School Academic Language Demands for Understanding Functional Relationships: A Design Research Project on the Role of Language in Reading and Learning

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ABSTRACT

Acquiring conceptual understanding of functions is far from being trivial for most students, especially language learners. The article reports on a design research project with students in Grades 8-11 (n = 94) that fostered academic language learners' development of conceptual understanding in the interplay of different semiotic representations. Theoretical and qualitative analyses of students' learning pathways and obstacles allowed the specification of school academic language demands based on concept demands for dealing with functional relationships. The strong interplay between concept and language demands can be described by the correspondence of conceptual compaction of conceptual facets and the language-related condensation of their verbalizations.

Key words: communicative and epistemic role of language, conceptual understanding, design research, functions, topic-specific academic language demands

INTRODUCING the PRACTICAL PROBLEM and the THEORETICAL QUESTIONS

Language proficiency is well known to influence mathematics achievement, but not only due to reading demands. In this article, the role of language in processes of developing conceptual understanding is investigated for the mathematical concept of functional relationship. Figure 1 shows an example from a high stakes test in grade 10 (MSW NRW 2012, p. 2) that illustrates interconnected reading and concept demands in a concrete way. Of course, this item contains reading challenges in the lexical dimension (e.g., the meaning of mileage and condensed expressions such as "mileage for a speed of"), but its main challenge is the conceptual understanding of functional relationships:

State of the literature

- In general, inadequate school academic language has been identified as an obstacle to learning for both monolingual and multilingual students with low language proficiency, especially with regard to conceptual understanding. However, the specific academic language demands emerging during conceptual development in mathematics learning have not yet been well specified.
- As many empirical studies show students' difficulties with the mathematical concept of functional relationships, teaching approaches for fostering its conceptual understanding have been developed. So far, however, the role of language in the processes of conceptual development has not been sufficiently investigated.
- Teaching approaches for fostering language learners have been criticized for being confined to the lexical dimension rather than supporting the syntactical or discursive dimensions.

Contribution of this paper to the literature

- In a design research project, school academic language demands for dealing with functional relationships are specified empirically.
- A teaching approach is developed for fostering language learners' conceptual understanding of functional relationships and investigated empirically with respect to the interplay of the topic-specific concept and language demands in the learning processes.
- In this way, the paper contributes to theorizing on the role of academic language for mathematics learning and to empirically grounding design principles for language-sensitive classrooms.

In order to mathematize the problem, students need to know that a function always connects two variables. Once the first variable and the dependent second variable are identified, the challenge in items (1) and (2) is reduced to finding out which quantity is given and which one is wanted and solving the given equation. However, many multilingual and monolingual students with low language proficiency could not activate this conceptual understanding in order to solve items of this type (Prediger, Wilhelm, Büchter, Benholz, & Gürsoy, 2015a).

This phenomenon was the starting point for a design research project that intended to foster

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Mileage problem. "The average mileage of the Wacker family's car (in liters/100 km) can be approximately calculated in dependency of its speed (in km/h) by the following equation:
$f(x) = 0.0005 \cdot (x - 40)^2 + 4.5262$.
(1) What is the average mileage for a speed of 150 km/h?
(2) What is the speed when 9.0 l for 100 km are needed?"

Figure 1. Reading demands and concept demands interrelated in a high stakes test item (Grade 10) ("In dependency of" is literally translated from German, it refers to functional relationships).

students' conceptual understanding of functional relationships in a content- and language-integrated teaching-learning arrangement. In order to develop a theoretical and empirical foundation for this practical need, the role of language in students' learning processes towards functional relationships has to be understood deeply, including the specification of topic-specific language demands. Thus, the intent of the design research project was not only to solve a practical problem (how to foster students' understanding) but also to contribute to two theoretically important overall research questions (to be refined in Section 2):

- Which language demands appear in processes of developing conceptual understanding?
- How can students be enabled to master both, the concept and the language demands?

In approaching these research questions, this article introduces the theoretical background on the roles of the school academic language register for conceptual understanding (Section 1) and then sketches the specific mathematical topic of functional relationships (Section 2). The research methodology of the project is briefly outlined in Section 3. Section 4 presents selected results of the qualitative analysis of concept and language demands in dealing with functional relationships while reading and solving word problems. Section 5 provides insights into processes of enhancing students' conceptual understanding based on a content- and language-integrated teaching-learning arrangement.

THEORETICAL BACKGROUND: THE ROLES OF SCHOOL ACADEMIC LANGUAGE FOR CONCEPTUAL UNDERSTANDING

Language Gaps in Conceptual Understanding and Conceptual Development

When achievement gaps between privileged and underprivileged students are reported, researchers mostly choose socio-economic status and immigrant background or family language background as indicators of privilege (Haag, Heppt, Stanat, Kuhl, & Pant, 2013; Mullis, Martin, Foy, & Arora, 2012; OECD, 2007; Secada, 1992). These indicators can easily be used to measure the issue of privilege using students' self-reports or existing school data, such as free school meals; thus, they are used more often than language proficiency. This trend also applies to the recent PISA report on low performers' backgrounds (OECD, 2016). However, when language proficiency in the language of instruction is also controlled, it turns out to be the factor with the strongest connection to mathematics achievement, stronger than multilingualism, immigrant background, or socio-economic status (Prediger et al., 2015a; Heinze, Reiss, Rudolph-Albert, Herwartz-Emden, & Braun, 2009). We thus agree with Hirsch (2003) that a "chief cause of the achievement gap between socio-economic groups is a language gap" (p. 22). This language gap occurs for multilingual as well as monolingual learners. Hence, for this article, the term *language learner* refers not only to second language learners but also to all students with low academic language proficiency in the language of instruction (which in this study is German). This focus is in line with Moschkovich's claim that "studies should focus less on comparisons to monolinguals and report not only differences between monolinguals and bilinguals but also similarities" (Moschkovich, 2010, p. 11).

The strong connection between mathematics achievement and language proficiency is often investigated with respect to language biases in tests (Abedi, 2006; Haag, Heppt, Roppelt, & Stanat, 2015) and constraints in reading proficiency (Abedi & Lord, 2001; Hirsch, 2003). In these studies, language is mostly treated in its *communicative role* and tends to be considered as external to the core of mathematics.

However, beyond reading challenges, many students with low language proficiency encounter other serious obstacles: in two recent studies, items that provided statistically unexpected difficulties (i.e., differential item functioning) for students with low language proficiency were those with high concept demands, such as conceptual understanding, not those items with reading obstacles (Ufer, Reiss, & Mehringer, 2013; Prediger et al., 2015a). This finding resonates with many qualitative studies, which show possible language obstacles in the processes of conceptual development (Moschkovich, 2010; Prediger & Krägeloh, 2015).

These findings call for taking into account not only the communicative role of language, but also its *epistemic role* in the processes of knowledge construction as a medium of thinking (Heller & Morek, 2015; Vygotsky, 1978). Students with low language proficiency might not only be hindered by reading obstacles (*communicative role*) in showing their competencies in tests but also be constrained throughout their individual school history, especially with respect to developing conceptual understanding (Prediger et al., 2015a; Moschkovich, 2015).

Three Roles of the School Academic Language for Conceptual Understanding

In order to explain the statistically evident connection between language proficiency and conceptual development, we draw upon the sociolinguistic distinction between school academic register and everyday register (Cummins, 2000; Snow & Uccelli, 2009; Schleppegrell, 2004). A register is defined as a "set of meanings, the configuration of semantic patterns that are typically drawn upon under the specific conditions, along with the words and structures that are used in the realization of these meanings" (Halliday, 1978, p. 23). Hence, registers are characterized by the types of communication situations, their fields of language use, the discourse styles, and modes of discourse. The school academic language is the register "that is used by teachers and students for the purpose of acquiring new knowledge and skills . . . , imparting new information, describing abstract ideas, and developing students' conceptual understanding" (Chamot & O'Malley, 1994, p. 40). Thus, the school academic language register has the *role of an important learning medium*, used in the mode "communicate to learn" (Lampert & Cobb, 2003; Pimm, 1987).

The sociolinguistic relevance of the school academic register lies in its second role, as an *unequally distributed learning prerequisite:* Whereas all children can acquire basic communication skills in the everyday language in their families, only socially privileged families also provide learning opportunities for aspects of the academic register (Snow & Uccelli, 2009).

An educational consequence can be drawn immediately: If the school academic register serves as a learning medium, it is a learning prerequisite for all students. If this prerequisite

cannot be taken for granted for all students, it is a matter of equity to treat it as *learning goal* in classrooms (from "communicating to learn" to "learning to communicate"; cf. Lampert & Cobb, 2003; similarly Schleppegrell, 2004; Thürmann, Vollmer, & Pieper, 2010).

For treating the school academic register as a learning goal in mathematics classrooms, its relevant features have to be well specified. Linguists have described the general differences between everyday language and school academic language in the *lexical dimension* (e.g., by specialized vocabulary, composite or unfamiliar words, and specific connectors) and in the *syntactical dimension* (e.g., long and syntactically complex sentences, passive voice constructions, and long noun phrases and prepositional phrases). Beyond the lexical and syntactic dimension, the school academic register can be characterized on the *discursive dimension* through specific discursive practices (e.g., arguing and explaining why), which are also not equally offered in all families (Bailey, 2007; Heller & Morek, 2015; Thürmann et al., 2010).

Although there is a consensus on these lexical, syntactical, and discursive features in general, there is still a research gap in specifying the specific school academic language demands that are *most relevant for learning specific mathematical topics,* for instance, the development of a conceptual understanding of functional relationships examined in this study (Moschkovich, 2015; Bailey, 2007). As each mathematical topic requires specialized language means to think and communicate about it, this specification needs to be topic specific. This article intends to contribute to this specification, because it provides a theoretically grounded and empirically based foundation for a focused language support. As topic-specific academic language demands are not separable from technical language on the micro level, we subsume both under academic language demands.

In order to *specify academic language demands*, most existing studies choose the method of analyzing textbooks and other curriculum materials (e.g., Bailey, Butler, Stevens, & Lord, 2007; Thürmann et al., 2010). Although this approach is insightful (and is also used in our preparatory work in Section 1.3), it risks the tendency to prioritize written language over oral communication and to restrict the focus mainly to the communicative role of language. In order to take into account the epistemic role of language in the three functions of (1) learning medium, (2) unequally distributed learning prerequisite, and (3) learning goal that requires further topic-specific specification, we extend the approach to analyzing (oral) learning processes on the micro level. As most regular classrooms do not provide conceptual learning opportunities, these learning processes have to be initiated by specifically designed learning arrangements. Thus, the research methodology of choice for this research is *topic-specific design research with a focus on learning processes,* which allows the researchers to overcome the deficit focus on language learners' obstacles by focusing on subtle resources in processes (see Section 3).

Moschkovich (2010) pleads for a research focus on students' processes of developing conceptual understanding and claims that "in order to focus on the mathematical meanings learners construct, rather than the mistakes they make, researchers will need frameworks for recognizing the mathematical knowledge, ideas, and learning that learners are constructing in, through, and with language" (Moschkovich, 2010, p. 12). In order to provide a systematic base for these empirical tasks, we briefly report on the language demands as far as they

could be specified theoretically.

First Specification of Lexical, Syntactical, and Discursive Demands
for Functional Relationships

The first step of the study involved specifying academic and technical language demands in the *language reception* on functional relationships in a preliminary textbook analysis (Zindel, 2013). Table 1 shows excerpts from the (incomplete) collection of used phrases for functional relationships that occur in word problems. The *lexical variety* of three different phrases for the same concept (three lines in the table) appears to be less critical than the *syntactical complexity* given by the German grammar with at least two to four grammatical variations for each phrase (in the six cells). Subtle syntactical constructions (grammatical cases) allow different orders for subject and object in the sentence without changing the sense. This is challenging for many students (even for those with high language proficiency) because the subtle syntactical differences and commonalities require language awareness.

All phrases in Table 1 describe functional relationships in a very condensed way and have to be interpreted by the students in order to decode the texts. However, many students do not even identify their relevance in a problem text (Zindel, 2013), as this discursive demand of identification requires conceptual understanding of functions. This conceptual understanding can become visible when students are able to relate different representations (in word problems, mainly the verbal and symbolic representation), which again requires their interpretation.

Table 1. First steps towards receptive language demands: German phrases for functional relationships (Zindel, 2013)

f(A) = B	Active Sentence Structure	Passive sentence structure
Dependency B of A	*The function indicates B in dependency of A.* • Die Funktion gibt B in Abhängigkeit von A an. • Die Funktion gibt das von A abhängige B an. • Die Funktion gibt B an, das von A abhängig ist. • Die Funktion gibt B an, das von A abhängt.	*In the function, B is given in dependency of A.* • B wird in Abhängigkeit von A angegeben. • Es wird das von A abhängige B angegeben. • Es wird B angegeben, das von A abhängig ist. • Es wird B angegeben, das von A abhängt.
Assignment A→ B	*The function assigns each A to a B.* • Die Funktion ordnet jedem A ein B zu. • Die Funktion ordnet ein B zu jedem A zu.	*Each A is assigned to a B.* • Jedem A wird ein B zugeordnet. • Ein B wird jedem A zugeordnet.
Implicit description by prepositions	*The function gives a B for [to] each A.* • Die Funktion gibt **für** jedes A ein B an. • Die Funktion gibt **zu** jedem A ein B an.	*For [to] each A, a B is given.* • Es wird **für** jedes A ein B angegeben. • Es wird **zu** jedem A ein B angegeben.

Summing up, the theoretical analysis of previous research and the textbook analysis allowed the specification of four main discursive demands (denoted by capital letters) in dealing with word problems of functions in language production and reception, which are strongly intertwined:

- READING COMPLEX TEXTS (in this study: word problems involving functions) is the discursive demand that requires managing the presented syntactical complexity.
- It first involves IDENTIFYING the relevant but highly condensed phrases in which the information about the functional relationship is coded.
- Decomposing the condensed phrases then involves the language production with the discursive demand of INTERPRETING TEXTS OR SYMBOLS.
- Of course, interpreting the texts is only possible after having developed conceptual understanding of the core concept functional relationship, and most important to the development of this understanding is the productive discursive demand of EXPLAINING THE MEANING of concepts (see Prediger & Wessel, 2013).

Because each of these discursive demands also requires conceptual understanding of functional relationships, the next section focuses on conceptual understanding.

CONCEPTUAL UNDERSTANDING OF FUNCTIONAL RELATIONSHIPS: STATE OF RESEARCH AND RESEARCH NEEDS

State of Research on Functional Relationships: Perspectives and Representations

The functional relationship is considered "one of the most fundamental and significant" concepts, applied in many inner- and extra-mathematical situations (Niss, 2014, p. 239). Although the approaches for specifying necessary elements for its conceptual understanding vary (see Niss, 2014; Carlson & Oerthmann, 2005; Leinhardt, Zaslavsky, & Stein, 1990), there is a common core related to representations and basic meanings, which are distinguished, for example, by the following perspectives:

- The *correspondence perspective* on functions conceptualizes how each value x in a functional relationship $y = f(x)$ is assigned to a unique value y (Vollrath, 1989; Confrey & Smith, 1994). Thompson refers to this perspective as a kind of static perspective, explained as seeing an "invariant relationship between two quantities whose values vary" (Thompson, 2011, p. 46).
- In contrast, the *covariation perspective* focuses on the way in which two varying quantities change together (Vollrath, 1989; Confrey & Smith, 1994; Carlson & Oerthmann, 2005). Thompson (2011) outlines covariational reasoning as "the very operations that enable one to see invariant relationships among quantities in dynamic situations" (p. 46).
- The *holistic perspective* on the function mainly focuses an encapsulated object perspective (Vollrath, 1989).

Besides these perspectives, some scholars have suggested other distinctions (e.g., the action, process, and object perspective by Dubinsky & Harel, 1992), while others have suggested distinctions that are bound to single types of functions (e.g., linear and exponential) or single representations (e.g., algebraic representation in equations, numerical representation in tables, graphical representations in graphs, and verbal descriptions). In this paper, we try to consider the core of functional relationships relevant in all these four representations, and we focus on the correspondence and covariation perspective and on the need for students to coordinate them (Vollrath, 1989; Thompson, 2011).

Conceptual understanding of functional relationships has often been described as the ability to adopt different perspectives flexibly in all four representations and to coordinate them (as summarized by Niss, 2014). Since the 1980s, *connecting four representations* has been identified as a key activity for understanding functional relationships (Swan, 1985; Leinhardt et al., 1990; Duval, 2006), but often with some shortcomings: In spite of the claimed symmetry, most design and research projects have focused either on the relation between qualitative graphs and verbal descriptions (e.g., Swan, 1985) or on graphs, equations, and tables (e.g., Leinhardt et al., 1990; Moschkovich, Schoenfeld, & Arcavi, 1993; Romberg, Fennema, & Carpenter, 1993). Less attention has been spent so far on the *connection between equations and verbal descriptions*, such as in mathematizing word problems in functions expressed either in the everyday, school academic, or even technical register. Another shortcoming concerns the "translation" metaphor, which does not imply a one-to-one-translation: Even if all three perspectives (correspondence, covariation, and holistic) are relevant in each representation, the shift between representations mostly implies modifications of meanings (Moschkovich et. al., 1993, p. 72); this also applies to shifts in the language registers.

Reacting to students' documented difficulties with activities involving flexibly moving between representations, a huge variety of teaching approaches have been suggested (see Leinhardt et al., 1990; Carlson & Oerthmann, 2005). These findings all show that enhancing students' conceptual understanding is a possible but complex task with many different aspects: "The desired outcomes are not likely to occur by default with most students . . . and they come at a price: time and effort" (Niss, 2014, p. 240; more details in Carlson & Oerthmann, 2005).

This fact raises the need to specify the conceptual core demands for functional relationships common to all types of functions and in all representations. The empirically grounded facet model of this core is presented in Section 2.2 and examples are investigated for the connection between verbal and algebraic representations in Section 4.1.

Facet Model for Specifying Concept Demands for Functional Relationships

Because the wide consensus about relevant perspectives and representations for functions has turned out to be too general for the purpose of specifying language demands in dealing with functional relationships, we have constructed a refined model of conceptual facets for

functional relationships that provides a language for a finer-grained analysis of elements of students' conceptual understanding of the core of functional relationships. In the empirical part of this article (Section 4), this model will be used to identify the language demands when dealing with different facets of functional relationships.

In order to construct this model, we refer to Hiebert and Carpenter's (1992) definition of understanding as related to learning with meanings. A concept "is understood if it is part of an internal network. . . . The degree of understanding is determined by the number and the strength of the connections" (Hiebert & Carpenter, 1992, p. 67).

This conceptualization of understanding as consisting of a dense network of pieces of knowledge calls for refining the pieces of knowledge we call facets of knowledge. The construct of *understanding as a network of facets* was fruitfully combined with Aebli's (1981) construct of *compacting into denser concepts*: When learning new concepts, single facets of conceptual understanding have to be acquired and then related to each other. Once the network is mentally constructed, it can be compacted into more condensed facets. A deep understanding of a concept is reached when learners are able to flexibly switch between the compacted facets and to *unfold* them again into their more elementary facets (Drollinger-Vetter, 2011).

Based on the theoretical construct of Hiebert and Carpenter (1992) and Aebli (1981) of understanding as a *network of facets that are compacted into denser concepts* and on the preliminary empirical results of our research, we constructed the model of conceptual facets of understanding the core of functional relationships in Figure 2. It provides the language for describing and comparing students' resources, processes, and obstacles (see Section 4.1).

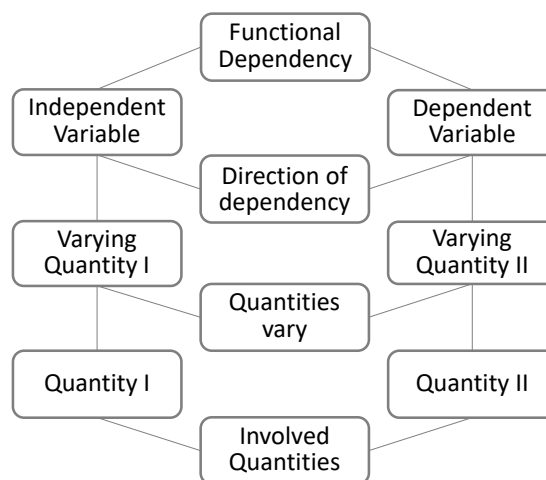In order to explain the facet model, we refer to the mileage problem in Figure 1. In this

Figure 2. Model of conceptual facets of understanding the core of functional relationships

In this problem, the (compacted) symbolic equation $f(x) = 0.0005 \cdot (x - 40)^2 + 4.5262$ has to be related to the (condensed) phrase "the average mileage . . . can be approximately calculated in dependency of its speed. . . ." The successful coordination of both representations is considered an indicator for understanding the compacted concept ‖functional dependency‖ (our denotation ‖ . . . ‖ marks a facet of the model in Figure 2 or additional facets that students address).

Students who understand this compacted concept can unfold it into the conceptual facets required for succeeding in this coordination of representations: Students need to know that each functional relationship connects two ‖involved quantities‖ and that these ‖quantities vary‖. The ‖direction of dependency‖ matters, so it is important to identify the speed as the ‖independent variable‖ and mileage as ‖dependent variable‖. This analysis resonates with Thompson (2011), who emphasized the high relevance of quantities as mental entities for understanding functional relationships and of quantitative reasoning. The facet model is the base for the following definition:

> Conceptual understanding of the core of functional relationships is defined as the ability to adopt different perspectives in different representations and to coordinate them by addressing the facets from Figure 2 flexibly and adequately. This requires flexible compacting and unfolding of conceptual facets, thus moving upwards and downwards in the facet model.

In this definition, "flexibly" marks the need to find different ways in different situations, and "adequately" refers to the specific situations given by a context problem, a teacher question, or a task. As the empirical analysis will show, the model allows unpacking of concept demands for compacting and unfolding complex concepts along with the specific language demands.

Fostering Language Learners' Conceptual Understanding

Once having specified the network of conceptual facets to be acquired by students, the question arises as to how this acquisition can be fostered, especially for language learners. Moschkovich (2013) has articulated four general recommendations for multilingual language learners that apply also to monolingual language learners:

#1: Focus on students' mathematical reasoning, not accuracy in using language.
#2: Focus on mathematical practices, not language as single words or vocabulary.
#3: Recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity.
#4: Treat everyday and home languages as resources, not obstacles. (Moschkovich 2013, p. 50)

The main mathematical practices we focus on are sense making and modelling, for which Moschkovich (2013) recommends "keep[ing] tasks focused on high cognitive demand, conceptual understanding, and connecting multiple representations" (ibid, p. 52). Thus,

connecting multiple representations is not only a learning goal but also an important design principle for achieving the goal.

The design principle of connecting algebraic, numerical, verbal, and graphical representations (e.g., Bruner, 1967) can be extended to the idea of relating language registers (everyday register, school academic register, and technical register). This has been theoretically justified (Prediger, Clarkson, & Bose, 2016) and investigated for the case of fractions (Prediger & Wessel, 2013). Rather than planning a unidirectional process from the everyday register and graphical representations to the technical register and symbolic representation, the design principle of relating registers and connecting representations pleads for repeatedly moving forward and backward, without assuming a hierarchy between the representations or registers.

Cognitive activities for connecting representations and registers have been described by Duval (2006): following Piaget's operative principle, he emphasizes the effectiveness of the activity of *systematic variation* in one representation and investigating its effects in a second representation (Duval, 2006, p. 125). In our teaching approach, we apply the activity of systematic variations of phrases, i.e., in the verbal representation (see Sections 4.3 and 5).

Research Questions

Based on these theoretical considerations and preliminary specifications, the research questions on specifying demands (RQ1) and on possible approaches for fostering students' conceptual understanding (RQ2) can be refined as follows:

(RQ1) Which concept and language demands arise for students when dealing with functional relationships and how are they interrelated while connecting representations?

(RQ2) How can the designed teaching learning arrangement with the design principle "relating registers by systematic variation of phrases" support students' learning processes towards mastering the interrelated concept and language demands?

METHODOLOGİCAL FRAMEWORK OF THE DESİGN RESEARCH PROJECT

Methodology of Topic-Specific Design Research with a Focus on Learning Processes

Since for this project, specifying the demands and learning goals is as important as investigating effects of design approaches, we choose the methodological framework of Topic-Specific Design Research.
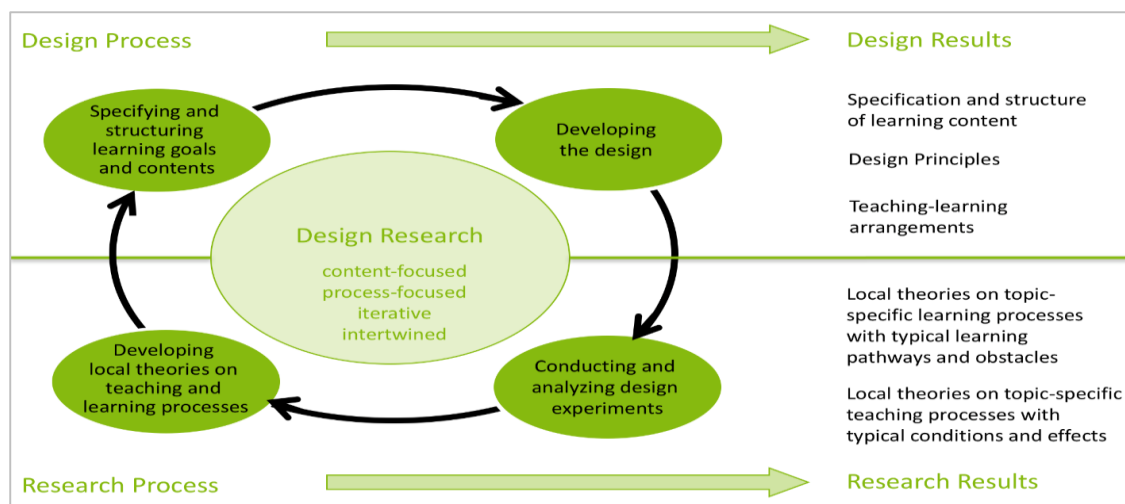
Figure 3. Four working areas for Topic-Specific Design Research (Prediger et al., 2012)

Like many approaches within the methodology of design research with a focus on learning processes (Gravemeijer & Cobb, 2006; Prediger, Gravemeijer, & Confrey, 2015b), our framework of Topic-specific Design Research relies on the iterative interplay between designing teaching-learning arrangements, conducting design experiments, and empirically analyzing the processes. Its four working areas and typical design and research results are depicted in Figure 3 (Prediger et al., 2012).

The design outcomes of the reported project comprise a further elaboration of the specified and structured learning content (in this case, concept and language demands for developing conceptual understanding of functional relationships), refined design principles (in this case, connecting representations and systematic variation of phrases; see Prediger et al., 2016), and a prototype learning arrangement. The research outcomes consist of empirical insights and contributions to local theories on learning and teaching processes of the treated topic (in this case, the role of the school academic language in processes of developing conceptual understanding of functions).

Design Experiments as the Method for Data Collection

Design experiments are considered the methodological heart of design research studies as they allow in-depth investigations of learning *processes* rather than only learning *outcomes* (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Gravemeijer & Cobb, 2006).

In the overarching project, we conducted three design experiment cycles (19 design experiments in 1-3 sessions each) in laboratory settings with 18 pairs of students and one single student (one student's partner was ill) in Grades 8-11 (14-17 years old). The fourth design experiment cycle took place with students in three whole classes in three classroom settings (n = 57), with each class lasting for 45 minutes each. In total, 42 design experiments each lasting 30-60 minutes were completely video-recorded (1890 minutes of video) and

partly transcribed. At the beginning of the first cycle, a textbook analysis and clinical interviews with think-aloud protocols were conducted in order to identify typical obstacles with problems such as the one in Figure 1. Based on this material, the teaching-learning arrangement was developed and iteratively elaborated using design experiments in four cycles.

The case studies presented in the following chapters use data from Cycle 1 (clinical interviews dealing with RQ1) and Cycle 3 (design experiments dealing with RQ2) in which the design experiments in laboratory settings were led by the second author. The students in the case studies reported from Cycle 1, Manuel, Luisa und Dennis, were in Grade 10 and were 15-16 years old. The case study from Cycle 3 involved Fynn and Svenja, who were 15 years old and from a Grade 9 class in a comprehensive school. These students were selected as cases because they had shared monolingual German language backgrounds with further language learning needs in the school academic language register, but had contrasting profiles in mastering the concept and language demands.

Qualitative Methods for Data Analysis

The qualitative analysis of selected transcripts of interviews and design experiments was conducted with the aim of specifying concept and language demands in the processes of problem solving or acquiring conceptual understanding.

By employing a turn-by-turn analysis of the selected transcripts, students' conceptual thinking was captured in Vergnaud's (1996) framework of students' individual concepts- and theorems-in-action. Vergnaud defines a theorem-in-action as "proposition that is held to be true by the individual subject for a certain range of situation variables" (Vergnaud, 1996, p. 225). Theorems-in-action are indicated using "< . . . >", e.g., <For identifying the dependent quantity, it suffices to consider the unit of rate of change.> These theorems-in-action are shaped by concepts-in-action, defined as "categories . . . that enable the subject to cut the world into distinct . . . aspects and pick up the most adequate selection of information" (ibid.); in this study they are ‖involved constants‖ and ‖dependent variable‖. In the first step of data analysis, students' theorems-in-actions were inferred from their utterances and actions. Vergnaud's framework allows extrapolation of the underlying concepts-in-action. In a second step, categories for concepts- and theorems-in-actions were built by systematically comparing and contrasting the different cases of students' thinking. In the preliminary work, the systematization of concepts-in-action resulted in the model of conceptual facets (as presented in Section 2.2). Thus, facets of the model are typical concepts-in-action, but other concepts-in-actions can also be inferred by the open data analysis procedure. In the third step presented here, the model was used as an open categorical scheme, and the extrapolated uses of facets were related to the language forms in which they appeared.

Together, these analytic procedures allowed the researchers to unpack the conceptual and language-related sides of demands in both situations of reading word problems (Section 4) and design experiments for developing conceptual understanding (Section 5).

CONCEPT AND LANGUAGE DEMANDS IN DEALING WITH FUNCTIONAL RELATIONSHIPS WHILE READING AND SOLVING WORD PROBLEMS

The empirical specification of concept and language demands started with analysis of three cases with respect to concept demands (Section 4.1) and language demands (Section 4.2).

Revealing Concept Demands in the Interplay of Representations

The three cases show the processes of three students, Manuel, Luisa, and Dennis, when trying to solve the mileage problem from Figure 1. The case of Manuel represents a successful process in connecting the given symbolic and verbal representation. After reading the mileage problem (in Figure 1), he immediately thinks aloud:

7   Manuel   In each case, you have the function, which anyway assigns . . . the mileage to the speed—here . . . .

20 Manuel   When one factor changes, . . . that the other factor changes . . . The function [*tells*] you only . . . for which speed you have which mileage.

The analysis of Manuel's thinking process is visualized by the facet model in Figure 4 in which adequately addressed facets or connections are framed by green lines and inadequately addressed facets by red dashed lines.

In Line 7, identified the ‖functional dependency‖ adequately and reformulated the text of the problem: "anyway assigns . . . the mileage to the speed." He seemed to transform "in dependency of" into the alternative (but equally condensed) phrase "assigns to" (Table 1). We interpret his flexible descriptions for the highest level in the facet model as an indicator

Lines 7, 20: Symbolic Representation ←→ Verbal Representation
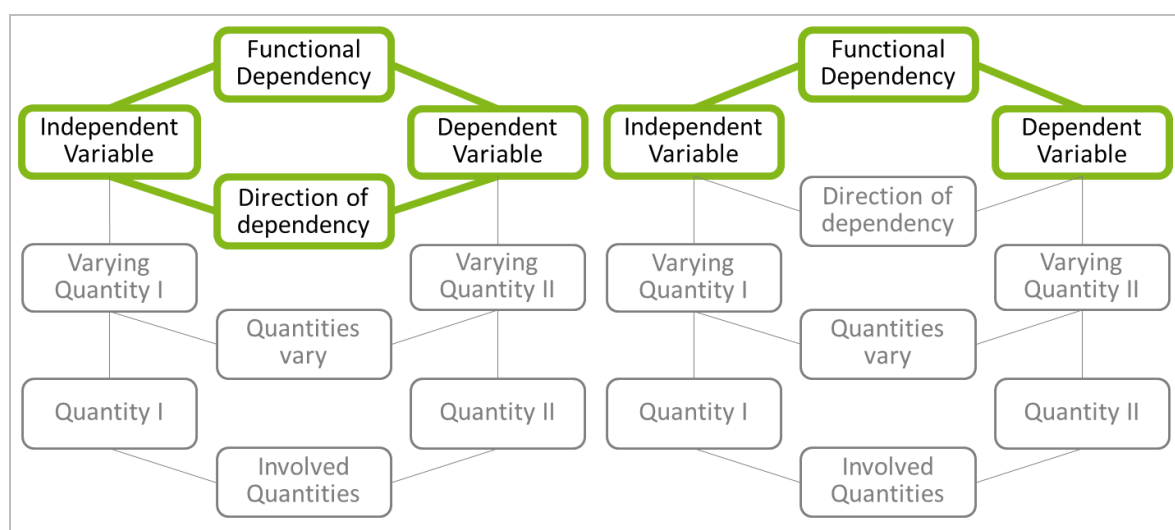

Figure 4. Reconstruction of Manuel's addressed conceptual facets

of his highly developed conceptual understanding.

The analysis of Line 20 supports this interpretation. Building on the insight that there were two varying quantities, Manuel realized that the ‖direction of dependency‖ mattered: "when one factor changes, . . . the other factor changes" (Line 20). This allowed him to reformulate the verbal representation in a highly condensed form: "The function [*tells*] you only . . . for which speed you have which mileage." For this translation, he implicitly identified the ‖independent variable‖ and the ‖dependent variable‖ adequately. Hence, he unfolded the functional relationship on the medium level of the facet model (marked in green in Figure 4) successfully with respect to the symbolic representation. One can assume that he would have been able to unfold it also on the lowest level, but this was not necessary for him.

In contrast, many other students encountered serious difficulties in the design experiments. The facet model allows the identification of sources of their obstacles, as it did for Luisa (15 years old).

17 Luisa   Thus, we have here three numbers [*hints to the constants in the equation*].
19 Luisa   But here [*in the text*], there are only two, driven kilometers [*per hour*] and mileage. Any [*of the three*] must be of something completely different.

Luisa's theorem-in-action, <The three parameters in the equation belong to the quantities in view>, indicates a deviant coordination of the ‖involved quantities‖ in the verbal representation with the ‖involved constants‖ of the symbolic representation, without focusing the phrase "in dependency of."

Her attempt to coordinate the ‖involved constants‖ in one representation and the ‖involved quantities‖ in the other representation is visualized in the facet model in Figure 5. It indicates the urgent need to enhance her conceptual understanding of functional relationships beyond decoding the text.

In Dennis's (15 years old) case, the model allows identifying his understandings that are not yet conceptually viable and capturing his successive process of cracking the word problem. Dennis started as follows:

5 Dennis   They have only given the information for the mileage and the speed.
6 Dennis   That is now the question; if there is x, x is probably the mileage, because "in dependency of the speed" is then—oh, probably simply the 40 or the 4.5462.

In clarifying the meaning of the problem, in Line 5 Dennis identified the two ‖involved quantities‖ (see Figure 6). So far, this facet was treated in an isolated way, without yet addressing the ‖direction of dependency‖, for example.

In Line 6, Dennis identified an inadequate ‖independent variable‖ in the symbolic representation and constructed a deviant meaning for it in the verbal representation: His implicit theorem-in-action, <In order to identify the value of the independent variable, one can search among the constants of the equation>, led him to consider a single value rather than a (possibly varying) quantity. This is interpreted as an indicator of a not yet accomplished understanding of the facet ‖dependent variable‖ and as a reason why he related the phrase "in dependency of the speed" to an appropriate part of the equation.

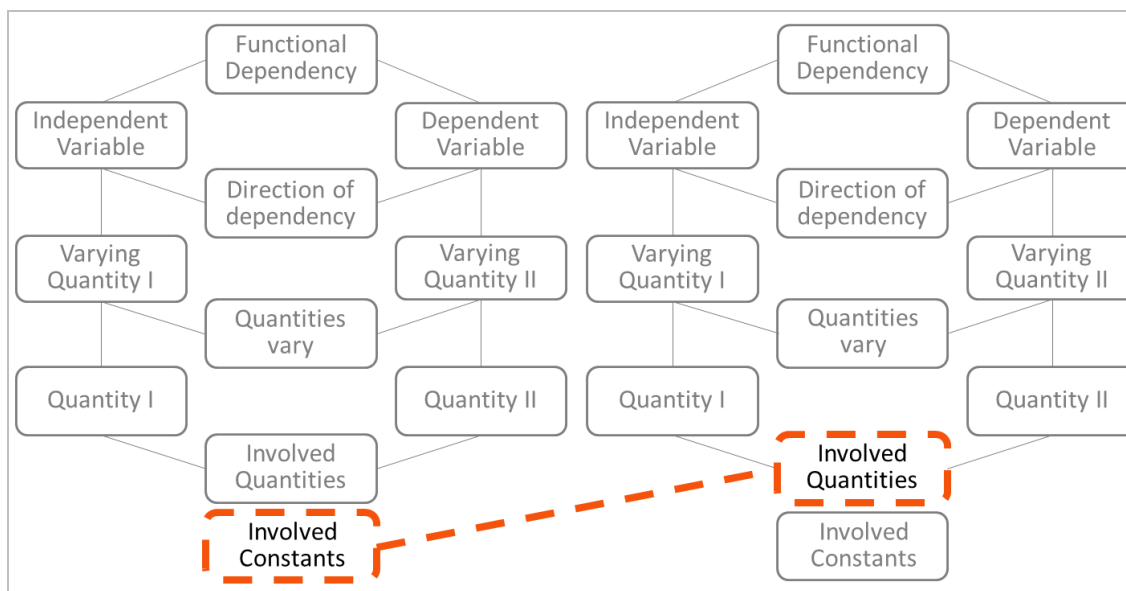Lines 17,19: Symbolic Representation ⟷ Verbal Representation

Figure 5. Reconstruction of Luisa's addressed conceptual facets

In a much later step, he corrects himself:

101 Dennis   x is the speed, because—the mileage is now—don't know exactly what this will be—but x is the speed, so that you can always insert something else.

For the ||independent variable||, he activated an appropriate theorem-in-action: <The independent variable is the one that can be evaluated for different values>. Using this theorem, he unfolded the ||functional dependency|| but isolated the ||independent variable|| from the ||dependent variable||. This isolation was the source of the difficulty in identifying the role of the mileage.

With some more support of the design experiment leader, he could finally succeed in relating the different conceptual facets to each other (compacting) and thereby in decoding the problem.

These small excerpts from the cases of Manuel, Luisa, and Dennis show the *concept demand of coordinating and connecting the different facets in both representations*: all conceptual facets can become relevant for succeeding in coordinating the symbolic equation and the phrase "in dependency of" (literally translated from German), as they have to be adequately addressed, combined, and related between representations. Obstacles appear when students:

    (a) focus too exclusively on one facet (e.g., as Dennis in Line 5),
    (b) address a mismatching facet (e.g., as Luisa referring to the constants),
    (c) mismatch one facet in different representations (e.g., as Luisa in Line 19), or
    (d) show structural misunderstanding of a facet (e.g., as Luisa or Dennis in Line 6).
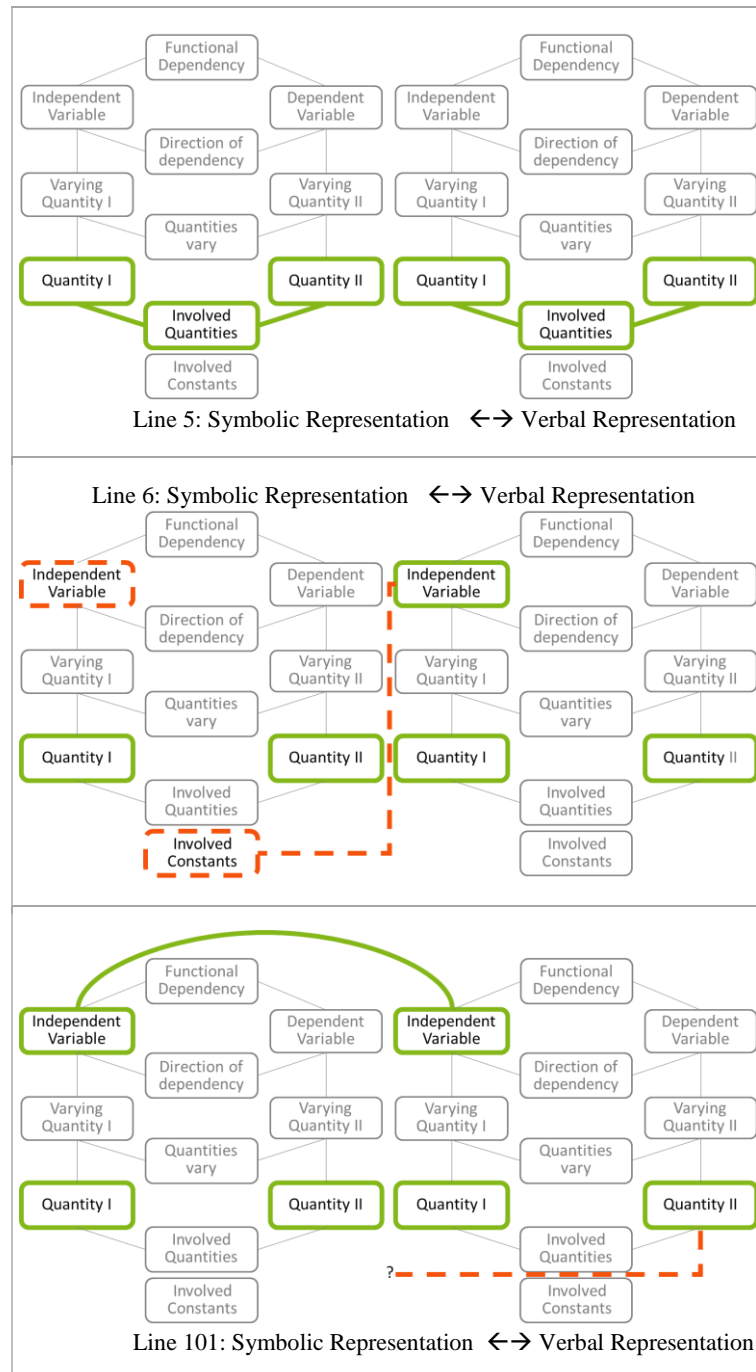
Figure 6. Reconstruction of Dennis' addressed
conceptual facets

Whereas mode (d) indicates conceptual misunderstandings, modes (a)-(c) could also only indicate a strategic flaw in decoding the concrete text in spite of existing understanding of

the concept. In either conceptual obstacles or strategic reading obstacles, the model of conceptual facets (Figures 4-6) allows the empirical unpacking of the complex underlying cognitive phenomena.

Revealing Receptive and Productive Language Demands

These case studies can now be discussed with respect to the occurring language demands: the case of Luisa exemplifies the *receptive language demands* anticipated in Section 2.2 in the communicative role of language: Luisa failed in READING COMPLEX TEXTS as she missed IDENTIFYING the condensed phrase "the mileage in dependency of the speed."

Beyond that, the empirical analysis in Section 4.1 provides insights into *demands in students' language production* occurring with the demanded language decomposition of the highly condensed phrase for ‖functional relationships‖ that refer to the epistemic role of language: As the complex phrase contains all other conceptual facets in a compacted form, condensing syntactically (e.g., by nominalizations or prepositional constructions; see Jorgensen, 2011) can be considered the language-related counterpart of the conceptual process of compacting in Aebli's sense (1981).

This *correspondence of conceptual compacting and language-related condensing* is visualized in Figure 7. Thus, for INTERPRETING and UNDERSTANDING the phrase, it must be unfolded into its facets on the lower levels of the model, and this process of unfolding requires language production on the lower levels. The corresponding de-composing of nominalizations brings much longer sentences for the four facets. Manuel's decomposed explanation activates if-then clauses (Lines 7-20, typical for the covariation perspective) and expresses the the ‖direction of dependency‖ as well as the two ‖varying quantities‖. Isolated identification of quantities on the lowest level, as in Dennis's case, sometimes goes along with language challenges to express the dependency in relational words; this is another prototypic example
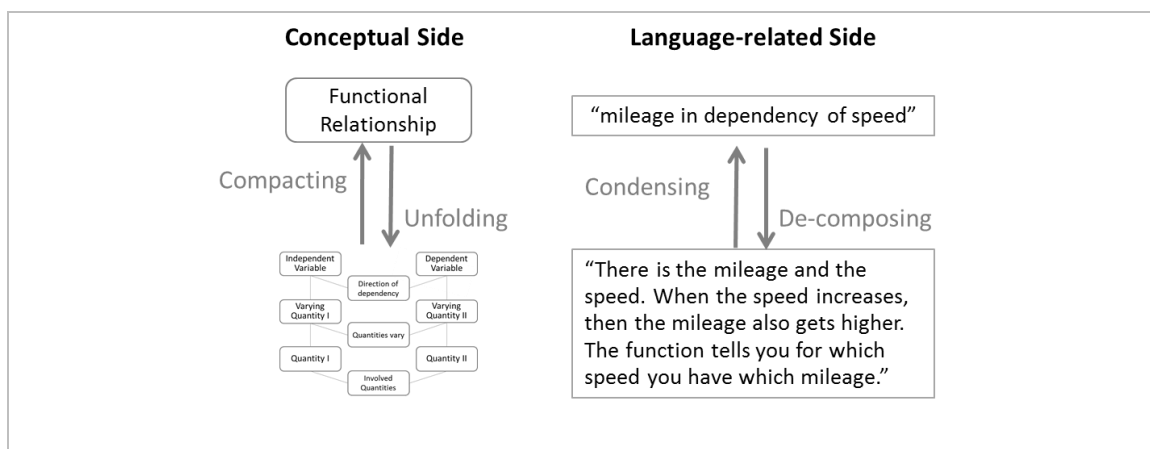

Figure 7. Correspondence of conceptual compaction and language-related condensation

for the epistemic role of academic language. In addition, having conceptual understanding is necessary to be able to address conceptual facets verbally. We summarize the main findings for this topic:

Receptive and productive demands occur in the communicative and epistemic role of language. The strong interplay between concept and language demands can be described by the correspondence of conceptual compaction of conceptual facets and the language-related condensation of their verbalizations.

Consequences for the Teaching-Learning Arrangement for Understanding Functional Relationships

The refined specification of concept and language demands outlined in Section 4.2 constituted the starting point for redesigning the learning arrangement. Due to the findings on the necessity of relating conceptual facets, the redesign followed a new design principle: *focus on coordinating and relating the conceptual facets*. This coordination of conceptual facets is triggered by the design principles of relating registers and systematic variation of texts (see Section 2.3).

Figure 8 shows one central activity from the designed learning arrangement in Design Experiment Cycle 3. In Question 1, students are asked to compare two offers for online streaming: DreamStream and Streamox3. When working on such tasks, students usually refer to the ||rates of change|| and the ||start values|| for the comparison. In order to answer Question 2, students calculate values in the table. The tables can be read vertically (in a covariation perspective) or horizontally (correspondence perspective). The covariation perspective emphasizes the meaning of the ||involved constants||, while the correspondence perspective underlines the ||involved quantities||. The intent of Questions 3 and 4 about the equation is to enhance students' focus on the ||involved quantities||.

In order to find the equation, students need to coordinate all facets, ||involved quantities||, ||quantities vary||, and the ||direction of the dependency||, and, in this case, even the ||involved constants|| are important. Question 5, by deciding and explaining which phrases match or mismatch to the equations, addresses different facets explicitly, because they vary systematically in one of these facets.

In this way, the activity is designed to foster conceptual understanding by dealing with unfolded facets and re-compacting them. This is especially necessary for those students who did not understand single facets structurally or those who are not able to compact them without prompts. Furthermore, comparing the descriptions aims at initiating reflection and sensitizing for details in the formulations (thus enhancing some language awareness).

Table 2 summarizes some of the decisions in the design of Cycle 3 that were made based on consequences from previous cycles. Without assuming any automatism in how design elements can enhance the overcoming of obstacles, Table 2 roughly sketches hypothesized connections. Empirically, the potential of the principle of systematic variation of phrases for overcoming conceptual obstacles will be shown in the next section.

Comparing Streaming Offers

(1) Compare the different offers. Which one would you choose?
(2) Which offer is better after how many months?
(3) What is the total price, if you use the offer for 12 months?
(4) Find the equation that describes the general relationship.
(5) Which description matches which of your equations?

DREAMSTREAM
In our online video store you can rent a film for a flat rate of 19,99 EUR per month. For this amount, you can rent as many films as you like every month. There is an additional one-time registration fee of 5 EUR.

Number of months	Total price

$$f(x) = 19{,}99 \cdot x + 5$$

Description A: The equation indicates the total price in dependency of the number of months.

STREAMOX3
Watch our complete library of films and series conveniently on your television with our new Streamox3-TV! There is a one-time fee of 49 EUR, with a monthly membership fee of only 9.99 EUR!

Number of months	Total price

$$f(x) = 9{,}99 \cdot x + 49$$

Description C: The equation indicates the number of months in dependency of the total price.

Description B: With the equation, I can - in dependency of the number of months - calculate the total price.

Figure 8. Activities from the learning arrangement in Design Experiment Cycle 3, realizing the design principles of relating registers and systematic operative variation of phrases (Descriptions A-C literally translated from German)

Table 2. Overview of consequences of previous cycles' effect on the design of Cycle 3: Design elements for different obstacles

Potential conceptual obstacles	Design elements in the learning arrangement
(a) Focus too exclusively on one facet	Systematic variations of phrases triggers focus on other facets
(b) Address a mismatching facet (constants)	Structure of the intended learning pathway shifts the attention from the constants to the involved quantities
(c) Mismatch of one facet in different registers	Enhance language awareness by reflecting the systematic variations of phrases
(d) Show structural misunderstanding of a facet	Develop conceptual understanding by working on the missing facets

Potential language obstacles	Design elements in the learning arrangement
(e) No attention to key phrases such as "in dependency of"	Enhance language awareness by reflecting on the systematic variation of phrases
(f) Phrase is focused, but inappropriately interpreted due to missing strategic focus on relations	Finding equations triggers to search for the quantities, thus fix meaning of variables

CONCEPTUAL AND LANGUAGE-RELATED PROCESSES WHILE DEVELOPING CONCEPTUAL UNDERSTANDING OF FUNCTIONAL RELATIONSHIPS

The following two transcripts from Svenja's case offer empirical insights into how the redesigned learning arrangement in Section 4.3 helps students to master the intertwined concept and language demands (RQ2).

Svenja (15 years old) worked with Fynn and the design experiment leader (in this case, the teacher) in Cycle 3 in attempting to reflect the meaning of Description A (in Figure 8). They provide insights into the intertwinement of students' conceptual and language-related learning pathways. The first transcript shows how the receptive and productive language demands are interrelated. The transcript starts when Svenja's partner Fynn tried to explain whether Description A matched the streaming offer from DreamStream (Question 5 in Figure 8).

340	Fynn	Uh. First, the equation doesn't indicate anything [*reading Description A*]. Well, in the end it does, but [*simultaneously*] one shall calculate it.
341	Svenja	[*simultaneously*] . . . It doesn't indicate a price. So.
342	Fynn	Exactly.
343	Svenja	But . . . what one, uh, has to pay.
344	Teacher	[*approvingly*] Mhm.
345	Svenja	It isn't a fixed price; um, well, so total price, because one doesn't know now how many months, because . . . see as months. That's why . . . .
346	Teacher	. . . does it match?
347	Svenja	Um. "In dependency of the months." So this here [*points to the functional equation of the DreamStream offer*]. This . . ., let's say here, dependency are five months.
348	Teacher	[*approvingly*] Mhm.
349	Svenja	So that one is able to calculate the price — the total price one has to pay after five months.

Fynn had difficulty identifying the phrase that was relevant to deciding whether the description matched or not. He justified his first judgment that it mismatched by saying that "the equation doesn't indicate anything" (Line 340).

Svenja (for whom the analysis is depicted in Figure 9) elaborated Fynn's utterance with respect to the ||dependent variable|| and asserted that the equation did not indicate one fixed price (Lines 341-345). She approximates this idea in three steps: "it doesn't indicate a price" (Line 341), "but what one has to pay" (Line 343), and, finally, "it isn't a fixed price . . . because one doesn't know now how many months" (Line 345).

After an incoherent utterance in Line 343, she started to address several facets, with more language coherence in Line 345: She compacted the ||varying quantity II|| and explained the ||dependent variable|| by relating it to the ||independent variable||. With her utterance "one doesn't know now how many months," she addressed the ||direction of dependency||.
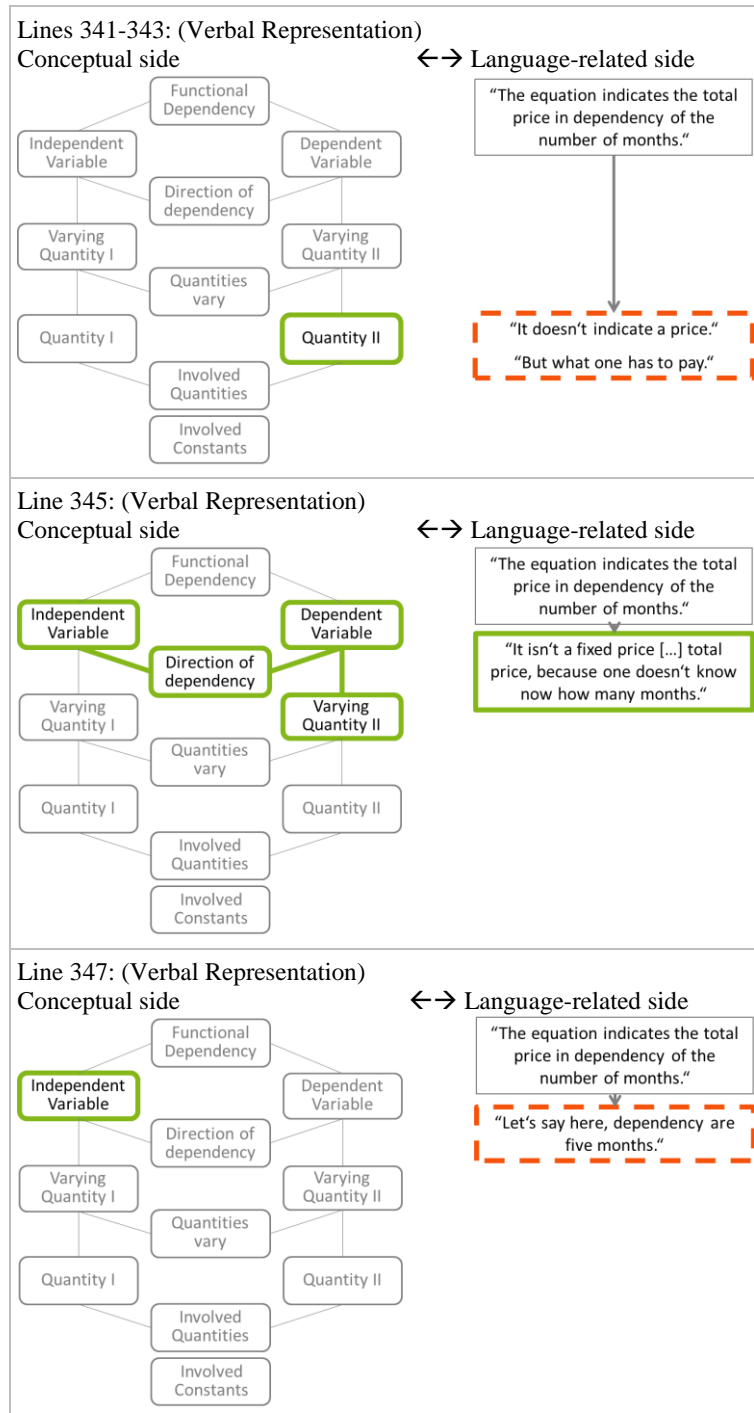
Lines 341-343: (Verbal Representation)
Conceptual side   ←→ Language-related side

Functional Dependency	"The equation indicates the total price in dependency of the number of months."
Independent Variable / Dependent Variable	
Direction of dependency	
Varying Quantity I / Varying Quantity II	
Quantities vary	
Quantity I / **Quantity II**	"It doesn't indicate a price." "But what one has to pay."
Involved Quantities	
Involved Constants	

Line 345: (Verbal Representation)
Conceptual side   ←→ Language-related side

Functional Dependency	"The equation indicates the total price in dependency of the number of months."
Independent Variable / **Dependent Variable**	"It isn't a fixed price [...] total price, because one doesn't know now how many months."
Direction of dependency	
Varying Quantity I / **Varying Quantity II**	
Quantities vary	
Quantity I / Quantity II	
Involved Quantities	
Involved Constants	

Line 347: (Verbal Representation)
Conceptual side   ←→ Language-related side

Functional Dependency	"The equation indicates the total price in dependency of the number of months."
Independent Variable / Dependent Variable	"Let's say here, dependency are five months."
Direction of dependency	
Varying Quantity I / Varying Quantity II	
Quantities vary	
Quantity I / Quantity II	
Involved Quantities	
Involved Constants	

Figure 9. First part of the reconstruction of
Svenja's learning pathway in the model of conceptual facets

Line 349: (Verbal Representation)
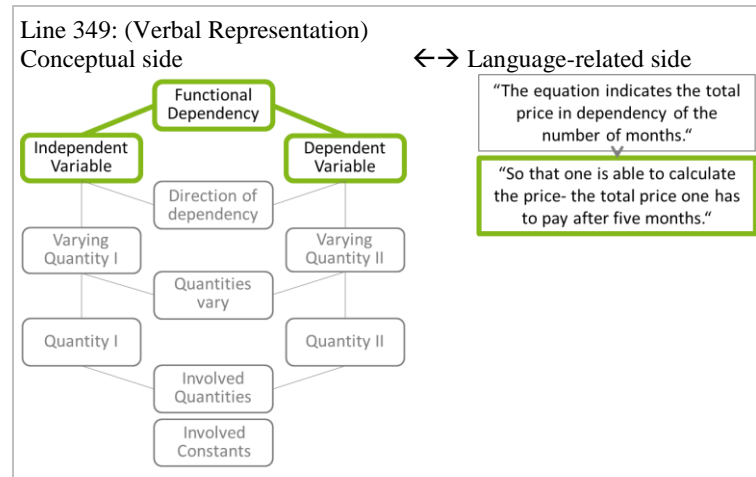Conceptual side ←→ Language-related side

Figure 9. contiued

Having unfolded the necessary conceptual facets in this way, she condensed them again to the given phrase "in dependency of the months" (Line 347). When she intended to evaluate the function for value 5 she chose as an example, she articulated this intention by saying "dependency are five months" (Line 347) as a not yet completely adequate but highly condensed phrase. The last utterance, "the total price one has to pay after five months" (Line 349), is a perfect description of her example. Thus, she adequately addressed the ||direction of dependency|| with reference to the ||independent variable|| and the ||dependent variable|| on a high level of compaction.

Within Lines 341-349, Svenja decomposed the condensed phrase and successively described its facets. The process shows how much language production is necessary for the conceptual process of unfolding. All four discursive demands (READING, IDENTIFYING, INTERPRETING, and EXPLAINING) are involved here and mastered with respect to the relation between verbal and symbolic representation.

Some minutes later, Svenja and her partner Fynn have assigned all matching phrases to the DreamStream offer. The transcript below starts when Svenja explained why the same phrases match Streamox3 (in Figure 8):

377 Svenja Because these 9,99 € are per month
378 Teacher Mhm
379 Svenja [*Points to the Streamox3 offer*] and there one time this unique [the basic rate] as here these unique 5 € [*looks at the DreamStream offer*], one has to pay.
Hence, these as well [*points to the phrases they have assigned to the DreamStream offer*] . . .
for both the same.

Svenja (Figure 10) activated the deviant concept-in-action ||involved constants|| in order to justify matching both descriptions. This hindered her from justifying the match using the

||involved quantities||, which were compacted in the ||functional dependency||. Svenja also changed her reading strategy:

385	Svenja	Let's say we have five months again [*points to the Streamox3 offer*], then one calculates five times this amount that one has to pay per month.
386	Teacher	Mhm
387	Svenja	Uh. Plus this starting amount that you have to pay generally when you buy this box.
388	Teacher	Mhm
389	Svenja	That's the same as here, um, when one subscribes, one has to pay always these five Euros [*points to the DreamStream offer*] and . . . five months—we take, um, this 20 multiplied by 5.
390	Teacher	Mhm
391	Svenja	Then . . . this price per month multiplied by five plus this one-time five Euros. That is exactly the same as here [*points to the Streamox3 offer*], so to speak.

In Lines 385-391, Svenja connected the two ||involved constants|| and the ||independent variable|| by the calculation rule and thereby justified the match of the phrases using the
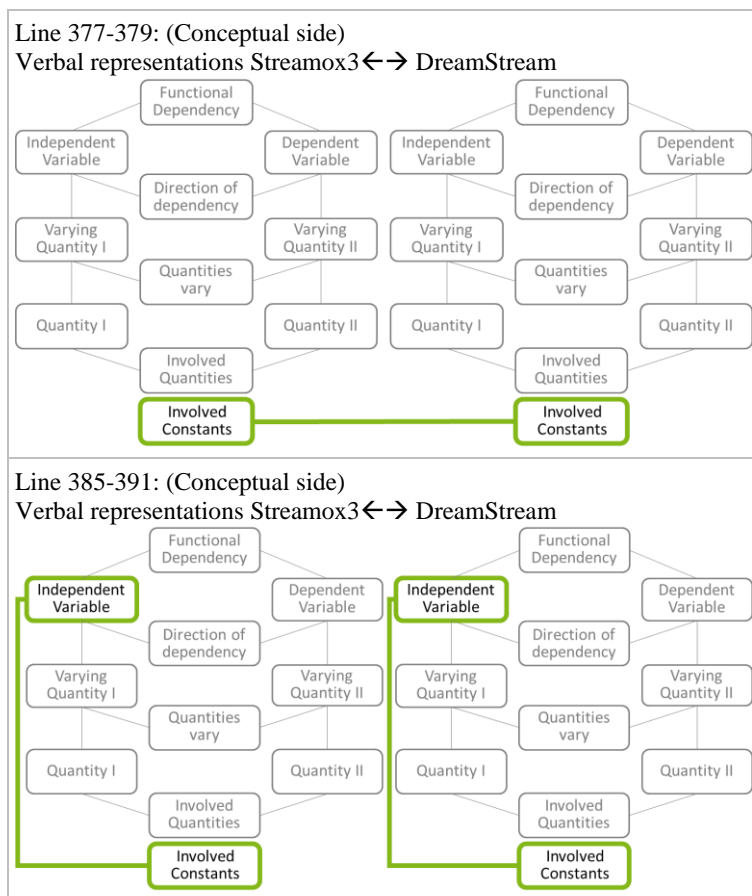

Figure 10. Second part of the reconstruction of
Svenja's learning pathway in the model of conceptual facets

4180

theorem-in-action, <For controlling the match of phrases to two different equations, the meaning of the start value, rate of change, and the independent quantity can be compared>. Nevertheless, so far, she has not related the ||independent variable|| to the more compacted facet ||functional dependency||, as she has not referred to the ||dependent variable||. Moreover, interestingly, the phrases she used all referred to calculation rules, not directly to the conceptual facets and their meaning in the context. The teacher asked her to reconsider the tables.

392	Teacher	Uh, so, what does that description have to do with the tables? You have always looked at the equations . . . How can I find something in the tables that matches this here well [*hints to Description A*] . . .
393	Svenja	[*4 sec break*] Well, here we have the months [po*ints to the left column of the DreamStream table*] and here we have the total price, [*points to the right column of the DreamStream table*] we have to pay every month. . . . again an example of five months.
394	Teacher	Mhm
395	Svenja	Having multiplied this . . . by 20, we have these 500 Euros, which we have to pay for five months [*points to the DreamStream table*]. This means we have already calculated the price here. And here it is also the same [*points to the Streamox3 table*]. One has here the total price when one would pay for five months.
394	Teacher	Yes
397	Svenja	That is why that matches somehow, because it is in dependency of the months. When you subscribe for two or five months, it is thus always the total price [*points to the right column of the Streamox3 table*].

The table headers seemed to steer Svenja's attention to the ||involved quantities||, even if only implicitly addressed by the deictic "here" in Line 393 (analyzed in Figure 11). Including the ||involved constants|| in her calculation led her to think of the total prices as ||dependent variable|| in Line 395. Svenja activated the phrase "in dependency of" in order to justify the match between the two phrases using the identical ||functional dependency||. For doing so, she also addressed the independent variable|| and the ||dependent variable||.

This analysis of Svenja's pathway shows how the successive activities with varying phrases can support Svenja in addressing several conceptual facets and relating them to each other. Without going to the highest level, she succeeded in unfolding the compacted concept. The tables played a key role not only in Svenja's but also in other students' learning pathways as they scaffolded the comparison of texts with respect to the involved quantities.

For Svenja as well as for the 36 students in the other design experiments, the empirical analysis of students' learning pathways has proven the analytic power of the facet model as an analytical tool for extrapolating students' conceptual pathways in dealing with functional relationships and the connected language demands.
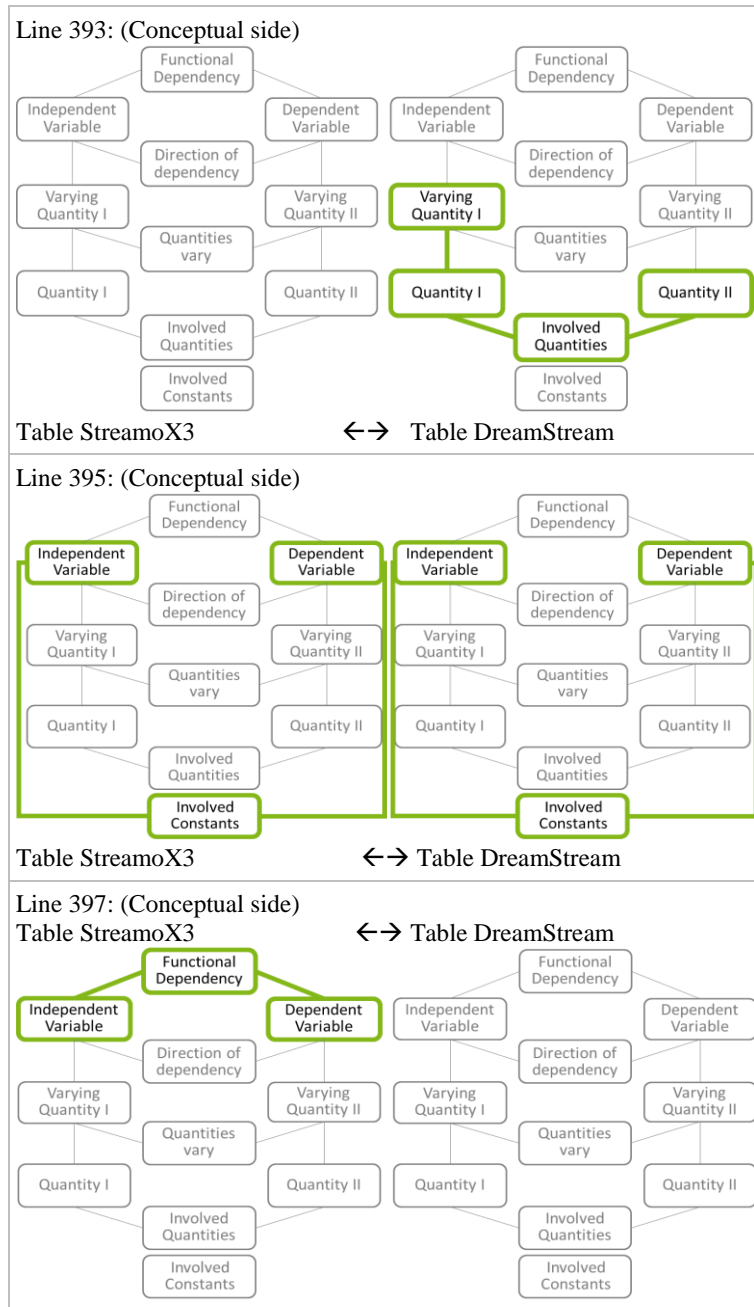
Figure 11. Third part of the reconstruction of
Svenja's learning pathway in the model of conceptual facets

Furthermore, the analysis has provided empirical insights into the functioning of two highly important design principles: connecting representations and systematic variation of phrases, which both have the potential to initiate students' discursive activities and deepen their conceptual understanding (Prediger et al., 2016).

DISCUSSION

Statistical results showing that social achievement gaps can be traced back to language gaps have shifted the attention from specific challenges of multilingual learners to the wider demands of the school academic language register (including the technical register) for multilingual as well as for monolingual students (Hirsch, 2003; Prediger et al., 2015a). Taking into account the epistemic role of language, three functions of the academic language register must be taken into consideration: (1) as underestimated learning medium, (2) as unequally distributed learning prerequisite, and (3) as learning goal, which thus has to be specified more concretely (e.g., Lampert & Cobb, 2003; Thürmann et al., 2010).

In order to enhance language learners' pathways towards language learning goals, the relevant academic language demands have to be specified in more detail and for different mathematical topics (Bailey, 2007). It was the aim of this design research project to contribute to this topic-specific specification of academic language demands in both, the lexical, and the syntactical and discursive dimension (Moschkovich, 2002). The empirical analysis of students' reading processes (in Cycle 1) and then students' learning processes (in Cycle 3) provided insights into the complexities of academic language demands in their lexical, syntactical, and discursive dimensions.

For the analysis of students' reading processes, the often described activity of connecting representations (Duval, 2006; Swan, 1985) was differentiated in detail in order to locate the obstacles on the micro level. The resulting model of conceptual facets (Figure 2) follows Aebli's (1981) idea of concepts being flexibly unfolded or compacted (see Figure 7). Like every specification of demands, the model can be used analytically to describe typical processes, learning pathways, and obstacles (Sections 4.1, 4.2, and 5). Beyond this, it serves as a prescriptive orientation for designing the learning arrangement; in our case, the activities were focused on coordinating specific conceptual facets with each other (see Section 4.3). The conceptual facet model also allows for the analysis of language demands, as they were revealed to be strongly connected to the facet model: In order to be able to address conceptual facets from the model, language means that describe these facets on each level are necessary; in this way, the facet model allows differentiating the language means:

Language Demands Arising When Dealing with Functional Relationships (RQ1)

Four discursive demands were specified theoretically when dealing with functional relationships (always marked in capital letters): READING COMPLEX TEXTS (in this study, word problems of functions), IDENTIFYING the relevant but highly condensed phrases in which the information about the functional relationship is coded, INTERPRETING TEXTS OR SYMBOLS, and, for developing the necessary conceptual understanding, EXPLAINING THE MEANING of functional relationships. The empirical analysis shows that these demands occur simultaneously, together with the simultaneous relevance of the communicative and the epistemic role of language. Syntactical demands are mainly receptive ones, appearing with highly condensed phrases in given texts. The productive language demands appear in the processes of making sense of the texts as well as in situations of conceptual development.

Most important with respect to the epistemic role of language are the findings of systematic parallelism of conceptual learning processes of both unfolding and compacting with the language-related processes of decomposing and condensing on the micro level of students' processes. This observation has immediate practical consequences as it leads to specifying possible scaffolds in the lexical and syntactical dimension in the processes for different facets. Beyond this, it might be theoretically relevant as it coordinates density (as a typical characteristic of academic language) with a typical characteristic of mathematics: Mathematical concepts are highly compacted constructs of complete networks of facets. However, once compacted they receive a new ontological status as reified objects. In this way, we found an empirical foundation of Sfard's (2008) theoretical assumption of inseparability of concept reification and language condensation. As argued in Section 4.2 and empirically illustrated by the case studies of Luisa (Section 4.1) and Svenja (Section 5), these processes of unfolding and compacting on the conceptual side as well as decomposing and condensing on the language-related side are intertwined. This gives another empirically grounded explanation as to why and which kind of academic language is required for learning mathematics.

Along the investigated learning pathways (e.g., in the case of Svenja), students meet various language demands in the lexical, syntactical, and discursive dimensions when trying to verbalize their emerging insights, and the parallel processes of compacting/unfolding on the conceptual side and condensing/decomposing on the language-related side are striking. Additionally, the increasing precision and explicitness of students' language could be found as relevant for explicitly addressing each facet.

In these case studies, language appears in all three functions: as learning prerequisite and learning goal. At the same time, it is a learning medium for students' reflection on topic-specific phrases, which sensitizes students for language demands and for mastering concept demands. The analysis of the case of Svenja illustrates how she successfully mastered the decoding of the complex texts and how she successfully elaborated her language production for this purpose. In particular, the reflection of systematically varied phrases leads her to address and combine all relevant conceptual facets before compacting them again to one concept.

Effects of the Design Principle "Relating Registers by Systematic Variation of Phrases" to Support Students in Mastering the Interrelated Concept and Language Demands (RQ2)

In a similar way, the design principle of systematic variation of phrases also proved its situative potential for deepening the conceptual understanding in other cases (not presented here). From these findings, we draw the first evidence for the hypothesis that reflecting topic-specific complex phrases and their relation to each other can be an appropriate support for deepening students' conceptual understanding. The classroom design experiments of Cycle 4 (not reported here) suggest that this also seems to apply for more language-proficient students.

Limitations and Further Research Needs

Necessarily, the case studies presented here have limitations, especially with respect to the small sample of students (n = 4). Even when taking into account all 97 students in the sample of the overarching design research project, the results are limited by the specific topic-specific activities in view. Further research will be required to expand the scope of the results to more students, to other activities with functions (especially in the covariation perspective), and to other mathematical topics beyond functions.

However, the study substantially contributes to the research discourse on content- and language-integrated learning of functions by offering practical solutions for classrooms and enhancing the theoretical discourse on the role of academic language for mathematics learning. As emphasized by Barwell (2012), Moschkovich (2015), and others, language demands cannot be reduced to the lexical or syntactical dimension, as its discursive dimension always shows the most relevant complexities in the learning process. Research designs that allow for the investigation of learning processes can provide insights into the complex intertwinement of the communicative and epistemic role of the school academic language, being both learning medium and learning goal at the same time. Further research will be required to unpack these complexities for further mathematical topics.

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