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Teacher-student alignment to develop a valuing pedagogy: A case study of cultural dialogues for mathematical problem-solving

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Abstract
This study aims to develop effective pedagogies for addressing teacher and student values for mathematical problem-solving or valuing pedagogies (VPs) for mathematics. The aim is achieved by teacher-student alignment. The case study was conducted with a mathematician (the teacher) teaching an educational psychologist (the student) how to solve problems like a mathematician. The teacher taught the student by engaging in problem-solving activities like a one-to-one tutorial, immediately followed by their oral reflections. The formal teaching (including hand-writing) of teacher-student interaction was recorded. Follow-up oral and written dialogues continue between the teacher and the student in the later data analysis and paper process. Qualitative data analysis finds that the process generates three patterns of value alignment: teacher values transmission to students, teacher-student negotiations, and bridging teacher-student gaps by reform with teacher reflection and restructuring of pedagogies. The expert dialogues serve as an innovative methodology for identifying in-depth mechanisms for teacher-student value alignments and effective VPs.

Keywords: mathematical problem-solving, value, pedagogy, value alignment

INTRODUCTION

Value serves as the guide for human actions, including education. Value is one of the guiding factors in the learning compass for 2030 (Organization for Economic Cooperation and Development [OECD], 2019). The major actors in education settings are students and teachers. Values alignment between teachers and students in content and congruence would benefit student self-esteem (Benish-Weisman et al., 2020). However,

> "educators and their respective students come to class with their own aspects of valuing and these are likely to be different. If values alignment helps to maintain a functioning classroom environment, then educators and students will want to see one another's values aligned and in harmony" (Kalogeropoulos et al., 2021, p. 329).

While mathematics may be viewed as a relatively common language across cultures, values in society are a cultural phenomenon. For example, despite common values in learning mathematics among Chinese students (e.g., achievement, relevance, practice, communication, feedback, and information and communications technology), Taiwanese and Chinese mainland students value communication more than Hong Kong students (Zhang et al., 2016). Cultural issues mixed with the concern of teacher-student value alignment may generate student, teacher, and cultural issues in implementing pedagogies for mathematics, as we will see in this study.

This study uses an innovative research method: a Western mathematician teaching an Eastern educational psychologist, who has no formal degrees in mathematics. Both experts' teaching/learning practices allow full teacher-student dialogues at equal status, and all the student and teacher voices can be fully retrieved and documented. This will contribute to the knowledge of the values and valuing processes of students, teachers, and their alignment.

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Contribution to the literature

- The study introduces valuing pedagogies (VPs) as a framework for linking teacher and student values in mathematical problem-solving.
- The study identifies three patterns of teacher–student value alignment: transmission, negotiation, and reform through reflective pedagogical restructuring.
- The study demonstrates expert dialogue as a methodological innovation for examining mechanisms of effective VPs in mathematics education.

Students' Values and Valuing Process of Learning or Problem-Solving

Student values

Values are part of the affective factors in learning. Values, however, are not included in McLeod's (1988, 1992, 1994) framework of affect in mathematics, where affect includes beliefs, attitudes, and emotions. Based on McLeod's (1994) and Mason et al.'s (1996) theories, Gomez-Chacon (2000) identified two levels of affect: 'local' and 'global'. 'Local affect' is concerned with emotional responses, while 'global affect' relates to 'values and beliefs', with values and beliefs as the origins of emotional responses. A similar conception is addressed by Hannula's (2002) valuation structure and Seah's (2019) definition of values: values (including morality and ethics) guide student emotions, attitudes, beliefs, and actions in mathematical problem-solving processes.

Students' valuing process of mathematical problem-solving as affect-cognition interaction

Mathematical problem-solving is not only a cognitive issue but also an emotional issue. Burton (1994) defines mathematics as a subject of "the study of patterns and relationships" (p. 12), in striving for progression to enhance capacity, students experience anxiety and wish to exercise their creativity and control over their mathematics learning.

The interaction between students' cognitive and affective responses in mathematical problem-solving is complex. Based on theories about affect (e.g., McLeod, 1994), Hannula (2002) developed a framework of four evaluation processes: emotions in a situation, emotions associated with a stimulus, expected consequences, and relationships between a situation and personal values. The four evaluations are an interaction between cognition and affect in the process of mathematical problem-solving. The emphasis, however, focuses on 'evaluations' and the link to the problem-solving process may need further clarification.

Students need to conscientiously solve problems, reflect on prior experiences, and link actions and feelings in the problem-solving process. Key moments in mathematical problem-solving include getting started, getting involved, mulling over, keeping going, insight, being skeptical, and contemplating (Mason et al., 1996). These key moments include related cognitive experiences (e.g., understanding) and affective dialogues (e.g., frustration) in students' minds (Marmur & Koichu, 2021). Gomez-Chacon's (2000) study identified the five most frequent emotional responses to mathematical problem-solving: calmness, confidence, cheerfulness, just great (a state similar to 'insight' in Mason et al.'s, 1996 study), and blocked.

In addition to problem-solving process, mathematical tasks or problems play essential roles in students' responses. Students' initial interests are developed from their previous learning experiences and then determine their perception of how fun, important, changing an activity is, the degree to which students perceive an activity as personal choice and appropriate difficulty. These interest arousal and control beliefs further determine whether students engage in the long-term problem-solving process. This situational interest will further impact students' development of long-term interest in engaging in related activities in the future (Hidi & Renninger, 2006; Krapp, 2002). Value-added activities (e.g., Liebendörfer & Schukajlow, 2020) and effective teacher feedback (Pinger et al., 2018) are needed to maintain students' interest in future problem-solving activities.

Solving non-routine problems requires a significant amount of time and intense feelings (McLeod, 1994). Solving realistic mathematical problems provides more chances of failure (Middleton & Spanias, 1998). When teachers value mathematician-like problem-solving, the prediction is that students will experience many more challenges in cognitive and affective aspects.

Most studies successfully link mathematical problem-solving processes and students' responses (e.g., Gomez-Chacon, 2000; Mason et al., 1996). However, the links tend to focus on student responses. For teaching practices, the teaching process and teacher values play an important role, which will be the focus of this study: to identify valuing pedagogies (VPs) for the process of mathematical problem-solving.

Teachers' Values and Valuing Pedagogies

Teachers' values

Teachers' values inevitably contribute to their teaching practices. Teachers' every action (including the problem posed, the questions asked, and the activity enacted) can be viewed as a representation of their explicit and implicit values. Values explicitly addressed by teachers in teaching can increase student interest in mathematics (Acee & Weinstein, 2010; Liebendörfer & Schukajlow, 2020).

An ecological perspective on pedagogical design suggests that teachers' values tend to be multidimensional (Chiu, 2020). Mathematics teachers' values can focus on mathematics, general education, and mathematics education (Bishop, 1988). For student learning, mathematics and science teachers value three dimensions: the ideological dimension focuses on rationalism and empiricism; the attitudinal or sentimental dimension emphasizes control and progress; and the sociological dimension focuses on openness and mystery (Bishop et al., 2006), consistent with the affective, cognitive, and sociocultural approaches to mathematics teaching (Seah & Wong, 2012).

Teachers' valuing pedagogies

Pedagogy refers to teaching techniques through teacher-learner interactions to enable students learning (Siraj-Blatchford et al., 2002). Valuing pedagogy for mathematics (VPM), therefore, can be defined as pedagogies that allow students to understand the values perceived by teachers.

There appears to be no past study that has addressed VP or VPM formally. This study might be one of the pioneering studies to address this research topic or issue.

Research Questions

This study utilizes the process of a mathematician teaching an educational psychologist as a context to advance the knowledge of VPs or VPM. For research purposes, VPM is operationally defined or understood by teacher and student perceived, implemented, and received curricula (Chiu, 2016), as addressed by the following three research questions (RQs).

1. For perceived curricula, what do the teacher and student view as important before teaching and learning? (What are the values in the mind of the teacher and student before engaging in the teaching and learning process?)
2. For implemented curricula, what is the teacher process (including key moments and issues raised) for addressing the teacher's values? (How is the VPM implemented?)
3. For received curricula, what are the teacher and student reflections on the implemented curriculum? What assessment does the teacher use to assess the student's learning outcome? What is the student's response to the assessment? Further, how is the VPM reflected, negotiated, or aligned between the teacher and student?

A synthesis of the answers to RQs 1-3 will generate a framework of VP or VPM in the Discussion section. For educational practice, the framework would smooth the process of teacher-student value alignment and help student mathematics learning.

METHOD

Methodology

This study used self-study as the major methodological approach. "Self-study of teacher education practices (S-STEP)" emerged as a movement in the 1990s for teachers to explicitly question their practice and advance their knowledge through qualitative (action) research methods, practitioner reflection, collaborative interaction (supportive community), and exemplar-based validation of trustworthiness (Vanassche & Kelchtermans, 2015). S-STEP benefits teachers and others through professional development and enhancement for teaching and learning (Pithouse-Morgan, 2022).

S-STEP is especially suitable for teachers or teacher educators to study teaching practices or pedagogy, as in this study. In the process of self-study, tension arises, identity emerges, and professional development is achieved (Bullock & Ritter, 2011; Petrarca & Bullock, 2014). Self-study has been used by teachers teaching diverse disciplines such as mathematics (Alderton, 2008; Marin, 2014), science, and extracurricular activities (O'Dwyer et al, 2020). Not only for the cognitive aspect of teaching practices, but self-study is also applicable for teacher belief research (Lovin et al., 2012), as in this study. Through self-study, teachers link knowledge and practice, advance and share knowledge, and develop empathy (sense of beyond self) for both teachers themselves and agents in diverse communities (e.g., schools, students, and scholars) (Loughran, 2007, 2010).

Participants

The major participants were two academic scholars in higher education (also authors of this paper). One mathematician (David) from Australia serves as the teacher in this study. One educational psychologist (Mitchell) from Taiwan acted as the student taught by David. They also worked to triangulate the data analysis and interpretation of the data analysis result. In other words, David and Mitchell served as academic and critical friends for each other in this collective self-study (LaBoskey & Richert, 2015) (all the names are pseudonyms in the paper submission and review process.

No ethical review was needed for this study. The reason was that all the participants were the authors of this paper, agreeing to participate in this study and co-author the paper.

Data Collection

The basic empirical data came from the interactions between David and Mitchell. The idea exchanges and teaching sections lasted three months (from August to October 2023). The frequencies of formal teaching ended until no further teaching was needed in terms of mathematical problem-solving perceived by the teacher and the student. This resulted in the three teaching lessons, teaching 1 (October 2), teaching 2 (October 9), and teaching 3 (October 27), being fully video-recorded with 21-page hand-writing documents in the teaching process. Each teaching lasted 1-2 hours. The teacher (David) taught the student (Mitchell) in each teaching lesson. They engaged in problem-solving activities like a formal tutorial, followed by oral reflections based on their roles as a teacher and student. After each lesson, David and Mitchell reflected on the teaching they had experienced and had dialogues. They expressed and exchanged their ideas through emails, writing a Word or Google document file for supplementary purposes after the teaching, and co-editing the documents with comments and suggestions. The reflection and dialogue ended until no issues were raised for the teaching.

English was the major language used in the whole teaching and research process. In addition to the three lessons, David wrote four written documents (mainly on mathematical problem-solving and related pedagogies) and Mitchell wrote one (mainly on her pedagogical design on affect-focused mathematical teaching). Field notes documented the exchanges and communications between David and Mitchell, starting before the three formal teachings and lasting three months. These data are formed by the main analysis materials in the Results section. Follow-up oral and written dialogues continued among David and Mitchell in the later data analysis and paper writing process.

Data Preparation and Analysis

All the teaching lesson records were video-recorded and transcribed (using Google Meet). The hand-writing documents in the lessons were photographed and electronic communications (e.g., emails, MS Word files, and Google Documents files) were fully documents. All the process data were stored and shared among the researchers for later data analysis. The data analysis (including triangulation) continued until the paper was published.

Qualitative methodology is the major method to analyze the data. While general qualitative data analysis methods are the basic methodology (Miles & Huberman, 1994), phenomenography (Marton, 1981) was an additional method used due to its emphasis on teacher and student perception toward their teaching and learning experiences. In the final writing-up process, Mitchell's open coding started the data-analysis process (Charmaz, 2015). Constant comparison was also made in the process to clarify the meaning of the codes and categorize the codes into themes. David double-checked the coding. Disagreement was resolved by discussion. Three sets of coding schemes were especially used in the process as follows:

1. Perceived (intended) curricula, implemented curricula, and received curricula.
2. Teaching, reflection, and thoughts.
2. Data extracted with affective, problematic, and interactive elements (Bullock & Ritter, 2011; Idris et al., 2022).

Intending to develop an effective VPM in mind, the analysis focuses on teacher-student value negotiation, reflections, and alignment. The Results section is structured to answer the RQs. RQ1 utilized data mostly before the three formal teaching lessons when David and Mitchell focused on dialogues of making 'academic' friends, not teaching and learning. Data from the three formal, video-recorded teaching lessons and related communication documents were used to answer RQ2 (i.e., David's instructions and Mitchell's behaviors) and RQ3 (i.e., David's assessment and the reflections of David and Mitchell). For answering RQ2 and RQ3, Mitchell and David served as critical friends in the whole process as in general self-study research (LaBoskey & Richert, 2015). They looked at each of the data collected after each interaction and provided immediate feedback as collective self-study or interaction until all concerns were fully addressed (Petrarca & Bullock, 2014). Detailed concrete examples of extracts (including text and images) from the collected data were used in the results section to increase the trustworthiness (Vanassche & Kelchtermans, 2015) of this study.

RESULTS

Perceived Curricula

Formal writing documents tended to best represent David's and Mitchell's perceived curricula before the teaching activity.

David

David, as a mathematician, uses a story-telling way to address the life of a mathematician.

> At home we have a larder and a fridge. When we ask the question what can I have for tea, we go to these places for a way of answering the question. Mathematicians at work trying to solve problems

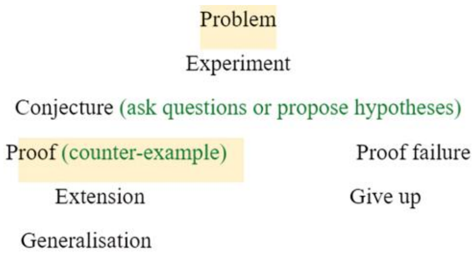


Figure 1. David's outline for solving mathematics problems (the text inside brackets is what Mitchell does not understand and added comments) (Source: Authors' own elaboration)

> also have a larder and fridge: It's the collection of all number and algebra and all of the other things that mathematicians have produced over the years, including theorems like Pythagoras'. All these results are available to be used on the next problem, though sometimes new mathematics has to be solved along the way. When Wiles solved Fermat's last theorem, he first had to prove a result about elliptic curves, a matter that everyone else hadn't thought of doing to help him. But this larder and fridge [are] certainly where students should look in their quest for solving problems (David writing on August 21).

For Mitchell, this writing appears to be a story from an outside world, not from this world. Even though Mitchell knows Pythagoras' theorem, she learned it in middle school, and it appears to be easy. 'Wiles solved Fermat's last theorem' is completely out of her mind. She feels a vivid picture of 'a larder and a fridge', which, however, cannot link with Pythagoras' theorem. What is the larder? Is the Pythagoras' theorem a fridge, a metaphor for the final product?

Further, David provides Mitchell with his 'outline for solving mathematics problems' (**Figure 1**) (Holton, 2010, 2025). Mitchell does not know what **Figure 1** means. She revises and comments on David's written documents:

1. How can students create problems and ask further questions or create further relevant problems? This is about mathematics creativity.
2. What is proof? Is this in terms of mathematical proof or just daily life proof (evidence)? (cf. Mitchell's comments on David's writing document, **Figure 1**)

Mitchell understands only the terms 'problem,' 'experiment,' and 'failure/give up' in **Figure 1**. Mitchell perceives 'problems' as routine problems in mathematics classrooms and examinations, and 'experiments' as utilizing diverse problem-solving strategies to achieve correct solutions to the problems. These perceptions appear to be different from those in David's, as explained after the first formal teaching lesson.

> What a mathematician does is when they meet a problem for the first time. They experimented with it … You just experiment, and mathematicians do that. And that stage. If you're lucky you can see that there might be a conjecture and two things happen. You either prove it or you find a counterexample … (October 02 video record).

Mitchell

As an educational psychologist, Mitchell's research and interest focus on the affective emotional, or non-cognitive aspects of teaching and learning mathematics. Mitchell believes gamifying mathematics lessons with technology support can at least partly resolve both the affective and cognitive issues of learning mathematics. She shows her pedagogical framework and one of the apps her project developed (on prime and composite numbers) as an example to Derek. The framework focuses on four phases of teaching, termed "the four-seasons islands" on the app (Chiu, 2024). The teaching emphasizes the geometrical formation (as a way to find factors) of prime and composite numbers.

After looking at Mitchell's writing of the framework, David writes:

> "… Look at a range of areas and list all the lengths of the sides. Doing this, students will find all of the factors in the numbers in the areas. This will give them the unique factorization of numbers result and that will add to the fact that 1 isn't prime. But it also will give them a way to find the different factors of a number. For example, $2^3 5^6$ has (3 + 1)(5 + 1) factors."

David further proposes:

> "Why four seasons? Why not five seasons?"

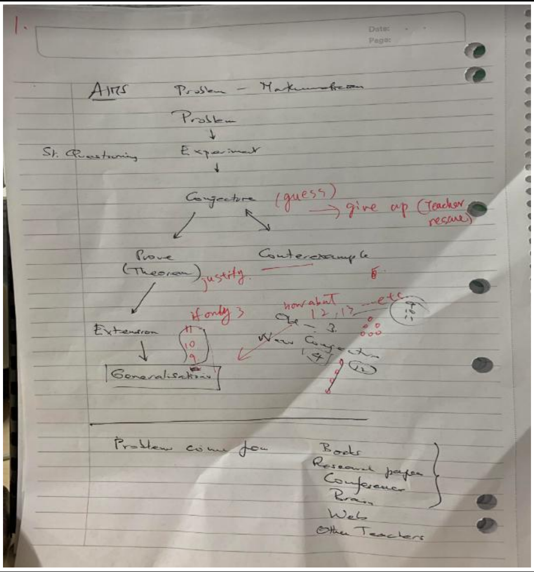
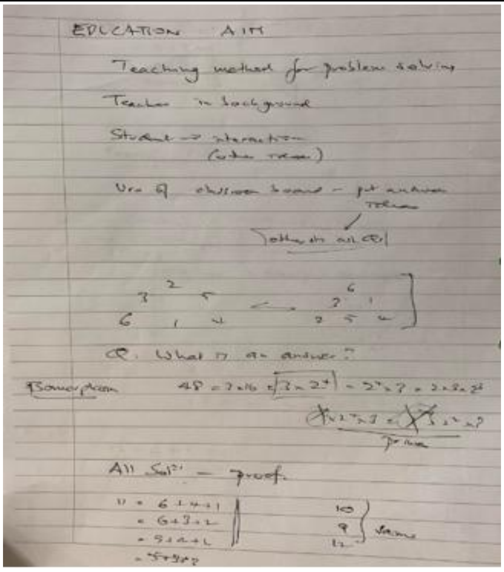
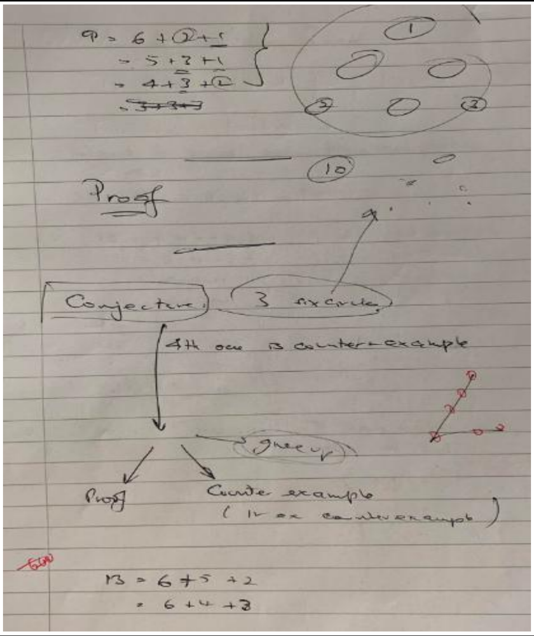
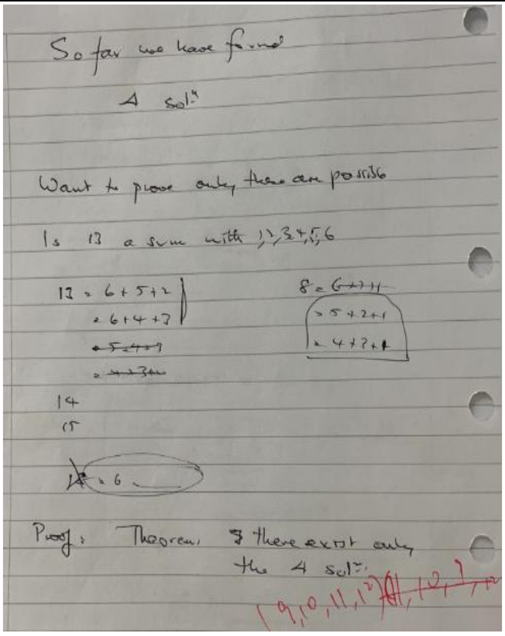
David's two questions raise Mitchell's interest, with a surprise and smile on her face, and asks:

> "What will be the fifth season? It's so ~~ creative. I like this idea–Why not five seasons?"

David smiled in response.

Mitchell and David now find their commonality: creativity. Mitchell wishes to know how mathematicians create knowledge, which will add something new to Mitchell's original framework. Mitchell expresses her wish to understand how a mathematician solves problems creatively, which is not routine problem-solving in typical mathematics classrooms. David expresses his agreement with this idea.

Table 1. Teaching 1: David's direct teaching his pedagogy with an example

1. David's pedagogy	2. David's education aim and the six-circles problem
	
3. David's explanation 1	4. David's explanation 2
	

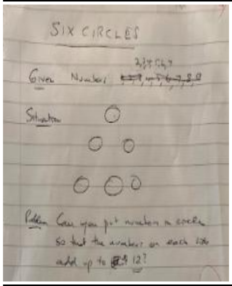
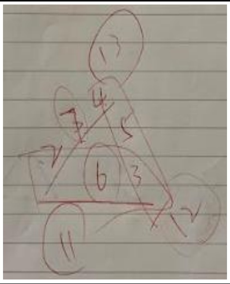
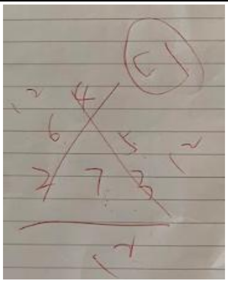
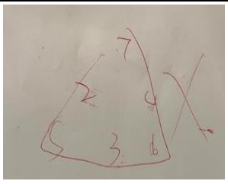
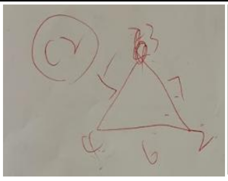
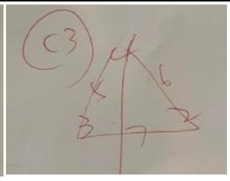
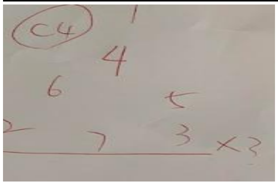
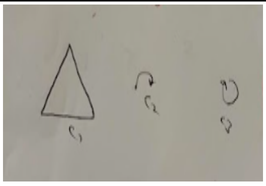
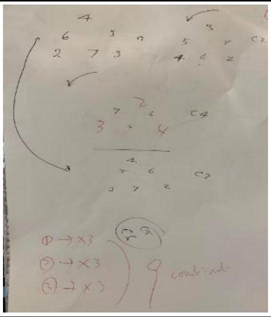
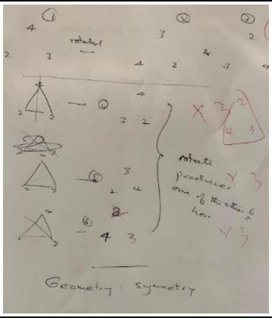
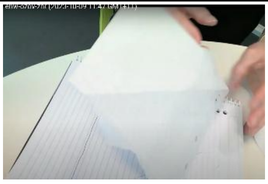
Implemented Curricula

Teaching 1. David's direct teaching

David directly emails Mitchell about what he is going to do. In the email, David mentions for the first time the pedagogy (part 1 in **Table 1**) and the six-circles problem (part 2 in **Table 1**) he will propose for the teaching.

> "I hope that you can see what I'm trying to do. My main ideas are to (1) show why problem-solving is valuable both in and outside mathematics and (2) to establish a pedagogy of problem-solving. I am sure that there is a lot in the literature in both of these and we will use that to support the argument. But I need the problem as an example to help us understand what we are trying to say.
>
> To give you some idea of where I'm going I'll list some of the things I have in mind.
>
> 1. I'll ask students to choose their own different set of 6 (different?) numbers and a sum to see if they can create their own six-circle situation . Put any results on the board. This is a good use of the whiteboard as students who are struggling can get ideas from what they see. It's also a chance for everyone to be creative.
>
> 2. I'll bring the problem to a conclusion by getting students to show that any number bigger than 9 has a solution (It might be useful for you to try step 1 and step 2. What difficulties might students have?).

Table 2. Teaching 2: Hand-writing and video samples (9 October)

1. David's 2nd 6-circle problem	2. Mitchell try 1	3. Mitchell try 2	4. Mitchell try 3	5. Mitchell try 4	6. Mitchell try 5
					
7. Mitchell try 6	8. David help 1	9. David help 2	10. David help 3	11. David help 4	
					

3. Summarize the steps and ideas that have been produced.

4. Discuss the table (problem, experiment, etc.) in fine detail.

5. Section on pedagogy.

6. At some later stage I want to also extend the problem even further. There are many possibilities but I won't do them all in any great detail. They will just be examples that can help the section in 4 above."

Then, in the meeting, David directly tells Mitchell what the pedagogy looks like (part 1 in **Table 1** or **Figure 1**) and uses a problem (the six-circles problem) (part 2 in **Table 1**) to explain the pedagogy.

Mitchell records the teaching process through videos and written documents. Most of the record focuses on the procedure of this study, including paper writing. For example,

1. *Results: The narratives of the teaching, including the reflection.*
2. *Discussion: The pedagogies for the 5th island* (cf. Chiu's, 2024 framework is 'four-seasons islands').
3. *Which journals to submit the paper to?*

Teaching 2. David's inquiry/problem-based teaching

David used number 1-number 6 in the six-circle problem in teaching 1. In this teaching 2, David starts with a different six-circle problem:

"Given numbers 2, 3, 4, 5, 6, and 7, can you place numbers on the circle so that the numbers on each side add up to 12?"

He wrote and drew on a piece of paper (part 1 in **Table 1**).

After posing the problem, David hands over the paper to Mitchell. Mitchell feels a bit anxious because she is not good at this kind of thinking and her estimation is that this problem looks easy but is actually quite difficult. A voice comes to Mitchell's mind:

"I cannot make it completely and can only solve it by trial and error. The end cannot be seen" (Mitchell signs quietly).

Mitchell started to draw and calculate the numbers on each side. The try 1 and try 3 of Mitchell's experimenting are wrong, even though try 2 is correct (part 2-part 4 in **Table 2**). She feels a bit worried because two of the three tries are wrong. Mitchell, by intuition, finds an easy way to do it correctly and draws try 4, try 5, and try 6 (part 5-part 7 in **Table 2**). She feels very happy because she does three correct tries continuously.

However, David decides to intervene:

What if I say there are several more?

Mitchell: Perhaps.

David: You can say, there's only this one here and I can say, I can think of two.

Mitchell: OK. How do you do it?

David: Oh, you're the student here and my philosophy of problem-solving is the student does the work. It's like riding a bike. I can hold you up for a while to keep you balanced, but in the end, it's your work.

Mitchell: So I have to figure out how another way is just to keep the large number in between but turn others around.

David: Go on!

Mitchell keeps on trying. In the process, she has many negative utterances, such as

"I don't think I can make a different one,"

"I don't think I just turned it around,"

"Impossible,"

and

"So hard."

David keeps on using positive, encouraging statements and guiding questions, such as

"All right, if we can get a C3,"

"Are there only three different ones in this sense of being different?,"

and

"What else can we do?"

David draws help 1-help 2 (part 8 and part 9 in **Table 2**), aiming to help Mitchell find something:

"You've rotated that once and then you've rotated it twice. And this was the C1, and this is the C2. And this was a C3 What can you do to an equilateral triangle? That sends it back on itself."

Finally, Mitchell discovers:

"I think they are the same."

(A voice comes to Mitchell's mind: 'rotating once and twice' means 'copying', which implies 'stupid', only focusing on easy, quick solutions, which is a surface approach (Platow et al., 2013). Mitchell, as an educational psychologist, has a stereotype that 'deep approaches' are good and 'surface approaches' are bad).

It's a mixed feeling about 'rotating' for Mitchell: copy (negative) and discovery (positive). The two "helps" disappoint Mitchell because David's drawings show that Mitchell's continuously successful tries are just by 'rotating', which means 'copy' to Mitchell (Note. David views Mitchell's tries were "doing real mathematics in the process to simplify her work"). Even shameful, Mitchell finds that try 2 is completely the same as try 6. At the same time, When the term 'rotate' comes out, it's easy for Mitchell to find for each 'original formation' (e.g., try 2), there are three rotating or similar formations as a group (e.g., tries 2, 4, and 5). The reason is that it's a triangle. Mitchell is happy with this quick finding.

David keeps on asking:

"This is an equilateral triangle. What can we do to an equilateral triangle?"

Mitchell (think for a while and laugh):

"I don't think I can do it."

David starts to make a triangle using a piece of paper to show (help 4; part 11 in **Table 2**). This impresses Mitchell. She writes down the term 'geometry symmetry' (a good representation transfer from geometry to number).

Mitchell:

"Oh! I now understand. It's smart!"

(A voice comes to Mitchell's mind. Why can David, or mathematicians, can think out this solution method, 'geometry symmetry,' to overcome 'copying' or 'rotating?'. Another sign in Mitchell's mind is, 'No wonder I won't be a mathematician and they, like David, become).

Teaching 3. David's teaching for generalization

David starts by posing a problem: Is it possible to find 6 numbers so that they make a side sum of 30? (part 1 in **Table 3**). Then, David suggests two ways to solve the problem (part 2 in **Table 3**):

"Give numbers → find sum; give sum → find numbers."

The process will generate (and write) on the paper:

"(1) Fun, by exploring and interesting"

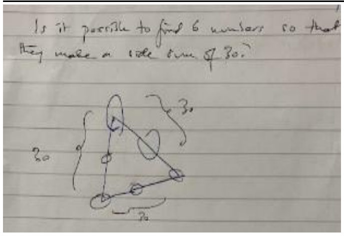
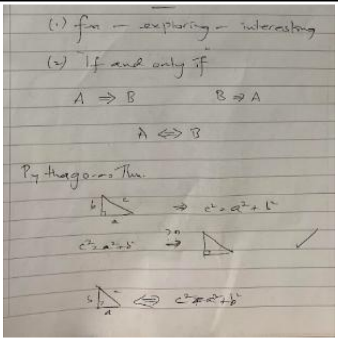
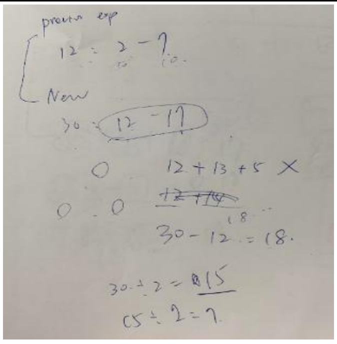
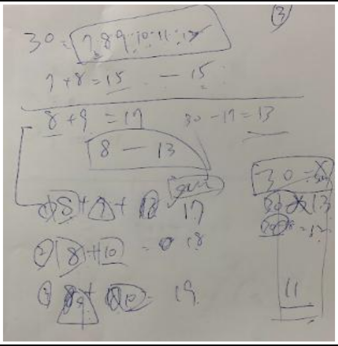
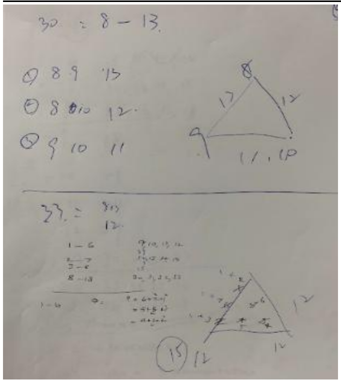
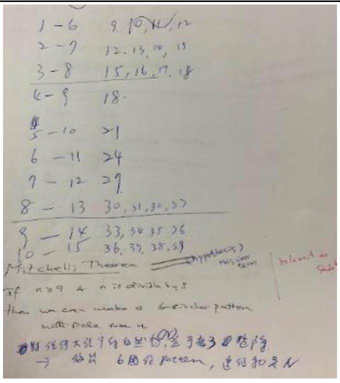
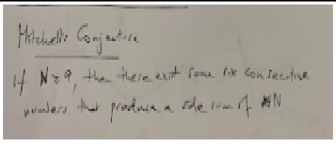
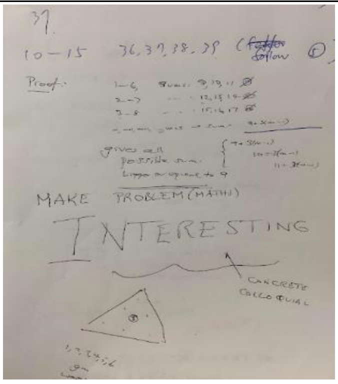
"(2) If and only if"

David shows Mitchell what is 'if and only if' by logical symbols and using the Pythagorean theorem as an example (part 2 in **Table 3**). After this, David writes

"Important for me: Show students how mathematics works."

Mitchell is not sure why this is important but David says, 'what is important for him,' which appears to concentrate Mitchell on the task, *'important'* to the instructor (Note. The value of David and Mitchell aligns).

Table 3. Teaching 3: Written documents (David in black & Mitchell in blue)

1. The problem	2. If and only if	3. Mitchell's experiment	4. Mitchell's starting with number 7
			
5. Mitchell's starting with number 7	**6. Mitchell's theorem**	**7. Mitchell's conjecture**	**8. Use '37' as a proof**
			

David's teaching the problem, indicating the 'if and only if', and the direct saying '*important*' interest Mitchell. Mitchell starts to explore with fun, like a child exploring a new world.

This task looks not difficult because David reminds Mitchell that she's done 12 by number 2-number 7 in the previous experiment, and now 30 by another set of numbers. Mitchell starts with 30 by number 12-number 17, which does not work; then, Mitchell finds 7 may be the starting number with 30 divided by 2, and then 15 divided by 2 (part 3 in **Table 3**). Starting with number 7 gives Mitchell an incorrect solution, linking to the previous teaching (part 4 in **Table 3**), but starting with number 8 works (part 5 in **Table 3**).

Next, David prompts Mitchell to synthesize the relationships between the side sum and the likely numbers on the triangle. David rewards Mitchell by naming Mitchell's finding as 'Mitchell's theorem: If $n \geq 9$ and n is divisible by 3, then we can make a 6-circle pattern with side sum n.' David asks Mitchell to write the theorem using her native language (traditional Chinese characters) (Mitchell writes 'hypothesis' along 'theorem' because, in Mitchell's term or field, she uses 'hypothesis' in similar circumstances).

To prove, David wrote down Mitchell's theorem again but using the term 'Mitchell's conjecture' (part 7 in **Table 3**). David asks Mitchell to use 37 as an example to prove 'Mitchell's conjecture.'

Finally, David explains his explicit teaching principles. For example, go slow, put children in groups, and don't expect teachers to know everything. Further, David's meta-teaching principles include explaining definitions by knowing and feeling, and mathematician and student approaches are the same.

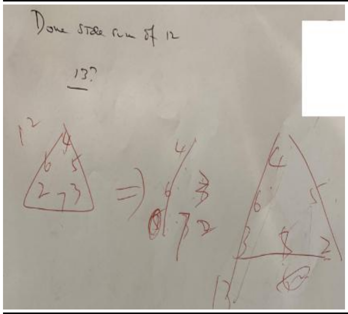
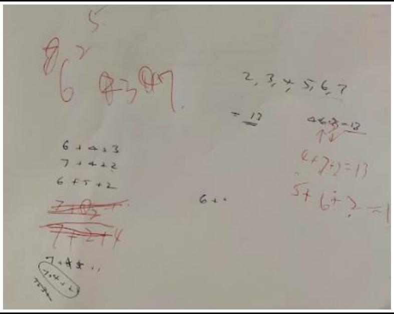
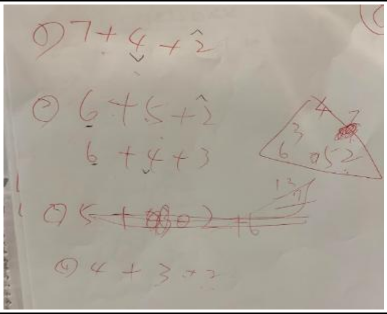
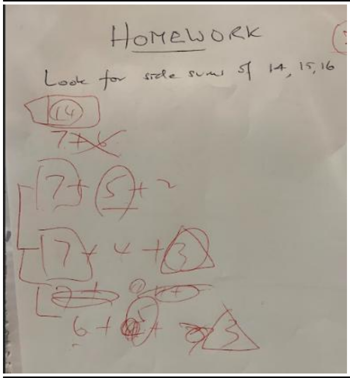
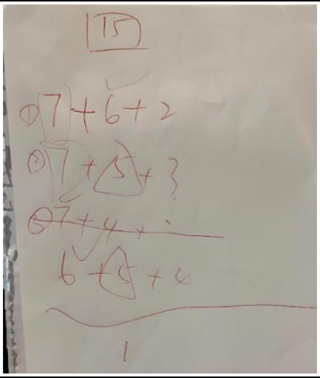
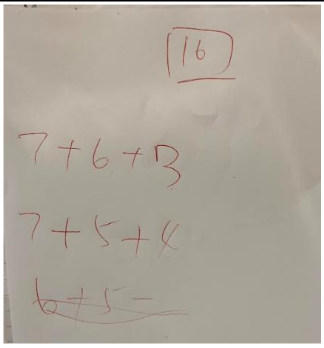
Received Curricula

Teaching 1

David's reassure key content to the student: David's 1st teaching focused on David's pedagogy (**Figure 1**) and initially introduced the 'six-circles' problems. As such, David did not provide Mitchell with an assessment but wished to ensure the key content of the teaching to Mitchell. David emailed Mitchell four days after teaching 1.

> "To give you some idea of where I'm going, I'll list some of the things I have in mind.
>
> 1. I'll ask students to choose their own different set of 6 (different?) numbers and a sum to see if they can create their own six circle situation. Put any results on the board. This is a good use of the whiteboard as students who are struggling can get ideas from what they see. It's also a chance for everyone to be creative.

Table 4. The 2nd assessment section (9 October) (David in black & Mitchell in red)

1. Mitchell draws randomly	2. David demonstrates writing in a way	3. Mitchell mimics David's writing
		
4. Mitchell's try of side sum, 14	5. Mitchell's try of side sum, 15	6. Mitchell's try of side sum, 16
		

2. I'll bring the problem to a conclusion by getting students to show that any number bigger than 9 has a solution (it might be useful for you to try step 1 and step 2. What difficulties might students have?).
3. Summarize the steps and ideas that have been produced.
4. Discuss the table (problem, experiment, etc.) in fine detail.
5. Section on pedagogy.
6. At some later stage I want to also extend the problem even further. There are many possibilities but I won't do them all in any great detail. They will just be examples that can help the section in 4 above."

Mitchell's thoughts: The 1st teaching appears to be no stress for Mitchell. The major reason may be that the content is more related to pedagogies, which fit Mitchell's expertise. Mitchell wrote in her field note,

> "Case studies to explore the opportunities to add mathematics creativity to … related studies."

Mitchell appeared to focus on her concerns on research at this stage.

Teaching 2

David's assessment: David asks the first follow-up question:

> "We've done the side sum of 12. Can it be the sum of 13?"

Mitchell draws by adding one to a number of a side from 12 (= 4 + 6 + 2) to 13 (= 4 + 6 + 3) (part 1 in **Table 4**) and surely fails. Next, Mitchell keeps drawing randomly and David models writing in order (part 2 in **Table 4**). Mitchell feels that she has learned 'something' and starts to mimic David's behavior: write by repeating the numbers in the corner of the triangle.

David, then, gives homework formally: Look for side sums of 14, 15, and 16.

> Mitchell: Can I do it now?

(Mitchell feels a motive to test what she's just 'learned' although she does not know what she really has learned, no terms, words, or languages to express her thinking in mind).

> David: Yes.

Mitchell is so happy that David allows her to do the homework now ('I'm lazy to do it as homework,' another thought coming to her mind). She continuously tried 14 (part 4 in **Table 4**), 15 (part 5 in **Table 4**), and 16. Mitchell finds that the 'rule' (linking one repeated number to the next triple number) is not so hard, and the tests can reflect her learning.

Mitchell's thoughts: Unlike the 1st teaching, Mitchell experiences much emotional arousal during this teaching. Two reasons are that Mitchell perceived this 'six-circles' problem as 'simple' but got stuck twice:

(1) using trial-and-error, with Derek's support, and finding 'rotating' was such a surface approach to solving the problem and

(2) failing to figure out any strategy for finding the other solutions, with Derek's support, not only explanation but also using folding papers to show 'geometrical symmetry.'

However, around 4 hours after returning home, Mitchell was aware of how much she learned from the process: Yes. I did things by trial and error, which was a waste of time (David commented afterward, 'Not a waste of time. It's a general way to start solving (experiment). Give other and better ways to proceed'). Perhaps I need to do things by finding 'patterns' by intension, which is 'a deep approach,' advocated by the educational psychology research community.

David's reflections: David emailed Mitchell his reflection, as a response to Mitchell's thoughts at the end of the teaching.

> "Yes the problem should have been set in some interesting situation. I failed step 1 of problem-solving there–Presenting a problem that has some motivation for the solver. That could have easily been fixed. Actually the second problem I mentioned, about the stamps, did have a story around it, but I didn't know how good your algebra is … The difficulty with a lot of the problems that I use is that they require a certain amount of collecting data before they can be solved–Before you can see patterns. I'm sorry about the tedious calculations, but in a classroom this would have been helped by the competition to put answers on the board … Note also how there was a need to go off into geometry to show that all the possible ways to show the answers were essentially the same. In class these sideways looking pieces of the mathematics could be taught in a prior lesson. I thought you did very well in getting on top of the symmetry of the triangle … I'm sorry that I missed that your third answer was not a rotation of the previous one. It's important to note that teachers will also miss things or even make errors or not be sure of the answer. This has happened to me from time to time. No one is perfect. This is an important message to get over. … You also did well to pick up the efficient and systematic way of finding the sums of three numbers and seeing that the numbers that were repeated needed to be in the corners."

This email serves as a good communication with Mitchell. The reason is that Mitchell felt her voices were listened to and her difficulties were accepted.

David also indicated after teaching 3,

> "Problems may need more than one part of mathematics before they can be solved ... Note that the symmetry's discussion introduces ideas in geometry and so shows that the problem isn't only a problem about arithmetic."

The term 'multiple representations' comes to Mitchell's mind. The common pedagogy (though with slightly different terms or expressions) aligns David and Mitchell.

Teaching 3

David's reflections: David wrote to Mitchell on the 2nd day after teaching 3,

> "Conjecture … when I asked the problem, can every number be the side sum of some six different (whole) numbers, we had a discussion that came to a conjecture on whether it could or not. I then scaffolded by asking can the side sum 30 be produced this way. As expected, Mitchell experimented with some numbers to try to see if 30 could be found. Any number would do but 30 was chosen because (1) it is a small number and (2) it is divisible by 3. I thought that Mitchell would approach the problem by looking at successive sets of six consecutive numbers, adding one at a time to all of these numbers. This would gradually increase the numbers 9, 10, 11, 12, that can be found from the set {1, 2, 3, 4, 5, 6} to 12, 13, 14, 15; 15, 16, 17, 18; and so on until the number 30 is reached. Not long after that, Mitchell could see that any multiple of 3 from 9 on, couldn't be a side sum."

Mitchell replied,

> "It took me a "long" time, subjectively, because the process is "difficult". I have to use my brain energy a lot. Thanks to myself. I have my previous writing at hand, which scaffolded me to find the pattern. If no such an aid, it certainly would take me much more time and more frustration … David guided me to write down (as David's writing).

Set	Side sums
{1, 2, 3, 4, 5, 6}	9, 10, 11, 12
{2, 3, 4, 5, 6, 7}	12, 13, 14, 15

> This helped a lot. I'm thinking about how can a teacher do this in a typical classroom with around 20-30 students."

(David comments: "Groups–Seed the groups separately when they are ready for it").

Table 5. Summary of teacher-student value or VP alignments

Curriculum	Cognition/ affect	Teacher values/VPs	Student values	Student affective states	Teacher (T)–Student (S) value/VP alignment
Perceived	Cognition	MPS	Routine school mathematics	Goal: Bridging the gap between school and mathematicians' mathematics	T ⇒ S: T giving and S expecting to receive
	Affect	Why not 'the 5th island'?	Mathematics creativity	Interest: Imagination	T ⇄ S: T & S the same interest
Implemented	Cognition	1. Content and process knowledge of MPS	1. Knowledge of mathematics education research	Goal: Writing a unique paper on mathematics education	T ⇄ S: T & S negotiating to find commonalities
		2. Posing mathematician mathematics problems (from easy to hard)	2. School mathematics; pattern finding	Confidence: Knowledge construction	T ⇎ S: Due to T & S expertise gap, hard to achieve by short-term teaching
		3. Scaffolding questions, multiple representations	3. Progress on MPS	Confidence: Sense of achievement	T ⇒ S: T's VP to create S progress
	Affect	1. Assuring students' responsibility	1. Accepting responsibility	Value: Accepting	T ⇒ S: T value to S by communicating
		2. Indicating values, 'important for me'	2. Accepting teacher values automatically	Value: Transmitting	T ⇒ S: T value statement and S acceptance
		3. Indicating student achievement, 'you have done' and student name 'theorem', explained by student native language	3. Successful experiences	Confidence: Achievement sense of belonging	T ⇒ S: T response to S achievement
Received	Cognition	Pedagogies for MPS	General pedagogies	Goal: Pedagogy for MPS, with general pedagogies in mind	T ⇎ S: T & S differences in domains
	Affect	Teaching for improving toward long-term achievement	Short-term achievement	Goal: Short-term and long-term achievements	T ⇒ S: T communication aiming for S improvement and long-term achievement

Mitchell's thoughts: Teaching 3 ended positively, completing David's teaching process (**Figure 1**) with David's reflection on teaching 3. However, five days after teaching 3, Mitchell felt something unfinished or unresolved. Mitchell reviewed David's previous writing about generalization and proof of a result, and wrote

> "How to prove it? This is so hard. I checked our 2023/10/26 (teaching 3) discussion. The proof was written by David (part 8 in **Table 3**). For me, it's a better form of a "mathematical theorem", compared to my initial theorem (part 6 in **Table 3**) … This proof definitely needs practice and revision supported by teachers, from my view. I could not write it out as David did because I did not know how to write it mathematically, as in the teaching. I would keep the last sum "12" in the sums "9-12" for the number 1-number 6 (part 8 in **Table 3**)."

David replied,

> "But you had the idea. My method is something that would come by practice. I think that proofs are important and students should be helped to improve with time. I have no problems with rough edges. I put a lot of things in that are unnecessary for the students, but they were there to say what I was trying to achieve in the long term in a class" (To Mitchell's 'I did not know how to write it mathematically', David especially comments, 'This is a matter of practice!!!').

Mitchell replied, 'OK' to David's reply.

Mitchell's underlying thought is 'Perhaps I had a misconception, like most students, that learning is an easy, simple process. I'm too anxious or too impatient to learn something so hard, a mathematician's problem-solving.' Mitchell felt embarrassed because she felt like her post-graduate students' impatience, feeling able to achieve higher-order thinking quickly of an educational psychologist (Mitchell's expertise).

DISCUSSION

Patterns of Teacher-Student Value or Valuing Pedagogy Alignments

The teaching by a mathematician for an educational psychologist to learn a mathematician's problem-solving skills generates three patterns of teacher-student alignments of value or VP (**Table 5** last column). The three patterns involve ten key moments (Marmur & Koichu, 2021), suggesting key VPs for practices.

The value alignment involves cognitive and affective aspects, which is consistent with past theories and empirical study findings (e.g., Gomez-Chacon, 2000; McLeod, 1994).

Pattern 1. Teacher value/VP transmitting to students (T ⇒ S) (n = 6)

This pattern shows a picture that teachers possess higher status or authority than students. Although this is a 'sad' situation, it appears to be a general practice in real K-12 classrooms, where the teachers have more professional competencies than students. The systemic factors contributing to this situation may include institutional pressures or cultural norms within mathematics education or general education systems worldwide. This occurs in 6 key moments, which can be found in all three stages of the perceived, implemented, and received curricula) and in two cognitive and four affective aspects.

(VP 1) Focused topics (perceived cognition): The teacher has mathematician problem-solving (MPS) competence, but the student has routine school mathematics skills only. To build MPS competence needs over several years of schooling, going from simple to hard problems. The weaker tend to learn from the stronger, a human nature. They assume that the teacher has higher status or more expertise in MPS, and can 'teach' students, who expect to learn. This pattern occurs in six out of the 10 alignments.

(VP 2) Scaffolding student learning (implemented cognition 3): The teacher's VP (including scaffolding questions and multiple representations) helps the student make progress on MPS skills. The positive learning outcome increases the student's confidence or sense of achievement.

(VP 3) Assuring student responsibility (implemented affect 1): Active learning is a key component for successful learning outcomes. However, students may over-rely on teachers. Teachers' reminding students to take responsibility for learning may help direct student attention, autonomy, and deep approaches to learning (Stefanou et al., 2004) and avoid rote learning. Relocating scaffolding agencies gradually, progressively from experts (teachers) to students equips students to become lifelong, self-control learners (Holton & Clarke, 2006), who take up their responsibility for learning.

(VP 4) Addressing teacher values (implemented affect 2): Teachers' directly stating their values has found to impact students' value appraisal and interest (Acee & Weinstein, 2010; Liebendörfer & Schukajlow, 2020). This study identifies the mechanism: Teachers with higher status directly transmit their values to students. Students perceive teachers' values as their values without many debates or negotiation but directly accept teachers' values.

(VP 5) Manifesting student achievement (implemented affect 3): Student achievements are raised by the teacher's three sub-VPs:

(1) listing the concrete content of student accomplishment,

(2) using students' names to name a mathematical phenomenon (e.g., Mitchel's theorem), and

(3) letting students explain using their (own) native languages.

These immediate (Fokides & Alatzas, 2023), process feedback (Harks et al., 2014) and acknowledgement of performance (Luzzo et al., 1999) accomplishment has found effective VPs.

(VP 6) Aiming for long-term achievement (received affect): The teacher directly tells the student that the teaching aims for a long-term achievement or improvement, rather than a short-term achievement. Even though the student aims at both short- and long-term achievement (able to solve a mathematical problem after teaching), the teacher's direct statement on focusing on a long-term achievement or improvement appears to impact, direct the student on the right track. Emphasis on long-term achievement, improvement, progress, or growth (O'Keefe et al., 2023) has long been advocated in educational psychology, and this attitude should be taken up by both students and teachers. Teachers' reminding is still important for students, even for an expert in educational psychology.

The picture of teacher value/VP to students appears to be accompanied by the picture: Student negative affect rises with cognitive difficulties and positive affect rises with teachers' effective valuing and helping students to learn by repeated problems and aids. Achievement-related emotions rise (e.g., self-concept and self-efficacy (Chiu, 2012). These signals of student negative affects need to be detected by teachers or technology if computer assisted system is used. Teachers need to have a repository of pedagogical skills at hand to support student learning adaptively.

Pattern 2. Teacher-student value negotiations (T ⇄ S) (n = 2)

Negotiations between teachers and students tend to occur at the start of the teaching. The negations aim to find or reach commonalities. Two of the 10 alignments bear this pattern.

(VP 7) Finding commonality (perceived affect): The teacher and student have open discussions to find common interests (e.g., mathematics creativity in this case study). Having common values appears to be the key to collaboration, including the business of teaching and learning. Common interest (becoming part of personality) appears to be more stable than situational interest (Hidi & Renninger, 2006; Krapp, 2002).

(VP 8) Exercising complementation (implemented cognition 1): Different from the perceived affect focusing on finding commonality, different expertise can compensate each other. In this case study, one has content knowledge (know what) and procedure knowledge (know how) in mathematics and one has in educational psychology. This cross-disciplinary collaboration is essential for creative work. Research has found that diversity in a team relates to creative, high-quality innovations (Hülsheger et al., 2009).

Teacher-student value alignment by equal-status, common interests, values, or intellects appears to be an ideal but rare phenomenon in school settings. Value negotiations in perceived affect at the start of teaching, however, can be achieved and advocated to initiate a positive classroom atmosphere, which may relax students' mind for challenging tasks.

Value negotiations in implemented cognition, needs cross-disciplinary collaboration. This can be done by pedagogies to invite student cooperations by cooperative learning (e.g., group discussion and group project) (Ding et al., 2007). This appears to be rare between teachers and students as mostly education is for teachers to teach students, as the first pattern (teacher value/VP to student). Teachers' being teammates with equal status to students may be also an effective pedagogy to create a positive classroom and engage students in learning.

Pattern 3: Teacher-student value reform (T ⇎ S)

Even though teachers and students negotiate on equal status to find common values, interests, or competencies, there is still a gap. Normally this is a tough experience for students (except for highly able students in mathematics) because it's hard to reach the cognitive level of the teacher.

(VP 9) Bridging content gap by knowledge construction (implemented cognition 2): Even with teacher problem-posing from easy to hard, the student with only conventional learning experiences of school mathematics still failed to fully obtain the MPS skill or pattern finding, from the student's term. The process, however, increases students' confidence in terms of knowledge construction or being able to solve mathematicians' problems successfully.

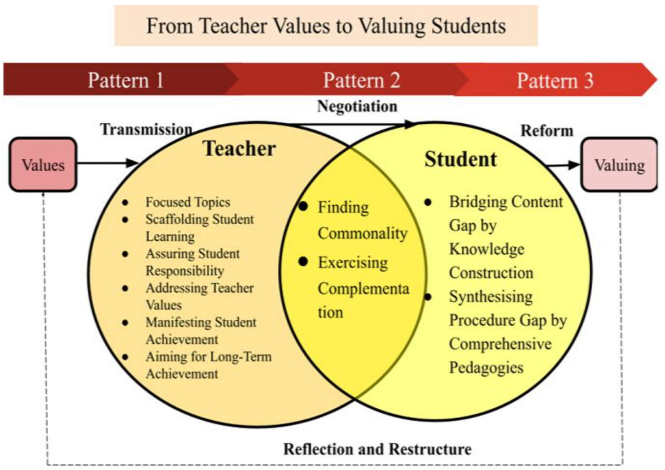


Figure 2. A mechanism from teacher values to valuing students (Source: Authors' own elaboration)

(VP 10) Synthesizing procedure gap by comprehensive pedagogies (received cognition): Domain-specific pedagogies have some overlaps with the general pedagogy (e.g., scaffolding using multiple representations). When teaching students to solve mathematicians' (or advanced) problems, domain and general pedagogies still fail to merge together. Pedagogical content knowledge still dominates in teaching advanced mathematics.

The teacher-student value dis-alignment in the long run may be rooted in teacher-student professional gaps. Diversity in students' mathematics talents, abilities, or competencies also matters in reaching teachers' levels. Slow learners may need more support to transform their experiences into constructive or productive struggles (Kalinec-Craig & Rios, 2024) perhaps by the ten VPs proposed. For example, teachers can cover gaps as mathematical problems develop to scaffold student learning (VP2).

The Mechanism From Values to Valuing

The above analysis of teacher-student value alignment generates a mechanism addressing the process from teacher values to valuing students (Figure 2). As discussed in the content of the three patterns, the traditional teaching process starts with teacher values, although the teacher's role in transmitting values to the student still dominates (discussion/patterns …/pattern 1). Negotiations between teachers and students are needed for collaboration to achieve educational purposes. For the persistent gap in content and procedure, there is a need for reform to valuing students' needs. Teacher reflections are needed to restructure their pedagogical system, by which new values are built, and then start the process from transmission and negotiation, to reform.

CONCLUSION

Contributions

This study contributes to the knowledge of teacher-student relationships or value alignment in the teaching process. Values, given their characteristics of guiding actions (Seah, 2019), serve as a platform to facilitate communication, debate, or learning.

There are three patterns of alignments, each including effective VPs. Pattern 1, teacher values transmit (or VPs) to students by focusing on a specific topic, scaffolding student learning, assuring student responsibility, stating teacher values, manifesting student achievement, and aiming for long-term achievement (or improvement). Pattern 2, teachers and students negotiate to reach commonality or collaborative complementation. Pattern 3, persistent, unresolved teacher-student gaps may be bridged by constructivism and collaborations.

The second major contribution of this study is to use an innovative, creative data collection method that allows for an "expert" student (majoring in education psychology, who has a relatively deep understanding and broad knowledge of learning and deep reflections on the process). Experts teaching experts appears to be a new methodology in the literature and deserves further investigation.

Limitations of This Study and Suggestions For Future Research

This study is a qualitative study, with one case (one teacher teaching one student) only. Later large-scale experimental/control design will validate the findings of this study with more research participants. MPS appears to be an advanced mathematical thinking. Whether MPS should become a routine teaching topic in mathematics curriculum deserves further research and government leaders to discuss: The time to formally infuse MPS in mathematics classrooms needs to consider the general aim of learning mathematics, becoming a person with mathematical skills and with mathematical thinking. As the results show, MPS is a long-term aim, and problems can be chosen over a year (or longer) to lessen the development of weak and strong, which may increase the possibility of infusing MPS in schooling at earlier stages.

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