

Statistical knowledge of future secondary mathematics teachers in reading and arguing graphical information

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Abstract

This article describes the levels of reading and the arguments provided by future secondary mathematics teachers from two Chilean universities when decoding the information presented in a histogram and a box plot. A qualitative methodology was followed, based on content analysis. As a data collection technique, an instrument with two activities was applied. The 36 participants were selected through non-probabilistic sampling. Among the main results, it was found that participants perform basic readings of graphical representations. The arguments provided are conceptual in nature and are not related to the activity's context. Additionally, future teachers do not refute the conclusion formulated based on the box plot, which indicates that they do not connect the representation to the context; those who do fail to provide sufficient justification. The study concludes that there is a need to strengthen argumentation skills to foster statistical literacy and statistical thinking.

Keywords: argumentation, statistical thinking, statistical literacy, pre-service teachers, histogram, box plot

INTRODUCTION

The ability to read graphical representations is one of the key skills associated with statistical literacy. In this context, several researchers have focused their studies on understanding the skills and knowledge acquired by students at different educational levels to extract information from statistical graphs, as well as to make predictions or identify trends, such as Curcio (1989), Díaz-Levicoy et al. (2021), Izagirre et al. (2023), and Monteiro and Ainley (2006).

In the school curriculum of several countries, including Chile (Ministry of Education [MINEDUC], 2012, 2015), graphical representations are introduced in primary education, covering those associated with both qualitative and quantitative data. For qualitative variables, bar graphs (simple, grouped, or subdivided) are commonly studied, whereas for quantitative variables, the primary focus is on histograms and box plots. These types of graphs allow for the identification of data distribution patterns, as well as measures of

central tendency and variability, providing essential information depending on the context. Additionally, box plots help visualize unusually distant data points, known as outliers (Triola, 2006). This type of graph, introduced by Tukey (1977), has sparked debate both due to its presence in primary and secondary education textbooks in Chile (Rodríguez-Alveal et al., 2021) and its interpretation (Armah, 2025; Bakker et al., 2005; GAISE, 2016; Gea et al., 2017; González, 2021).

Studies on statistical graphics have provided evidence that pre-service teachers have primarily developed procedural skills when working with graphical representations (Bernal-Valdés et al., 2025; Martins & Carvalho, 2018). In this context, rather than focusing on construction activities or calculating measures of central tendency, variability, and distribution shape, the emphasis is on studying skills related to reading and argumentation about graphical representations associated with quantitative data, which are closely linked to statistical thinking and statistical literacy (Garfield & Ben-Zvi, 2008).

Contribution to the literature

- This article analyzes the levels of reading demonstrated by pre-service mathematics teachers in activities related to histograms and box plots.
- This study provides insights into the argumentation process undertaken by pre-service mathematics teachers when interpreting information presented in a histogram and a box plot, a skill emphasized in both international and national guidelines.
- This study serves as a foundation for future research on graphical representations associated with continuous quantitative data, with the aim of influencing the training of future mathematics teachers for the school system.

Table 1. Learning objectives related to the argumentation of graphical representations in the Chilean primary and secondary education curriculum (MINEDUC, 2015, 2019)

Level	Grade (age)	Learning objectives
Primary education	7 th (13)	Represent data obtained from a sample using absolute and relative frequency tables, employing appropriate graphs either manually or with educational software.
	8 th (14)	Justify the choice of a graph for a given situation and its corresponding dataset.
Secondary education	3 rd and 4 th (17-18)	Make informed decisions based on statistical evidence and/or the evaluation of results obtained from a probabilistic model.
		Argue using symbolic language and various representations to justify the truth or falsehood of a conjecture and assess the scope and limitations of the arguments used.

Based on this premise, the present study aimed to analyze the reading levels and argumentation of pre-service mathematics teachers regarding the information presented in histograms and box plots. To achieve this general objective, the following specific objectives were established:

1. Categorize the reading levels of pre-service mathematics teachers concerning the information presented in a histogram and/or box plot.
2. Describe the types of argumentation used by pre-service teachers when interpreting information in a histogram and/or box plot.

THEORETICAL FRAMEWORK**Histogram and Box Plot in the School Curriculum**

Depending on the nature of the data—qualitative or quantitative—various graphical representations can be used to summarize their distribution (Devore, 2016). In the case of quantitative data, the histogram is defined as “a bar graph where the horizontal scale represents classes of data values, and the vertical scale represents frequencies. The heights of the bars correspond to frequency values, and the bars are drawn adjacent to each other” (Triola, 2006, p. 51). This type of graph allows for the visualization of key data characteristics such as skewness, central tendency, and variability. The box plot (Tukey, 1977) provides similar insights while also identifying extreme data points that deviate significantly from the main body of observations. These outliers are defined as data points that fall below $Q_1 - 1.5(Q_3 - Q_1)$ or above $Q_3 + 1.5(Q_3 - Q_1)$. However, Chilean school textbooks use the expressions $x_{\min} - 1.5(Q_3 - Q_1)$ and $x_{\max} + 1.5(Q_3 - Q_1)$, which, in some

cases, do not indicate the presence of outliers (Dodge, 2008).

In summary, statistical graphs not only represent data distribution—fundamental components of statistical literacy and statistical thinking (Wild & Pfannkuch, 1999)—but also illustrate key statistical concepts (Boels et al., 2019), such as variability, a fundamental notion in statistics (Watson & Callingham, 2003), as well as skewness and measures of central tendency. These concepts help explain the phenomenon under study within a given contextual situation.

In the Chilean school curriculum, particularly for 7th and 8th grade students (age 13 and age 14), skills related to graphical representations, decision-making in non-deterministic situations, and statistical data analysis are promoted (MINEDUC, 2015). **Table 1** presents the learning objectives related to graphical representations.

In the Chilean secondary education curriculum, there are no learning objectives related to graphical representations in 1st year and 2nd year courses. However, in 3rd year and 4th year courses, references are made to probabilistic models and graphical representations without specifying their types.

On the other hand, the Chilean standards for secondary mathematics teacher training explicitly state that teachers should: “Explore and describe the behavior of univariate and bivariate data using statistical methods, graphical and tabular representations, and technological tools to develop exploratory data analysis skills” (MINEDUC, 2021, p. 8). This is directly related to statistical literacy, as reading and interpreting graphical representations are essential skills within this construct (Garfield, 2002).

Reading Graphs and Its Relationship With Statistical Literacy and Statistical Thinking

One of the key skills associated with statistical literacy and statistical thinking is the ability to read and interpret tables and graphs, which are in alignment with the GAISE (2005, 2016) guidelines. In this context, the present study adopts Aoyama's (2007) adaptation of the structure of observed learning outcomes taxonomy by Biggs and Collis (1982, 1991). This adaptation allows for a more detailed classification of the skills associated with the reading of graphical representations while integrating statistics with context, which enables data to be made meaningful, an essential component of statistical thinking (Wild & Pfannkuch, 1999). This framework consists of five progressive levels, ranging from the most basic to the most complex, as described below:

1. *Idiosyncratic*: At this level, values are not read, nor are trends identified in graphical representations. Incorrect values are provided, or questions related to the graphs remain unanswered.
2. *Basic graph reading*: At this level, values and trends in graphs are identified, but their contextual meanings or observed characteristics are not explained.
3. *Rational/literal*: At this level, specific values and trends are read, and their contextual meanings are explained literally, based on the characteristics shown in the graph.
4. *Critical*: At this level, graph values are read and contextual variables are understood. Additionally, students can evaluate the reliability of the contextual meaning described in the graph and question the presented information.
5. *Hypothesis and model development*: At this level, students can read and interpret the represented information, accept and evaluate data, and formulate their own hypotheses or models to explain the findings. This level implies a more sophisticated understanding and the ability to propose alternative explanations for the characteristics of the data within the context of the problem situation.

Statistical Argumentation

Argumentation is conceived as a discursive process that enables individuals to discover new ideas and critique others' reasoning in order to persuade them of the validity of a conclusion (Rumsey & Langrall, 2016). According to Larios et al. (2018), argumentation can be used to validate the knowledge an individual constructs or acquires—in this case, knowledge related to statistical representations during their formative process. In this context, interpretation and argumentation are key skills associated with statistical thinking, as noted by (Wild &

Pfannkuch, 1999), in which language plays an essential role recognizing that statistics, like any discipline, has a specialized language that allows for the communication of ideas and knowledge. Like any discipline, statistics has a specialized language that facilitates the communication of ideas and knowledge. Furthermore, Pfannkuch and Ben-Zvi (2011) highlight the importance of formative feedback in developing argumentation skills, emphasizing its role in the learning process.

In Chile, MINEDUC (2016) emphasizes that "communicative and argumentative skills are central to expressing ideas clearly and are essential for understanding the reasoning behind each problem, result, or concept" (p. 9). However, as noted by Goizueta et al. (2023), the Chilean curricular guidelines do not provide an explicit definition of the ability to communicate and argue. Nevertheless, the curricular framework for students from 7th grade in primary education to 2nd grade in secondary education (MINEDUC, 2015) states that:

The ability to argue is primarily developed when trying to convince others of the validity of obtained results. It is important for students [...] to establish the difference between intuitive argumentation and mathematical argumentation and to be able to interpret and understand chains of logical implications. This enables them to make effective predictions in various situations and formulate conjectures, hypotheses, examples, and conditional statements. They are expected to develop their ability to verbalize their intuitions and reach correct conclusions, as well as learn to identify erroneous statements, absurdities, or overgeneralizations (p. 98).

Similarly, the curricular framework for 3rd and 4th years of secondary education explicitly states that:

The ability to communicate and argue is primarily exercised when students have the opportunity to express themselves orally and in writing in various ways on mathematical matters. These range from explaining the basic properties of familiar objects, calculations, procedures, and results, to identifying patterns and trends in data, as well as more complex ideas and relationships, including logical relationships. Reflecting on their own or others' procedures, comparing them, or engaging in discussions on problematic mathematical situations enhances the learning process (MINEDUC, 2019, p. 315).

From this, it can be inferred that argumentation, at different educational levels, is a process in which oral and written expression play a fundamental role, guiding students from intuitive argumentation toward mathematical argumentation (Goizueta et al., 2023). For

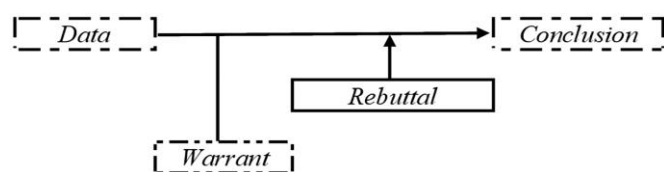


Figure 1. Adaptation of Toulmin's (2003) model by Conner (2008)

their part, Rumsey and Langrall (2016) conceive mathematical argumentation as a discourse process that enables the discovery of new ideas and also allows for the critique of others' reasoning in order to convince others of the validity of a conclusion—something that can be extended to statistics.

In this regard and based on Toulmin's (2003) model as adapted by Conner (2008), argumentation is structured around four elements (see **Figure 1**). In this regard, *data* refers to the evidence or information that supports a claim; *conclusion*: the thesis to be proven by the arguer; *warrant*: it presents statistical rules, patterns, properties, among others; and *rebuttal*: it serves to refute a part of the argument (Cervantes-Barraza & Cabañas-Sánchez, 2022).

METHODOLOGY

The research is qualitative, exploratory-descriptive, and the responses provided by pre-service mathematics teachers were analyzed using content analysis (Krippendorff, 1997), following Aoyama's (2007) taxonomy and the adaptation of Toulmin's (2003) model by Conner (2008).

Context and Participants

The study was conducted with 36 pre-service secondary mathematics teachers from two teacher training institutions in the central-southern region of Chile. Participants were selected through non-probabilistic convenience sampling (McMillan & Schumacher, 2011). It is important to note that all participants have completed and passed the required statistics courses in their academic curriculum. Also, their participation was voluntary and was confirmed through the signing of an informed consent regarding the use of the data obtained from the instrument application. This process was carried out in accordance with the Ethics Committee guidelines of the university to which the primary author is affiliated. To ensure confidentiality, participants were assigned alphanumeric codes. For example, (PFUN°, N°) represents a pre-service teacher from university 1 or university 2, and N° shows their position in the database.

Data Collection Instrument and Analysis Procedure

For the purposes of this study, an instrument consisting of two activities, each containing open-ended

questions, was applied (see **Figure 2**). Activity 1 is based on a news article published in a Chilean media outlet in 2021, within the context of the COVID-19 pandemic. Activity 2 relates to the mathematics assessment scores from the Sistema de Medición de la Calidad de la Educación (SIMCE), a standardized test taken by all eighth-grade students (age 13) in Chile. The purpose of SIMCE is to assess students' content acquisition and skill development as outlined in the national curriculum.

In activity 1, pre-service mathematics teachers were expected to provide arguments regarding the income trends of individuals in 2021, considering data distribution shape, variability, and central tendency measures present in the histogram. Their analysis was expected to go beyond a literal interpretation. In activity 2, participants were presented with a conclusion based on the behavior of SIMCE scores, represented in a box plot. They were required to either refute or accept the conclusion, using the information provided in the graph as justification.

The information provided by the participants was analyzed using content analysis, following the steps outlined by Mayring (2000). To achieve this, categories were developed cyclically and inductively, allowing for the identification of convergences and divergences in the responses to the questions guiding the tasks. Additionally, arguments were classified according to Aoyama's (2007) taxonomy and the adaptation of Toulmin's (2003) model (**Figure 1**).

ANALYSIS AND RESULTS

Classifications of Participants' Interpretations According to Aoyama's (2007) Taxonomy

The future mathematics teachers surveyed, regardless of the teacher education institution to which they belong, primarily demonstrate, according to Aoyama's (2007) taxonomy, skills associated with the basic reading of information provided by the histogram. That is, they report values and highlight trends present in the graph, but they do not delve into the contextual situation implicit in the activity prompt, such as the global impact of COVID-19. As an example, some interpretations made by the participants are presented below.

People who earn less than the median income are the ones most likely to be employed. The least employed individuals fall within the income range of 1,000,000 to 2,000,000 (PFU2, 6).

More than half of the employed population has an income between 0 and \$700,000. At least 10% of employed individuals earn between \$1,000,000 and more than \$3,000,000. Most employed people have a median income (PFU1, 9).

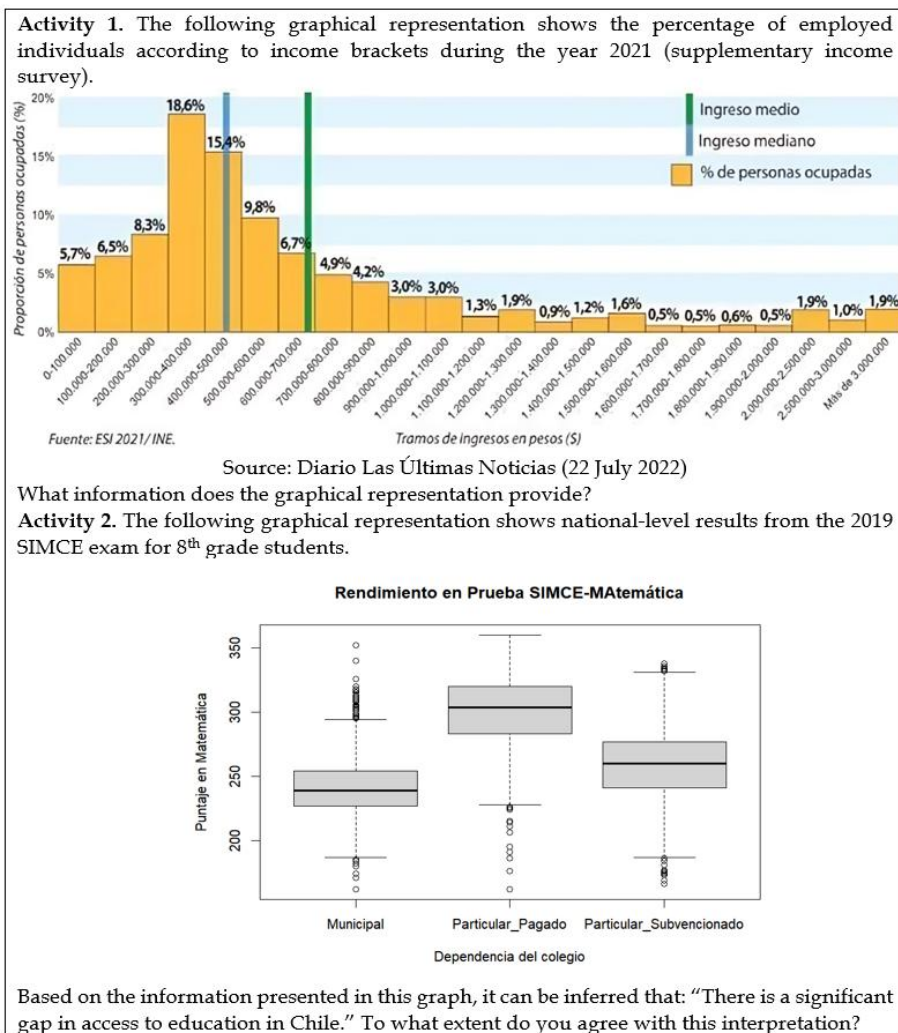


Figure 2. Activities presented to preservice mathematics teachers (Diario Las Últimas Noticias, 22 July 2022)

On the other hand, there is a low frequency of future teachers from both institutions who fall into the Idiosyncratic level, given that they do not interpret the statistical measures related to the box plot (quartiles) in connection with the contextual situation. This reveals the complexity they face when extracting information based on the context of the problem situation. An example of this is the following quote:

It can be observed that private schools have higher scores than municipal and subsidized private schools, showing the gap in access to education (PFU2, 4).

I find it difficult to interpret this representation (PFU1, 13).

The graph shows that private schools achieve better results in the mathematics test, while municipal schools obtain the lowest scores, and subsidized private schools fall in between, neither too low nor too high (PFU1, 15).

In contrast, only one future teacher from university 1 and another from university 2 provided arguments that

can be classified in the rational/literal category, according to Aoyama's (2007) taxonomy, as shown in the following excerpts:

It is possible that people whose income is above the average have online or part-time jobs, as these are well paid and require less time. In the range where people earn between 300,000-400,000, there may be labor overexploitation, meaning that the work performed far exceeds what should be remunerated (PFU1, 5).

According to the graph, there is a large number of people who receive less than \$400,000 per month. There is a very small number of people who earn more than one million. Based on this, there is a left skew, which reflects income inequality (PFU2, 1).

In line with these observations, **Table 2** presents the reading levels demonstrated by participants when interpreting contextualized information in histograms and box plots.

It is noteworthy that, in general, participants—regardless of their training institution—exhibited basic-

Table 2. Arguments of pre-service mathematics teachers according to Aoyama's (2007) adapted taxonomy for graph reading

Taxonomic levels	Histogram		Box plots	
	University 1	University 2	University 1	University 2
Idiosyncratic	-	2	3	3
Basic graph reading	23	9	18	6
Rational/literal	1	1	3	3
Critical	-	-	-	-
Hypothesis and model development	-	-	-	-

Table 3. Distribution of Toulmin's (2003) model elements present in the arguments made by pre-service teachers in activity 1

Elements	University 1		University 2	
	Yes	No	Yes	No
Data	12	12	5	4
Warrant		23	1	8
Rebuttal	5	18		9
Conclusion		24		9

level reading skills for both histograms and box plots. Additionally, it is significant that six participants (three from university 1 and three from university 2) were classified at the idiosyncratic level, meaning they were unable to interpret the box plot, which may suggest that its structure poses an inherent challenge for them.

Arguments of Pre-Service Mathematics Teachers

Table 3 shows the presence of the phases of Toulmin's (2003) model in the responses provided by the future teachers in the task related to the income of Chileans during the year 2021, as summarized in the histogram (activity 1).

Table 3 shows that, regardless of the teacher education institution, the future teachers refer to the data phase, mentioning statistical concepts such as median and mean income, but without relating them to the problem situation or to the skewness of the data, as observed in the following excerpts:

It can be concluded that fewer people have higher incomes. As income exceeds the median, the number of employed individuals decreases. It can also be concluded that the highest percentage of employed people earn a median income (PFU1, 23).

If the percentage of employed individuals in 2021 was to be shown, it should have displayed the percentage per month. The median income has a higher percentage of employed people compared to the mean income (PFU2, 5).

In these responses, statistical arguments based on the mentioned measures (median and arithmetic mean) are not provided. That is, participants did not apply the "warrant" element from Toulmin's (2003) adapted

model. However, one participant from university 1 (PFU1, 10) implicitly linked the highest bar in the histogram to the mode, while another participant (PFU1, 11) explicitly mentioned the mode, identifying it in the \$300,000-\$400,000 interval, but without calculating the class mark (midpoint of the interval width) to estimate the most frequent income in Chile during 2021. The following excerpts illustrate these observations:

The highest percentage of people earn between \$300,000 and \$400,000. The graph does not have a symmetric distribution (PFU1, 10).

The mode is between \$300,000 and \$400,000. Very few people earn more than \$900,000 (PFU1, 11).

Additionally, only two pre-service teachers (one from each institution) referenced the shape of the histogram, as evidenced in the following responses:

According to the graph, a significant number of people earn less than \$400,000 per month. There is a very small percentage of individuals earning over one million pesos. Based on this, the distribution is left-skewed, which reflects income inequality (PFU1, 2).

The left-skewed distribution means there is a greater tendency for people to earn lower wages. The percentage of individuals earning over one million pesos is very low. The most employed individuals are those earning the least (PFU1, 1).

In the previous cases, even though the participants' reading of the graph is relevant, they are unclear about the direction in which the tail of the distribution should extend for it to be classified as positively or negatively skewed (warrant). This is noteworthy, as this topic is introduced in primary education in Chile (Muñoz et al., 2016) and is also covered in specialized textbooks such as Triola (2006) and Devore (2016).

Similarly, it is important to highlight that the participants do not relate the position of the measures of central tendency (mode < mean < median) in the histogram to data skewness according to Pearson's skewness coefficient $CA = \frac{3(Mediana - \bar{x})}{s}$ (Canavos, 1981; Devore, 2008). This indicates that income distribution follows a positively skewed distribution. In line with

Table 4. Distribution of Toulmin's (2003) model elements present in the decision regarding the premise related to the graph in activity 2

Elements	University 1		University 2	
	Yes	No	Yes	No
Data	5	18		9
Warrant	12	12	5	4
Rebuttal		23	1	8
Conclusion		24		9

this, only one pre-service mathematics teacher identified that income distribution in Chile during 2021 exhibits positive skewness. However, while they used the data phase, they did not provide warrants, as they failed to explain the significance of this finding within the context of the problem situation. This is illustrated in the following excerpt:

The majority of employed individuals in 2021 had an income between [300,000-400,000]. The graph shows right skewness. At least 50% of employed individuals in 2021 had an income below [400,000-500,000] (PFU2, 12).

In activity 2, participants analyzed a box plot displaying mathematics scores from the SIMCE exam for Chilean school students, categorized by school type (municipal, subsidized private, and fully private schools). Pre-service mathematics teachers were asked whether they agreed or disagreed with the statement: "There is a significant gap in access to education in Chile". **Table 4** shows the number of participants according to the elements of Toulmin's (2003) model.

Table 4 shows that, in general, 18 out of 24 respondents from university 1 and university 9 out of 12 from university 2 do not refute the given statement, providing arguments such as:

I agree, because as we can see in the box plots, there is a clear difference between municipal and private schools, as well as between subsidized private and fully private schools. This makes it evident that in order to receive a good education and achieve good results, one must pay, which not everyone can afford. This creates a major gap between individuals from lower and higher socioeconomic backgrounds (PFU1, 9).

I agree with this information because it shows that private schools achieve better scores than municipal and subsidized private schools, which highlights the gap in access to education (PFU2, 4).

A plausible explanation for these types of responses is that the pre-service teachers focused only on comparing score distributions across school types, as summarized in the box plot, without directly relating them to the statement about educational access. In other

words, they did not make use of the data to critically evaluate the claim. However, five participants from university 1 did refute the statement, justifying their responses based on the data element of Toulmin's (2003) adapted model (**Figure 1**) but without incorporating theoretical background (warrant) to support their arguments. An example is the following:

It cannot be said that there is an educational access gap because the graph does not present this issue. What is shown are the mathematics SIMCE scores, meaning it displays test results, not the quality of students' education (PFU1, 1).

In this response, the participant points out that the box plot only summarizes students' test scores by school type and does not necessarily reflect disparities in access to education. In contrast, another pre-service teacher (PFU1, 14) refuted the premise but did not reference the data provided in the box plot:

I don't fully agree with this interpretation because it is somewhat difficult to determine the educational gap between municipal, private, and subsidized private schools. What is clear is that there is a gap within private schools, but it is not that large. I would have chosen a different type of graph to represent these results (PFU1, 14).

Additionally, one participant challenged the conclusion by shifting the focus from access to education to a broader issue of national concern: the quality of mathematics education.

Rather than access to education, I would argue that the real issue is the quality of education. A municipal school will never provide the same quality of education as a private school (PFU1, 11).

From this, it can be inferred that, overall, the future mathematics teachers surveyed have not yet acquired skills aimed at refuting assertions made based on the information provided by a graphical representation specifically, a box plot. This may indicate that they have not yet developed skills such as interpretation, which aligns with statistical thinking.

CONCLUSION

Reading and statistical argumentation are fundamental skills in both statistical thinking and statistical literacy, enabling the teaching of statistics to move beyond procedural aspects. These skills align with the demands of international reports, such as those published by GAISE (2005, 2016). In this context, statistical graphs play a crucial role, as they are widely used in media communication and serve as essential tools for conveying complex messages (Izagirre et al., 2023; McConway, 2016). Among these graphs,

histograms and box plots are prominently featured in primary, secondary, and tertiary education textbooks worldwide, including Chile.

In this regard, the study's results provide evidence indicating that the future mathematics teachers who participated in the study generally do not use the warrant and data elements of Toulmin's (2003) model, revealing that they have not acquired conceptual skills related to statistical thinking (Garfield & Ben-Zvi, 2008).

Overall, the group of future mathematics teachers surveyed tend to perform basic readings of the information presented in the histogram, according to Aoyama's (2007) taxonomy. However, they do not connect their interpretations to real-world contexts, such as COVID-19 or national and international economic trends. It is worth noting that in 2021, the world was gradually returning to normal after two years of global lockdowns. This trend has been documented in national studies on graphical representations (e.g., Díaz-Levicoy et al., 2021; Rodríguez, 2017; Rodríguez-Alveal y Sandoval, 2012), as well as in research indicating that statistics education is predominantly approached from a procedural perspective (Estrella, 2017).

Another analyzed representation was the box plot, in which pre-service mathematics teachers were expected to critically evaluate the given conclusion. However, most participants did not challenge the statement, revealing a lack of transition from real-world systems to statistical systems (Shaughnessy & Pfannkuch, 2002). Those who did refute the claim incorporated the data element but failed to provide warrants, meaning they lacked theoretical and conceptual justifications related to statistical thinking. In summary, they only used three of the four elements from Toulmin's (2003) adapted model. Moreover, the surveyed pre-service teachers use measures of central tendency and variability without relating them to the problem situation, which clearly suggests that the study group received primarily procedural instruction—that is, teaching focused on calculating measures of central tendency, shape, and variability, and constructing number-based rather than data-based graphs (Cobb & Moore, 1997).

The findings of this study underscore the need for teacher training programs to develop statistical skills that go beyond procedural aspects. Specifically, future educators should be encouraged to strengthen their interpretation and argumentation skills, which are essential components of statistical thinking and statistical literacy. Enhancing these abilities will help prepare students to critically analyze the information circulating in the media.

One of the study's limitations concerns the number of participants and the type of sampling used. In this sense, it would be valuable to apply the instrument to future teachers from other teacher education institutions in

order to gather more information about the levels of argumentation they have developed during their training, using probabilistic sampling. Additionally, it would be advisable to interview university instructors regarding the didactic and/or pedagogical strategies they have implemented in statistics courses to help future mathematics teachers acquire argumentative skills, in line with the curricular guidelines established by the MINEDUC.

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AI statement: The authors stated that they have not used AI in the development of this article.

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