



# Research on Mathematical Animation Using Pascal Animation as an Example

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## ABSTRACT

Most students thinking mathematics is a difficult subject. This study aims to enhance students' motivation and efficiency in learning mathematics. This study developed 3D animation on the binomial theorem with historical stories of mathematics as the plot. It also examined the effect of animation on students' learning willingness and comprehension.

And using questionnaire to collect data. The results showed that enhance the students' comprehension of the binomial theorem, suggesting that 3D animation could effectively improve mathematical learning. Moreover, the results showed that the students' willingness to learn mathematics also increased, suggesting that 3D animation can be used as an auxiliary learning tool for students. The results demonstrated that using 3D animation can improve students' learning willingness, comprehension, and application of mathematics.

**Keywords:** Pascal animation, mathematics, learning tool

## INTRODUCTION

The multimedia study by Höffler, Prechtel, and Nerdel (2010) explored whether animation or pictures could effectively help students to improve their comprehension of mathematics (Lowe & Rieber, 1994; Schnotz, 2008; Tversky & Morrison, 2002). The rapid developments in animation in recent years have rendered animation popular. Therefore, this study aimed to introduce animation into mathematical teaching to provide students with substantial assistance in learning.

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### **State of the literature**

- This paper uses mathematics teaching as an example of binomial theorem, because it is a subject that is difficult for numerous students, and this paper employs the development of 3D animation teaching materials according to the plots of historical stories.
- This paper also examines the influence of the proposed 3D animation on student comprehension and willingness to learn.

### **Contribution of this paper to the literature**

- Providing historical examples of mathematical concepts by using 3D animation enhances mathematics education.
- Verifying 3D animation can effectively help students to understand the unit on the binomial theorem. Additionally, improved learning willingness and comprehension positively influence learning performance in mathematics.
- 3D animation is promising for use in mathematics education.

Meserve (1983) stated that the historical evolution of mathematics is the resource that most assists students in understanding mathematics and its application. For example, the ancient Egyptians used the diagonal of the original square as the side length of the new square to solve the problem of “creating a square with doubled area.” By expanding the distributive law of  $(a + b)^2 = a^2 + 2ab + b^2$  and employing a graphic description, the same result such as “used the diagonal of the original square as the side length of the new square to solve the problem of “creating a square with doubled area.”” can be obtained. Such presentations of a geometric concept are easy to understand and demonstrate the relationship between geometry and algebra. This example demonstrates that modern mathematics can provide solutions to problems; however, these example are not the only solutions, as ancient people have used simpler, more direct methods to solve the same problem. Through historical examples, we can reveal explanations that are more suitable for students (Yang, 1992). Liu (2004) designed a calculus course by using a historical approach to convey the nature and humanistic concern of mathematical knowledge and observed the influences of the course on students’ mathematical thinking, mathematical knowledge, problem-solving skills, and learning behaviors. Wu (2011) integrated the history of science in teaching to promote students’ understanding of scientific concepts, thus enhancing their interest in science and their understanding of the nature of science. Clearly, introducing historical stories is effective for teaching.

According to the aforementioned discussion, the purposes of this study were to develop 3D animation teaching materials on the basis of the plots of historical stories and to further examine the influence of the proposed 3D animations on student comprehension and learning attitudes. The research questions were as follows:

- (a) Can learning mathematics with animation improve student comprehension of mathematical concepts?

- (b) Does stronger learning willingness in mathematics affect student comprehension of mathematical concepts?
- (c) Does comprehension of mathematical concepts affect student learning willingness in mathematics?
- (d) Does comprehension of mathematical concepts affect student mathematics learning?
- (e) Does learning willingness in mathematics affect student performance in mathematics?

Students' preferred subjects are typically those that are easy to understand or supplemented with examples. By contrast, subjects that students commonly dislike typically involve abstract concepts and are inherently difficult to understand. Mathematics is a subject that numerous students find challenging, and it therefore may be categorized as a commonly disliked subject.

Chiang and Chang (2008) applied the principle and theory of affective computing in the human-computer interface design of large-screen 3D games to determine the key factors in the human-computer interface that cause affective changes in users when they play 3D games. Korakakis, Pavlatou, Palyvos, and Spyrellis (2009) combined 3D animation with written narratives to enhance science courses, finding that 3D animation could improve student learning interest and reduce cognitive load. Yang (2011) produced teaching films that employed 3D virtual reality for special teaching units, improving student understanding of mathematical concepts.

Compared with the 2D perspective, the 3D perspective produces the rotational visual effect of 3D vectors, enabling students to visualize the differences of spaces. The sensory stimulation may attract student attention to the current teaching progress and teaching content. In addition to improving learning outcomes, it could reduce student review time at home and enhance comprehension in class. This study used 3D technology to produce animation for mathematical teaching and explored the resulting effect on student learning willingness and comprehension.

## METHOD

Using the subject of mathematics as the primary example, this study developed 3D animation on the binomial theorem with historical stories of mathematics as the plot. It also examined the effect of animation on student learning willingness and comprehension. The 3D animation of the binomial theorem is displayed in [Figure 1](#).

The questionnaire contains the following dimensions:

- (a) Most/least favored subject
- (b) Favorite unit in the math course
- (c) Preference for animation

- (d) Whether learning mathematics with animation could improve learning willingness
- (e) Whether learning mathematics with animation could improve comprehension
- (f) Whether greater learning willingness in mathematics influences the comprehension of mathematical concepts
- (g) Whether comprehension of mathematical concepts affects learning willingness in mathematics
- (h) Whether learning willingness in mathematics influences mathematics learning performance
- (i) Whether comprehension of mathematical concepts affects mathematics learning

### **Analysis Tools**

Analysis was conducted using IBM SPSS 20 Statistics Data Editor.

### **Measuring Tools**

A 5-point Likert scale was adopted for the questionnaire. Specialists and scholars of Mathematics Education were invited to discuss and provide feedback on the test. After revision, nine questions remained on the questionnaire. Validity and reliability analyses were used to assess content errors. The validity analysis was conducted to examine the compliance between testee comprehension and the dimensions of the questionnaire are matched. The reliability analysis was performed to determine whether the dimensions of the questionnaire were consistent with repeated measurements. Factor loading greater than or equal to 0.5 was applied as the assessment standard in the validity analysis; Cronbach's  $\alpha$  value greater than or equal to 0.7 was used as the assessment standard in the reliability analysis (Hair, Black, Babin, & Anderson, 2010). **Table 1** displays that the Cronbach's  $\alpha$  value in this study was 0.904, whereas the factor loadings were between 0.697 and 0.931. The test results all exceeded the assessment standard. Therefore, the questionnaire was determined to have acceptable convergent validity and reliability.

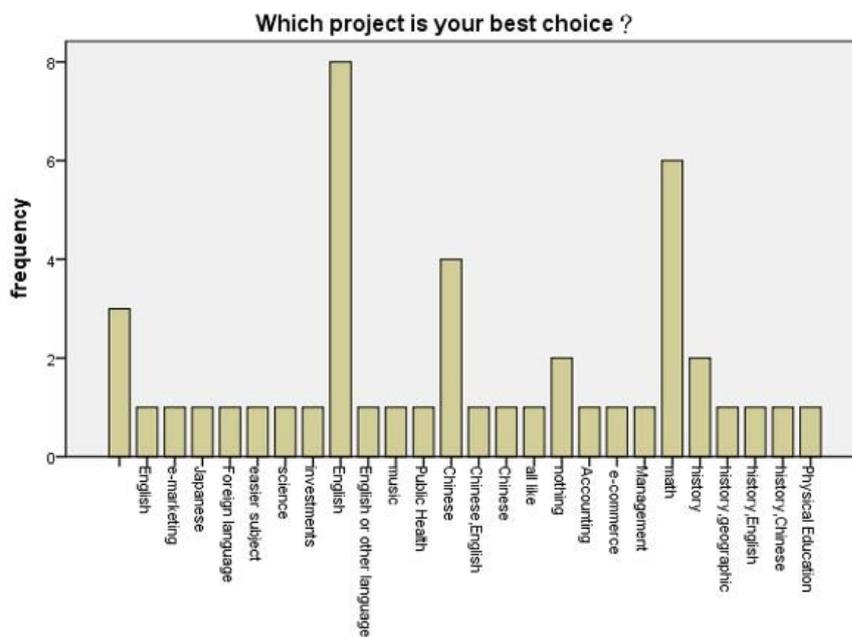
### **Implementation Procedure**

In this study, the participants were high school students from southern Taiwan who were enrolled in mathematics courses that covered the binomial theorem. The implementation stages were as follows:

- (a) Start the class.
- (b) Begin the 3D animation to assist with the course content.
- (c) Students touch screen, flip and resize objects.
- (d) Request that the students complete the questionnaire after viewing the 3D animation.
- (e) Collect and analyze the questionnaires.

**Table 1.** Results of reliability and validity analyses

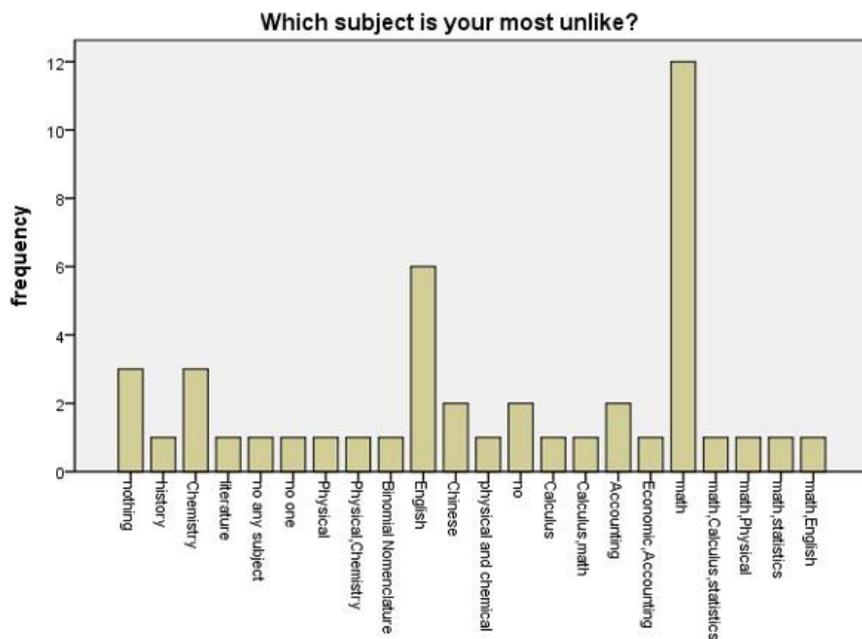
Item	Factor Loading	Cronbach's $\alpha$ value
After viewing this animation, you are more interested in learning mathematics.	0.884	.904
The current learning method can improve your learning willingness.	0.697	
Learning mathematics with 3D mathematical animation can generate more ideas.	0.909	
The current method can improve your understanding and application of mathematics.	0.889	
Using mathematical 3D animation can improve your learning willingness in mathematics.	0.887	
Using mathematical 3D animation can improve your mathematical understanding and application.	0.778	
Using mathematical 3D animation can improve your comprehension or mathematical application.	0.931	
Using mathematical 3D animation can improve your learning willingness in mathematics.	0.758	
Using the mathematical 3D animation method can effectively enhance your mathematical learning.	0.876	



**Figure 1.** Most favorite subjects

### Basic Data and Pilot Study

The participating subjects consisted of 48 college students. After six invalid questionnaires were eliminated, 42 valid samples remained (14 male and 28 female student samples). The statistical results pertaining to the most favored course subjects are displayed in **Figure 2**, and those of the least favored subjects are provided in **Figure 3**.



**Figure 2.** Least favorite subjects

Most students preferred languages and humanities courses. The language subjects included English, Chinese, and Japanese, and the humanities subjects included history and geography. These courses were preferred because language subjects are generally easy to understand and contain content that is familiar based on daily life. Therefore, the students had more interest in these subjects.

Furthermore, the students mostly preferred subjects in mathematics. This is because subjects in mathematics and science are generally difficult to understand, include complicated test questions, and involve formulas that are difficult to memorize.

As displayed in **Figure 3**, 29 of the 42 students enjoyed the animation, whereas only one disliked the animation. This largely favorable disposition reflects the fact that animation is generally perceived as lively and interesting. Therefore, integrating animation into teaching may increase learning willingness and attentiveness, especially among young learners.

As displayed in **Figure 4**, more than half of the students (32) had never viewed mathematical animation, even though most had viewed 2D mathematical animation. This illustrates that 3D mathematical animation is still rare in teaching. However, on the basis of student feedback, 3D animation evidently possesses great potential for teaching.

The results of the preliminary pilot study verified that students found the most difficult mathematical units to be binomial theorem, permutation and combination, and calculus, due to their complicated formulas and overall difficulty. Therefore, this study involved the production of an animation on the binomial theorem for students. A questionnaire survey was conducted after the students viewed the animation.

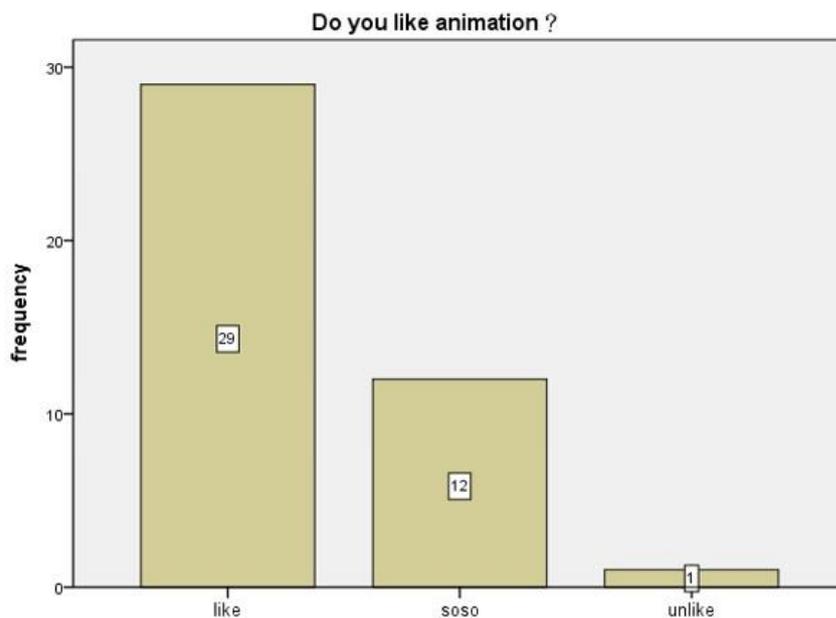


Figure 3. Preference of animation

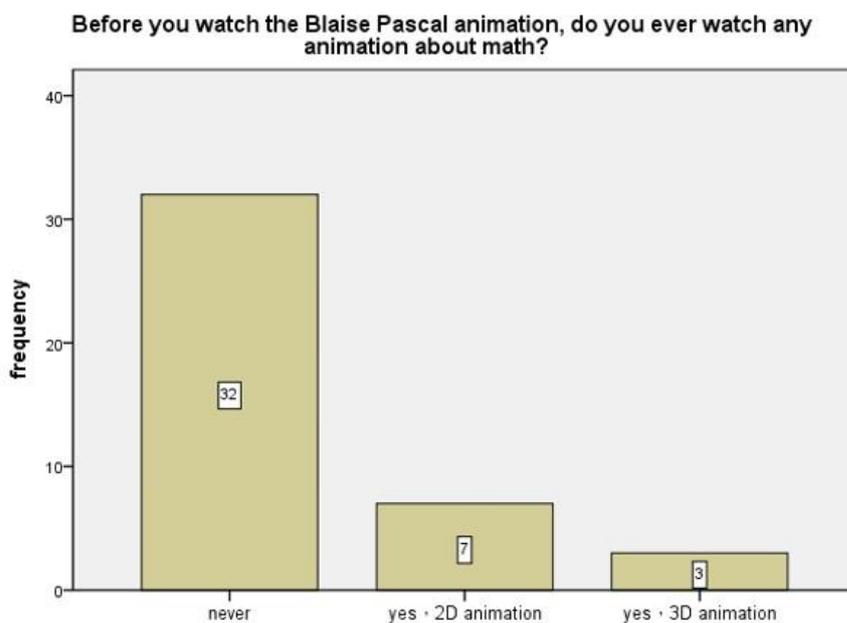


Figure 4. Students who have previously watched mathematical animation

**Table 2.** Analysis result regarding "Whether learning mathematics with animation could improve learning willingness."

Model	Non-Standardized Coefficient		Standardized Coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
After viewing this animation, you are more interested in learning mathematics (constant)	2.374	.443		5.354	.000
	0.479	.117	.545	4.108	.000

**Table 3.** Analysis result regarding "Whether learning mathematics with animation could improve comprehension."

Model	Non-Standardized Coefficient		Standardized Coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
I think learning Mathematics using 3D mathematical animation can generate more ideas. (constant)	2.638	.340		7.761	.000
	.355	.085	.550	4.161	.000

## RESULT

In the regression analysis, for the questionnaire dimension of "Whether learning mathematics with animation could improve learning willingness," as displayed in **Table 2**, the independent variable was given as "After seeing this animation, you are more interested in learning mathematics," and the dependent variable was given as "The current learning method can improve your learning willingness." The result demonstrated a significance level of  $.00 < .05$ , indicating a positive influence. The predictive power was .479, suggesting that learning with animation could increase students' interest in learning mathematics and their learning willingness. The influence was therefore determined to be positive.

Regarding the questionnaire dimension of "whether learning mathematics with animation could improve comprehension," as provided in **Table 3**, the independent variable was given as "Learning mathematics with 3D mathematical animation can generate more ideas," and the dependent variable was given as "The current method can improve your understanding and application of mathematics." The result demonstrated a significance level of  $.00 < .05$ , indicating a positive influence. The predictive power was .355, suggesting that learning with animation could generate more ideas as well as increase students'

**Table 4.** Analysis result regarding “Whether stronger learning willingness in mathematics influences the comprehension of mathematical concepts.”

Model	Non-standardized coefficient		Standardized coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
Using mathematical 3D animation can improve your learning willingness in mathematics. (constant)	2.247	.411		5.463	.000
	.279	.111	.370	2.518	.016

**Table 5.** Analysis result regarding “Whether comprehension of mathematical concepts affects learning willingness in mathematics.”

	Non-Standardized Coefficient		Standardized Coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
Using mathematical 3D animation can improve your comprehension or mathematical application. (constant)	1.962	.649		3.022	.004
	.490	.194	.370	2.518	.016

understanding and application of the binomial theorem and their comprehension of the probability unit. The influence was thus determined to be positive.

For the questionnaire dimension of “Whether stronger learning willingness in mathematics influences the comprehension of mathematical concepts,” as displayed in **Table 4**, the independent variable was given as “Using mathematical 3D animation can improve your learning willingness in mathematics,” and the dependent variable was given as “Using mathematical 3D animation can improve your mathematical understanding and application.” The results revealed a significance level of  $.016 < .05$ , indicating a positive influence. The predictive power was  $.279$ , suggesting that learning with animation could increase learning willingness and in turn improve comprehension of the binomial theorem and the probability unit. The influence was therefore determined to be positive.

Regarding the questionnaire dimension of “Whether comprehension of mathematical concepts affects learning willingness in mathematics,” as provided in **Table 5**, the independent variable was given as “Using mathematical 3D animation can improve your comprehension or mathematical application,” and the dependent variable was given as “Using mathematical 3D animation can improve your learning willingness in mathematics.”

**Table 6.** Analysis result regarding “Whether comprehension of mathematical concepts affects mathematics learning.”

Model	Non-Standardized Coefficient		Standardized Coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
Using the mathematical 3D animation method can effectively enhance your mathematical learning. (constant)	1.813	.710		2.556	.015
	0.429	.177	.358	2.426	.020

**Table 7.** Analysis result regarding “Whether learning willingness in mathematics influences mathematics learning performance.”

Model	Non-Standardized Coefficient		Standardized Coefficient	t	Significance
	B's estimated value	Standard error	Beta distribution		
Whether learning willingness in mathematics influences mathematics learning performance. (constant)	1.946	.338		5.754	.000
	.471	.103	.586	4.570	.000

The result demonstrated a significance level of  $.016 < .05$ , indicating a positive influence. The predictive power was  $.490$ , suggesting that learning with animation could improve comprehension and in turn increase students’ confidence and improve their learning willingness. The influence was thus determined to be positive.

For the questionnaire dimension of “Whether comprehension of mathematical concepts affects mathematics learning,” as displayed in **Table 6**, the independent variable was given as “Using mathematical 3D animation can improve your comprehension or mathematical application,” and the dependent variable was given as “Using the mathematical 3D animation method can effectively enhance your mathematical learning.” The result revealed a significance level of  $.02 < .05$ , indicating a positive influence. The predictive power was  $.429$ , suggesting that learning with animation could improve comprehension and in turn enhance learning performance regarding the binomial theorem and probability unit. The influence was therefore determined to be positive.

Regarding the questionnaire dimension of “Whether learning willingness in mathematics influences mathematics learning performance,” as provided in **Table 7**, the independent variable was given as “Using mathematical 3D animation can improve your learning willingness in mathematics,” and the dependent variable was given as “Using the

mathematical 3D animation method can effectively enhance your mathematical learning.” The result demonstrated a significance level of  $.00 < .05$ , indicating a positive influence. The predictive power was .471, suggesting that learning with animation could improve learning willingness in mathematics and in turn enhance the students’ learning performance regarding the binomial theorem and probability unit. The influence was thus determined to be positive.

## DISCUSSION

According to Fornell and Larcker (1981), we can use the R-square value as a predictor of the discrimination value. The R-square value should exceed 0.3. A higher R-square value indicates more accurate impact predictions. In this study, the predictive power regarding the enhancement of learning willingness was .471, and the predictive power of promoting understanding and application was .355, suggesting that learning with animation can improve student learning willingness in mathematics. Learning with animation can also enhance student performance regarding the binomial theorem and the probability unit. This supports the findings of earlier studies.

Animation has previously been demonstrated to promote student mathematical learning and achievement (Forster, 2006; Philpot, Hall, Hubing, Flori, Oglesby, & Vikas, 2003; Stoffa, 2004; Taylor, Pountney, & Malabar, 2007; Yang & Li, 2013). The findings of this study indicate that integrating mathematics history and mathematical concepts through 3D animation can reduce student mathematics learning pressure. This paper suggests that reducing the stress of learning can produce two crucial effects: (a) it can enhance learning willingness, and (b) it can promote mathematical understanding and application. These findings support the results of earlier studies (Chiang & Chang, 2008; Korakakis, Pavlatou, Palyvos, & Spyrellis, 2009; Liu, 2004; Wu, 2011; Yang, 1992; Yang, 2011).

This study developed 3D animation teaching materials by combining stories of mathematical history and mathematical concepts to reduce student learning pressure during mathematics learning. This paper suggests that reduced learning pressure can produce two crucial effects: (a) enhancing learning willingness and (b) promoting mathematical understanding and application. The current findings are consistent with those of previous studies (Chiang & Chang, 2008; Korakakis, Pavlatou, Palyvos, & Spyrellis, 2009; Liu, 2004; Wu, 2011; Yang, 1992; Yang, 2011). This study adopted regression analysis to analyze the prediction accuracy of the effects. According to Fornell and Larcker (1981), the R-square value can be adopted as the discriminant value of predictability. This value should exceed 0.3, and a higher R-square value indicates greater accuracy of impact predictions. In this study, the predictive power regarding the enhancement of learning willingness was .479, whereas the predictive power regarding the improvement of understanding and application was .355. These two values each exceeded the suggested value. Therefore, if mathematical teaching is conducted using teaching materials and concepts similar to those employed in this study, students may improve their learning willingness, understanding, and application.

Furthermore, given these probability values, the 3D animation digital teaching materials developed in this study were confirmed to have a positive influence on improve their learning willingness, understanding, and application. Regarding the relationship among learning willingness, understanding, and application, the probability of .490 confirms that “understanding the mathematical application of solving math issues positively influences learning willingness in mathematics.” Furthermore, the probability of .279 verifies that “improving learning willingness positively affects the understanding of the mathematical application of solving math issues.” However, the value of the notion that “improving learning willingness positively affects the understanding of the mathematical application of solving math issues” failed to reach the threshold, whereas the probability of the concept that “understanding the mathematical application of solving math issues positively influences learning willingness in mathematics” was higher. Regarding the influence of learning willingness, understanding, and application on learning achievement, the probability of .429 confirms that “understanding mathematical application of solving math issues positively influences learning achievement in mathematics.” Moreover, the probability of .471 confirms that “learning willingness in mathematics affects learning achievement in mathematics.” On the basis of previous research findings, this study proposed a method for verifying design, proving that when the methods which provide 3D animation are used as course materials and applied in teaching, their effects can be classified into the categories of (a) the effect on improving learning willingness and (b) the probability that impact prediction improves understanding and application.

### CONCLUSIONS

The results of this study showed that 3D animation can effectively help students to understand the unit of binomial theorem. Moreover, improved learning willingness and comprehension has positive influence on learning performance in mathematics. Hence, 3D animation can be used in mathematics education with promising potentials.

### SUGGESTIONS

Future studies can explore the following aspects: after viewing the animation, whether the learning willingness in mathematics has significant differences in self-control and self-learning; whether comprehension of mathematical concepts has significant differences in self-control and self-learning; whether self-control and self-learning have significant differences in learning performance.

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