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Spatial visualization skills present in items of the Brazilian high school national exam

Carmen Vieira Mathias [1*], Cristian Martins da Silva [2], Fábio Luiz Borges Simas [3]

[1] Universidade Federal de Santa Maria, Santa Maria, BRAZIL
[2] Universidade Franciscana, Santa Maria, BRAZIL
[3] Universidade Federal do Estado do Rio de Janeiro, Rio de Janeiro, BRAZIL

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Abstract

This article aims to analyze, categorize, and quantify the Brazilian high school national exam (ENEM) questions that require spatial visualization skills (SVS) for their resolution. This is a documentary research with a qualitative-quantitative approach whose corpus of analysis is ENEM exams carried out from 2009 to 2022, in the subject of mathematics and its technologies. There are four main results of this research: (1) spatial geometry (SG) questions that require SVS for their resolution (39.7%) are proportionally more frequent in ENEM than in four most used textbooks in Brazil (<20.0%), (2) some categories of SVS are much more frequent than others in ENEM questions, (3) the proportion of geometry questions in ENEM mathematics exam (23.0%) is considerably higher than the corresponding proportion of geometry-related skills in ENEM skills framework (13.0%), and (4) approximately half of ENEM geometry questions address SG.

Keywords: spatial visualization skills, the Brazilian high school national exam, geometry

INTRODUCTION

Spatial visualization skills (SVS) refer to the ability we have to create and manipulate mental images of what we see around us. Specifically, SVS are the human capacities to represent, transform, create and remember symbolic and non-linguistic information (Linn & Petersen, 1985). It is widely acknowledged that people with limited abilities in this field face difficulties in several areas such as architecture, astronomy, biology, cartography, chemistry, engineering, geology, mathematics, music, and physics (Hartman & Bertoline, 2005). Given the significant impact of SVS, research on factors that may influence their teaching is crucial, as in the case of geometry questions in the Brazilian high school national exam [Exame nacional do ensino médio] (ENEM) and in textbooks.

ENEM has a great influence on high school curricula and also on the teaching practices of teachers who, in general, plan their classes based on the most frequent contents in the exam, in order for their students to achieve good results (Stadler & Hussein, 2017). Curriculum recommendations, national exams for student evaluation and textbooks used directly affect classroom teaching. According to Apple (2013), these factors influence relevant teacher decisions, for example, about which knowledge is a priority.

It should be noted that there is a certain discrepancy between what the Brazilian national common core curriculum [*Base nacional comum curricular*] (BNCC) recommends, what is taught in school, what is in textbooks and the questions on ENEM. BNCC does not directly mention SVS and there is no mention of visualization in the mathematics curriculum component. However, the document mentions "spatial thinking" and states that this is associated with intellectual development that integrates knowledge not only from geography, but also from other areas (such as art and literature, mathematics, and science) (Brasil. Ministério da Educação, 2018b, p. 359).

In this context, the research by Settimy and Bairral (2020) pointed out the need to implement activities that focus on visualization and spatial representation in the school environment. These authors emphasize the importance of visualization as a fundamental skill of mathematical thinking that must be taught, since it is not innate. Similarly, Silva et al. (2012) highlighted the positive correlation between spatial skills and performance in high school and argue that the

✉ carmen@ufsm.br **(*Correspondence)** ✉ martinsdasilvacristian@gmail.com ✉ fabio.simas@uniriotec.br

Contribution to the literature

- This article documents the current misalignment between assessment (ENEM), textbooks and curriculum (BNCC) in Brazilian High School with regard to Spatial Visualization Skills.
- This study contributes to establishing parameters and language for analyzing geometry teaching materials for Basic Education.
- Although this is not the main issue here, this article stimulates curricular discussion about the role of spatial visualization skills in school geometry. New studies should consider neuroscience research related to brain development.

development of these skills since elementary school is important to prepare students for solving practical problems, reinforcing the national guidelines.

According to the 2014 guide of the national textbook program [*Programa nacional do livro e do material didático*] (PNLD) for mathematics, traditional high school textbooks have not been effective in promoting the improvement of the skills of drawing and visualization of geometric objects. Given this scenario, it is essential to explore different perspectives, projections, cuts, unfoldings, and other resources of representation of objects to stimulate the development of these very important skills (Brasil. Ministério da Educação, 2014b). By doing so, we can help students to understand and apply geometric concepts more easily and efficiently, and better prepare them for future studies and careers in fields that require these skills.

Despite this suggestion in PNLD, Mathias and Simas (2021), when analyzing exercises of mathematics textbooks that require use of SVS to be solved, concluded that there is an excessive emphasis on area and volume calculations to the detriment of the development of spatial reasoning in students. Thus, this article aims to analyze, categorize and quantify ENEM questions that require SVS for their resolution. To do this, the following sections contain a theoretical framework about ENEM and SVS, the methodology used, the analysis of the results and the final considerations.

BRAZILIAN HIGH SCHOOL NATIONAL EXAM

ENEM is a voluntary diagnostic assessment applied annually to students who are graduating or have graduated from high school. According to Brasil. Ministério da Educação (2017b, p. 40), the results of the exam serve as a unique, alternative or complementary mechanism for access to higher education, especially that offered by federal higher education institutions. Also, the main aim of this exam is to provide a reference for self-assessment, based on the competencies and skills that structure it (Brasil, 2005, p. 7). These competencies and skills are described in the exam framework (Brasil. Ministério da Educação, 2012b). According to Bonamino (2014):

> The term *framework* (in Portuguese, *matriz de referência*) is used specifically in the context of large-scale evaluations to indicate skills to be assessed at each educational stage and to guide the development of test and exam items, as well as the construction of proficiency scales that define *what* and *how much* the student accomplishes in the context of the assessment.

In the context of ENEM, the framework is the guiding document for the elaboration of the assessment items, popularly called questions. Such questions assess the development of a skill in a particular area of knowledge. According to Instituto Nacional de Estudos e Pesquisa [National Institute of Studies and Research] [INEP] (2010, p. 9):

> The item must be structured in such a way that it is a unit of proposition and covers a single skill of the framework. To this end, coherence and cohesion between its parts (main text, statement and options) must be ensured, so that there is an articulation between them, and a single problem situation and a homogeneous content approach are made explicit.

This means that each of the questions that make up the exam is linked to a single skill of the framework, has a supporting text that presents the information necessary to solve the proposed problem situation and is multiple choice. In the particular case of the subject area mathematics and its technologies, the 45 questions that make up the assessment of this area must be linked to one of the 30 skills, as described in Brasil (2012b). The competence that refers to geometrical knowledge as well as the associated skills are highlighted in **Table 1**. We

Table 1. Competences & skills related to geometry (adapted from Brasil, 2012b)

Competences	Skills
Using geometrical knowledge to read & represent reality & act on it	Interpreting location & movement of people/objects in 3D space & their representation in 2D space
	Identifying characteristics of plane or spatial figures proposed as a solution to everyday problems
	Solving a problem situation involving geometrical knowledge of space & shape
	Using geometrical knowledge of space & shape in selection of arguments

noticed that, of the seven competences listed in Brasil (2012b), only one mentions geometrical knowledge, and of 30 skills listed, only four are linked to this competence. But skills listed in **Table 1** have a connection with object of this study, that is, SVS.

In terms of BNCC, the geometrical knowledge at middle and high schools relates to basics of plane and spatial geometry (SG). SVS can be charged in ENEM at most (if not all) of this geometric content. For the sake of completeness, in plane geometry students should learn congruent and similar triangles, parallel lines and angles, circles and triangles, angles inscribed in circles, areas of circles and polygons and some of isometric and homothetic transformations and some of tailings. In space geometry, students are taught orthogonal and perspectives views, prisms, pyramids, cylinders, cones, spherical surfaces them area and volume formulas and cartographic projections.

SPATIAL VISUALIZATION SKILLS

SVS can be described as "the ability or skill drawn upon to mentally transform or manipulate spatial properties of an object" (Lowrie et al., 2019, p. 2). As the object of study in a growing number of studies since the mid-twentieth century in various fields of research, in particular in the area of psychology and cognition from the school perspective, there is no single term to refer to SVS (Gutiérrez, 1996). In this regard, it is common to notice, in studies dedicated to the teaching and learning of geometry, the frequent use of the terms spatial reasoning, spatial thinking and spatial visualization to refer to SVS (Mathias & Simas, 2021).

Gonzato et al. (2011) analyzed tasks in textbooks and considered that the ability to visualize and orient an object is not limited only to the ability to see objects and spaces, but also involves the ability to reflect on their possible representations, relationships between their parts and to examine the possible transformations (such as rotation, cross section and unfolding) that the object can undergo.

Initially, the authors discuss static orientation tasks of the subject and objects and define spatial orientation (SO), as an action present in

> [...] tasks that require understanding the body schema, identifying and using its polarities: up-down, left-right, front-back, using this language to describe the position of one's own body, or that of another observer, in relation to objects or other people and the positions of objects in relation to other objects. In these activities, we consider that objects and people are immobile (Gonzato et al., 2011, p. 100, our own translation).

An example of a task that requires SO to be solved is in **Figure 1**, where children appear in different positions and the individual is required to paint the left shoes one

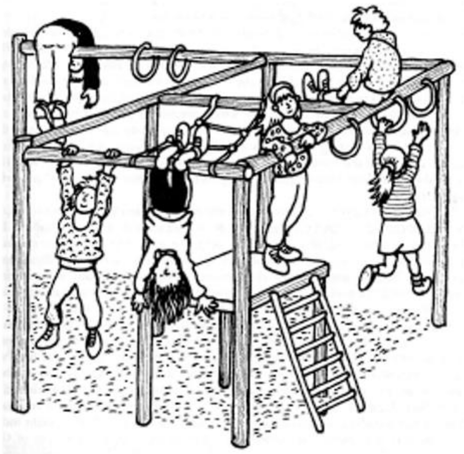

Figure 1. Task, where SO is required (Wiegand, 2006, p. 107)

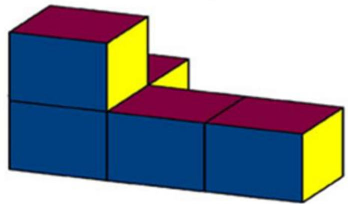

Figure 2. Task, where it is necessary to change type of representation (Pittalis & Christou, 2010, p. 209)

color and the right shoes another. Thus, the subject needs to interpret the orientation of each child and realize which is the left shoe and which is the right.

In addition, these authors describe five actions present in tasks for interpreting perspectives of three-dimensional objects. The authors call the first visualization action "change the type of representation". This action involves representing a physical object with a flat representation, constructing a three-dimensional object based on its flat representation or converting flat representations of different types (Gonzato et al., 2011, p. 101). This type of action is contemplated in the task illustrated in **Figure 2**, where there is the representation of a solid and it is required to represent it using orthogonal projections.

The second visualization action is called "rotate an object" and involves the ability to rotate the object or parts of the object, or, equivalently, to change the perspective mentally (Gonzato et al., 2011, p. 101). An example of an activity that exercises this action is illustrated in **Figure 3** in which the person must imagine which of the options represents the object after being rotated.

Gonzato et al. (2011) call the third visualization action "folding and unfolding", which refers to the concept of unfolding. This action involves the act of folding a net to form a three-dimensional object (physical or represented), or vice versa, unfolding the object to obtain one of its unfoldings (ibid, p. 101). An example of this action is illustrated in **Figure 4**, where unfoldings are

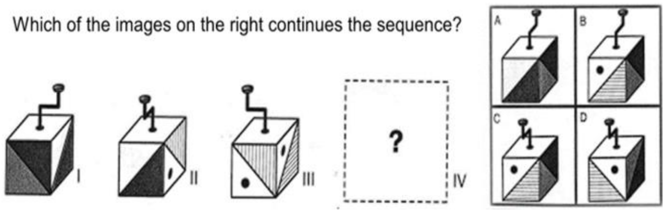

Figure 3. Task, where it is necessary to rotate an object (adapted from Ferrero, 2008, p. 172)

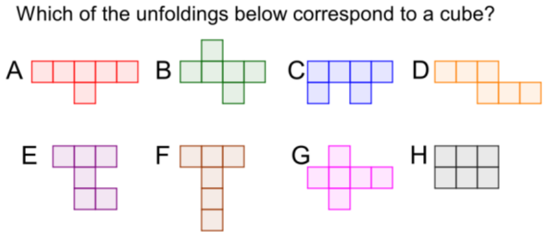

Figure 4. Task, where folding & unfolding is required (adapted from Almodovar & García, 2009, p. 197)

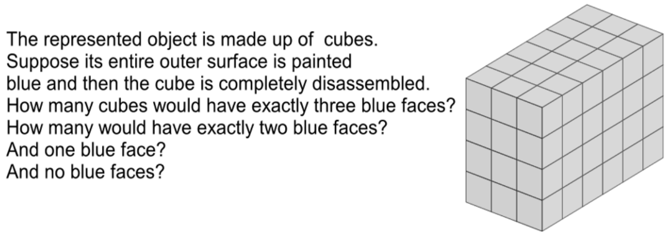

Figure 5. Task that requires composing & decomposing into parts & counting elements (Bishop, 1983, p. 187)

given and it is required to recognize which of them are derived from a specific three-dimensional solid.

The fourth visualization action the authors listed is called "compose and decompose into parts". This action involves considering two or more parts of a three-dimensional object and asking that they be composed to form a solid, or vice versa (Gonzato et al., 2011, p. 101).

Finally, the last visualization action addressed by Gonzato et al. (2011) is called "count elements". This action involves counting the elements that make up a given solid, such as volume units, faces, edges, vertices, among others (ibid, p. 101), as the name itself suggests.

These last two actions are necessary to solve the task illustrated in **Figure 5**, where the individual is required to separate the block into several cubes and then count the parts based on a specific criterion.

In this regard, when analyzing Brazilian high school mathematics textbooks, Mathias and Simas (2021) used the five actions present in tasks of interpretation of perspectives of three-dimensional objects (Gonzato et al., 2011) to categorize actions required in tasks that demanded SVS for their resolution, as shown in **Table 2**. We observed that SO was not explored in Mathias and Simas (2021), although it was mentioned in Gonzato et al. (2011), since tasks that require this type of element are generally addressed in the early years of elementary school.

MATERIALS & METHODS

According to Gil (2017, p. 25), in order to evaluate the quality of the results of a research, it is necessary to know how the data were obtained, as well as the procedures adopted in their analysis and interpretation. In this context, regarding the approach of the problem, this study can be classified as quali-quantitative. According to Creswell and Creswell (2021), "quali-quantitative" or "mixed methods" research consists of a combination of qualitative and quantitative aspects.

In this study, in each question, the authors identify if there is a visualization task and, if so in which visualization actions the analyzed questions belong. Subsequently, a quantification is performed to facilitate the comparison between the selected editions of the exam. In addition, this investigation consists, in terms of design, in a documentary research, whose main characteristic is that the source of data collection is restricted to documents, written or not, that constitute what is called primary sources. In this case, the *corpus* of analysis was ENEM exams from 2009 to 2022.

For the documentary collection of ENEM exams, we accessed the collection made available by INEP on its web portal (https://www.gov.br/inep/pt-br/areas-de-atuacao/avaliacao-e-exames-educacionais/enem/provas-e-gabaritos), through download. We selected ENEM exams that contained the area of mathematics and its technologies in the period from 2009 to 2022, then we identified and analyzed the questions related to geometry and which had some SVS involved.

Table 2. Visualization actions (adapted from Mathias & Simas, 2021, p. 7-9)

Action	Description
CIV	Exercises that depend on flat representations of object, which require some projection system to be carried out.
RS	Exercise in which student is asked to mentally rotate a given solid. Usually, image of a solid is shown &, after rotation, it is necessary to identify image of same solid.
GSR	Exercises in which flat representations of object & axis of rotation are presented & student needs to recognize solid during its rotation.
DS	Exercises, where solid is represented in a flat way & spatial representation is requested, or vice versa.
CDS	Exercises, where solid (or one of its representations) is presented & its identification as decomposed into two or more parts is requested (or vice versa).
ISS	Exercise, where a solid & a plane intersecting it are given & identification of figure formed at intersection is requested. Or, conversely, intersections of solid with planes are given & people are asked to identify it.

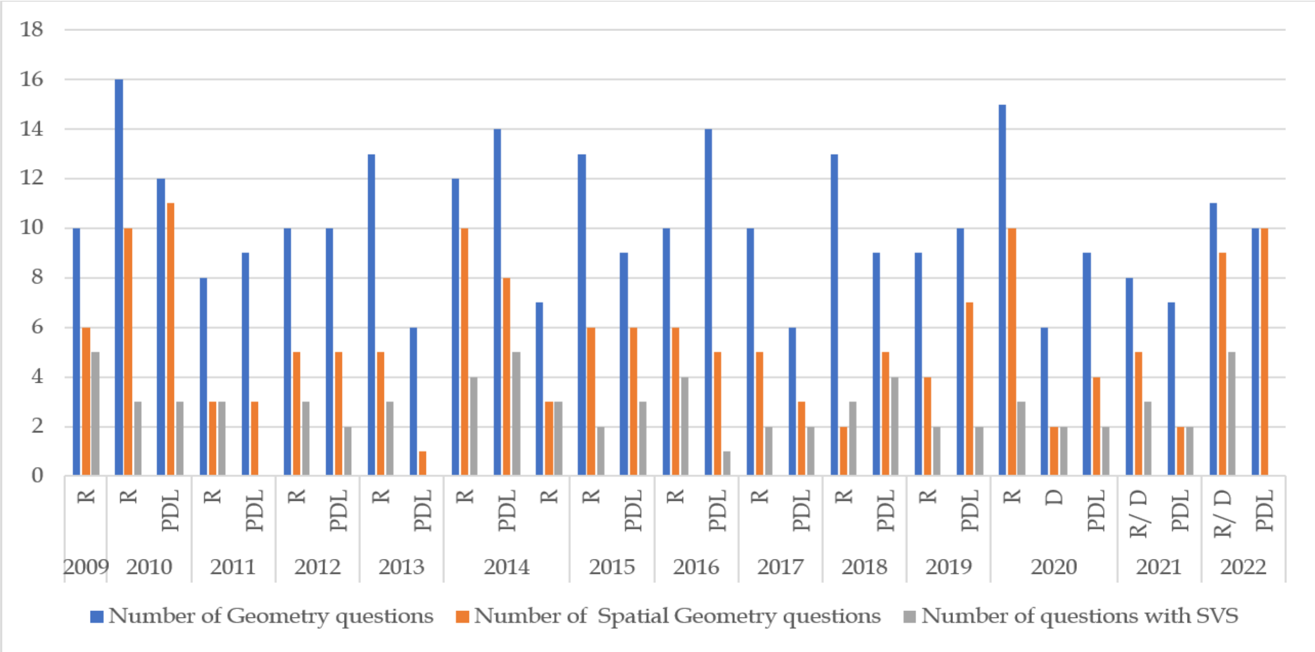

Figure 6. Overview of exams & questions (Source: Authors' own elaboration)

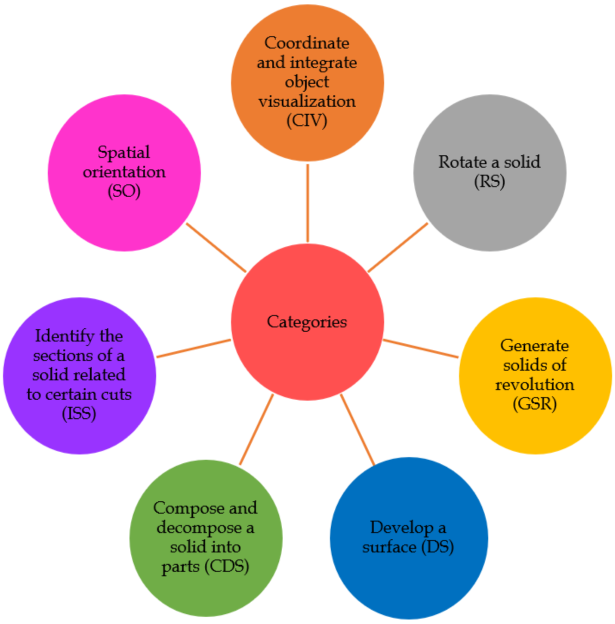

Figure 7. Categories used to analyze ENEM questions (Source: Authors' own elaboration)

It is noteworthy that, in addition to the regular exams, we also analyzed the exams of editions in which ENEM had double application due to operational problems, those applied to persons deprived of liberty and the exams carried (R) out digitally (D).

Figure 6 shows the number of geometry questions per year of application, the color of the notebook, the application situation and the number of questions considered for analysis. We can see that the number of mathematics questions every year is always 45.

Subsequently, we analyzed the questions according to the criteria established in Mathias and Simas (2021) and SO described in Gonzato et al. (2011), as illustrated in **Figure 7**.

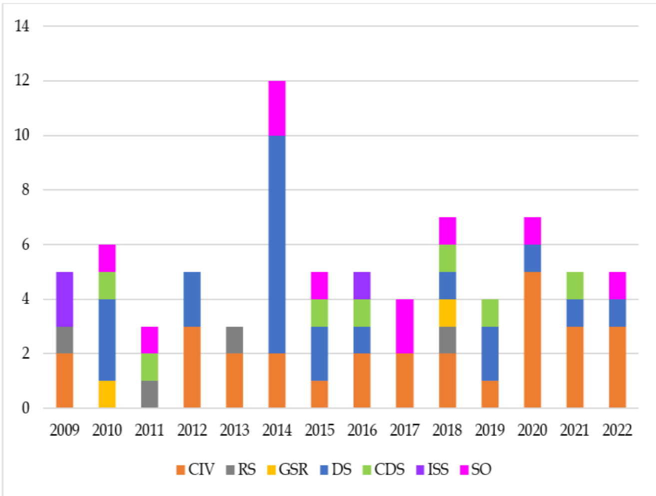

Figure 8. Number of questions analyzed per category & year of application (Source: Authors' own elaboration)

The classification was performed while reading the questions when we observed in which category they fit.

ANALYSIS OF RESULTS

Based on the selected exams, the questions were classified according to the methodology described before. After categorization, we made a quantitative summary of results, as shown in **Figure 8** and **Table 3**.

In quantitative and general terms, it is noteworthy that the prevalence of questions requiring SVS among those of SG in the years of ENEM application that we analyzed was 39.7%. This is considerably higher than the analogous proportion in the four textbooks analyzed by Mathias and Simas (2021), which ranged from 14.0% to 19.8%. This suggests that ENEM item (question) developers are more aligned with the role of SG in basic

Table 3. Number of questions analyzed per category & year of application

	Mean	Median	Min	Max
Geometry (G)	10.21	10.00	6.00	16.00
Spatial geometry (SG)	5.38	5.00	1.00	11.00
SVS	2.62	3.00	0.00	5.00
SVS in SG	2.21	2.00	0.00	4.00
G/total (total=45)	0.23	0.22	0.13	0.36
SG/G	0.51	0.50	0.15	0.92
SG/total	0.12	0.11	0.02	0.24
(SVS in SG)/SG	0.40	0.40	0.00	1.00

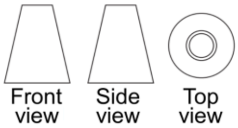

Figure 9. Views of tower presented in question 145 (Brasil. Ministério da Educação, 2020a)

education than textbooks, with regard to SVS (Brasil. Ministério da Educação, 2014b; Gaulin, 1985).

In specialized literature, there are several indications and statements that geometry has lost its prominence in school mathematics, especially SG (Pavanello, 1989). But ENEM seems to contribute to moving this "pendulum" to the side of geometry since, on average, 23.0% of ENEM questions address geometry (the minimum was 13.0% and the maximum 36.0%). This proportion is considerably higher than the share of geometry-related skills (four) in ENEM framework (30), approximately 13.0%. This can serve as a motivation to increase the prominence of geometry in school mathematics.

SG makes up approximately 51.2% of the geometry questions (therefore, 12.0% of the total exam questions) from which one can infer (being optimistic) that there is a valorization of SG in the whole of geometry. But it is also worth reflecting (being pessimistic) that, from a curricular point of view, SG constitutes almost the entire high school geometry, which the exam aims to assess. Looking specifically at SVS, we can observe that the tasks coordinate and integrate visualizations (CIV) and develop a surface (DS) are the most frequent, composing, respectively, 37.0% and 29.0% of the questions that require SVS. At the opposite extreme are the tasks identify solid sections (ISS) and generate solids of revolution (GSR) with frequencies of two and three questions, respectively (approximately 4.0% and 5.0% of the total 76 questions requiring SVS). Next, in this section, we present the qualitative analyses based on the listed categories. As the name implies, CIV category relies on the coordination and integration of views to identify a solid, a clear example of which is given below.

Question 145

In technical drawing, it is common to represent a solid through three views (front, side, and top), resulting from projection of solid on three planes, perpendicular two to two. **Figure 9** shows views of a tower.

Based on the provided views, the student had to choose, among the ones illustrated in **Figure 10**, which one best represents the tower.

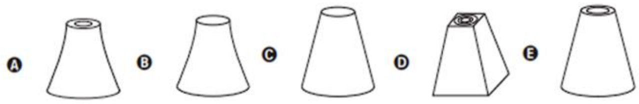

Figure 10. Options presented in question 145 (Brasil. Ministério da Educação, 2020a)

Being able to assess these three views together is essential to interpret and recognize orthogonal projections, for example, if a student only considers the top view, options "A" or "E" could be considered correct. This type of strategy, of using only one of the views, is explored in Gutiérrez (1998), who states that the inability to coordinate the different projections is something that persists for most students in basic education when trying to construct a solid represented by its projections.

As previously mentioned, CIV consists of activities that depend on some kind of projection to be carried out, as in the following example.

Question 154

The access between the two floors of a house is via a circular staircase (spiral staircase), as shown in **Figure 11**. The five points A, B, C, D, and E on the handrail are equally spaced, and points P, A, and E are on the same line. On this staircase, a person walks while sliding their hand over the handrail from point A to point D, as illustrated in **Figure 11**.

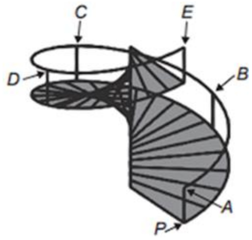

Figure 11. Staircase with handrail presented in question 154 (Brasil. Ministério da Educação, 2014a)

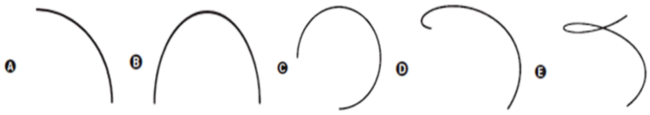

Figure 12. Options presented in question 154 (Brasil. Ministério da Educação, 2014a)

The student is then asked to think about which one best represents the orthogonal projection, on the floor of the house (plane), of the path that person's hand has followed, options of which are illustrated in **Figure 12**.

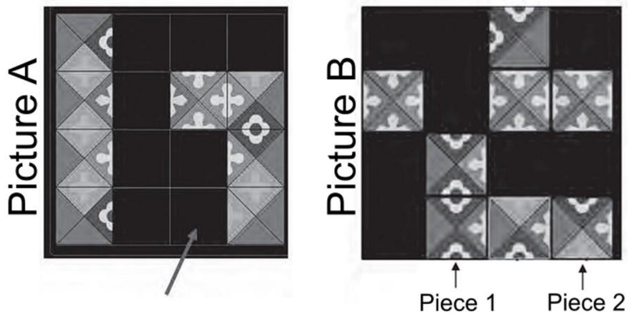

Figure 13. Boards presented in question 145 (Brasil. Ministério da Educação, 2009)

Figure 14. Anti-clepsydra presented in question 170 (Brasil. Ministério da Educação, 2018b)

This type of task, there is a need to identify the path along which the point passes through the solid and to determine the orthogonal projection of the path taken. Thus, we classified this activity in CIV category.

As for rotate a solid (RS) action, Mathias and Simas (2021) observed that it was not present in any of the analyzed textbooks. This is probably due to the understanding that this type of task is more common in elementary school (Mathias & Simas, 2021). In this regard, it is noticeable that the Brazilian educational system does not develop or evaluate SVS in a broad way, since, according to Gonzato et al. (2011) and Gorgorió (1998), this type of task requires visual processing strategies. That is, in order to solve it, it is necessary to imagine the context of the task, or a rotation of the object, or the change of position of the subject or object. However, four ENEM questions addressed this category, one of which was the following.

Question 145

Figure 13 shows a part of a jigsaw puzzle that is being assembled. Note that the pieces are square and there are eight pieces on the board in figure A and eight pieces on the board in figure B. The pieces are removed from the board of picture B and placed on the board of picture A in the correct position, so as to complete the drawings.

It is possible to correctly fill the space indicated by the arrow on the board in picture A by placing one of the pieces according to the options below:

1. One after rotating it 90° clockwise.
2. One after rotating it 180° counterclockwise.
3. Two after rotating it 90° clockwise.
4. Two after rotating it 180° clockwise.
5. Two after rotating it 270° counterclockwise.

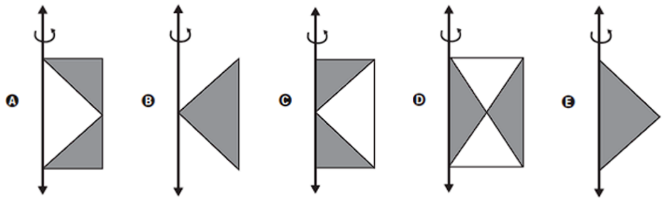

Figure 15. Options presented in question 170 (Brasil. Ministério da Educação, 2018b)

In the categorization of the questions, only two were perceived as GSR. In one of them, the solid of revolution is given and it is necessary to identify which one was rotated to generate it, as in the following example.

Question 170

Figure 14 shows an anti-clepsydra, which is a geometric solid obtained by removing two cones opposite each other by the vertices from an equilateral cylinder whose bases coincide with the bases of that cylinder. The anti-clepsydra can also be considered as the solid resulting from the rotation of a plane figure around an axis.

Based on this, the question is "which plane figure rotates around the indicated axis to generate an anti-clepsydra like the one in the figure above?", according to the options illustrated in **Figure 15**.

According to Gonzato et al. (2011), when solving this type of task, students need to work with a flat representation of a three-dimensional body and, in addition, use knowledge about axis of rotation and rotation of a figure around an axis and the structure of the resulting solid.

It is noticeable that the categories RS and GSR had a close number of occurrences, which possibly occurs because both actions can be considered as a derivation of mental rotation (MR). According to Maier (1996), MR involves the ability to rotate a 2D or 3D figure. Thus, RS involves the ability to rotate a 3D figure, while, in the case of GSR, it is necessary to rotate a two-dimensional curve mentally, under a given axis, in order to determine the solid. This means that this category is also part of MR. According to Gonzato et al. (2011), by varying the structure of the body and its type of representation (initial stimulus), combining several rotation axes and changing the type of answer (for example, drawing), it is possible to formulate different rotation tasks.

Also, Gonzato et al. (2011) point out difficulties when solving tasks that require DS in their resolution, and this category was the second most addressed in the editions of ENEM, with one of the examples described below.

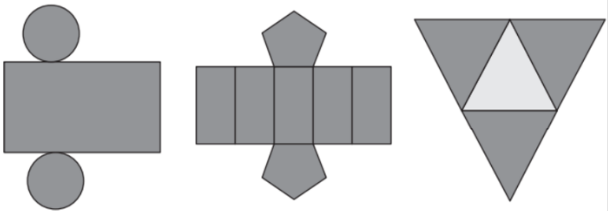

Figure 16. Unfoldings presented in question 141 (Brasil. Ministério da Educação, 2012a)

Question 141

Maria wants to innovate in her packaging store and has decided to sell boxes with different shapes. **Figure 16** shows the unfoldings of these boxes.

Which geometric solids will Maria get from these unfoldings?

1. Cylinder, pentagonal prism, and pyramid.
2. Cone, pentagonal prism, and pyramid.
3. Cone, truncated pyramid, and pyramid.
4. Cylinder, truncated pyramid, and prism.
5. Cylinder, prism, and truncated cone.

In this case, flat representations of three geometric solids commonly studied in basic education are given, and the student is required to identify the solids formed from these unfoldings.

Despite its apparently simple resolution, one of the difficulties faced in this type of task comes from the fact that, in these representations, there is a duplication of some elements of the three-dimensional body. For example, one edge of the tetrahedron can be represented by two edges in the unfolding (Gonzato et al., 2011).

We also note that, since 2009, there were only seven questions in compose and decompose a solid into parts (CDS) category, where a solid is given and students are asked to compose or decompose it into two or more parts. In one of these questions, a block composed of cubes in layers is given, and the student must identify which of the options fits into the first solid to form a complete 4×4×4 cube, as described below.

Question 146

Minecraft is a virtual game that can help develop knowledge related to space and shape. It is possible to create houses, buildings, monuments and even spaceships, all in full scale, by stacking cubes. A player wants to build a cube with dimensions 4×4×4. He has already stacked some necessary cubes, as shown in **Figure 17**.

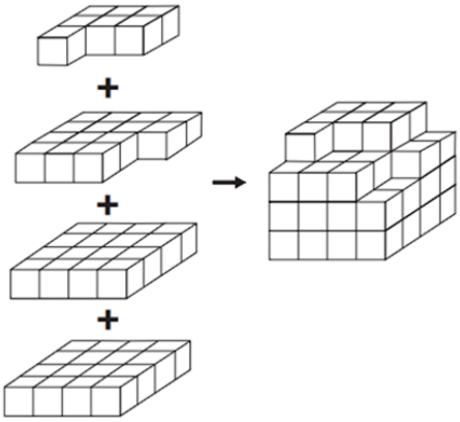

Figure 17. Division into layers of block presented in question 146 (Brasil. Ministério da Educação, 2018a)

The cubes that remain to be stacked to finish the construction of the cube, together, form a single piece, enough to complete the task. The shape of the piece that completes 4×4×4 cube is one of the shapes in **Figure 18**.

This type of task is interesting because of the way it can be solved. For example, the student can use the separation of the solid into layers to count how many cubes are missing in each one and compare with the alternatives. This process of decomposing a solid made of cubes in layers is one of the mental processes that require CDS that strongly requires visualization (Reyes et al., 2013).

ISS is one of the categories least covered in ENEM editions. In this type of activity, it is necessary to identify the figure formed when a given solid is sectioned by a plane or vice versa, as in the following example.

Question 162

It is common for visual artists to appropriate mathematical entities to produce, for example, shapes and images through manipulations. In one of his works, a visual artist wants to depict the various polygons obtained by the intersections of a plane with a regular square pyramid. According to the classification of polygons, which of them are possible for the artist to obtain?

1. Squares only.
2. Triangles and squares only.
3. Triangles, squares, and trapezoids only.
4. Triangles, squares, trapezoids, and irregular quadrilaterals only.
5. Triangles, squares, trapezoids, irregular quadrilaterals, and pentagons only.

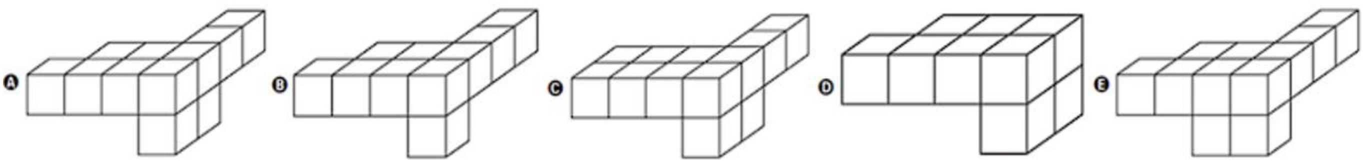

Figure 18. Options presented in question 146 (Brasil. Ministério da Educação, 2018a)

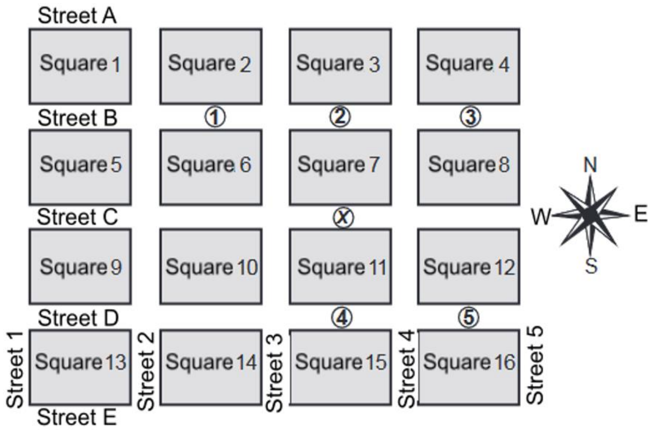

Figure 19. Map presented in question 158 (Brasil. Ministério da Educação, 2017a)

In this question, there is no visual representation of the square pyramid, and the student must imagine and identify the cuts that a plane can make in this solid. According to Blanco (2014), in this type of task SVS is involved when it is necessary to establish the relationship between the plane that makes the cut and the solid considered. And it is exactly the relationship between the inclination of the plane and the solid, which establishes the number of vertices of the resulting figure. ISS is applied to be able to recognize the figure or the elements of the pyramid that remain after making the cuts (mentally).

Finally, SO category refers to tasks, where it is necessary that the student recognize his/her position in a space and his/her possible movements. This action can be seen in the example of **Figure 19** in which we have a starting point, and it is necessary to move around the map with information such as right and left from the cardinal points (north, south, east, and west).

SO is related to one of the skills of ENEM framework: Interpret the location and movement of people/objects in three-dimensional space and their representation in two-dimensional space (Brasil, 2012b). Thus, this was the category with the third highest occurrence, covered in a total of 10 questions of which we highlight the following example.

A boy has just moved to a new neighborhood and wants to go to the bakery. He asked for help from a friend who gave him a map with numbered spots (**Figure 19**), which represent five places of interest, among which is the bakery. In addition, the friend gave the following instructions: from the point, where you are, represented by the letter X, walk west, turn right at the first street you see, go straight and turn left at the next street. The bakery will be just around the corner.

The bakery is represented by the spot with the number:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

We emphasize that, according to Lowrie et al. (2017), SO, together with spatial visualization and MR, is one of the three pillars of spatial reasoning, which is fundamental for geometric reasoning, which focuses, among other aspects, on articulating axiomatic properties.

CONCLUSIONS

Researchers such as Jirout and Newcombe (2015) state that SVS are important for students to succeed in science, technology, engineering, and mathematics. In this context, spatial skills play an essential role in performing many everyday tasks as well as in many subjects, especially mathematics and geometry. Our aim in this manuscript was to analyze, categorize and quantify ENEM questions that require SVS in their resolution, allowing us to outline a profile of them, based on the categories listed in Mathias and Simas (2021).

When quantifying the results, we realized that, although the number of questions focused on SG in ENEM varied between six and 16, those requiring spatial visualization did not exceed five in any edition, and in some exams, there were not any tasks requiring SVS. Thus, we understand that this assessment method fails to a certain degree regarding spatial thinking, a skill of widely discussed importance for cognitive development.

Regarding the visualization action categories, it is noteworthy that CIV was the one with the highest number of occurrences, followed by DS, which shows that questions related to projection systems and solid unfoldings are the most frequently asked in ENEM. There is also a considerable number of questions focused on SO, following the recommendation of the framework (Brasil, 2012b). However, the exam did not assess the other visualization actions in an expressive way, as GSR, ISS, and RS categories varied between two and four questions in total, over 14 editions of the exam and 29 applications.

Thus, we hope that this study encourages discussions in this field of research, such as evidencing this discrepancy between spatial visualization actions in ENEM questions, and that future research may propose ways to circumvent this issue.

In addition, this study is further evidence, as well as the research by Mathias and Simas (2021), that SVS are relevant in basic education, even though they are not widely developed by the guidelines for national curricula, such as BNCC, textbooks and ENEM itself. Finally, we suggest teachers to consider SVS as an explicit needed content, not just as a goal of opportunity. Seeing is not an easy task, so teachers should know how to identify and overcome students' misconceptions. Although some studies address this aspect, we consider that the field need more solid research on it.

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