please specify how many times you want to flip coins: 20
No. Outcomes
1 HT
2 HT
3 TT
4 TT
5 HH
6 TT
7 TH
8 HH
9 TH
10 TH
11 HH
12 TT
13 HH
14 TH
15 TH
16 TT
17 HT
18 HH
19 HH
20 HH
Number of HH = 7
P({HH})= 0.35
Number of HT= 3
P({HT})= 0.15
Number of TH= 5
P({TH})= 0.25
Number of TT= 5
P({TT})= 0.25
```

Figure 3. Simulation results of two coins being simultaneously flipped

When learners gradually improve their ability to complete activities on their own, the complementary learning process shifts and declines (Schukajlow et al., 2015a; Stender & Kaiser, 2015; Tropper et al., 2015). A scaffolding process for learning activities to simulate the probabilistic behavior of random events can be thought of as discussed below.

Classroom Scaffolding

Classroom scaffolding involves simultaneously augmenting learning by everyone in the classroom. The instructor explains the concepts and procedures for simulating the probabilistic behavior of random events with Python. Then, he demonstrates how to use Python to simulate probabilistic behavior and walks learners through the process. Learning is arranged utilizing a Facebook group as a host of the M30225 Modeling technology course while performing research on the spread of COVID-19. After all students have joined the Facebook page, they are able to get news and crucial information regarding the courses here. Instructors post instructional materials and a link to a Zoom classroom on the Facebook group as part of the learning management process. The teacher then administers the course using Zoom, which makes emulating the Python process simple using Google Collaboratory. While the teacher instructs, shows, and guides the students in practice, every phase of simulating the probabilistic behavior of random events with Python is recorded from Zoom. The teacher posts the recorded video on the Facebook group for learners to review after the session.

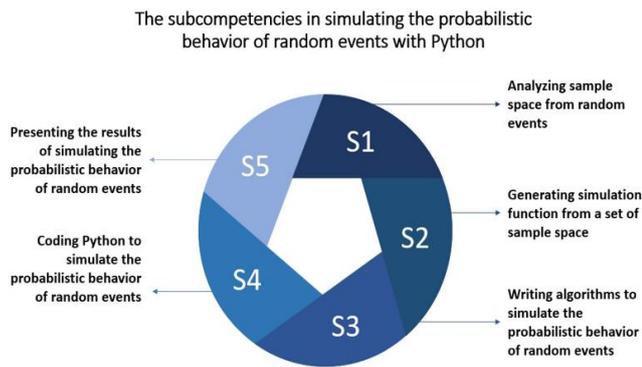


Figure 4. The process of simulating probabilistic behavior of a random event and the sub-competencies required

Group Scaffolding

Group scaffolding involves augmenting learning through group activities. Here, the teacher guides groups of learners. The outcome of this cooperative learning exercise will be shared so that students may learn together. Instructors publish instructional materials and post a Zoom classroom link on the Facebook group, after which he runs the course using Zoom. An instructor introduces students to objects that behave in a probabilistic manner such as coins, dice, and cards. The learners are placed in groups in the Zoom break-out room, each with its own set of probabilistic situations to simulate. Probabilistic media is supplied by the instructor or other probabilistic media encountered by the learner is used to design a simulation. Students show their work via video clips and submit it using Google forms, which the instructor then posts to the Facebook group. Then, he fixes the presentation footage and posts it to the Facebook group for everyone to learn. Students work in the Zoom break-out room to define a random event, create a sample space, and create simulation functions during this portion of the learning process. Writing a simulation method, coding it in Python for simulation, and generating video presentations are all outside-of-class activities.

Individual Scaffolding

Individual scaffolding is intended to supplement the learning of individuals in this setting. Since the students have previously learned about simulating the probabilistic behavior of random events with Python, the instructor is prepared to examine each student individually in this step to determine which students require additional instruction. A small number of students will require assistance at some point during the simulation, so the teacher will personally assist them. Instructional materials are posted as a link to the Zoom classroom in the Facebook group as part of the learning management process. After that, instructors schedule their lessons through Zoom. Probabilistic behavioral occurrences are identified. Then, using Python, each student should create a method for simulating the

probabilistic behavior of a given random event. When they finish, they present video recordings of worksheets based on the instructor's assignments and submit them via a Google form that includes a link to submit the assignment to the Facebook group. After that, he goes over the work one-on-one with students to provide feedback and information. In this section, learners use Zoom to complete the learning process, which includes designing a random event, building a sample space, and developing a simulation function. Throughout the lesson, students request additional assistance via Facebook Messenger.

Evaluation Criteria

Considering the previously presented academic concept of assessing modeling competency (Ferri, 2017; Hidayat et al., 2022; Lingefj rd & Holmquist, 2005), an assessment framework demonstrating competency and student thinking is proposed for simulating the probabilistic behavior of random events with Python. At each level of the procedure, there should be evidence of competency and critical thinking. Evaluation is separated into five sub-competency phases, as follows:

1. **S1:** Competency in analyzing sample space of random events.
2. **S2:** Competency in generating simulation functions from a sample space.
3. **S3:** Competency in writing algorithms to simulate the probabilistic behavior of random events.
4. **S4:** Competency in coding Python to simulate the probabilistic behavior of random events.
5. **S5:** Competency in presenting the results of simulating the probabilistic behavior of random events.

Since this is an evaluation of an activity, rubric scoring is used, which reflects each student's level of competency based on their performance. A Likert-scale questionnaire was employed as an instrument for analyzing the enhancement of enjoyment, perception of value, interest, and self-efficacy of each learner. Based on the previously presented academic concepts (Krawitz & Schukajlow, 2018), four questions were used to summarize these factors as follows:

1. **Q1:** I enjoy solving the problem of simulating the probabilistic behavior of random events with Python.
2. **Q2:** I think it is important to be able to solve problems by simulating the probabilistic behavior of random events with Python.
3. **Q3:** It would be interesting to solve problems of simulating the probabilistic behavior of random events with Python.
4. **Q4:** I am confident that I can solve problems of simulating the probabilistic behavior of random events with Python.

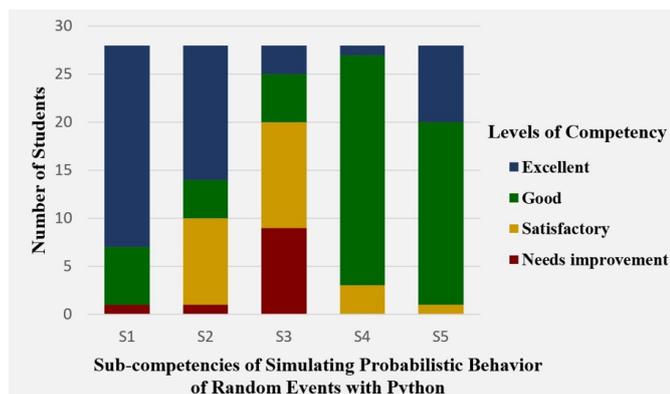


Figure 5. Number of students at each competency level for each sub-competency of simulating probabilistic behavior of random events with Python

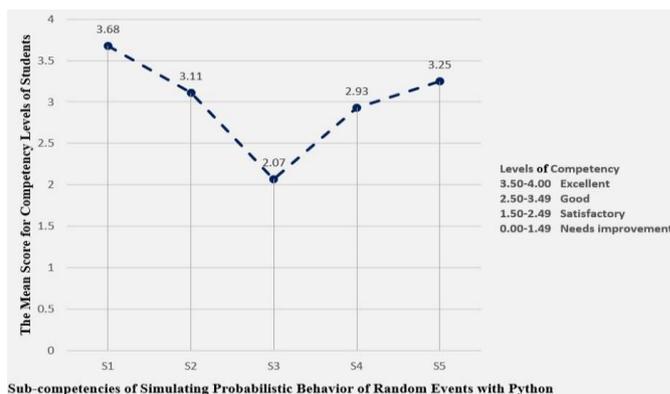


Figure 6. Mean score for each sub-competency of simulating probability behavior of random events with Python

RESEARCH METHODOLOGY

Participants in the Research

The participants in this investigation were secondary school students who were enrolled in a science classroom in a university-affiliated school project. There was a total of 28 students in grade 12 who were between the ages of 17 and 18 years old.

Research Instruments

The research tools used in the current study included three instruments. The first was a learning activity plan for simulating the probabilistic behavior of random events with Python. A competency test with an evaluation rubric criterion was used for simulating the probabilistic behavior of random events with Python. Finally, a Likert-scale questionnaire was employed to assess the enjoyment, perceived value, interest, and self-efficacy of the activities.

Methods of Data Analysis

Our approach to data analysis uses the results of a rubric assessment of competency in simulating the probabilistic behavior of random events with Python. The frequency of students at each level of competency for each sub-competency is presented. Students' mean scores are used to summarize the overall level of competency in each sub-competency. The mean scores and standard deviations from the Likert-scale questionnaire are presented to determine the impact of the activity on learner enjoyment, perceived value, interest, and self-efficacy.

Procedure

The study started with developing and preparing research tools. This was followed by preparing the target audience, running the activity by simulating the probabilistic behavior of random events with Python with the target audience according to the scaffolding

strategy. Finally, the results were evaluated and analyzed to draw conclusions.

RESULTS AND DISCUSSION

The following results of a competency test and questionnaire evaluation are presented. This is to establish that the developed activities can enhance learner competency in simulating probabilistic behaviors of random events with Python in a way that students find enjoyable, valuable, interesting, and supportive of their self-efficacy. Students were evaluated in their ability to simulate probabilistic behavior of random events with Python. They simulated the probabilistic behavior of a three-coin toss to determine the probability of all three tosses being heads. The rubric evaluation results can distinguish the level of competency for each sub-competency, as depicted in **Figure 5**.

From **Figure 5**, it can be surmised that the competency assessment of S1 showed that most students had excellent competency, followed by good competency, while the satisfactory and needs improvement levels were almost nonexistent. As for S2, most of the students had excellent competency, followed by the good level, with a small number of students at the satisfactory and needs improvement levels. For S3, many students were at the needs improvement and satisfactory level, while only a few were good or excellent. In S4, almost all students were at the good level. A few students were at excellent and satisfactory levels. Finally, in S5, the majority of students were at the good level, followed by excellent, with only a few students scoring satisfactory and none scoring needs improvement.

The scores generated by individual rubrics can be used to provide an overview of the student competency in each of the sub-competencies needed for simulating the probabilistic behavior of random events with Python, as shown in **Figure 6**.

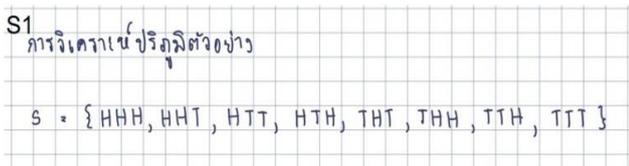


Figure 7. A graphical representation of a sub-competency in analyzing a sample space for random events

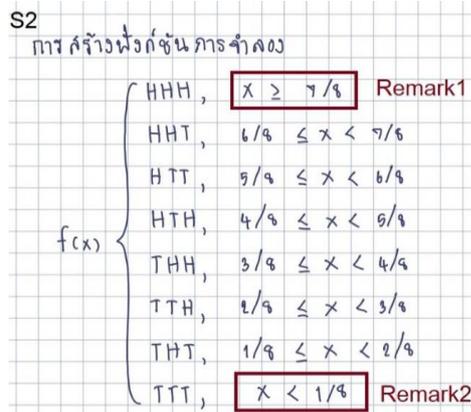


Figure 8. A schematic representation of sub-competency in generating a simulation function from a set of sample spaces

Figure 6 shows that the mean sub-competency scores for S1 were 3.68. Those for S2, S4, and S5 were 3.11, 2.93, and 3.25, respectively. For S3, the mean was 2.07. Based on the mean scores, the overall level of learner competency in each sub competency for simulating the probabilistic behavior of random events with Python could be described as follows. The competency in analyzing sample spaces of random events was excellent while that for generating simulation functions from a set of sample spaces was good. Writing algorithms to simulate the probabilistic behavior of random events was satisfactory and competency in coding Python to simulate the probabilistic behavior of random events was good. Finally, the competency in presenting the results of random events was good.

Figure 7 depicts an illustration of the outcomes of analyzing a sample space from random events. The letter H depicts an instance of a heads coin toss while T represents tails. The student's work in the figure is completely accurate. In accordance with the criteria of the rubric, the assessment results were given a score of 4, which corresponds to the level of excellent.

Figure 8 depicts an example of the output of generating a simulation function from a set of sample spaces. It is a satisfactory level for student work because assigning $7/8 \leq x < 1$ values of the HHH simulation function based on remark 1 may result in errors since the simulation function cannot produce a result at $x=1$. So, it should be revised to $7/8 \leq x \leq 1$, and for the same reason, $x < 1/8$ should be revised to $0 \leq x \leq 1/8$ in remark 2. As a result, its score is 2 according to the rubric criteria.

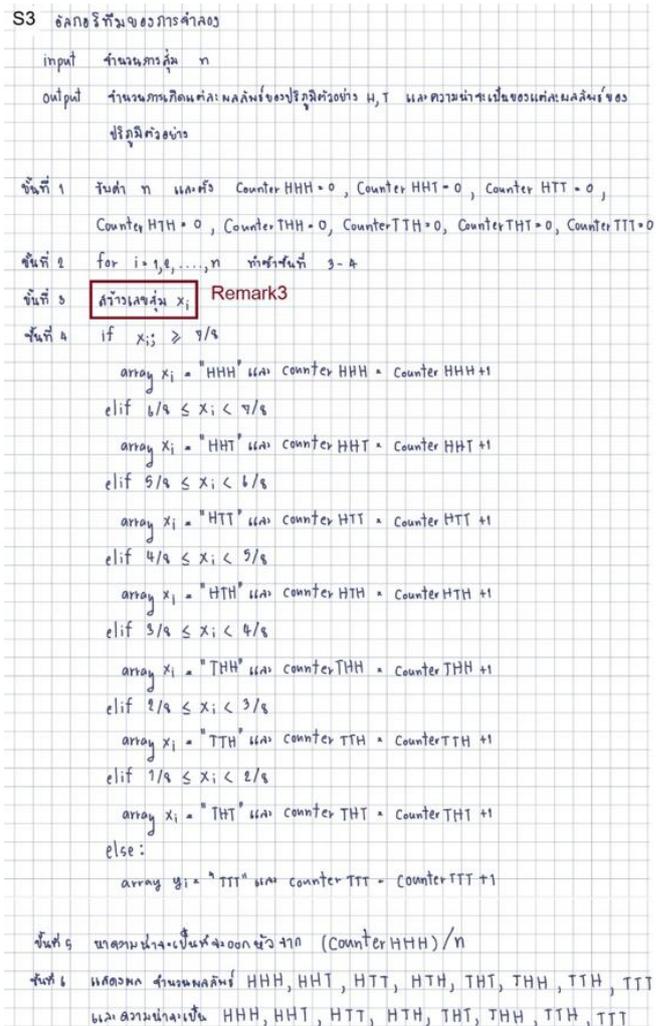


Figure 9. A schematic representation of the sub-competency in writing algorithms to simulate the probabilistic behavior of random events

However, the majority of the student assessments were at a good level. This reflection is presented as a guideline for making suggestions for improving student performance.

Figure 9 depicts an example of the outcomes of writing algorithms to simulate the probabilistic behavior of a random event. Considering remark 3, the algorithm is written in steps ordered to generate a random number x_i but lacks a specified range for x_i . This should be revised in step 3 to generate a random number, x_i , where $x_i \in [0, 1]$ evaluates the result. Its score is 2 points on according to the rubric, which is a satisfactory level. This is due to a lack of explainability indicating that additional practice and development is required. Aside from that, students are unaware of the significance of writing algorithms. They are unaware that a good algorithm can be used to develop other programming languages and improve programming skill. As a result, there is a lack of attention to detail in writing good quality algorithms. When teachers bring the importance of writing good algorithms to the attention of students, it should lead to development of improved algorithms.

S4 ตัวอย่างโปรแกรม

```

from random import seed
from random import random
x=float
y=str
counterHHH=0
counterHHT=0
counterHTT=0
counterTHH=0
counterTTH=0
counterTHT=0
counterTTT=0
q=[]
d=[]
print('การจำลองการโยนเหรียญสามครั้ง')
print('จำนวนครั้งที่โยนเหรียญ')
n=int(input())
for i in range(0,n):
    x=float(random())
    if x>=7/8:
        Remark4
        y="HHH"
        counterHHH=counterHHH+1
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 6/8<=x<7/8:
        y="HHT"
        counterHHH=counterHHH
        counterHHT=counterHHT+1
        counterHTT=counterHTT
        counterTHH=counterTHH
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 5/8<=x<6/8:
        y="HTT"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT+1
        counterTHH=counterTHH
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 4/8<=x<5/8:
        y="HTH"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH+1
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 3/8<=x<4/8:
        y="THH"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH+1
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 2/8<=x<3/8:
        y="TTH"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH
        counterTTH=counterTTH+1
        counterTHT=counterTHT
        counterTTT=counterTTT
    elif 1/8<=x<2/8:
        y="THT"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH
        counterTTH=counterTTH
        counterTHT=counterTHT+1
        counterTTT=counterTTT
    else:
        y="TTT"
        counterHHH=counterHHH
        counterHHT=counterHHT
        counterHTT=counterHTT
        counterTHH=counterTHH
        counterTTH=counterTTH
        counterTHT=counterTHT
        counterTTT=counterTTT+1
    q.append(y)
    d.append(i)
print('ครั้งที่','\t','ผลลัพธ์')
for i in range(1,n):
    print(d[i],'\t',q[i])
print('จำนวน HHH ที่เกิด=',counterHHH)
print('ความน่าจะเป็นของการเกิดหัว 3 ครั้ง=',counterHHH/n)
print('จำนวน HHT,HTH และ THH ที่เกิด=',counterHHT+counterHTH+counterTHH)
print('ความน่าจะเป็นของการเกิดหัว 2 ครั้ง=',counterHHT+counterHTH+counterTHH/n)
print('จำนวน HTT,TTH และ THT ที่เกิด=',counterHTT+counterTTH+counterTHT)
print('ความน่าจะเป็นของการเกิดหัว 1 ครั้ง=',counterHTT+counterTTH+counterTHT/n)
print('จำนวน TTT ที่เกิด=',counterTTT)
print('ความน่าจะเป็นของการเกิดหัว 0 ครั้ง=',counterTTT/n)
    
```

Remark5

Remark6

Figure 10. A schematic representation of the sub-competency of coding Python to simulate the probabilistic behavior of random events

S5 ผลลัพธ์ ข้อสรุป

```

การจำลองการโยนเหรียญสามครั้ง
จำนวนครั้งที่โยนเหรียญ
ครั้งที่ ผลลัพธ์
1 TTT
2 THT
3 THT
4 THH
5 HTH
6 THH
7 HTT
8 HTT
9 HTT
10 TTH
11 HHT
12 HHT
13 HHT
14 THH
15 HTH
16 HTT
17 HHH
18 TTH
19 HHT
จำนวน HHH ที่เกิด= 1
ความน่าจะเป็นของการเกิดหัว 3 ครั้ง= 0.05
จำนวน HHT,HTH และ THH ที่เกิด= 8
ความน่าจะเป็นของการเกิดหัว 2 ครั้ง= 5.15
จำนวน HTT,TTH และ THT ที่เกิด= 10
ความน่าจะเป็นของการเกิดหัว 1 ครั้ง= 8.1
จำนวน TTT ที่เกิด= 1
ความน่าจะเป็นของการเกิดหัว 0 ครั้ง= 0.05
    
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สรุป

พบว่าเมื่อทำการจำลองการโยนเหรียญสามครั้ง 20 ครั้ง พบว่า
 จำนวนครั้งที่เกิดพร้อมกันหัว 3 เหรียญ อยู่ 1 ครั้ง แสดงความ
 น่าจะเป็นของการเกิดหัว 3 ครั้งเท่ากับ 0.05

Figure 11. A schematic representation of sub-competency in presenting results of simulating the probabilistic behavior of random events

that the simulation sequence is 1, 2, 3, ..., n. The statement “y=“THT” and “counterTHT=counterTHT+1” can be left in the remark 5 condition. The other positions in the if ..., elif ..., and else conditions can be modified in the same way. Finally, the position of Remark 6 should be changed from (1, n) to (1, n+1) because (1, n) will execute loops 1, 2, 3, ..., n-1, failing to execute that last cycle. However, this is a minor error. As before, when we simulate large n values, better simulation results are achieved. This is completely an aesthetic factor, so, the assessment results in a score of 3 points according to the rubric, which is at a good level.

Figure 11 depicts an example of presenting the results of simulating the probabilistic behavior of a random event. Remark 7 reveals that students used only 20 simulations to reach a conclusion. A greater number of simulations with large n values will result in better results. As a consequence, the assessment score is 3 points based on the rubric criteria, which is a good level.

The results reflect the success of activities to promote learners’ competency in simulating the probabilistic behavior of random events with Python. The factors that led to this result include, first and foremost, the learners’ ability to perform activities. This is because only academically gifted students are chosen to participate in the science classrooms in a university-affiliated school project. Therefore, they are academically exceptional. The second critical aspect is scaffolding strategic planning, which involves pre-planning scaffoldings, both at the micro- or macro-levels.

Figure 10 depicts an example of coding Python to simulate the probabilistic behavior of a random event. Given the position of remark 4, the code should be changed from (0, n) to (1, n+1) so that the emulation begins at 1 and ends at n due to Python’s unique functionality that does not iterate a loop for the last number of a specified range. The rank, (0, n), is evaluated as 0, 1, 2, ..., n-1. So, index must be adjusted to (1, n+1) so

Table 2. The results of the questionnaire on learning activities for simulating the probabilistic behavior of random events with Python on learner enjoyment, perceived value, interest, and self-efficacy

Item for evaluation	Average	SD	Implication
I enjoy solving problems simulating the probabilistic behavior of random events with Python.	4.29	0.95	Agree
I think it is important to be able to solve problems by simulating the probabilistic behavior of random events with Python.	4.50	0.72	Strongly agree
It is interesting to solve problems simulating probabilistic behavior of random events with Python.	4.08	1.06	Agree
I am confident that I can solve problems simulating probabilistic behavior of random events with Python.	3.63	1.31	Agree

Anticipating the challenges that students will face when simulating the probabilistic behavior of random events with Python is necessary. It has a significant impact on increasing competence to achieve goals. As a result, the success of promoting competency in simulating the probabilistic behavior of random events with Python is affected by the development of a good scaffolding strategy. This is consistent with research on the significance of scaffolding strategies in promoting mathematical modeling competencies (Geiger et al., 2022; Greefrath & Vorhölter, 2016; Schukajlow et al., 2015a). The final, but very important element is Python, a mathematical tool used to simulate the probabilistic behavior of random events. Greefrath and Siller (2017) emphasized the significance of embracing digital technology. It is used in mathematical modeling processes, particularly simulation modeling, which takes advantage of Python's ease simulating the probabilistic behavior of random events. Furthermore, Rodríguez Gallegos (2015) and Rodríguez Gallegos and Quiroz Rivera (2015) concluded that selecting the appropriate digital technology for mathematical modeling activities can improve the modeling performance of learners. The use of Python for simulating the probabilistic behavior of random events resulted in positive outcomes for learner competency, as data analysis of the activities revealed. All of these elements are essential to learning activities designed to promote learner competency in simulating the probabilistic behavior of random events with Python.

The mean and standard deviation of the analysis of the developed activities on enjoyment, perceived value, interest, and self-efficacy from the Likert scale questionnaire were computed to determine the effect of this activity on these four domains, as shown in **Table 2**.

Based on their responses to the questionnaire, the students agreed that the activity was enjoyable. In terms of value, the majority of students strongly agreed that this activity was worthwhile. The majority of students agreed that the practical activities piqued their interest. When asked about their self-efficacy, they all agreed that the activities gave them confidence in their ability to use Python to simulate the probabilistic behavior of random events. The results of the questionnaire indicate the success of the activities in simulating the probabilistic behavior of random events with Python. Additionally, it positively affected student enjoyment, perceived value,

interest, and self-efficacy. Instructional management that focuses on learners is very important. These characteristics are consistent with organizing learning activities through group processes, learning assistance, and empowerment strategies (Davadas & Lay, 2018; Schukajlow et al., 2012).

CONCLUSIONS

The goal of developing learners with fundamental competency and knowledge for simulating the probabilistic behavior of random events with Python was achieved. This is a significant model with unique characteristics that require effective application to real-world problems. In these cases, there is uncertainty regarding the probability of the observed behavior. To drive the required baseline competency for students to accomplish simulation modeling at a higher level, learning activities about simulating the probabilistic behavior of random events with Python were developed. These activities encourage students to practice the simulation process until it becomes a competency so that they can continue in the study of simulation modeling at a higher level and to apply simulation modeling in real-life situations. The 28 grade 12 students in the science classrooms of a university-affiliated school project served as the study's target group for examining the effects of activities on competency development. Students completed a subjective test using a rubric at the end of the activity to assess how well the activity affected their competency in simulating the probabilistic behavior of random events with Python. Additionally, the students provided information via a Likert-scale assessment that asked them to reflect upon how the activity affected their enjoyment, perceived value, interest, and self-efficacy. The assessment results revealed that learning activities involving simulating probabilistic behavior with Python were positively evaluated in both the cognitive and affective domains. Student competency in simulating the probabilistic behavior of random events using Python was more than satisfactory. They agreed that practicing activities increases their enjoyment, perceived value, interest, and self-efficacy. All of these are necessary for continued development of learners in simulation modeling. The activities of the current study are deemed successful because they achieved the goal of developing learners.

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