MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2023, 19(12), em2363
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/13836

Strategies and representations used by early childhood education students in a functional thinking task: A case study

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Received 06 September 2023 ▪ Accepted 16 October 2023

Abstract

This article presents a study that addressed the functional relationships that two early childhood education students (five-six years old) evidenced, as well as the representations they used when solving a functional thinking task. The proposed task involved the function $f(x)=2x$, with six questions on particular cases and one on generalization. The data was collected through a semi-structured interview to each of the students and a qualitative analysis of their answers was carried out in each of the questions of the task. The results suggest that the two students are capable of approaching the proposed task through different strategies, such as additive and multiplicative correspondence relationship, and covariation. Also, it was found that they use systems of varied representations, being the verbal representation to express the generalization of the functional relationship one that stands out. It is concluded that early childhood education students may be able to tackle tasks that involve algebraic notions that focus on functional thinking.

Keywords: early algebra, early childhood education, functional thinking, representations

INTRODUCTION

The curricular proposal early algebra seeks to promote modes of algebraic thinking from the first educational levels as an alternative to the traditional teaching of algebra at schools. Particularly, it seeks to promote algebraic ways of thinking from previously adapted mathematical tasks, rather than the abrupt and decontextualized step based on the mechanization of procedures (Molina, 2009). These modes of algebraic thinking are intended for students to focus on the study of arithmetic patterns and structures, relationships between quantities, generalization, and the use of increasingly sophisticated representations (Brizuela & Blanton, 2014; Molina, 2009).

From early algebra, functional thinking is considered as an approach that allows the development of algebraic modes of thinking from the first school years (Cañadas & Molina, 2016). It is intended that through tasks that involve relating quantities, students can establish general mathematical relationships using varied representations that may lead them to make such relationships (Soares et al., 2005). The tasks linked to functional thinking emphasize the relationships between quantities, bringing the student closer to the concept of function in an intuitive way (Blanton, 2008; Smith, 2008). The advantages attributed to functional thinking lie in the fact that students can identify mathematical patterns and structures, being able to develop skills focused on the transfer of varied representations, reaching the generalization of a relationship between variable quantities (Cañadas et al., 2016; Torres et al., 2021).

Tasks linked to functional thinking have been considered as central curricular elements in the math classroom, given the algebraic skills and knowledge that it promotes (Rico, 2006). For this reason, countries such as Australia, Canada, Chile, China, Japan, Korea, Portugal, Spain, and the United States have included objectives associated with this type of thinking into their curricular guidelines, starting from the first educational levels (Merino et al., 2013; Ministry of Education of Chile [MINEDUC], 2012). For example, the Chilean primary education curriculum (ages six-14) states that students must be able to identify relationships between quantities in such a way that they must explore how the change of

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Contribution to the literature

- In this article, we present evidence of functional thinking of two early primary education students (five-six years old) when solving a task associated with the concept of function. Both students used strategies focused on the multiplicative correspondence relationship and were able to verbally generalize the functional relationship involved in the research task.
- We highlight importance of this work in the research community, given that, according to our background information, research on functional thinking in early primary education is in an incipient state.
- Through this study, we provide relevant findings that could be useful for the characterization of functional thinking of students at this educational level. We believe that the results we present here complement the existing body of knowledge in functional thinking from the first educational levels.

one quantity affects the other (MINEDUC, 2012). Whereas the early childhood education curriculum (ages zero-six) proposes that students recognize patterns, work with quantifying elements to compare quantities, and establish correspondence relationships (MINEDUC, 2018). In this article we focus on the early childhood education level, since students from a very early educational age have amazing mathematical skills and their development will have a direct relationship with the success they will have in later studies, including those related to algebra in school contexts (Castro et al., 2017; Clements et al., 2013). The above makes this research relevant, since the teaching of algebra in early childhood education has not penetrated much into the topic of functional thinking and, despite curricular efforts, its teaching has focused particularly on tasks alluding to classifications, serializations and qualitative patterns. Additionally, regarding research, it is in an incipient state (Morales & Parra, 2022).

In accordance with the previous considerations, this research aims to describe strategies and representations used by two early childhood education students when they solve a functional thinking task.

CONCEPTUAL FRAMEWORK

Next, we describe the theoretical concepts that support this research.

Functional Thinking

Functional thinking is understood as a type of algebraic thinking that seeks the construction, description, representation, and reasoning with and about the functions and the elements that constitute them (Cañadas & Molina, 2016). Specifically, this type of thinking focuses on how quantities vary together as well as on the emphasis on the correspondence between values of the variables involved in the functional relationship, in addition to the use of different representations in a problem-solving context (Canadas & Molina, 2016). For functional thinking, the generalization of the functional relationship is an important element, as well as its justification and the use of representations that can encompass verbal language, as well as pictorial, tabular, graphic, symbolic or algebraic. Therefore, in a functional thinking task, the student is expected to be able to reason fluently through such generalized representations in order to understand and predict the behavior of the function (Blanton et al., 2015). Thus, a student demonstrates functional thinking when focused on describing the relationship between two or more covariate quantities (Confrey & Smith, 1991; Smith, 2008).

Functional relationships

A functional thinking task seeks, among other learning, the identification of functional relationships that can be of two types: covariation and correspondence (Blanton & Kaput, 2005; Confrey & Smith, 1991; Smith, 2008). In this sense, the covariation relationship implies that the change of one variable has incidence in another, thus, it refers to the simultaneous change between two variables that is produced by the existence of a relationship between them (Gómez, 2016, p. 170). A student identifies a covariation relationship when they is able to identify those changes that occur between the amount of the independent variable and its incidence in the amount of the dependent variable or vice versa (Blanton et al., 2011; Blanton & Kaput, 2005). For example, in the tabular representation of **Figure 1**, the covariation relationship is given in the sense that, if the value of the independent variable (x) increases (e.g., by one), the value of the dependent variable (y) also increases in the same quantity (see curved dates).

In addition, correspondence relationship associates each quantity of the independent variable with a quantity of the dependent variable (Clapham, 1998), and is established between the corresponding pairs of the quantities of both variables (Confrey & Smith, 1991;

	Independent variable	Dependent variable	
+1	1	6	+1
+1	2	7	+1
+1	3	8	+1
	4	9	

+ 5

Figure 1. Functional relationship of covariation & correspondence (Source: Authors' own elaboration)

Smith, 2008). Thus, identifying correspondence implies focusing on that pattern that determines a single value of the dependent variable given a value of the independent variable (Blanton et al., 2011). For example, in **Figure 1**, the correspondence relationship is determined by the pattern of adding five to each of the independent variable quantities to find the value of the corresponding dependent variable (horizontal arrow).

Generalization

Generalization is a key element in functional thinking, regarded as the heart and initiator of algebraic learning (Mason, 1996; Strachota, 2016). In the context of functional thinking, many of the definitions of generalization emphasize that, in order to achieve it, it can be done through particular cases. For example, Kaput (2000) indicates that generalizing comprises:

> deliberately extending the range of one's reasoning or communication beyond the case or cases considered, explicitly identifying and exposing commonality across cases, or lifting the reasoning or communication to a level, where patterns across and relations among cases or situations become the focus, rather than the cases or situations themselves. Appropriately expressed, the patterns, procedures, relations, structures, etc., can become the objects of reasoning or communication (p. 6).

Cañadas and Castro (2007), drawing on Polya's work, propose that generalization can be achieved through work that organizes different particular cases, which lead to the identification of a pattern and its subsequent generalization. Mason (1996) argues that generalization can be achieved by means of a single example or particular case with certain characteristics, that is known as a generic example. Functional thinking tasks that promote generalization can be expressed by diverse representations, each time promoting more sophisticated ones such as symbolic representation.

Representations

In the context of functional thinking, the verbal, pictorial, tabular, graphic, and symbolic representations become important since they help students understand the behavior of the function and make it possible to demonstrate the presence of functional thinking (Blanton et al., 2011; Cañadas & Molina, 2016; Cañadas et al., 2016). The verbal representation system is the one that refers to the natural language to express mathematical concepts, which can be either oral or written (Cañadas & Figueiras, 2011). The pictorial representation system refers to visual resources, e.g., drawings, that allow expressing mathematical relationships and which are essential because they are the original representations of the subjects who solve the mathematical tasks (Blanton et al., 2011; Cañadas & Figueiras, 2011). Symbolic representation system is alphanumeric in nature, whose syntax is described by a series of rules and procedures (Rico, 2009); this system involves mathematics symbols and signs that allow expressing with accuracy and precision the quantities of the variables as well as the variables themselves in a functional relationship task. This representation system requires sophisticated mathematical thinking to express a functional relationship (Azcárate & Deulofeu, 1990; Blanton, 2008).

BACKGROUND

There is substantial evidence regarding the ability of primary education students to solve tasks that involve functional relationships (Morales et al., 2018; Torres et al., 2021). For example, Blanton et al. (2015) showed that American students aged six were able to transition from a recursive pattern to a covariation and correspondence relationship in functional relationship tasks such as $f(x)=mx$ and $f(x)= x+b$. Canadas et al. (2016), showed that six-seven year old students approached the functional relationship task $f(x)=2x$ through two approaches: a recursive pattern (counting by twos) and another based on a correspondence relationship (doubling). Merino et al. (2013) reported that students aged 10-11 made use of verbal, pictorial and numerical representations to express the generalization of functional relationships. In the same line, Pinto et al. (2018) showed that students aged nine to 11 used covariation and correspondence in a task whose function was $f(x)=2x+6$ and generalized it using algebraic representation and verbal representation. Similarly, Cetina-Vázquez and Cabañas-Sánchez (2022) showed that students of similar ages, in tasks related to functions, were capable of expressing generalizations using verbal representation. Research in functional thinking in early childhood education is incipient and the current data show that students at this level are capable of establishing relationships between quantities. For example, Blanton and Kaput (2004) reported that children between four-five years old evidenced the covariation and correspondence relationship associated with parity, additive structure and multiplicative structure, by determining the number of eyes and tails when asked for a certain number of dogs, in a task involving the functions $f(x)=2x$ and $f(x)=3x$. Castro et al. (2017) reported that in a group of 12 students aged five-six, they were able to tackle a task that involved the function $f(x)=x$ and the function $f(x)=2x$ through various strategies such as recursive, i.e., they identified regularities between the values of a variable; furthermore, they also used the functional relationship of correspondence and covariation. The authors state that there were even students who verbally generalized this correspondence relationship.

Additionally, the study by Warren et al. (2013) informs that early childhood education students were able to relate input elements (independent variable) with output elements (dependent variable), thus determining a correspondence relationship and its generalization.

METHODOLOGY

Methodological Approach

This research is qualitative, exploratory, and descriptive (Hernández et al., 2007). We conduct this type of research given that, as discussed in the background section, functional thinking in early childhood education students has not been extensively explored. Also, we approach this research through a case study, being our focus to investigate in depth the particular work that each student performs when solving tasks that imply functional relationships (Stake, 1999).

Participants

Two early childhood education students (hereinafter, *St1* and *St2*, respectively), aged five-six, participated in this research. At the time of the investigation, these students were studying in a private-subsidized school in the Maule Region of Chile. For the purposes of the investigation, the consent of the school and their parents was requested, along with the consent of each student. The participants were intentionally selected, according to the criteria of accessibility and availability to participate in the research. As reported by their math teacher, both students had an advanced performance level and had some prior knowledge necessary for this research, namely, working with patterns and serialization, basic operations (addition and subtraction up to 20), and experiences with one-to-one correspondence.

Semi-Structured Interviews & Task

We designed a task that sought to promote functional thinking in early childhood education students whose functional relationship was given by the function $f(x)=2x$, which was validated by expert judgment. We consider this function as a way of understanding the strategies that students used when solving the tasks provided. We presented the task to each student in a semi-structured interview context. The statement of the task that we proposed to them was framed in a context close to them and referred to the relationship between the number of grandchildren that a grandmother had (independent variable) and the number of balloons (dependent variable) that each child would receive from her, as we show below:

> Grandma Lita is inviting all her grandchildren to celebrate her birthday, and she will give each one two balloons so they can play. Let's help granny Lita give balloons to her grandchildren.

After presenting this statement to each student, seven questions we previously elaborated were proposed, following the guidelines of the inductive reasoning by Cañadas and Castro (2007). This is that, based on questions about small particular cases, large particular cases and a general case, the generalization of the functional relationship was sought. In **Table 1**, we show the seven questions that we proposed to each student.

Also, each student had manipulatives to represent children and balloons. They were given different representations of the balloons: pictorial (colored

Table 1. Description of task questions

Case type	Description	Questions
Consecutive small case	It refers to questions in which participant is requested to find first cases of functional relationship, according to numerical scope with which student is working. These are consecutive.	1. If Gustavo is first to arrive at Grandma Lita's house, how many balloons will Gustavo receive in total? 2. If Noemi, her other granddaughter, arrives now, how many balloons in total will Grandma Lita give to her two grandchildren? 3. If another grandchild arrives now, how many balloons will Grandma Lita give away in total to her three grandchildren?
Non-consecutive small case	It refers to those questions in which participant is requested to find first cases of functional relationship, according to numerical field in which student is working. These are non-consecutive.	4. If now there are five grandchildren in Grandma Lita's house, how many balloons will Grandma Lita give among all of them?
Non-consecutive large case	It refers to those questions that ask participant to find large & isolated cases in functional relationship, which require identification of a pattern (functional relationship).	5. If a total of 10 grandchildren arrives, how many balloons will Grandma Lita give among her 10 grandchildren? 6. If 20 grandchildren arrive at Grandma Lita's house, how many balloons will Grandma Lita give to her 20 grandchildren?
General case	It is that question that seeks generality of functional relationship.	7. If we do not know number of grandchildren Grandma Lita has, can we find out number of balloons needed to give among all grandchildren? How?

Figure 2. Manipulatives used by students in task (Source: Authors' own illustration)

circles), concrete (tangible balloons), and symbolic (numerals) to use when answering the questions in the task, as well as pictorial representations of boys and girls representing Grandma's grandchildren. **Figure 2** shows manipulative material used in the task.

Analysis Techniques & Categories

For the analysis of the information, we used the technique of content analysis (Fernández, 2002). The units of analysis that we considered in this research came from two sources of information:

(a) transcripts of the interview related to the answers of each student to each question of the task and

(b) photographs of the productions of the students to each of the task questions.

These units were analyzed according to categories that we show in **Table 2**. The categories were built considering following elements: strategies and representations that two students manifest regarding research task, main elements of objective of this research; conceptual framework of this research; the background; and through an a priori analysis of the information. Strategies are understood as the students' actions on mathematical tasks and are sequences of procedures carried out on concepts and relationships (Rico, 1997). In this article, strategies are actions executed by students, which focus on arithmetic calculations, patterns, and evidence of functional relationships.

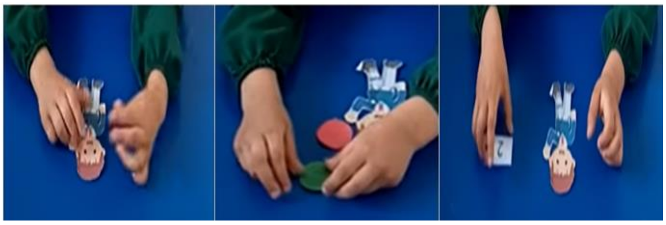

Figure 3. *St1's* response with pictorial & symbolic representation (Source: Authors' own illustration)

RESULTS

Firstly, we show the results of student 1 (*St1*) and, later, the results of student 2 (*St2*), for each of the seven questions in the task. Finally, we show a comparison of the work done by both students.

Student 1's Analysis of Work

Question 1

Once the interviewer presents the material to *St1*, they asks question 1: Gustavo is the first to arrive at Grandma Lita's house, how many balloons will Gustavo receive in total? *St1* takes an image of a child and places it on the table, representing Gustavo (left image, **Figure 3**).

Next, they verbally states "two […] because it is the double" alluding to two balloons that they quickly represented pictorially next to the image of the grandchild (central image, **Figure 3**).

The interviewer (*Int*, hereinafter), to inquire further about this response, continued the interview, as follows:

> *Int*: Two, why will she give him two balloons? How do you know?
>
> *St1*: Because it is the double.
>
> *Int*: It's the double. And who said that she was going to give him the double?
>
> *St1*: Granny.

Table 2. Categories of data analysis

Categories	Sub-category	Description
Strategies	Non-functional	No evidence of functional relationship: Application of operations with no relationship between quantities.
		Recursive pattern: Add two by two, but without relationship between quantities.
	Functional	Covariation: Relationship between how change in amount of independent variable affects amount of dependent variable.
		Incipient correspondence: Indications of a functional correspondence relationship is observed, although it is not manifested as such.
		Additive correspondence: Adding quantity of independent variable to itself.
		Multiplicative correspondence: Doubling amount of independent variable.
Representations	Concrete	Using concrete elements (balloons).
	Verbal	Communicating interventions or justifications through oral or written language.
	Pictorial	Using drawn circles to express quantities.
	Symbolic	Using cards with representations of numerals.

Figure 4. *St1*'s response with pictorial & symbolic representation (Source: Authors' own illustration)

Figure 5. *St1's* response with pictorial & symbolic representation (Source: Authors' own illustration)

We observe that *St1* answered this question using the incipient correspondence functional relationship strategy, since *St1* understands that to find the number of balloons (dependent variable) there is another variable that must be doubled (line 3 and line 4), and everything suggests that they is referring to the number of grandchildren, even if it is not explicitly said. In addition, in the previous extract, we observe that *St1* used different representations to answer question 1. On the one hand, *St1* used the verbal representation since they orally mentioned "two" and "double"; they also used pictorial representations because they used two circles (central image, **Figure 3**); and a symbolic representation since they also used the numeral "2" (right image, **Figure 3**).

Question 2

Int asks *St1* question 2, "If Noemi, her other granddaughter, arrives now, how many balloons in total will Grandma Lita give to her two grandchildren?" *St1* placed on the table, next to the image of Gustavo (question 1), an image that represented the granddaughter Noemi (see left image, **Figure 4**).

Given the question, *St1* verbally indicates that Grandma Lita will give Noemi four balloons. *Int*, to deepen *St1*'s answer, asked, as we read below:

> *Int*: If there are two grandchildren, how many balloons will granny Lita give away?
>
> *St1*: Four, because there are two for each one.
>
> *Int*: So, how did you know it would be four?
>
> *St1*: Because it is the double.
>
> *Int*: The double of what?
>
> *St1*: The balloons are the double of the number of grandchildren.

In the previous extract, we observe that *St1* used a multiplicative correspondence relationship strategy, since *St1* understands that each grandchild is entitled to two balloons and, therefore, four balloons would be needed for two grandchildren. *St1* found the number of balloons (dependent variable) by doubling the number of grandchildren (independent variable), as shown in line 10. *St1* responded through various representations. On the one hand, *St1* used the verbal representation since they mentioned "four" and "double" in their answer; *St1* also uses a pictorial representation, since they placed four circles between the two grandchildren (central image, **Figure 4**); as well as a symbolic one when using the numeral "4" (right image, **Figure 4**).

Question 3

Int raised question 3, if there are three grandchildren now, how many balloons will Grandma Lita give in total? *St1* immediately says "six balloons". Given this response, *Int* continued to ask, as follows:

> *Int*: How many balloons will there be in total?
>
> *St1*: Three [St1 says it in a low voice thinking], six!
>
> *Int*: And how do you know that it is six?
>
> *St1*: Because there are two for each one and it is the double.

In the previous extract we identified that *St1* used, as in the response to question 2, the multiplicative correspondence relationship strategy, since they doubled the number of children to find the number of balloons, thus obtaining six balloons for three grandchildren. *St1* used different representations: verbal representation as they orally expressed the answer "double" and "six"; pictorial representation since they located two circles for each of the three grandchildren (right image, **Figure 5**); and a symbolic representation through the card with the numeral "6" (left image, **Figure 5**).

Question 4

Int asked *St1* question 4, related to a non-consecutive close case, "If now there are five grandchildren in Grandma Lita's house, how many balloons will Grandma Lita give among all of them?" *St1* located the images that represent the grandchildren (left image, **Figure 6**). In addition, they placed two colored circles next to each image of the grandchildren (left image, **Figure 6**) and immediately responded "it's 10".

Given this answer, and for further understanding, *Int* asked "how did you know that was the amount?" To which *St1* replied "it's 10 […] because it's the double". Once again, it is observed that *St1* used a functional

Figure 6. *St1*'s response with pictorial representation (Source: Authors' own illustration)

Figure 7. *St1*'s response with pictorial representation (Source: Authors' own illustration)

strategy of multiplicative correspondence, because to find the number of balloons (dependent variable), they doubled the number of grandchildren (independent variable). Also, *St1* used the verbal representation, since they referred to "10" and "double"; and the pictorial one, because they placed two circles next to the image of each one of the grandchildren to represent the number 10.

Question 5

Int asks *St1* question 5, related to a non-consecutive close case: How many balloons will Grandma Lita give away when 10 grandchildren arrive? Subsequently, *St1* placed on the table five extra images of grandchildren right next to the five images used in the previous question, thus leaving 10 images of grandchildren (left image, **Figure 7**). Also, *St1* said "20" and placed two circles for each grandchild (right image, **Figure 7**).

In the following interview extract we identify the justification for *St1* 's answer to question 5.

> *Int*: You calculate it mentally!
>
> *St1*: Yes!
>
> *Int*: And how do you know that it's 20?
>
> *St1*: Because it is the double.
>
> *Int*: Is it the double of 10?
>
> *St1*: Yes!

Through the previous extract, we observe the student used the functional strategy of multiplicative correspondence relationship strategy; since to find the number of balloons (dependent variable) *St1* doubled the number of grandchildren (independent variable). In addition, this answer confirms that *St1* is doubling the number of grandchildren, since when *Int* asks, is it the double of ten? *St1* answers "yes". According to **Figure 7** and the interview extract, we see that *St1* used the verbal representation, by saying "twenty" and "double", as well as the pictorial representation of these same quantities, since they placed two circles for each one of the ten grandchildren (right image, **Figure 7**).

Figure 8. *St1's* response with pictorial & symbolic representation (Source: Authors' own illustration)

Question 6

Int asked question 6 to *St1*, related to a non-consecutive distant case: How many balloons will Grandma Lita give away for 20 grandchildren? *St1*, without even taking any of the manipulatives that were available to support the solution of the question, verbally states that there are "forty"–referring to the number of balloons–. To delve into the answer, *Int* asked "how do you know?", generating the following dialogue.

> *St1*: There are forty, it's the double.
>
> *Int*: And how do you know?
>
> *St1*: Because it is the double.

In the interview extract we observe that *St1* used the idea of double to find the number of total balloons for the 20 grandchildren. Therefore, *St1* used the functional strategy of multiplicative correspondence, since, to find the number of balloons for 20 grandchildren, they doubled this number. On the other hand, they used verbal representations since they orally mentioned "forty"; pictorial, by using circles to represent the total number of balloons and corroborate what was indicated in their answer; and symbolic, by using the card "40" (see **Figure 8**).

Question 7

Finally, *Int* proposed question 7, which addressed the generalization: If we do not know the number of grandchildren Grandma Lita has, how can we find out the number of balloons needed to give among all the grandchildren? When asked, *St1* indicates that it will be twice the number of grandchildren, as shown in the following interview fragment.

> *Int*: If we do not know the number of grandchildren that will arrive at the party, can we

Figure 9. *St2's* response with concrete, pictorial, & symbolic representation (Source: Authors' own illustration)

Figure 10. *St2*'s response with concrete representation (Source: Authors' own illustration)

know how many balloons Grandma Lita will need?

St1: Yes.

Int: Why?

St1: Because it is the double.

Int: Oh! The double, for example, if we do not know how many grandchildren; What would happen?

St1: The balloons would be the double of the number of grandchildren.

Through this answer, we observe that *St1* generalized the multiplicative correspondence relationship strategy, given that, to find the number of balloons when not knowing the number of grandchildren, *St1* answered that "the balloons would be the double". Therefore, *St1* found the number of the dependent variable (number of balloons) doubling the number of the independent variable (number of grandchildren). *St1* expressed the generalization through verbal representation (see line 29).

Student 2's Analysis of Work

Once *Int* presented student 2 (*St2*) with the material and the statement of the problem, they asked question 1.

Question 1

"If Gustavo is the first to arrive at Grandma Lita's house, how many balloons will Gustavo receive in total?" *St2* placed an image of a child representing Gustavo (left image, **Figure 9**) and next to it two balloons, two circles and the numeral 2 (right image, **Figure 9**). In addition, they said "two", alluding to the balloons that the grandmother gave to her grandson Gustavo. *Int* elaborated on *St2*'s answer, as follows in the following extract:

Int: Why did you decide to do it that way?

St2: Because you can do two of three different things.

Int: Three different things, but what do they represent?

St2: Two.

Int: And why two?

St2: Because it's just one child.

Int: And how many balloons does one child have?

St2: Two.

Given the previous extract, and as we can see in **Figure 10**, *St2* responded using the incipient correspondence functional relationship strategy; despite the relationship itself is not explicitly evoked, *St2* established the relationship that a grandchild is entitled to two balloons, such as it can be seen in line 33 to line 37. On the other hand, *St2* used different representations. Initially, they used the verbal representation when saying "two", as well as concrete (two balloons), pictorial (two circles) and symbolic (numeral 2) representations, as we can see in right image of **Figure 9**.

Question 2

Next, *Int* proposed question 2. *St2* places on the table an image that represents the granddaughter Noemi, next to Gustavo's image (question 1) (see left image, **Figure 10**). Also, they verbally responded "four balloons", and placed four balloons next to the images of the grandchildren (right image, **Figure 10**). *Int* deepened on *St2*'s response, as evidenced in the following fragment:

Int: Why four?

St2: Because there are two for each grandchild.

Int: How can you show me?

St2: Like this, [indicates with their fingers the four balloons that they placed next to the images of the two grandchildren (right image, **Figure 10**)].

Int: How many are they in total?

St2: Four.

From the above, we see that *St2* used an incipient correspondence functional relationship strategy, because there are hints of a relationship *St2* made between the number of grandchildren and the number

Figure 11. *St2*'s response with concrete & pictorial representation (Source: Authors' own illustration)

Figure 12. *St2*'s response with concrete & pictorial representation (Source: Authors' own illustration)

of balloons, since they allocated four balloons to the two grandchildren.

In addition, *St2* responded through various representations: verbal representation, since they orally mentioned "four"; and the concrete one because they placed four balloons next to the images of the grandchildren (see right image, **Figure 10**).

Question 3

Subsequently, when faced with question 3, *St2* added the image of a child to the previous ones and immediately responded "six balloons" (left image, **Figure 11**). Also, they placed six balloons next to the three images of the children, which they later removed, and replaced with six circles (central and right image, respectively, **Figure 11**).

Int elaborated on *St2*'s answer, as follows in the following extract:

> *Int*: And how do you know that there are six?
>
> *St2*: Because at home I practice adding.
>
> *Int*: Adding, and how do you add?
>
> *St2*: Two plus two is four, two plus four is six.
>
> *Int*: Oh, of course, because you had four before and you added two, because the number of grandchildren increased by one.
>
> *St2*: [Nods].

From the above, we see that *St2* used two strategies. On the one hand, they used the recursive pattern, since they added two by two, whose quantities allude to the values of the dependent variable (number of balloons), without mentioning the quantities of the independent variable, as can be seen in the line 47. The other strategy they used was the covariation relationship, because when *Int* mentioned "because you had four before and you added two", *St2* agrees (line 49), which means that when the amount of the independent variable (number of grandchildren) increases by one, the dependent increases by two (number of balloons). In addition, *St2* used: the verbal representation, since they answered "six [...] two plus two is four, two plus four is six"; the concrete one, since they placed six balloons next to the grandchildren (central image, **Figure 11**); and pictorial, because they placed six circles next to the grandchildren (right image, **Figure 11**).

Question 4

In relation to question 4, *St2* placed two images of grandchildren next to the previous three (previous question) (left image, **Figure 12**). Also, they immediately said "eight". Given this answer, we understand that the student used a strategy based on the recursive pattern, because they added two more balloons to the number of balloons that were in the previous answer (six balloons), focusing exclusively on particular consecutive cases.

Next, *Int* asked if they was sure of that, leading *St2* to place two balloons on top of each grandchild (central image, **Figure 12**, two balloons are not seen in the image), and after counting one by one, *St2* pointed out that they are "10 balloons".

Int elaborated on *St2*'s answer, as shown in the following extract:

> *Int*: So, how many balloons did Grandma Lita give to five grandchildren?
>
> *St2*: 10.
>
> *Int*: How did you know that it's 10?
>
> *St2*: I forget sometimes, but five plus five is 10.
>
> *Int*: Ah, so you mean that the number of grandchildren is repeated to find out the number of balloons.
>
> *St2*: Yes.
>
> *Int*: Are you sure?
>
> *St2*: Yes.

From the previous extract we observe that *St2* used an additive correspondence relationship strategy, since to find the number of balloons (dependent variable) they added up the number of grandchildren by itself (independent variable), this is shown when they said, "five plus five is 10". In addition, we observe that *St2* used the verbal representation when they referred to "ten", pointing out: "five plus five is 10"; the concrete one, because they placed two balloons for each image of the grandchildren (see central image, **Figure 12**); and the

Figure 13. *St2*'s response with concrete, pictorial, & symbolic representation (Source: Authors' own illustration)

pictorial one, when they replaced the ten balloons with 10 circles (see right image, **Figure 12**).

Question 5

Regarding question 5, *St2* placed next to the five images from the previous question, other five, thus leaving 10 images of grandchildren (left image, **Figure 13**). Furthermore, they verbally responded "20". *St2* assigned 20 balloons, two for each grandchild, which they later removed to place 20 circles and the card with the number 20 (see central and right image, **Figure 13**).

To go deeper into the answers of *St2*, the following dialogue is developed:

> *Int*: How do you know it's 20?
>
> *St2*: Because 10 plus 10 is twenty.
>
> *Int*: Sure, so again you are adding twice the number of grandchildren to find the number of balloons.
>
> *St2*: [Nods].

From the above, we observe that *St2* employed the additive correspondence relationship strategy; since to find the number of balloons (dependent variable) *St2* added the number of grandchildren by itself (independent variable), that is "10 plus 10 is 20", as can be seen in line 59 and which they later reaffirmed in response to *Int*'s question (see line 60 and line 61). Furthermore, St2 used: the verbal representation, since they verbally mentioned "20", which they justified saying "because 10 plus 10 is 20"; the concrete one, by assigning two balloons for each grandchild; the pictorial representation when they located 20 circles in total (see central and right image, **Figure 13**); and the symbolic one, because they used a card with the number 20 (see central and right image, **Figure 13**).

Question 6

Regarding question 6, without taking any of the elements that were available to support the solution, *St2* verbally stated, "there are 40", alluding to the number of balloons. Subsequently, they placed the number of grandchildren corresponding to the question on the table together with two circles for each of the grandchildren (see **Figure 14**). Finally, they counted the circles one by one to check their answer.

Figure 14. *St2*'s response with pictorial representation (Source: Authors' own illustration)

Given this answer, *Int* asked "how do you know?" and *St2* replied "because I added this", referring to the images of the grandchildren. This suggests that *St2* used the functional strategy of additive correspondence, since, to find the number of balloons for 20 grandchildren, they added the number of grandchildren to itself (independent variable), resulting in 40 balloons. In addition, we observe that they used: verbal representation, since they verbally mentioned "forty" and justified saying "because I added this"; pictorial representation, since they used circles (two for each grandchild) to represent the total number of balloons (see **Figure 14**).

Question 7

Finally, regarding question 7, *St2*'s answer and justification are shown in the following extract:

> *Int*: If we do not know number of grandchildren that will arrive at the party, can we know how many balloons Grandma Lita will need?
>
> *St2*: It's the double […], two for each grandchild.
>
> *Int*: So, if we do not know the number of grandchildren, how do you solve the task?
>
> *St2*: Because I add.
>
> *Int*: And how do you add?
>
> *St2*: It's the double.
>
> *Int*: It's the double, and what is the double?
>
> *St2*: It's twice the number of grandchildren.
>
> *Int*: So, if I did not know the number of grandchildren, how would I know the number of balloons?
>
> *St2*: Because it is the double.

From this answer we observe that *St2* generalized the functional relationship given in the task, using the multiplicative correspondence relationship strategy. This is that, to find the number of balloons, they doubled

Table 3. Summary of responses from student 1 & student 2

Category	Number of questions						
	1	2	3	4	5	6	7
				Student 1 (*St1*)			
Strategy	I.F.R.	M.Cr.R.	M.Cr.R.	M.Cr.R.	M.Cr.R.	M.Cr.R.	M.Cr.R.
Representation	V.R. P.R. S.R.	V.R. P.R. S.R.	V.R. P.R. S.R.	V.R. P.R.	V.R. P.R.	V.R. P.R. S.R.	V.R.
				Student 2 (*St2*)			
Strategy	I.F.R.	I.F.R	R.P./Cv.R.	A.Cr.R.	A.Cr.R.	A.Cr.R.	M.Cr.R.
Representation	V.R. C.R. P.R. S.R.	V.R. C.R.	V.R. C.R. P.R.	V.R. C.R. P.R.	V.R. C.R. P.R. S.R.	V.R. P.R.	V.R.

Note. N.F.R.: No evidence of functional relationship; R.P.: Recursive pattern strategy; Cv.R.: Covariation relationship strategy; I.F.R.: Incipient correspondence functional relationship strategy; M.Cr.R.: Multiplicative correspondence relationship strategy; A.Cr.R.: Additive correspondence relationship strategy; V.R.: Verbal representation system; C.R.: Concrete representation system; P.R.: Pictorial representation system; & S.R.: Symbolic representation system.

the number of grandchildren. We observe this at the time when *Int* asked that if the number of grandchildren is not known, how many balloons would Grandma Lita need to which the student replied, "it's the double […], two for each grandchild".

Likewise, we observe that *St2* generalized this strategy using verbal representation (lines 53, 67, and 71).

Synthesis of Results of Student 1 & Student 2

In **Table 3** we summarize the work done by *St1* and *St2* in each of the seven questions of the task. Subsequently, we describe those common and different responses observed in both students.

From **Table 3**, we see that *St1* used the incipient correspondence functional relationship strategy only in question 1 and the multiplicative corresponding relationship strategy in the remaining questions. We observe that in all the questions *St1* used the verbal and pictorial representation, except in question 7 in which they only used the verbal one. Regarding the symbolic representation, *St1* used it in questions 1, 2, 3, and 6. Finally, we observe that this student generalized the multiplicative correspondence relationship and did so through verbal representation.

St2, on the other hand, used the incipient correspondence functional relationship strategy in the first two questions. In the third question, they used two strategies, one based on the recursive pattern and the other on the covariation relationship. In questions 4, 5, and 6, *St2* used an additive correspondence relationship strategy and in question 7 a strategy based on multiplicative correspondence relationship. Moreover, we observe that in all the questions they used the verbal representation strategy; a concrete representation in the first five questions; and a pictorial one in questions 1, 3, 4, 5, and 6. Also, *St2* used the symbolic representation in question 1 and question 5. Finally, they generalized the relationship strategy of multiplicative correspondence through verbal representation.

Student 1 & student 2 responses in common

From **Table 3**, we observe coincidences between both students regarding the strategy used in questions 1 and 7; while in the first they used the incipient correspondence functional relationship, in the last they used the multiplicative correspondence relationship.

Furthermore, both students used verbal representation in all their responses; they used pictorial representation in questions 1, 3, 4, 5, and 6; as well as symbolic representation in question 1 and question 5. Finally, both students verbally generalized the multiplicative correspondence relationship strategy.

Student 1 & student 2 different responses

Regarding the differences, we see that, according to **Table 3**, both students differ in the strategies used in the answers to questions 2, 3, 4, and 5. For example, St1 used the multiplicative correspondence relationship strategy while *St2* used the incipient correspondence functional relationship strategy.

In addition, *St1* continued to apply in the responses to questions 3, 4, 5, and 6 the multiplicative correspondence relationship strategy, while *St2* responded to question 3 with the recursive pattern and covariate relationship strategies, while for questions 4, 5, and 6 did so with the additive correspondence relationship strategy.

Regarding the representations used, we observed that only *St2* used the concrete one in question 1 to question 5, *St1* in question 2 used the pictorial representation, while *St2* did not. Regarding the symbolic representation, *St1* used it in questions 1, 2, 3, and 6, while *St2* did so in the answers to question 1 and question 5.

DISCUSSION & CONCLUSIONS

In this study, despite the fact that the results are not generalizable, we have shown that students at early school ages (aged five-six) are capable of giving correct answers when solving algebraic tasks focused on functional thinking. The two students in this research were able to approach the task through different strategies, highlighting those focused on the functional relationship of correspondence. For example, *St1* used the multiplicative correspondence relationship in all the questions except for the first in which he used incipient correspondence. On the other hand, *St2* used various strategies during the interview, given that, in the first questions, he used the incipient functional relationship, the recursive pattern, and covariation. But, as interview progressed, this student was able to use strategies focused on a functional correspondence relationship, both additive and multiplicative. These findings are in line with those found by Castro et al.'s (2017) research, where it is evidenced that students of these ages address similar tasks through covariation and correspondence. This is significant since these types of studies provide greater evidence to the emerging body of knowledge about abilities that early learners have to engage with algebraic notions through functional thinking.

In this line, we provide more evidence on the abilities of early age students when solving tasks in a functional context of school algebra. Despite the above, and given the size of the sample, we believe it is necessary to continue investigating the abilities and the way in which students of these ages respond to tasks similar to ones proposed here. This would make it possible to consolidate an important body of knowledge to determine the real abilities of these students as a way of impacting the teaching for the learning of mathematics in early childhood education.

It is noteworthy that both students answered the questions of the task through the multiplicative structure, with the notion of double. These students found the amount of the dependent variable (number of balloons) by doubling the amount related to the independent variable (number of grandchildren). This approach calls our attention due to two relevant aspects:

(a) similar results have been found, but with older students (Blanton & Kaput, 2004; Cañadas et al., 2016) and

(b) from a curricular context, the multiplicative structure is approached with students of ages later than those of this study (seven-eight years old onwards, MINEDUC, 2012) as is the case of multiplication by two.

This finding can be explained because the students, at the time of the interview, had previous knowledge regarding the notion of double. We believe that the results of this research are important for the teaching of mathematics in the first educational levels, since they show how students are able to become familiar with concepts associated with the multiplicative structure. This finding can be useful for future research as a way to confirm that the multiplicative structure can be approached before primary education from a functional approach to school algebra. In addition, the arithmetic calculations were easily solved by the two students, which implies that the quantities involved in each question of the task can help to promote adequate calculations in the context of the multiplicative structure.

We highlight the way in which *St2* responded to question 3, because they approached the task using two functional strategies: covariation and correspondence. This suggests that a functional thinking task can be approached from these two relationships, being a consistent indication of the manifestation of functional thinking (Morales et al., 2018). Although both students used arithmetic calculations to find the quantities requested in the different task questions, the students managed to go further, since they generalized the correspondence relationship evidenced. Specifically, they verbally stated that to find the number of balloons it was necessary to double the number of children, that is, they generalized the functional relationship. Thus, it can be deduced that these students move from arithmetic to algebraic, considering that generalization is a central element of algebraic thinking. For their part, the students were able to generalize the correspondence relationship using the idea of double in the context of the multiplicative structure. Apparently and as observed in previous research (e.g., Morales et al., 2018; Pinto & Cañadas, 2018), the correspondence relationship is easier to generalize in a functional thinking task.

Regarding the representations used by the students, we observed that they used all those proposed in this research (verbal, concrete, pictorial, and symbolic). However, we highlight that the verbal representation was the most used by both students, since they expressed a verbal answer to each question; even to address the generalization, both students did so through this system of representation. This is not extraordinary given that these representations are used by students from a very early age and in some cases, such systems are the ones they are most used to employing. On the other hand, we highlight that the pictorial, concrete and symbolic representations were used by the two students only to corroborate the answers given verbally after the mental calculation was done.

Finally, despite its exploratory nature and limitations this study offers some insight into the abilities that students of early educational ages may have to adequately respond to tasks that imply a functional relationship. Thus, this work allows us to broaden the perspective on real abilities of students in mathematics and especially those related to algebra such as functional thinking (Alsina, 2020; Clements & Sarama, 2007).

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that informed consent, voluntary participation and confidentiality were respected during this research study. The purpose of the study and all stages of the study/procedure were explained in detail to the students, their teachers and parents. Names were not noted in the students´ manuscripts so that there would be no traceable data until the end of the study.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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