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Specialized knowledge and statistical thinking: Points of convergence in the education of pre-service mathematics teachers

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Abstract

The aim of this study is to explore the stochastic knowledge demonstrated by pre-service mathematics teachers in their development of a statistical project based on the investigative cycle, from the perspective of the mathematics teacher's specialized knowledge (MTSK) model. The participants were 16 pre-service mathematics teachers selected through non-probability sampling. These participants completed a statistical project aligned with an experimental design to study the effect of light on the growth of bean seedlings. Following a qualitative methodology, content analysis was used to identify the predominant ideas that emerged across the different phases of the experiment. The findings indicate that, while participants carry out all phases of the research cycle, they exhibit limited interpretive skills, revealing weaknesses in the development of competencies associated with statistical thinking. These findings suggest limited development of the disciplinary knowledge required to effectively support the teaching and learning of statistics in school settings.

Keywords: investigative cycle, statistical thinking, specialized knowledge, pre-service mathematics teachers, MTSK-stochastic, disciplinary knowledge

INTRODUCTION

The knowledge of mathematics teachers has been the subject of research since the 1980s (Padilla-Escorcia et al., 2023) with the aim of "identifying the characteristics that distinguish them from other disciplines requiring advanced mathematical knowledge" (p. 161), such as mathematics, engineering, and statistics. Among the less explored areas is knowledge of statistics and probability, whose applications encompass virtually all fields of knowledge as well as many aspects of everyday life (Rao, 1975; Wild & Pfannkuch, 1999). Moreover, such knowledge is fundamental, as it serves as a structural framework in nearly all disciplines (Madaki, 2021).

In the context of primary and secondary education, the National Council of Teachers of Mathematics (2000) proposed the incorporation of topics related to statistics and probability into school curricula at the international level more than two decades ago. This recommendation has been reflected in the curricular frameworks of the United States and Spain, as well as in several Latin American countries, including Chile (MINEDUC, 2015), Peru (MINEDU, 2018) and Argentina (Kelmansky, 2009). Students are expected to develop, from an early age, the ability to manage, summarize, interpret, and provide arguments based on contextualized data (Engel, 2019). Of particular interest is argumentation because it closely aligns with the interpretation of results and statistical thinking (Garfield & Ben-Zvi, 2008).

To achieve these educational objectives, project-based statistics instruction is an appropriate instructional approach, as it promotes active learning through the use of authentic, contextualized data. Furthermore, this approach involves the use of statistical summary methods in relation to the nature of the data, as well as their interpretation in context (Batanero, 2021; Kirk, 2007), thereby promoting students' use of fundamental statistical concepts (Batanero & Díaz, 2011).

However, the literature provides empirical evidence that teachers tend to avoid teaching statistics, while those who do tend to primarily address it from a procedural perspective (Estrella, 2017). This orientation limits opportunities for the development of statistical thinking by focusing on essentially arithmetical tasks,

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Contribution to the literature

- This study analyzes the stochastic disciplinary knowledge demonstrated by pre-service mathematics teachers when responding to an experimental task situated within the investigative cycle.
- It provides information about the arguments used by pre-service mathematics teachers to justify changes in data representation and the use of graphical and/or numerical methods when addressing a contextualized statistical problem.
- The findings highlight the need to assess the stochastic knowledge (SK) acquired by pre-service mathematics teachers through the MTSK-stochastic framework.

resulting in conceptual deficiencies and difficulties in connecting statistical notions, representations, and concepts. This has been revealed in studies on teacher education both in Chile (Rodríguez-Alveal et al., 2019) and Spain (Muñiz-Rodríguez et al., 2020), among other contexts.

Addressing these types of difficulties highlights the need to identify the specific knowledge that mathematics teachers should have acquired, and this has led to the development of theoretical-analytical models such as the *mathematics teacher's specialized knowledge* (MTSK) framework introduced by Carrillo-Yañez et al. (2018). Given that its presence in the specialized literature dates back no more than a decade, empirical studies using this model to describe the knowledge of statistics teachers remain scarce, particularly in the Chilean context.

One of the strengths of this model lies in its specificity for analyzing teachers' knowledge across particular domains and subdomains, making it possible to examine how mathematics teachers solve statistical tasks involving definitions, knowledge, procedures, and possible connections among them. Nevertheless, most studies focus on procedural aspects and skills related to statistical literacy, without providing a deeper examination of the knowledge mobilized by teachers. For example, research on the investigative cycle proposed by Wild and Pfannkuch (1999) as a framework for statistical work has provided evidence that pre-service mathematics teachers have not developed skills associated with statistical literacy (Guven et al., 2021). Similarly, Rodríguez-Alveal and Aguerrea (2025a) note that this group exhibits conceptual errors related to the selection of graphical representations and provides implausible arguments regarding such selections, revealing weaknesses in the development of skills associated with statistical literacy and statistical thinking. In addition, it should be noted that, in general, studies have also failed to perform a deeper examination of the connections among the statistical concepts underlying the tasks involved in the different phases of the investigative cycle, which can be examined through the specific lens of the MTSK framework. This absence constitutes a gap that motivates the present study, whose purpose is to explore the statistical knowledge of prospective mathematics teachers as they carry out the different phases of a project based on the investigative cycle.

THEORETICAL FRAMEWORK

The MTSK model was originally introduced by Carrillo-Yañez et al. (2018) and has been used to examine the knowledge of teachers who teach mathematics at different educational levels (primary, secondary, and tertiary). Its versatility and applicability can be observed across different contexts within mathematics education, as documented in studies on geometry (Advíncula-Clemente et al., 2022; Climent et al., 2021), algebra (Pantoja-Agreda et al., 2025), calculus (Bustos & Ramos, 2022; Montes & Carrillo, 2017), and linear algebra (Vasco & Climent, 2018), among others.

For the purposes of this study, we focus on the specialized knowledge of prospective mathematics teachers who will teach statistics in their professional practice. In this regard, Vidal-Szabó and Estrella (2020) have proposed extending the MTSK model to the statistical domain by replacing the word mathematics with statistics in the labels of the domains and subdomains. Likewise, Seguí and Alsina (2023) developed a questionnaire to explore primary school teachers' knowledge of statistics and probability under the designation MTSK-stochastic, based on the dimensions of the original model.

However, from the authors' perspective, the adjective "stochastic" is more appropriate than "statistical," since statistics cannot be taught without reference to probability, as noted by Batanero and Borovcnik (2016). A further theoretical justification is provided by Groth (2007), who argues that statistics is not simply a branch of mathematics, but rather a discipline with its own object of study. Accordingly, the designation MTSK-stochastic provides an appropriate analytical framework for examining the knowledge of teachers who teach statistics and probability in school settings. In this case, it is appropriate to distinguish the following domains: SK, stochastic pedagogical content knowledge, and beliefs about the teaching and learning of stochastics, which permeate both domains (**Figure 1**).

In particular, the SK domain is subdivided into the knowledge of stochastic topics (KoT-stochastic) subdomain, which encompasses teachers' knowledge of

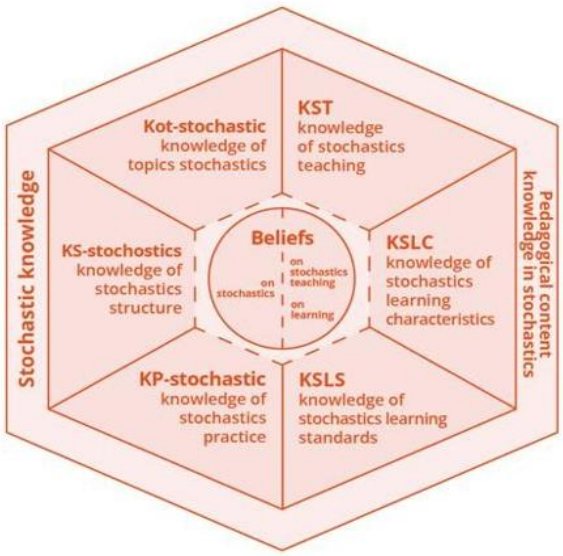


Figure 1. Adaptation of the MTSK model to the MTSK-stochastic model (adapted from Carrillo et al., 2018)

concepts, properties, and procedures associated with stochastics. This subdomain also includes the category of phenomenology, which is related to the application of stochastic phenomena in both stochastic and non-stochastic contexts. For example, it encompasses the control of variability in experimental designs within fields such as agronomy, engineering, and medicine.

The knowledge of stochastic structure (KS-stochastic) subdomain, following Montes et al. (2016), refers to teachers' understanding of how different stochastic concepts are interrelated. These examples include the connections between variability and randomness—the two fundamental concepts underlying statistics and probability—as well as the conceptual relationships among measures of center, dispersion, and shape, the distributional characteristics of data, and the links between graphical representations such as histograms and continuous probability distributions. In sum, this subdomain encompasses simplification, complexification, transversal connections, and auxiliary connections, all of which require the development of statistical thinking and statistical reasoning (Garfield & Ben-Zvi, 2008).

Finally, the third subdomain, knowledge of stochastic practice (KP-stochastic), refers to teachers' knowledge of stochastics from a syntactic perspective. Unlike the KoT-stochastic and KS-stochastic subdomains, KP-stochastic is not organized into specific categories or subcategories. However, as Espinoza (2020), argues, teachers' knowledge cannot be understood independently of practice; that is, the different categories and subdomains of the MTSK-stochastic model become evident through practice.

In the present study, teacher knowledge is examined through the different stages of the investigative cycle proposed by Wild and Pfannkuch (1999), which make it possible to identify the knowledge and connections among statistical and probabilistic content that pre-service mathematics teachers draw upon when responding to a contextualized problem. It should be noted that data and their context are fundamental to statistics education (Pfannkuch et al., 2018), as context provides meaning to the interpretation of the variability that is ubiquitous in data. Based on the above, Wild and Pfannkuch (1999) argue that everything begins with a problem situation from which the questions motivating an investigation emerge. Answering these questions involves developing an analysis plan and interpreting the data within the context of the problem situation. This process has been termed the investigative cycle, whose acronym is problem-plan-data-analysis-conclusion (PPDAC).

The PPDAC cycle therefore constitutes a procedural scaffold through which individuals articulate statistical and probabilistic concepts, knowledge, and procedures, as well as the connections that give rise to other concepts, such as the relationship between measures of central tendency and the shape of a data distribution, among others. In this regard, the data phase is associated with how data are obtained, which is related to the type of sampling and the scope of the data for decision-making (KoT-stochastic and KS-stochastic). *Data analysis* is related to the selection of methods for summarizing data graphically and numerically in accordance with the purpose of the research study (KoT-stochastic and KS-stochastic). In this regard, Chick (2003) proposes transnumeration techniques that enable movement from the real system to the statistical system in order to reveal what is hidden in the data and communicate it convincingly. This process is fundamental to the development of statistical thinking (Wild & Pfannkuch, 1999), as it enables meaning to be constructed from data (Gea et al., 2022; Lee et al., 2014). **Table 1** presents the techniques associated with graphical representations and measures of central tendency and dispersion.

The *conclusion* phase plays an important role in reading and interpreting tables, graphs, numerical measures, and inferential statistical tests, which are skills associated with statistical thinking (Wild & Pfannkuch, 1999). This aligns with the KS-stochastic and KP-stochastic subdomains. It should be noted that Garfield and Ben-Zvi (2008) refer to these phases as the statistical process, which is consistent with the concept of statistical thinking, understood as "the procedures through which a [professional] statistician works and what the statistician thinks to learn more in the realm of context" (Pfannkuch & Wild, 2004, p. 41).

METHODOLOGY

This study employed a case study design (Yin, 1994). It was conducted within an interpretive paradigm (McMillan & Schumacher, 2011) and adopted a qualitative approach, as its purpose was to explore the

Table 1. Some transnumeration techniques (adapted from Chick, 2003)

Technique	Description
Graphing/tabulating	Variables can be represented using graphs or frequency distribution tables, depending on the nature of the data.
Calculation of measures of central tendency	A numerical measure of central tendency is used to summarize the behavior of each variable. Examples include the arithmetic mean, median, and mode.
Calculation of measures of dispersion	A measure of dispersion is selected to describe the spread of values associated with a numerical variable. Examples include the range, standard deviation, interquartile range, and coefficient of variation, which are statistics used to describe the variability of a variable.

Table 2. Example table

Variables	In absence of natural light				In presence of natural light			
	Day 1	Day 2	…	Day 14	Day 1	Day 2	…	Day 14
Height								
Diameter								
Number of leaves								

stochastic disciplinary knowledge demonstrated by pre-service mathematics teachers while completing a statistical project based on the investigative cycle proposed by Wild and Pfannkuch (1999).

Context and Participants

The participants were 16 prospective mathematics teachers enrolled at a university in south-central Chile. Their mean age was 25 years, and 68.0% were male. They were selected through non-probability convenience sampling (McMillan & Schumacher, 2011), based on access conditions at the university and the participants' willingness to take part in the study. All participants were in the eighth semester of their ten-semester degree program and had successfully completed two compulsory statistics courses. The first covered topics related to descriptive statistics, basic probability concepts, and linear regression. In the second course, they studied random variables, discrete and continuous probability distributions, parametric statistical inference, and goodness-of-fit tests. In addition, they had completed an elective course on statistical thinking designed to explore and discuss the appropriateness of data analysis methods for contextualized situations. To ensure participant anonymity, each individual was assigned an alphanumeric code in the format PMTn, where PMT stands for pre-service mathematics teacher, and n identifies the participant in the dataset. For example, PMT5 refers to the fifth participant in the dataset. Those who signed an informed consent.

Data Collection and Analysis Procedure

Data were collected during the elective course on statistical thinking, which took place between August and December 2025 and formed part of the participants' degree program. These non-experimental conditions place the exploration within the study of MTSK in professional development (Montes et al., 2021). Specifically, the 16 prospective mathematics teachers were required to conduct an experiment based on the phases of the investigative cycle proposed by Wild and Pfannkuch (1999). The purpose was to answer the following research question: How do sunlight and shade affect the morphological characteristics of bean seedlings?

To address this question, the participants were asked to place bean seeds on cotton wool, used as a growing medium, inside 12 transparent containers made of glass or plastic. The aim was to observe germination, root development, stem growth, and leaf production. Based on this experimental design, the participants were required to complete the following tasks:

a. Record the daily growth of the bean seedlings after germination, paying particular attention to morphological characteristics such as height, number of leaves, and stem diameter, in the presence and absence of natural light, using a table such as the one shown in **Table 2**.
b. Identify the statistical methods to appropriately address the research question, considering both the nature of the variables and how the data are recorded.
c. Discuss and justify the appropriateness of the statistical methods proposed in task (b).
d. Answer the research question and formulate a statistical conclusion within the context of the experimental problem.

Subsequently, the participants' responses were analyzed following the procedures proposed by Mayring (2000). First, the responses from the pre-service mathematics teachers were transcribed verbatim into an Excel matrix, organizing the information into four columns corresponding to the assigned tasks (a), (b), (c), and (d). Second, the content of the responses for each task was reviewed to identify episodes that clearly provided evidence of the participants' SK; otherwise, the segments were discussed among the authors in order to reach a consensus. In the case of task (a), aspects regarding the execution of the experiment, the

procedure for measuring the morphological characteristics of the seedlings, and the recording of data in the provided worksheet were identified. In task (b) and task (c), indications were sought regarding the proposals and/or rationales provided by participants on the use of statistical methods: either descriptive (measures of central tendency, shape, and variability) or inferential (hypothesis testing, analysis of variance [ANOVA], etc.). Similarly, in task (d), the arguments provided by participants to statistically answer the question of what lies behind the experiment were analyzed. Finally, in third place, the responses to task (b) and task (c) were analyzed for indications of the domains knowledge of topics (KoT-stochastic) and KS-stochastic. For example, if a participant refers to the mean, average, arithmetic mean and the median as measures of central tendency in their response to task (b), that segment is assigned to the concepts category of the KoT-stochastic subdomain. Conversely, if a prospective mathematics teacher merely mentions that using the median is preferable without providing any justification, that paragraph is not assigned to the properties category within the KoT-stochastic subdomain, given that participants were required to justify their assertions—a condition not observed in this case. On the other hand, when a participant indicates that the arithmetic mean synthesizes the behavior of the seedlings' morphological characteristics, but notes that the median should be preferred in the presence of outliers, this segment is placed in the properties category of the KoT-stochastic subdomain. Another example is when the response to task (c) explains how to calculate both measures (mean and median), which is classified under the procedures category. If they provide the calculated values of the arithmetic mean and median—demonstrating the ability to transition from a real-world system (data obtained) to a statistical system—it is categorized under change in representation. Additionally, the connections participants made to stochastic content taught during their teacher training program were identified through the KS-stochastic category. For instance, when participants refer to data variability and nature using semiotic representations such as histograms or box plots, among others, alongside measures of central tendency, variability, and shape, they connect prior knowledge—such as the positioning of central tendency measures with the shape of the data distribution, or sample statistics with parameters in hypothesis testing to make decisions, among other connections.

Finally, task (d) delves into KP-stochastic. In this case, the language used by participants and the statistical notation employed to summarize measures of central tendency, shape, and variability were observed. Thus, the completion of each task generated a set of data inputs whose records constituted the corpus of analysis framed by the subdomains of the MTSK-stochastic model.



PMT6 PMT12 PMT3

Figure 2. Images of the experiment conducted by the pre-service mathematics teachers (the authors' own elaboration)

ANALYSIS AND RESULTS

This section presents an analysis of the participants' responses. First, the 16 prospective mathematics teachers operationalized the research problem by conducting the experiment (problem phase). Specifically, they placed a bean seed in cotton, used as a growing substrate, inside each clear cup. **Figure 2** shows images of this process.

Then, during the plan phase, the prospective mathematics teachers identified the variables and measurement instruments to be used for data collection. Regarding the variables, most participants (14/16) considered the measurement of seedling height, stem diameter, and number of leaves. Only two participants also considered daily temperature as a variable of interest. With respect to the selection of measurement instruments, 15 of the 16 prospective mathematics teachers reported using a ruler without necessarily questioning the suitability of the instrument. One participant, however, commented:

> The measurements in this experiment were rather 'rough,' as I did not have a suitable instrument for measuring the seedlings. I used a ruler. [PMT9]

Only one participant reported using a Vernier caliper, a high-precision instrument more suitable for measuring the morphological characteristics of seedlings, such as stem diameter and height, in centimeters or millimeters.

In the next phase (data), all participants (16/16) recorded data for the variables of interest from day 1 through day 14, distinguishing between the two treatments (sunlight and shade). In general, they used the template table provided, as shown in **Table 3**.

The data show that the variables of interest exhibit variability between the presence and absence of sunlight. This variability is the statistical phenomenon that participants must analyze to answer the research question, thereby creating a need to consider statistical methods for comparing the behavior of the variables under each of the treatments being studied. This task is related to the process of transnumeration, which is associated with statistical thinking and the KoT-stochastic and KS-stochastic subdomains (analysis phase). In this sense, 8 of the 16 pre-service mathematics

Table 3. Example of data documenting the morphological characteristics of bean seedlings measured by the participants

Variables		Week 1						Week 2							
		D 1	D 2	D 3	D 4	D 5	D 6	D 7	D 8	D 9	D 10	D 11	D 12	D 13	D 14
PMT3															
Sunlight	Height (cm)	0	0	0.4	1.3	2.7	4.2	5.1	6.4	7.9	9.1	10.6	12.2	13.1	14.6
	N	0	0	0	0	0	1	1	2	2	2	3	3	3	4
	Diameter (mm)	0	0	1	1.3	1.5	2.0	2.2	2.5	2.7	3.0	3.2	3.5	3.8	4.0
Shade	Height (cm)	0	0	0.7	2.3	4.8	7.2	9.8	11.9	13.7	15.3	16.4	17.2	17.5	17.5
	N	0	0	0	0	0	0	0	0	1	1	1	1	1	1
	Diameter (mm)	0	0	1	1	1	1.2	1.2	1.4	1.4	1.6	1.6	1.6	1.6	1.6

Note. N: Number of leaves; D: Day

teachers mentioned measures of central tendency (arithmetic mean and median), measures of variability (standard deviation), and graphical representations (simple and grouped bar charts, histograms, line graphs, boxplots and scatterplot). In general, the participants provided arguments demonstrating conceptual knowledge, as illustrated by the following excerpts:

> To analyze seedling growth […] we propose calculating the arithmetic mean […] in order to determine the average growth of the plants under sunlight and shade. The standard deviation provides information about the variability of the measurements within each treatment; that is, whether the plants grow uniformly or whether they show substantial differences. [PMT2]

> The descriptive methods are the mean and standard deviation. These statistics allow us to assess the homogeneity of each distribution and determine how representative the data are. It would also be interesting to compare them in a graph. [PMT4]

> Calculate the mean, median, range, variance, and other descriptive statistics to analyze data dispersion. Construct a bar chart to represent the data graphically. [PMT5]

> We could use graphs, such as a scatterplot. This could be done by plotting height as a function of stem diameter. [PMT14]

On the other hand, five of the 16 participants mentioned descriptive and inferential methods, including mixed models for repeated measures, ANOVA, student's t-test, and the Mann-Whitney test, among others. Of these, only one pre-service mathematics teacher noted that the seedling measurements were not independent, commenting on the necessity of the following:

> Repeated-measures analysis to compare growth curves (measurements on day 7 and day 14), because measurements taken from the same container or individual are correlated. Mixed models allow all observations to be used and make it possible to model the time-by-treatment effect, as well as to include the block or replicate as a random effect if necessary. [PMT9]

The comment reflects that PMT9 makes a connection of increasing complexity (KS-stochastic), considering that they mention a model that allows for the study of correlated observations. This demonstrates knowledge that goes beyond the learning outcomes typically expected in an undergraduate statistics course. Furthermore, 4 out of the 16 participants mention using ANOVA and/or the two-sample student's t-test, without placing emphasis on the dependence of observations within the experimental unit. As observed in the following quotes:

> ANOVA to compare two groups (sunlight and shade), considering different variables such as stem, leaf, and root. [PMT7]

> The Welch t-test is appropriate in this case because the main objective is to compare two groups (sunlight and shade), and this method works well when sample sizes are small and variances are unequal. [PMT13]

> A hypothesis test using student's t-test, since we first want to test the hypothesis of whether light affects germination, and student's t-test allows us to compare two groups (specifically, their means). [PMT16]

These responses suggest that the pre-service mathematics teachers demonstrate KS-stochastic. Specifically, the comments made by PMT7 and PMT16 reveal simplification connections, as they relate concepts concerning samples, populations, statistics, and parameters (KoT-stochastic), as well as connections associated with statistical inference (KS-stochastic). Nevertheless, the study participants did not articulate the statistical connections necessary to justify the appropriateness of the proposed methods in the context of the experimental design or to explain the effects of the treatments (Peña, 2010). These omissions suggest a limited understanding of complexification connections and transversal connections between parametric and nonparametric statistical methods in the context of

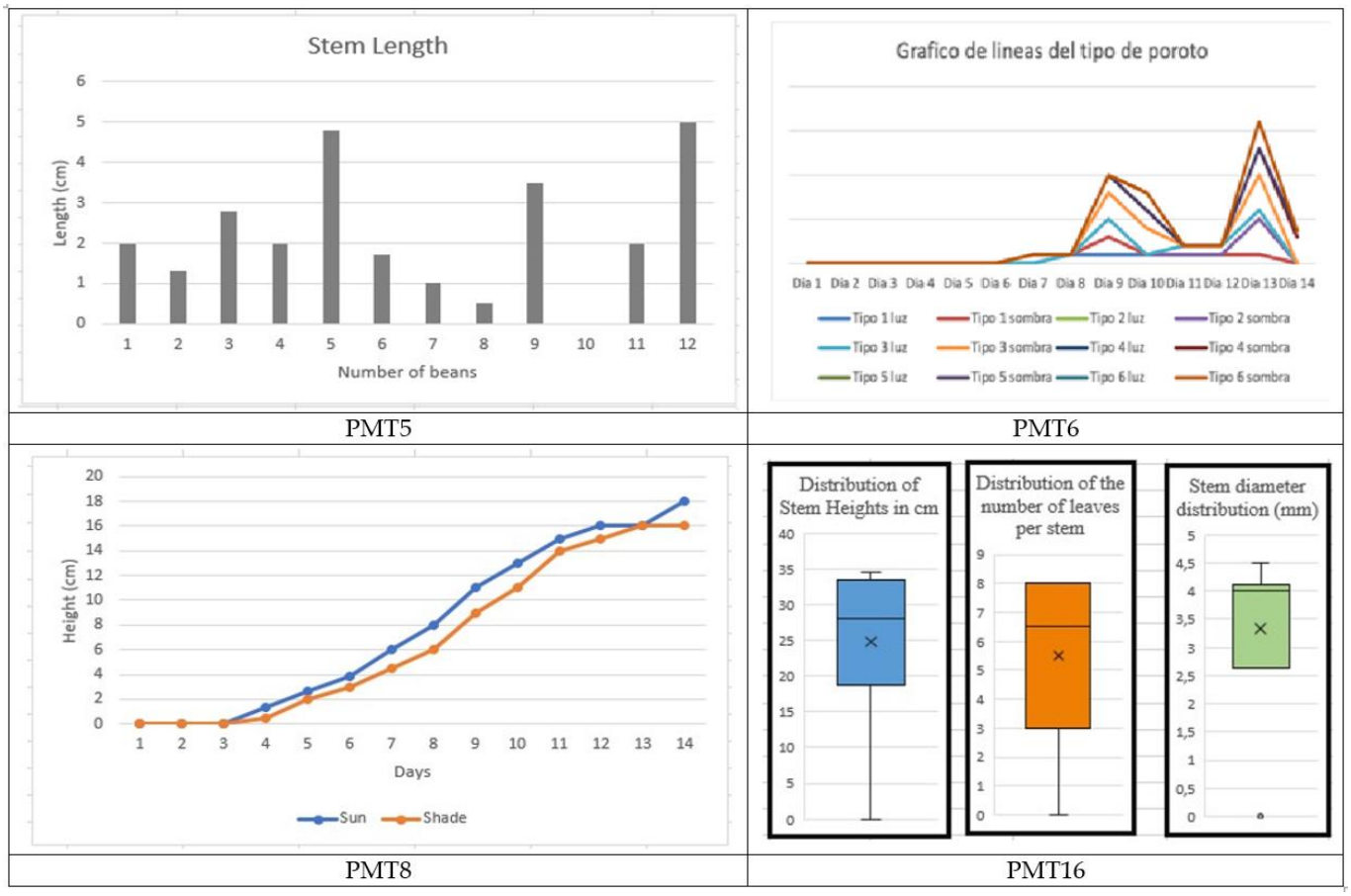


Figure 3. Examples of graphical representations produced by the participants (the authors' own elaboration)

statistical inference. Given the participants' academic preparation, they were expected to refer explicitly to the assumption of normality underlying the data distribution, a concept associated with the KoT-stochastic subdomain. Such assumptions directly affect the robustness of hypothesis tests and, consequently, the validity of statistical decision-making.

As mentioned in relation to the graphical and numerical statistical methods discussed above, it should be noted that only four participants (4/16) provided graphical summaries of the behavior of the variables (seedling height, stem diameter, and number of leaves), offering evidence of skills associated with the process of transnumeration (KoT-stochastic). As illustrated in **Figure 3**, PMT5 represented stem length over time using a bar chart, whereas PMT6 and PMT8 represented seedling height over time using line graphs. Only PMT16 used a boxplot to visualize the distributional behavior of the study variables.

Another representation used is the bar chart to represent the behavior of plant height. This graph is appropriate for the nature of the variable. However, despite this, authors such as Rodríguez-Alveal and Aguerrea (2025a), Muñiz-Rodríguez et al. (2020), and delMas et al. (2005) have questioned linking bar charts with discrete quantitative data. Likewise, it is striking that participants do not provide arguments justifying their choice of graphical representations. In contrast, PMT16 used boxplots to display the distributions of seedling height, stem diameter, and number of leaves as an appropriate graphical choice for illustrating the distributions of the experimental variables. In general, participants make use of appropriate graphical representations to show the distributional behavior of the variables of interest in the experiment (KoT-change of representation). However, errors were observed in the construction of both the line graph (PMT6) and the boxplots (PMT16), as neither participant labeled the axes nor identified the variables represented. These shortcomings have already been documented by Arteaga et al. (2016) and Rodríguez-Alveal and Izagirre (2025b) in similar populations.

In addition to graphical representations, the pre-service mathematics teachers used numerical measures of central tendency and variability to summarize the data (**Table 4**). Only PMT1 summarized these measures in a summary table, facilitating comparisons of seedling height and stem diameter under both treatments. On the other hand, PMT16 simultaneously employed measures of central tendency and variability to examine the behavior of the variables, thereby providing evidence of KS-stochastic.

On the other hand, the nature of the tasks involved in the investigative cycle made it possible to examine the effects of the treatments beyond mere description. However, only 2 out of the 16 preservice teachers

Table 4. Examples of numerical summaries produced by the participants

PMT1	Arithmetic average		
	Height		Diameter
Sunlight	0.46		0.08
Shade	1.44		0.23
PMT16	Height	N	Diameter
M	24.7	5.5	3.3
SD	12.6	3.1	1.7
Rank	34.5	8.0	4.5

Note. M: Arithmetic mean; SD: Standard deviation; N: Number of leaves

performed a hypothesis test regarding the behavior of the seedlings' morphological characteristics under light and shade environments. In this regard, PMT16 uses the parametric student's t-test:

H_0. μ trait *i*, sunlight = μ trait *i*, shade

H_1. μ trait *i*, sunlight ≠ μ trait *i*, shade

In this formulation, *i* refers to the morphological characteristics of the seedlings considered in the study (height, diameter, and number of leaves). The participant explained the following:

> Here, trait *i* refers to the variables under study. Student's t-test is used to obtain the p-values for stem height, number of leaves, and stem diameter, respectively: 0.269, 0.929, and 0.934. Since p > 0.05 in all cases, it can be concluded that there is insufficient statistical evidence to assert that sunlight or shade produces a significant difference in the characteristics studied, at either the 95% or 99% confidence interval (CI). [PMT16]

The arguments use concepts such as the p-value and confidence (KoT-stochastic), but do not explicitly explain their relationship to hypothesis testing (KS-stochastic). Likewise, the statistical assumptions underlying student's t-test, such as normality, independence, and homoscedasticity, are omitted. Explicitly addressing these assumptions would demonstrate an understanding of the connections among statistics, parameters, and sampling distributions involved in decision-making, which entail both simplification and complexification connections (KS-stochastic).

PMT12, in turn, proposed the nonparametric Mann-Whitney U test without establishing connections between data normality and its relationship to the parametric student's t-test (Triola, 2018), without using any specialized software. To illustrate this, **Figure 4** shows the procedure (KoT-stochastic) followed by the prospective mathematics teacher:

It should be noted that PMT12 confuses the concept of significance level with CI, where the former is used in hypothesis testing and the latter in the construction and interpretation of CIs. Furthermore, failing to correctly formulate the statistical hypotheses—particularly the alternative hypothesis—evidences a conceptual inaccuracy (KoT-stochastic).

Using the Mann-Whitney U test:

- Sample size: n_1 (sunlight group) = 6, n_2 (shade group) = 6
- Significance level: alpha = 0.05 (the standard CI in the sciences).
- Type of test: two-sided (or two-tailed), because the alternative hypothesis (H_a) states that the groups are *different* ($\mu_{Sunlight} \neq \mu_{Shade}$), without specifying in advance which group has the higher values.

First, all height measurements (in mm) are pooled, and a rank is assigned to each value, from the lowest (rank 1) to the highest (rank 12).

Environment	Bean	Final height (mm)	Rank
Sunlight	4	1	1
Sunlight	5	3	2
Shade	7	4	3
Shade	12	6	4
Shade	8	7	5.5
Shade	10	7	5.5
Shade	11	10	7
Sunlight	6	15	8
Sunlight	2	19	9
Shade	9	20	10
Sunlight	1	40	11
Sunlight	3	50	12

Next, we separate the ranks by group and calculate the sum of ranks (W) and the U statistic for each group:

Height	Rank
1	1
3	2
15	8
19	9
40	11
50	12
Sum of ranks (W1)	43

The Mann-Whitney U test statistic is the lesser value of U_1 and U_2. Therefore, U = 14. According to the Mann-Whitney rule, the null hypothesis (H_0) is rejected if $U_{Observed}$ is less than or equal to $U_{Critical}$.

Since 14 > 5

- The null hypothesis (H_0)is not rejected.
- There is insufficient statistical evidence at the alpha = 0.05 significance level to conclude that the height growth of the bean plants under sunlight is significantly greater than (or different from) their growth under shade.

Although the mean height under sunlight (21.33 mm) is greater than that under shade (9.00 mm), the lack of uniformity in initial growth and the small sample size mean that, according to this nonparametric test, the differences observed cannot be ruled out as resulting from random variability among the bean plants.

Figure 4. Examples of the nonparametric Mann-Whitney U test procedure (elaborated by PMT12)

Finally, 14 of the 16 participants made claims about the behavior of the seedlings in the presence and absence of sunlight in an effort to answer the research question during the final phase of the investigative cycle (*conclusion*). Of these, four participants noted that seedlings grown in the absence of sunlight achieved a greater average height, as illustrated by the following excerpts:

> The light and dark environments influenced seed germination. Those without sunlight grew faster and more successfully than those exposed directly to light. [PMT7]

> Darkness causes bean plants to grow taller more quickly, but with less stem thickness and fewer leaves. Sunlight results in slower but more balanced plant growth, with a greater number of leaves and a thicker stem. [PMT3]

As these excerpts show, the prospective teachers draw conclusions without referring to measures of central tendency or variability that would enable them to support their comments using the empirical results obtained from the experiment. In contrast, nine of the 16 pre-service mathematics teachers did base their conclusions on numerical measures, such as the mean, standard deviation, and maximum heights reached by the seedlings. This is illustrated in the following excerpts:

> The results show that the shaded environment had a significant effect on seedling height growth. The mean growth rate of plants exposed to sunlight was 0.46 mm per day, whereas those placed in shaded conditions grew 1.44 mm per day […]. Regarding the standard deviations, the values were relatively low (6.02 for height and 0.77 for stem diameter) […]. Seedlings not exposed to sunlight generally showed lower standard deviations. [PMT1]

> According to the data, […] in darkness, the seedlings grew taller, in some cases exceeding 30 cm, but developed thin stems and fewer leaves. This pattern is associated with the phenomenon of etiolation, whereby a plant elongates rapidly in search of light and allocates its resources to stem growth rather than leaf development. In contrast, plants grown under light conditions reached shorter heights (between 7 and 10 cm) but developed more leaves (between 2 and 4) and stronger stems. A comparison of the means shows that seedling height was greater in darkness, whereas the number of leaves and stem thickness were greater under light conditions. [PMT6]

One noteworthy aspect of these arguments is that PMT1 does not use the concept of variability to contextualize the values associated with the standard deviation. PMT6, in turn, explains the results concerning seedling behavior in the presence and absence of sunlight by referring to concepts such as etiolation and leaf development. This provides evidence of phenomenological knowledge (KoT-stochastic), as the participant connects the statistical findings with disciplinary knowledge related to agronomy. Even though PMT12 and PMT16 apply inferential statistical methods (student's t-test and the nonparametric Mann-Whitney U test), only PMT16 addresses the experiment from an inferential perspective, commenting that:

> Based on the information collected above and despite the preliminary analyses of the characteristics of the groups, it is concluded that there is insufficient statistical evidence to assert that sunlight or shade produces a significant difference in the characteristics studied, at either the 95% or 99% confidence level. The initial differences may be due to a number of factors beyond the study's control […] or even to the small sample size. [PMT16]

In this regard, it is striking that PMT16 does not indicate whether the differences pertain to the average or to the variability of the seedlings' morphological characteristics. Furthermore, they confuse concepts such as the level of significance with the level of confidence, revealing a lack of conceptual knowledge (KoT-stochastic) and connections of simplification (KS-stochastic).

CONCLUSION

This study was based on an experiment adapted from the GAISE (2016) report to assess the statistical and probabilistic (stochastic) knowledge of pre-service mathematics teachers. Specifically, data were collected on the development of a statistical project following the phases of the investigative cycle proposed by Wild and Pfannkuch (1999). The incorporation of this cycle into teacher education has been recommended by Guven et al. (2021), among others, as an effective approach to developing skills associated with statistical thinking.

Overall, the findings provide evidence that the participants had knowledge related to data collection and proposed analyses focused primarily on descriptive methods, referring to measures of central tendency and variability, as well as graphical representations (KoT-stochastic). By contrast, references to inferential tests, such as student's t-test, ANOVA, and the nonparametric Mann-Whitney U test, were scarcely addressed by the prospective mathematics teachers who participated in the study, revealing a conceptual gap regarding statistical inference.

Another finding to note is that, when justifying the suitability of their chosen methods, most participants provided arguments demonstrating conceptual knowledge of descriptive statistics (KoT-stochastic). This was not observed in their discussions of inferential methods. One possible explanation is their familiarity with descriptive statistics topics, which have been included in the Chilean curriculum since primary school.

With regard to the final phase of the investigative cycle (conclusion), there are indications that the prospective teachers demonstrated limitations in

interpreting numerical summaries in the context of a problem situation, whose abilities would be reflecting limited skills associated with the KP-stochastic subdomain. This is evidenced by the fact that their interpretations generally consisted of literal readings of the information presented in tables and graphs, restricting in this case the scope of their conclusions within the context of the investigative task. One plausible explanation is the arithmetic-oriented teaching of statistics at both school and university levels, as noted by Estrella (2017). Furthermore, the participants followed a similar pattern when working with measures of central tendency and variability. This finding is consistent with evidence that "prospective mathematics teachers tend to be cognitively anchored in algorithmic calculation processes, which prevents them from going beyond numerical calculation or summarizing information in context to decode the statistical meaning of the data represented" (Rodríguez-Alveal et al., 2016, p. 948).

These findings evidence proved that the pre-service mathematics teachers who completed the experiment have shown procedural knowledge within the KoT-stochastic subdomain, enabling them to communicate statistical ideas at a basic level. This result is consistent with the findings reported by Batanero (2000). However, they are also the experiment was an instance to gather information about the limited conceptual knowledge of statistical concepts and changes in representation (KoT-stochastic), particularly when defining variables and understanding their role within the context of the research problem. Furthermore, they appeared to require greater familiarity with the fundamental principles underlying the design and construction of statistical graphs (Gal, 2002), as well as with the connections between descriptive and inferential methods (KS-stochastic). Putting this knowledge into practice would demonstrate a better understanding of statistical and probabilistic concepts, which are essential to the development of statistical thinking.

In summary, this study provides empirical evidence regarding the stochastic disciplinary knowledge of prospective mathematics teachers who will teach statistics in the school system, using the MTSK-stochastic model as an analytical framework. Specifically, it reports findings on the connections between descriptive and inferential methods as participants addressed a problem situation involving the investigative cycle proposed by Wild and Pfannkuch (1999).

Limitations

The following considerations call for a cautious interpretation of the results. First, the sampling method used does not allow for broad generalizations to be drawn from the findings. Likewise, the small number of participants limits the extent to which the results can be extrapolated to similar populations. A second limitation is that the qualitative findings are based primarily on the written responses provided by the pre-service mathematics teachers, whose arguments were often brief. The lack of more detailed explanations makes it difficult to explore their explicit and implicit ideas in greater depth.

Recommendations

Based on the findings of this study, several recommendations can be made, as outlined below. Given the participants' difficulties in moving from the real system (data) to the statistical system, teacher educators should design learning activities that require analysis and argumentation, as well as the justification and explanation of changes in representation, thereby strengthening the KoT-Stochastic subdomain. Similarly, teacher education programs should incorporate learning experiences that explicitly emphasize the connections between data and the process of transnumeration, in accordance with the KS-stochastic subdomain. Along the same lines, greater emphasis should be placed on formulating comparative and inferential questions that can be answered using data, thereby promoting higher levels of statistical reasoning and statistical thinking.

From a research perspective, future studies should explore statistical argumentation in greater depth to help pre-service mathematics teachers move beyond simply describing data and interpret statistical evidence within the context of the investigative problem. Particular attention should also be paid to the development of specialized statistical vocabulary to strengthen the KS-stochastic and KP-stochastic subdomains of the MTSK-stochastic model. In this sense, conducting interviews could provide an alternative means of exploring disciplinary knowledge and statistical reasoning in greater depth as prospective teachers solve statistics and probability tasks, using the phases of the investigative cycle as the analytical framework.

Finally, given that the findings of this study may inform decisions regarding teacher training, it is recommended that degree programs for pre-service mathematics teachers incorporate additional courses focused on statistics and probability (stochastics). These courses should aim to strengthen disciplinary knowledge of statistical methods and techniques for addressing authentic, context-based problems, with particular emphasis on developing statistical thinking.

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