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Teachers' leading mathematical whole-class discussion: Productive networks that develop in lesson study

Filipa Alexandra Baptista Faria [1,2,3*], João Pedro da Ponte [1,2], Margarida Rodrigues [4,5], Marisa Quaresma [1,2]

[1] Instituto de Educação da Universidade de Lisboa, Lisbon, PORTUGAL
[2] UIDEF, Lisbon, PORTUGAL
[3] Escola Superior de Educação, Instituto Politécnico de Setúbal, Setúbal, PORTUGAL
[4] Ci&DEI, Lisbon, PORTUGAL
[5] Escola Superior de Educação, Instituto Politécnico de Lisboa, Lisbon, PORTUGAL

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Abstract

This study aims to understand how the practice of planning and leading whole-class discussions in mathematics may develop in lesson study (LS). To achieve it, we looked at teachers' final reflections searching for productive networks that illustrate that relationship. This study is qualitative and interpretative. We analyze groups of middle school teachers (grade 5 and grade 6) by using discourse analysis. Data was collected between 2022 and 2023 and excerpts were selected focused on planning, leading and reflecting on whole-class discussions. We identified that teachers' discourse is linked to three discursive marks: students' mathematical activity, lesson structure and interventions and LS catalysts. The results show that the first two marks are associated with seven productive networks: sharing challenges, facing challenges, sharing knowledge, accessing students' mathematical activity, actively listening, working with tasks and, finally, planning the lesson. The third mark is associated with facilitator attitude, group collaboration and the observation/reflection of the research lesson.

Keywords: mathematics teaching, whole-class discussion, lesson study, professional development, productive network

INTRODUCTION

Over the years, research in mathematics education sought to understand aspects related to the social construction of students' knowledge, namely through whole-class discussions in an exploratory approach of the subject (Duarte et al., 2024). Despite the contribution of this research, mathematics teaching and learning remains tendentially traditional (Mosvold, 2024), which means teachers mostly expose mathematical ideas and processes to their students, who in turn are expected to replicate them.

Although in Portugal there have been changes in the curricular orientations that point towards a more exploratory teaching, such as the *programa de matemática do ensino básico* [mathematics program for elementary education] (Ponte et al., 2007) and the current *aprendizagens essenciais* [essential learning] (Canavarro et al., 2021), there is a need to support teachers continuously, particularly through collaborative processes that prompt the development of their teaching practice. According to Mewald and Murwald-Scheifinger (2019) "change [of practice] is therefore a difficult endeavor which has a better chance to succeed if embodied in dynamic and mediated interactions between tutors, teachers and their students through collaborative teaching and learning as the social construction of knowledge" (p. 219). Just reforming the curriculum or diffusing research results in education does not automatically guarantee improvements in teaching and learning (Stigler & Hiebert, 1999) and, therefore, teachers need regular support to deal with challenges and try different approaches (Takahashi, 2021).

According to Foucault's (1980), power relations can be understood as productive networks that run through

✉ filipa.faria@edu.ulisboa.pt **(*Correspondence)** ✉ jpponte@ie.ulisboa.pt ✉ margaridar@eselx.ipl.pt ✉ mq@campus.ul.pt

Contribution to the literature

- LS promotes productive networks that link planning and leading whole-class discussions, such as sharing and facing challenges together, having access to students' mathematical activity and having a facilitator present.
- The productive networks identified can support future lesson studies to promote a development process focusing on supporting teachers to teach considering the leading of whole-class discussions, namely within an exploratory approach.

the whole social body, which produce knowledge and discourses itself, not necessarily associated with hierarchical relationships. It is expected that in a collaborative setting where teachers work together, these productive networks emerge. Lesson study (LS) is a teachers' professional development process based on a collaborative and reflective environment, with exchange of ideas and co-construction of teaching material (Stigler & Hiebert, 1999). LS may foster power relations, that is, productive networks since discourses and knowledge are formed from the interactions of the participating teachers. Research by Warwick et al. (2016) contributed to clarify what makes interactions that occur in LS to have impact in teaching practice by indicating that "teachers were seen to be engaging in dialogic interactions that involved such features as requesting information, giving reasons, providing evidence, making supportive comments and articulating shared ideas" (p. 566). Besides, Warwick et al. (2016) considered that although LS is acknowledged in a growing number of countries, there is still very scarce evidence on how and what teachers learn through this process. To deepen the knowledge about these productive networks, this study aims to contribute to the understanding of how the practice of planning and leading whole-class discussions in mathematics may develop in LS. To achieve this aim, we look closely at teachers' final individual and group reflections, searching for productive networks that develop in this teacher education process.

Lesson Study and Mathematics Teaching and Learning

LS is a teacher professional development process originated in Japan, focused on improving students learning from the development of teaching practice. This process can be organized in a cycle form made up of five phases (Fujii, 2018): defining the learning aim, planning the research lesson, conducting and observing the lesson, doing a post-lesson discussion and subsequent reflection. Re-teaching the discussed and revised lesson to a new group of students can also be considered within a LS cycle (Murata, 2011).

This process, whether in continuing or initial teacher education, requires a group of teachers who voluntarily work in collaboration. With in-service teachers, this group can be made up of teachers from a single or from different schools, from the same level of education or from different levels. In European countries, it is common for LS to be led by a facilitator, who prepare sessions, select resources to explore, anticipate teachers' contributions, lead the sessions, orient teachers toward the thinking of others, and adjust the LS sessions plans considering the participants' needs (Gomes et al., 2023). The facilitator needs to have knowledge about content and pedagogy (Hourigan & Leavy, 2021), but he/she cannot script completely the sessions because teachers' assessments of their students' needs and their teaching challenges must be considered (Lewis, 2016) and teachers' contributions may open many unforeseen learning opportunities.

According to Fujii (2018) and Takahashi (2021), LS and teaching through problem-solving are inseparable. Teaching through problem-solving is very close to the exploratory approach, an approach already well recognized by mathematical research in Portugal (Duarte et al., 2024). The common foundation of these approaches is that students should be active agents in their learning process. Mathematics, which is often stigmatized by society, should not be perceived by students as a subject in which the presentation and validation of knowledge belong exclusively to the teacher. In an exploration approach, teachers prepare their lessons in advance, create, adjust or select tasks, structure the lesson and anticipate students' strategies, difficulties and questions to pose to promote a social construction of mathematical knowledge. According to Ponte (2017), lessons planned under the exploratory approach may have the following structure: introduction of the task; autonomous work by the students, whole-class discussion and final synthesis. This is similar to the problem-solving structure used in Japan (Fujii, 2018).

The aim of such approaches is "not for students to merely get an answer to the problem, but rather to teach them mathematical ways of thinking" (Fujii, 2018, p. 14). Therefore, deciding on what tasks to use and reflecting on how well they promote students' learning is an important activity for teachers engaged in LS. Regarding the question "do teachers really need to plan a lesson in detail?", Ponte et al. (2015) indicate that the answer is yes. These authors consider that detail and reflection during the planning phase leads to a greater ability to adjust the lesson plan and improvise, if needed, promoting a more confident teaching practice. For

example, within the planning phase, preparing mathematically relevant questions can promote students' engagement and competence in problem-solving (Mewald & Murwald-Scheifinger, 2019).

When participating in LS, teachers observe research lessons and reflect on them. According to Kager et al. (2022), those reflection moments must include looking back, becoming aware and creating solutions. Having teachers observe and take notes in research lessons helps to make the students' mathematical activity visible, creating opportunities for teachers to discuss the tasks, the lesson structure and students' particular strategies and difficulties based on concrete evidence (Fuji, 2018).

This collaborative environment promotes broader professional development (Benedict et al., 2023). According to Mewald and Murwald-Scheifinger (2019), teacher learning is a self-directed process that is influenced by personal beliefs, experiences and cultural norms. In line with this, Pan et al. (2023) state that teachers' learning is influenced by personal attributes, but also by their context. Factors such as the teachers' professional identity or their activities contribute to the development of their learning power. For the authors, teachers' learning power is "characterized as a form of consciousness that involves specific attitudes, values, and dispositions toward learning" (p. 229). In addition, Kager et al. (2022) state that "dissonance may also emerge during the planning phase when teachers have diverging opinions on a topic, or during the research lessons when instructions in the classroom do not work as expected" (p. 3). Based on Foucault's (1980) lens of power, we consider that participants' collaboration in a LS process has power to produce discourses and knowledge about mathematics teaching and learning, resulting in productive networks that can shape mathematics teacher practice.

Whole-Class Discussion Within Mathematics Teachers' Practice

A whole-class discussion in an exploratory approach of mathematics is understood as a dynamic and collaborative talk on a mathematical subject, facilitated by the teacher and based on students' mathematical thinking, that seeks to support the exploration and negotiation of mathematical ideas, concepts and procedures, thus enhancing the development of students' learning in a socio-constructivist perspective grounded in a mathematical discourse (Faria et al., 2024; Kooloos, 2022; Pirie & Schwarzenberger, 1988; Ponte, 2017). Considering the learning aim, tasks and the flow of the discussion, Takahashi (2021) identifies four types of whole-class discussion in teaching through problem-solving: to develop a new idea by examining the progression of thought; to compare multiple approaches to learn a new concept; to address misconceptions through debate; and to compare various solutions to an open-ended problem.

According to van Zoest et al. (2017), "responding effectively to student mathematical thinking requires careful attention to the content of the thinking to discern the underlying mathematical idea and what it might offer as the object of a class discussion (p. 51)". For example, correct answers may just lead to continuing the discussion or they may be an opportunity to engage students in synthetizing important mathematical ideas or procedures (van Zoest et al., 2017). Discussing errors can be a productive opportunity to build new concepts or understand mathematical ideas, therefore, errors constitute a fundamental basis for students' learning in a whole-class discussion (Kaufmann et al., 2022). Professional development process should support teachers not only to anticipate opportunities to build on students' thinking, but also to support them to recognize when such opportunities emerge in a whole-class discussion and to support teachers' reflection about this lesson moment.

Preparing a whole-class discussion includes several aspects, such as choosing the task with adequate cognitive challenge and structure the lesson and its flow (Duarte et al., 2024), as well as monitoring students' work, resulting in the selection and sequencing of possible ideas and strategies to address in the discussion (Stein et al., 2008). Regarding the moment of discussion itself, Duarte et al. (2024) highlights that the teacher needs to intervene according to the learning aim, analyze the students' work with a focus on mathematical ideas, promote students' participation and conclude the discussion with a final synthesis carried out in collaboration with the students and anchored in their work. In their research, Rüede et al. (2023) identify a positive relationship between teachers' productive talk moves, such as asking for more information or challenging to deepen their reasoning, and student justifications, showing that the frequency of these justifications is positively associated with the effectiveness of orchestrating classroom talk, in which the whole-class discussion is included.

Therefore, if students are rarely asked to negotiate and construct mathematical meanings in collaboration with their peers, listening or thinking with others will not be significantly present in the whole-class discussion. This may be a consequence of professional development which needs to evolve (Rüede et al., 2023). Kooloos et al. (2023) identified that teachers' practice moves from the professional development process to the classroom and back as an "arrow:" part of what happened in the professional development process appears in the classroom and then reappears in the development process. Even though teachers' development regarding planning and leading a whole-class discussion is personal, lesson preparation "is not a one-dimensional arrow but can involve a cyclic process and can include rounds of feedback" (p. 11), which is in line with LS, in which the cyclic process of lesson

Table 1. Information on the LS groups

Lesson study group	Timeline	Grades	Teachers that led research lessons
A–4 teachers	January to June 2022	6	Patrícia
B–9 teachers	January to June 2022	6	Marta and Diana
C–3 teachers	September to December 2022	5	Marta and Diana
D–2 teachers	January to June 2023	5	Marta and Diana

preparation is more explicitly planned, based on collaboration and reflection (Kooloos et al., 2023).

According to Web et al. (2019), teachers' practices that are most effective for supporting students may be different in the multiple classroom structures, such as small group work, student-student dialogue or whole-class discussion. Therefore, there is a need to support teachers in engaging students considering the particularities of each structure, including whole-class discussion (Mosvold et al., 2024; Webb et al., 2019). Teacher moves, teacher noticing and teacher mathematical thinking are three sub-practices identified by Kooloos et al. (2023). In Kooloos et al.'s (2023) research, teacher noticing, understood as the teachers' attention for student thinking, grew and became more explicit. In addition, they increasingly connected student thinking to their own moves and their own mathematical thinking. Teachers realized that they needed to put in mathematical thinking, such as thinking through multiple solution methods in their preparation, to be able to notice and work with students' thinking and solution methods. Over the course of Kooloos et al.'s (2023) research, teachers demonstrated to be progressively linking students' thinking to their own practice as mathematics teachers, recognizing that engaging in mathematical thinking themselves was crucial for responding to students, and consequently to promote student participation in the whole-class discussion.

Research in mathematics education has already contributed significantly to the understanding of whole-class discussions and to teachers' practice to promote students' engagement in such discussions. Still, there is a need to better understand how to support teachers so they can best engage their students in a whole-class discussion.

METHODS

This study follows a qualitative and interpretative approach given its aim and the nature of data, and, for data analysis, uses discourse analysis (Fairclough, 1992; Foucault, 1980). It was conducted in Portugal, with four groups of middle school mathematics teachers who participated in LS between January 2022 and June 2023. All names presented are fictitious and this research respected educational ethical principles, namely the informed consent (AERA, 2011).

Four teachers participated in group A, all from the same school, and nine in group B, from five different schools, all based in the same city. Two of the teachers who participated in group B decided to continue in their own school between September to December with another colleague, forming group C. Between January and June of 2023, these two teachers decided to continue participating in LS, therefore, forming group D (**Table 1**). The participating teachers had more than 15 years of professional experience. Most held degrees in primary and middle school education, with a specialization in teaching mathematics and science, except for two teachers who graduated in biology and physics-chemistry and later completed in-service professionalization. Regarding their school contexts, the teachers reported that they undertook peer cooperation, mostly by sharing resources. The average number of students per class was approximately 30, and all teachers initially reported that most of their students experienced learning difficulties in mathematics.

The structure was similar in all lesson studies, having about 12 sessions, each with two hours, inspired in Fujii's (2018) cycle and considering the possibly to re-teach (Murata, 2011). Even though the research lessons were led by Patrícia, Marta, and Diana, there was a shared responsibility among participants in collaboratively planning, observing, and discussing. The educational project of the school of group A gave high value to collaboration among teachers. Marta and Diana, from group B who worked in the same school, also used to collaborate with each other. In both cases, collaboration was about sharing classroom materials, therefore they did not plan or discuss the preparation of lessons together.

For this study, the data analyzed was collected in group A and group B final written reflections (WR), and in group C and group D was collected from the videotaped reflection session (RS), since teachers in these groups did not write individual reflections. The first author was the facilitator of these four lesson studies and the second author participated in some post-lesson discussions as a knowledgeable other. To ensure credibility and trustworthiness, the authors discussed the results, negotiating and validating the interpretations of the data, the discursive marks and the productive networks.

A discourse analysis was inductively carried out through an articulation of Fairclough's (1992) critical discourse analysis and Foucault's (1980) concept of power. Discourse is understood as a mode of action, even if only intended (Fairclough, 1992). The idea of intertextuality, present in the analyses of Fairclough

(1992) and Foucault (1980), stems from the notion that all discourses are anchored in previous ones, since it refers to the condition whereby all communicative events draw on earlier events (Fairclough, 1992), with no "free, neutral, and independent enunciation" (Foucault, 1980, p. 114). Thus, considering intertextuality when analyzing teachers discourse in reflexive moments allows us to link the excerpts of their discourse to the LS activities or aspects of it. In addition, considering Foucault's (1980) meaning of power relations as productive networks, we analyze teachers' reflections to identify productive networks linking the LS phases, which includes planning the lesson, to the leading of whole-class discussion itself. Given the Foucault's (1980) concept of power, we consider that participants' interventions have power to produce knowledge, therefore, forming productive networks, since power "forms knowledge, produces discourse. It needs to be considered as a productive network which runs through the whole social body" (p. 119). Accordingly, Jorgensen and Phillips (2002) admit that "power does not belong to particular agents such as individuals or the state or groups with particular interests, rather, power is spread across different social practices... Thus, power provides the conditions of possibility for the social" (p. 13), namely teaching and learning as a social practice, in which teachers are permeable to their professional development opportunities.

We thus consider that the teacher's discourse during individual or group reflection about whole-class discussion is an intertext that integrates the teacher's prior understanding of whole-class discussions and the work done in LS, making visible the productive networks that have contributed to the development of their teaching practice. Regarding the individual final WR (group A and group B) and the post-lesson discussions (group C and group B), we selected excerpts associated with the preparation of the whole-class discussion, its leading and reflection. We noted when these excerpts could be linked to a specific discourse mark or could relate to it. These discursive marks made it possible to organize the content of the teachers' discourses and are aligned with Faria et al. (2025):

(1) students' mathematical activity,

(2) lesson structure and interventions, and

(3) LS catalysts.

In line with these discursive marks, productive networks were identified.

RESULTS

Group A

Group A worked at the same school, teaching grade 5 and grade 6 classes in mathematics and natural sciences. Initially, Patrícia volunteered to conduct the research lesson, and the rest of the group agreed. The research lesson was observed by the other three teachers and the facilitator. The aim of the research lesson was to *recognize direct proportional situations and to indicate the proportion constant, explaining its meaning*.

In a final WR, Carla reflects on the students' mathematical activity when engaging in the tasks explored in the LS sessions, even though she did not lead the observed lesson. The competences she highlighted are aligned with the curriculum orientations and promoted students' participation in the whole-class discussions, such as mathematical communication:

> In my practice, when I applied the tasks [planned in the LS], I realized that the students were involved in diverse and meaningful learning activities, as they promoted problem-solving, mathematical connections, mathematical communication and the power of argumentation (A6).

In Carla's reflection, it was also clear which aspects of the LS were catalysts for her involvement in the process. Regarding the facilitator's attitude, she emphasized that "the facilitator often valued the spirit of observation, initiative, critical capacity and scientific curiosity. She listened attentively to our opinions, intervening to organize the discussion, encourage participation, stimulate cooperation between everyone and avoid dispersion" (C1). Regarding the collaboration in the group and the observation of the lesson, Carla felt that collaboration, particularly with feedback from her colleagues, made her more confident in her practice, even though she did not lead the research lesson:

> Collaborative work gives us greater confidence to move forward with our work and to take on new challenges. We had the reflection of our colleagues and their feedback to improve our teaching practices. An outstanding aspect of this LS was undoubtedly the spirit of mutual help and teamwork that was established, the systematic sharing of knowledge and worries, dialogue ... (C2, C3).

Patrícia, the teacher who led the research lesson in group A, emphasized the importance of sharing and facing challenges throughout the LS sessions, by reflecting on her practice and the need to actively listen to the students, asking questions and promoting their participation, stating that she "had to be aware of the teaching-learning processes we would provide for the students, as well as the opportunity to reflect on the best way to enhance them, questioning/evaluating my role in this learning process above all" (AB1, AB2, AB5).

Regarding listening actively to the students, Patrícia developed this reflection further, relating it to the application of the task and accessing the students' mathematical activity as it emerged from the task:

> My interventions during the task were aimed at asking for specific justifications for what they had recorded, clarifying some ideas, or introducing a 'doubt' so that they could argue or look for other answers, taking care to ensure that it was the students, not me, who decided what was right or wrong (AB4, AB6).

More specifically, regarding the whole-class discussion, Patrícia stated that it was prepared during planning, leading her to reflect on her role as someone who supports or questions the students to promote their participation, mathematical authorship and justifications:

> As a teacher, my role at this stage was to decide which group to 'invite' to start their conclusions. This decision was based on the criteria defined in the planning of the task carried out by us and after this start, my role was to support or question the students' interventions by asking them to justify their ideas, promoting communication between the students and gradually leading them to validate their answers (AB3, AB5, AB6, AB7).

Regarding the LS, Patrícia said that it was significant for her that she had her lesson observed by her colleagues, considering that this observation was an effective feedback on her students' mathematical activity. It supported Patrícia to improve her teaching practice, namely when making decisions on tasks to use:

> Peer observation allowed me to have more effective feedback on the processes/strategies used by my students when carrying out group tasks, which I believe can help me choose/adapt other strategies so that they are not only appropriate, but also motivating and challenging, improving not only students' learning but also my teaching practice (AB1, AB2, C3).

Rute, in the same vein, felt that the work done on the tasks and the lesson plan encouraged the students' involvement, making them more active in their learning process, especially by developing their communication competence, in which she included students' critically analyzing what their peers shared:

> This LS has made it possible to create learning situations where students can use their skills to discover their learning path; to develop a classroom culture where it is essential that students get involved in presenting, explaining and defending their ideas, that they react to and comment on their colleagues' interventions, that they critically analyze what they hear (AB6, AB7, A5).

By the time of this final reflection, Rute also planned for her future the development of a classroom culture where errors are seen as progress, as she proposed "developing a new classroom culture where students have a central role in their learning, where mutual respect prevails, where they do not see mistakes as a 'failure' but a means to progress" (AB2).

Amália also focused on the work carried out around the tasks, namely the concern with teacher's active listening to the students' mathematical activity during the whole-class discussion. The task is also associated with the structure and planning of the lesson, including the whole-class discussion:

> I applied tasks that had already been constructed and that came with the textbooks, but reconstructing or deconstructing them was very positive. Defining the objectives, preparing the task properly, determining the time to devote to each part of the development of the task, paying particular attention to whole-class discussion, because it is the students' responses that make it possible to explore the learning to be developed (B3, B5, AB6, B7).

Amália continued to reflect on the students' mathematical activity, particularly the sharing of knowledge during the sessions, which made her more attentive to students' reasoning and how to support their learning:

> Trying to predict students' answers makes the task richer and more effective because it helps us to be even more attentive to the different reasoning they present, so that through their answers, whether complete or incomplete, correct or not, we can draw conclusions and support knowledge. We can have a very good task, but if we do not know how to lead it, it may not serve the aims we set ourselves (A3, A4, AB6, AB7).

In general, these reflections in group A highlight the observation and reflection of the research lesson, recognizing the feedback from their peers, particularly on the whole-class discussion, as an enriching aspect. This proved to be important for all teachers, although only one of them led the research lesson including the whole-class discussion. Regarding the students' mathematical activity and the structure of the lesson and interventions, the teachers' reflections focused mainly on the work carried out around the tasks and lesson planning, namely exploring, applying and reflecting on the tasks, as well as planning the lesson in detail and preparing questions to ask the students during the discussion. Actively listening to colleagues and actively listening to students are productive networks also identified in these teachers' reflections.

Group B

In group B, pairs of teachers were from the same school, except for a teacher. Two of the nine teachers were working in primary school, while the rest were teaching mathematics and natural sciences to grade 5 and grade 6. Initially, Marta and Diana, who were from the same school, volunteered to conduct research lessons, and the rest of the group agreed. Marta led the first research lesson, observed by all the other teachers. Adjustments were made collaboratively to the lesson plan and the task so that Diana could also lead the lesson in her class. The aim of the research lesson defined by the teachers was to *recognize the direct proportion ratio between the perimeter and diameter of a circle and to designate π as the proportion constant.*

Even at the end of the LS, Catarina praised the concerns that guided the work carried out in this group, especially regarding whole-class discussions. These concerns illustrate challenges teachers shared and faced regarding whole-class discussions while planning the research lesson, such as "When to start the discussion? Which is more correct? What is completely wrong? What about the groups that leave the sheet blank? Should all the groups be asked for their work, even if their reasoning is similar?" Catarina stated that these reflections allowed for debate and discussion of points of view (B1, B2).

Marta brought to her reflection the impact of the planning phase activities in her practice of leading whole-class discussions, especially when anticipating and analyzing possible students' responses to a specific task:

> This analysis allowed us to see that different ways of communicating a result can be correct or contain information that the teacher should value … All forms of communication should be considered and complemented by sharing in a large group … This analysis was important for me to reflect on the way I lead my lessons. It highlighted aspects that the teacher should pay attention to in order not to condition the students' performance (A4, AB7, C3).

Marta also considered that in the lessons, teachers were able to see that "that the quality of the lesson is closely linked to the way the teacher organizes it, especially with regard to planning" (AB4, AB7). Regarding the whole-class discussion, she added that "the whole-class discussion was what deserved my commitment and attention the most, as they were perhaps the most innovative factors in my teaching practice" (B6). Specifically, about the whole-class discussion, Marta added:

> This is a real challenge for me because it deconstructs the concept I had of student participation in lessons. This concept of whole-class discussions brings to mathematics lessons the possibility that, on the one hand, the students will participate more actively with their contributions, and, on the other hand, the teacher will know how to guide these contributions to organize knowledge (B2, AB4, AB5).

For Diana, the opportunity to observe another colleague's lesson, as well as to have her own lesson observed, was fundamental, especially to restructure the task, an aspect that strongly influenced students' participation in the whole-class discussion, but also Diana's confidence to explore students' interventions:

> The observation by my colleagues was very collaborative, as they were there in a context of mutual help, and their records were very useful for later analysis, serving as a basis for restructuring the task for the next lesson observation ... The observation of the first lesson allowed me to be more confident about leading my class, to explore more of the students' mathematical comments/observations and build the desired knowledge with them (C3, AB2, AB4, AB5, B7).

On the same hand, Ana, who did not conduct a research lesson, considered observation a very challenging but enriching activity:

> We weren't supposed to intervene in any way or help them work out the reasoning behind what they were being asked to solve as a group. This part was a real challenge … However, our role as 'just' observers in these research lessons proved to be fundamental in this process, as everything we saw and recorded was explored in the 'post-lesson discussion' session (C3, AB1, AB2).

Diana also mentions all these elements–task selection, lesson structure and whole-class discussion–and emphasizes their importance as an innovative aspect of her practice regarding the development of students' reasoning:

> Organizing the lesson and leading the whole-class discussion was crucial in my teaching practice. Previously, I selected tasks and contextualized them to make them more appealing so that all students could do them without too much difficulty or, on the other hand, modifying them in a playful way so that they didn't become bored through repetition. I recognize that these situations are not very interesting or challenging from the point of view of developing mathematical reasoning (AB2, AB6, AB7).

The planning to which Diana refers also includes the anticipation of students' strategies, which is expected in LS activities. Still, besides indicating her increased confidence, Diana broadened her reflection on the impact on students' learning, stating that they could participate creatively in mathematics lessons, a necessary condition to enrich whole-class discussions:

> Detailed planning, in which the diversity of possible responses on the part of the students is also considered, allows for greater confidence in leading the lessons. LS allows students to explore the task freely, creatively and in a more participatory way (A3, A4, A7).

Violeta's final reflection was very aligned with these contributions of the planning phase, stating that LS "reinforced the importance of good lesson planning … It is more demanding for the teacher, as they have to plan in great detail and manage lessons more carefully, but it is also more rewarding, as learning is more meaningful" (B2, B7). For Inácia, even though she did not lead the research lesson, the anticipation and organization of students' strategies made communication more efficient and productive in her lessons, a fundamental aspect for the quality of whole-class discussions, as she recognized that she "did it in a disorganized way, which did not fully enhance learning. So, while participating in LS, I realized the importance of anticipating and organizing so that communication is more efficient and productive" (AB3, AB4).

For Ana, the facilitator's attitude and the group disposition to share and face challenges promoted collaboration, as she stated that "the factors that most promoted collaboration were our facilitator's assertive handling of the sessions and the willingness of each of the participants to assimilate something that could be different, stimulating and challenging for our students in learning" (C1, C2, AB1, AB2). For Sara, the facilitator' attitude was also relevant, specifically because she "proved to be knowledgeable in the training areas covered, always encouraging us to try out new techniques and to include the students' personal experiences in teaching practice" (C1). In the same vein, Sofia considered that, as the focus of the group was student learning, the group collaboration allowed "interaction between colleagues from other educational establishments, making this formative process even more important for the improvement and innovation" of her lessons (C2).

In general, these reflections of the teachers in group B emphasized the collaboration of the group as a determining factor in their involvement in the process, highlighting the role of collaboration, giving or building on ideas to deal with challenging aspects of teaching, such as Catarina stated about when to start a whole-class discussion. Group B also showed a strong emphasis on the facilitator's attitude, namely by encouraging the group to try out new approaches and to include the students' activity in their teaching practice. The contribution of observation and consequent reflection to their practice was also the subject of much reflection, even by teachers who had not led a research lesson. The detailed writing of the lesson plan, as well as access to the students' mathematical activity, in particular the reflection on episodes of dialogue between the lesson led by Marta and the lesson led by Diana, were remarkable productive networks. Planning more carefully and promoting a learning environment more meaningful for students, by selecting tasks for example, has a direct impact on the effectiveness of the whole-class discussion.

Group C

In group C, although they were all from the same school, Marta and Diana were still teaching grade 5 and 6, while Petra was teaching grade 9. The lesson was planned for grade 6 and led in Marta and Diana's classes with the aim of *understanding the concept of angle and to understand that the amplitude of an angle can be measured.* The task used in the research lesson was built on Marta and Diana' students' errors and misconceptions collected in a diagnosis task.

In the RS, Petra recalled her fear about discussing errors in a whole-class discussion, stating that she believed, in the beginning of the LS, that "errors don't always have to be depreciative, but I had the idea of an old-fashioned teacher. I was afraid of letting errors show up" (AB1). From her side, Diana reflected about the contribution of applying diagnostic tasks, which enable teachers to have a more detailed understanding of the possible misconceptions or ineffective strategies students may use during the research lesson to first address the aim defined. Therefore, Diana considered diagnostic tasks a good instrument to access students' mathematical activity before the research lesson:

> I think it was in line with what we had anticipated might happen, but perhaps we were at an advantage because we had what they had done in the diagnostic task. So, we ended up anticipating more easily or we had more anticipation (AB1, AB2).

Diana, still about the type of task used in this research lesson, stated that:

> The task used was challenging because the students needed to write their justifications. In the future, I think we must use this type of task, where they can evaluate given solutions, some of them with errors or wrong ideas. Although it's difficult for us to lead whole-class discussions and manage

> time with this type of task, it's exciting to realize that the students are involved.

In this LS, when reflecting on the research lesson, Marta considered that students were actively listening to their peers, which is also a significant illustration of the teachers being able to listen and promote students' interventions:

> I think that the moment of the whole-class discussion went very well. I noticed that the students listened to each other and tried to make their own contributions. They learned. The most challenging part for us is the management of time, because we don't want to cut off the students' participation either.

In Marta' reflection, time management appears to be a challenge that teachers need to face when planning and leading whole-class discussions. In her reflection, she assumed the need to act on this challenge:

> An aspect to improve is always time' management. We must give them a set amount of time to finish the task, which is for them to improve their work pace... And then, maybe, instead of us repeating things, we should continue the lesson (B1, B2).

Regarding the group collaboration and activities done, Marta emphasized a different way of collaborating with her colleagues:

> There was mutual help before, during and after the lesson and real collaboration. Even though I was conducting the lesson, I felt that it was the result of everyone's work, which gave me greater comfort. The collaborative work carried out in the sessions proved to be more fruitful than that what is usually done in school (C2, C3).

These reflections in group C mainly illustrate that identifying challenges and facing challenges were very productive in the LS. These productive networks seem to be associated with the fact that a diagnostic task was undertaken in this LS and, as a result, the task for the research lesson was built from the students' solutions with errors and misconceptions identified in the diagnostic task. The group collaboration consisted of mutual help, which was also identified to be powerful.

Group D

Group D was formed by Marta and Diana, and both teachers once again led a research lesson, this time with the following learning aim: *to establish relationships between fractions, decimals and percentages in the context of problem solving and to relate percentages to fractions with a denominator of 100.*

In Diana's stated two aspects emerge: the impact of the task on the students' participation and mathematical activity but also the challenge it represents for teachers to create such tasks and to be prepared to guide a discussion with diverse mathematical ideas:

> In the applet part, I was surprised because I wasn't expecting so much diversity. This means that the students were curious, they enjoyed getting involved in the task, it was an exploratory task … We used to do huge tasks. It's still difficult to create an exploratory task, but oriented, so that when they ask us questions, we can support them without giving them the answer, and then discuss it ... They're interested; they wanted to create ways of finding out. When we see their enthusiasm and commitment to mathematical challenges, we get motivated too (AB2, AB6).

Lesson structure and interventions were also an aspect of Diana reflection, she stated that "interrupting the lesson to clarify questions is not beneficial. We can clear up questions about interpretation during the introduction of the task... But bring the rest to the whole-class discussion". Regarding the LS aspects, even though it was not the first time Diana was participating in the process, she highlighted how the collaborative environment and the presence of a facilitator was important for her:

> LS is a huge help because we prepare things, we feel prepared, but it takes time ... There has to be collaborative work and there must be external support, support from a facilitator. That way we can build, and joint reflection helps to optimize the work (C1, C2).

When reflecting on the lesson, Marta focused on the organization of the board as a main aspect of preparing the whole-class discussion and as support for student learning:

> It was quite curious, because the facilitator always focused a lot of the sessions on organizing the board. I really had the feeling that having thought about and organized the board, what it would look like when we stuck things on the board, was very important … So, the fact that we had already rehearsed and thought about it was good, because I stuck mine in the middle and then asked the students where they should stick theirs. The way I called students out was also even more intentional (C1, AB7).

The sequence of tasks, its possible solutions and the interventions prepared in advance were also objects of reflection. Marta stated: "I realized that I hadn't been working with the students in previous years like this. They understood nothing. Now, with the questions I've

Table 2. Discursive marks and productive networks identified in teachers' final reflection linked to the whole-class discussion

Discursive marks	Productive networks
A) Students' mathematical activity	**1 Sharing challenges** - Introducing a new discourse by sharing a challenge - Reproducing a discourse by agreeing with a challenge identified by a colleague **2 Facing challenges** - Assuming the need to act on a challenge identified - Giving or building on ideas to deal with challenging teaching-learning aspects **3 Sharing knowledge** - Anticipating students' strategies and difficulties, supported by previous experience or literature - Suggesting tasks or adaptations to tasks **4 Accessing students' mathematical activity** -Analyzing an answer to a diagnostic task, before the research lesson - Analyzing an answer to a research task, after the research lesson - Reflecting on episodes of classroom dialogue
B) Lesson structure and interventions	**5 Listening actively** - Listening, posing questions and building up on other teachers' interventions during LS sessions - Listening, posing questions and promoting students' interventions during the research lesson **6 Working with tasks** - Exploring/applying tasks - Reflecting on tasks after their application **7 Planning the lesson** - Writing the lesson plan - Planning questions to pose to students - Planning the board - Reflecting on the lesson plan, questions posed and the board after the lesson
C) LS catalysts	**1 Facilitator attitude** **2 Group collaboration** **3 Lesson observation and reflection**

been asking and the tasks we've prepared, they understand. It was key to a lot of learning".

These reflections in group D illustrate that reflecting on the tasks brings the teachers to broaden their reflections regarding their practice. Therefore, exploring, applying and reflecting on tasks in a LS also constitutes a productive network. Regarding planning, Marta reflection illustrates how important it was for her to plan the board in advance, and well as to prepare questions to pose to the students. As stated in her reflection, planning the board was prompted by the facilitator, but not an imposed chore. Diana also highlighted the role of external support and collaboration to optimize work.

DISCUSSION

Table 2 organizes the productive networks associated with each type of discursive mark. It should be emphasized that given the intertextuality, it is common for an excerpt to have more than one discursive mark. When more than one discursive mark appears and several productive networks are identified, it is a good illustration that teachers are in fact relating the structure of the lesson and the interventions to the students' learning and their practice and identifying already the contribution of the LS features and activities in the development of their teaching practice.

As already indicated, we identified that teachers' discourse in the final reflection was linked to three discursive marks. In their reflections, regarding students' mathematical activity, teachers focused on the mathematical content or focused on mathematical competences such as communication, establishing connections and argumentation. Regarding the lesson structure and interventions, teachers related their decisions on these dimensions to their students' mathematical activity, reflecting in such aspects as what questions to pose, regarding the tasks, to promote students learning. Considering the LS aspects as catalysts, teachers connected, for example, the lesson observations and reflection to the enrichment of their knowledge about the students' mathematical activity. Therefore, in several excerpts of these teachers' reflections there are interconnections between the three identified discursive marks, which we identified to be evidence of an understanding of how the teachers' practice, promoted by a process of professional development, could enhance students' learning. In turn, the productive networks also identified are commonly intersected in the teachers' discourse throughout the final reflection, an inevitable consequence of intertextuality (Fairclough, 1992). Looking into more detail each productive network will clarify the relationship between whole-class discussions in mathematics and LS.

Sharing and Facing Challenges

LS is a collaborative and reflective process at its roots, so sharing and facing challenges is to be expected (Stigler & Hiebert, 1999). From teachers' reflections, it is possibly to link the group collaboration and the facilitator's attitude to their availability to share challenges associated with their teaching practice. Therefore, we state that these two aspects should be safeguarded and promoted throughout the process, so that the teachers establish trust relationships by collaboration and under the presence of a facilitator. It is by sharing their real challenges that conditions are created for their learning power to develop (Pan et al., 2023), promoting other productive networks that enhance teacher development (Foucault, 1980).

As Kager et al. (2022) indicate, dissonance can also be identified during the planning phase, which can lead to critical reflection, leading in turn to creating solutions. This aspect was noticeable in teachers' reflections when they referred to the planning phase, namely recalling moments of creating solutions to adapt a task, planning the lesson or preparing questions to pose to students (Benedict et al., 2023; Warwick et al., 2016). In general, their final reflections illustrate that teachers looked back at their practice before engaging in LS, as well as at the LS process itself, consciously identifying the contribution of planning lessons considering an exploratory approach. In their final reflections, teachers also shared suggestions for their future practice that can highly influence planning and leading whole-class discussions, such as promoting communication between the students and gradually leading them to validate their answers (Duarte et al., 2024; van Zoest et al., 2017).

Therefore, teachers faced challenges by creating solutions during planning, but also as they reflected. Teachers often reported greater confidence in conducting the lesson and whole-class discussions, asking questions or in managing unforeseen events, linking their confidence to the detailed lesson planning, which supported them to face these challenges (Mewald & Murwald-Scheifinger, 2019, Mosvold, 2024).

Sharing Knowledge

Sharing knowledge is itself a materialization of an intertext, since this knowledge was anchored on previous discourses. In this specific case, the knowledge shared by teachers was also visible in their anticipation of the students' strategies or difficulties, whether through their previous experience or the resources they explored during the sessions (Fairclough, 1992). In this sense, sharing knowledge in a collaborative context is also an open door for new knowledge to form and is inevitably a productive network according to Foucault's (1980) definition of power. As indicated by Warwick et al. (2016), "by combining their intellectual resources, members of a group can address a shared problem and pursue a common goal more effectively than they could alone. In using language to make joint sense of their experience, those people may create new understandings which each individual could not have achieved" (p. 556).

In teachers' reflections there seems to be a clear link between the catalyzing aspects of LS and knowledge sharing. The teachers praise the group collaboration, referring to the mutual help, as well as the facilitator's management of the dialogue, in which the teachers say that their ideas were valued and cordiality was fostered (Hourigan & Leavy, 2021; Lewis, 2016). Observing and reflecting on the research lesson was also widely referred to as a moment of knowledge sharing (Benedict et al., 2023): those who observed the lesson had access to and reflected on the students' mathematical activity through their own lenses, bringing other ideas or points of view.

Accessing Students' Mathematical Activity

Accessing the students' mathematical activity proved to be highly powerful. In the case of group C, their reflections focus on the importance of having applied a diagnostic task, based on which the task for the research lesson was then developed. By understanding students' errors and misconceptions, the aim of the lesson also became clearer, and the teachers decided to tackle one of their challenges: discussing errors and misconceptions in the whole-class discussion. This is evidence that, in fact, the work carried out during the LS also made it clearer to the teacher what kind of whole-class discussion they were looking to promote, considering the aim set for the lesson. In group C, the planned and conducted whole-class discussion was closer to a discussion to address misconceptions through debate. For group A, it was closer to a whole-class discussion to develop a new idea by examining the progression of thought (Takahashi, 2021). Therefore, accessing students' mathematical activity by using a diagnosis task may have supported them to plan a whole-class discussion more aligned with the particular aim of the lesson.

On the other hand, during the research lesson itself, the structure of the lesson and the interventions of both teachers and students are decisive to access students' mathematical activity. The exploratory or problem-solving approach, by definition, provides space for students to share their ideas and negotiate mathematical meanings, namely through whole-class discussions. It is at the moment that teachers have significant access to the students' mathematical activity (Fujii, 2018; Kooloos et al., 2023; Ponte, 2017), if the structure and flow of the discussion allows it (Faria et al., 2024), which will better inform teachers about their students learning. The students' activity teachers had access during the research lesson is then the subject of reflection by the participating teachers in the LS, contributing to making LS a process that seeks to connect teachers' professional development with their real teaching-learning contexts

(Benedict et al., 2023; Ding et al., 2024), a widely recognized necessity (Mosvold, 2024).

Actively Listening

Listening actively to their students in the context of the lesson, prompted by the structure of the lesson, also gives teachers access to their mathematical activity. Thus, when the impact of their practice on students' learning is tangibly by noticing difference in their participation, it becomes clearer to teachers that their prior preparation and all the actions carried out in the lesson had a positive outcome. Questioning the students, promoting their participation in the discussion, selecting and sequencing solutions to be discussed are examples that contribute or are a consequence of actively listening to students present in teachers' reflections. Those are aligned with Rüede et al. (2023) when it comes to promoting a positive relationship between teachers' productive talk moves and leading whole-class discussions.

Actively listening to other teachers were also illustrated in teachers' reflections, namely linked to collaboration. Listening, posing questions and building up on other teachers' interventions during LS sessions is mutually connected to the concept of intertext and power relations (Foucault, 1980; Fairclough, 1992), which promoted teachers to look back, become aware of and create solutions for teaching-learning mathematics (Kager et al., 2022).

Tasks

During their reflections, teachers were more aware of the relevance of not only choosing but also applying tasks with very clear aims and well-prepared lesson, as stated by Amália. Teachers often reinforced how the task had great impact on the students' mathematical activity, making them more involved in discussions and showing a greater and deeper mathematical understanding (Fujii, 2018; Ponte, 2017).

Regarding the tasks and the students' activity around them, a greater understanding of the importance of discussing errors and misconceptions, and how these errors can constitute a significant learning lift (Kaufmann et al., 2022; van Zoest et al., 2017), became evident in the teachers' reflections, especially in group A and group C. The development of this greater understanding and the willingness to change their practice seems to be linked to the teacher's appropriation of the tasks, and the contribution of collaboration in fostering anticipation of possible solutions and student difficulties.

Planning the Lesson

Working on tasks and lesson planning brought the teachers closer to exploratory teaching, namely by planning and leading whole-class discussions. Planning in detail made Marta, Diana, and Patrícia feel more confident. This detailed planning, including the questions to pose to the students, was widely referred to by the teachers as something that encourages them to lead whole-class discussions, because it turned out tangible for them that this moment promotes students to construct their mathematical knowledge.

Lesson planning was fundamental for the teachers to feel confident (Mewald & Murwald-Scheifinger, 2019, Ponte et al., 2015). According to Mosvold et al. (2024) and Webb et al. (2019), it is essential that a professional development process considers the specificities of leading whole-class discussions so that it can effectively contribute to this dimension of the teacher's practice. In their reflections, teachers emphasized how the detail of the lesson plan allowed them to feel more confident and to know what questions to ask their students. In group D, for example, Marta's reflection indicated how planning on the board made the teacher feel more confident, taking more conscious actions to manage whole-class discussion and, consequently, to promote significant students' learning (Kooloos, 2023; Web et al., 2019).

Facilitator Attitude, Group Collaboration, and Lesson Observation and Reflection

Regarding the facilitator's attitude, several teachers identified the facilitator's actions as promoting their involvement in the LS. Despite referring to the facilitator's knowledge (Hourigan & Leavy, 2021), the teachers' discourse shows that they felt listened to and valued, an aspect emphasized by Lewis (2016) as necessary for the teachers' engagement in the process. The participating teachers also recognized the work involved in preparing and leading the whole-class discussions, identifying the facilitator's actions that were largely related to the whole-class discussion, namely by suggesting a structure for the lesson plan, the questions raised, or directing their attention to the board planning (Gomes et al., 2023).

Group collaboration was also very present in teachers' reflections. They see it as mutual help and feedback from her colleagues, and as an established, systematic sharing of knowledge, worries and dialogue. Teachers also stated that it made them more confident in her practice and more confident also to take on new challenges like leading a whole-class discussion. The collaborative environment allowed the research lesson to be seen as the result of everyone's work (Benedict et al., 2023; Fujii, 2018; Murata, 2011), making teachers more comfortable with the lesson, namely when leading the moment of the whole-class discussion. Lesson observation and reflection was an aspect that teachers also linked to the feedback from her colleagues. Teachers considered this to be effective feedback on her students' mathematical activity. It also supported them to improve

their teaching practice, namely when making decisions on tasks to use, which has a huge impact on the students' activity, enriching the whole-class discussion.

CONCLUSION

Teachers' leading mathematical whole-class discussion is fostered when they participate in LS because power relations are being established as a consequence of this process with a collaborative and reflective nature, but also because this process is deeply anchored in the practice developed in real lessons and their observation by the participating teachers and an external facilitator. Those power relations are made visible in teachers' final reflections as a form of productive networks, contributing to understanding how the practice of planning and leading whole-class discussions in mathematics may develop in LS.

Power relations are productive networks that run through social relationships. There are powerful social relationships that work as productive networks for developing, in this case, the teachers' practice regarding whole-class discussions. When teachers shared challenges and actively listened to their colleagues, for example, they took key steps towards developing their knowledge and teaching practice: putting the relationship between students' learning and teachers' practice into question. It is as if the LS made visible the gap between mathematics teaching and mathematics learning, and this gap is not only filled with student participation, but with the intentional and consistent promotion of this participation by teachers, in which the LS supports by engaging teachers with activities such as elaborating mathematical tasks and planning in detail.

The productive networks identified can inform the facilitators of a mathematics LS about aspects that enhance teachers' development in planning and leading whole-class discussions in this process. Therefore, the activities associated with these productive networks must be guaranteed and carried out based on a collaborative and reflective environment, without imposing practices, but instead anchored on teachers' real practice.

A limitation of this study is that it does not allow us to understand how these productive networks evolve over time within the same working group. For future studies, it would be also worth understanding if these productive networks remain sustainable in the absence of an external facilitator.

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