



# Sociomathematical norms and the teacher's mathematical belief: A case study from a Korean in-service elementary teacher

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The beliefs of teacher regarding their students' learning in mathematics are an essential factor due to their importance in developing a desirable mathematics classroom culture. The purpose of this study is to investigate the mutual relationship between the construction of sociomathematical norms in classroom mathematical culture and teachers' beliefs. We observed 13 sessions of a fourth grade elementary mathematics class for two periods totaling one year, and conducted interviews with the elementary teacher and students who participated in this study. The study results revealed that the mathematical belief of the elementary school teacher was reflected in the decision making for mathematics instruction, and greatly influenced the purpose, contents and methods of the mathematics class, which contributed to creating sociomathematical norms, or repeated patterns appearing between the teacher and students.

*Keywords:* elementary mathematics classroom, mathematical belief, mathematics classroom culture, sociomathematical norms

## INTRODUCTION

The Korea 2009 revised mathematical curricula suggested that, when students work together to solve a mathematical problem, including explaining their own thinking in a convincing way, paying attention to and making an effort to understand others' opinions, and actively engaging in discussion, they can attain a better understanding of the subject and the ability to express their opinions with greater clarity and sophistication. The researchers in charge of analyzing the compliance with the Korea 2007 revised mathematical curricula in mathematics classes found that most classes were still textbook-and teacher-oriented (Hong et. al, 2009; Kim, 2012). As an alternative, we can consider creating a mathematics classroom culture that promotes active participation and interaction from the students. In other words, we

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should create a classroom culture in which various ideas can be exchanged and shared, through discussion, to solve questions (Park, 2004). Cobb & Yackel (1996) introduced the term “sociomathematical norm” in their study in which they observed a mathematics classroom and analyzed the classroom culture. The establishment of sociomathematical norms at the classroom level can contribute to enhancing the quality of mathematics education (Song & Yim, 2007).

Many researchers argued that since elementary school mathematics classes rely heavily on the role of the teacher, teachers’ beliefs and behaviors heavily impact the decision-making capabilities of their students. For instance, Thompson (1984), who studied teachers who held specific beliefs and views about mathematics and teaching, also claimed that teachers’ mathematical beliefs are a deciding factor in the process of the construction of the sociomathematical norms between teachers and their students. From a cultural and anthropological perspective (Cho, 2001), the sociomathematical norms observed in mathematics classrooms were said to be closely related with the beliefs and values of the surrounding society.

In a mathematics classroom community, it is the teacher’s belief that serves as an important medium in negotiating social interaction and mathematical meaning. Therefore, it has been believed that the norms that could promote social interaction in mathematics classrooms both implicitly and explicitly were closely connected with a teacher’s belief about mathematics and its teaching and learning. Cobb and colleagues (2011) argued that these beliefs could have a psychological relationship with sociomathematical norms; thus, teachers’ individual beliefs could grow alongside the classroom’s sociomathematical norms. Mathematical beliefs are the criteria for teachers to select and set classrooms’ sociomathematical norms. Therefore, the purpose of the present study is to analyze how the sociomathematical norm, or the process of promoting the understanding of mathematical concepts and principles through the interaction between teacher and students, is established in relation to the teacher’s mathematical beliefs in a fourth-grade mathematics class.

## BACKGROUND

### Sociomathematical norm

A norm is a sociological construct that refers to the understandings or interpretations that become normative, or are understood to be shared by a group (Yackel, 2001). Thus, a norm is a collective rather than an individual notion. Cobb et al. (1992) introduced the term “norms” to designate the reciprocal expectations that are established in the classroom, through the interactions between the teacher and the students. In other words, both teachers and students subconsciously knew how to act appropriately in specific situations as they arose, even though neither had a

### State of the literature

- A norm is a sociological construct that refers to the understandings or interpretations that become normative, or are understood to be shared by a group.
- A sociomathematical norm is the consideration of a mathematically acceptable explanation in conjunction with an understanding of what has been mathematically different.
- Teachers’ individual mathematical beliefs and values develop concomitantly with the classroom sociomathematical norms.

### Contribution of this paper to the literature

- This paper reviews the concepts and research trends of norm, sociomathematical norm, and mathematical beliefs.
- This paper analyzed the mutual relationship between the construction of sociomathematical norms in classroom mathematical culture and teachers’ beliefs.
- This paper showed that the mathematical belief of the elementary school teacher was reflected in the decision making for mathematics instruction by creating sociomathematical norms, or repeated patterns appearing between the teacher and students.

blueprint of the interaction patterns (Cobb, Yackel, & Wood, 2011). They mentioned that norms are values that are established through the interaction between the teacher and students, and that they appear and are taken as shared through interaction, depending on the argument and purpose of students and the teacher (Kim, 2001). Yackel & Cobb (1996) also attempted to demonstrate that these norms are not predetermined criteria that are introduced into the classroom from the outside; instead, these normative understandings are continually regenerated and modified by the students and the teacher through their interactions. Essentially, norms are not short-term individual concepts but repetitively assembled collective concepts, and are referred to as expectations which may be established through interaction.

A sociomathematical norm is the consideration of a mathematically acceptable explanation in conjunction with an understanding of what has been mathematically different. Yackel and Cobb (1996) mentioned sociomathematical norms as being normative understandings of what counts as mathematically different, mathematically sophisticated, efficient, and elegant in a classroom. Bowers and colleagues (1999) indicated that examples of sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, an insightful mathematical solution, and an acceptable mathematical explanation. The individual correlates of sociomathematical norms consist of the teachers' and students' specific mathematical beliefs and values that constitute their mathematical dispositions. Therefore, we advance the notion of sociomathematical norms- that is, normative aspects of mathematical discussions that are specific to students' mathematical activity (Lampert, 1990; Voigt, 1996; Yackel & Cobb, 1996), and we conclude that they can be defined as acceptable mathematical explanations, mathematical difference, mathematical effectiveness, or mathematical insights.

### **Teachers' mathematical beliefs**

Beliefs are more cognitive, are felt less intensely, and are more difficult to change than are attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction, and they are not consensual (Philipp, 2007). Many scholars have attempted to research teachers' belief. Researchers who study teachers' knowledge, beliefs, and affect related to mathematics teaching and learning are still attempting to tease out the relationships among these constructs and to determine how teachers' knowledge, beliefs, and affect relate to their instruction (Philipp, 2007).

Additionally, it is apparent that we took mathematical beliefs and values in school to be the psychological correlates of sociomathematical norms (Cobb, Stephan, McClain, & Gravemeiger, 2011, p.5). Thus, teachers' individual mathematical beliefs and values develop concomitantly with the classroom sociomathematical norms (Yackel et al., 2000).

Teachers' behaviors and decision-making processes during instruction rely heavily on their mathematical beliefs. Koehler et al. (1993) studied the influence of teachers' mathematical beliefs on their decision making. We advance the notion of sociomathematical norms, i.e., normative aspects of mathematical discussions that are specific to students' mathematical activities (Cobb et al., 1991; Yackel & Cobb, 1996). We analyzed the collective norms of the entire classroom rather than those of individuals within the class community, and the focused on the relationship between individual and group activities. Analyses of sociomathematical norms regard psychological correlation about norms as an individual factor. We consider the relationship between the sociomathematical norms that are constituted in the

classroom and the students' and teacher's mathematical beliefs and values to be reflexive (Yackel et al., 2000).

As such, the conjectured relation between sociomathematical norms and mathematical beliefs implies that a teacher who initiates and guides the renegotiation of sociomathematical norms is simultaneously supporting individual teacher/student reorganization of the corresponding beliefs (Cobb & Yackel, 1996). The mathematical beliefs of a teacher are considered general beliefs that she or he attained in the process of teaching and learning mathematics. Therefore, the negotiation of sociomathematical norms gives rise to learning opportunities for both teachers and students (Yackel & Cobb, 1996). Sociomathematical norms determine the quality of teaching and learning activities in a mathematics classroom, as well as guiding and promoting the students' mathematical activities.

### Beliefs about the nature of mathematics

The term 'mathematical belief' encompasses beliefs about the nature of mathematics and about teaching and learning (Nam et. al, 2008). Ernest (1989) describes three philosophies of mathematics, instrumentalist, Platonist and problem solving, whereas TEDS-M<sup>1</sup>(2008) called the four components: mathematics as structural (belonging to the instrumentalism and traditionalism perspectives); mathematics as forma structural (similar to the Platonist view); mathematics as procedural (related to the problem-solving perspective); and mathematics as applied (see Table 1). A more diverse approach that involves TEDS-M's (2008) beliefs about the nature of mathematics model has been discussed. Tatto and colleagues (2008) mentioned "belief about the nature of mathematics" in TEDS-M (2008) by citing the study by Grigutsch et al. (1998). Tatto and colleagues (2008) mentioned "belief about the nature of mathematics" in TEDS-M (2008), by citing the study done by Grigutsch et al. (1998). They mentioned the term "belief about the nature of mathematics", from formalism, scheme, process, and application perspectives, which are referred to as 'mathematics as formal', 'mathematics as structural', 'mathematics as procedural', and 'mathematics as applied', respectively, in TEDS-M (2008). In TEDS-M(2008), which was used as the analysis criteria in this study (See Table 1), the term "belief about the nature of mathematics" includes questions about how teachers see mathematics as a school subject.

**Table 1.** Beliefs about the nature of mathematics in TEDS-M(2008)

Types	Concerns Regarding Beliefs
Mathematics as Formal	<ul style="list-style-type: none"> <li>• It is similar to the Platonist view (Ernest, 1989).</li> <li>• Mathematics is perceived as an exact science developed deductively from axiom.</li> <li>• Mathematical thinking is determined by abstraction and logic.</li> </ul>
Mathematics as Structural	<ul style="list-style-type: none"> <li>• It belongs to the instrumentalism and traditionalism perspectives.</li> <li>• Mathematics involves the remembering and application of definitions, formulas, mathematical facts, and procedures.</li> <li>• Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.</li> <li>• Mathematics is a set of procedures and rules intended to discover a correct problem solving method.</li> </ul>
Mathematics as Procedural	<ul style="list-style-type: none"> <li>• It is related to the problem-solving perspective (Ernest, 1989).</li> <li>• Mathematics is understood to be a science which consists of the identification process of structure and patterns, and problem solving processes.</li> <li>• If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts).</li> </ul>
Mathematics as Applied	<ul style="list-style-type: none"> <li>• Mathematics is considered to be a science related to society and life.</li> <li>• Mathematics helps solve everyday problems and tasks</li> </ul>

<sup>1</sup>Teacher Education and Development Study in Mathematics

## Teacher's mathematical belief about teaching and learning

Cobb and Yackel (1996) argued that reflexivity between sociomathematical norms and individual beliefs implies that a teacher who initiates and guides the renegotiation of sociomathematical norms is simultaneously supporting individual students' reorganization of the corresponding beliefs. Teacher is at the center of the interaction with, and the negotiation of, mathematical meaning in a mathematics classroom. Sociomathematical norms, which may promote interaction in a mathematics class, are closely connected with teacher's mathematical beliefs about teaching and learning.

Teachers' beliefs about mathematics teaching are related with their beliefs about the nature of mathematics (Thompson, 1984). The content strands of beliefs about mathematics learning in TEDS-M (2008) include questions about the purpose of mathematics, students' cognitive processes, and the appropriateness of specific instructional activities, which were used as the analysis criteria of this study (See Table 2).

## RESEARCH METHODS

Subjects and instructional themes observed in this study.

During the 1<sup>st</sup> and 2<sup>nd</sup> semester of 2012, we observed 13 sessions of Teacher K's elementary mathematics class for a period of one year. The themes for each lesson unit are summarized in Table 3.

Following Kim (2001)'s recommendation for a study on teachers' beliefs (beyond formal beliefs) through interpersonal conversation, we conducted an interview with Teacher K in order to explore the formation of the classroom society norm and

**Table 2.** Beliefs about mathematics teaching and learning in TEDS-M (2008)

Content Strands	Belief
Purposes of Mathematics as a School Subject	<ul style="list-style-type: none"> <li>• The best way to do well in mathematics is to memorize all the formulas.</li> <li>• It doesn't really matter if you understand a problem, if you can get the right answer.</li> <li>• To be good in mathematics, you must be able to solve problems quickly.</li> <li>• In addition to getting a right answer, it's important to understand why an answer is correct.</li> </ul>
Appropriateness of Particular Instructional Activities	<ul style="list-style-type: none"> <li>• Teachers should allow pupils to figure out their own ways to solve a mathematical problem.</li> <li>• Non-standard procedures should be discouraged because they can interfere with the correct procedure.</li> <li>• It is helpful for pupils to discuss different ways to solve particular problems.</li> <li>• Time used to investigate why a solution to a mathematical problems works is time well spent.</li> </ul>
Student's Cognitive Processes	<ul style="list-style-type: none"> <li>• Pupils need to be taught exact procedures for solving mathematical problems.</li> <li>• Pupils learn mathematics best by attending to teacher's explanations.</li> <li>• When pupils are working on mathematical problems, more emphasis should be placed on getting the correct answer than the process followed.</li> <li>• Pupils can figure out a way to solve a mathematical problem without the teacher's help.</li> <li>• Hands-on mathematical experiences aren't worth the time and expense.</li> </ul>

**Table 3.** The number of each lesson session, and content strands according to the theme of the lesson units

Theme of Lesson Units	# of Lesson Sessions	Content Strands
Multiplication and Division	4	Numbers and Operations
Finding Patterns	2	Patterns and Problems Solving
Addition and Subtraction of Fractions	2	Numbers and Operations
Addition and Subtraction of Decimals	2	
Quadrilateral and Polygons	1	Figures
Perimeter and Area of Plane Figure	1	Measurement
Finding Patterns and Problem Solving	1	Patterns and Problem Solving

sociomathematical norm. K was a woman in her mid-fifties. Prior to her 28-year career as a teacher, she had completed her masters in the graduate school of education. The interview with Teacher K was recorded.

### Data collection

Lodico and colleagues (2010) suggested that for scientific and unbiased research, the researcher must be systematic in the data collection process, and record the data with accuracy. Therefore, in this study the mathematics classroom conducted by K was observed and monitored by recording, and the students' lesson activities sheets were collected after each class. Both the students and K were interviewed.

The interview structure facilitated the overall arrangement of questionnaires that were drawn adaptively within the contents of the category. Depending on the occasion of the interview, the sequence of the questions was varied, and those that were beyond the themes of study were either modified or removed (See Table 4). The interview questions examined broad subjects such as school environment, personal information, earlier teaching experience, interest in mathematics, interest in the subject of mathematics, beliefs about the nature of mathematics, beliefs about mathematics teaching and learning, and mathematics instruction.

### Data analysis

In this study the collected data were categorized and filed, through documentation, for the data analysis. Our researchers repeatedly read through all of the organized data to analyze the data in depth. The meanings of responses and behaviors of the students were analyzed. Additional questions or fuzzy points were recorded by associating them with the researcher's perspective, after which the characteristic features were extracted by reading the entire results again.

#### *Sociomathematical norm*

The sociomathematical norm presented in the aforementioned studies associated with the mathematics classroom culture (e.g., Cobb et al., 1991; Yackel & Cobb, 1996) were modified and complemented, and coded into the following five components (See Table 5): 'an understanding of what counts as mathematically acceptable explanations: Acceptable Explanations (AE)', 'an understanding of what counts as mathematical difference: Mathematical Difference (MD)', 'an understanding of what counts as mathematical effectiveness: Mathematics Effectiveness (ME)', 'an understanding of what counts as mathematical insight in a classroom: Mathematical Insight (MI)', and 'Others (O)'. 'Others (O)' was eventually deleted because it identified no frequency; the analysis was carried out with the other four codes (AE, MD, ME, and MI).

**Table 4.** Questions asked to teacher K.

Category	Question
School Environment	• What is the percentage of students from a low-income bracket who are subsidized for internet communication expenses, compared to the entire student body?
Personal Information	• What is your teaching background and experience?
Interest in mathematics	• Were you interested in mathematics?
Interest in the Subject of Mathematics	• Do you have subjects that you specialize in?
Early Teaching Experience	• Was there any success or failure in your early teaching career?
Belief About the Nature of Mathematics	• Question of beliefs about the nature of mathematics from TEDS-M (2008)
Belief About the Teaching and Learning of Mathematics	• Question of beliefs about mathematics teaching and learning from TEDS-M (2008)

**Table 5.** The Coding Scheme of Sociomathematical Norm used in this Study

Code	Definition	Descriptions of Codes
Acceptable Explanations (AE)	An understanding of what counts as mathematically acceptable explanations.	Cases in which a teacher or student would question, or make reference to, how a certain problem is thought to be mathematically acceptable or understood.
Mathematical Difference (MD)	An understanding of what counts as mathematical difference.	Cases in which a teacher or student would question, or explain, how other students' mathematical ideas differ from others' or could be compared to others'.
Mathematics Effectiveness (ME)	An understanding of what counts as mathematical effectiveness.	Cases in which a teacher would question which problem solving strategies (among several presented by the teacher) are most effective, or easiest to apply, for each student.
Mathematical Insight (MI)	An understanding of what counts as mathematical insight	Cases in which the mathematical justification of a higher level is achieved through insightful interaction, as well as discussion on mathematical topics presented by either the teacher or students.
Others (O)		Cases in which a new or unfamiliar sociomathematical norm, which did not appear in the theoretical review, would be disclosed and observed.

**Table 6.** The Elements Comprising the Mathematical Beliefs of a Teacher

Belief about nature of mathematics	Belief about teaching and learning of mathematics
<ul style="list-style-type: none"> <li>mathematics as formal</li> <li>mathematics as structural</li> <li>mathematics as procedural</li> <li>mathematics as applied</li> </ul>	<ul style="list-style-type: none"> <li>Purpose of mathematics as a school subject</li> <li>Appropriateness of particular instructional activities</li> <li>Students' cognition processes</li> </ul>

### *A teacher's mathematical belief*

The teacher's mathematical belief, which is the subject knowledge connected to the sociomathematical norm and formed by their experiences within the classroom culture, can be separated into two beliefs about the nature of mathematics and about the teaching and learning of mathematics, based on the framework of TEDS-M (2008)(See Table 6). Data, in the form of responses and behaviors gleaned from the teacher or students during the interviews or while being monitored in the classroom, were collected and individually analyzed.

### *Inter-coder agreement*

The primary correlation between the two independent codes in the 'Coding Scheme' was obtained. The assistant holding an academic degree in elementary education was asked first to prepare preliminary coding works on each classroom, which were based on the 'Coding Scheme', and which demonstrated that the agreement between two coders (the teacher and the researcher) was slightly over 50%. The results of similar coding work done by another peer demonstrated that the agreement increased to 62%. The initial coding scheme showed slightly matching and agreement between the coders. Thereafter, the 'Coding Scheme' was continuously modified and supplemented. Based on the finalized 'Coding Scheme', the final inter-coders' concordance was 72% on average (See Table 7).

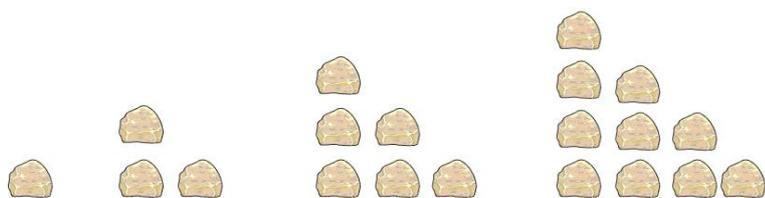
In order to satisfy the standard, the researchers analyzed the activity sheets of the students and reviewed whether the transcriptions of researchers and the intention of the analysis were in agreement with those of students and the teacher. That is, this review was done to ensure a non-biased approach, since the prejudices of the researcher may appear in the qualitative research. A peer participated in the study process to reduce methodical, analytical, and procedural errors.

**Table 7.** Inter-coder agreement on sociomathematical norms

Codes	Inter-coders' Reliability	
	Primary Coding	Secondary Coding
MD (Mathematical Difference)	51%	70%
ME (Mathematical Effectiveness)	53%	72%
AE (Acceptable Explanations)	60%	65%
MI (Mathematical Insight)	72%	81%

**Table 8.** Classroom Episode 1: 2<sup>nd</sup> Semester Lesson 7. 'Finding Patterns and Solving Problems' - Observation 13 Session

100 Teacher K: We go up to 120. Let's read it together?  
 101 Students: The ancient Greeks placed stones representing the pattern of the number of stones. Tell me what number will come fifth...



...  
 109 Teacher K: Tell me what can come fifth, S1?  
 110 S1: 15  
 111 S2: (In a loud voice) Why? 14.  
 112 Students: (rumble on )  
 113 S3: (*S2's counterpart*) That's 10 more because in the next five.  
 114 S3: uh....that's right.

## RESULTS

Sociomathematical norm in a teacher K's elementary math classroom - The mathematical beliefs of the teacher

*Mathematics as formal - 'Acceptable Mathematical Explanation (AE)': Logical justification.*

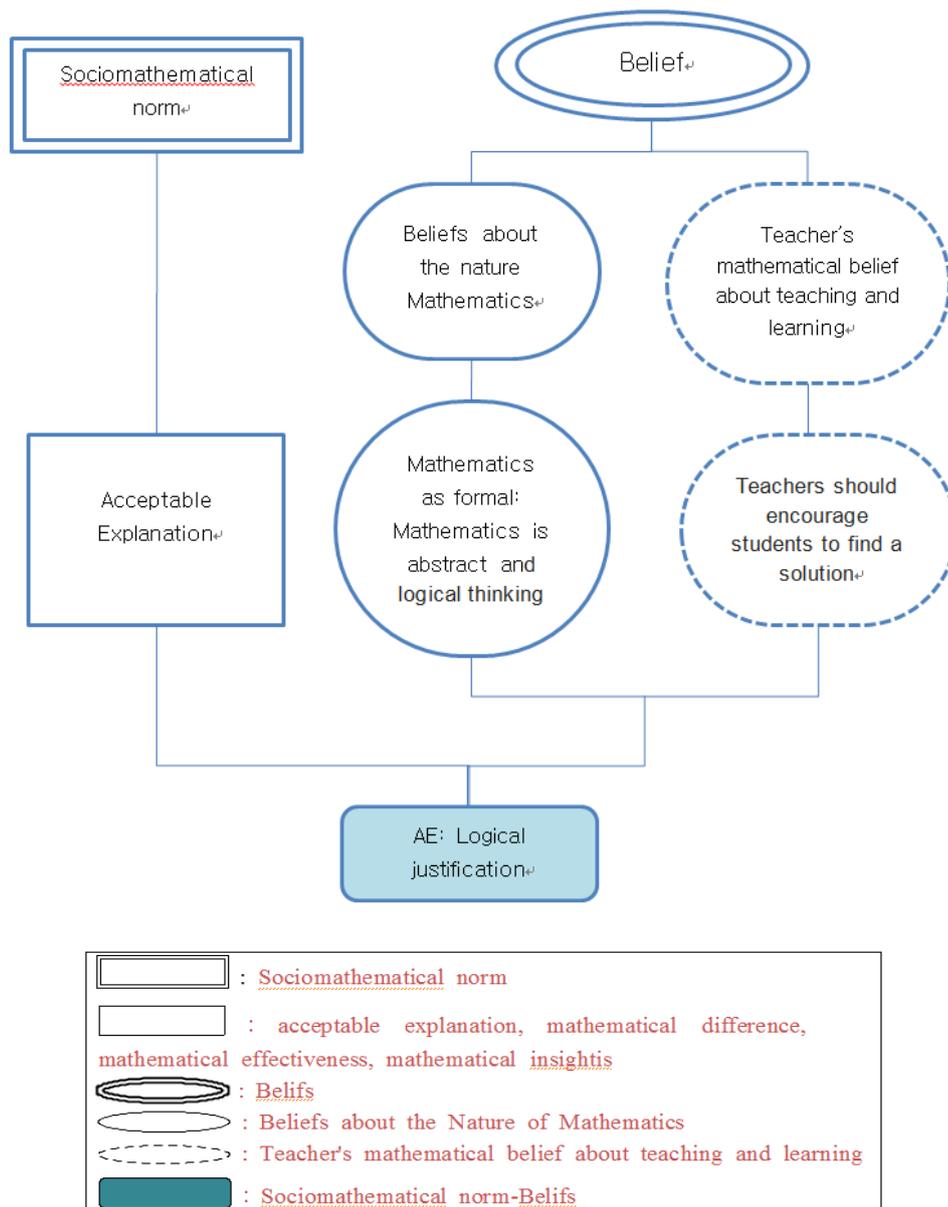
Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI

K-1-5-14<sup>2</sup> Math....Math....is logical.  
 15 So, we have to solve problem logically.

Teacher K had a formal point of view that this requires a logical explanation about the nature of mathematical knowledge, because mathematical knowledge varies with age. The lesson above indicates a number of students coming fifth in the activity, finding that the stones had been placed in a regular pattern. Hence, the entire class was guided by conceptual reason to discuss why the result should be 15 and not 14.

In the classroom, we were able to look at the pattern of repeated mathematical interaction between teacher and students, as demonstrated in Figure 1. Teacher K

<sup>2</sup>(Interview with a Teacher K) K-1-5-14: Teacher K-1<sup>st</sup> Semester-5<sup>th</sup> interview-14 sentence

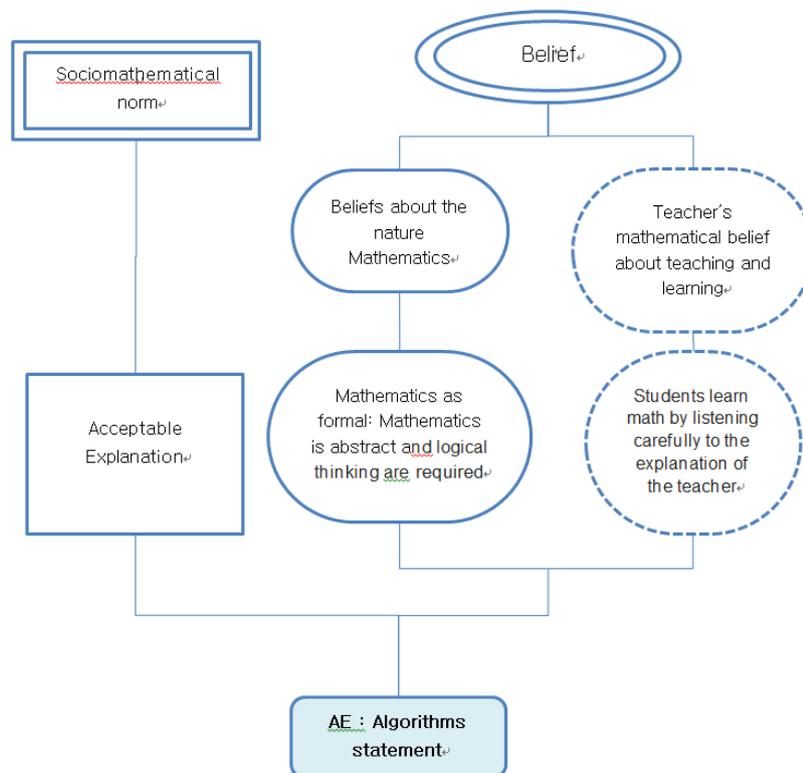


**Figure 1.** Mathematics as formal - 'acceptable mathematical explanation (AE)': Logical justification

showed a strong formal and procedural point of view, in regard to their respective beliefs about the nature of mathematics, which was associated with the sociomathematical norm. Teachers' mathematical beliefs and values can develop concomitantly with the classroom's sociomathematical norms (Yackel et al., 2000). In the illustration below, "belief" is indicated by the circles, while "norm" is indicated by the squares.

**Table 9.** Classroom Episode 2: 2<sup>nd</sup> Semester Lesson 2. ‘Multiplication and Division’ - Observation 1 session

AE	<p><i>115 Teacher K: Right. I multiply one digit number by the tens digit first, then I'll sum after multiplying the one-unit digit.</i></p> <p>116 S4: Yielded 694 multiplied by place of work, had no place in the ten</p> <p>117 Teacher K: Right, 57 469 times</p>
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**Figure 2.** Mathematics as formal - ‘acceptable mathematical explanation (AE)’: Algorithms statement

*Mathematics as formal - ‘Acceptable Mathematical Explanation (AE)’: Algorithms statement.*

Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI
[Diagrammatic connection line]							

K-2-1-22 We also have to just remember the elementary level.

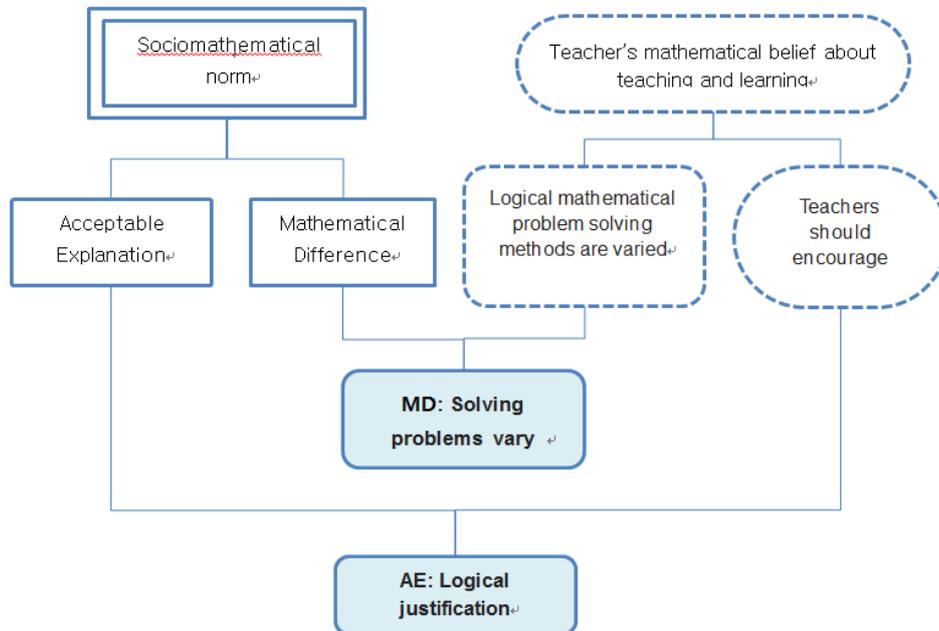
23 What could explain it, but solving? From memory, you require just a simple level.

24 But the answer is not to change. Math is truth...I think it is better to do so for elementary school students.

Teacher K’s belief about the nature of mathematics showed formal point of view. Math algorithms for such a basic level of knowledge deal with the real truth that was needed to solve the problem.

**Table 10.** Classroom Episode 3: 1st Semester Lesson 8. 'Finding Pattern' - Observation 5 Session

AE	132 Teacher K: Let's find the rule of the Greek stones.
MD	... 136 S5: <i>Number increases like 2, 3, 4 by adding 1. Then, 5 must come next. So it is 15.</i>
AE	137 Teacher K: <i>Any other mathematical methods?</i> 138 Yes, S5 raised his hands. 139 S6: <i>Look at the figure. The right side (number of diagonal shape) is added on.</i>



**Figure 3.** Mathematics as formal - 'acceptable mathematical explanation (AE)': Logical justification - 'Mathematical Difference (MD)': Various problem solving methods

*Mathematics as formal - 'Acceptable Mathematical Explanation (AE)': Logical justification - 'Mathematical Difference (MD)': Various problem solving methods.*

Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI

1-1-41 Math did the logical.

42 Mathematical knowledge is logical.

43 All knowledge is fundamental and recognized.

As evident from this review, Teacher K [137] asked if there was another problem solving strategy for what was being drawn on the blackboard. S5 was asked to justify why they chose this strategy. Students were asked to logically explain how to solve a problem that can be solved in a number of ways.

K-

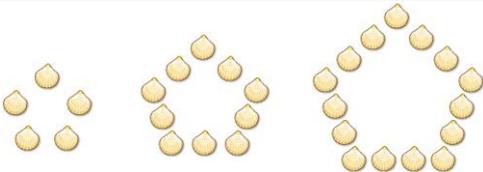
*'Acceptable Mathematical Explanation (AE)': Logical justification-'Mathematical Insight (MI)': Generalization.*

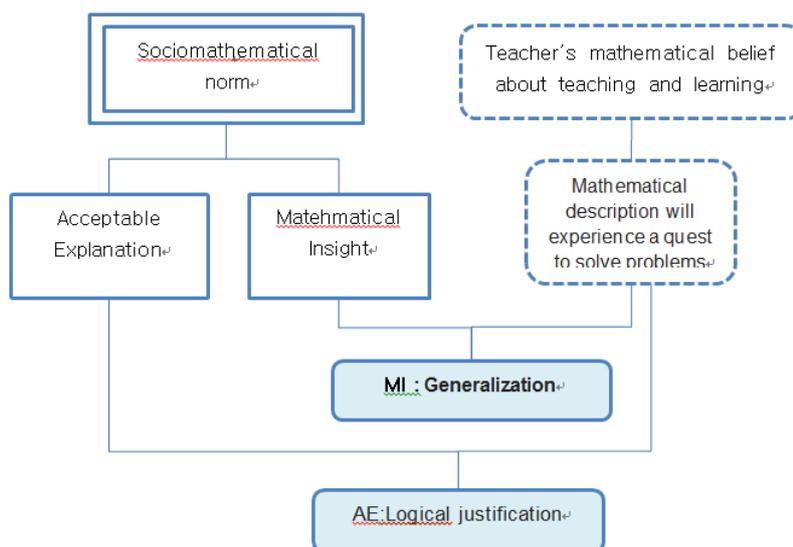
Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI

K-2-2-20 When children show an excellent way, and are surprised  
 21 Never thought that I know the way.  
 22 When children say the new rules or something makes you do unwittingly 'Oh!' Admiration.

S7 saw a stone placed in a triangle (the multiplication of 3); to see the stones placed in the pentagon increases the multiplication number to 5, using a pattern. So, entering one stone at a time on each side of the triangle (3, 6, 9), and the regularity of the way... In this way he did it by thinking creatively. Acceptable mathematical explanation (AE),

**Table 11.** Classroom Episode 4: 2<sup>nd</sup> Semester Lesson 7. 'Finding pattern and solving' - Observation 13 session

AE	<p>136 Teacher K: Would you tell me more specifically?                      137 S7: <i>When we count the number such as 5, 10, and 15. It's like the multiplication table.</i></p> 
MI	<p>138 This triangle is 3, 6, 9, and 12... It's 3 times table.                      139 Well, this pentagon means 5, 10, and 15.                      140 <i>Now that is a triangle 3, a pentagon is 5</i>                      141 S8: <i>Well. That's right, triangles have three sides increasing terms. So it's 6. Wow!</i></p>



**Figure 4.** 'Acceptable Mathematical Explanation (AE): Logical Justification-Mathematical Insight (MI): Generalization

as a mathematical insight (MI), is assumed to have been that which was created as the means of solving the problem.

*“Mathematical Difference (MD)’: Various problem solving methods - ‘Mathematical Effectiveness (ME)’: Simply the number.*

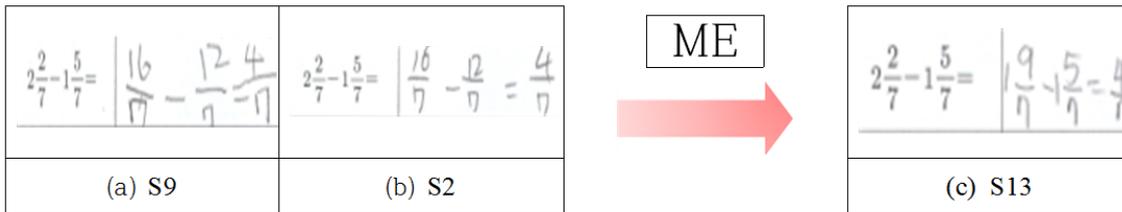
Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI

K-1-9-25 Sometimes I explain the answer directly. But when I ask students to answer a question, they bring out various solution methods.

26 When they come to solve the problem, I try to encourage them to compare their own solution methods. Then they are pleased to share their ideas with each other.

**Table 8.** Classroom Episode 5: 1<sup>st</sup> Semester Lesson 2. ‘Multiplication and Division’ - Observation 2 session

MD	<p><u>113 Teacher K: Is there another way?</u>  <u>114 S9: I solved it with improper form changing 1.</u>                  115 I think it might be wrong because all the calculations have yet to change improper fractions.                  116 What that's easy for me.                  117 Teacher K: At some point, what do you do?  <u>118 S9: When you borrow 1, it only gets easier.</u>  <u>119 S2: Ah...this is easier because numbers are not complicated ...</u></p>
ME	



**Figure 5.** S9, S2, and S13's  $2\frac{2}{7} - 1\frac{5}{7}$  problem solving: Mathematical Effectiveness (ME)

In [Figure 5], whereas mixed fractions are converted to improper fractions by S9 and S2, the fractions are converted to mixed fractions by S13. When the denominator cannot be subtracted during the addition and subtraction of numbers with the same mixed fraction, borrowing one to teach arithmetic as seen in this workaround is easier and more efficient.

*Mathematics as procedural - ‘Mathematical Difference (MD)’: Various problem solving methods.*

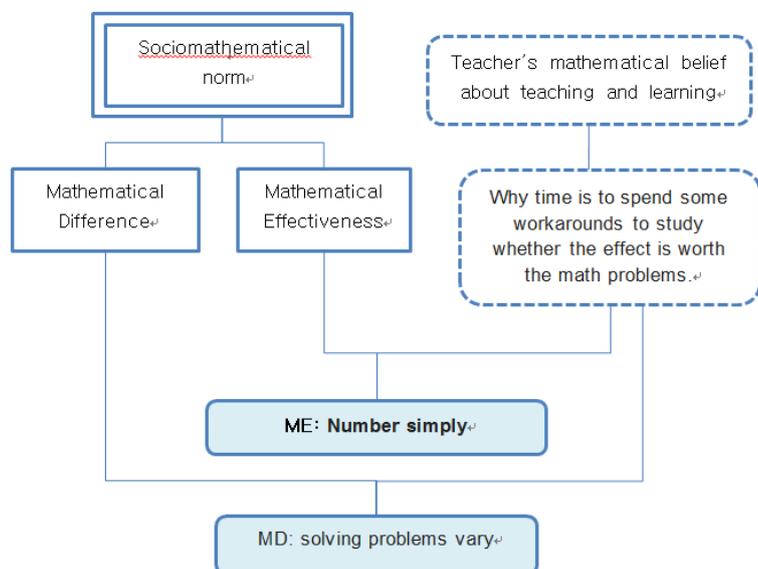
Belief about nature of Mathematics				Sociomathematical Norm			
Mathematics as formal	Mathematics as structural	Mathematics as procedural	Mathematics as applied	AE	MD	ME	MI

K-2-6-11 When teaching just one solving method, students easily receive certain knowledge passively.

12 Although the elementary school students, they have this ability.

13 They solve the way I did not know.

14 They can find anything you like ... new rules.

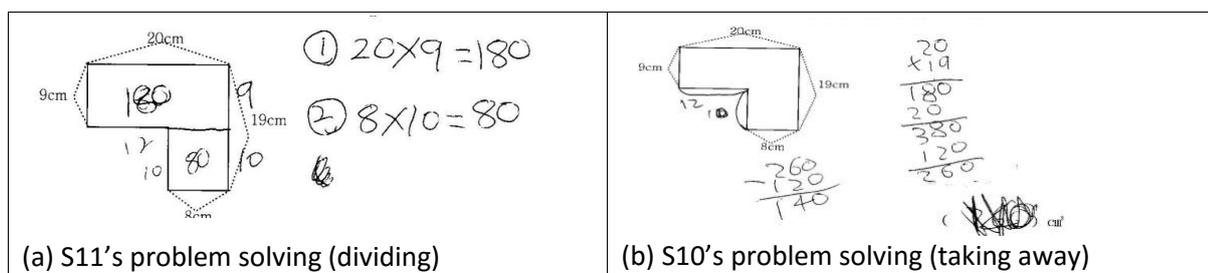


**Figure 6.** ‘Mathematical Difference (MD)’: Various Problem Solving Methods - ‘Mathematical Effectiveness (ME)’: Simply the Number

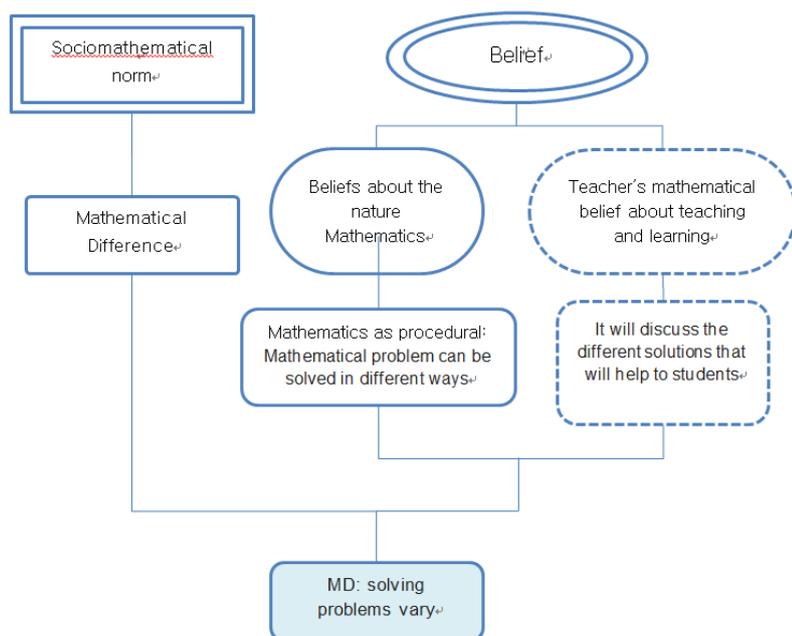
**Table 9.** Classroom Episode 6: 2<sup>nd</sup> Semester Lesson 5. Perimeter of the floor plan type and extent’ - Observation 12 session

MD	<p>160 Teacher K: It will help you solve a variety of shapes in different ways? Others?</p> <p>161 S10: I put a small square (width) in the large square (width)</p> <p>162 Teacher K: How can you solve it?</p> <p>163 S10: 19 to 20 times minus 12 times 10.</p> <p>164 S11: (Nods his head)</p> <p>&lt;S11’s interview&gt;</p> <p>7 Interviewer: Earlier, S11 said that he learned that at a private educational institute? We’re gonna solve various shapes cut into squares.</p> <p>...</p> <p>10 S11: Yeah, I did not know what I'm saying, so I think it would also be released.</p>
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Teacher K showed new connections to the resolution of issues about the nature of mathematics. Regarding the nature of mathematics, she appeared to consider mathematics procedural. Varied shapes are divided, removed, etc., and there may be several different solutions (See Figure 7). In an interview after the lesson the teacher was trying to properly acknowledge the effort that students have a full issue, while another student, rather than the correct answer, expressed the belief that it is more important for the teacher to solve problems in a variety of ways.



**Figure 7.** Method for measuring of figures of students, S11 and S10

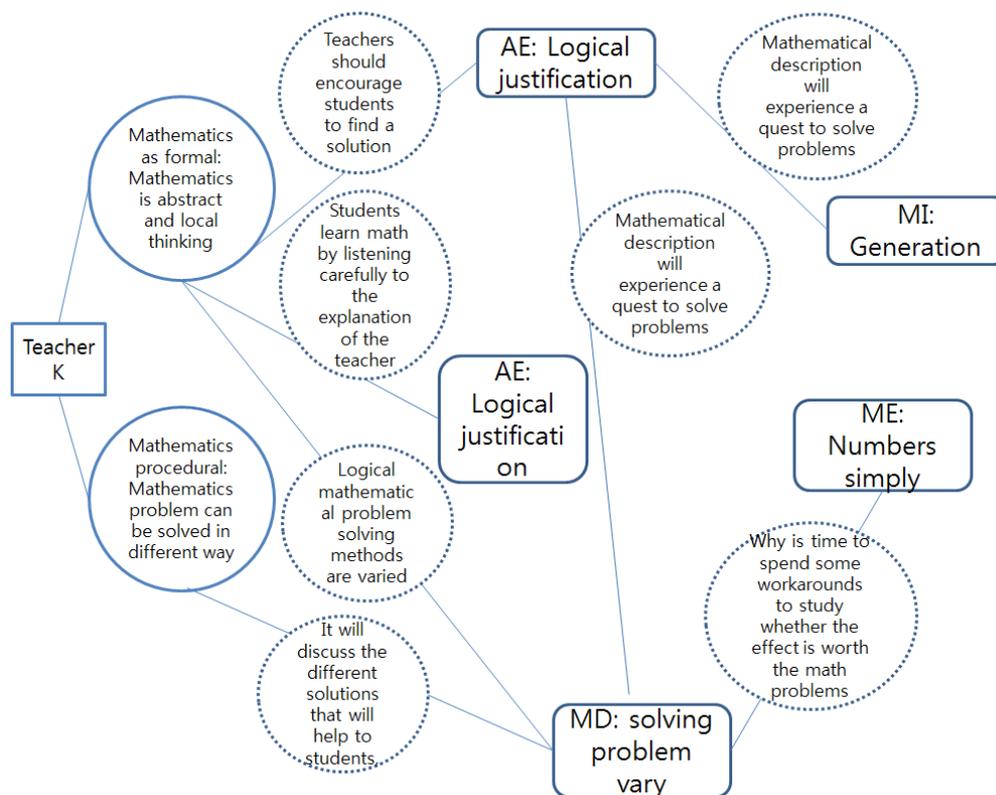


**Figure 8.** Mathematics as Procedural - 'Mathematical Difference (MD)': Various Problem Solving Methods

### SUMMARY

The formation of sociomathematical norms seen in mathematical beliefs of Teacher K is shown in Figure 9. Among the various beliefs about the essence of mathematics, Teacher K highlighted both the view of 'Mathematics as formal,' which argues that mathematics requires logical thinking, and the view of 'Mathematics as procedural,' which claims that a mathematics problem allows for multiple solutions. Teacher K's viewpoint of 'Mathematics as formal' is related to the logical justification of Acceptable Explanations [AE], which leads to a belief that teacher should encourage students to search for alternative solutions. Consequently, its connection with the statement about the algorithm of Acceptable Explanations [AE] leads to a belief that students can learn mathematics if they pay attention to the teacher's explanation.

The 'Mathematics as formal' stance is related to the various problem solving methods incorporated in the view of 'Mathematical Difference [MD].' As the logical justification of [AE] is connected to [MD], it is related to a belief that a math problem can allow for various logical solutions. When students are experiencing inquiry activities, the generalization of [MI] can provide a mathematical explanation about a problem in the process of logical justification of [AE]. The relationship of the view of 'Mathematics as procedural' to [MD] leads to a belief that a realistic problem can be



**Figure 9.** Teacher K's beliefs and mathematical form of the sociomathematical norm

solved by various methods. Based on our belief in the value of spending time to determine why a certain method can be an effective solution to a math problem, this is related to a belief that it is efficient to simplify those numeric figures used in the problem solving process.

Cobb and other researchers (Bowers, Cobb, & McClain, 1999; Cobb & Bowers, 1999; Cobb, Stephan, McClain, & Gravemeiger, 2011; Cobb & Yackel, 1996; McClain & Cobb, 2001; Yackel, 2001) discussed how teachers encourage students to form their own sociomathematical norms by allowing students to negotiate mathematical explanations. In other words, the differences and efficiency do not directly suggest how norms form in math class, which is what we thought as we looked at the decision-making process based on the teacher's mathematical beliefs and, thus, the sociomathematical norms.

## CONCLUSIONS

A teacher's mathematical belief is reflected in his or her decision making at every moment of a math class, and greatly influences teaching and learning, specifically, the instructions, objectives, content and methods involved in their classroom environment. Furthermore, a teacher's mathematical belief is related to the formation of sociomathematical norms between a teacher and his or her students. Based on the results, the following discussion arose.

First, Yackel and Cobb (1996) argued that community mathematics norms are not created automatically, so that students in class can express and share mathematical ideas while the teacher acts as a mediator. In contrast to those authors' perspective, the present study results revealed that the teacher developed her thoughts into norms

through decision-making about mathematical beliefs. In this study, we can see that students can form sociomathematical norms in the classroom based on their teachers' mathematical beliefs (See Figure 9). This stems from beliefs about the nature of mathematics, and sociomathematical norms and beliefs on teaching and learning can influence each other. In other words, teachers' differences in mathematical beliefs influenced the formation of sociomathematical norms during a math class in an elementary school. Cho (2001) argued that the norm of mathematical classroom culture is related to the belief and values of the math classroom, from a cultural anthropological viewpoint.

As Yackel and Cobb (1996) argued, because sociomathematical norms are not automatically formed, a teacher should act as a mediator by encouraging students to express their mathematical ideas and share them with their classmates in a classroom setting. A sociomathematical norm is a norm that is developed from the activities performed by a teacher and students, from a mathematical viewpoint, which demonstrates that the teacher's role is very important in enhancing the mathematical quality of the classroom environment. Given that teachers play a critical role in improving the mathematical quality of a classroom environment, future research must be targeted at exploring the types of discourses and the teachers' teaching methods.

The kind of norms and meanings a teacher chooses in regard to the class progression, mathematical concept, rule, principle and teaching method can significantly affect the math activity patterns. Therefore, more detailed and extensive research must be carried out on the role of the teacher in forming sociomathematical norms in an elementary school mathematics class. When a teacher chooses the rules of the classroom and interactive activities based on his or her mathematical beliefs, he or she can thereby influence the interaction between the teacher and the students, the interaction between two or more students, and their class participation structure. More research should be targeted at determining the appropriate level of teacher intervention and the desirable role of the teacher.

#### **AUTHORS' NOTE**

This material is based on parts of a dissertation titled, "A study on constructing sociomathematical norms and teachers' beliefs of elementary school in two Mathematics class," by the main author.

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