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Teacher profiles in rural contexts: Multivariate characterization of algebra teaching and its configuration in differentiated praxeologies

Victoria Alejandra García Fajardo [1*], Diana Carolina Burbano González [1], Cristhian Leonardo Urbano-Leon [2]

[1] Facultad de Educación, Grupo de Investigación CIEDUS, Universidad Santiago de Cali, Cali, COLOMBIA
[2] Facultad de Ciencias Contables, Económicas y Administrativas, Universidad del Cauca, Popayán, COLOMBIA

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Abstract

The development of algebraic thinking from an early age is a priority in contemporary mathematics education, especially in rural contexts due to their structural and sociocultural conditions. Grounded in the Anthropological Theory of Didactics and hermeneutic phenomenology, this study aims to identify and characterize teacher profiles based on their algebra teaching practices and their perceptions of different types of mathematical thinking. A descriptive-exploratory multiple case study was conducted with empirical data from 16 teachers at a rural institution in western Colombia, analyzed using multivariate statistical techniques (cluster analysis, multiple correspondence analysis, and principal components analysis). Two teacher profiles were identified: one focused on digital technologies and problem-solving, and another that prioritizes traditional strategies. The findings underscore the importance of considering institutional and territorial conditions when designing contextualized pedagogical proposals that foster algebraic thinking in rural areas. Findings are transferable to similar contexts but not statistically generalizable to the broader teaching population.

Keywords: algebraic thinking, praxelogies, ATD, hermeneutic phenomenology, multivariate analysis

INTRODUCTION

The development of algebraic thinking constitutes a priority in contemporary mathematics education, as it fosters forms of generalizing, analytical, and structural reasoning that are essential from the early school years (Kaput, 2008; Kieran, 2007; Radford, 2010). This teaching process becomes significantly more complex in rural contexts, where educational conditions are often marked by material, symbolic, and institutional limitations, posing specific challenges for teaching practice. In this scenario, teaching depends not only on the curriculum. This contrasts with the deep inequalities that persist in rural education in Latin America, reflected in the precariousness of school infrastructure, limited connectivity, restricted access to didactic resources, and discontinuity in teacher training (Duarte et al., 2011; Mora Naranjo et al., 2023).

In Colombia, the gap in mathematics performance between rural and urban students exceeds 30 points and has deepened with the COVID-19 crisis (Mauris, 2022). Added to this are the difficulties in consolidating a qualified teaching body in these contexts, which limits professional development and affects educational quality (Bautista Macia, 2019). These conditions are also reflected in countries such as Ecuador, where the lack of technological and didactic resources in rural schools directly affects students' cognitive development in the field of mathematics (Cedeño et al., 2023; Mora Naranjo et al., 2023).

In this context, recognizing differentiated teacher profiles becomes particularly relevant, not only as an analytical category but also as a decisive factor in understanding how pedagogical practices are configured in highly complex environments such as rural contexts. The competencies and strategies that teachers mobilize in response to the demands of their

✉ victoria.garcia01@usc.edu.co **(*Correspondence)** ✉ diana.burbano02@usc.edu.co ✉ cristhianleon@unicauca.edu.co

Contribution to the literature

- The study points out that differentiated teacher profiles are identified in algebra teaching in rural contexts.
- The multivariate analysis in this research reveals variation in didactic praxeologies.
- Empirical evidence of the article highlights the territorial impact on algebra teaching.
- Teaching practices provided in this study shape the configuration of differentiated praxeologies.

environment are not homogeneous; rather, they reflect both structural conditions and their own professional trajectories.

Therefore, it is necessary to study the teaching of algebra from a perspective that acknowledges both the material and formative limitations faced by rural teachers and the pedagogical potential that emerges from their adaptive strategies, local knowledge, and experience.

To analyze this complexity, the Anthropological Theory of the Didactic (ATD) makes it possible to understand mathematical activity and teaching as social and human systems, using the concept of praxeology (Bosch & Gascón, 2014; Chevallard, 1999) to analyze teaching practices in precarious contexts. Complementarily, phenomenological–hermeneutic approaches are valuable for deeply interpreting how teachers make sense of their daily work in adverse settings, providing access to the subjective and experiential dimension of rural pedagogy (Méndez, 2014; Farfán et al., 2023).

Despite the importance of these approaches, there remains a knowledge gap regarding how specific contextual and institutional factors shape differentiated teacher profiles and their pedagogical practices in the teaching of algebra in rural settings.

This research is situated precisely within that need: to understand what teacher profiles emerge in a rural Colombian context based on teaching practices related to the development of algebraic thinking, in terms of their praxeologies and lived experiences, and how the didactic strategies aimed at developing algebraic thinking are characterized among teachers classified within different profiles.

In relation to these research questions, the main objective is to identify and characterize teacher profiles according to their educational level, professional trajectories, and their relationship with the didactic praxeologies employed in the teaching of algebra, as well as to examine how they value different types of mathematical thinking.

For this purpose, the research adopted a descriptive–exploratory case study design (see Stake, 2020) conducted at a rural educational institution located in the western region of the Cauca Department, Colombia. To identify these teacher profiles, rigorous multivariate statistical techniques were employed, such as Cluster Analysis using the k-means algorithm, Multiple Correspondence Analysis (MCA), and Principal Component Analysis (PCA). Specifically, the cluster analysis clearly distinguished two teacher profiles: one group composed of teachers under 50 years of age, holding master's degrees, open to integrating digital technologies, and promoting problem-solving–based learning environments; and another group consisting of teachers with initial training (undergraduate or specialization degrees), with longer professional trajectories, whose practices emphasize the use of concrete materials, repetitive techniques, and institutionalized tasks.

The results are discussed by integrating the foundations of the Anthropological Theory of the Didactic (ATD) and hermeneutic phenomenology to provide a comprehensive analysis of the institutional configurations of teaching practices and the lived experiences within the context of algebraic thinking instruction in rural areas.

The findings of this study contribute to a deep and contextualized understanding of the educational phenomenon, offering relevant insights for the design of effective and context-sensitive teacher training processes capable of recognizing the pedagogical and epistemological diversity present in Latin American rural settings, such as the case of Colombia.

THEORETICAL FRAMEWORK

The Anthropological Theory of the Didactic (ATD) and Teaching Praxeologies

The ATD is formulated by Chevallard (1999) and later expanded by Bosch and Gascón (2009, 2014), Bosch et al. (2006), conceives the teaching of mathematics as an institutionalized human activity shaped by social and cultural norms. From this perspective, every educational practice is analyzed as a complex system in which academic knowledge—referred to as "scholarly knowledge"—undergoes profound transformations to become "taught knowledge" through the process of didactic transposition.

The central concept is praxeology, a model that describes mathematical activity in terms of tasks, techniques, technologies, and theories. Praxeologies are determined by social and institutional constraints, a crucial condition in teaching, since knowledge is not transferred to the school setting in a neutral way but is

instead selected and transformed according to social norms and educational objectives.

In the context of the present study, the ATD provides crucial conceptual tools to analyze how rural conditions such as structural precariousness, teachers' discontinuous training trajectories, and culturally situated practices directly influence the configuration of teaching praxeologies. In other words, ATD makes it possible to understand how the teaching of algebra is shaped by the complex institutional and territorial constraints that are specific to a rural school.

Situated Development of Algebraic Thinking

The development of algebraic thinking constitutes one of the fundamental pillars of contemporary mathematics education, understood not simply as the learning of symbolic manipulation techniques but as the progressive construction of capacities to generalize, represent, and reason about mathematical relationships (Kaput, 2008; Kieran, 2004; Radford, 2010). This conception emerges in response to traditional approaches that relegated algebra to the later stages of schooling, instead proposing an early and gradual integration of algebraic reasoning modes in the first levels of education.

Authors such as Kaput (2008), Kieran (2004), and Radford (2010, 2014) have been key in formulating this perspective. From different approaches, they converge in asserting that algebraic thinking involves not only operating with symbols but also identifying patterns, constructing multiple representations, and establishing generalizations grounded in diverse and socially mediated cognitive activities.

Within this framework, it is proposed that the development of algebraic thinking should be based on three essential dimensions: the generalization of patterns and structures, the progressive formalization of symbolic representations, and argumentation and justification as practices inherent to mathematical work from early ages (Radford, 2014; Molina & Cañadas, 2020). This conception shifts teaching away from an exclusive focus on algorithms toward a model that promotes processes of meaning construction.

From this perspective, analyzing teaching practices becomes essential to understanding how institutional dynamics and pedagogical strategies in rural areas with their resource limitations and situated knowledge affect the construction of these models of algebraic thinking among students. Adopting this framework ensures that the research does not focus solely on deficiencies but also highlights teachers' adaptive strategies.

Lived Experience and Pedagogical Understanding from a Hermeneutic Phenomenological Perspective

Hermeneutic phenomenology constitutes a methodological and epistemological approach aimed at understanding human experiences as they are lived, interpreted, and imbued with meaning within situated contexts (Ricoeur, 2002; Van Manen, 2003). In the educational field, this perspective allows for the analysis of teaching practices not merely as technical acts but as manifestations of subjectivity, intentionality, and relationships with the world. Authors such as Husserl (1913) and Gadamer (1960) laid the foundations for interpreting pedagogical understanding as inherently shaped by lived experience and context.

In the field of educational research, Van Manen (2003) adapts these principles to explore pedagogical experiences, seeking to capture their deeper meaning beyond superficial descriptions.

The integration of this approach is fundamental to this case study, as it enables access to the subjective and experiential dimension of teachers (Farfán et al., 2023; Méndez, 2014). This is essential for moving beyond the mere description of their praxeologies and toward understanding the deeper meaning they attribute to their decisions and strategies within a complex and challenging environment such as the rural context.

METHODOLOGY

This study is developed under a mixed descriptive exploratory case study design (Stake, 2020), framed within the interpretive paradigm. It was carried out in a public rural educational institution located in the western region of the Cauca Department, Colombia, characterized by its territorial complexity, with seven campuses distributed across remote rural areas that are difficult to access. This geographical condition made it possible to capture the situated nature of rural teaching practices to meet the objective of identifying and characterizing teacher profiles according to their educational level, professional trajectories, and their relationship with the didactic praxeologies employed in the teaching of algebra, as well as to examine the value they assign to different types of mathematical thinking.

A mixed-methods approach was adopted, integrating qualitative analysis with quantitative multivariate techniques. This combination is not merely an addition of methods but rather an intentional triangulation designed to address the complexity of the object of study.

- Structural Perspective (ATD and Quantitative): The ATD, through the notion of praxeology, provides the structural lens to examine what: how the tasks, techniques, and didactic technologies involved in the teaching of algebra are structured within the rural context. To classify these structures, multivariate analysis techniques (Cluster Analysis, MCA, and PCA) were applied to identify patterns and establish significant groupings of teacher profiles.

- Subjective Perspective (Phenomenology): This classification is complemented by hermeneutic phenomenology, which guides the understanding of the why highlighting how teachers assign meaning to and justify their pedagogical decisions. This approach makes it possible to go beyond behavioral indicators (quantitative data) to interpret teaching practices as situated expressions of knowledge and lived experience.

The combination of these analytical perspectives made it possible to interpret the data not merely as indicators of teaching behavior but as situated expressions of knowledge, practices, and stances toward mathematical understanding. The articulation of both perspectives enables a comprehensive analysis: the profiles (quantitative/structural) are interpreted and further explored considering lived experiences (qualitative/subjective), ensuring a rigorous and contextually grounded understanding of the phenomenon.

Data Collection

The target population consisted of mathematics and primary education teachers affiliated with the focal rural school of the study. Due to the small size of the population and the difficult access conditions, a convenience sampling strategy was employed, including 16 teachers who were formally employed and actively teaching during the data collection period. The aim was to capture the experience that lived in its phenomenological depth rather than to obtain a statistically representative sample. Thus, the interpretative value that each participant contributes to understanding the educational phenomenon in its situated context (Van Manen, 2003) is what gives relevance to the sample.

Data collection was carried out through a self-administered questionnaire, completed in person under the direct supervision of the researchers between January and March 2025. The instrument was designed by an interdisciplinary team composed of researchers in mathematics education and statistics, ensuring its theoretical and contextual coherence. For validation purposes, the questionnaire was reviewed by experts in statistical analysis and by a specialist familiar with the specific rural context in which the research was conducted.

The instrument consisted of 42 questions organized into thematic sections: sociodemographic characteristics, teaching practices, instructional difficulties, pedagogical strategies, and teachers' assessment of different domains of mathematical thinking (numerical, spatial, metric, random, and variational). It included closed-ended, dichotomous, open-ended, and numerical scale items (ranging from 0 to 10), all carefully worded in a clear and contextually adapted manner, considering the rural cultural, linguistic, and educational environment in which it was administered.

Regarding reliability, a technical review was conducted to eliminate or reduce formulation bias, ambiguity, or social desirability effects, ensuring a homogeneous interpretation of the items. Additionally, a pilot phase was implemented with teachers from an institution with similar characteristics, allowing for the refinement of the questionnaire's language and format prior to its final administration. Since the study follows a descriptive-exploratory approach and does not seek to make statistically generalizable inferences, inferential biases were not considered in the data treatment. The purpose of the instrument was not to establish correlations or generalizations but rather to describe relevant patterns and provide a comprehensive understanding of the contextualized pedagogical practices of the participating teachers.

Data Processing and Descriptive Analysis

The data obtained from the survey were digitized and organized into a database for processing using the statistical software R (version 4.3.1). Variable names were standardized through functions from the janitor package, followed by the transformation of certain dichotomous responses into binary format (1 = yes, 0 = no) to facilitate analysis. Since no unanswered questions were identified, all observations were included, and no data cleaning or imputation techniques were required.

The descriptive analysis included the calculation of frequency distributions for categorical variables and measures of central tendency and dispersion for quantitative variables. The results are presented through visualizations such as histograms, box plots, horizontal bar charts, and word clouds to enhance interpretability. Given the focus of the study, particular attention was paid to variables related to teachers' educational level, the use of specific pedagogical practices, and their perceptions of difficulties in teaching mathematics.

Textual Analysis

The open-ended responses were analyzed through an exploratory textual treatment that involved stopword filtering and frequency analysis, allowing for the visualization of the most recurrent terms using a word cloud. The tidytext package in R was employed for this process.

Exploratory Multivariate Analysis

For the analysis of teaching practices and reported difficulties, multivariate statistical techniques were applied with a strictly exploratory and descriptive focus. Given the small size of the participant population, no inferential or general objectives were pursued; instead, the aim was to identify relevant patterns that would

Table 1. Sociodemographic characteristics of the teaching staff

Sociodemographic variables	Category	Frequency	Percentage
Gender	Female	14	87
	Male	2	13
Age	39-54	6	38
	55-70	10	63
Primary education	Elementary	13	81
	Secondary and high school	3	19
University education	Vocational education	5	31
	Specialization	7	44
	Master's degree	4	25

allow for a deeper understanding of the internal diversity within the group of teachers analyzed. Multivariate techniques were selected because they enable the exploration of patterns and sequences across groups or subgroups.

The techniques employed included Cluster Analysis (CA), Multiple Correspondence Analysis (MCA), and Principal Component Analysis (PCA). In this study, variables were organized into thematic matrices. Patterns of use were explored for six key teaching practices: problem-solving in contextualized situations, sequencing, multiple representations, use of concrete materials, collaborative work, and educational technology. Frequencies were calculated in both absolute and relative terms, at the general level and across subgroups identified in the cluster analysis.

Cluster Analysis

Based on information about teaching practices, an input matrix was built to perform cluster analysis. The k-means algorithm was used, selecting the optimal number of clusters through the elbow method on the within-group sum of squares, using the fact extra package in R. Each cluster was then characterized in terms of sociodemographic variables and the valuation of mathematical thinking types. For methodological background on this technique, see Everitt et al. (2011) and Cuadras (1996).

Multiple Correspondence Analysis

The pedagogical practices coded in the dataset are categorical in nature (use/non-use type). Therefore, another multivariate analysis technique described by Greenacre (2007) and Cuadras (1996) was applied MCA which enabled the visualization of association patterns between pedagogical practices and teachers. This technique also allowed for the simultaneous representation of both response modalities and individuals within a two-dimensional factorial space, thus facilitating the interpretation of conceptual proximities. The MCA was performed using the FactoMineR package, and visualizations were generated with fact extra. In applying this technique, the first two dimensions jointly explained more than 50% of the total variability, which was considered acceptable given the exploratory nature of the research.

Analysis of Numerical Ratings and PCA

The numerical responses (ranging from 0 to 10) concerning the perceived importance of different types of mathematical thinking were analyzed using descriptive measures and boxplot visualizations. Subsequently, to detect underlying patterns in these ratings, a multivariate technique known as PCA was applied (see Cuadras, 1996; James et al., 2021). The use of this technique resulted in an explanation of more than 70% of the total variance in the first principal component, suggesting a common axis of "general importance attributed to mathematical thinking." These results were visualized through biplot graphs and individual projections, which illustrated both global consistency and individual divergences among teachers.

RESULTS

The following section presents the results, focusing on the sociodemographic characterization, the identification of teacher profiles derived from multivariate analysis, and the meaning of teaching practices based on cluster analysis. It also addresses the difficulties associated with mathematics teaching and the dimensions of mathematical thinking promoted through teaching practices. These results will be further discussed in detail in the Discussion section.

Sociodemographic Characterization of Teachers

The characterization presented in **Table 1** reveals a predominantly female composition, with 87% women and 13% men.

Regarding age distribution, it was observed that 63% of the teaching staff are between 55 and 70 years old, while 38% are between 39 and 54 years old. The average age is 54.06 years, with a median of 56.50 years, indicating a teaching workforce with extensive professional experience and long-standing trajectories in the field of education. This can be interpreted as a sedimentation of meanings and professional experiences that directly influence how educational practice is conceived and carried out.

With respect to professional trajectory, 81% of teachers reported having developed most of their careers in primary education, while only 19% have worked in secondary or upper-secondary education. This finding is key, as it places the focus of the transition from arithmetic to algebra at the early stages of schooling, where the first approaches to algebraic thinking are formed.

In terms of academic qualifications, 44% of teachers hold a specialization degree, 25% have a master's degree, and 31% possess only an undergraduate or bachelor's degree. Although these figures reflect progress in teacher qualification processes, they also highlight challenges related to access to advanced training, especially in rural contexts. These limitations likely have an impact on the development of didactic strategies that foster sustained and reflective practices for the advancement of algebraic thinking.

Teaching Practices and Difficulties in Mathematics Instruction

Figure 1 shows the frequency with which the participating teachers reported using various teaching practices aimed at developing algebraic thinking. Collaborative work stands out as the most frequently mentioned practice, with a total of 13 references, followed by using concrete materials, which received 12 mentions. Didactic sequences and problem solving were each mentioned 11 times, while multiple representations (symbolic, graphical, and verbal) were reported on 10 occasions. Conversely, the use of digital technologies was the least frequent practice, with only 4 mentions.

Figure 2 presents the main difficulties reported by teachers regarding the teaching of mathematics in their school context. The most frequent difficulty, mentioned by 10 teachers (62.25% of the sample), refers to the lack of adequate teaching materials to support the teaching–learning process. In second place, with 8 mentions, is the insufficient prior knowledge of students, suggesting

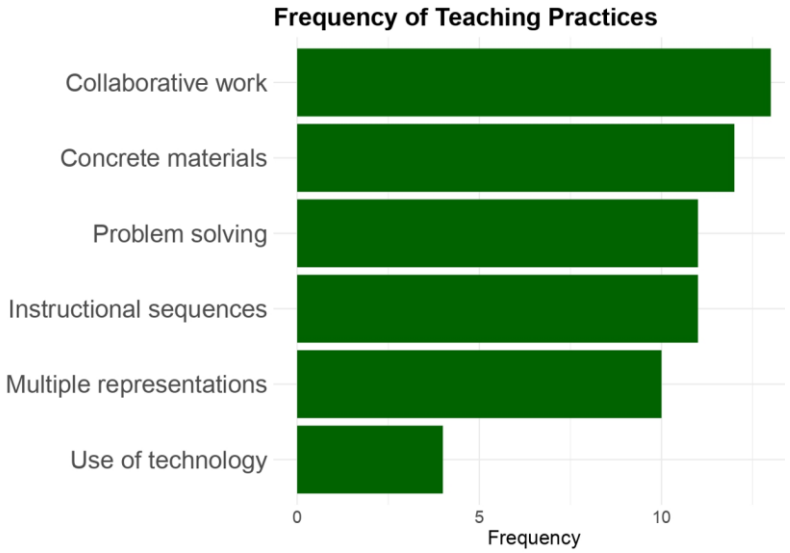

Figure 1. Frequency of use of teaching practices (Source: Authors' own elaboration)

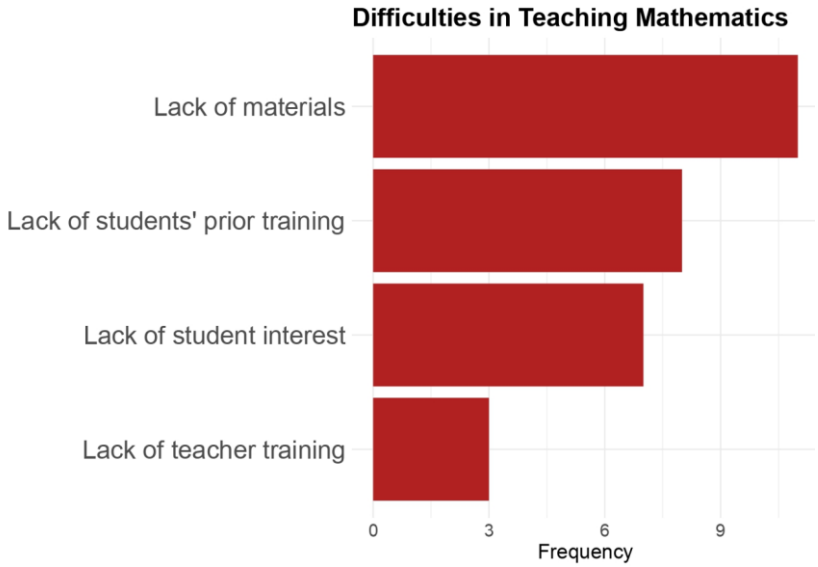

Figure 2. Difficulties in teaching mathematics (Source: Authors' own elaboration)

Figure 3. Word cloud on difficulties in mathematics teaching and strategies to address them (Source: Authors' own elaboration)

significant conceptual gaps that hinder the construction of new learning.

Lack of student interest was also reported as a significant difficulty, with seven mentions revealing tensions in the relationship between students and mathematical knowledge. Finally, three teachers pointed out the lack of specialized teacher training as a limitation that hinders their ability to effectively address classroom challenges.

These results provide a characterization of structural challenges that go beyond the individual will of teachers, highlighting the material and training precariousness that mediates the teaching of mathematics in the rural contexts participating in the study. The subsequent discussion will address these difficulties from theoretical perspectives that help us to understand them as phenomena of the educational field, rather than merely as isolated problems.

Analysis of Open-Ended Responses: The Meaning of Pedagogical Strategies

Figure 3 shows the textual analysis of the open-ended responses provided by teachers to the question regarding the difficulties in teaching mathematics and the strategies they use to overcome them.

First, the prominence of terms such as material, didactic materials, use, and implement reveals a recurring concern regarding the availability, appropriateness, and effective use of teaching resources. This emphasis reinforces the findings of the previous quantitative analysis, in which the lack of adequate materials was identified as one of the main difficulties.

Teacher Profiles: Cluster Analysis

The number of clusters was determined using the elbow method, through which two clusters were selected, as this is the point where the first sharp change in the slope occurs, as shown in **Figure 4**.

The k-means method identified two teacher groupings (clusters), which can be defined as profiles. The first cluster is characterized by being composed of teachers with master's degrees; in the study it was called innovative profile. This group is mostly younger and shows a high frequency in the use of digital technologies, collaborative work, multiple representations, and an orientation toward problem solving. In contrast, the second cluster is composed mainly of teachers with

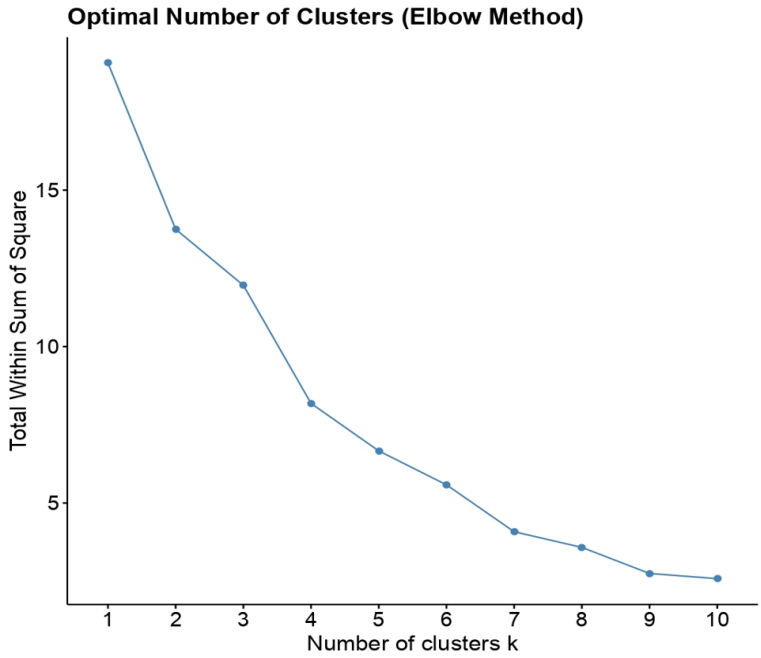

Figure 4. Optimal number of clusters (Source: Authors' own elaboration)

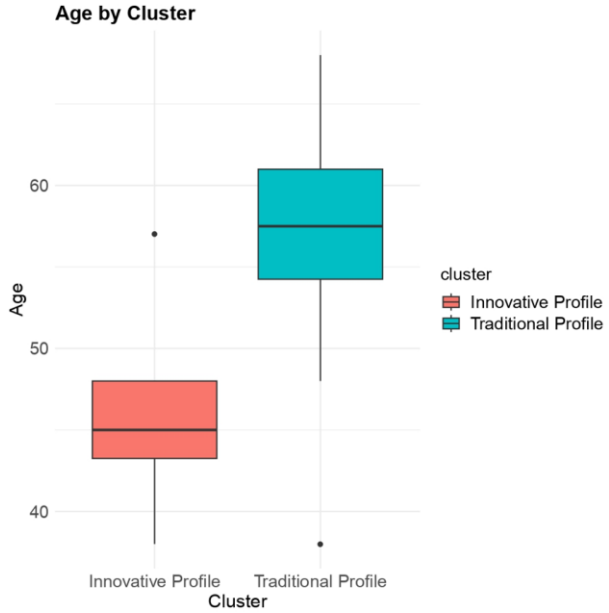

Figure 5. Age by cluster (Source: Authors' own elaboration)

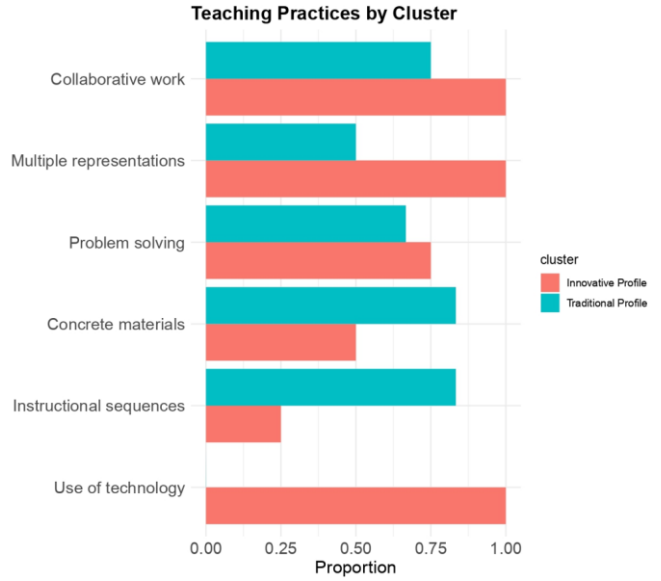

Figure 6. In red, the innovative profile; in blue, the traditional profile (Source: Authors' own elaboration)

professional or specialization degrees, who were called traditional profile.

They tend to employ more conventional strategies, such as the use of concrete materials, predefined didactic sequences, and limited use of technology in the classroom.

Figure 5 shows a comparative boxplot of the ages of the teachers grouped by cluster. It can be observed that the innovative group, represented by the red cluster, consists of younger teachers, with a median age around 45 years, while the traditional group, represented by the blue cluster, includes older teachers, with a median age close to 58 years. This suggests different professional trajectories that may influence the ways of approaching algebra teaching and the configuration of their didactic praxeologies. The profiles identified here are described in detail in **Table 2**.

In **Figure 6**, the traditional group appears to operate from functional type praxeologies, oriented toward immediate resourcefulness within an environment that imposes structural constraints, whereas the innovative group tends to adopt praxeologies with stronger theoretical grounding and greater openness to innovation, possibly mediated by more recent formative experiences.

Likewise, **Figure 7** shows the proportion of teachers grouped according to their level of academic training. Cluster 1, corresponding to the innovative group, is composed exclusively of teachers holding a master's degree, whereas Cluster 2, corresponding to the traditional group, presents a more diverse composition, mostly made up of teachers with a specialization or undergraduate professional degree. This pattern clearly defines two well-differentiated profiles among the group of teachers participating in the study.

Relationship Between Teaching Practices and Teacher Profiles: Multiple Correspondence Analysis (MCA)

The MCA is another multivariate methodology employed in this study. Its purpose goes beyond comparing individual teachers; it seeks to compare the

Table 2. Characteristics by profile

Variable	Traditional profile	Innovative profile
Average age	58 years	45 years
Academic training	Diverse (mainly specialization or undergraduate)	Exclusively with master's training
Theoretical orientation	Little theoretical foundation	High theoretical foundation
Strategies used	Oral explanation, conventional practices	Problem solving, didactic innovation
Use of digital technologies	Little or no	Frequent and with pedagogical meaning
Relationship with concrete materials	Frequent	Less use, more focused on the symbolic
Use of multiple representations	Infrequent	Active use
Focus on problem solving	Limited	Strong presence
Collaborative work	Moderate	Strong presence
Teaching sequences	Frequent use of structured sequences	More flexible and adaptive use

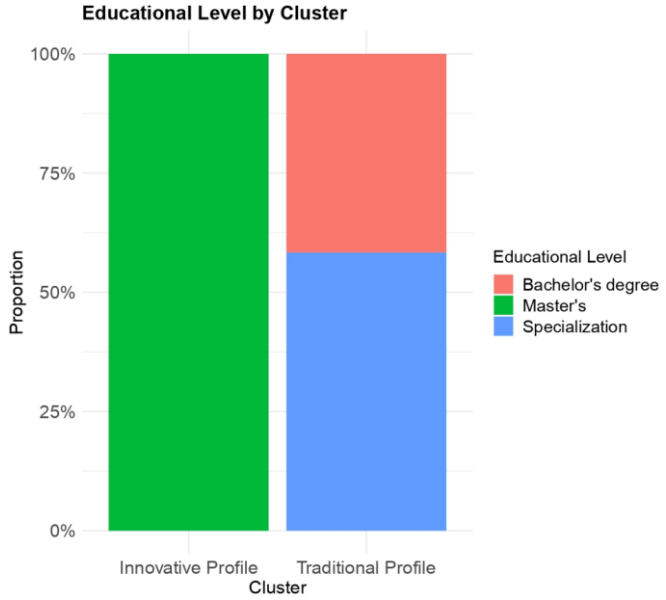

Figure 7. Academic level by cluster (Source: Authors' own elaboration)

groups of teachers identified and their behavioral patterns regarding teaching practices.

Figure 8 expands the understanding of the teacher profiles obtained through cluster analysis by graphically representing the association between categorical variables related to pedagogical practices and the characteristics of the identified profiles. Two main dimensions are revealed, which explain a significant portion of the total data variability: Dimension 1 accounts for 33.5%, and Dimension 2 for 22.8%.

In the two-dimensional plane, a clear separation between the two groups of teachers is evident, confirming and visually supporting the profiles described in the cluster analysis. Group 1, located in the upper right quadrant, is associated with younger teachers, those holding a master's degree, and those whose practices are centered on pedagogical innovations such as the use of digital technologies, problem-solving, multiple representations, and collaborative work. In contrast, Group 2, situated in the lower left quadrant, is linked to teachers with lower academic qualifications, greater accumulated experience, and a preference for more conventional practices, such as the use of concrete materials, traditional didactic sequences, and oral explanations.

To deepen the analysis of pedagogical practices oriented toward the development of mathematical thinking, the same analysis was applied, focusing on how teachers evaluated various approaches and strategies: problem-solving, use of concrete materials, didactic sequences, collaborative work, multiple representations, and technology. This made it possible to identify clusters of coexisting variables within practice that reflect different ways of structuring mathematical thinking in the classroom, as illustrated in **Figure 9**.

The results show that the first two dimensions of the MCA jointly explain more than 50% of the total

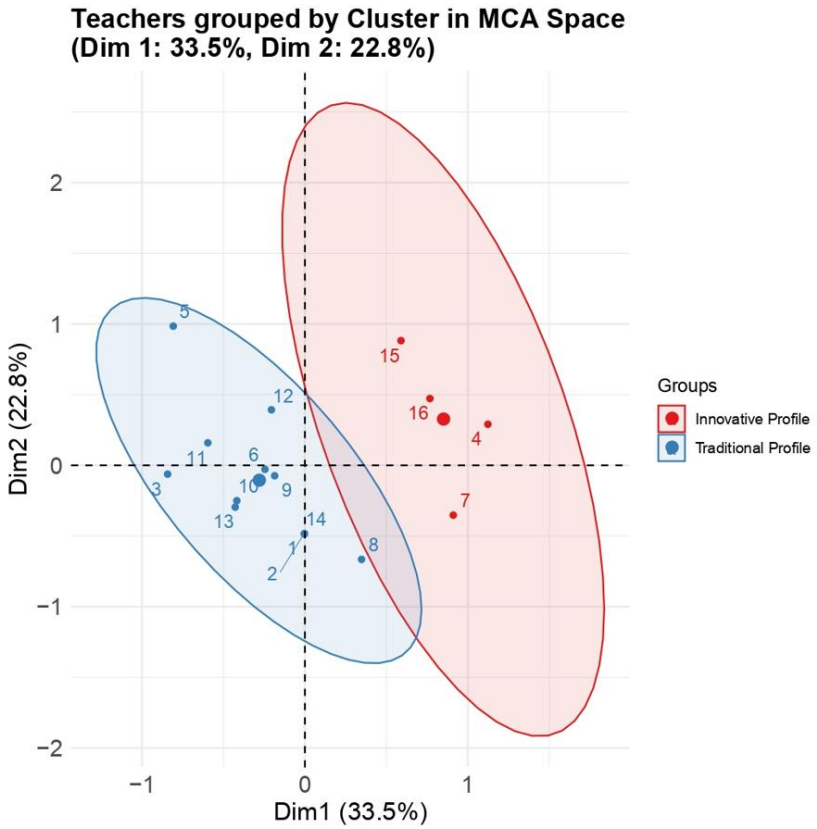

Figure 8. Correspondence analysis between academic background and teaching profiles (The red oval represents the innovative profile, and the blue oval represents the traditional profile) (Source: Authors' own elaboration)

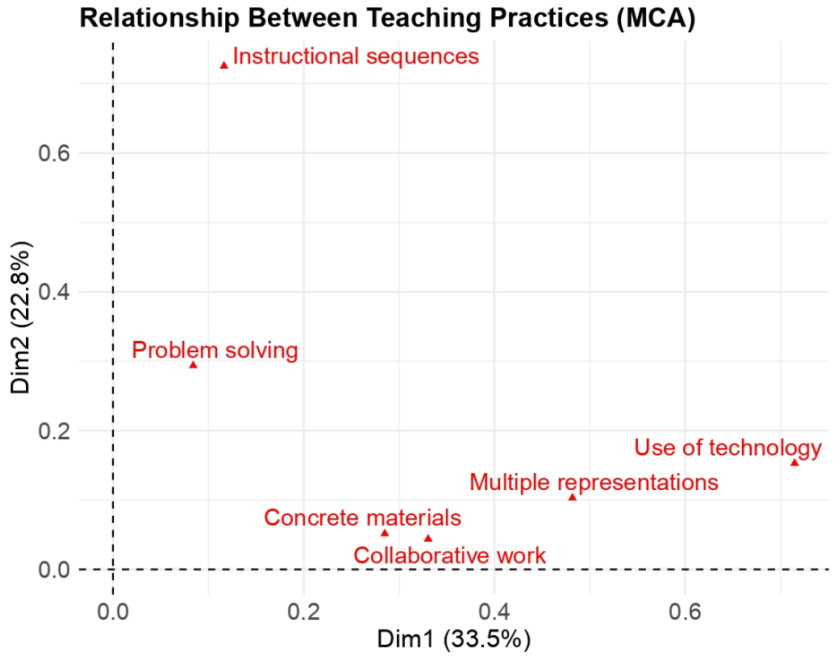

Figure 9. Correspondence analysis between pedagogical strategies and profiles (Source: Authors' own elaboration)

variability, allowing a clear differentiation between the two groups of teaching practices (profiles) aligned by cluster. Dimension 1 is associated with strategies that have a strong symbolic and representational component (problem-solving, multiple representations, and use of technology), whereas Dimension 2 is linked to more situated and material-oriented strategies (use of concrete materials, collaborative work, and structured sequences).

These findings provide evidence of an association between teacher education and the types of tools they use, showing a relationship among age, academic background, pedagogical strategies used in classroom practice, and educational level. It also reveals that teaching practices are not merely repetitive technical actions, but rather meaningful choices shaped by teachers' formative trajectories, institutional experiences, and relations of power, knowledge, and discourse within the school context.

Teachers' Assessment of Types of Mathematical Thinking

One of the aspects addressed in the questionnaire was the importance teachers assign to different types of mathematical thinking: numerical, spatial, metric, random (probabilistic), and variational. Teachers rated each of these domains on a 0-10 scale. **Figure 10** presents the overall results through a boxplot diagram.

The numerical, metric, and spatial types of mathematical thinking were the most highly valued by the group of teachers, with high medians (between 8 and 10) and low dispersion, suggesting a broad consensus regarding their relevance in mathematics teaching. The variational thinking dimension was also positively

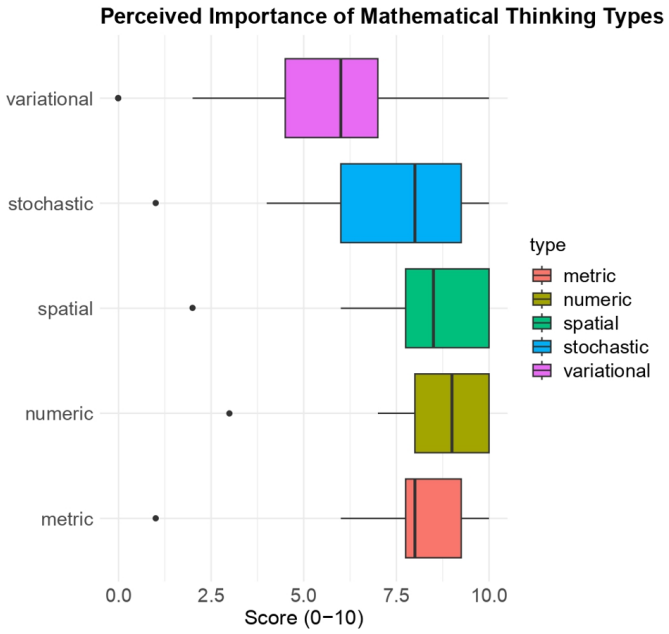

Figure 10. Importance assigned to types of mathematical thinking (Source: Authors' own elaboration)

valued, though with greater variability (some teachers rated it as low as 2), which may reflect differences in understanding or classroom implementation.

In contrast, random (probabilistic) thinking displayed the widest dispersion and several low scores, which may indicate limited familiarity, less frequent application in teaching practice, or weaker formal preparation in this domain.

This pattern is also evident in **Figure 11**, which shows a strong positive correspondence among most types of mathematical thinking, especially between numerical and metric, as well as metric and variational. Conversely, the perception of random thinking exhibits

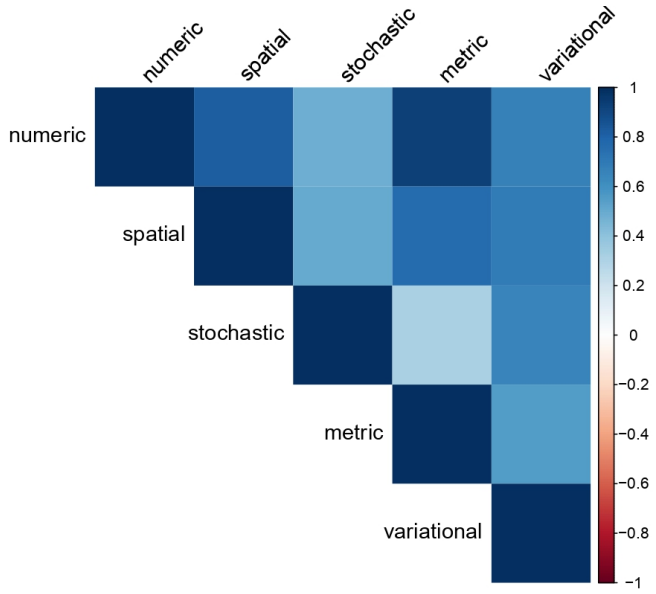

Figure 11. Correlation map among types of mathematical thinking valued by teachers (Source: Authors' own elaboration)

weaker correlations with the others, suggesting greater divergence in how teachers value this dimension. This trend will be analyzed in greater detail in the Principal Component Analysis (PCA) presented in the following section.

Subsequently, a PCA was applied to the scores assigned to each type of mathematical thinking, as shown in **Figure 12**. The first component explains 72.1% of the variance, suggesting the presence of a common axis of overall valuation across the different types of thinking.

The second component (16.6%) indicates a slight opposition between random (probabilistic) thinking, located on the negative side, and the numerical, metric, and spatial types of thinking, positioned on the positive side of the axis.

Teacher 8, located farther from the main group, may represent an atypical profile, possibly due to extreme or unbalanced scores favoring a specific type of mathematical thinking.

Overall, the PCA results suggest that there is a strong internal consistency in how teachers value the different types of mathematical thinking as indicated by the alignment of the vectors while also revealing some individual differentiation, particularly at the extremes of the first component.

When differentiating the results by teacher cluster, as shown in **Figure 13**, significant contrasts were observed. Both groups assigned high ratings (median above 7) to all five types of mathematical thinking; however, the innovative group (Cluster 2) gave higher average scores, particularly for variational and numerical thinking. In contrast, the traditional group (Cluster 1) showed greater dispersion and a lower median in their responses, especially regarding random thinking, suggesting a more heterogeneous valuation or less consensus within the group. This may indicate lower familiarity or a less structured understanding of this dimension of mathematical thinking.

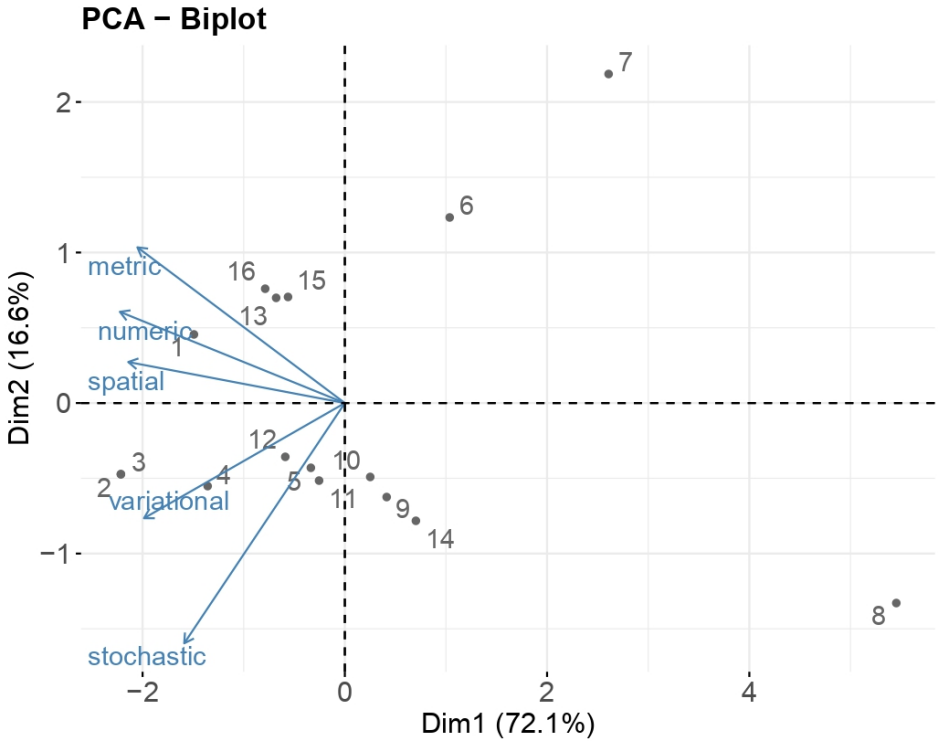

Figure 12. PCA applied to the five important variables. The arrows represent the directions of maximum variability for each type of mathematical thinking (Source: Authors' own elaboration)

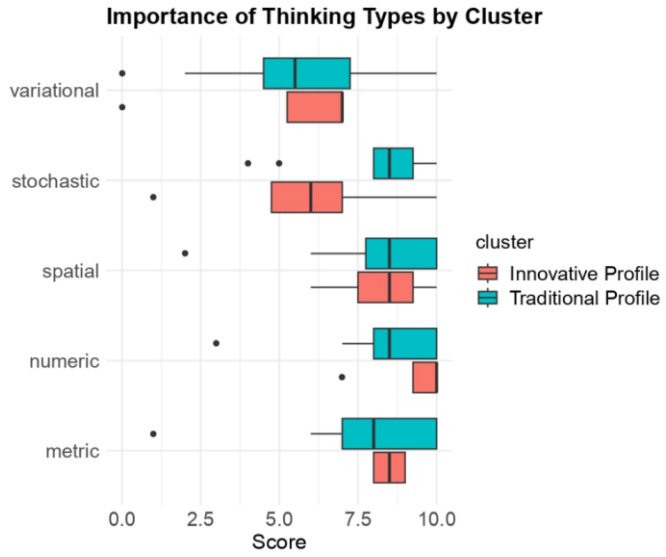

Figure 13. Distribution of scores assigned by teachers to the types of mathematical thinking, differentiated by cluster (Source: Authors' own elaboration)

These graphs reveal an overall positive perception among teachers regarding the different types of mathematical thinking. However, there are notable variations between teacher groups and certain individual differences, particularly concerning random thinking. These differences could serve as valuable inputs for designing differentiated teacher training strategies, tailored to the profiles and professional trajectories identified in this study.

DISCUSSION OF RESULTS

As presented in the methodology section, the use of multivariate analysis was carried out for strict exploration and descriptive purposes, in line with the study's qualitative approach. This strategy allowed the identification of significant groupings of teachers whose practices reflect differentiated configurations of didactic praxeologies linked to the teaching of algebra.

In this sense, the discussion is structured around the teacher profiles identified through multivariate analysis, revealing two configurations of algebra teaching practices. This integration of quantitative data and qualitative interpretation drawing on the Anthropological Theory of Didactics (ATD) and hermeneutic phenomenology goes beyond a mere description of practices to understand their situated meaning and the formative, institutional, and epistemological dynamics that underpin them.

Teacher Profiles: The Tension Between Innovation and Tradition

Cluster analysis allowed the identification of two clearly differentiated teacher profiles, reflecting distinct configurations of their algebra teaching practices (praxeologies). The innovative group consists of teachers with higher academic qualifications (master's degree), younger age (< 50 years), and openness to integrating digital technologies and problem-solving environments. Their practices are closer to a complex reconstruction of knowledge, articulating techniques and technologies with greater didactic grounding. In contrast, the traditional group includes teachers with lower formal education (undergraduate/specialization) and longer professional trajectories (> 54 years), whose practices focus on the use of concrete materials, repetitive techniques, and pre-established institutional tasks. Their praxeologies tend to be routine based, relying more on experience than on explicit didactic theory.

The coexistence of these profiles highlights the internal diversity of the rural teaching community and the need to move beyond homogenizing perspectives. The traditional group, while operating with less systematized techniques, has built its practice in contexts of scarcity and vulnerability, where practical knowledge is crucial to sustaining teaching. The innovative group, on the other hand, represents a pedagogical agency that seeks experimentation and knowledge dialogue (Arzarello et al., 2012; Boaler, 1993).

Structural Constraints and Teaching Practices

The findings reaffirm that structural limitations (lack of resources, low connectivity, insufficient specialized training) directly impact the configuration of algebra teaching (Bautista Macia, 2019; Mora Naranjo et al., 2023). From the perspective of ATD, this reality translates into the simplification of teacher praxeologies: the need to prioritize classroom management over knowledge construction leads to functional techniques such as repetition or the use of improvised materials.

This tension is especially visible in the traditional group. The predominantly female sample, with long professional trajectories (average age over 54), implies a sedimented professional habitus (Bourdieu, 2002). While these trajectories are valuable due to their contextual knowledge, they may also generate resistance to adopting new digital mediations, a phenomenon that can be interpreted as a structural effect, where lower cultural capital hinders access to updated didactic technologies (Bolea et al., 2001).

Didactic Mediations and Lived Experience

Algebra teaching in rural contexts is characterized by the adaptive creativity of teachers. One of the most relevant findings of the study lies in how rural teachers structure their pedagogical practices for teaching algebra, particularly regarding mediations, representations used, and strategies for mathematical generalization. The analysis of open-ended responses, combined with the practice patterns identified through clustering, reveals a heterogeneous landscape that highlights both teacher creativity and the structural

limitations of the rural educational system. This is manifested in the use of local materials, games, recycled resources, and dialogue with families; this orientation toward the everyday and accessible demonstrates a pedagogical disposition focused on connection with the territory and students lived realities. Through handcrafted resources, concrete examples, and playful dynamics, teachers adapt school content to the reality of their students, mediating from a situated rationality, yet the study confirms a key difference in the use of mediations.

The innovative group employs more elaborate strategies, incorporating multiple representations (graphic, symbolic) and digital resources, which is essential for the transition from arithmetic to algebraic reasoning (Kaput, 2008). Their praxeologies are more systematic and theoretically grounded. The traditional group focuses on empirical experience and direct transmission; their practices reflect an "implicit technology" (Chevallard, 1998), where decisions are practical but lack an explicit formal didactic theory.

These differences should not be read as deficiencies but as lived experiences, interpreted through a phenomenological lens (Van Manen, 2003), reflecting how teachers give meaning to their practice in rurality. The use of handcrafted resources is a way of sustaining and nurturing mathematics in contexts of vulnerability.

The contrast between both profiles highlights the internal diversity of the rural teacher cohort as well as the need to move beyond homogenizing views that treat rural contexts as uniform blocks. Each cluster reveals not only differences in terms of techniques but also divergences in the ways teachers give meaning to teaching, relate to mathematical knowledge, and configure their professional roles in these contexts.

As Boaler (1993) emphasizes, it is not only about what content is taught, but how the context deeply conditions the interpretation and application of mathematical knowledge. Thus, the traditional group should not be seen as deficient but as carriers of practices coherent with their conditions of possibility. Similarly, the innovative group represents a disposition oriented toward didactic openness, favored by greater availability of cultural and institutional capital. In this sense, teaching practices should not be read as isolated technical units, but as deeply situated configurations, influenced by the environments in which they develop and by the lived relationships they construct. In the context of this study, it is important to note that these interactions effectively shape the clearly differentiated teacher profiles identified in the findings.

Taking this into account, the MCA allows not only the identification of quantitative differences but also the revelation of structures of thought and action that traverse the teacher cohort, which must be interpreted from an integral perspective that articulates theory, subjectivity, and field.

Implications and Emerging Questions

The coexistence of differentiated profiles in algebra teaching presents significant challenges for teacher professional development in rural contexts. How can professional development programs be designed to value the experience of the traditional group while simultaneously promoting critical reflection and the incorporation of robust didactic frameworks? What tensions arise when institutional discourses promote practices that do not always align with contextual realities?

These findings invite reflection that goes beyond the purely technical plane and situates itself within the debate on curricular justice and epistemic equity in historically marginalized territories. Understanding these profiles, mediated through ATD and phenomenology, allows for advancing pedagogical proposals that are simultaneously structurally relevant and humanly meaningful.

CONCLUSION

The study allowed the identification of two differentiated teacher practice profiles, demonstrating the internal diversity of the teaching staff. The innovative profile is characterized by the systematization of strategies, the intentional use of technological resources and multiple representations, and the coordinated planning of mathematical tasks. The traditional profile relies on experience, practical knowledge, and the use of local materials and contextually adapted pedagogical strategies, albeit with less epistemological grounding, exhibiting operational practices adapted to resource scarcity.

Far from establishing a hierarchical opposition between the two groups, this distinction emphasizes that didactic practices are deeply conditioned by structural limitations and curricular tensions in the rural environment.

This analysis, carried out through multivariate techniques and phenomenological interpretation of lived experiences, is fundamental for designing more contextually relevant teacher development processes, capable of respectfully engaging with existing practices rather than imposing homogeneous models disconnected from the context.

In relation to the types of mathematical thinking, the results show a broadly positive valuation of numerical, metric, and spatial thinking, contrasted with greater dispersion around random and variational thinking. This finding is crucial for guiding the content and approaches in future teacher training.

Furthermore, this study allowed an approach to the lived experiences of rural teachers as legitimate sources of knowledge. Far from reducing teaching to a set of applied techniques, this perspective highlighted the meaning teachers give to their practice, their ethical connection with mathematical knowledge, and their sustained effort to create learning conditions in contexts with challenging geographical and social conditions.

The dialogue between the multivariate approach, employed strictly for exploratory-descriptive purposes rather than inferential ones, and the phenomenological-hermeneutic interpretation of practices, provided a deeper and more situated understanding of algebra teaching from rural teaching practices, emphasizing the need to recognize and strengthen local praxeologies as a foundation for teacher professional development and for designing contextualized guidelines that facilitate the development of algebraic thinking.

Finally, the results obtained in this study go beyond the description of a particular educational institution and provide a solid foundation for designing effective and context-sensitive interventions. In this sense, the main value of this study lies in the following aspects:

1) Teacher recognition: The need to recognize and strengthen the strategies and local knowledge of rural teachers is highlighted as a starting point for any professional development process;
2) Situated curriculum design: The findings constitute an essential input for developing curricular proposals and didactic guidelines that are more sensitive to the real conditions and resources of rural environments;
3) Focus on training: It is necessary to design accompaniment programs that promote critical reflection on traditional practices, facilitating the transition towards the integration of didactic technologies and the management of multiple representations, essential elements for the development of algebraic thinking.

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REFERENCES

Arzarello, F., Ferrara, F., & Robutti, O. (2012). Mathematical modelling with technology: The role of dynamic representations. *Teaching Mathematics and its Applications: An International Journal of the IMA, 31*(1), 20-30. https://doi.org/10.1093/teamat/hrr027

Bautista Macia, M. (2019). La formación en servicio de los maestros rurales de Colombia [In-service training of rural teachers in Colombia]. *Revista de la Universidad de La Salle, 1*(79), 67-89. https://doi.org/10.19052/ruls.vol1.iss79.4

Boaler, J. (1993). Encouraging the transfer of 'school' mathematics to the 'real world' through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*, 341-373. https://doi.org/10.1007/BF01273906

Bolea, P., Bosch, M., & Gascón, J. (2001). La transposición didáctica de organizaciones matemáticas en proceso de algebrización: El caso de la proporcionalidad [The didactic transposition of mathematical organizations in the process of algebraization: The case of proportionality]. *Recherches en Didactique des Mathématiques, 21*(3), 247-304. https://revue-rdm.com/2001/la-transposicion-didactica-de

Bosch, M., & Gascón, J. (2009). Aportaciones de la teoría antropológica de lo didáctico a la formación del profesorado de matemáticas de secundaria [Contributions of the anthropological theory of didactics to the training of secondary mathematics teachers]. In *Investigación en educación matemática XIII*, pp. 89-114. Dialnet. https://dialnet.unirioja.es/servlet/articulo?codigo=3628647

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs, & S. Prediger (eds.), *Networking of theories as a research practice in mathematics education. Advances in mathematics education* (pp. 67-83). Springer International Publishing. https://doi.org/10.1007/978-3-319-05389-9_5

Bosch, M., García, F. J., Gascón, J., & Ruiz Higueras, L. (2006). La modelización matemática y el problema de la articulación de la matemática escolar. Una propuesta desde la teoría antropológica de lo didáctico [Mathematical modeling and the problem of the articulation of school mathematics. A proposal from the anthropological theory of the didactic]. *Educación Matemática, 18*(2), 37-74. https://doi.org/10.24844/EM1802.02

Bourdieu, P. (2002). *Campo de poder, campo intelectual* [Field of power, intellectual field]. Montressor Jungla Simbólica. https://extension.intecunigua jira.edu.co/pluginfile.php/29130/mod_resource/content/1/Campo%20de%20poder%2C%20campo

%20intelectual%20%28Bourdieu%2C%202002%29.pdf

Cedeño-Molina, M. F., Pazmiño- Pazmiño, H., Mina-Quiñonez, A. A., Raptis-Estupiñán, K. V., & Revelo-Chicaiza, P. A. (2023). Diferencias de prácticas pedagógicas y aprendizajes del área de matemática entre zonas urbanas y rurales del Ecuador [Differences in pedagogical practices and mathematics learning between urban and rural areas of Ecuador]. *MQRInvestigar, 7*(1), 36-53. URL: https://doi.org/10.56048/MQR20225.7.1.2023.36-53

Chevallard, Y. (1998). *La transposición didáctica: Del saber sabio al saber enseñado* [The didactic transposition: From scholarly knowledge to taught knowledge] (3rd ed.). Aique Grupo Editor. http://yves.chevallard.free.fr/spip/spip/rubrique.php3?id_rubrique=3

Chevallard, Y. (1999). El análisis de las prácticas docentes en la teoría antropológica de lo didáctico [The analysis of teaching practices in the anthropological theory of the didactic]. *Recherches en Didactique des Mathématiques*, *19*(2), 221-266. http://yves.chevallard.free.fr/spip/spip/article.php3?id_article=60

Cuadras, C. M. (1996). *Métodos de análisis multivariante* [Multivariate analysis methods] (3rd ed.). EUB.

Duarte, J., Gargiulo, C., & Moreno, M. (2011). Infraestructura escolar y aprendizajes en la educación básica latinoamericana: Un análisis a partir del SERCE [School infrastructure and learning in Latin American basic education: An analysis based on SERCE]. BID. http://doi.org/10.18235/0010286

Everitt, B. S., Landau, S., Leese, M., & Stahl, D. (2011). Cluster analysis (5th ed.). John Wiley & Sons. https://doi.org/10.1002/9780470977811

Farfán Pimentel, D. E., Huerto-Caqui , E., Asto-Huamaní, A. Y., Sanabria-Rojas, L. G., Sánchez-Glorio, J. F., Lizandro-Crispín, R., Fuertes-Meza, L. C., & Farfán-Pimentel, J. F. (2023). Aporte de la hermenéutica y la fenomenología en la investigación: Una reflexión teórica [Contribution of hermeneutics and phenomenology in research: A theoretical reflection]. *Ciencia Latina Revista Científica Multidisciplinar*, *7*(3), 4064-4075. https://doi.org/10.37811/cl_rcm.v7i3.6466

Gadamer, H. G. (1960). Verdad y método [Truth and method]. Ediciones Sígueme. https://sonocreatica.org/wp-content/uploads/2021/02/Gadamer-Verdad-y-Metodo-II.pdf

Greenacre, M. (2007). *Correspondence analysis in practice* (2nd ed.). Chapman and Hall/CRC. https://doi.org/10.1201/9781420011234

Husserl, E. (1913). Ideas relativas a una fenomenología pura y una filosofía fenomenológica [Ideas concerning a pure phenomenology and a phenomenological philosophy] (1st ed.). Editorial Universitaria, Madrid. https://books.google.com.co/books?id=ptdVnwEACAAJ

James, G., Witten, D., Hastie, T., & Tibshirani, R. (2021). An introduction to statistical learning (2nd ed.). Springer. https://doi.org/10.1007/978-1-0716-1418-1

Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 703-756). Information Age Publishing. https://doi.org/10.4324/9781315097435-2

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator*, *8*(1), 139-151. https://scholar.google.com/citations?view_op=view_citation&hl=en&user=pCxClnYAAAAJ&citation_for_view=pCxClnYAAAAJ:W7OEmFMy1HYC

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Information Age Publishing. https://bibliotecadigital.mineduc.cl/handle/20.500.12365/17680

Mauris, L. (2022). Liderazgo pedagógico en la educación rural colombiana: Los desafíos, retos y oportunidades generadas por la crisis sanitaria del covid-19 [Pedagogical leadership in Colombian rural education: The challenges, challenges, and opportunities generated by the COVID-19 health crisis]. *Revista Estudios Psicológicos, 2*(1), 74-87. https://doi.org/10.35622/j.rep.2022.01.005

Méndez A., G. E. (2014). Fenomenología – hermenéutica en paul ricoeur. una integración para el abordaje de lo real [Phenomenology – hermeneutics in Paul Ricoeur. An integration for the approach to the real]. GACETA DE PEDAGOGÍA, (33), 85-101. https://doi.org/10.56219/rgp.vi33.531

Molina, M., & Cañadas, M. C. (2020). La noción de estructura en early algebra [The notion of structure in early algebra]. In Teaching mathematics: A tribute to professors Francisco Fernández and Francisco Ruiz (pp. 129-141). Universidad de Granada. https://produccioncientifica.ugr.es/documentos/6526e265a884a20a1ae89783

Mora Naranjo, B. M., Basurto Cobeña, J. V., Rosales Macas, F. J., Reyna Rodriguez, D. C., & Cedeño Loor, M. A. (2023). Recursos didácticos en centros educativos de ecuador [Didactic resources in educational centers in Ecuador]. *Ciencia Latina Revista Científica Multidisciplinar, 7*(6), 5600-5618. https://doi.org/10.37811/cl_rcm.v7i6.9105

Radford, L. (2010). Layers of generality and types of generalization in pattern activities. *PNA, 4*(2), 37-62. https://doi.org/10.30827/pna.v4i2.6169

Radford, L. (2014). The progressive development of early embodied algebraic thinking. *Mathematics Education Research Journal, 26*, 257-277. https://doi.org/10.1007/s13394-013-0087-2

Ricoeur, P. (2002). Del texto a la acción: Ensayos de hermenéutica II [From text to action: Essays on hermeneutics] (2 ed.). Fondo de Cultura Económica.

Stake, R. E. (2020). Investigación con estudio de casos (2nd ed., Ediciones Morata, Madrid. Obra original publicada en 1995 como The Art of Case Study Research.

Van Manen, M. (2003). Investigación educativa y experiencia vivida. Idea Books, Barcelona, España, Traducción al castellano.