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# Secondary school learners' reasoning about quantum randomness in the context of single-photon interferometer and double-slit experiments

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**Abstract**

Despite formal instruction, secondary school students often struggle to move beyond classical interpretations when reasoning about quantum phenomena, especially concerning the statistical nature of quantum physics. This study explores how students conceptualize quantum randomness following instruction centered on two key experiments: the Michelson interferometer (MI) with single photons and the double-slit experiment. Qualitative content analysis of interviews with 25 students revealed three distinct reasoning types: (1) statistical ensemble interpretation, (2) ensemble reasoning involving hidden variables or spatial limitations, and (3) a focus on localization and spatial uncertainty. Notably, students referencing the MI context in their reasonings more often demonstrated advanced ensemble-based reasoning, distinguishing quantum probabilities from classical ones. In contrast, students who relied primarily on the DSE context were more capable of framing their reasoning in terms of spatial uncertainty. These findings highlight the pivotal role of experimental context in fostering students' quantum thinking and point to promising directions for future research.

**Keywords:** quantum physics, secondary school, photon, double-slit experiment, interferometer, qualitative analysis, thinking types

## INTRODUCTION

Quantum physics teaching at schools generally mirrors historical developments in terms of content. Typical topics include Bohr's atomic model, the Franck-Hertz experiment and the photoelectric effect. In contrast, many physics education researchers promote educational pathways that aim to promote a quantum physical way of thinking among learners through focusing on quantum effects that cannot be described semi-classically. These often focus on experiments, also referred to as *key experiments*, involving a variety of aspects of quantum physics (Küblbeck & Müller, 2002; Müller & Wiesner, 2002; Scholz et al., 2020; Weber, 2020). The key experiments are characterized by the fact that they

(1) can be easily carried out and interpreted in a classical framework but can also be easily reinterpreted at the quantum level,

(2) when performed at a quantum level, they demonstrate many aspects of quantum physics in a short period of time, without having to use semi-classical models, and

(3) at the quantum level, there are experimental results that cannot be interpreted classically, creating a cognitive conflict for learners (Müller & Mishina, 2021; Waitzmann et al., 2020), which can therefore initiate a conceptual shift from mechanistic to quantum physics-based ideas (Kalkanis et al. 2003).

So, using key experiments, students are guided through the exploration of quantum by only investigating a few experimental situations. Examples of such key experiments include:

(1) double-slit experiment (DSE) (Feynman et al., 1965; French & Taylor, 1978; Leisen et al., 2000),


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**Contribution to the literature**

- We have explored and categorized students' reasoning about the statistical behavior of photons in the experimental context of the MI and the DSE.
- We found the following types: (1) ensemble interpretation (which corresponds to the minimalistic interpretation of quantum mechanics), (2) ensemble with limitations (when students hold hidden variable ideas), and (3) focus on localization or spatial aspects.
- We have demonstrated that learners can benefit from learning about quantum interferometers, since this context helps them to develop the ensemble interpretation thereby enhancing their quantum physical way of thinking.

(2) interferometers, for example Michelson interferometer (MI) (Bitzenbauer, 2020; Bitzenbauer & Meyn, 2020, 2021; Hennig et al., 2024a; Maries et al., 2020) or Mach-Zehnder interferometer (Maries et al., 2020; Marshman & Singh, 2016), and

(3) Polarization experiments (Michelini et al., 2000, 2004; Tóth et al., 2024; Zuccarini & Michelini, 2023),

and this list is far from complete (Faletič, 2020; McIntyre, 2002; Sakurai & Napolitano, 2011).

Despite the existence of several teaching-learning paths based on key experiments, empirical studies that investigate the learning processes of students in experiment-based learning environments for quantum physics in detail are still scarce. In this paper, we contribute to filling this gap by investigating secondary school students' reasoning about one of the characteristics of quantum physics, *the statistical behavior of quantum objects* following instruction centered on two key experiments, namely the *single-photon MI and the single-photon DSE*. For this purpose, we addressed two research questions within a teaching-learning sequence (Erlangen concept, see Bitzenbauer, 2020; Bitzenbauer & Meyn, 2020) that includes both MI and DSE.

**RQ1.** What types of conceptions regarding the statistical interpretation of quantum physics do secondary school learners hold?

**RQ2.** Which experimental contexts do secondary school learners refer to when describing the statistical behavior of quantum objects?

The paper is based on data from one of the authors (P. B.), collected during his earlier work (see Bitzenbauer, 2020, 2022), which has been reanalyzed, extended, and discussed, providing a new focus on students' reasoning in comparison to the current literature.

# RESEARCH BACKGROUND

## Key Experiments in Quantum Physics

By utilizing key experiments, fundamental laws of quantum physics can be presented at the secondary-school level without relying on semi-classical models, delving into mathematical formalism or using overly complex technical terminology (Bitzenbauer, 2020; Michelini et al., 2002; Müller & Mishina, 2021; Tóth & Tél, 2023; Tóth et al., 2024). These basic features of quantum physics are outlined and emphasized in several articles and can be summarized using four reasoning tools (Müller & Mishina, 2021).

Bitzenbauer (2021) found that in a key experiment-based instructional approach, such as the so-called Erlangen concept, students outperformed those taught using a traditional approach in terms of improving their conceptual understanding of quantum physics at the secondary level. In addition, Bitzenbauer et al. (2024) demonstrated that, in the context of quantum measurement, three distinct two-state approaches focusing on key experiments were more effective than the traditional method in secondary school settings. The research findings indicate that educational approaches centered on key experiments merit further investigation.

## Secondary School Students' Conceptions of Quantum Concepts

### *Students' conceptions in quantum physics*

Students often describe quantum phenomena in terms of classical physics concepts and frameworks (Bitzenbauer et al., 2024; Bouchée et al., 2022; Fischler & Lichtfeldt, 1992; Greca & Freire, 2003; Johnston et al., 1998; Kalkanis et al., 2003), which is called as the *classical way of thinking* by Michelini and Stefanel (2021). Students instinctively strive to visualize quantum effects (Bitzenbauer & Ubben, 2025; Ubben & Bitzenbauer, 2022; Ubben & Heusler, 2021; Tóth & Tel, 2023), but a single quantum object cannot be observed until the moment of measurement, and there is no way to describe or represent its spatial and temporal continuous motion. This is a major challenge in secondary school, and it is no wonder that students often develop misconceptions (Krijtenburg-Lewerissa et al., 2017). For example, photons are imagined as having well-defined properties (all physical quantities have sharp values), as hard balls (Mashhadi & Woolnough, 1999) moving along a predictable trajectory (Marshman & Singh, 2017; Sayer et al., 2017; Thacker, 2003; Tóth et al., 2024; Ubben & Bitzenbauer, 2022, 2023; Ubben & Heusler, 2021); the statistical description is due to a lack of knowledge and inaccuracy of the measuring instruments. According to the theory of conceptual change (Posner et al., 1982), students often find it difficult to abandon their classical physics worldview and to adopt, at least partially, a quantum perspective (Ireson, 1999; Stefani & Tsaparlis, 2009; Zuccarini & Michelini, 2023).

Researchers have explored, in addition to classical and quantum thinking, an intermediate or classical-quantum *mixed thinking* scheme is in presence (Bitzenbauer et al., 2024; Michelini & Stefanel, 2008, 2021; Michelini et al., 2022; Ubben & Bitzenbauer, 2023), which is called the hidden variable way of thinking by Michelini and Stefanel (2021). Students who think in this way continue to attribute classical properties to quantum objects, but they emphasize that these properties cannot be experimentally accessed. They draw a distinction between the empirically testable domain and a theoretical realm in which quantum objects possess hidden, well-defined properties. According to their reasoning, photons move like classical mass points; however, their trajectories cannot be measured because any attempt at measurement would disturb the system. As a result, they argue, we are compelled to rely on probabilistic descriptions. Bitzenbauer et al. (2024) found that these three thinking types (classical, mixed and quantum mechanical way of thinking) are appropriate to describe and evaluate students' conceptual understanding of quantum measurement in different teaching approaches.

### *Students thinking about quantum randomness*

Prior research has shown that students often struggle to interpret quantum randomness. Singh and Marshman (2015) reviewed students' difficulties in quantum physics, highlighting a widespread confusion between the probability of measuring a particle´s position and the expectation value of its position. In a task, students were asked to give the probability of finding an electron in a given position in a potential well, but instead, students tried to find the expectation value of the position. Passante et al. (2015) reported that learners were confused between the concepts of a mixed state and a superposition state, showing that classical and quantum randomness are not clearly separated in students' minds. Michelini et al. (2022) found that students are often unable to understand quantum concepts because they are not fully familiar with the corresponding classical concepts; and Tóth et al. (2024) found that even if students preserve the classical motion, they are able to accept and understand the probabilistic nature of quantum physics. In a study by Mannila et al. (2002), the authors report that fewer than 30% of learners appear to have any awareness of the statistical interpretation of the DSE. The diverse research findings indicate that students' understanding of quantum randomness is highly context-dependent and highlight the need for further research to characterize students' reasoning patterns better.

Based on the literature review regarding students' general way of thinking in quantum physics, three types of thinking about quantum randomness can be distinguished by deductively:

1. **Classical type:** When students believe that the statistical description is due to a lack of knowledge and inaccuracy of the measuring instruments.
2. **Mixed type:** When students think that certain (hidden) information about the system is very hardly or even not available, and that forces us to use statistical description in practice.
3. **Quantum type:** It is usually impossible to predict a single measurement outcome on a single quantum object, but the collective behavior of quantum objects can be predicted statistically.

However, due to the different grain sizes, these thought patterns are not necessarily sufficiently precise in terms of the students' way of thinking that are formed of a specific concept in a specific educational approach.

Against the background of these documented student conceptions, the question arises as to what differences can be observed in a successful teaching method of quantum physics, called Erlangen concept of quantum optics (Bitzenbauer, 2020; Bitzenbauer & Meyn, 2020), which covers two key experiments covered in the

(1) MI and then

(2) DSE.

We have chosen this teaching–learning pathway because Tóth et al. (2024) found that the majority of students were able to comprehend the probabilistic behavior of photons at a beam splitter utilizing an MI-based approach, which closely aligns with the Erlangen concept. Empirical findings regarding this question could contribute

(1) to the identification of the central key experiments for quantum physics teaching or

(2) provide information on how both experiments should be optimally combined in the classroom in order to promote a conceptual shift towards a quantum thinking among students.

In addition, our research helps us to unpack broad categories of students' thinking along a specific educational pathway. To clarify such questions, however, different studies with different study designs and data collection instruments seem necessary. In this paper, we report the results of an exploratory interview study that we use to specify the question raised above.

# MATERIALS & METHODS

## The Teaching-Learning Sequence

All participants learned about quantum physics utilizing the Erlangen concept, which is described in more detail in Bitzenbauer and Meyn (2020), and Bitzenbauer (2020). The teaching-learning sequence is structured into five main parts:

(1) detecting non-classical light,

(2) preparation of single-photon states,

(3) anti-correlation of photons at a beam splitter,

(4) investigating single-photon interference using an MI (the experiment by Grangier et al., 1986), and

(5) following the Erlangen concept, all students examined a single-photon DSE.

## Study Design and Sample

In this paper we report the results of an exploratory interview study on secondary school students' ways of thinking about the statistical behavior of quanta. Twenty-five upper-level German secondary school students volunteered to participate in the study. The study has been conducted as part of the summative evaluation of a newly developed teaching-learning sequence on quantum physics at the secondary school level, which introduced learners to quantum concepts through quantum optics experiments with single photons (Bitzenbauer, 2020, 2021; Bitzenbauer & Meyn, 2020). In this paper, we re-analyze the data from this interview study to shed new light on how students use different experimental contexts (in particular, MI and DSE) to reason about the statistical behavior of single photons (Bitzenbauer, 2020, 2021).

## Interview Guideline

During the interviews, three questions focusing on the statistical behavior of quantum objects were posed to students:

(1) What do you understand by quantum random?

(2) Describe an experiment in which the probabilistic nature of quantum physics appears!

(3) Describe and explain the following statement: "In quantum mechanics, usually only statistical predictions are possible!"

### *Data analysis carried out to answer RQ1*

The students' answers were evaluated on the basis of deductively and inductively formed categories utilizing qualitative content analysis (Mayring, 2000). In this study, students are considered to hold a particular conception if they make at least one statement during the interview that can be classified under the corresponding category. During the coding process, all categories are treated equally, and one response can be assigned to different categories simultaneously. Repeated instances of the same category within a single participant's transcript are not coded again, as recurring expressions or similar explanations do not yield additional insights into the student's conceptions. Students whose responses reflected similar patterns, based on identical category assignments, were grouped together. To ensure interrater reliability of the coding, all coding were re-encoded by an independent person and with Cohens kappa of $\kappa = 0.89$, a high interrater reliability was shown. A frequency analysis was subsequently conducted to determine the occurrence of each category, supporting the investigation of **RQ1**. Based on these findings, we derived three independent thinking types that characterize students' conceptions of the statistical behavior of photons.

### *Data analysis carried out to answer RQ2*

To address **RQ2**, we analyzed which of the two experimental contexts (DSE or MI) students used when explaining the probabilistic behavior of quantum objects. Since both experiments were covered during the instructional sequence, our analysis focused specifically on these two cases. If a student used both experiments, they were included in both categories.

# RESULTS

## Results Regarding Research Question RQ1: Categorization of Students' Answers About the Statistical Behavior of Quantum Objects

The students' conceptions of the statistical behavior of quantum objects, identified from the interview protocols through qualitative content analysis are summarized in **Table 1**.

**Table 1.** Summary of students' responses regarding quantum randomness, categorized according to their reasoning patterns

| Category | Description | Anchor example (translated from German) | AF (%) |
|---|---|---|---|
| C1. Statistical predictions for an ensemble | This category is assigned if students state that in order to be able to make statistical statements in quantum physics, it is always necessary to consider an ensemble of quantum objects. | *"Well, In my opinion, this* [the statistical statements] *refers to the fact that there are real coincidences in quantum mechanics and you can just say that if you shoot a large number of single photons one after the other at the beam splitter, for example, they will be transmitted and reflected in the end according to the law of large numbers, but you can't determine for a single photon whether it will be transmitted or reflected."* | 21 (84%) |

**Table 1 (Continued).** Summary of students' responses regarding quantum randomness, categorized according to their reasoning patterns

| Category | Description | Anchor example (translated from German) | AF (%) |
|---|---|---|---|
| C1. Statistical predictions for an ensemble | This category is assigned if students state that in order to be able to make statistical statements in quantum physics, it is always necessary to consider an ensemble of quantum objects. | *"Well, In my opinion, this* [the statistical statements] *refers to the fact that there are real coincidences in quantum mechanics and you can just say that if you shoot a large number of single photons one after the other at the beam splitter, for example, they will be transmitted and reflected in the end according to the law of large numbers, but you can't determine for a single photon whether it will be transmitted or reflected."* | 21 (84%) |
| C2. Probabilities cannot be avoided by principle | This category is assigned if learners explicitly acknowledge that quantum probabilities are fundamentally unavoidable. | *"If I now have a dice […], I can eventually say, after a long simulation, what the outcome should be, but since I don't know what a quantum particle does, I can't calculate anything."* | 21 (84%) |
| C3. Uncertainty of the spatial location | This category is assigned if interviewees talked about probabilities in the context of spatial information. | Interviewers: "[Describe] *simply an experiment in which quantum probabilities are used."*<br>*"Yes, I would perhaps just start from the double-slit experiment, because you can see an interference pattern there, where you can't determine exactly where the quanta are afterwards or through which slit they really fly […]."* | 7 (28%) |
| C4. Relative frequencies | This category is assigned if respondents identify probabilities with relative frequencies, i.e., for a given number of photons emitted, the relative frequency of each measurement outcome can be predicted. | *"[…] so there's always a probability, and from these probabilities you get an average value, which roughly tells you how likely or how often an event occurs – accordingly, one can say which event is now most probable in a quantum experiment."* | 6 (24%) |
| C5. Hidden variable | This category is assigned if students express doubts as to whether there might be information that could be used to resolve the need for a statistical description of quantum phenomena. Such statements were assigned to a category called *hidden variables*. | *"I'm more of a determinist and I would say that we don't have enough knowledge, so I would say that it's a thing and you could theoretically predict this thing if you had all the information. It's just that we can't read this information […]."* | 3 (12%) |
| C6. No idea | This category is assigned if students are not able to give any interpretation about the statistical behavior of quantum objects. | | 1 (4%) |

Note. AF (%): Absolute frequencies (percentages)

There was only one student who was in category C1 but not in category C2, all the others achieved both categories together. While this remains to be confirmed with a larger sample, it can be assumed that categories C1 and C2 do not tend to be anchored independently of each other in the learners' perceptions. Learners who are aware that statistical statements do not relate to a single measurement outcome also seem to think of the unavoidability of quantum probabilities.

## Results Regarding Research Question 1

Our analysis revealed three categories of students' thinking about the statistical behavior of quantum objects in MI a DSE (see **Table 2**). We have removed students #14 (see **Table 2**) from thinking types since this student is associated with category C6 (no idea). All of the results (categories and thinking types) are represented in **Figure 1**.

### *Type A. Statistical ensemble*

**Table 2** and **Table 3** clearly indicate that a group of students is simultaneously associated with categories C1 (statistical predictions for an ensemble), C2 (probabilities cannot be avoided), and C4 (relative frequencies), motivating the creation of a distinct student thinking type. They hold a minimalistic interpretation of quantum probabilities and express in 100% of the cases the necessity of considering ensembles of quantum objects in order to make statistical statements. All distinguish quantum probabilities from the classical probabilities, such as dice rolling. None of the learners in this type reflect on potentially hidden variables. Instead, they all focus on the analysis of relative frequencies that

**Table 2.** The categorization of students' into types

| Student | C1 | C2 | C3 | C4 | C5 | C6 | Type |
|---|---|---|---|---|---|---|---|
| ID-1 | x | x | | x | | | A |
| ID-2 | x | x | x | | | | C |
| ID-3 | x | x | x | | | | C |
| ID-4 | x | x | | | | | B |
| ID-5 | x | x | | | x | | B |
| ID-6 | x | x | | | | | B |
| ID-7 | x | x | | x | | | A |
| ID-8 | x | x | | x | | | A |
| ID-9 | x | x | x | | | | C |
| ID-10 | x | x | | | | | B |
| ID-11 | | | | | x | | B |
| ID-12 | x | | | | | | B |
| ID-13 | x | x | | | | | B |
| ID-14 | | | | | | x | - |
| ID-15 | x | x | | x | | | A |
| ID-16 | x | x | x | | x | | C |
| ID-17 | | x | | | | | B |
| ID-18 | x | x | | | | | B |
| ID-19 | x | x | | x | | | A |
| ID-20 | x | x | x | | | | C |
| ID-21 | x | x | | | | | B |
| ID-22 | x | x | x | | | | C |
| ID-23 | x | x | | x | | | A |
| ID-24 | x | x | | | | | B |
| ID-25 | | | x | | | | B |
| Absolute frequency | 21 | 21 | 7 | 6 | 3 | 1 | |
| Percentage | 84% | 84% | 28% | 24% | 12% | 4% | |

Note. If a student falls into a category, then a symbol "x" appeared in the corresponding cell & the last column presents which type is assigned to the students which also represented by colors: green, yellow, and blue correspond to types A, B and C, respectively

**Table 3.** The absolute frequency of students per type and the amount of categories appeared in each type regarding the statistical behavior of photons

| Type | Amount | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|---|
| A | 6 | 100% | 100% | 0% | 100% | 0% |
| B | 12 | 83% | 75% | 8% | 0% | 17% |
| C | 6 | 100% | 100% | 100% | 0% | 17% |

result from different outcomes when repeated measurements are performed, and they relate this to the concept of probability–sometimes even referring to the law of large numbers. Six of the participants are assigned to this type, corresponding to 24% of the sample.

### *Type B. Statistical ensemble with limitations*

We associate this type of reasoning with students who hold limited or incomplete ideas regarding statistical ensembles. As **Table 3** presents, none of the learners in this type refer to relative frequencies (C4). Seven students are associated with C1 and C2 but not C4 categories, therefore, these seven students cannot be put into thinking type A. In addition, two (17%) students (#5 and #11) hold hidden variable (C5) ideas (student #5 is also associated with C1 and C2). Three students were assigned to either C1, C2, or C3. Student #25 who are only associated with C3 (spatial aspects) cannot be put into the next thinking type (type C) too, since it reflects a limited mindset compared to that group. Ten out of 12 students assigned to this thinking type are aware of the necessity of considering ensembles of quantum objects

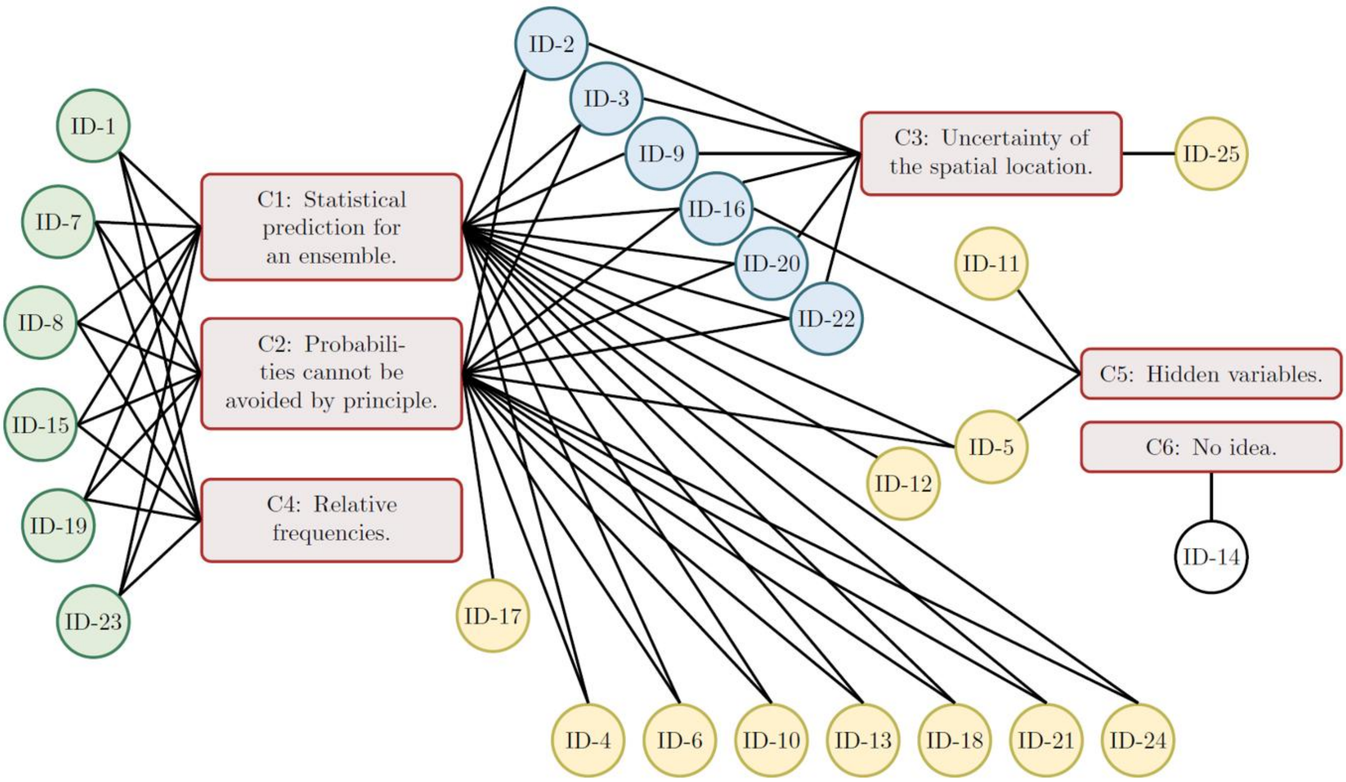


**Figure 1.** Visualization of the results (each student is represented by a small circle labeled with a number, the color of each circle indicates the student's thinking type [A, B, and C are represented by green, yellow, and blue, respectively], and a line connects a student to a category if the student used an argument that fits that category (Source: Authors' own elaboration)

to make statistical statements (83%). The distinction between quantum probabilities and classical probabilities is explained by the vast majority (75%). This type includes 12 of the participants, which represents 48% of the total sample.

### *Type C. The context of localization*

Based on the data (as presented in **Table 2** and **Table 3**), it is reasonable to identify a new thinking type in which C1 (ensemble), C2 (probabilistic nature), and C3 (spatial aspects) appear simultaneously. Only one student (#16) also belongs to category C5 (hidden variables). Due to the distinctive presence of category C3 in this group, we refer to this thinking type as "the context of localization." These students use probabilistic statements for localizing quantum objects. Their focus is on spatial aspects, on the unavoidable quantum probabilities, and on the need to know where quantum objects are, whose location is perceived as uncertain. Six participants are assigned to this type, which corresponds to 24% of the sample.

We note that only three students were associated with category C5 (hidden variables), although prior research, e.g., Michelini and Stefanel (2021), suggests the existence of this category. It is possible that this thinking type (hidden variables) would emerge with a larger sample size. In this study, we did not include this type in our analysis.

## Results Regarding Research Question RQ2

A noticeable structure can be seen from students' answers (see **Table 4**): All learners from type C (when students use probabilistic statements for localizing quantum objects and they focus is on spatial aspects, on the unavoidable quantum probabilities, and on the need to know where quantum objects are, whose location is perceived as uncertain) used the DSE in their explanations of *statistical behavior*, e.g., *"It is random at which point a single electron appears on the screen."*

Conversely, all respondents from type A (when students use the minimalistic interpretation of quantum physics via ensembles of photons) argued using the behavior of a single photon at a beam splitter cube, as it appears in the interferometer, and additionally partly used the DSE, e.g., *"If you now send a large number of single photons one after another into the beam splitter, about half will be transmitted and half reflected, but you simply cannot determine for a single photon whether it will be transmitted or reflected."* The data certainly implies that way of thinking about the statistical behavior of quantum objects.

# DISCUSSION

The literature uses a threefold division of students' thinking patterns: classical, hidden variable or mixed, and quantum mechanical way of thinking (Bitzenbauer et al., 2024; Michelini & Stefanel, 2021; Tóth et al., 2024).

**Table 4.** Distribution of students' responses (N = 25) in different types according to the experimental context mentioned in their explanations during the interviews (we have marked two types [A and B] in **bold** because in those types, each of the students listed a specific experimental context)

| | | Type A | Type B | Type C |
|---|---|---|---|---|
| Number of answers | | 6/25 | 12/25 | 6/25 |
| Experimental context | DSE | 2/6 | 9/13 | **6/6** |
| | MI | **6/6** | 5/13 | 0/6 |

Our findings extend these earlier results by providing a more nuanced analysis within the Erlangen teaching approach, specifically regarding the concept of quantum randomness. We found that the students' thinking about quantum randomness is based on the context and the experimental situation what they are thinking. The results indicate that students with more elaborate views (type A) seem to be more likely to use MI or MI and DSE than only DSE. So, they can justify their thoughts with the MI (and DSE) while others stick only to DSE. It seems that the Erlangen concept of quantum optics helps students to understand the statistical behavior of quantum objects. However, we did not compare instructional sequences focusing exclusively on either the DSE or the MI, nor did we control sequence effects (i.e., whether MI or DSE was taught first). Participants were first faced with MI, then DSE.

We think that the DSE imposes a greater conceptual burden on students, because in addition to the statistical behavior of photons, the experiment requires them to understand the absence of classical motion by interpreting quantum interference, while also introducing the effect of measurement and incompatibility. The MI, on the other hand, can be broken down into parts and the different features of quantum physics can be analyzed separately: to discover the statistical behavior of photons, it is enough to first consider only a beam splitter experiment, which is remarkably simple. In this way, students do not have to acquire an abstract conceptual understanding of the lack of classical motion at the beginning of their learning. In line with the literature (see e.g., Hennig et al., 2024b; Marshman & Singh, 2016; Sayer et al., 2017; Thacker, 2003; Ubben & Bitzenbauer, 2022, 2023; Ubben & Heusler, 2021) overcoming classical intuitions about motion, i.e., interpreting quantum interference, is one of the biggest conceptual challenges for students and requires mostly a conceptual change. A promising approach to foster a quantum thinking in learners (Michelini & Stefanel, 2021), may lie in the mathematical description of quanta. In this context, the challenge of understanding statistical results through a continuous distribution, as required in the DSE, is greater than that of analyzing two-state systems with only two possible measurement outcomes, as exemplified by the MI.

## LIMITATIONS

It is important to highlight that despite alignment with existing quantum physics education literature, the relatively small sample size (N = 25) limits the generalizability of these conclusions. New thinking types regarding quantum randomness (e.g., a hidden variable perspective) might emerge with a larger sample. We only asked students to describe quantum randomness and observed that–despite equally long treatments in class–students with more elaborate views tended to refer to the MI, whereas students with more classical views more frequently referred to the DSE. However, we did not compare instructional sequences focusing exclusively on either the DSE or the MI, nor did we control sequence effects (i.e., whether MI or DSE was taught first). Furthermore, we restricted the scope of the intervention to these key experiments while further ones might exist. Moreover, it is not always clear whether a given answer is based on the student's own reasoning or merely reproduces memorized content. For instance, follow-up interviews conducted after a certain time interval could help to assess the durability and depth of students' conceptual understanding. Another promising approach would be the use of a carefully constructed and well validated concept test specifically targeting key ideas in quantum optics.

## CONCLUSION

We analyzed secondary school students' reasoning about quantum randomness through a novel lens by employing the Erlangen teaching-learning approach to quantum optics. By utilizing both inductive and deductive qualitative content analysis, twenty-five students' responses were categorized into 5 categories and identified three types along these categories, which were archetypes describing students' beliefs about the statistical behavior of quantum objects. The three types are

(1) Type A. The statistical ensemble,

(2) Type B. Statistical ensemble with limitations, and

(3) Type C. The context of localization.

We investigated students who started their quantum physics studies with

(1) the single-photon MI and then

(2) the single-photon DSE.

We found that those who thought about the experimental context of MI are more inclined to interpret quantum physics randomness more appropriately (type A), presumably due to the DSE's overemphasis on the absence of classical motion and the difficulty of interpreting a continuous probability distribution instead of two random variables.

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**AI statement:** The authors stated that Generative AI tools (e.g., ChatGPT by OpenAI) were used to check the English language clarity of the manuscript only. No content generation was performed by AI.

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