MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(3), em2798
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/18073

Recognition and conversion of electric field representations: The case of the algebraic notation

Genaro Zavala [1,2], Kristina Zuza [3], Jenaro Guisasola [4], Esmeralda Campos [5*]

[1] Institute for the Future of Education, Tecnologico de Monterrey, Monterrey, MEXICO
[2] Facultad de Ingenieria, Universidad Andres Bello, Santiago, CHILE
[3] Department of Applied Physics, University of the Basque Country (UPV/EHU), San Sebastian, SPAIN
[4] School of Dual Engineering, IMH Campus-UPV/EHU, Elgoibar, SPAIN
[5] Austrian Educational Competence Center Physics, University of Vienna, Vienna, AUSTRIA

Received 04 December 2025 ▪ Accepted 24 January 2026

Abstract

This conceptual understanding article is part of a series where we analyze the recognition and conversion of representations of the electric field concept; in this article, we present the case of algebraic notation. We conducted a study with introductory and upper-division physics students taking electricity and magnetism courses in a large private Mexican university to learn how students recognize the electric field's main characteristics in the algebraic notation of the field and how they convert to and from different representations. We refer to the theory of registers of semiotic representations as a theoretical framework and use a phenomenographic approach to analyze data. We explored students' recognition and conversion abilities through interpretation and construction tasks for the electric field's algebraic notation. We found that the main difficulties of interpreting and constructing the algebraic notation are related to separating the mathematical expression from the situation's physical meaning. Sometimes, students referred only to the physical meaning without using algebraic notation. In other cases, they construct algebraic notation without explicitly describing the physical meaning. Another source of difficulty is the treatment process because some students make mistakes or misinterpretations that they carry throughout. We recommend that introductory and upper-division electricity and magnetism instructors and physics education researchers in higher education be aware of the difficulties that some interpretation and construction tasks may present to students learning the electric field concept.

Keywords: conceptual understanding, physics education research, electricity and magnetism, higher education, educational innovation, STEM education

INTRODUCTION

The use of mathematical representations is critical for understanding physical phenomena. These representations can take many forms, such as mathematical expressions, graphs, and geometric diagrams. Algebraic notation, referred to as symbolic expressions (Bollen et al., 2017) or mathematical formalisms (Albe et al., 2001; Wilcox et al., 2013), is widely used across the different branches of physics because it allows the representation of a wide range of physical situations, such as the representations of scalar and vector quantities. As versatile as algebraic notation may be, it is crucial to consider the physical context and not focus solely on the equations. However, the relationship between mathematical expression and physical meaning, which is necessary for understanding the physical concept, is not always achieved (Redish, 2006). Even in integrated physics-mathematics courses that explicitly foreground modeling activities, students may construct mathematically consistent expressions but still struggle to apply them appropriately to real

✉ genaro.zavala@tec.mx ✉ kristina.zuza@ehu.eus ✉ jenaro.guisasola@ehu.eus
✉ esmeralda.campos@univie.ac.at **(*Correspondence)**

Contribution to the literature

- This study provides a fine-grained, empirically grounded characterization of students' difficulties with the algebraic notation of the electric field (e.g., treatment errors and dissociation between mathematical expressions and physical meaning) across introductory and upper-division levels.
- It shows that the role of algebraic notation as source vs. target representation systematically shapes recognition and conversion difficulties, supported by comparative category results across conversion tasks.
- It advances a combined theoretical-methodological approach by integrating Duval's (2006) theory of semiotic registers with phenomenography to link descriptive categories to levels of recognition and conversion.

experimental situations, which underscores the persistent challenge of coordinating symbolic expressions with their physical referents (Dominguez et al., 2024). These difficulties raise questions about how students interpret and use algebraic representations in specific physics domains.

In introductory university-level electricity and magnetism courses, students encounter a new level of abstraction when learning about the electric field (McDermott et al., 1999). The electric field is a vector field that explains electrical interactions. It is a physical quantity that cannot be accessed directly; it can only be accessed through semiotic representations (i.e., the algebraic notation, the vector field diagram, and electric field lines, among others) (Campos et al., 2020). When the electric field is expressed in algebraic notation, the resulting expressions follow all the rules and associations that are intrinsic to algebraic representations. At the same time, their interpretation must consider the context of electromagnetism, because it conveys the physical meaning of the electric field: it represents the existence of a source of field and the appropriate boundary conditions for the physical system. An incorrect interpretation of a mathematical relationship in the electromagnetic context may lead students to difficulties intrinsic to the representation (Rainson et al., 1994). In this article, we present a study on students' understanding of the algebraic notation in electromagnetism, specifically in representing the electric field concept.

This article is the last of a series of exploratory studies in which we identify students' abilities to recognize and convert between three representations of electric fields: algebraic notation, vector field diagrams, and electric field lines. Identifying students' difficulties in recognizing and converting among these three representations is essential to designing instructional materials that improve students' conceptual understanding of the electric field. In the previous publications of this series, we have presented a pilot study of students' conversion difficulties between these three representations (Campos et al., 2020), a fine-grained analysis of students' difficulties of recognition and conversion focused on the electric field lines diagram (Campos et al., 2023), and a similar analysis focused on the vector field diagram (Campos et al., 2026). In this study, we present a fine-grained analysis of students' difficulties of recognition and conversion focused on the algebraic notation of the electric field. We individually analyzed students' understanding of algebraic notation through interpretation and construction tasks and examined the difficulties that arise when it is the source or target representation.

We frame our analysis using Duval's (2006) theory of registers of semiotic representation and a phenomenographic approach to students' conceptions of electric and magnetic interactions (Guisasola et al., 2023; Hernandez et al., 2022; Marton, 1981). This combined theoretical lens allows us to characterize how students experience algebraic representations of the electric field and how they recognize and convert between different semiotic registers, including conversion processes between representations (also called transductions) (Svensson & Campos, 2022). In the next sections, we present this theoretical framework in more detail and formulate the research questions (RQs) that guide our study. While prior research has documented students' difficulties with electric fields, superposition, field-line diagrams, vector field diagrams, and vector calculus, much less is known about how algebraic notation for the electric field functions as both source and target in conversion tasks. This study addresses that gap by using the combined theoretical framework of semiotic representations and the phenomenographic approach to characterize recognition and conversion difficulties specific to algebraic notation and to examine how these difficulties differ when algebraic notation is used alone or in combination with other representations.

LITERATURE REVIEW

Several studies have investigated the relations between mathematical expressions and understanding the physical meaning in fundamental topics of introductory physics at the high school and university levels. One of the most common problem-solving strategies is to plug and chug quantities into equations without understanding the physical meaning (Tuminaro

& Redish, 2007). When analyzing the cognitive structure of Newton's second law equation and the system concept, Robertson (1990) found that some students had difficulty understanding the system concept and solving problems involving this law. In a study about motion and Newton's second law, Bagno et al. (2008) found that, when prompted to describe the components of a formula with physical concepts, students provided vague descriptions and ignored some components. When analyzing equations involving circuit elements, such as resistors and capacitors, students have difficulty relating multiple equations to a single physical quantity (Li & Singh, 2012). Taken together, these studies show that students often manipulate equations successfully while only partially connecting them to the underlying physical concepts.

Uhden et al. (2012) argued that the interrelationship between physics and mathematics runs two ways: the physical phenomena drive the creation of mathematical tools, and, simultaneously, the mathematization of physical theories serves as a means of representing the physical theory and as a reasoning guide for understanding new physical phenomena. In line with this argument, Hull et al. (2013) proposed that evaluating students' problem-solving skills with rubrics needs to consider the physical meaning at every step of the mathematical computation to discern expert-like and algorithmic solutions. Karam (2014) distinguished between the technical (instrumental or procedural) and structural (relational or organizational) roles of mathematics in physics. He studied the didactical strategy of a single professor to identify the interplay between physics and mathematics. In his categorization, he presented the visual representations as essential for connecting the mathematical aspects with their physical meanings. Karam and Krey (2015) proposed that the connection between physical meaning-making and mathematical computation should come from instruction. Together, these works emphasize that meaningful physics learning requires coordinated attention to both mathematical techniques and physical interpretation.

The difficulties of understanding mathematical expressions and relating them to physical phenomena are also evident when learning the concept of the electric field at the university level (van Kampen & De Cock, 2023). Some of the difficulties university students have in understanding the electric field concept are confusing electric field and electric force (Furió & Guisasola, 1998; Garza & Zavala, 2013; Saarelainen et al., 2007), which are related in the algebraic notation through Lorentz's equation of force and Coulomb's law. This difficulty is explained by a conceptual change that should occur when introducing electrical fields to explain interactions using Lorentz's equation of force and may be associated with an incomplete understanding of the electric field as a vector field (Saarelainen et al., 2007). The evidence found in several studies suggests that students have difficulties applying the superposition principle (Campos et al., 2021; Rainson et al., 1994; Viennot & Rainson, 1999) and tend to think that only the nearest charges contribute to the net electric field or that other electrical objects can block the electric field (Garza & Zavala, 2013; Li & Singh, 2017; Singh, 2005). When focusing on electric field-line diagrams, Campos et al. (2019) similarly found that many students' interpretations of field lines are inconsistent with the superposition principle, for instance by assuming that added charges or conductors can locally "erase" or block existing contributions to the net field. These difficulties persist even after instruction in advanced undergraduate courses (Maries et al., 2022). Analogous patterns have been identified when students reason about electric flux and magnetic circulation in Gauss's and Ampere's laws, where confusion between field and flux (or between field and circulation) and a blocking interpretation of Gaussian surfaces and Amperian loops hinder the correct application of the superposition principle (Hernandez et al., 2025). Rainson et al. (1994) reported that one of the sources of difficulty for students when using the superposition principle for the electric field is the misinterpretation of a mathematical relationship. Albe et al. (2001) found that mathematical formalisms are complex for students because they do not relate the representation to the physical concepts and models. Even for advanced engineering students, it is challenging to relate mathematical calculations to physical ideas, consider the physical situation when performing calculations, and use the appropriate mathematical tools, which contributes to their difficulties in electricity and magnetism (Pepper et al., 2012; Wilcox et al., 2013). Collectively, these results indicate that students' difficulties with the electric field and related quantities are closely tied to how they interpret and use algebraic expressions within the electromagnetic context.

Other studies have focused on the difficulties students have with multiple representations, including the algebraic notation of vector fields. Barniol and Zavala (2014) reported several challenges students face with the unit-vector notation and the transition to graphical notation. Some studies identified students' difficulties in electromagnetism when moving between algebraic notation and vector field diagrams, specifically with the dot product and the cross product (Bollen et al., 2015, 2016). Bollen et al. (2017) applied a questionnaire to identify difficulties in switching between vector field representations, specifically algebraic notation, vector field diagrams, and field lines. The questionnaire showed that students struggle with vector sums and find it challenging to choose an appropriate coordinate system when building mathematical expressions of vector fields. Building on these findings, Hahn and Klein (2023) implemented multi-representational learning

tasks that combined algebraic expressions, vector field diagrams, sketching activities, and interactive simulations, and showed that such tasks can enhance students' conceptual understanding of vector field concepts while managing cognitive load. At the secondary level, Hoyer and Girwidz (2024) found that students' ability to interpret unit vector representations of electric and magnetic fields lags behind their ability to draw them. They showed that contrasting conventional and centered unit vector plots can support the development of representational competencies. These studies point to the need for a deeper understanding of how students recognize and convert between different representations of vector fields, including algebraic notation.

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

We refer to the theory of registers of semiotic representations (Duval, 2006) as a theoretical framework for this study. A representation stands for something else, and within this definition, semiotic representations use symbols, rules, and associations to represent an object. According to this theory, mathematical objects can only be accessed through semiotic representations; therefore, the role of semiotic representations of mathematical objects is to represent the object and allow cognitive activity in the form of transformations of registers. A register of semiotic representations is a representational system that describes the object and can be transformed. The transformations can be treatments when they occur within the same register, and conversions when they occur between two or more registers.

Duval (2006) proposed that treatments and conversions are sources of difficulty that affect understanding the mathematical object. One source of difficulty is recognizing the object's characteristics in the representation. This difficulty is more critical in conversions because it is necessary to recognize the characteristics in two registers that do not represent the object using the same symbols, rules, and associations. Another challenge is the direction of conversion, since students can successfully convert from one representation to another, but not vice versa. The third source of difficulty is that students associate the representation of an object with the object itself, so dissociation is necessary to promote conceptual understanding of the mathematical object. For dissociation between the representation and the object to occur, it is necessary to have synergy in the conversion of representation registers.

The theory of registers of semiotic representations has been widely used in Mathematics education research (Deliyianni et al., 2016; McGee & Martinez-Planell, 2014; Moreno-Arotzena et al., 2021; Trigueros & Martinez-Planell, 2010), linking recognition and conversion difficulties with cognitive activity and understanding. In physics education research, a few studies have included this theory as a theoretical framework (Campos et al., 2020; Ceuppens et al., 2018). In this study, we refer to the theory of registers of semiotic representations to approach the following RQs:

RQ1. What difficulties do students have in recognizing the main characteristics of the electric field in the algebraic notation?

RQ2. What difficulties do students have when converting from the algebraic notation to the vector field diagram and the electric field lines diagram?

RQ3. What difficulties do students have when converting from the vector field diagram and the electric field lines diagram to the algebraic notation?

The electric field concept is the mathematical object in this study. The three registers of semiotic representations we consider are the algebraic notation, the electric field lines diagram, and the vector field diagram. These representations are relevant in learning electromagnetism at the university level, as reported in other studies (Bollen et al., 2017; Campos et al., 2020). This article focuses on algebraic notation and explores its role in the conversion not only to, but also from, the vector field diagram and the electric field lines diagram.

METHODOLOGY

The study took place at a private Mexican university, where 295 engineering physics students (210 in introductory and 85 in upper-division courses) completed the electricity and magnetism and electromagnetic theory courses, respectively. The introductory electricity and magnetism course was calculus-based and used a standard textbook (Young & Freedman, 2013) and tutorials (McDermott et al., 2002). Both courses used the three representations in lectures and activities, so participants were familiar with them at both levels. However, there was no explicit instruction on how to convert between them.

In the introductory electricity and magnetism course, the students attended one 3-hour lecture session and one 90-minute laboratory session per week. The lecture sessions were an active learning course that included peer instruction and cognitive scaffolding in a SCALE-UP environment (Campos et al., 2016; Zavala, 2019). The peer instruction and cognitive scaffolding strategies took approximately two thirds of the lecture sessions. During peer instruction, the professor would present between 5 and 10 conceptual multiple-choice questions to the students and prompt them to discuss their answers, until they would convince their peers about the correct option. This strategy works because the students with correct reasoning make stronger arguments, and students with

Table 1. Definition of the objective of the items included in each questionnaire

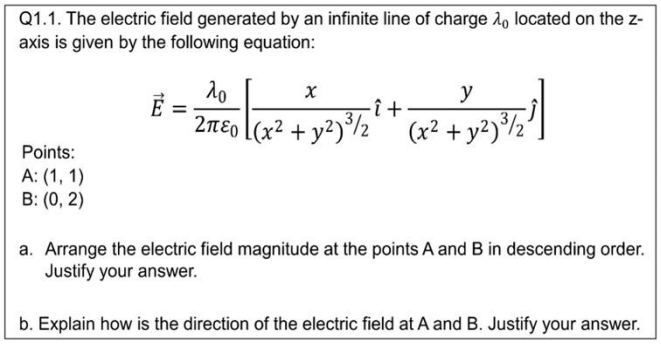
Objective	Interpretation	Construction
Recognition of the algebraic notation	Q1.1	Q2.1
Conversion (source: algebraic notation)	Q1.2 & Q2.2	
Conversion (target: algebraic notation)		Q1.3 & Q2.3

Note. The objectives are the recognition of the algebraic notation and conversion with the algebraic notation as a source or target representation and each item describes an interpretation or construction task

erroneous conceptions tend to have weaker arguments (Crouch & Mazur, 2001). The cognitive scaffolding activities included between 3 to 5 worksheets that students would answer in small teams. The main purpose of the cognitive scaffolding activities is to help students to reflect, think and conceptualize the right ideas to solve a problem in predefined steps (Zavala, 2019). The topics covered to teach the electric field included Coulomb's law, the superposition principle, electric potential and Gauss' law.

The upper-division electromagnetic theory course followed a traditional approach, with some active learning activities, and used a standard textbook (Griffiths, 2017). The students attended two 90-minute sessions per week oriented to solving end-of-chapter problems and tutorials designed for upper-division electromagnetism (Chasteen, 2012). The purpose of the tutorials was to help students develop metacognitive strategies and for the instructor to model problem-solving strategies (Chasteen, 2012). The topics covered to teach the electric field included Coulomb's law, superposition principle, electric potential and Gauss' law. The main difference is that instruction in this course requires advanced mathematical proficiency to solve special methods, like separation of variables to solve the Laplace equation, method of images, and multipole expansion.

The instruments for the full implementation were two open-ended questionnaires with 12 items in total. In this study, we focused on six items: three for interpreting and three for constructing the algebraic notation of the electric field. Each item's objective was either the recognition of the characteristics of the field in the algebraic notation or the conversion from or to the algebraic notation, as detailed in **Table 1**. The conversion items (Q1.2, Q2.2, Q1.3, and Q2.3) were analyzed and refined in the pilot study (Campos et al., 2020). The recognition items (Q1.1 and Q2.1) had not been analyzed previously. In this contribution, the analysis was performed with a larger number of participants, which enabled category saturation and a fine-grained analysis focused on the implications of using the algebraic notation of the electric field, both individually and in conversion tasks.


Q1.1. The electric field generated by an infinite line of charge λ_0 located on the z-axis is given by the following equation:

$$\vec{E} = \frac{\lambda_0}{2\pi\varepsilon_0}\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{\imath} + \frac{y}{(x^2+y^2)^{3/2}}\hat{\jmath}\right]$$

Points:
A: (1, 1)
B: (0, 2)

a. Arrange the electric field magnitude at the points A and B in descending order. Justify your answer.

b. Explain how is the direction of the electric field at A and B. Justify your answer.

Figure 1. Item Q1.1 targets the recognition of the electric field's magnitude and direction in the algebraic notation (Source: Authors' own elaboration)

We collected data after the instruction. The questionnaires were administered randomly to all participants. Each student answered only one version of the questionnaire to avoid interference between the objectives. Before answering the test, the students were informed of the study's objective, that their participation was voluntary, and that their answers would not impact their grades. Any identifying information was not collected, and the data was treated anonymously. All the data collection and analysis were done in Spanish; in the following section, we present translations of the items, and in the results section, we present translations of the students' answers.

Instrument

We present the instrument in two sections:

1. Interpretation tasks: the items that require an interpretation of the electric field's algebraic notation, and
2. Construction tasks: the items that require constructing a mathematical expression for the electric field in the algebraic notation.

Interpretation tasks

Item Q1.1 targets the recognition of the electric field's magnitude and direction in the algebraic notation, as presented in **Figure 1**. It is an interpretation task because the student needs to interpret an equation of a physical situation to conclude the electric field's characteristics. Students interpret the electric field's magnitude in part (a) and the direction in part (b). In part (a), the correct answer to this question is for students to identify that it is necessary to substitute the values in the equation and solve the field's magnitude at each point through treatment. They develop a treatment to identify the magnitude of the field in both positions and compare them. They conclude that the magnitude of the field in A is greater than that of B. They explain that it makes sense since point A is closer to the line of charge than point B. Students link mathematical treatment within the algebraic notation register with the electric field's physical sense through this reasoning. In part (b),

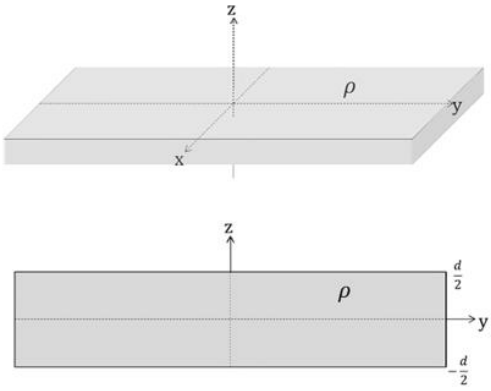
Q2.2 There is a very large, non-conductive plate with electric charge ρ uniformly distributed in its volume, and height d. The figure shows a cut of this plate.

The electric field generated by this plate has the form:

$$\vec{E} = \begin{cases} \frac{\rho}{\varepsilon_0} z\,\hat{k}; & -\frac{d}{2} < z < \frac{d}{2} \\ -\frac{\rho}{\varepsilon_0}\frac{d}{2}\,\hat{k}; & z < -\frac{d}{2} \\ \frac{\rho}{\varepsilon_0}\frac{d}{2}\,\hat{k}; & z > \frac{d}{2} \end{cases}$$

a. Draw in the following figure a vector field plot to describe the electric field inside and outside the plate.

b. Explain your drawing in terms of magnitude and direction of the electric field. Justify your answer.



Note: in item Q1.2, the instruction (a) is different:

a. Draw in the following figure electric field lines to describe the electric field inside and outside the plate.

Figure 2. Item Q1.2 and item Q2.2 aim to convert from the algebraic notation to the vector field diagram or the electric field lines diagram (Source: Authors' own elaboration)

Q2.1 There is a very large and thin plate on x=0. The surface has positive charge density $+\sigma_1$ evenly distributed.

a. Write a mathematical expression for the electric field generated by the plate. Explain your answer in terms of magnitude and direction of the electric field. Justify your answer.

b. A second very large and thin plate with evenly distributed negative charge density $-3\sigma_1$ is placed on x=d. Write a mathematical expression for the electric field generated by the system. Explain your answer in terms of magnitude and direction of the electric field. Justify your answer.

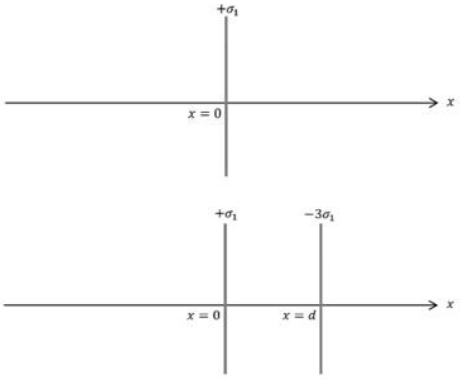


Figure 3. Item Q2.1 targets recognizing the electric field's characteristics in applying the superposition principle in the algebraic notation as a construction task (Source: Authors' own elaboration)

students recognize that in point A (1, 1), the field's direction is on the diagonal and express it as the sum of the Cartesian components $\hat{\imath} + \hat{\jmath}$. They also recognize that at point B, the direction is vertical towards $+\hat{\jmath}$. They explain that the direction is radial and outward in the x-y plane since the wire is on the z-axis and has a positive charge.

The items of conversion where the algebraic notation is the source representation are Q1.2 and Q2.2. The electric field presented in both items is the same; in Q1.2, students convert to the electric field lines diagram, and in Q2.2, to the vector field diagram. Both conversions are done independently. The item is shown in **Figure 2**. This is an interpretation task since the algebraic notation is the source representation. Students should recognize the characteristics of the electric field in the mathematical expression and be able to represent the characteristics of the field in a different register of representation (i.e., a vector field diagram or an electric field lines diagram) to convert efficiently from the algebraic notation to either representation. The correct interpretation requires students to recognize that the electric field varies within the plate with z and remains uniform outside the plate. Students should identify the three regions with their differences in behavior, both in magnitude and direction. They should describe that the field points up in $z > 0$ and down in $z < 0$. They should also state that the field in $z > \frac{d}{2}$ and $z < -\frac{d}{2}$ is uniform, whereas in $-\frac{d}{2} < z < \frac{d}{2}$, its magnitude varies with z. In item Q1.2, students are expected to draw electric field lines representing the variations of magnitude through the density of field lines inside and outside the plate and with an adequate direction. In item Q2.2, students are expected to draw enough vectors to represent the electric field's variations inside the plate and the uniformity of the field outside with the appropriate direction.

Construction tasks

Item Q2.1 targets recognizing the electric field's characteristics in applying the superposition principle in the algebraic notation, as presented in **Figure 3**. It is a

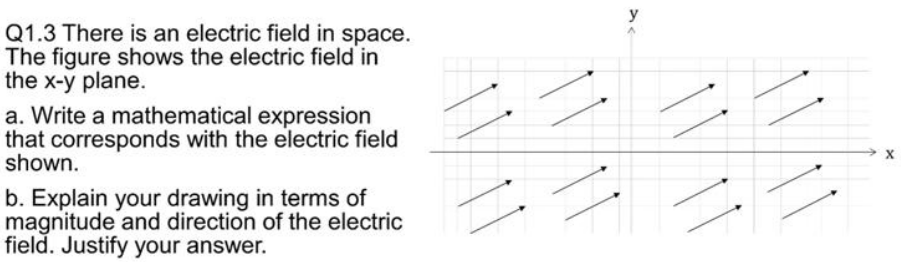


Figure 4. Item Q1.3 aims to convert from the vector field diagram to the algebraic notation (Source: Authors' own elaboration)

construction task because the student needs to create a mathematical expression of the electric field in algebraic notation using superposition. Students write a mathematical expression in the algebraic notation of a single charged plate in part (a) and of two charged plates in part (b). In part (a), students are expected to interpret the physical situation, describe the electric field on both sides of the plate, and write an algebraic expression representing the magnitude and direction of the field. Students should indicate that on the left side, the field is in the direction of $-\hat{x}$ and on the right side, in the direction of $\hat{x}$, or some variation of this notation. They should describe that the magnitude corresponds to $\sigma/2\varepsilon_0$, justifying this by applying Gauss's law to get the field in this situation. In part (b), students are expected to apply the superposition principle to explain and describe the electric field in the different electric field regions of the problem. Students should identify three different regions where there is an electric field: to the left of the two plates ($x < 0$), between the two plates ($0 < x < d$), and to the right of the plates ($x > d$). They write an expression that correctly describes the field in all three regions, in terms of magnitude and direction, using the principle of superposition. Besides, they explain that fields are added up or canceled in certain regions.

The items of conversion where the algebraic notation is the target representation are Q1.3 and Q2.3. In item Q1.3 (**Figure 4**), students convert from the vector field diagram, and item Q2.3 (**Figure 5**) from the electric field lines diagram. The two items are a construction task because the algebraic notation is the target representation. To convert efficiently to algebraic notation, students should recognize the electric field's characteristics in vector field diagrams or electric field lines diagrams and be able to represent those characteristics in algebraic notation. In item Q1.3, the correct construction requires the students to express a sum of constants in Cartesian directions. Constants can be a and b, 4 and 2, E_x and E_y, or any other pair of different constants. Besides, students should relate their responses to the electric field's characteristics, such as mentioning that it is uniform. In item Q2.3, students should recognize that the field's magnitude decreases with the distance from the center in a counterclockwise direction. For correct construction, students should generate a mathematical expression that includes a

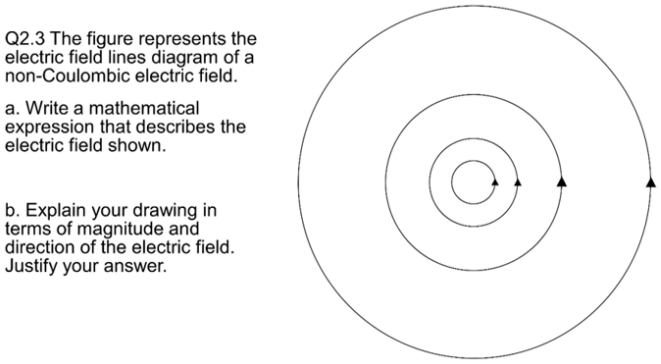


Figure 5. Item Q2.3 aims to convert from the electric field lines diagram to the algebraic notation (Source: Authors' own elaboration)

proportionality constant, an inverse dependence on the radius, and an angular direction in polar coordinates.

Data Analysis

We analyzed data employing a phenomenographic approach. This method helps identify different ways the participants interpret an aspect of the physical world by describing, analyzing, and understanding people's experiences. The researchers create descriptive categories that explain a collective understanding, focusing on variations in participants' experiences (Marton, 1981); these categories are the main result of the phenomenographic approach. In this study, we identified descriptive categories to explain students' abilities to recognize the characteristics of the electric field in algebraic notation, both when interpreting and constructing it, and their abilities to convert from algebraic notation to vector field diagrams and electric field lines diagrams, and vice versa.

In the first data analysis phase, all the researchers familiarized themselves with the data and created descriptive categories for 20 students' responses. We considered the students' drawings, equations, and explanations in their responses to create descriptive categories. We validated the first set of categories through the researchers' agreement. In the second phase, we iteratively analyzed the remaining participants. We validated the second phase with Cohen's kappa, attaining an inter-rater reliability of 0.90. The descriptive categories have a hierarchical sequence that reflects the

Table 2. Correct interpretation for items Q1.1, Q1.2, and Q2.2, along with an example and the percentages of introductory (Intro) and upper-division (UD) students who answered each item correctly

Characteristics	Item	Example	Intro	UD
Recognition of magnitude, treatment, and physical meaning	Q1.1 (a)	Answer: First, the students replaced the points in the equation and simplified it. Then, they compared the two magnitudes to generate a conclusion, "$\|E_A\| > \|E_B\|$." Explanation: "*it was calculated, and that conclusion was reached, but it makes sense because the distance to point A is less than the distance to point B, and the E depends on* $1/r^2$". (introductory, student UD12-I25)	38%	51%
	Q1.1 (b)	The direction in A: "*In the direction of* $\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$". The direction in B: "*In the direction of* $\hat{y}$" Explanation: "*Can be seen from calculations or also considering that the field always points out of the z-axis.*" (upper-division, student UD12-A15)	22%	35%
Recognition of magnitude and direction, conversion to electric field lines	Q1.2	Drawing: Electric field lines that increase in density in the region within the plates and whose density remains constant outside the plates. Explanation: "*Outside the plates, the field lines are equally spaced and with the direction of* $\bar{E}$ *to show that it is a uniform field. Within the plate, the direction is equal, but closer to z=0, there is less density of field lines, and these increase in density per unit area as z increases, as well as the magnitude of* $\bar{E}$". (upper-division, student UD12-A13)	37%	53%
Recognition of magnitude and direction, conversion to vector field diagram	Q2.2	Drawing: drew vectors up in region z > 0 and down in z < 0. Demonstrates the field's variation inside the plate through the arrow's size, while on the outside, the arrows are consistently the same length. Explanation: "*The direction is up when z is positive and down when negative. Within the volume, the field increases according to z until it reaches d/2; from there, the magnitude remains constant.*" (upper-division, student UD12-A13)	28%	60%

most common skills and difficulties students present in their responses, thereby describing the outcome space for the categories. It is important to note that the categories were identified, validated, and classified by experts in the field to determine the hierarchy of descriptive categories.

After establishing the phenomenographic descriptive categories, we connected them with the students' level of recognition and conversion through a three-step procedure. First, for each item, we coded students' responses into qualitative categories that captured how they interpreted or constructed the electric field in the algebraic notation. Second, we assigned each category a level of recognition and a level of conversion (high, medium, or low), based on whether students correctly identified the main characteristics of the electric field in the source and/or target register and whether they produced an appropriate representation in the target register. Third, we combined these recognition and conversion levels into four global categories (A, B, B', C, and D), which summarize students' performance across items and are later used to compare conversion processes when algebraic notation is the source or the target representation.

The recognition items (Q1.1 and Q2.1) are differentiated by the type of task: interpretation or construction of the representation. The item of interpretation of the magnitude and direction in each representation (item Q1.1) was categorized independently for each characteristic. The item involving a construction task (item Q2.1) was analyzed for a single electric field source and for the system with two sources, independently. We identified the different patterns in students' answers, their interpretation and construction strategies, and their application of the superposition principle. The descriptive categories that emerged in the conversion items (Q1.2, Q2.2, Q1.3, and Q2.3) were classified into a theoretical structure reflecting students' abilities of recognition and conversion as a framework for the descriptive categories (Campos et al., 2020). All questions had the same theoretical structure of categories to allow for comparing the conversion tasks. Due to the nature of the questions, the descriptive categories differed, but they shared a theoretical structure based on recognition and conversion abilities.

RESULTS

Interpretation of Algebraic Notation

In this section, we first present examples of students correctly interpreting questions Q1.1, Q1.2, and Q2.2, and the percentages of introductory and upper-division students who answered each item correctly in **Table 2**. As shown in **Table 2**, the recognition percentages of electric field characteristics are close to 30% for introductory students and 50% for advanced students. All questions qualitatively showed increased recognition skills when moving from an introductory profile to upper division. Identifying these differences in the correct answer is essential, as they show the experience students gain in transiting from novices to experts. The increase in converting from algebraic notation to the vector field diagram is relatively high and

Table 3. Answers that imply difficulties in recognizing the electric field's characteristics in the algebraic notation in items Q1.1, Q1.2, and Q2.2

Characteristics	Item	Example	Intro	UD
Treatment errors	Q1.1 (a)	Answer: Performed the treatment calculations appropriately, concluded "B>A." Explanation: "*The magnitude of B is k*1/4, and that of A is k*1/2, given by the formula.*" (introductory, student UD12-I7)	21%	23%
Alternative explanations of the direction	Q1.1 (b)	The direction in A: "45° between the x- and y-axes." The direction in B: "*Vertically in y.*" Explanation: "*A has the same component in i and j; B only has a component in j.*" (upper-division, student UD12-A9)	41%	40%
Physical meaning, without using the equation	Q1.1 (a)	Answer: "E_A>E_B." Explanation: "*Point A is closer to the origin of the electric field.*" (upper-division, student UD12-A3)	12%	14%
	Q1.1 (b)	The direction in A: "*Radial.*" The direction in B: "*Radial.*" Explanation: "*The electric field of an infinite wire is radial; if you put a Gaussian cylinder, it is demonstrated.*" (upper-division, student UD12-A6)	10%	14%
Confusion of interpretation of the magnitude	Q1.2	Drawing: Arrows that start at the bottom of the plate and extend to the top, pointing upward. Outside of the plate, smaller arrows point upward over the plate and down below it (see part a in **Figure 6**). Explanation: "*Inside the field goes in the z-direction as for z>d/2. Below the plate goes down, but above the plate, its magnitude changes, in the middle and down is equal.*" (upper-division, student UD12-A6)	46%	41%
	Q2.2	Drawing: Arrows that start at the bottom of the plate and extend to the top, pointing upward. Outside of the plate, smaller arrows point upward over the plate and down below it (see part b in **Figure 6**). Explanation: "*In the center of the plate, the field goes in the positive direction of k and is not divided by any factor; below y/2, the field is negative and on it, positive, both multiplied by a factor of ½.*" (introductory, student UD12-I32)	48%	29%

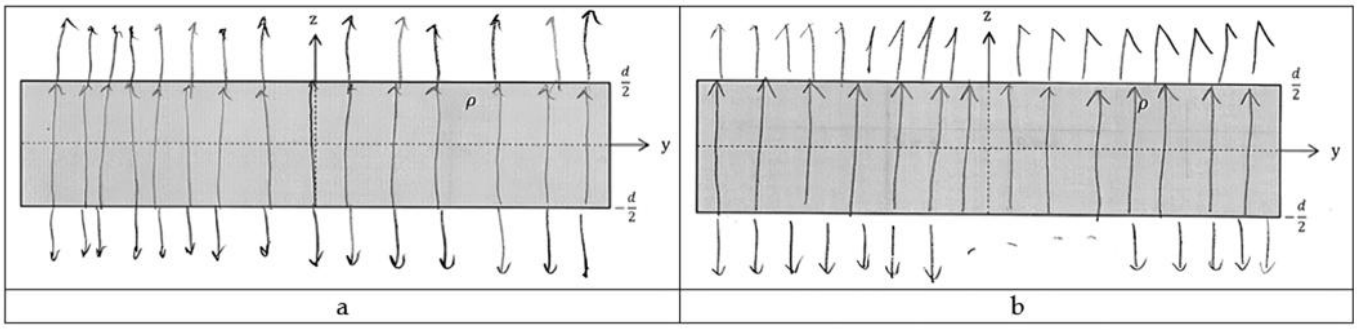


Figure 6. Example of difficulties recognizing the algebraic notation when converting to (a) electric field lines and (b) vector field diagrams in items Q1.2 and Q2.2, respectively (Source: Authors' own elaboration)

provides evidence of the high synergy between these two representations.

Alternative interpretations of the algebraic notation

Several alternative interpretations emerged in the items interpreting algebraic notation. Among students' most relevant difficulties are errors in the treatment of algebraic notation, in interpreting equations without physical meaning, and vice versa. In **Table 3**, we present the categories that emerged in items Q1.1 (a), Q1.1 (b), Q1.2, and Q2.2, where students had several difficulties explaining their answers.

The "treatment errors" category emerged in item Q1.1 because it is the only question in the questionnaire that required treatment. Students needed to replace values in the equation and develop an algebraic treatment to reach a conclusion. Three main errors were identified during this task. The first type of treatment error was that students compared both magnitudes and concluded that the magnitude of the field in B was greater than in A. This error is exemplified in **Table 3**. The second treatment error was that students substituted the values in the equation and applied the treatment without getting the magnitude. The third treatment error was that students miscalculated during the treatment steps.

The "alternative explanations of direction" category included students who correctly calculated the direction at points A and B but did not explain the physical meaning of the direction in item Q1.1 (b). Students

recognized that at point A (1, 1), the field's direction is on the diagonal and expressed it as the sum of the components $\hat{\imath} + \hat{\jmath}$ and that at point B, the direction is vertical towards $+\hat{\jmath}$. The students explained the vector components of the equation.

The "physical meaning, without using the equation" category included students who used their physical senses rather than interpreting the algebraic equation. In item Q1.1 (a), some students concluded that the field in A is greater than in B by noting that position A is closer to the field source than position B. In item Q1.1 (b), students explained that the electric field is radial because it is a line of charge. These students did not use the mathematical expression for the electric field or describe the field direction at each marked position.

The "confusion of interpretation of magnitude or direction" category included students who misinterpreted the electric field's magnitude in the conversion items. These recognition difficulties were shared in conversions to the electric field lines diagram and the vector field diagram in items Q1.2 and Q2.2. Some students failed to realize that the field varies with z inside the plate and is constant outside. In some cases, the magnitude inside was interpreted as twice the magnitude outside. In other cases, the magnitude inside was assumed to be constant. Other students identified that the field had an upward direction throughout the plate's internal region, which makes it difficult to recognize the direction of the electric field in algebraic notation.

Table 3 shows that there are two types of alternative interpretations. On the one hand, some interpretations do not pose a significant difficulty in understanding the concept of the electric field. For example, item Q1.1 makes sense that there are treatment errors since the need for treatment in this question adds difficulty (Duval, 2006). Alternative explanations of direction do not lead students to incorrect conclusions. Students who resorted to physical sense without using the equation did not interpret the algebraic notation. However, this did not prevent them from understanding the concept of the electric field and the physical situation presented. These minor difficulties persisted after the experience gained from introductory students to upper division.

On the other hand, there are interpretation difficulties in cases where the algebraic notation is the initial representation in a conversion process. These difficulties create conflict in understanding the electric field since the conversion process requires recognizing the characteristics of the electric field in two representations, and, in this case, the recognition in algebraic notation creates difficulty. **Table 3** shows that when converting to the electric field lines diagram (item Q1.2), students' difficulties in recognition persisted from an introductory to an upper-division profile. However, fewer upper-division students had these difficulties when converting to the vector field diagram (item Q2.2). This result could mean that the target representation (in this case, the electric field lines diagram) makes it more difficult to interpret the algebraic notation. Therefore, the synergy between the source representation and the target representation can impact students' skills in recognizing the characteristics of the electrical field in the source representation.

Construction of Mathematical Expressions of the Electric Field in Algebraic Notation

Students' construction skills of algebraic expressions for the electric field allow us to identify how they use the algebraic notation characteristics to represent the electric field's magnitude and direction. This is one of the steps necessary to perform a successful conversion to algebraic notation and demonstrate recognition of the characteristics of the electric field in this representation. Students are asked to build algebraic expressions of the electric field in items Q2.1, Q1.3, and Q2.3. In Q2.1(a), students need to interpret a physical situation and write an algebraic expression for the electric field. In part (b), the situation is modified, and students need to construct a new algebraic expression, applying the principle of superposition. In items Q1.3 and Q2.3, students convert from an initial representation (vector field diagram and electric field lines diagram, respectively) to the algebraic notation. **Table 4** shows the correct construction of the algebraic notation for each item, an example, and their respective percentages.

Based on the results in **Table 4**, we observe that introductory students have less than 30% of the correct

Table 4. The correct construction for items Q2.1, Q1.3, and Q2.3

Characteristics	Item	Example	Intro	UD
Construction of algebraic notation for a physical system	Q2.1 (a)	Answer: "$\mathrm{E} = \frac{\sigma}{2\varepsilon_0}\hat{x}$ *for x>0 and* $\mathrm{E} = -\frac{\sigma}{2\varepsilon_0}\hat{x}$ *for x<0.*" Explanation: "*You can make a Gaussian cylinder and use Gaussian's law in its integral form to obtain it*," $\int E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$ (upper-division, student UD12-A11)	0%	24%
Application of the principle of superposition	Q2.1 (b)	Answer: "$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$, *x<0;* $\vec{E} = \frac{2\sigma}{\varepsilon_0}\hat{x}$, *d>x>0;* $\vec{E} = \frac{\sigma}{\varepsilon_0}(-\hat{x})$, *x>d.*" Explanation: "*With superposition of the fields generated by the plates, you reach those expressions. The field will be larger between the plates.*" (upper-division, student UD12-A10)	8%	26%

Table 4 (Continued). The correct construction for items Q2.1, Q1.3, and Q2.3

Characteristics	Item	Example	Intro	UD
Conversion: Vector field plot to algebraic notation	Q1.3	Answer: "$E = a\hat{\imath} + b\hat{\jmath}$, *where a and b are positive constants.*" Explanation: "*The electric field in the drawing seems to be uniform, so it does not depend on any position variable. Besides, it goes in the direction* $\hat{\imath}$ *and* $\hat{\jmath}$ *positive, so the components must be positive constants.*" (upper-division, EM2018-A2-1)	31%	45%
Conversion: Electric field lines to algebraic notation	Q2.3	Answer: $\vec{E} = \frac{A}{r}\hat{\phi}$ Explanation: "*Direction, the field circulates, so it goes in* $\hat{\phi}$*; Magnitude, there is a greater magnitude to the center, so it decreases to a greater radius*." (upper-division, EM2018-A1-2)	8%	28%

Table 5. The answers that imply difficulties constructing the algebraic notation "mathematical features, no physical meaning" in items Q2.1, Q1.3, and Q2.3

Characteristics	Item	Example	Intro	UD
Mathematical features, no physical meaning	Q2.1 (a)	Answer: "*For x>0,* $\mathrm{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathrm{x}}$ *and for x<0,* $\mathrm{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathrm{x}}$" Explanation: "*By border conditions, the sum of the normal components to the surface must be* σ/ε_0*. In this case, it is distributed symmetrically.*" (upper-division, student UD12-A13)	10%	17%
	Q1.3	Answer: $\vec{E} = E_0\left[\cos\left(\tan^{-1} 1/2\right)\hat{x} + \sin\left(\tan^{-1} 1/2\right)\hat{y}\right]$ Explanation: "*Constant magnitude and direction.*" (upper-division, EM2018-A2-6)	22%	21%
	Q2.3	Answer: $\vec{E} = \frac{\mu_0 I}{2r}\hat{\phi}$ Explanation: "*As it moves away from the center, intensity decreases, as seen in factor 1/r. The field 'circulates,' so it goes in the direction of* $\hat{\phi}$ *in the proposed expression*." (upper-division, EM2018-A1-3)	4%	10%

Table 6. The answers that imply difficulties constructing the algebraic notation "dependency on distance" in items Q2.1, Q1.3, and Q2.3

Characteristics	Item	Example	Intro	UD
Dependency of distance	Q2.1 (a)	Answer:$\vec{E} = \frac{4\pi}{\varepsilon_0}\int \frac{\sigma}{\lvert r-r'\rvert^2}\hat{r}dr$ Explanation: "*It has an electric field in the* $\hat{x}$ *direction, and its magnitude is defined by σ and r.*" (upper-division, student UD12-A3)	33%	19%
	Q1.3	Answer: $\vec{E} = \frac{\rho}{\varepsilon_0}x\hat{x} + \frac{\rho}{2\varepsilon_0}x\hat{y}$ Explanation: "*The electric field travels diagonally in a positive direction of* $\hat{x}$ *and* $\hat{y}$*. However, it has magnitude on the axis* $\hat{y} \propto \frac{1}{2}x$." (upper-division, EM2018-A2-15)	19%	14%
	Q2.3	Answer: $\vec{E} = r\cos\theta\,\hat{x} + r\sin\theta\,\hat{y}$ Explanation: "*The mathematical expression tells us that the field has a tangent direction and becomes weaker as we separate from the origin.*" (upper-division, EM2018-A1-12)	5%	12%

answers to all questions, implying that they need help representing the electric field's characteristics in algebraic notation. Qualitatively, their skills improved incrementally as they moved to the upper-division profile in all cases. The conversion between vector field diagrams and algebraic notation promotes students' success in constructing mathematical expressions in algebraic notation.

Difficulties in constructing the algebraic notation of the electric field

In analyzing the items that call for constructing algebraic expressions of the electric field, we found some categories that reflect students' difficulties. The predominant ones were that, in some situations, students represented the mathematical characteristics of the electric field without considering their physical meaning; they described only one part of the field, either magnitude or direction; some interpreted the field as distance-dependent or used rote learning resources. In this section, **Table 5**, **Table 6**, and **Table 7** show these difficulties.

The "mathematical features, no physical meaning" category included students who represented the electric field using algebraic notation but did not explain its physical meaning. It is inferred that this behavior comes from a sufficient familiarity with the representation of algebraic notation but illustrates difficulties relating this representation to the physical meaning of the electrical field or related laws and principles. In item Q2.1 (a), the students presented a correct algebraic equation for the given situation but did not explain that it was obtained

Table 7. The answers that imply difficulties constructing the algebraic notation "description of magnitude and direction, only" in items Q2.1 and Q2.3 and "memorization" in item Q2.1

Characteristics	Item	Example	Intro	UD
Description of magnitude or direction, only	Q2.1 (a)	Description of magnitude: Answer: $E = {}^{\sigma}/_{2\varepsilon_0}$ Explanation: "*If we apply Gauss' law with a box as a Gaussian surface, we find that the area through which the field passes are two faces of the box while the surface of the plane enclosed by the same box is equal to the area of one of the faces of the box.*" (upper-division, student UD12-A19) Description of direction: Answer: $\vec{E} = \lvert\vec{E}\rvert\hat{x} + 0\hat{y}$ Explanation: "*The plate only generates a field parallel to the x-axis, so it only has a component in* $\hat{\imath}$." (introductory, student UD12-I34)	11%	17%
	Q2.3	Answer: $\vec{E} = E(r)\hat{\varphi}$ Explanation: "*The magnitude changes according to the radius (distance from the center), and the direction is always azimuth.*" (upper-division, EM2018-A1-1)	5%	2%
Memorization	Q2.1 (a)	Answer: $E = \begin{cases} {}^{\rho}/_{\varepsilon_0}\hat{x}\ x > 0 \\ -{}^{\rho}/_{\varepsilon_0}\hat{x}\ x < 0 \end{cases}$ Explanation: "$\nabla \cdot E = {}^{\rho}/_{\varepsilon_0}$I*f we use Gauss' law for an infinite plate.*" (upper-division, student UD12-A12)	19%	19%

through Gauss' law. In item Q1.3, some students indicated that the field is a sum of constants a and b in Cartesian directions $\hat{\imath}$ and $\hat{\jmath}$, but did not explain that the vector field diagram provides information about a uniform electric field. In item Q2.3, some students associated the non-conservative electric field with a magnetic source. Students in this category attempted to guess the source of this field type and associate it with a magnetic source. Students can be identified as associating a magnetic field source when they include current indicators and the permeability constant in their mathematical expressions.

The "dependency on distance" category included students representing the electric field with a distance dependence. Items Q2.1 and Q1.3 show uniform electric fields in both cases, while item Q2.3 has an inverse distance proportionality. In item Q2.1, some students wrote an expression that depends on some distance variable (x, l, r). In many cases, the expression is similar to the law of Coulomb and somehow involves charge density. Students who exhibited these characteristics failed to represent the field's magnitude as constant, as they had difficulty understanding the physical situation. In item Q1.3, some students struggled to recognize the magnitude of the field in the vector field diagram, as they indicated that the electric field varies. This behavior is reflected in their mathematical expression or explanation. They also had difficulty converting to algebraic notation, as their expression depended on the magnitude of the electric field and the variables of this system (x, y). In item Q2.3, some students interpreted a linear dependence between the electric field and radius. They wrote an expression where the magnitude is proportional to the radius or some distance variable.

In the "description of magnitude or direction only" category, students described one of the electric field characteristics: in some cases, only the magnitude of the field, and in others, only the direction. This behavior poses a difficulty, since the electric field is a vector quantity; it must be represented by both magnitude and direction to provide a complete description. If only the magnitude is represented, it can be confused with a scalar quantity, whereas if only the direction of the field is represented, not enough information about its intensity is available. In item Q2.1, some students identified the field's direction on both sides of the plate but did not write an expression of the magnitude. Other students identified only the magnitude but not the direction. In item Q2.3, some students correctly interpreted the field direction in the field line diagram and expressed a mathematical relationship in terms of radius and angular direction in polar coordinates. Although they identified that the field changes with the radius, they did not specify how the electrical field depends on the distance.

The "memorization" category included students who described that the magnitude of the electric field is σ/ε_0, which corresponds to twice the magnitude of the field on each side of the plate in item Q2.1 (a). Sometimes students justified that this was achieved by applying Gauss' law to obtain the field in this situation. It is inferred that this reasoning may be the memorization of applying Gauss' law. Students may be familiar with the case of two evenly distributed, opposite-sign parallel plates, which have the same magnitude as they reported in their answers.

From the results in this section, we observed that when constructing mathematical expressions in algebraic notation, some difficulties persist as students

Table 8. Results of recognition and conversion with the algebraic notation as a source or target representation

Source representation			Algebraic notation		Algebraic notation		Vector field plot		Electric field lines	
Target representation			Vector field plot		Electric field lines		Algebraic notation		Algebraic notation	
Cat	Rec	Con	Intro	UD	Intro	UD	Intro	UD	Intro	UD
A	High	High	16%	21%	2%	5%	30%	45%	12%	29%
B	High	Med	11%	40%	35%	47%	3%	2%	0%	0%
B′	Med	High	0%	0%	0%	0%	22%	21%	4%	10%
C	Med	Med	48%	29%	46%	41%	19%	14%	45%	35%
D	Low	Low	25%	10%	17%	7%	26%	19%	39%	26%
Total			100%	100%	100%	100%	100%	100%	100%	100%

Note. The results for each conversion process are according to the theoretical structure of categories (Cat) that determine the success in recognition (Rec) or conversion (Con) for introductory (Intro) and upper-division (UD) students

move from introductory to upper division. Students explained the mathematical characteristics without involving the physical sense of the situation, describing only the magnitude or direction, or using rote learning resources. However, the difficulty "dependency on distance" does present a qualitatively significant decrease in the situation of parallel plates (item Q2.1) but not in conversion questions (item Q1.3 and item Q2.3). Combined with the correct-response analysis, this analysis shows that the identified difficulties are persistent, while the correct response increases, suggesting a decrease in non-classifiable responses.

Conversion Between Representations of the Electric Field

The previous sections presented and analyzed the descriptive categories that emerged in the conversion items. These categories were further analyzed within a theoretical framework to denote the levels of recognition and conversion (high, medium, or low). This theoretical structure allows for item comparison, even when the conversion tasks differ. The conversion categories follow a hierarchical order, where category A means a high level of recognition and conversion; in category B and category B′, either the recognition or the conversion presented difficulties (identified as medium). In category C, both recognition and conversion presented difficulties. Category D groups the students whose difficulties could not be identified in the categories mentioned above or who did not answer the question (identified as low). We present the results of conversions, with the algebraic notation as the source or target representation, in **Table 8**. The results are presented for all the conversion processes for both introductory and upper-division profiles. The first two columns correspond to the algebraic notation as a target representation, and the last two as a source representation.

Table 8 shows that recognition difficulties occur in about 60% of introductory students and 40% of upper-division students (categories B′, C, and D) when the algebraic notation is the source representation. This result implies that 40% to 60% of students can interpret algebraic notation effectively during conversion processes. The previous results show that the main difficulties in recognizing algebraic notation were similar across target representations. These difficulties were that students interpreted the electric field as pointing upward inside the plate, or as having a constant, greater magnitude there. Some students interpreted the electric field outside as varying with distance. These difficulties relate to interpreting the physical situation more than the electric field's mathematical expression in this case.

As shown in **Table 8**, when the algebraic notation is the target representation, and students make a successful recognition, they tend to make a successful conversion (category A and category B). In this case, the conversion difficulties in the C and D categories are related to the source representation. When the source representation is the vector field diagram, conversion difficulties are around 40% of students, while for the electric field lines diagram, more than 60% of students. The most important outcome when the algebraic notation is the target representation is that we have category B′, which means that students can convert to algebraic notation while still struggling to recognize the electric field in the target representation. We can see that this tendency is higher when the vector field diagram is the source representation, which means that, when converting from the vector field diagram to the algebraic notation, students can make a conversion without explicitly recognizing the electric field's characteristics in the source representation.

DISCUSSION

To address the RQs, we discuss our findings on the recognition and conversion of algebraic notation for the electric field. First, we examine students' recognition of key field characteristics within algebraic notation, including both their interpretation of given expressions and their construction of expressions from stated field features (RQ1). Next, we analyze students' difficulties when converting from algebraic notation to vector field diagrams and electric field lines diagrams (RQ2). We then discuss students' difficulties when converting vector field diagrams and electric field lines diagrams to algebraic notation (RQ3). Finally, we compare whether

these difficulties differ when algebraic notation is analyzed alone versus in combination with other representations.

Recognition of the Algebraic Notation

Analyzing students' interpretation of the algebraic notation for the electric field, we observed that about 30% of introductory students and 50% of upper-division students successfully recognized the electric field's characteristics in this representation. As presented in **Table 2**, 38% of introductory students and 51% of upper-division students correctly interpreted the magnitude of the electric field in the equation in item Q1.1 (a). Interpreting the direction of the electric field proved more challenging than its magnitude, as only 22% of introductory and 35% of upper-division students made a correct interpretation. The main difficulties students had interpreting this representation related to treatment errors (in **Table 3**, 21% of introductory students and 23% of upper-division students in item Q1.1 [a]), alternative explanations about the direction of the field in the different positions (41% introductory and 40% upper-division students in item Q1.1 [b]), and the use of the physical meaning of the situation without interpreting the equation provided in the algebraic notation (12% introductory and 14% upper-division students in item Q1.1 [a], and 10% introductory and 14% upper-division students in item Q1.1 [b]).

From these results, we can see that more upper-division students provided correct interpretations of both the magnitude and direction than introductory students. However, the percentages of difficulties are very similar across both populations, suggesting that these difficulties persist among more experienced students. The nature of the difficulties reinforces the idea that students have difficulty relating mathematical expressions to their physical meanings, consistent with other studies (Albe et al., 2001; Pepper et al., 2012; Wilcox et al., 2013). On the one hand, both the difficulty of identifying the magnitude of treatment errors and the difficulty of alternative explanations of the direction are more inclined towards the algorithmic use of the algebraic notation, without considering the physical meaning. In the examples shown, students seem to use only algebraic notation to explain their answers, without relating the representation's characteristics to those of the field. On the other hand, the difficulties of using physical meaning without interpreting the equation are more inclined towards a qualitative analysis of the physical situation presented, without recognizing the characteristics of the electric field in the algebraic notation and working with it. From the difficulties that we identified with this analysis, it seems that the bridge between the qualitative analysis of the physical problem and the algorithmic use of the algebraic notation was not fully stable in these students, so they would stay on either side of the bridge.

Prior work on the role of mathematics in electromagnetism suggests several complementary sources for the pattern we observe. First, students may frame tasks involving algebraic notation primarily as procedural treatments (i.e., manipulating symbols to obtain an answer) rather than as opportunities for sense-making about the physical situation. This type of "chug-and-plug" framing can promote algorithmic use of equations without coordinating symbolic structure with physical meaning (Bagno et al., 2008; Tuminaro & Redish, 2007). In this perspective, students may focus on surface features (e.g., a distance dependence or a familiar functional form) while leaving implicit or omitting entirely the conceptual meaning of terms, signs, and unit-vector directions, consistent with studies emphasizing difficulties in explaining and interpreting equations in electromagnetism (Albe et al., 2001; Hull et al., 2013; Karam, 2014; Karam & Krey, 2015).

In addition, converting between algebraic notation and field representations requires representational fluency with vectors (magnitude and direction, unit vectors, and spatial dependence) and, especially at upper-division level, resources from vector calculus. Persistent difficulties with these ideas have been documented in electrodynamics, including interpreting and sketching vector fields and coordinating symbolic and graphical representations (Bollen et al., 2015, 2016, 2017; Hoyer & Girwidz, 2024). From this standpoint, some "template-based" responses in our categories, such as interpreting the field inside the charged plate as uniform and capacitor-like, can be understood as overgeneralization from canonical instructional cases when the invariant physical conditions of the problem are not fully recognized (Furió & Guisasola, 1998; Li & Singh, 2017; Maries et al., 2022). Within Duval's (2006) framework, these mechanisms can prevent recognition of invariant characteristics across registers and encourage support on familiar surface cues rather than coordination of representations through meaning-based conversion.

Analyzing students' construction of mathematical expressions in algebraic notation, we observed that introductory students were unable to represent the electric field's characteristic algebraically successfully. The evidence in **Table 4** suggests that none of the introductory students could correctly portray the field of a single charged plate, and only 8% could represent the field of two charged plates. Still, their skills improved in the upper-division profile, as 24% correctly represented the field for one charged plate and 26% for the system with two charged plates. The analysis of construction difficulties will focus only on the case with one plate, since the case with two plates was analyzed to assess students' ability to apply superposition rather than construct the representation. The main difficulties in constructing mathematical expressions in algebraic notation (item Q2.1 [a] in **Table 5**, **Table 6**, and **Table 7**)

were representing the characteristics of the algebraic representation without analyzing the physical meaning of the electric field (**Table 5**, 10% introductory and 17% upper-division students), the partial description of the electric field in terms of magnitude or direction (**Table 6**, 11% introductory and 17% upper-division students), and rote learning resources such as the dependence on distance (**Table 5**, 33% introductory and 19% upper-division students) and a rote application of Gauss' law (**Table 7**, 19% in both populations), which could be interpreted as a "chug and plug" approach (Tuminaro & Redish, 2007). We generally observed that these difficulties are independent of context or physical situation and do not change considerably when moving from an introductory to an upper-division profile. When constructing the representation, students could create mathematical expressions that adequately described the electric field without explicitly describing the physical meaning. We found evidence that students' difficulties in relating mathematical expressions to their physical meanings are not only present in interpretation tasks but also in constructing the algebraic notation for the electric field.

This subsection discussed the students' difficulties in interpreting and constructing the algebraic notation for the electric field (item Q1.1 and item Q2.1). This analysis showed that when working with the algebraic notation alone, students have difficulty relating the mathematical expressions to their physical meanings, both in interpretation and construction tasks. In the following subsection, we will discuss whether similar or different difficulties arise in conversion tasks where the algebraic notation may interact with other semiotic representations, such as the vector field diagram and the electric field lines diagram.

Conversions of the Algebraic Notation as a Source or Target Representation

When the algebraic notation was the source representation, the results presented in **Table 8** show that around 30% of introductory students and more than 50% of upper-division students had a high level of recognition in both conversions (adding category A and category B). When converting to a vector field diagram, 28% of introductory and 60% of upper-division students correctly recognized the characteristics of the electric field, even though some had difficulty representing them in the vector field diagram. When converting to the electric field lines diagram, 37% of introductory and 53% of upper-division students made a correct interpretation of the mathematical expression, even though the vast majority presented difficulties representing the electric field's characteristics in the electric field lines diagram. As this part of the discussion focuses on the ability to recognize the algebraic notation, the difficulties of constructing electric field lines have been analyzed elsewhere (Campos et al., 2023).

The main recognition difficulties (category C in **Table 8**) were concentrated on an alternative interpretation of the electric field inside the plate: that it has an upward direction and a constant magnitude in this region of the system. These difficulties accounted for 46% of introductory and 41% of upper-division students when converting to the electric field lines diagram, and 48% of introductory and 29% of upper-division students when converting to the vector field diagram (item Q1.2 and item Q2.2 in **Table 3**). This alternative interpretation may result from rote learning, as students might confuse the presented system (a sizeable non-conductive plate with a uniform volumetric charge distribution) with that of an electric capacitor (which, effectively, the electric field inside presents the characteristics students in this category explain). Such overgeneralization from canonical capacitor-field examples, and associated difficulties with electric-field reasoning and superposition, have been widely reported in the literature (Furió & Guisasola, 1998; Li & Singh, 2017; Maries et al., 2022).

When the algebraic notation was the target representation, we analyzed students' ability to construct this representation after interpreting either a vector field diagram or an electric field lines diagram. In analyzing the results presented in **Table 8**, we observe that category B′ emerged when the algebraic notation was the target representation in this study. Recalling the definition of categories, B′ means successful conversion skills but difficulties in recognition. This result is impressive because, in both cases, it is converted from a graphical, visual representation (the electric field lines diagram and the vector field diagram) to a symbolic, algebraic register (algebraic notation). This category suggests that students do not need to fully recognize the electric field's characteristics in source representations to convert to an algebraic notation that adequately represents those characteristics. The outcome suggests that students can memorize situations and resort to alternative interpretations of source representations to perform the conversion.

When converting from the electric field lines diagram to algebraic notation, some students resort to an alternative interpretation as they attempt to associate the field line diagram with a magnetic source (4% of introductory and 10% of upper-division students in Q2.3 in **Table 5**). In this case, since the electric field is non-Coulombic, the algebraic expression presented by the students is a good conversion because it includes a proportionality constant (specifically, μ_0) and an inverse dependence of the radius. However, students include an electric current as the field source in their equations (usually I or I0), not included in the problem. This difficulty implies that students associate the field line diagram with circular symmetry with the typical magnetic field problem generated by a current line, implying some memorization of magnetism problems.

When converting from the vector field diagram to the algebraic representation, around a fifth of students appropriately convert to algebraic notation without relating the characteristics of the representation to the electric field (22% of introductory and 21% of upper-division students, in item Q1.3 in **Table 5**). Students do not explicitly interpret that the electric field is uniform or that the characteristics of both records reflect those of the electric field. This category implies that students are familiar with converting between vector field diagrams and algebraic notation in other contexts and can do so without accessing the object being rendered.

This behavior was more prominent when converting from the vector field diagram to the algebraic notation than when converting from the electric field lines diagram. This implies more synergy between the algebraic notation and the vector field diagram, as reported in Campos et al. (2020). Students may be more familiar with this type of conversion from previous physics and mathematics courses, and they can make it without explicitly accessing the electric field concept. We acknowledge that students may access the mathematical object in the conversion process, but do not explicitly describe the characteristics that represent the electric field in each register. In the other case, when converting from the electric field lines diagram to the algebraic notation, we did observe that students could convert efficiently without explicitly accessing the object, but at a lower rate. This result is expected since interpreting the electric field lines diagram conveys its difficulties (Campos & Zavala, 2017; Li & Singh, 2017, Törnkvist et al., 1993).

Because our study relies on open-ended written responses (rather than interviews), we cannot directly observe students' real-time reasoning; therefore, we infer "access to physical meaning" when students explicitly coordinate the physical situation (source and region/symmetry conditions) with the representational structure (e.g., direction/unit vectors, spatial dependence, and magnitude trends) when interpreting or constructing the algebraic notation. Conversely, we infer limited access when students produce a formally plausible conversion but do not articulate the corresponding physical conditions or meanings in their explanations. The clearest evidence of this pattern is category B′ in **Table 8**, which captures successful conversion with only medium recognition when algebraic notation is the target representation: for example, 22% of introductory students and 21% of upper-division students converted from a vector field plot to an adequate algebraic expression while still displaying difficulties in recognizing and describing the field's characteristics in the source representation. A smaller but analogous pattern occurs when converting from electric field lines to algebraic notation (4% introductory; 10% upper-division). Consistent with this, we also observed responses in which students imported familiar templates or sources (e.g., capacitor-like uniform fields or current-based expressions in circular patterns) even when those sources were not present in the problem context. Taken together, these patterns justify the conclusion that a nontrivial subset of students can carry out conversions as a representation-to-representation mapping, without explicitly evidencing meaning-based coordination between registers in their written work.

CONCLUSION

In the present study, we combined the theory of registers of semiotic representations as a theoretical framework with the phenomenographic approach to data analysis to link difficulties in recognition and conversion between representations to students' conceptual understanding of the electric field. The study methodology allows students to identify students' difficulties in recognizing and converting the algebraic notation of the electric field using the phenomenographic approach and linking these difficulties to the conceptual understanding of the electric field through the theory of registers of semiotic representations. The main findings are that students' difficulties in interpreting and constructing the algebraic notation include disassociating the physical meaning and the algebraic representation itself and using rote learning strategies. This behavior was observed in recognition and conversion processes, although different in nature.

We acknowledge the limitations of this study. Opting for open-ended questionnaires was important for analyzing a larger number of participants, but it compromised the depth of the analysis. Conducting interviews could provide a deeper understanding of students' difficulties, but it would be possible to work with fewer students. Another possible limitation is that looking into the use of semiotic representations is not the only way to approach students' understanding of the electric field concept. Other students' abilities can be considered, such as qualitatively analyzing a problem before solving it and the recourse to physical principles and energy considerations, which are out of the scope of this study.

In the teaching and learning of science, it is necessary to understand students' difficulties to identify possible teaching approaches that guide them in overcoming them. Our results suggest that explaining the conversion process between different representation systems is crucial to contributing to teaching and learning in science. Physics instructors should also consider that a strong synergy among representations is necessary for conceptual understanding of abstract properties. Dedicating time and practice to coordinating various representations of the electric field concept is essential to promoting the conceptual understanding of this physical

quantity. Students need to be able to dissociate the electric field concept from the representations through the conversion tasks. At the same time, they need to be able to relate the representation to its physical meaning. We observed that a substantial subset of students could carry out conversions while not explicitly articulating (in their written explanations) the physical meaning of the electric field concept or the physical conditions corresponding to the expressions they produced (see, for example, category B′ patterns in **Table 8**). Based on these research results, professors and researchers in the field can consider the difficulties posed by certain recognition and conversion tasks; thus, this work contributes to discussions of the teaching-learning processes of the electric field concept at the university level.

More concretely, our findings indicate that instructors should design explicit conversion tasks that require students to move in both directions between algebraic notation and graphical representations (especially vector field diagrams and electric field-line diagrams), rather than only translating algebraic expressions into diagrams. In addition to performing algebraic manipulations, students should be asked to explain the physical meaning of each term in an expression for the electric field, so that they practice coordinating symbolic structure with physical interpretation. Instruction can also explicitly target common mis-associations identified in this study, such as treating any uniform field as a capacitor field or interpreting circular field patterns exclusively as magnetic fields, by contrasting correct and incorrect examples and discussing the underlying physical conditions. Finally, it may be beneficial to incorporate problems that require students to use both algebraic and qualitative reasoning within the same task, for example, by first predicting the qualitative behavior of the field and then constructing and interpreting an algebraic expression consistent with that behavior. These activities align with our analysis of recognition and conversion categories and can help students build a more flexible, conceptually grounded use of algebraic notation in electromagnetism.

Overall, our study contributes to research on semiotic registers in physics by providing a detailed characterization of how university students recognize and convert algebraic representations of the electric field, and by showing that the role of algebraic notation as source or target representation systematically shapes the kinds of difficulties that emerge.

Author contributions: GZ: writing – original draft, supervision, funding acquisition, resources, validation, methodology, investigation, formal analysis, data curation, conceptualization; **KZ:** writing – review & editing, validation, formal analysis; **JG:** writing – review & editing, investigation, conceptualization, methodology; **EC:** writing – review & editing, validation, methodology, formal analysis, data curation, conceptualization, project administration.All authors sufficiently contributed to this study and agreed with the results and conclusions.

Funding: This study is a product of a project funded in the Challenge-Based Research Funding Program 2022 Project I010-IFE002-C1-T2-T by Tecnologico de Monterrey.

Acknowledgments: The authors would like to acknowledge the financial support of Writing Lab, Institute for the Future of Education, Tecnologico de Monterrey, Mexico, in the production of this work.

Ethical statement: The authors stated that the study was approved by the Institutional Review Board at Tecnologico de Monterrey. Written informed consents were obtained from the participants.

AI statement: The authors stated that the generative AI tool ChatGPT was used to enhance the clarity of the English language in the manuscript when necessary, and Grammarly was utilized for proofreading.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Albe, V., Venturini, P., & Lascours, J. (2001). Electromagnetic concepts in mathematical representation of physics. *Journal of Science Education and Technology, 10*(2), 197-203. https://doi.org/10.1023/A:1009429400105

Bagno, E., Berger, H., & Eylon, B. S. (2008). Meeting the challenge of students' understanding of formulae in high-school physics: A learning tool. *Physics Education, 43*(1), 75-82. https://doi.org/10.1088/0031-9120/43/1/008

Barniol, P., & Zavala, G. (2014). Test of understanding of vectors: A reliable multiple-choice vector concept test. *Physical Review Special Topics–Physics Education Research, 10*, Article 010121. https://doi.org/10.1103/PhysRevSTPER.10.010121

Bollen, L., van Kampen, P., & De Cock, M. (2015). Students' difficulties with vector calculus in electrodynamics. *Physical Review Special Topics–Physics Education Research, 11*, Article 020129. https://doi.org/10.1103/PhysRevSTPER.11.020129

Bollen, L., van Kampen, P., Baily, C., & De Cock, M. (2016). Qualitative investigation into students' use of divergence and curl in electromagnetism. *Physical Review Physics Education Research, 12*, Article 020134. https://doi.org/10.1103/PhysRevPhysEducRes.12.020134

Bollen, L., van Kampen, P., Baily, C., Kelly, M., & De Cock, M. (2017). Student difficulties regarding symbolic and graphical representations of vector fields. *Physical Review Physics Education Research, 13*, Article 020109. https://doi.org/10.1103/PhysRevPhysEducRes.13.020109

Campos, E., Hernandez, E., Barniol, P., & Zavala, G. (2021). Phenomenographic analysis and comparison of students' conceptual understanding of electric and magnetic fields and the principle of

superposition. *Physical Review Physics Education Research*, *17*(2). https://doi.org/10.1103/physrevphyseducres.17.020117

Campos, E., & Zavala, G. (2017). A Look into Students' Interpretation of Electric Field Lines. In *Advances in educational technologies and instructional design book series* (pp. 342-364). https://doi.org/10.4018/978-1-5225-2026-9.ch017

Campos, E., Silva, L., Tecpan, S., & Zavala, G. (2016, July 20-21). Argumentation during active learning strategies in a SCALE-UP environment. Paper presented at *Physics Education Research Conference 2016*, Sacramento, CA. https://doi.org/10.1119/perc.2016.pr.011

Campos, E., Zavala, G., Zuza, K., & Guisasola, J. (2019). Electric field lines: The implications of students' interpretation on their understanding of the concept of electric field and the superposition principle. *American Journal of Physics, 87*(8), 660-667. https://doi.org/10.1119/1.5100588

Campos, E., Zavala, G., Zuza, K., & Guisasola, J. (2020). Students' understanding of the concept of the electric field through conversions of multiple representations. *Physical Review Physics Education Research*, *16*(1). https://doi.org/10.1103/physrevphyseducres.16.010135

Campos, E., Zuza, K., Guisasola, J., & Zavala, G. (2023). Recognition and conversion of electric field representations: The case of electric field lines. *Physical Review Physics Education Research*, *19*(2). https://doi.org/10.1103/physrevphyseducres.19.020117

Campos, E., Zuza, K., Guisasola, J. & Zavala, G. (2026). Recognition and conversion of electric field representations: The case of vector field plot. *Eurasia Journal of Mathematics, Science and Technology Education, 22*(1), Article em2759. https://doi.org/10.29333/ejmste/17633

Ceuppens, S., Deprez, J., Dehaene, W., & De Cock, M. (2018). Design and validation of a test for representational fluency of 9th grade students in physics and mathematics: The case of linear functions. *Physical Review Physics Education Research, 14*, Article 020105. https://doi.org/10.1103/PhysRevPhysEducRes.14.020105

Chasteen, S. V. (2012). Teasing out the effect of tutorials via multiple regression. *AIP Conference Proceedings, 1413*(1), 143-146. https://doi.org/10.1063/1.3680015

Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics, 69*(9), 970-977. https://doi.org/10.1119/1.1374249

Deliyianni, E., Gagatsis, A., Elia, I., & Panaoura, A. (2016). Representational flexibility and problem-solving ability in fraction and decimal number addition: A structural model. *International Journal of Science and Mathematics Education, 14*, 397-417. https://doi.org/10.1007/s10763-015-9625-6

Dominguez, A., De la Garza, J., Quezada-Espinoza, M., & Zavala, G. (2024). Integration of physics and mathematics in STEM education: Use of modeling. *Education Sciences, 14*(1), Article 20. https://doi.org/10.3390/educsci14010020

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1-2), 103-131. https://doi.org/10.1007/s10649-006-0400-z

Furió, C., & Guisasola, J. (1998). Difficulties in learning the concept of electric field. *Science Education, 82*(4), 511-526. https://doi.org/10.1002/(SICI)1098-237X(199807)82:4<511::AID-SCE6>3.0.CO;2-E

Garza, A., & Zavala, G. (2013). Contrasting students' understanding of electric field and electric force. *AIP Conference Proceedings, 1513*(1), 142-145. https://doi.org/10.1063/1.4789672

Griffiths, D. J. (2017). *Introduction to electrodynamics* (4th ed.). Cambridge: Cambridge University Press. https://doi.org/10.1017/9781108333511

Guisasola, J., Campos, E., Zuza, K., & Zavala, G. (2023). Phenomenographic approach to understanding students' learning in physics education. *Physical Review Physics Education Research*, *19*(2). https://doi.org/10.1103/physrevphyseducres.19.020602

Hahn, L., & Klein, P. (2023). The impact of multiple representations on students' understanding of vector field concepts: Implementation of simulations and sketching activities into lecture-based recitations in undergraduate physics. *Frontiers in Psychology, 13*, Article 1012787. https://doi.org/10.3389/fpsyg.2022.1012787

Hernandez, E., Campos, E., Barniol, P., & Zavala, G. (2022). Phenomenographic analysis of students' conceptual understanding of electric and magnetic interactions. *Physical Review Physics Education Research, 18*(2), Article 020101. https://doi.org/10.1103/PhysRevPhysEducRes.18.020101

Hernandez, E., Campos, E., Barniol, P., & Zavala, G. (2025). Students' understanding of electric flux and magnetic circulation and the role of the superposition principle in Gauss's and Ampere's laws. *Physical Review Physics Education Research, 21*(1), Article 010120. https://doi.org/10.1103/PhysRevPhysEducRes.21.010120

Hoyer, C., & Girwidz, R. (2024). Vector representations and unit vector representations of fields: Problems of understanding and possible teaching strategies. *Physical Review Physics Education Research, 20*,

Article 010150. https://doi.org/10.1103/PhysRevPhysEducRes.20.010150

Hull, M. M., Kuo, E., Gupta, A., & Elby, A. (2013). Problem-solving rubrics revisited: Attending to the blending of informal conceptual and formal mathematical reasoning. *Physical Review Special Topics–Physics Education Research, 9*, Article 010105. https://doi.org/10.1103/PhysRevSTPER.9.010105

Karam, R. (2014). Framing the structural role of mathematics in physics lectures: A case study on electromagnetism. *Physical Review Special Topics–Physics Education Research, 10*, Article 010119. https://doi.org/10.1103/PhysRevSTPER.10.010119

Karam, R., & Krey, O. (2015). Quod erat demonstrandum: Understanding and explaining equations in physics teacher education. *Science & Education, 24*(5-6), 661-698. https://doi.org/10.1007/s11191-015-9743-0

Li, J., & Singh, C. (2012). Students' difficulties with equations involving circuit elements. *AIP Conference Proceedings, 1413*(1), 243-246. https://doi.org/10.1063/1.3680040

Li, J., & Singh, C. (2017). Investigating and improving introductory physics students' understanding of the electric field and superposition principle. *European Journal of Physics, 38*(5), Article 055702. https://doi.org/10.1088/1361-6404/aa7618

Maries, A., Brundage, M. J., & Singh, C. (2022). Using the conceptual survey of electricity and magnetism to investigate progression in student understanding from introductory to advanced levels. *Physical Review Physics Education Research, 18*, Article 020114. https://doi.org/10.1103/PhysRevPhysEducRes.18.020114

Marton, F. (1981). Phenomenography—Describing conceptions of the world around us. *Instructional Science, 10*(2), 177-200. https://doi.org/10.1007/BF00132516

McDermott, L. C. & Redish, E. F. (1999). Resource letter: PER-1: Physics education research. *American Journal of Physics, 67*(9), 755-767. https://doi.org/10.1119/1.19122

McDermott, L. C., Shaffer, P. S., & Physics Education Group (2002). *Tutorials in introductory physics*. Upper Saddle River: Prentice Hall Inc.

McGee, D. L., & Martinez-Planell, R. (2014). A study of semiotic registers in the development of the definite integral of functions of two and three variables. *International Journal of Science and Mathematics Education, 12*, 883-916. https://doi.org/10.1007/s10763-013-9437-5

Moreno-Arotzena, O., Pombar-Hospitaler, I., & Barragués, J. I. (2021). University student understanding of the gradient of a function of two variables: An approach from the perspective of the theory of semiotic representation registers. *Educational Studies in Mathematics, 106*, 65-89. https://doi.org/10.1007/s10649-020-09994-9

Pepper, R. E., Chasteen, S. V., Pollock, S. J., & Perkins, K. K. (2012). Observations on student difficulties with mathematics in upper-division electricity and magnetism. *Physical Review Special Topics–Physics Education Research, 8*, Article 010111. https://doi.org/10.1103/PhysRevSTPER.8.010111

Rainson, S., Tranströmer, G., & Viennot, L. (1994). Students' understanding of superposition of electric fields. *American Journal of Physics, 62*(11), 1026-1032. https://doi.org/10.1119/1.17701

Redish, E. F. (2006). *Problem solving and the use of math in physics courses*. arXiv. https://doi.org/10.48550/arXiv.physics/0608268

Robertson, W. C. (1990). Detection of cognitive structure with protocol data: Predicting performance on physics transfer problems. *Cognitive Science, 14*(2), 253-280. https://doi.org/10.1207/s15516709cog1402_3

Saarelainen, M., Laaksonen, A., & Hirvonen, P. E. (2007). Students' initial knowledge of electric and magnetic fields—More profound explanations and reasoning models for undesired conceptions. *European Journal of Physics, 28*(1), 51-60. https://doi.org/10.1088/0143-0807/28/1/006

Singh, C. (2005). Student understanding of symmetry and Gauss's law. *AIP Conference Proceedings, 790*(1), 65-68. https://doi.org/10.1063/1.2084702

Svensson, K., & Campos, E. (2022). Comparison of two semiotic perspectives: How do students use representations in physics? *Physical Review Physics Education Research, 18*, Article 020120. https://doi.org/10.1103/PhysRevPhysEducRes.18.020120

Törnkvist, S., Pettersson, K. A., & Tranströmer, G. (1993). Confusion by representation: On student's comprehension of the electric field concept. *American Journal of Physics, 61*(4), 335-338. https://doi.org/10.1119/1.17265

Trigueros, M., & Martínez-Planell, R. (2010). Geometrical representations in the learning of two-variable functions. *Educational Studies in Mathematics, 73*, 3-19. https://doi.org/10.1007/s10649-009-9201-5

Tuminaro, J., & Redish, E. F. (2007). Elements of a cognitive model of physics problem solving: Epistemic games. *Physical Review Special Topics–Physics Education Research, 3*, Article 020101. https://doi.org/10.1103/PhysRevSTPER.3.020101

Uhden, O., Karam, R., Pietrocola, M., & Pospiech, G. (2012). Modelling mathematical reasoning in physics education. *Science & Education, 21*(4), 485-506. https://doi.org/10.1007/s11191-011-9396-6

van Kampen, P., & De Cock, M. (2023). Students' understandings of electricity and magnetism. In M. F. Taşar, & P. R. L. Heron (Eds.), *The international handbook of physics education research: Learning physics* (pp. 5-1-5-24). Melville: AIP Publishing. https://doi.org/10.1063/9780735425477_005

Viennot, L., & Rainson, S. (1999). Design and evaluation of a research-based teaching sequence: The superposition of electric field. *International Journal of Science Education, 21*(1), 1-16. https://doi.org/10.1080/095006999290804

Wilcox, B. R., Caballero, M. D., Pepper, R. E., & Pollock, S. J. (2013). Upper-division student understanding of Coulomb's law: Difficulties with continuous charge distributions. *AIP Conference Proceedings, 1513*(1), 418-421. https://doi.org/10.1063/1.4789741

Young, H. D., & Freedman, R. A. (2013). *University physics with modern physics* (13th ed.). San Francisco: Pearson.

Zavala, G. (2019). The design of activities based on cognitive scaffolding to teach physics. In *Upgrading physics education to meet the needs of society* (pp. 169-179). Springer International Publishing. https://doi.org/10.1007/978-3-319-96163-7_11