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Recognition and conversion of electric field representations: The case of vector field plot

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Abstract

This conceptual understanding article is part of a series where we explore the recognition and conversion between representations of the electric field concept; in this article, we present the vector field plot's case. We conducted a study with higher education introductory and upper-division Physics students in a large private Mexican university, intending to learn how students recognize the electric field's main characteristics in the vector field plot independently and as a source or target representation in conversion processes. We used the theory of registers of semiotic representations and a phenomenographic approach as a framework to analyze data. We explored students' recognition and conversion abilities through interpretation and construction tasks. We found that the main difficulties of interpreting and constructing the vector field plot are related to the representation's surface features, often delimited by physical considerations. In the interpretation tasks, this was observed as the interpretation of the physical symmetry, while in construction tasks, it was evident by the confinement of vectors and other associated difficulties. In conversion processes, we observed that there might be an interaction between the source and the target representation, which may result in difficulties in interpreting the vector field plot. We recommend that higher education introductory and upper-division electricity and magnetism instructors and physics education researchers be aware of the difficulties that some interpretation and construction tasks may present to students learning the electric field concept.

Keywords: conceptual understanding, physics education research, electricity and magnetism, higher education, STEM education, educational innovation

INTRODUCTION

In university-level physics, the comprehension of abstract physical phenomena depends on interpreting mathematical representations expressed through diverse modes, including symbolic, graphical, and verbal forms. Within electromagnetism, mastering the concept of the electric field requires understanding its representation through vector field plots. Because this type of representation also appears in other domains of physics, such as kinematics and mechanics, students are expected to be acquainted with vector field plots before or during introductory and advanced electromagnetism courses. Nonetheless, prior research has shown that specific representational features can mislead students as they attempt to make sense of the electric field (Gire & Price, 2014), largely due to its abstract nature (McDermott & Redish, 1999). Therefore, recognizing and addressing students' conceptual challenges related to the mathematical representations of the electric field is essential for designing effective instructional strategies (Bollen et al., 2018).

This work forms part of a series of exploratory investigations designed to examine students' capacity to

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Contribution to the literature

- This study identifies specific challenges physics students face in recognizing and constructing vector field plots of electric fields, particularly when interpreting surface features and applying the superposition principle. It contributes insights into persistent learning obstacles in electromagnetism.
- By examining how students convert between vector field plots, algebraic notation, and electric field line diagrams, this research reveals the nuanced ways in which interactions between these representations can lead to non-scientific conceptions, especially in recognizing field magnitude and direction.
- The findings support the need for explicit instructional strategies that address the interpretation and conversion between electric field representations, suggesting practical approaches to improve conceptual understanding in physics education.

recognize and translate among three representations of the electric field: algebraic notation, vector field plots, and electric field line diagrams. According to prior research, students' skills in recognizing and converting across these representations are closely tied to their conceptual grasp of the topic (Campos et al., 2020; Duval, 2006). Understanding the specific challenges students face in both recognition and conversion processes is essential for developing instructional strategies that effectively address them. In previous studies, we reported preliminary findings on students' difficulties when performing conversion tasks involving these three representations (Campos et al., 2020), as well as a detailed analysis centered on the electric field line diagram (Campos et al., 2023). The present study extends that work by focusing on students' comprehension of the vector field plot–examining their interpretation and construction of this representation in isolation and identifying the challenges that emerge when the vector field plot functions as either the source or target in conversion activities.

Our analysis was grounded in the theory of registers of semiotic representations (Duval, 2006) and employed a phenomenographic approach (Marton, 1981). In this theoretical framework, semiotic representations are understood as symbolic systems used to express abstract concepts. A register refers to a representational system that enables the transformation of information across different forms, while conversion denotes a process akin to transduction (Svensson & Campos, 2022). Throughout this paper, these terms are used consistently to ensure alignment with the theoretical perspective adopted. A more extensive discussion of these constructs, along with a graphical illustration of their relationships, can be found in our previous publications (Campos et al., 2020, 2023).

Examining students' difficulties when interpreting representations independently versus during conversion tasks provides valuable insight, as these two types of challenges differ in nature. When a representation is analyzed in isolation, students may successfully identify and describe the key features of the electric field depicted. However, once that same representation must be related to another form in a conversion process, additional difficulties often emerge–a phenomenon previously observed in our study centered on the electric field line diagram (Campos et al., 2023). In this work, we hypothesize that a comparable pattern will arise when examining the vector field plot. Conducting this comparative analysis of students' understanding in both isolated and conversion contexts enables a more comprehensive view of the problem and supports the design of instructional strategies that specifically address these challenges.

In this paper, we begin by reviewing the main challenges associated with learning the electric field concept, with particular attention to the difficulties students encounter in understanding the vector field plot. We then describe the methodological foundations of the study, outlining the theoretical perspective of the theory of registers of semiotic representations and the phenomenographic approach used for data analysis. The results are presented in two major sections that correspond to students' interpretation and construction of the vector field plot within the context of the electric field. Finally, we discuss the implications of these findings for teaching and learning the electric field concept at the university level.

LITERATURE REVIEW

Physics education research has long documented the principal conceptual challenges university students face in grasping the electric field concept (van Kampen & De Cock, 2023). A recurring issue is the confusion between the notions of electric field and electric force (Furió & Guisasola, 1998; Garza & Zavala, 2013; Saarelainen et al., 2007). This misunderstanding stems from the conceptual shift required to justify the existence of electric fields when explaining interactions through the Lorentz force equation (Furió & Guisasola, 1998) and may also indicate an incomplete comprehension of the electric field as a vector quantity (Saarelainen et al., 2007; Zuza et al., 2018). Additional studies have shown that students frequently struggle to apply the principle of superposition (Rainson et al., 1994; Viennot & Rainson, 1999). Some believe that only nearby charges contribute to the resultant electric field, or that other charged bodies can obstruct it (Garza & Zavala, 2013; Li & Singh, 2017;

Singh, 2005). Considerable attention has also been devoted to how students interpret representations of the electric field, most notably the electric field line diagram (Campos et al., 2019; Cao & Brizuela, 2016; Törnkvist et al., 1993). Findings from these works indicate that students often treat field lines as tangible objects and experience difficulty linking the field's magnitude to line density. Furthermore, research has shown that such misconceptions persist even after conventional instruction in advanced undergraduate courses (Maries et al., 2022).

Representations are fundamental to grasping the electric field concept and examining the challenges students face with the vector field representation offers valuable insight into the recognition and conversion difficulties explored in this study. Previous research has shown that students often struggle to apply their understanding of vector field representations within the context of the electric field, which hinders the development of a complete conceptual framework (Moynihan et al., 2020). The vector field plot serves as a natural depiction of the electric field, as the field itself is inherently vectorial. However, unlike other vector fields in physics, the electric field is accessible only through semiotic representations (Duval, 2006), including algebraic notation, vector field plots, and electric field line diagrams (Campos et al., 2020). As with any representational system, the vector field plot entails both advantages and constraints. Its main strengths lie in the proportionality between arrow length and field magnitude–allowing for direct evaluation of the field–and in the arrow orientation, which clearly indicates direction and supports the use of the superposition principle. Depicting fields with arrows also tends to activate students' spatial–intuitive reasoning by connecting visual and physical aspects of the representation (Heckler & Scaife, 2015). Nonetheless, this format allows the visualization of the field only at discrete points and merges two distinct spaces: one corresponding to the vector arrows that depict the field and another representing the physical plane where those arrows are placed (Gire & Price, 2013). The use of arrows has also been found to elicit difficulties in performing vector operations at both introductory (Barniol & Zavala, 2014; Heckler & Scaife, 2015) and advanced physics levels (Bollen et al., 2015, 2016). In the specific context of electric fields, the representation can further mislead students because arrows near charge locations appear disproportionately large (Hoyer & Girwidz, 2024). It is also important to distinguish between interpreting and constructing vector representations, as these processes demand distinct cognitive skills (Hoyer & Girwidz, 2024). Recent studies highlight that thoughtfully designed, multi-representational learning environments can strengthen students' representational fluency with vector fields. For example, Hahn and Klein (2023) incorporated sketching exercises and interactive simulations into lecture-based recitations in electrodynamics, demonstrating that such interventions enhanced students' visual interpretation of vector field diagrams and reduced cognitive load during problem-solving. These findings underscore the value of integrating explicit representational practices with traditional instruction to mitigate enduring conceptual challenges.

In introductory-level physics courses, it is essential to provide explicit instruction and repeated practice with vectors to help students develop fluency with this type of representation (Knight, 1995). Despite frequent exposure to vectors across various topics, many students still struggle to connect conceptual understanding with the graphical and mathematical features of vector representations. For instance, some fail to recognize that the quantities in Newton's second law possess vector character (Flores et al., 2004). Research indicates that students experience difficulties not only with the graphical aspects of vectors–such as magnitude, direction, and components–but also with their procedural operations, including vector addition, scalar multiplication by a negative constant, and the vector product (Barniol & Zavala, 2014; Heckler & Scaife, 2015). Such challenges often result in incorrect applications of the superposition principle, even in electricity and magnetism courses where students have previously encountered multiple opportunities to engage with vector reasoning (Nguyen & Meltzer, 2003). Heckler and Scaife (2015) further observed that when working directly with arrow-based representations, students can misinterpret the visual cues and erroneously add vectors where subtraction is required.

At the advanced level, Gire and Price (2014) reported that the graphical form of the vector field plot equips students with key elements for applying the superposition principle effectively. Nevertheless, even at this stage, students often experience difficulties grasping the concepts of divergence and curl, as well as understanding their corresponding physical meanings (Bollen et al., 2015). Research using eye-tracking methodologies has shown that exposure to multiple representations and targeted visual strategies can enhance students' comprehension of divergence and curl in vector field plots (Klein et al., 2018, 2019). Despite this, many advanced students continue to conflate the features of vector field plots with those of field line diagrams (Küchemann et al., 2021; Nguyen & Meltzer, 2003). Moreover, some students construct hybrid representations that merge elements from both the vector field and field line diagrams, as observed in studies addressing electric and magnetic field contexts (Campos et al., 2021a).

Table 1. Definition of the objective of the items in each questionnaire

Objective	Interpretation	Construction
Recognition of the vector field plot	Q1.1	Q2.1
Conversion (source: vector field plot)	Q1.2 & Q2.2	
Conversion (target: vector field plot)		Q1.3 & Q2.3

Note. Each item describes an interpretation or construction task

DEFINITION OF THE STUDY

This research is grounded in the theory of registers of semiotic representations (Duval, 2006), which posits that conceptual understanding of a mathematical object arises from the coordination and interaction among multiple representational systems. In this context, the mathematical object under study is the electric field, represented through three primary forms: the vector field plot, the electric field line diagram, and the algebraic notation of the field. The investigation concentrates on how students recognize the defining characteristics of the electric field within the vector field plot and examines how this representation interacts with the other two–electric field lines and algebraic notation–when functioning as either the source or the target in conversion tasks.

The theory of registers of semiotic representations has been extensively applied in mathematics education research to analyze how learners engage with different representational systems (Bansilal, 2012; Deliyianni et al., 2016; McGee & Martinez-Planell, 2014; Trigueros & Martínez-Planell, 2010). Within the field of physics education research, its relevance has been recognized by numerous studies (Battaglia et al., 2013; Ceuppens et al., 2018; Fazio et al., 2013; Fredlund et al., 2014), and several have explicitly adopted it as their guiding theoretical framework (Campos et al., 2020; Ceuppens et al., 2018). In the present work, we likewise employ this theory as the foundation for the following research questions (RQs):

RQ1. What difficulties do students have in recognizing the electric field's main characteristics in the vector field plot?

RQ2. What difficulties do students have in converting from the vector field plot to the representation of electric field lines and the algebraic notation?

RQ3. What difficulties do students have in converting from the electric field lines diagram and the algebraic notation to the vector field plot?

METHODOLOGY

The research was conducted at a private university in Mexico and involved a total of 295 engineering physics students: 210 enrolled in an introductory electricity and magnetism course and 85 in an upper-division electromagnetic theory course. The introductory course, which was calculus-based, followed a standard textbook and tutorial structure (McDermott & Shaffer, 2001; Young & Freedman, 2013). It was implemented in a SCALE-UP classroom setting and incorporated peer instruction along with cognitive scaffolding as part of its active learning methodology (Campos et al., 2016, 2021b). In contrast, the upper-division course utilized a conventional textbook (Griffiths, 2017) and was delivered primarily through traditional lectures, supplemented by selected active learning techniques. In both courses, the three electric field representations were consistently employed during lectures and within instructional materials, although students did not receive explicit guidance on converting between them.

Two open-ended questionnaires comprising a total of twelve items served as the research instruments. For the purposes of this study, we focus on six of these items–three designed to assess students' interpretation of the vector field plot and three aimed at evaluating their ability to construct it. Each question targeted either recognition or conversion skills between representations, as summarized in **Table 1**. The conversion-related items (Q1.2, Q2.2, Q1.3, and Q2.3) were previously examined and discussed in an earlier publication (Campos et al., 2020), whereas the recognition items (Q1.1 and Q2.1) are analyzed here for the first time. In this expanded investigation, the inclusion of a larger sample enabled the achievement of category saturation and allowed for a more detailed, fine-grained analysis.

Data were collected over three academic semesters between 2018 and 2019, following the completion of instruction. The questionnaires were distributed randomly to all participants during regular class meetings as part of their scheduled coursework activities. Students were explicitly informed that the survey served a research purpose unrelated to course evaluation and that their participation was completely voluntary. All students chose to participate, and no withdrawals occurred, thereby reducing potential concerns about self-selection bias. The surveys were administered in person, using a paper-and-pencil format, and the completion time averaged around fifteen minutes.

Prior to completing the questionnaire, students were informed about the purpose of the study and their right to decline participation at any time. They were reminded that participation was voluntary and that their responses would have no effect on their course grades. No identifying information–such as names or student ID

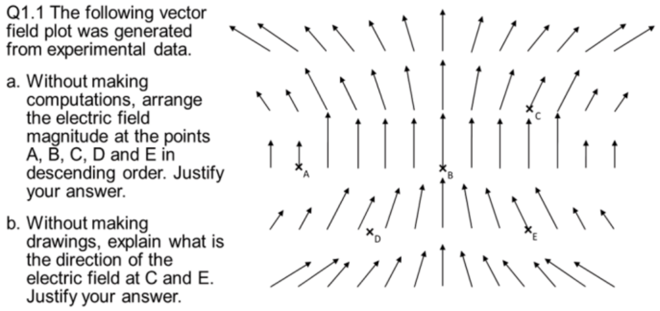

Figure 1. Item Q1.1 targets the recognition of the electric field's magnitude and direction in the vector field plot (Source: Authors' own elaboration)

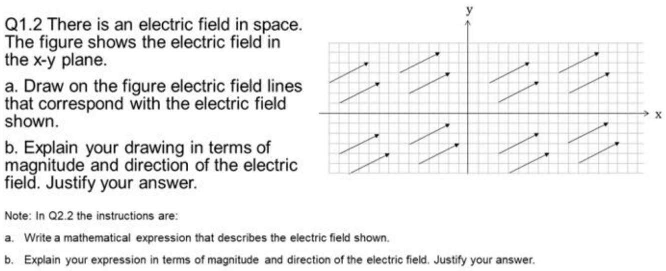

Figure 2. Item Q1.2 and item Q2.2 aim to convert from the vector field plot to the algebraic notation and the electric field lines diagram, respectively (Source: Authors' own elaboration)

numbers–was collected, ensuring complete anonymity in the dataset. To prevent any overlap between objectives, each participant responded to only one version of the questionnaire. The items are presented in the following section. All data collection and analysis were conducted in Spanish, and the items as well as students' responses are shown here in translated form.

Instrument

We present the instrument in two sections:

1. Interpretation tasks: The items that require an interpretation of the electric field's vector plot.
2. Construction tasks: The items that require constructing a vector field plot for the electric field.

Interpretation tasks

We consider three elements for measuring the vector field plot's interpretation: recognizing the field's magnitude (item Q1.1 a), direction (item Q1.1 b), and effectively recognizing the field's characteristics in conversions where the vector field plot is the source representation (item Q1.2 and item Q2.2).

Item Q1.1 was designed to assess students' ability to identify both the magnitude and direction of the electric field represented in a vector field plot, as illustrated in **Figure 1**. These two aspects were evaluated separately in part a and part b. In part a, students were asked to rank the magnitude of the electric field at five marked positions in descending order and to justify their reasoning. A correct response required ordering the magnitudes from greatest to smallest, with any of the following sequences accepted as valid: B > D > E = C > A; B > E = C > D > A; or B > D = E = C > A. Students were expected to base their explanations on the relative size of the arrows as indicators of field magnitude. Some students, however, referred to symmetry to reason about the field at position D–where no vector was drawn–acknowledging that an electric field still exists there. This reasoning highlights a key limitation of vector field diagrams: they display the field only at selected points. Students who recognized this limitation and invoked symmetry to justify the presence of a field at D demonstrated an alternative but valid interpretive approach.

In part b, students were required to describe the electric field's direction at two specific points and explain their rationale. The expected correct responses were to identify the field direction at position C as a diagonal pointing upward and to the right (e.g., northeast, 45° above the horizontal, or + î + ĵ), and at position E as a diagonal pointing upward and to the left (and equivalent descriptions). Students were expected to recognize that the direction of the electric field corresponds directly to the orientation of the arrows at each point in the vector field plot.

Item Q1.2 and item Q2.2 (illustrated in **Figure 2**) both use the vector field plot as the source representation. The same electric field configuration is shown in each item; however, in Q1.2, students are asked to convert it into an electric field line diagram, while in Q2.2, they must express it using algebraic notation. These two conversions are treated independently. Because the vector field plot serves as the source representation, both items are categorized as interpretation tasks. In these exercises, students analyze a vector field plot depicting a uniform electric field–constant in both magnitude and direction. The plot includes Cartesian coordinate axes, allowing students to identify the specific spatial characteristics of the field. They are expected to recognize that the field is uniform, directed diagonally, and that its magnitude can be represented through components along the horizontal and vertical axes.

To convert from the vector field plot to the electric field lines diagram effectively (item Q1.2), students should explain that vectors are tangent to field lines and that field line density describes the magnitude of the electric field. Also, it is necessary to relate uniform spacing between lines to vector size. To convert from the vector field plot to the algebraic notation (item Q2.2), students must recognize that the electric field is uniform and represent it through a sum of constants in Cartesian directions. The constants can be a and b, 4 and 2, E_x and E_y, or any other pair of different constants (e.g., $E_x\hat{\imath} + E_y\hat{\jmath}$ or $4\hat{\imath} + 2\hat{\jmath}$). Students should also relate their responses to

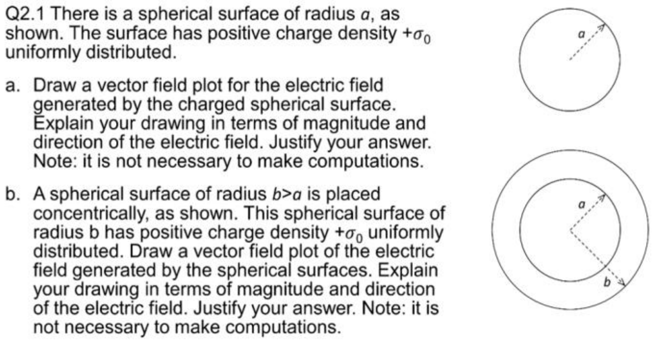

Figure 3. Item Q2.1 targets students' recognition of the electric field and applying the superposition principle in the vector field plot as a construction task (Source: Authors' own elaboration)

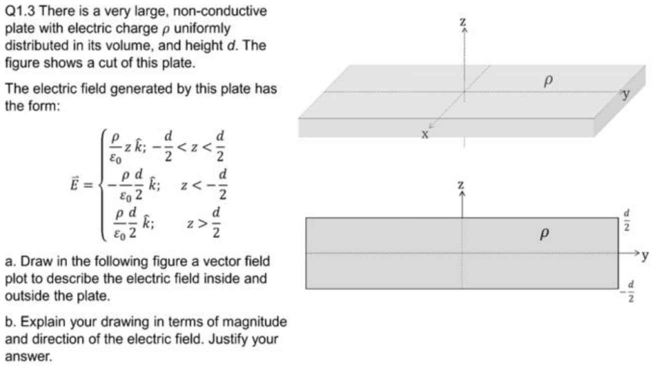

Figure 5. Item Q1.3 aims to convert from the algebraic notation to the vector field plot (Source: Authors' own elaboration)

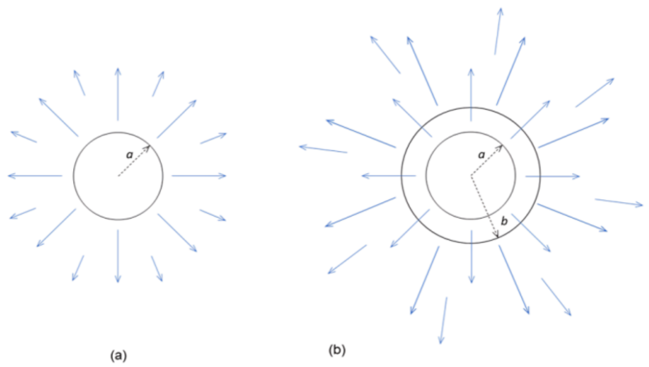

Figure 4. Expected construction of the vector field plot (blue) in item Q2.1. a and item Q2.1 b, respectively (Source: Authors' own elaboration)

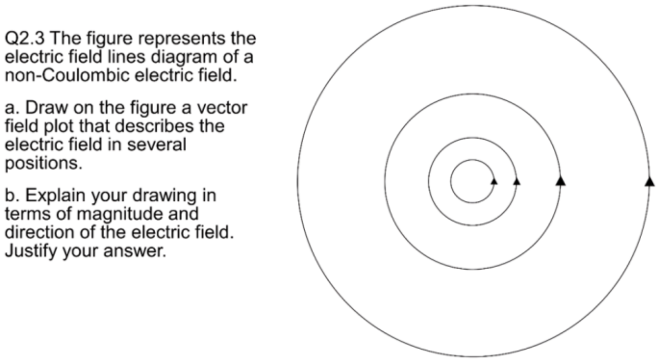

Figure 6. Item Q2.3 has the conversion objective from the electric field lines diagram to the vector field plot (Source: Authors' own elaboration)

the electric field's characteristics, mentioning that it is uniform in magnitude and direction.

Construction tasks

We consider two elements for measuring the vector field plot's interpretation: applying the superposition principle within the representation and effectively representing the field's characteristics in conversions where the vector field plot is the target representation.

Item Q2.1 targets the construction of the vector field plot by applying the superposition principle, as presented in **Figure 3**. This objective is a construction task because the student needs to create a vector field plot to represent the electric field using superposition. Students create a vector field plot for a single charged sphere in part a and two concentric spheres in part b. In part a, we expected students to interpret the physical situation, describe the electric field inside and outside the sphere, and draw field vectors representing the electric field's magnitude through the arrow's size and the field's direction, by the arrow's orientation, as shown in part a in **Figure 4**. Students should explain that the electric field outside the sphere is radial and decreases with distance; inside, it is zero. They should draw vectors directed radially outward from the sphere relative to its center. The vectors should be longer on the sphere's surface and decrease as they distance from it. Students should not draw vectors inside the sphere since this region's electric field is zero.

In part b in **Figure 4**, we expected students to apply the superposition principle to explain and describe the electric field in the three different regions: inside the two spheres (r < a), between the two concentric spheres (a < r < b), and on the outside of the two spheres (r > b). Students should explain that the electric field is zero in the r < a region. As in the previous case, students should explain that the electric field is radial and decreases with distance in the region a < r < b. In the region r > b, students should explain that the field is greater than the electric field in the previous case. The vectors they draw should be consistent in magnitude and direction with their explanations, as in part b in **Figure 4**. In the r > b region, the vectors should have direction radially outward; they should be larger on the sphere's surface and decrease as they move away. In the region a < r < b, the vectors should radiate outward and be smaller in size. Students should not draw vectors inside the r < a region, where the electric field is zero.

The conversion items where the vector field plot is the target representation are Q1.3 and Q2.3. In item Q1.3 (see **Figure 5**), students convert from the algebraic notation, and in item Q2.3 (see **Figure 6**), from the

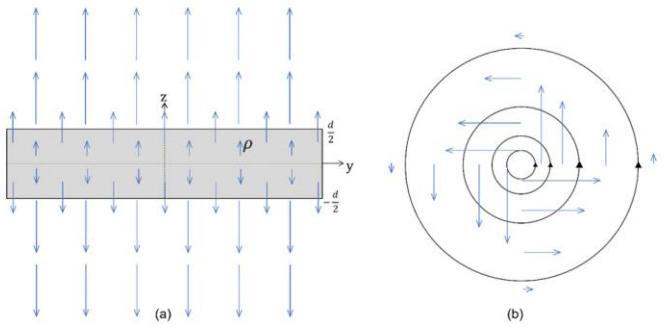

Figure 7. Expected construction of the vector field plot (blue) in conversion items: (a) Corresponds to the expected answer in Q1.3 & (b) is the expected answer in Q2.3 (Source: Authors' own elaboration)

electric field lines diagram. Item Q1.3 presents a construction task since the vector field plot is the target representation. The item shows a large plate with a charge density evenly distributed in its volume. The diagram displays the coordinate axes needed to understand the different regions of the system. The equation of the electric field emitted by this plate in algebraic notation is separated into three regions: under the plate ($z < -d/2$), inside the plate ($-d/2 < z < d/2$), and above the plate ($z > d/2$). Students should completely interpret the mathematical equation. The correct interpretation is when the students explain that the field's magnitude depends on z (or increases as it moves away from $z = 0$) inside and is constant outside the plate. The student should also mention that the field vectors point up to $z > 0$ and down to $z < 0$. The vectors should represent the appropriate directions in the different regions of the system and correctly represent the variation of magnitude inside and the field's uniformity on the outside with the arrow's length, as shown in part a in **Figure 7**.

Item Q2.3 presents a construction task since the vector field plot is the target representation. To convert efficiently from the electric field lines diagram to the vector field plot, the student should recognize that the magnitude of the field decreases with the distance from the center because the density of electric field lines decreases, and the direction is tangent to the electric field lines in a counterclockwise direction. Students should draw enough vectors over the same circle to visualize changes in the field direction. Also, in positions closer to the center, students should draw larger vectors that decrease as they move away from the center, as presented in part a in **Figure 7**.

Data Analysis

We employed a phenomenographic approach to analyze students' responses, as this method enables the identification of the various ways individuals perceive and make sense of meaningful aspects of reality through the description, interpretation, and understanding of their experiences (Marton, 1981, 2004). Within this framework, concepts and understandings are not treated as personal attributes but rather as descriptive categories that group together similar instances of individual reasoning. These categories are regarded as stable across different contexts and can be generalized, even though individuals may shift between them over time. Collectively, the set of categories reflects a shared or collective understanding (Marton, 1981). In this study, phenomenography guided both data collection and analysis, allowing us to derive descriptive categories based on students' explanations of their recognition and conversion of electric field representations (Guisasola et al., 2023). To ensure the reliability of the categorization process, we assessed inter-rater agreement between researchers' and students' classifications using Cohen's kappa coefficient (Leniz et al., 2017; Zuza et al., 2018).

The initial stage of data analysis involved reviewing the responses of 20 randomly selected students to develop preliminary descriptive categories for each question. In constructing these categories, we examined students' drawings, written equations, and accompanying explanations. Because each question addressed different aspects of recognition or conversion–classified as either interpretation or construction tasks–the resulting categories varied accordingly. Once the initial categories had been established, the remaining responses were analyzed through an iterative refinement process. Two researchers independently conducted the coding, and inter-rater reliability was assessed using Cohen's kappa, which yielded a value of $\kappa = 0.90$, reflecting an almost perfect level of agreement (Landis & Koch, 1977). The descriptive categories were organized hierarchically, not to classify answers as merely correct or incorrect, but to capture the most prevalent skills and difficulties demonstrated in students' reasoning. Furthermore, several subject matter experts participated in reviewing and validating the categories to ensure the appropriateness of the hierarchical structure.

The recognition items (Q1.1 and Q2.1) differed according to the nature of the task–whether it involved interpreting or constructing a representation. For the interpretation item focused on magnitude and direction (Q1.1), each characteristic was categorized separately. The construction item (Q2.1) was analyzed in two parts: first, for a single electric field source system, and second, for a system with two sources, each examined independently. Through this analysis, we identified patterns in students' responses, including their strategies for interpreting and constructing representations and their application of the superposition principle. The conversion items (Q1.2, Q2.2, Q1.3, and Q2.3) were analyzed following a structure that aligned with the recognition and conversion abilities outlined in the framework for descriptive categories (Campos et al., 2020). According to Duval's (2006) theory, the main sources of difficulty in

Table 2. Presenting the correct interpretation for items Q1.1, Q2.2, and Q1.2, along with an example and the percentages of introductory (intro) and upper-division (UD) students who answered each item correctly

Characteristics	Item	Example	Intro	UD
Recognition of magnitude	Q1.1 a	Answer: "*B > D = E = C > A*" Explanation: "*I ordered it according to the length of the arrows and by the symmetry of the field.*" Upper-division, student EM2019-A5	33%	48%
Recognition of direction	Q1.1 b	The direction in C: "*Up and right*" Direction in E: "*Up and left*" Explanation: "*You just see the direction of the vector describing the field at those points.*" Upper-division, student EM2019-A2	49%	69%
Conversion: Vector field plot as the source representation	Q1.2	Drawing: drew evenly spaced electric field lines in the correct direction (see part a in **Figure 8**). Explanation: "*The direction is always in the direction shown by the lines, and at each point of space, the magnitude is constant.*" Upper-division, student EM2018-A1-1	37%	50%
	Q2.2	Answer: "$E = a\hat{\imath} + b\hat{\jmath}$, *where a and b are positive constants.*" Explanation: "*The electric field in the drawing appears to be uniform, so it does not depend on any position variable. Besides, it goes toward i and j positive, so the components must be positive constants.*" Upper-division, student EM2018-A2-1	31%	47%

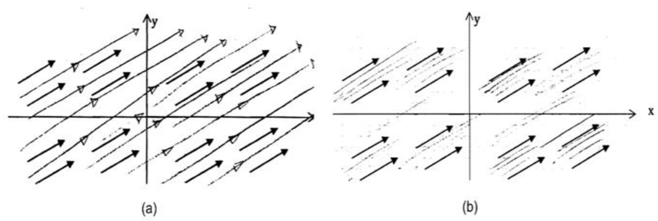

Figure 8. Examples of students' answers when converting from the vector field plot to the electric field lines diagram: Part a shows the vector field plot's correct interpretation and successful conversion to the electric field lines diagram & Part b shows an example of the difficulty of recognition, where students identified the density of vectors as an indicator of the magnitude (Source: Authors' own elaboration, from students' responses)

connecting representations to conceptual understanding arise from the need to recognize the same mathematical object across representations that differ in their features and to perform conversions between them. Although the specific descriptive categories varied across questions, all shared the same theoretical structure, allowing for comparison among conversion tasks. The theoretical organization underlying these conversion items is discussed in greater detail later.

RESULTS

Interpretation of the Vector Field Plot

This section presents examples of correctly interpreting questions Q1.1, Q1.2, and Q2.2 and the percentages of introductory and upper-division students who answered each item correctly (**Table 2**). We observe that the recognition percentage for introductory students was more than 30% on all questions, while for upper-division students, it was more than 45%. This result implies that students improved their recognition of the electric field's characteristics in the vector field plot when migrating from an introductory profile to an upper-division profile. This data provides qualitative evidence to establish that the constant use of a representation and the practice of relating the characteristics of representation to the electric field's characteristics allow students to learn and improve their transition from novices to experts. Additionally, we may consider that students understand the vector field plot and are familiar with this representation. For further reference, we present examples of students' correct interpretation when converting to electric field lines in part a in **Figure 8** and alternative interpretation in part b in **Figure 8**.

Alternative interpretations of the vector field plot

The students' answers provide evidence of some difficulties in interpretation related to the representation's surface features. Some students consider symmetrical elements for conclusions about the diagram without resorting to the arrows' size and direction. In some cases, the representation's surface features can lead students to interpret a quantity as a variable when it is not. **Table 3** presents the categories that emerged in items Q1.1, Q1.2, and Q2.2, where students had difficulties explaining their answers. In this section, we describe each category in more detail.

The category "elements of symmetry" includes students who justified their answers in item Q1.1 based on different symmetry elements for the magnitude and direction. In item Q1.1 a, some students correctly ordered the field's magnitude, but their explanation was not based on the characteristics of the representation. Students explained that they ordered the field's

Table 3. Presenting the examples and the percentages of introductory (intro) and upper-division (UD) students who answered each item and the corresponding difficulty

Characteristics	Item	Example	Intro	UD
Elements of symmetry	Q1.1 a	Answer: "*B > D = E = C > A*" Explanation: "*By symmetry, the magnitudes in C, D, and E are the same, since in the center there is a greater magnitude; therefore, in A will be smaller.*" Upper-division, student EM2019-A10	17%	24%
	Q1.1 b	The direction in C: "*Component y positive, component x positive.*" The direction in E: "*Component y positive, component x negative.*" Explanation: "*At all points in the field, the component y is positive. If we take B as the origin, the region y < 0 meets that E_x is equal in magnitude, but with the opposite sign as the region y > 0.*" Upper-division, student EM2019-A9	5%	5%
Misinterpretation of size	Q1.1 a	Answer: "*B > D > E > C > A*" Explanation: "*Vector size is larger at each point. I have doubts about the magnitude of vector D, which is not explicit, but because of its position between two vectors, it may be equal to the average of the two magnitudes.*" Upper-division, student EM2019-A7	21%	14%
Non-uniform field	Q1.2	Drawing: The student draws electric field lines closer together near the electric field vectors (see part b in **Figure 8**). Explanation: "*The lines go in the same direction as the electric field vectors, and the magnitude is greater where there are more lines.*" Upper-Division, Student EM2018-A1-15	2%	12%
	Q2.2	Answer: $\vec{E} = \frac{\rho}{\varepsilon_0} x\hat{x} + \frac{\rho}{2\varepsilon_0} x\hat{y}$ Explanation: "*The electric field travels diagonally in a positive direction of $\hat{x}$ and $\hat{y}$. However, it has magnitude on the y-axis, $\hat{y} \propto \frac{1}{2}x$*" Upper-division, student EM2018-A2-15	19%	14%

Note. Students' answers showed difficulties recognizing the electric field's characteristics in the vector field plot in items Q1.1, Q1.2, and Q2.2

magnitude because of its position relative to the diagram's center. They explained that the field is more intense in the center and that the other positions are relative to the center. The students did not mention the arrow's size to justify that the field near the center is the most intense, so we cannot determine if they considered this representation feature. In analyzing the field's direction in Q1.1 b, some students appropriately described the direction in position C and position E and justified their answer with the point's location in the diagram and the symmetry referring to the coordinate axes. The diagram shown in this question does not present the coordinate axes so that the student can place them arbitrarily in any region of the diagram. Generally, they locate the origin in the center (above position B).

The category "misinterpretation of size" emerged only in recognition of the electric field's magnitude. In item Q1.1 a, some students correctly explained that the arrow's size represents the magnitude of the electric field. However, when sorting the field's magnitude, they indicated that the field at position E is greater than at position C when the arrows have the same length. It is important to note that the electric field shown in this question is variable, but precisely at position C and position E, the field has the same magnitude.

The category "non-uniform field" includes students who have trouble recognizing the magnitude of the field in the vector field diagram. In item Q1.2, where students analyzed a uniform electric field and converted it to the electric field lines diagram, some interpreted vector density as an indicator of the electric field's magnitude, confused about the electric field lines' density. They related the field vectors' density to the magnitude rather than relating the arrow's size to the field's magnitude. When plotting the field lines, they did not have a uniform line spacing, so they did not show a uniform electric field, as presented in part b in **Figure 8**. In item Q2.2, where students analyzed a uniform electric field and converted it to the algebraic notation, they indicated that it varies with Cartesian coordinates in their mathematical expressions or explanations. Their mathematical expressions contained a dependency between the electric field's magnitude and the coordinate system (x, y).

We summarize the difficulties related to the surface features of the representation in **Table 3**. We observed that students identifying symmetry elements improve when moving from an introductory to an upper-division profile. This alternative explanation does not affect the conclusions about the electric field's magnitude in this physical situation. Another alternative interpretation is related to the misinterpretation of the size of the arrow and considering a non-uniform field. Some difficulties of interpretation decrease when moving from an introductory to an upper-division profile. However, the

Table 4. Presenting the correct construction for items Q2.1, Q1.3, and Q2.3, along with an example and the percentages of introductory (intro) and upper-division (UD) students who answered each item correctly

Characteristics	Item	Example	Intro	UD
Construction of vector field plot for a physical system	Q2.1 a	Drawing: Vectors with an outward radial direction decrease with the distance from the center (see part a in **Figure 9**). Explanation: "*The electric field vectors always point out of the center of the sphere, and their magnitude decays further from the origin. The field is completely symmetrical and is zero inside the sphere.*" Upper-division, student EM2019-A15	18%	42%
Application of the principle of superposition	Q2.1 b	Drawing: Longer vectors outside the sphere (r > b) that decrease as they move away from the center. They drew vectors in a < r < b that are less long than in the outer region. They did not draw vectors in the center (see part b in **Figure 9**). Explanation: "*There is no field within a (r < a), between a and b there is the same field and magnitude as in the previous fig. Outside of r > b, there is a field similar to that of the previous fig. in form and trend (decay as increments r), but the magnitude is double.*" Upper-division, student EM2019-A13	16%	40%
Conversion: Algebraic notation to vector field plot	Q1.3	Drawing: Upward vectors in the region z > 0 and downward in z < 0. Shows the variation of the field inside the plate with the length of the vectors, while outside the plate, the vectors have the same length (see part c in **Figure 9**). Explanation: "*The direction is up when z is positive and down when negative. Within the volume, the field increases according to z until it reaches d/2; from there, the magnitude remains constant.*" Upper-division, student EM2019-A13	16%	21%
Conversion: Electric field lines to vector field plot	Q2.3	Drawing: Several vectors inside the circles to represent the changes of direction. They drew several smaller vectors outside the circles to show the magnitude changes (see part d in **Figure 9**). Explanation: "*The magnitude decreases as you move away from the center because there is less density of lines, and the direction is circular.*" Upper-division, student EM2018-A2-11	19%	24%

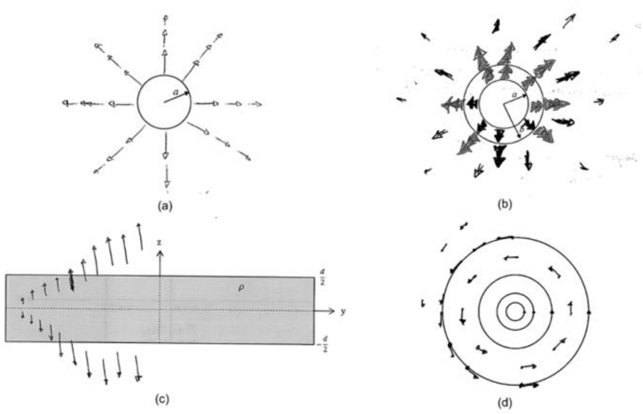

Figure 9. Examples of correctly constructing the vector field plot: (a) shows the correct construction for item Q2.1 a, (b) for item Q2.1 b, (c) for item Q1.3, and (d) for item Q2.3 (Source: Authors' own elaboration, from students' responses)

difficulty of confusing line density with vector density increased when moving from an introductory to an upper-division profile. This behavior is interesting because it only occurs when students convert from the vector field diagram to the electric field lines representation, implying that advanced students have a greater tendency to confuse characteristics of the vector field plot and the electric field lines diagram.

Construction of Vector Field Plot Diagrams for the Electric Field

Analyzing students' skills in constructing the vector field plot allows identifying how students use this register's characteristics to represent the electric field's magnitude and direction. This skill is necessary to successfully convert to the vector field plot and demonstrate the electric field's characteristics. The items that ask students to build a vector field plot for the electric field are Q2.1, Q1.3, and Q2.3. In part a of item Q2.1, students interpret a physical situation and present a vector field plot of the electric field. The situation changes in part b, and students construct a new vector field plot, applying superposition. In item Q1.3 and item Q2.3, students convert from an initial representation (algebraic notation and electric field line diagram) to the vector field plot. **Table 4** shows the correct construction of the vector field plot for each item, an example, and their respective percentages. For further references, we present examples from students' drawings in **Figure 9**.

In **Table 4**, we qualitatively observe that the skills to generate the vector field plots increase as students move from an introductory to an upper-division profile. This result is consistent with their ability to recognize the electric field's characteristics in the vector field plot. We observe a smaller improvement in the questions where there is a conversion (i.e., Q1.3 and Q2.3) compared to the questions where students interpret the physical

Table 5. Presenting the examples and the percentages of introductory (intro) and upper-division (UD) students who answered each item and the corresponding difficulty

Characteristics	Item	Example	Intro	UD
Identification of an external region	Q2.1 a	Drawing: Vectors with radially outward directions outside the sphere (see part a in **Figure 10**). Explanation: "*The electric field is radial and decays as we move away from the center of the sphere.*" Upper-division, student EM2019-A3	32%	28%
	Q2.1 b	Drawing: Vectors with a radially outward direction outside the two spheres decrease as they move away from the center (see part b in **Figure 10**). Explanation: "*The electric field remains radial and decays as we move away from the center of the spheres, but its magnitude is two times larger at each point for r > b.*" Upper-division, student EM2019-A3	22%	26%
Confinement of vectors	Q1.3	Drawing: Vectors that vary in size inside and are the same size on the outside. The vectors on the outside are much larger than on the inside (see part a in **Figure 11**). Explanation: "*All directions have the only direction in z. Within the plate, the smaller the magnitude is, the closer to z = 0. Outside the plate, the magnitude is always the same because the field is constant.*" Upper-division, student EM2019-A16	3%	19%
	Q2.3	Drawing: Vectors tangent to the circles, smaller in the inner circles and larger circles in the outer circles (see part b in **Figure 11**). Explanation: "*Greater magnitude near the center due to higher density of field lines; direction parallel to each point of the field lines.*" Upper-division, student EM2019-A10	7%	8%
Variation of magnitude	Q1.3	Drawing: Students drew vectors that show the variation of size inside the plate and vectors only on the surface outside the plate (see part a in **Figure 12**). Explanation: "*Outside the plate, the magnitude is constant. Inside, the magnitude increases as we move away from z = 0. The field takes the z-direction with a change of sign in z = 0.*" Upper-division, student EM2019-A9	9%	21%
Vectors as tangent lines	Q2.3	Drawing: Several vectors over different points in the same circle denote changes in direction and vectors in different circles to represent magnitude changes. The vectors make contact with the circle in the middle of the arrow (see part b in **Figure 12**). Explanation: "*The field is tangential to the field lines; where the lines' density is highest, the E field is greater in magnitude. The direction is given by the lines tangent to the field lines.*" Upper-division, student EM2018-A2-3	2%	5%

Note. Students' answers showed difficulties constructing the vector field plot in items Q2.1, Q1.3, and Q2.3

situation directly (i.e., Q2.1 a and Q2.1 b). This result is consistent with Duval's theory of registers of semiotic representation (Duval, 2006), as the conversion process requires recognizing the characteristics of the electric field in two representations that have no characteristics in common, which increases its difficulty. The following sections present students' difficulties with constructing vector field plots.

Difficulties constructing the vector field plot for the electric field

In analyzing the items requesting to draw vector field plots, some categories emerged that reflect students' difficulties making these diagrams, as presented in **Table 5**. The predominant ones were that students drew field vectors only in a region of the system, failed to represent the magnitude of the electric field employing the arrow's size confining the arrow to the space delimited by the drawing, represented the electric field only at one level of the region not allowing to visualize variations of the field, or they drew the vectors as if they were tangent lines.

The category "identification of external region" includes students who only constructed a vector field plot in the outer region of item Q2.1. In item Q2.1 a, the students explained that the electric field outside the sphere is radial and decreases with distance, but they did not explain that the electric field is zero inside the sphere. They drew vectors with directions radially outward at the surface of the sphere, and the vectors were larger on the sphere's surface and decreased as they moved away from it. Since students in this category did not explain that the electric field inside the sphere is zero, we cannot determine whether they recognized this characteristic. In item Q2.1 b, the students explained that the electric field outside of the two spheres (r > b) is larger than in the previous case, in some cases explicitly mentioning that it is double. They drew vectors with a radially outward

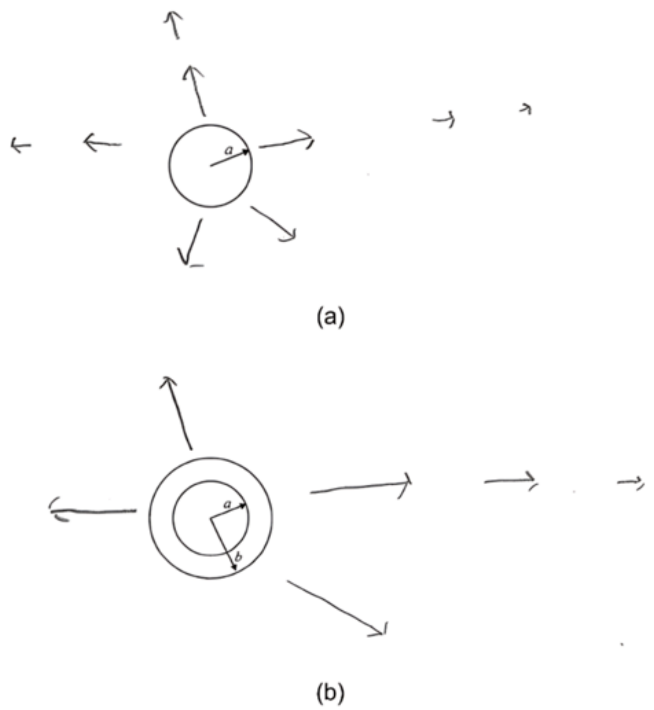

Figure 10. Difficulties in constructing the vector field plot: (a) shows the student identified only the outer region & (b) shows the same student only identified the outer region, failing to apply the superposition principle effectively (Source: Authors' own elaboration, from students' responses)

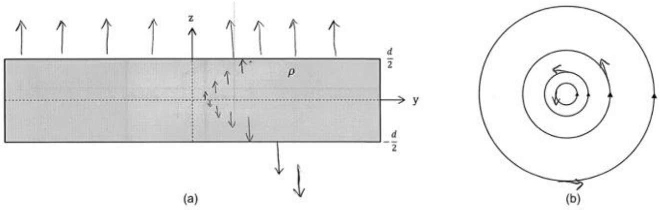

Figure 11. Difficulties in constructing the vector field plot related to the confinement of vectors: (a) shows the student confined the vectors inside the plate in item Q1.3 & (b) shows the student confined the vectors within the electric field lines in item Q2.3 (Source: Authors' own elaboration, from students' responses)

direction, larger on the sphere's surface, and decreased as they moved away. They did not draw vectors within the sphere, did not identify it as a separate region, and did not explain that the electrical field in r < a is zero, nor did they describe the electric field in the a < r < b region. The two difficulties are illustrated in **Figure 10**.

The category "confinement of vectors" includes students who did not allow vectors to cross specific barriers imposed by the system. In item Q1.3, the barrier is the plate's limits, as students prevent vectors from crossing from inside to outside the plate, as illustrated in part a in **Figure 11**.

This confinement means that the vectors inside are small, and the vectors outside are much larger, so the

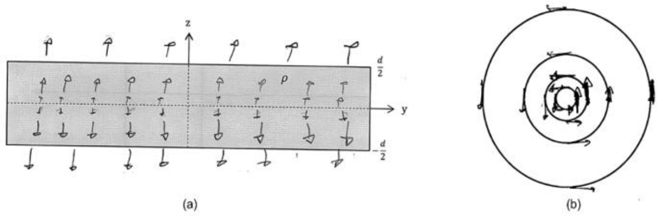

Figure 12. Difficulties in the construction of the vector field plot: (a) shows the difficulty of construction where students only draw one set of vectors and do not efficiently represent the electric field's magnitude & (b) shows the difficulty of constructing vectors as a tangent to the electric field lines (the vectors make contact with the field lines in the middle of the arrow) (Source: Authors' own elaboration, from students' responses)

arrow's size does not represent the magnitude of the electric field proportionately. In item Q2.3, some students prevented arrows from crossing the drawing boundaries (the circles that mark the electric field lines in this physical system, as illustrated in part b in **Figure 11**.

The category "variation of magnitude" emerged when converting from the algebraic notation to the vector field plot. In item Q1.3, some students did not represent variations in magnitude employing the arrows' size. This behavior was observed because students interpreted the equation correctly and explained all regions, but they drew vectors only on the surface outside of the plates. The category is illustrated in part a in **Figure 12**. This type of drawing does not show that the field on the plate's entire exterior is uniform. Some students repeated the same pattern inside, which does not represent the field's variation. It is important to note that these students explained that the electric field is variable inside and uniform on the outside. Still, they did not represent it in their diagram, which means this is a difficulty of construction and not recognition.

The category "vectors as tangent lines" emerged when converting from the electric field lines diagram to the vector field plot. In item Q2.3, some students drew vectors as lines tangent to the circles, as shown in part b in **Figure 12**. The main feature of this difficulty is that the point where the vector and field lines make contact is halfway through the arrow rather than at the beginning. This practice is common when drawing lines tangent to a curve; a straight line makes contact with the curve at a single point. Typically, the tangent line extends as needed. However, when drawing vectors tangent to a curve, we must be careful where the vector begins and ends.

From the results presented in **Table 5**, we qualitatively observe that some difficulties in constructing vector field plots increased when moving from an introductory to an upper-division profile. However, it is essential to note that the correct answer

Table 6. The recognition and conversion results

Source representation			Vector field plot		Vector field plot		Algebraic notation		Electric field lines	
Target representation			Algebraic notation		Electric field lines		Vector field plot		Vector field plot	
Cat	Rec	Con	Intro	UD	Intro	UD	Intro	UD	Intro	UD
A	High	High	30%	45%	15%	19%	16%	21%	19%	24%
B	High	Med	3%	2%	22%	31%	11%	40%	9%	12%
B′	Med	High	22%	21%	0%	0%	0%	0%	0%	0%
C	Med	Med	19%	14%	30%	29%	48%	29%	43%	36%
D	Low	Low	26%	19%	33%	21%	25%	10%	29%	28%
Total			100%	100%	100%	100%	100%	100%	100%	100%

Note. The results have the vector field plot as a source or target representation & we present the results for each conversion process according to the theoretical structure of categories that determine the success in recognition or conversion for introductory and upper-division students

percentages improved by moving from introductory to advanced profile in all questions, as presented in **Table 5**. Therefore, the increase in difficulties when drawing vector field plots implies that many students in the upper-division profile draw vector field plots, even if they present some difficulties. These students have more consistent responses that can be categorized and identified as difficulties, while in the introductory profile, many students give responses that cannot be categorized and do not show a specific understanding of the concepts.

Conversion Between Representations of the Electric Field

This section presents the results of the conversions with the vector field plot as the source or target representation (see **Table 5**). The descriptive categories that emerged in the conversion items were classified in a theoretical structure based on each process's low, medium, or high domain of recognition and conversion. A preliminary study presented this theoretical structure (Campos et al., 2020). The categories follow a hierarchical order, whereas category A means high recognition and conversion. In category B, students present a high recognition and some difficulties in the conversion (classified as a medium); in category B′, students had a high conversion ability but also difficulties in recognition (medium). In category C, students struggled with recognition and conversion and were classified as medium. Category D groups the students whose difficulties could not be identified in the categories or who did not answer the question, so they were classified as low. We present the results for all the conversion processes for both introductory and upper-division profiles; the first two columns correspond to the vector field plot as a source representation, and the last two as a target representation.

As observed in **Table 6**, in conversions where the vector field plot is the source representation, about 30% of introductory students and 50% of upper-division students had a successful recognition (category A and category B), regardless of the target representation. The recognition difficulties found are slightly different, so the nature of the difficulty might be associated with the target representation. When converting from the vector field plot to the electric field lines diagram, some interaction between the representations creates confusion. Specifically, some students confused vector density and field line density, as described in **Table 2** and part b in **Figure 8**. However, when converting from the vector field plot to the algebraic notation, this confusion does not happen. Instead, students identified that the electric field's magnitude is variable and included the variables corresponding to the coordinate system (x and y) in their algebraic expressions.

The results presented in **Table 6** suggest that students' skills converting towards the vector field plot (target representation) depend primarily on recognition skills in the source representation. We highlight that, once there has been recognition of the characteristics of the electric field in the respective source representations, there are still some conversion difficulties related to the construction of vector field plots; for example, the confinement of vectors to a region marked by the drawing and vectors to be drawn as tangent lines, as presented in **Table 5**. This explains the presence of category B, where students with high recognition may still achieve only medium conversion performance, due to these construction-related challenges. Moving from the algebraic notation to the vector field plot improves introductory students' conversion difficulties (category B). Upper-division students have better recognition of the source representation and, therefore, more space for conversion difficulties in the vector field plot representation.

DISCUSSION

The discussion is organized around the RQs, which are explicitly mentioned for greater clarity. The section is divided into two parts, each of which presents in detail the results concerning the recognition and conversion of the vector field plot within the context of the electric field concept, with the aim of addressing the RQs. We focus on students' difficulties of recognition of the electric field's characteristics when analyzing the vector field plot independently, both in interpretation and construction tasks. We discuss the difficulties that emerge when the vector field plot is the source or target representation in conversion tasks. We conclude the discussion by contrasting whether the analysis of the vector field plot individually and in conversion tasks elicit different difficulties for students.

Recognition of the Vector Field Plot

In this subsection, we approach RQ1 on the difficulties that students have in recognizing the electric field's main characteristics in the vector field plot. When students interpreted the vector field plot by itself, we observed that more than 30% of introductory students and more than 45% of upper-division students recognized the electric field characteristics in the vector field plot diagrams provided. As shown in **Table 2**, 33% of introductory students and 48% of upper-division students correctly interpreted the electric field's magnitude in the provided diagram (item Q1.1 a), while 49% of introductory students and 69% of upper-division students correctly recognized the direction of the electric field in item Q1.1 b. When interpreting the vector field plot individually, students' main difficulties were related to the surface features of the physical systems, including symmetry elements (in **Table 3**, 17% of introductory students and 24% of upper-division students in item Q1.1 a and 5% of introductory and upper-division students when interpreting the direction), and difficulties in accurately identifying the arrows' size (in **Table 3**, 21% of introductory and 14% of upper-division students in Q1.1 a). These difficulties depend on the physical system represented, while they could also be caused by the way vector field plots are often portrayed. Vector field plots are often symmetric, because presenting symmetry can be beneficial for physicists; the qualitative representations help physicists to create mathematical representations (Van Heuvelen, 1991). The representation of symmetry in vector field plots is important, but it would also be beneficial to aid students in transitioning from the qualitative symmetric representation to the quantitative algebraic representation. On the other hand, the misinterpretation of size can happen when arrows have similar lengths that cannot be determined quantitatively, or as in this case, when the arrows have equal length but opposite directions.

When students constructed the vector field plot alone, we observed that less than 20% of introductory students and about 40% of advanced students drew vector field diagrams considering both the direction and the arrow's size in enough positions to represent the electric field effectively In **Table 4**, 18% of introductory students and 42% of upper-division students constructed the vector field plot correctly (Q2.1 a), and 16% of introductory and 40% of upper-division students applied the superposition principle correctly with a vector field plot that reflected this principle (Q2.1 b). The main difficulties when constructing vector field plots by themselves were related to identifying the internal regions of the physical situation. This difficulty was shared by around 30% of introductory and upper-division students in Q2.1 a, and around 25% in Q2.1 b (**Table 5**). It is important to note that vector field plots merge several spaces in the same diagram. On the one hand, there is the physical space (e.g., the concentric spheres). On the other hand, there is the space that the vector uses (i.e., its length) to represent the magnitude of the electric field (Gire & Price, 2014). The physical situation can act as a barrier that leads to difficulty locating the problem's regions and accurately representing the electric field's magnitude. The internal and external regions of the physical space (i.e., the concentric spheres) delimit the difficulties in interpreting the field and constructing the vector field plot.

Analyzing the interpretation and construction tasks together, we can see that introductory students can perform better in interpretation (more than 30%) than in construction tasks (less than 20%), while for upper-division students the performance is similar in these two types of tasks (around 40% in both cases, though slightly higher in interpretation). When interpreting the direction of the vector field plot only, students had their best performance (49% introductory and 69% upper-division). The main reason for this could be that the direction of the arrow directly represents the direction of the field, a characteristic that is not as straightforward in other graphical representations of the field, as we have previously found for the electric field lines diagram (Campos et al., 2023).

Conversions of the Vector Field Plot as a Source or Target Representation

So far, we have discussed the difficulties in the interpretation and construction of the vector field plot when it is presented by itself (in item Q1.1 a and item Q1.1 b and item Q2.1 a and item Q2.1 b). We have yet to discuss the difficulties that arise in conversion tasks, when the vector field plot is presented with other representational systems, such as the electric field lines diagram and the algebraic notation of the electric field. This discussion will investigate whether the difficulties that emerge in conversion tasks are different to the difficulties of analyzing the vector field plot alone. In this subsection, we approach RQ2, when discussing the interpretation of the vector field plot, and RQ3 by discussing the construction difficulties when converting from the electric field lines diagram and the algebraic notation.

The correct interpretation of the vector field plot in conversion tasks had a similar pattern for the two target representations, 37% of introductory students and 50% of upper-division students when converting to electric field lines, and 31% introductory and 47% upper-division students when converting to the algebraic notation. This pattern is consistent with the interpretation of the magnitude when students analyze the vector field plot alone. However, we observed that the interpretation difficulties in conversion tasks differed for each target representation, implying that there is an interaction between the source and the target representation. In both cases, the students interpreted the field as non-uniform, but for different reasons that can be traced back to the target representations. When converting to the algebraic notation, 19% of introductory and 14% of upper-division students interpreted a uniform electric field in their mathematical expressions as variating. This result agrees with Ceuppens et al. (2018), who found that some students learned correlations between particular fields (in this case, electrical fields) and mathematical equations without a deep understanding of the physical situation. In our results, we can observe some level of memorization in this type of answer. When converting to the electric field lines diagram, 2% of introductory students and 12% of upper-division students drew field lines that were closer together, which can be related to the density of field lines. Students might remember that electric field line density describes the magnitude of the field. Since both are visual representations, students might confuse the specific characteristics of the two types of diagrams (Küchemann et al., 2021). The literature has reported examples of students blending characteristics of the vector field plot and the field lines diagram (Bollen et al., 2016), a feature related to the emergence of hybrid representations that combine elements of the vector field plot and the field lines diagram (Campos et al., 2021a).

The construction of the vector field plot in conversion tasks where the vector field plot acts as the target representation had a similar pattern for the two source representations. When converting from electric field lines, 19% of introductory and 24% of upper-division students did so successfully. While converting from the algebraic notation 16% of introductory and 21% of upper-division students converted correctly. This pattern differs with the construction of vector field plots individually for upper-division students (around 40% of upper-division students constructed vector field plots correctly). This difference may hint at a difficulty related to the conversion process. One of the difficulties found was that students confined the vectors within a barrier given by the surface features of the representation. When converting from the algebraic notation, there were 3% of introductory and 19% of upper-division students who presented this difficulty. When converting from the electric field lines diagram, the percentages were 7% for introductory and 8% for upper-division students. We attribute this difficulty to the spaces that merge in the same diagram when constructing vector field plots, the physical space and the space that the length of the vector occupies to represent the electric field (Gire & Price, 2014). The physical situation can act as a barrier that leads to difficulty in accurately representing the electric field's magnitude. When converting from the electric field lines diagram, the circular field lines can serve as a barrier that vectors do not cross. When converting from the algebraic notation, the barrier relates to the charged plate itself.

Another difficulty constructing the vector field plot was that students drew vectors as if they were tangent lines (they are in contact with electric field lines at the middle of the arrow instead of the beginning). This difficulty only emerged in a small amount (2% of introductory students and 5% of upper-division students) when converting from the electric field lines diagram. Furthermore, students only drew one set of vectors, which does not represent changes in magnitude effectively when converting from the algebraic notation to the vector field plot (9% of introductory and 21% of upper-division students). These two difficulties had been previously reported by Ceuppens et al. (2018) without a physical context and we identified that they keep happening in the electric field context. Both are examples of how the interaction between the source and the target representation can elicit difficulties in students.

The evidence suggests that the vector field plot and the algebraic notation have good synergy when the vector field plot is the source representation and the algebraic notation is the target (30% and 45%) but not the other way around (16% and 21%). Interestingly, in this same direction of conversion, category B′ (difficulties of recognition with successful conversion) was relevant (22% and 21%). This finding is particularly noteworthy, as it corresponds to category B′ (Rec medium, Con high) and suggests that, despite students' difficulties in recognizing the physical meaning of the vector field plot, they may still perform conversions from the vector diagram to algebraic notation with relative ease. This exception to the general pattern–where high recognition is usually a prerequisite for high conversion–can be explained by students' strong familiarity with both representations in mathematical contexts, where algebraic notation of a vector is often used as a procedural tool without necessarily being linked to a deep conceptual understanding. Van Deventer and Wittmann (2007) found that while students generally have little trouble generating vector representations, they often face significant difficulties when interpreting them. This distinction between procedural fluency and conceptual recognition helps explain why high conversion performance can sometimes occur despite only moderate recognition.

Combining categories A and B′ provides insight into the relationship between the vector field plot and the algebraic notation and how they interact. Heckler and Scaife (2015) emphasize the importance of the relationship between the vector plot and the algebraic notation, making the analogy with the relationship between graphs and equations. The synergy between the vector field plot and the field lines diagram presents more difficulties in recognition and conversion, despite both representations being visual. The synergy of at least three representations is crucial to achieve the dissociation between the concept and its representations to better understand the electric field concept. From these results, we can infer that the relationship between the algebraic notation and the vector field plot is more beneficial for students understanding of the electric field concept than between the vector field plot and the field lines diagram. Nonetheless, each of these relationships presents students with difficulties that need to be addressed by explicit instruction.

CONCLUSION

This study used the theory of registers of semiotic representation and a phenomenographic analysis to link the difficulties students have recognizing the vector field plot representation and converting to and from other electric field representations with their conceptual understanding of the electric field. Other studies in mathematics have analyzed the relationship between the difficulties of recognition and conversion with cognitive activity and understanding (Bansilal, 2012; McGee & Martinez-Planell, 2014; Trigueros & Martínez-Planell, 2010). As part of the phenomenographic analysis, the categories describe the main difficulties that arose from the students' explanations. The findings suggest that the interpretation of the vector field plot by itself and when converting to other representations has a similar success pattern. The difficulties that arise when interpreting the vector field plot alone can be related to the surface features of the representation. When converting to other representations, the emerging difficulties are related to the target representation, providing evidence of the interaction that may exist between the two representations. The construction of vector field plots had a smaller success percentage in introductory students, both while constructing vector field plots by themselves and in conversion tasks where the vector field plot was the target representation. In contrast, upper-division students had more ability to construct this representation alone (40%) but presented more difficulties in conversion tasks. The difficulties of constructing the vector field plot alone were found to be identifying all the regions of the physical situation, and the interplay between the physical and the vector space. In construction tasks, there were difficulties related to this same interplay between physical and vector space (such as the confinement of vectors, when converting from the algebraic notation), but there were other difficulties elicited by the interaction between representations (drawing tangent lines when converting from the electric field lines diagram).

The objectives of this study are exploratory in nature, and in subsequent research endeavors, we shall propose potential pedagogical methodologies predicated upon the findings garnered from this work as well as prior studies (Campos et al., 2020, 2023). This study has several limitations that should be considered when interpreting the results. First, the data were collected at a single private Mexican university, which may limit the generalizability of the findings to other institutional or cultural contexts. Second, the data was obtained across different semesters, during which different instructors taught the courses. Although all courses covered similar content, the instructors are the same for each semester, variations in teaching style and emphasis may have influenced students' preparation. Finally, while all three representations of the electric field were used in lectures and learning materials, there was no explicit instruction on conversion strategies; therefore, the observed performance may reflect students' spontaneous approaches rather than outcomes of targeted teaching. These limitations should be considered when extending the conclusions to other contexts or when designing future research to build on these findings. However, we note that they are congruent with research in disparate settings that elucidate the enduring challenges students face when employing vectorial language in the realm of electric fields. Furthermore, as delineated in the discussion section, the findings of this study contribute interpretative depth to these aforementioned challenges. It should also be noted that the scope of this study is confined to the vectorial interpretation of electric fields in a vacuum and does not consider other facets, such as energy or interactions with matter, which could be the subject of future developments.

In science teaching, knowing the difficulties of understanding is crucial to identify possible tools that help overcome difficulties. It is essential to explain the conversion process between different representation systems to contribute to teaching and learning in science (Maturano & Aguilar, 2009). The results of this research suggest that physics instructors embrace that a good synergy between representations is necessary for conceptual understanding of abstract properties, as proposed by Duval (2006). Dedicating time and practice to coordinating various electric field representations is essential to promote the dissociation between the concept and its representations and achieve a conceptual understanding of this physical quantity. Based on this research results, professors and researchers in the area can consider the difficulties posed by some conversion tasks, contributing to the discussion about the teaching-learning processes of the electrical field concept at the university level.

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