MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2023, 19(4), em2252
ISSN:1305-8223 (online)
OPEN ACCESS
Research Paper
https://doi.org/10.29333/ejmste/13108

Teachers' perceptions of the effectiveness of a planning framework on content sequencing for the teaching and learning of mathematics

Musarurwa David Chinofunga [1*], Philemon Chigeza [1], Subhashni Taylor [1]

[1] James Cook University, Cairns, QLD, AUSTRALIA

Received 11 October 2022 ▪ Accepted 09 March 2023

Abstract
Planning is an instrument for effective teaching and learning of mathematics, which can address the dropping enrolments of year 12 students studying advanced mathematics. This study investigated teachers' perceptions of how a planning framework on content sequencing from junior mathematical knowledge (years seven to 10) to senior mathematical knowledge (years 11 to 12) informs teaching and learning of mathematics in Queensland, Australia. This mixed methods study collected data through a survey and semi structured interviews with 16 high school mathematics teachers. The data reveals that the elements of the framework can enhance the process of content sequencing, promote an environment that enhances development of new knowledge from prior knowledge, and articulate the hierarchical nature of mathematics. The study found that the framework can enhance collaborative planning among teachers within and across year levels. The study argues that using the planning framework on content sequencing can be a significant tool that can play an important role in guiding teachers to plan and teach new mathematical knowledge building from prior mathematical knowledge.

Keywords: mathematics content sequencing, mathematics planning, collaborative planning, prior knowledge, essential concepts, hierarchical nature of mathematics

INTRODUCTION

Australian Mathematical Sciences Institute (AMSI) director Professor Tim Marchant warns that year 12 students studying advanced mathematics in Australia has dropped by 10% for the first time, mathematics enrolments have dropped to an alarming level and that action must be taken now (AMSI, 2022). With participation rates in advanced mathematics at senior secondary level declining in most western countries, that include the United Kingdom (Noyes & Adkins, 2016; Watt, 2007) and especially Australia (Bita & Dodd, 2022; Kennedy et al., 2014), planning for effective teaching and learning of mathematics needs renewed focus. Importantly, how teachers plan informs teaching and learning, which influences participation and achievement (Australian Institute for Teaching and School Leadership [AITSL], (2014). Moreover, the sequence of concepts and tasks teachers develop during planning are informed by several preparatory actions and is central to teaching and learning (Sullivan et al., 2013). Therefore, teachers' views on how content sequencing can inform teaching and learning of mathematics can assist planning at senior secondary level and influence student participation and achievement.

Planning is an instrument for effective teaching and learning of mathematics, which focuses on "how pupils learn mathematics; the structure of the mathematics curriculum; the specific content, skills and concepts you are teaching; the prior knowledge of the pupils; ways of teaching mathematics" (Jones & Edwards, 2017, p. 70). Planning informed by sequencing from fundamental to more complex content enhances teaching and learning (Fautley & Savage, 2014). However, limited research is available on how sequencing mathematics content and tasks inform the teaching and learning of mathematics (Sullivan et al., 2013). This study seeks to explore teachers' perceptions on how mathematics content sequencing, a key pillar of mathematics planning, can inform teaching of senior mathematics with the view to enhancing students' participation and achievement.

✉ david.chinofunga@my.jcu.edu.au **(*Correspondence)** ✉ philemon.chigeza@jcu.edu.au ✉ subhashni.taylor@jcu.edu.au

Contribution to the literature

- The study argues for mathematics content sequencing to build knowledge from junior to senior mathematics.
- The study introduces a mathematics content sequencing framework to aid transition from junior to senior concepts.
- The study evaluates the framework of content sequencing to enhance teaching and learning of mathematics.

Mathematics is hierarchical in nature (Nakamura, 2014). This means that sequential development of concepts fosters deeper mathematical understanding (Newton et al., 2020). In Japan and Thailand, the use of 'Bansho', which emphasizes making use of board space to sequence learning from prior knowledge has been hailed as an effective teaching and learning strategy (Kuehnert et al., 2018). Importantly, significant research (Duncan et al., 2007; Geary et al., 2013; Pagani et al., 2010; Schneider et al., 2011; Watts et al., 2014) indicates that prior mathematical knowledge enhances high achievement at upper grades. Similarly, creating a learning environment in which students' participation is anchored on creating skills and knowledge based on prior experience is one of the most effective pillars of a robust and effective teaching methodology (Ealy, 2018; Hailikari et al., 2008). Content sequencing by teachers maximizes their ability to set clear goals for the teaching and learning program (Smith et al., 2020). Therefore, sequencing of content enhances teaching and learning and content sequencing is key when planning for effective teaching and learning of mathematics as delivery should reflect planning. This article investigates teachers' perceptions of how a framework (Chinofunga et al., 2022) on content sequencing from junior prior mathematics knowledge (years seven to 10) to senior new mathematical knowledge (years 11 to 12) enhances teaching and learning of mathematics and is critical to enhance students' participation in mathematics.

Mathematics Planning

Planning sets the foundation and path for teaching and learning. Mathematics planning involves "imagining a learning trajectory" through sequencing content to be taught "in an order that is likely to lead learners to develop further" (Mousley et al., 2007, p. 466). Likewise, effective planning promotes development of coherent content and experiences that facilitate self-paced learning (Australian Association of Mathematics Teachers [AAMT], 2006). During planning, hypothesizing how students will engage with sequenced content helps teachers choose the most effective teaching and learning instruction and activities that will be utilized during lesson planning (Mousley et al., 2007; Simon, 1995). When content sequencing during planning is done collaboratively it builds teacher capacity through knowledge sharing and demonstrates that mathematics teachers across all year levels contribute to building students' mathematical knowledge (Chinofunga et al., 2022). Content sequencing informs mathematics lesson planning and sequencing, which is beneficial to teachers if done collaboratively.

Collaborative planning provides teachers with an opportunity to share knowledge and learn from each other (Gilbert & Gilbert, 2013). "If teachers spend time collaborating and providing critical feedback on their tasks with a goal of conceptual understanding, then their students have a better chance of developing mathematical understanding and increase interest in mathematics (Boyle & Kaiser, 2017, p. 406). Teachers need to have a deep understanding of the mathematics that students have to learn, which will help them to collaboratively determine a suitable progression of how concepts should develop to new knowledge (National Council of Teachers of Mathematics [NCTM], 2014). Collaborative mathematics planning "erases the inequities in student learning expectations that otherwise could exist across a grade level or course because teammates determine what students must know and be able to do, ensuring every student has a chance to learn (Schuhl et al., 2020). Hence collaborative planning can be used to enhance teacher efficacy and enhance teaching and learning.

Teachers are heavily involved in mathematics planning at school level in many countries. Official curriculum documents and in most cases centrally approved or endorsed resources such as textbooks are provided. However, teachers in most countries have the responsibility of sequencing content (Davidson, 2019) as well as contextualize official commercial (e.g., textbooks) or non-commercial (syllabus) documents to suit their classroom dynamics (Remillard, 2005). In China, while planning is heavily influenced by official nationally approved textbooks and curriculum and instructional materials, teachers still have to contextualize content to suit the needs of their students (Li et al., 2009). Similarly, in the United States of America, states develop the curriculum and provide suggested sequencing but mathematics teachers during planning decide on how content is sequenced and enacted in a classroom (Remillard, 2005). In Australia, Queensland mathematics teachers have the responsibility to sequence content during planning.

The Australian curriculum, developed by the federal government, sets the national curriculum from preparatory to year 10 (P-10) while each state or territory determines its own senior secondary curriculum (years 11 to 12). Long term planning such as teaching and learning plans or unit planning involve sequencing and contextualizing content to students' needs and learning experiences as schools' dynamics differ (Roche et al., 2014). Most curriculum bodies provide templates and exemplars that teachers can use as reference material during planning (Grundén, 2020). The framework on content sequencing, developed by Chinofunga et al. (2022), links the nationally designed Australian curriculum (prior knowledge) to state developed senior mathematics curriculum (new knowledge). The focus of this research is to evaluate the effectiveness of the framework for teaching and learning of mathematics especially at senior level. The framework emphasis on linking foundational concepts identified at junior level to concepts to be developed at senior level to promote the gradual and deeper understanding of mathematics to reduce students' cognitive overload.

Cognitive Load Theory

As students move from a junior to senior level in education there is an escalation in cognitive demands. The cognitive load theory focuses on prior knowledge playing a central role in lessening the cognitive burden. It emphasizes the importance of foundational knowledge in acquiring new knowledge (Sweller et al., 2011). Prior knowledge that is relevant and related to new knowledge makes learning the new knowledge less difficult (Paas & Sweller, 2012). Students who have acquired the necessary schema (foundational knowledge) have a better chance of deriving meaning from new knowledge and can use it as a building block to master a skill (Moreno & Park, 2010), thus learning follows a constructivist approach. Moreover, automation of lower level (foundational knowledge) schemas is critical for developing higher level (new knowledge) schemas (Sweller, 2010). Similarly, "skilled performance is developed through building ever greater numbers of increasingly complex schemas by combining elements of lower level schemas into higher level schemas" (Center for Education Statistics and Evaluation [CESE], 2017 p. 2). Sweller (2010) went further to note that students who possess the relevant lower level schemas in their long-term memory can learn and retain new knowledge effectively. Therefore, students who are highly skilled and can readily learn new knowledge have acquired enormous stores of schematic knowledge in their memory.

The long-term memory and working memory affect the cognitive load. Changes in the long-term memory store, that is, knowledge that has been learnt from others or through problem solving, happens slowly and gradually (Sweller, 2010). The working memory is activated when students are exposed to new information, which enables it to transfer available information from long term memory and keep it to enhance problem solving. However, the working memory has limited capacity when dealing with novel information and does not have the capacity to process more than four items (Cowan, 2001). Thus, burdening the working memory can impede learning (Martin & Evans, 2020). When familiar information is involved, few working memory resources are utilized. This freeing up of working memory increases the opportunity to learn and store information in existing schemas in long term memory (Roseshine, 2009). The framework on content sequencing facilitates the development of new knowledge from familiar concepts, which enables the activation of the working memory and promotes gradual changes in the long-term memory.

Framework on Content Sequencing from Junior to Senior Mathematics

The framework on content sequencing in **Figure 1** (Chinofunga et al., 2022) was developed to provide consistency and a broad understanding on how mathematics content can be sequenced from prior to new knowledge. The key objective was to promote collaborative planning among teachers through linking mathematics concepts from the national curriculum (P-10) to concepts at senior secondary (years 11-12). In Queensland, at senior secondary level students are required to choose mathematics subjects to pursue between calculus based and non-calculus-based options. Mathematical methods and specialist mathematics are calculus-based options. Students who previously achieved good results in junior secondary school (years 7-10) found themselves struggling to comprehend concepts in calculus-based subjects at senior secondary level (Bennett, 2019). Therefore, the framework on

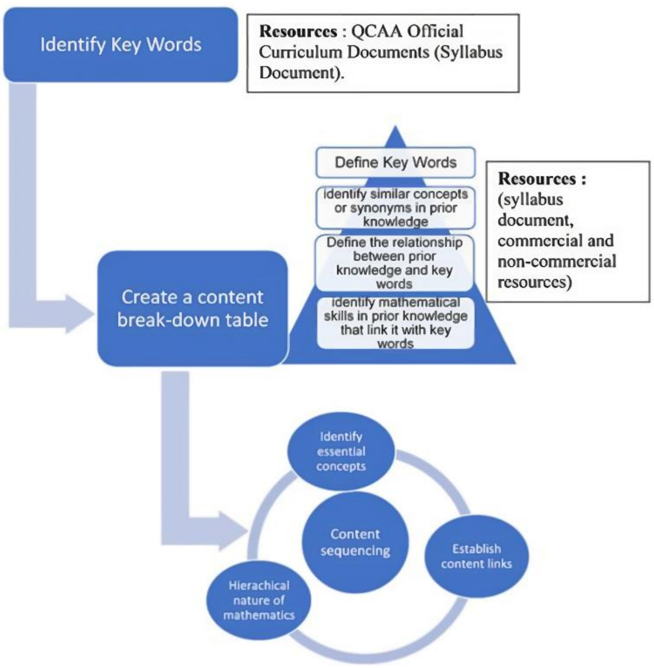

Figure 1. Framework on content sequencing (Chinofunga et al., 2022)

content sequencing demonstrates that prior knowledge (junior secondary mathematics concepts) are critical in developing new knowledge (senior secondary mathematics concepts).

Constructivists believe learners are active participants in their learning as they interpret the meaning of new knowledge and reference it to what they already know (Garbett, 2011). As a result, the study was conceptualized within a constructivist epistemology. Similarly, they emphasize that "knowledge is socially constructed through interaction of the researcher with research participants", as they share experiences (Tavakol & Sandars, 2014, p. 747). Therefore, the active interaction between the researcher and senior mathematics teachers and the sharing of experiences, beliefs and ideas played a vital role in evaluating the framework on content sequencing.

The framework "provides a step-by step systematic sequencing of curriculum content to promote interlinking, coherence and spiraling of mathematics concepts between lower- level and upper level topics" (Chinofunga et al., 2022, p. 3). It ensures that prior knowledge is central when mapping mathematics content from junior secondary to senior secondary level. Thus, the emphasis is on developing new knowledge from prior experiences. Such a framework supports the constructivist view that students learn by making sense of what is presented to them through the lenses of their prior knowledge and skills (Hu et al., 2011; Taber, 2019). Constructivism has been credited with reshaping the teaching and learning of mathematics over the years despite advocacy from traditional rote learning (Hu et al., 2011; Mallamaci, 2018; Simon, 1995; Stemhagen, 2016). Content sequencing helps to reduce the cognitive load of the official curriculum and make it familiar through linking new knowledge to prior knowledge.

Hence, evaluating how the framework informs teaching and learning of mathematics is key in realizing a critical part of mathematics planning that has been under researched.

The content sequencing framework informs the process of sequencing mathematics concepts from familiar to unfamiliar concepts, as described by Chinofunga et al. (2022). The framework is based on four elements, as described below:

The first element identifies and defines key words and their synonyms from the subject matter provided in the syllabus and is central to identifying skills and prerequisites of new knowledge. Keywords "provide significant clues for the main points about the sentence" (Li et al., 2020, p. 8196) in the content descriptions in the official curriculum documents. Similarly, key words give meaning to a sentence as dominant sentences are composed by important keywords (Domínguez et al., 2016; Wang, 2012). Importantly, by identifying key words teachers can identify the main concepts related to subject content provided in official curriculum documents (Chinofunga et al., 2021). The second element details how the prior skills and concepts link with new knowledge in the subject content and is central to content sequencing. Importantly, for deeper understanding students are expected to link mathematical concepts (Novak, 2010). Therefore, backward mapping using a concept break down table is critical in this process as it provides the opportunity to clearly link prior knowledge to new knowledge, which enhances teaching and learning of mathematics (Queensland Curriculum and Assessment Authority [QCAA], 2018). The third element identifies essential concepts. These are concepts that students are expected to return to at the end when the teaching and learning process is complete and this is done by grouping new knowledge and prerequisites into main concepts. Essential concepts are the key ideas in a unit (Schuhl, 2020), that enhances conceptual understanding (Hansen, 2011) and are to be retained long after the teaching and learning process (NCTM, 2014). The fourth and final element will follow the hierarchical nature of the identified main concepts and the sub-concepts under each main concept. "Mathematics is a hierarchical subject, where new learning builds on earlier learning in a highly connected way" (Australian Academy of Science, 2015, p. 17). Therefore, the framework takes into consideration the fact that mathematics concepts build in complexity as more teaching and learning take place.

This study focused on the following research questions:

1. What are teachers' perceptions of the effectiveness of a planning framework on content sequencing for the teaching and learning of mathematics?
2. What are teachers' perceptions of the effectiveness of collaborative planning using the planning framework for the teaching and learning of mathematics?

METHODS

This study will follow a convergent mixed methods approach. Mixed methods involve the use of quantitative and qualitative data in order to better understand the research problem as it builds on the strength of both types of data (Creswell, 2014; Creswell & Plano Clark, 2018). Importantly, convergent mixed methods provide the opportunity to converge or integrate data in a study (Fetters et al., 2013). A mixed methods approach helps to deepen (qualitative) and broaden (quantitative) the understanding of the phenomenon under study, hence providing opportunities for future research (McKim, 2017; Palinkas et al., 2013). Convergent mixed methods approach was used to explore mathematics teachers' perceptions and experiences with the framework on content sequencing. This involved obtaining quantitative data from Likert

Table 1. Likert scale responses showing participants perceptions of how framework on content sequencing & collaborative content sequencing enhances teaching & learning of mathematics

Question	SA	A	NS	D	SD
1. Content sequencing as outlined in the framework, is a critical component of mathematics planning and teaching as it provides a clear link between relevant and significant assumed prior knowledge and corresponding new knowledge.	(14) 87.5%	(2) 12.5%	0.0%	0.0%	0.0%
2. Content sequencing as outlined in the framework places assumed prior knowledge, skills and conceptual connections at the center of mathematics knowledge development.	(13) 81.3%	(3) 18.8%	0.0%	0.0%	0.0%
3. Content sequencing as outlined in the framework helps identify key concepts in a unit and hypothesizing effective delivery methods.	(13) 81.3%	(3) 18.8%	0.0%	0.0%	0.0%
4. Collaborative content sequencing as outlined in the framework reinforces teachers' responsibility of effective teaching of mathematics concepts at every level.	(13) 81.3%	(3) 18.8%	0.0%	0.0%	0.0%
5. Collaborative content sequencing as outlined in framework fosters a common agenda of focusing on how students develop mathematical knowledge.	(13) 81.3%	(3) 18.8%	0.0%	0.0%	0.0%
6. Collaborative content sequencing as outlined in the framework makes mathematics teaching a collective responsibility as students understanding and participation at higher levels depend on lower levels.	(14) 87.5%	0.0%	(2) 12.5%	0.0%	0.0%

Note. SA: Strongly agree; A: Agree; NS: Not sure; D: Disagree; & SD: Strongly disagree

scale items (**Table 1**) and qualitative data from open ended survey questions and semi-structured interviews (**Appendix A**).

Sample and Sampling Technique

Purposive sampling was used to select 16 high school mathematics teachers in Queensland, Australia. Purposive sampling involves identifying and selecting knowledgeable participants or those who have experienced the phenomenon of interest that are available and open to share their experiences and opinions (Bernard, 2011). Furthermore, in purposive sampling, participants are selected based on the researcher's background knowledge of the target population and the primary focus of the study (Monette et al., 2008). The inclusion criteria were teachers who are currently teaching or who have taught mathematics, especially calculus-based options at senior high school level (year 11 and 12) in Queensland (**Figure 1**). The teachers were also actively involved in mathematics curriculum planning at their schools and had at least five years of teaching experience.

The research participants took part in a 10-minute video presentation where they were shown how the framework on content sequencing could be used in planning forteaching and learning of mathematics. This was the most convenient way due to time constraints among participants stretched across Queensland. The mathematics content used to demonstrate how to use the framework was drawn from unit 1 in mathematical methods, with functions as a focus (Chinofunga et al., 2022). The study focused on functions because they are a major topic in mathematical methods at senior secondary, which is one of the calculus-based options with the highest dropout rate (Chinofunga et al., 2021). The participants were given a full term (10 weeks) to apply the framework in their planning sessions before data collection began.

Data Collection and Analysis

Data collection was conducted through a survey and semi structured interviews. The survey was made up of five-point Likert scale items and five open-ended questions. The semi-structured interview questions were open ended questions with the intention of prompting generative complex thoughts and responses(Bearman, 2019). The survey and semi structured interview questions were adapted from Abdeljaber (2015) and Truxaw et al. (2008). The survey instrument was also tested with a sample of 13 mathematics teachers at a regional Mathematics teachers conference to confirm reliability. The teachers confirmed the questions were clear, understandable, unambiguous, and needed approximately 10 minutes to complete. Similarly, the semi structured interview questions were pre-tested with two randomly selected teachers to establish flow and clarity of questions and approximate the interview time needed. The researcher and participants had follow-up and check-in sessions fortnightly via Zoom. The sessions were used to check on progress, challenges and if participants needed support or more information as they were applying the framework. Semi-structured interviews were conducted with eight of the sixteen participants. These were the participants who indicated availability to be interviewed online as the research was carried out during COVID-19 restrictions. The interviews provided opportunities for the interviewer to ask follow-up questions based on the interviewee's responses (Galletta & Cross, 2013; Kallio et al., 2016). Each interview took approximately 25 minutes.

Quantitative data from the 5-point Likert scale survey was collated and initial results tabulated. The mode and median responses for each question were determined. This is because Likert data are generally ordinal in nature and are best analyzed using modes and medians (Stratton, 2018). Thereafter, a table of questions and frequency of responses was created to summarize results. Data analysis of survey open-ended questions and interviews followed a thematic analysis. Thematic analysis aims to identify, investigate and reveal patterns found in a data set (Braun & Clarke, 2006). The transcribed interviews were shared with participants for checking. To ensure validity, the study used theory triangulation. It involves sharing qualitative responses among colleagues at different status positions in the field then comparing findings and conclusions (Guion et al., 2011). Coding was independently undertaken by the researchers on open ended survey responses and interview transcripts. This included independent initial identification of themes and data related to the themes, collaboratively reviewing findings, revising and discussing themes. The convergence of quantitative and qualitative data when using convergent mixed methods approach enhances the validity of the data (Creswell & Plano Clark, 2018). In this study quantitative and qualitative data converged to address the research questions.

RESULTS

The survey data collected using the five-point Likert scale was analyzed and the findings are presented in **Table 1**.

All participants strongly agreed or agreed that the collaborative content sequencing as outlined in the framework enhances teaching and learning of mathematics. In fact, at least 81.3% of participants strongly agreed that content sequencing informed by the framework linked development of new knowledge to prior knowledge. Likewise, at least 81.3% of participants strongly agreed that the framework highlighted the hierarchical nature of mathematics through collaborative content sequencing and mapping of concepts. The majority of participants strongly agreed with all the Likert scale items. This is further demonstrated by the mode and median of all items being five or strongly agree. The study strongly supports the importance of the framework on content sequencing in enhancing teaching and learning of mathematics. It further underpins the significance of collaboration during content sequencing in fostering mathematics teaching and learning and knowledge development and cohesion within and across levels.

The data from open ended survey questions and semi structured interview questions was analyzed. Although coding was done independently among researchers the results converged to only two themes. The following themes emerged, which captured the views of participants on:

- the utility of content sequencing framework in creating an environment that promotes development of new knowledge from prior knowledge and
- the utility of the framework on content sequencing in articulating the hierarchical nature of mathematics.

Theme 1: The Utility of Content Sequencing Framework in Creating an Environment That Promote Development of New Knowledge From Prior Knowledge

The general observations from participants in open ended survey questions showed that participants agreed that content sequencing as guided by the framework enhanced the development of new concepts from prior knowledge. Participants noted that the framework on content sequencing emphasized:

- sequencing content appropriately and logically to enhance student understanding,
- identifying skills needed to engage with new knowledge,
- linking prior knowledge to new concepts in the unit,
- breaking down concepts to determine fundamental concepts students need to understand or access new concepts,
- identify key concepts in the new unit and sequencing them in a logical way that links prior knowledge and builds on to new knowledge, thus develop new knowledge in small steps, and
- building from concrete to abstract.

These results demonstrated the importance of the framework on content sequencing in fostering how new and unfamiliar mathematics knowledge is developed from prior and familiar knowledge. Semi structured interviews supported the general observations but went further to include participants' perceptions on the four elements of the framework.

Semi structured interviews provided more detail on participants' views on element two of the framework. This aspect of the framework emphasized the importance of linking prior to new knowledge. The central role of prior knowledge in teaching and learning of mathematics was noted by participant 5 when he provided an example "*if you're doing measurement and geometry, make sure that the kids are good in numbers field, that number has to come before measurement.*" Thus, this provides students with an opportunity to participate and engage in the learning if they understand prior knowledge. The participant is emphasizing the importance of including in the planning and teaching, relevant and necessary prior knowledge to aid students' understanding of new knowledge. Participant 8 said

> "The proposed framework is very important because it provide guidelines and steps to follow when we are planning ... Expectations across each level are now uniform and teacher empowerment in different ways for example developing unit plans is being achieved."

The participant identified consistency in planning across levels as something that can be achieved by using the framework. The participant went further to say,

> "we really did not take content sequencing as so important until we become part of research participants but it's a weakness we are prepared to correct as we have realized it is very important for our students to develop their knowledge gradually from known to unknown."

The extract demonstrates that in some cases teachers might not have appreciated the importance of content sequencing, but the framework might highlight the benefits it brings to effective planning thus teaching. Participant 1 summed the content sequencing framework by saying

> "cut down an awful lot of time that we spend doing sequencing" and pointed out that "there is no document that I know of that links the current senior syllabus back to the knowledge that students need to know at P-10."

Therefore, the framework on content sequencing provides the basis of linking junior to senior curriculum.

Interestingly participants also highlighted how the framework on content sequencing helps to contextualize learning for different students depending on their capabilities. Participant 4 emphasized that

> "how we use sequenced content varies, depends on your local context and also conceptual and procedural connections between subject matters."

She went further to share her experience in two different schools when she said,

> "my second school, this is a more rural school, and students, their prior knowledge, has been observed, not as solid as in an urban school, so therefore content sequencing is helpful."

The participant is highlighting the key role content sequencing might play in a differentiated class. Participant 7 noted the importance of framework on content sequencing in a class when she said

> "I, myself personally feel that is best practice, that is an amazing opportunity to really customize for children."

She went further to say

> "We, we keep forgetting that every class has a specific group dynamic, every school has a specific context."

Importantly participant 3 noted that the framework could enhance teaching and learning after identifying

> "student ability level and their prior knowledge to see where we need to start."

Therefore, students operating at different levels of prior knowledge can significantly benefit as teachers have a pathway to follow, which is informed by planning. The different elements proposed in the framework play a significant role in content sequencing.

Participants also highlighted the importance of element 1 of the framework. This element emphasized the importance of identifying and defining key words and their synonyms from the subject content as central to identifying skills and prerequisites of new knowledge. Participant 6 appreciated the importance of key words in identifying prior knowledge when he said,

> "get those keywords that you talked about from the whole thing then from there, go back to your content, try to see which are the key concepts that I need to cover for students to understand new concepts."

Participant 2 went further to also include a benefit of identifying prior concepts when she said

> "identify prior concepts that you do need to teach for each particular topic using key words, this makes you think about the students' needs and what they already know."

Similarly, participant 8 pointed out that

> "key words help identify prior knowledge then fundamental and essential concepts that students have to master."

The participants emphasized how key words can provide a deeper insight into concepts. Identifying key words helps identify prior skills and concepts that are fundamental to develop new knowledge, however it is also important to explore and link the identified concepts to new knowledge.

Participants highlighted the importance of linking prior skills and concepts with new knowledge using a concept break down table as central to content sequencing. Participant 3 appreciated the framework by saying

> "the framework actually enhance content sequencing starting from prior experience through to a level content, and in fact I was keen to develop a content break down table when I saw it."

This was supported by participant 8 when he said,

> "it highlights the importance of content sequencing as it is central to any planning and demonstrate to teachers the importance of prior knowledge as demonstrated in the content breakdown table."

Importantly the participant went further to say,

> "Not that teachers are not aware of the importance of prior knowledge, but this goes deeper by including much more prior knowledge in our planning as in the content breakdown table so that our students can even correct prior knowledge misconceptions and increase their chances of understanding new knowledge with this clear and defined link."

Participant 4 had a similar view when she said,

> "building connection between prior experience and new knowledge using backward mapping in the content breakdown table is very important in systematically developing students' mathematics understanding."

Participants appreciated that linking of prior knowledge to new knowledge using a concept breakdown table provides more detail during planning on how new knowledge will be developed. The concept-break-down table plays a significant role in clearly defining how prior knowledge links with new knowledge.

Semi structured interviews also provided positive feedback on element 3 of the framework. This element helps in identifying essential or key concepts, which students are expected to return as they develop their conceptual understanding. Participant 6 noted that the framework on content sequencing helped to identify,

> "exactly the concepts that are very relevant and essential to teach …, which are the key concepts that I need to cover."

Participant 2 supported the view when she said,

> "very good in terms of identifying what are the key concepts."

Participant 5 gave an example of essential concepts when he said

> "depend on the matrix of the three big ideas algebra, geometry and number. If you're doing measurement and geometry, you make sure kids are good in numbers field."

The participant observed that foundational concepts may be key in developing higher order concepts and need to be included during planning to enhance teaching and learning. The key or essential concepts help build students' mathematics knowledge as they are the concepts that students need to retain or use as a foundation for conceptual understanding.

Theme 2: The Utility of the Framework on Content Sequencing in Articulating the Hierarchical Nature of Mathematics

Generally, results showed that participants agreed that collaborative content sequencing during planning illuminated the hierarchical nature of mathematics. From open ended survey results, participants' responses emphasized:

- hierarchical, spiraling and logical development of concepts,
- backward mapping to lower levels,
- link related concepts where one skill from a lower level can easily be transferred to the other unit at the level or above, and
- highlight the importance of teachers gaining a better understanding on how skills and the content they teach are prerequisites to learning new knowledge at a higher level.

Semi structured interviews showed positive views regarding element four of the framework. Participants agreed that use of the framework during collaborative planning articulated the hierarchical nature of mathematics across school levels. Participant 1 noted that,

> "collaborative content sequencing places the responsibility on teachers to make sure that their students know how to do this (apply a skill) because it's relevant down the track, whether it's the next topic or two three topics time, you know when a particular skill is important."

The participant went further to say,

> "teachers that never ever taught high level maths to see that okay, what I'm teaching here is really important out there, so I better do a really good job. And I really better make sure that my kids are doing or have mastered this because, it's then going to limit or they're going to limit themselves in being able to access higher learning of maths."

The participant emphasized the responsibility of teachers in determining that students understand junior concepts to be able to engage meaningfully with senior concepts. Participant 8 conclusively said

> "everyone is agreeing it remind teachers that mathematics is hierarchical therefore collaborative planning is more beneficial to everyone than individual planning."

Therefore, applying the framework collaboratively helps to foster the culture of collaboration at all levels and brings to the fore the understanding that mathematical concepts interlink and build on each other.

Participant responses also showed their agreement with the idea that exposure to more learning allowed concepts to develop and deepen for students. Participant 6 demonstrated the hierarchical nature of mathematics articulated by the framework as the basis of teaching and learning when he said,

> "when you move from one topic to another I always use some of the concepts that they did from previous lessons because if they suddenly jump and feel like there's a sudden jump, there's something that is very different from what they were doing on the previous lesson, it's is a hustle to get them to understand what needs to be done … I'll start with the basis, like the basics of the topic, so that at least I get the understanding of those students."

The participant went further to say,

> "you know the concepts that are relevant from other units or levels."

The participant's emphasis was on how the framework promotes linking of concepts to develop a web of knowledge that is coherent and developing in a gradual form. Similarly, participant 2 said the framework

> "draw the links between the topics …, which one comes first …, where we need to go to within that topic."

Participant 3 went on to say,

> "concepts are presented according to the level they are expected to be taught making content sequencing easy."

Participants appreciated that the framework on content sequencing could provide a foundation for effective planning. Participant 5 considered the broader hierarchy of mathematics when he said,

> "simple familiar content and build into complexity making sure they know the simple stuff and how to build it into, complex content."

The observation by the participant demonstrates that prior knowledge plays an important role in developing the understanding of complex concepts. Participants' responses show that the framework on content sequencing fosters the identification of prior concepts, development of new knowledge from prior knowledge, identification of key concepts and the hierarchical nature of mathematics.

DISCUSSION

The purpose of this study was to gain a better understanding of teachers' perceptions on how the framework on content sequencing from junior (years seven to 10) to senior level (years 11 to 12) can enhance planning, teaching and learning of mathematics. The results of this research provide supporting evidence that the framework places prior knowledge at the center of mathematics planning, teaching and learning. All participants agreed that the framework highlights that content sequencing is a critical component of mathematics planning and teaching as it links relevant and significant assumed prior knowledge and corresponding new knowledge. Qualitative data also supported the view as participants identified that the framework facilitated the systematic and logical linking of prior knowledge to new knowledge. Results align with previous findings by Hailikari et al. (2008) who posited that linking prior knowledge to new knowledge is key for effective mathematics teaching. The content sequencing framework focuses on including the prior knowledge at the planning stage and shows how it contributes to the development of new knowledge (Chinofunga et al., 2022). These results represent participants' support of the framework as an inherent part of planning that is key to teaching and learning of mathematics.

The identification of key words in the subject matter provided in official curriculum documents play a key role in identifying prior concepts and this is one of the critical processes advocated by the framework. Quantitative results show that at least 87.5% of participants strongly agreed that the framework facilitated the identification of prior knowledge and linked it to new content. Qualitative results provided further evidence that identification of key words in the syllabus was central to the identification of relevant prior knowledge. These results are consistent with the first stage of the framework that emphasizes that identification of key words from content as stated in official curriculum documents assists in identifying prior knowledge (Chinofunga et al., 2022). Moreover, these results are consistent with Li et al.'s (2020) work that emphasized that key words help decode the main focus of a sentence. After using key words in identifying prior knowledge, it is important to present how the prior knowledge links with new knowledge in the subject matter.

The use of concept breakdown tables in backward mapping concepts from junior to senior is also one of the key stages of the framework. Quantitative results from this study show that 81.3% of participants strongly agreed that the framework places assumed prior knowledge, skills and conceptual connections at the center of mathematics knowledge development. This in important because effective teaching and learning require students to have relevant prior knowledge to construct new knowledge and allow students to link concepts for deeper understanding (Novak, 2010). Moreover, open ended survey results showed that participants agreed with the view that it was important to break down new concepts using prior concepts so that student engagement with new concepts was enhanced. Semi structured interview results also highlighted the importance of concept breakdown tables in this regard and clearly identified the relationship as a gradual way for students to access new knowledge. These results are consistent with other research (Newton et al., 2020; QCAA, 2018) that suggest that a clear definition of the link between prior knowledge and new knowledge enhances teaching and learning of mathematics. Moreover, the results are consistent with the cognitive load theory in emphasizing the importance of prior knowledge in teaching new knowledge. The results of this research provide supporting evidence that the use of concept breakdown tables in the framework is vital in defining the relationship between prior knowledge and new knowledge thus enhancing teaching and learning of mathematics.

A vital feature of the framework is the identification of key or essential concepts that students should retain at the end of the teaching and learning process in order to build conceptual understanding. Both quantitative and qualitative results provide evidence that the framework contributes to conceptual understanding by facilitating the identification of key or essential concepts. Identification of key or essential concepts is important as it supports Hansen's (2011) and Schuhl (2020) findings that key concepts are key ideas in a unit and they are the ones that help students build conceptual understanding. Their identification helps teachers to focus on those concepts that students must retain long after the teaching and learning process (NCTM, 2014). Therefore, the opportunity that the framework affords to teachers in identifying the key concepts can enhance teaching and learning of mathematics.

Mathematics is a hierarchical subject and reflecting this in mathematics planning, teaching and learning can enhance understanding. Quantitative data from the Likert scale items showed that 87.5% of participants 'strongly agreed' that the framework reflected this hierarchical nature of mathematics, and this was confirmed by the qualitative results. These results are consistent with the understanding that mathematics is hierarchical (Nakamura, 2014), thus mathematical concepts build on each other as students are exposed to new concepts. The results from this study are also consistent with previous research undertaken by the Australian Academy of Science (20015), which show that new concepts build on earlier concepts because mathematics is highly interconnected. The hierarchical nature of mathematics makes collaborative planning the best way to apply the framework on content sequencing.

The hierarchical nature of mathematics sets the platform for collaborative content sequencing among teachers. Quantitative results show that at least 81.3% of participants strongly agreed that the framework on content sequencing from junior to senior mathematics emphasizes to teachers that understanding senior mathematics depends on how effectively concepts are taught at lower levels. Participants noted that the framework also highlights that effective teaching of mathematics at junior level is critical for students' participation at senior level. Similarly, qualitative results support that the framework stresses the interlinking of mathematics content within or across levels. The results of this research support Schneider et al. (2011) who posited that when students are taught well at junior levels and retain the knowledge, this enhances their chances of understanding senior level mathematics. Taken together, our findings indicate that the framework of content sequencing emphasizes the hierarchical nature of mathematics as a way mathematics can effectively be planned, taught and learnt.

CONCLUSIONS

In summary, teachers have a perception that the framework on content sequencing from junior to senior level mathematics can be an effective framework to use in identifying, linking and sequencing mathematics concepts. The results indicate that teachers believe that the stages in the framework can assist them to effectively sequence mathematics content in a way that promotes gradual development of new knowledge. Also, teachers noted that using the framework collaboratively appears to benefit teachers across all levels as the hierarchical nature of mathematics promotes the interconnection and interdependence of mathematics concepts. Importantly, the study provides a framework that teachers can use across schooling levels within a community of practice as they sequence content during planning. The study also highlights the importance of content sequencing during planning teaching and learning. This study supports the constructivist view of teaching mathematics that new knowledge is constructed from prior knowledge. Similarly, the study advocates for prior knowledge to be included during planning and linked to new knowledge, which could contribute towards conceptual understanding.

The study used teachers' perceptions as curriculum planners to evaluate framework on content sequencing from junior to senior concepts in mathematics. Although present results indicate that the framework on content sequencing can enhance teaching and learning of mathematics, it is appropriate to recognize that main limitation of the study is sample size. In terms of future research, it would be useful to extend current findings by examining impact of content sequencing using this framework on teacher instruction and student success.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: Authors stated that ethical approval was obtained from the Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

AAMT. (2006). Standards for excellence in teaching mathematics in Australian schools. *Australian Association of Mathematics Teachers*. https://aamt.edu.au/wp-content/uploads/2020/10/Standard-of-Excellence.pdf

Abdeljaber, S. R. (2015). *High school mathematics teachers' perceptions of mathematics education in northwest Florida* [Phd thesis, University of Phoenix].

AITSL. (2014). National professional standards for teachers. *Australian Institute for Teaching and School Leadership*. https://www.aitsl.edu.au/docs/default-source/national-policy-framework/australian-professional-standards-for-teachers.pdf?sfvrsn=5800f33c_74

AMSI. (2022). Maths crisis: Year 12 maths enrolment reach all-time low. *Australian Mathematical Sciences Institute*. https://amsi.org.au/2022/04/27/maths-crisis-year-12-maths-enrolments-reach-all-time-low/

Australian Academy of Science. (2015). Desktop review of mathematics school education. *Department of Education*. https://www.dese.gov.au/australian curriculum/resources/desktop-review mathematics-school-education-pedagogical approaches-and-learning-resources-june-2015

Bearman, M. (2019). Focus on methodology: Eliciting rich data: A practical approach to writing semi-structured interview schedules. *Focus on Health Professional Education, 20*(3), 1. https://doi.org/10.11157/fohpe.v20i3.387

Bennett, S. (2019). Kids claim new maths subjects too hard. *Courier mail*. https://www.couriermail.com.au/news/queensland/kids-claim-new-maths-subjects-too-hard/news-story/1214588829201ba7b603d551cd439483

Bernard, H. R. (2011). *Research methods in anthropology qualitative and quantitative approaches*. AltaMira Press.

Bita, N., & Dodd, T. (2022). Students shun maths as enrolments fall to all-time low. *The Australian*. https://www.theaustralian.com.au/higher-education/students-shun-maths-as-enrolments-fall-to-alltime-low/news-story/c08f2197fb24186768e8a05d591ca256

Boyle, D. J., & Kaiser, B. S. (2017). Collaborative planning as a process. *Mathematics Teaching in the Middle School, 22*(7), 406-419. https://doi.org/10.5951/mathteacmiddscho.22.7.0406

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101. https://doi.org/10.1191/1478088706qp063oa

CESE. (2017). *Effective reading instruction in the early years of school, literature review*. NSW Department of Education. https://www.cese.nsw.gov.au/images/stories/PDF/Effective_Reading_Instruction_AA.pdf

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2021). Senior high school mathematics subjects in Queensland: Options and trends of student participation. *PRISM: Casting New Light on Learning, Theory and Practice, 4*, 1. https://doi.org/10.24377/prism.ljmu.0401216

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2022). A framework for content sequencing from junior to senior mathematics curriculum. *EURASIA Journal of Mathematics, Science and Technology Education, 18*(4), em2100. https://doi.org/10.29333/ejmste/11930

Cowan, N. (2001). 'The magical number 4 in short-term memory: A reconsideration of mental storage capacity'. *Behavioral and Brain Sciences, 24*(1) 87-114. https://doi.org/10.1017/S0140525X01003922

Creswell, J. W. (2014). *Educational research: Planning, conducting and evaluating quantitative and qualitative research*. Pearson.

Creswell, J. W., & Plano Clark, V. L. (2018). *Designing and conducting mixed methods research*. SAGE.

Davidson, A. (2019). Ingredients for planning student-centered learning in mathematics. *Australian Primary Mathematics Classroom, 24*(3), 8-14.

Domínguez, A.-B., Carrillo, M.-S., González, V., & Alegria, J. (2016). How do deaf children with and without cochlear implants manage to read sentences: The key word strategy. *Journal of Deaf Studies and Deaf Education, 21*(3), 280-292. https://doi.org/10.1093/deafed/enw026

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L. S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428-1446. https://doi.org/10.1037/0012-1649.43.6.1428

Ealy, J. (2018). Analysis of students' missed organic chemistry quiz questions that stress the importance of prior general chemistry knowledge. *Education Sciences, 8*(2), 42. https://doi.org/10.3390/educsci8020042

Fautley, M., & Savage, J. (2014). *Lesson planning for effective learning*. McGraw-Hill Education.

Fetters, M. D., Curry, L. A., & Creswell, J. W. (2013). Achieving integration in mixed methods designs–Principles and practices. *Health Services Research, 48*, 2134-2156. https://doi.org/10.1111/1475-6773.12117

Galletta, A., & Cross, W. E. (2013). *Mastering the semi-structured interview and beyond: From research design to analysis and publication*. New York University Press. https://doi.org/10.18574/nyu/9780814732939.001.0001

Garbett, D. (2011). Constructivism deconstructed in science teacher education. *Australian Journal of Teacher Education, 36*(6), 36-49. https://doi.org/10.14221/ajte.2011v36n6.5

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PloS ONE, 8*(1), e54651-e54651. https://doi.org/10.1371/journal.pone.0054651

Gilbert, M., & Gilbert, B. (2013). Connecting teacher learning to curriculum. In A. M. Lindmeier, & A. Heinze (Eds.), *Mathematics learning across the life span* (pp. 337-344). PME.

Grundén, H. (2020). Planning in mathematics teaching–a varied, emotional process influenced by others. *LUMAT: International Journal on Math, Science and Technology Education, 8*, 1. https://doi.org/10.31129/LUMAT.8.1.1326

Guion, L. A., Diehl, D. C., & McDonald, D. (2011). *Triangulation: Establishing the validity of qualitative studies*. University of Florida Cooperative Extension Service, Institute of Food and Agricultural Sciences, EDIS. https://doi.org/10.32473/edis-fy394-2011

Hailikari, T., Katajavuori, N., & Lindblom-Ylanne, S. (2008). The relevance of prior knowledge in learning and instructional design. *American Journal of Pharmaceutical Education, 72*(5), 113. https://doi.org/10.5688/aj7205113

Hansen, E. (2011). *Idea-based learning a course design process to promote conceptual understanding*. Stylus Publishing.

Hu, Y., Wang, W., & Jiang, L. (2011). Teaching discrete mathematics with the constructivism learning theory. In *Proceedings of the 6th International Conference on Computer Science & Education*.

Jones, K., & Edawards, J. (2017). *Planning for mathematics learning*. Routledge. https://doi.org/10.4324/9781315672175

Kallio, H., Pietilä, A. M., Johnson, M., & Kangasniemi, M. (2016). Systematic methodological review: Developing a framework for a qualitative semi-structured interview guide. *Journal of Advanced Nursing, 72*(12), 2954-2965. https://doi.org/10.1111/jan.13031

Kennedy, J., Lyons, T., & Quinn, F. (2014). The continuing decline of science and mathematics enrolments in Australian high schools. *Teaching Science, 60*(2), 34-46.

Kuehnert, E. R. A., Eddy, C. M., Miller, D., Pratt, S. S., & Senawongsa, C. (2018). Bansho: Visually sequencing mathematical ideas. *Teaching Children Mathematics, 24*(6), 362-369. https://doi.org/10.5951/teacchilmath.24.6.0362

Li, H., Zhu, J., Zhang, J., Zong, C., & He, X. (2020). Keywords-guided abstractive sentence summarization. In *Proceedings of the 34th AAAI Conference on Artificial Intelligence* (pp. 8196-8203). https://doi.org/10.1609/aaai.v34i05.6333

Li, Y., Chen, X., & Kulm, G. (2009). Mathematics teachers' practices and thinking in lesson plan development: A case of teaching fraction division. *ZDM-Mathematics Education, 41*, 717-731. https://doi.org/10.1007/s11858-009-0174-8

Mallamaci, L. (2018). Constructivism in mathematics. *Vinculum (Parkville, Vic.), 55*(2), 20-21.

Martin, A. J., & Evans, P. (2020). Load reduction instruction (LRI): Sequencing explicit instruction and guided discovery to enhance students' motivation, engagement, learning, and achievement. In S. Tindall-Ford, S. Agostinho, & J. Sweller (Eds.), *Advances in cognitive load theory: Rethinking teaching* (pp. 15-29). Routledge/Taylor & Francis Group. https://doi.org/10.4324/9780429283895-2

McKim, C. A. (2017). The value of mixed methods research: A mixed methods study. *Journal of Mixed Methods Research, 11*(2), 202-222. https://doi.org/10.1177/1558689815607096

Monette, D. R., Sullivan, T. J., & DeJong, C. R. (2008). *Applied social research: A tool for the human services*. Thomson Brooks/Cole.

Moreno, R., & Park, B. (2010). Cognitive load theory: Historical development and relation to other theories. In J. Plass, R. Moreno, & R. Brünken (Eds.), *Cognitive load theory* (pp. 9-28). Cambridge University Press. https://doi.org/10.1017/CBO9780511844744.003

Mousley, J., Sullivan, P., & Zevenbergen, R. (2007). Keeping all students on the learning path. In *Proceedings of the 9th International Conference Mathematics Education in a Global Community* (pp. 466-471).

Nakamura, A. (2014). Hierarchy construction of mathematical knowledge. *Lecture Notes on Information Theory, 2*(2), 203-207. https://doi.org/10.12720/lnit.2.2.203-207

NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

Newton, K. J., Lange, K., & Booth, J. L. (2020). Mathematical flexibility: Aspects of a continuum and the role of prior knowledge. *The Journal of Experimental Education, 88*(4), 503-515. https://doi.org/10.1080/00220973.2019.1586629

Novak, J. D. (2010). Learning, creating, and using knowledge: Concept maps as facilitative tools in schools and corporations. *Je-LKS, 6*, 3. https://doi.org/10.20368/1971-8829/441

Noyes, A., & Adkins, M. (2016). Studying advanced mathematics in England: Findings from a survey of student choices and attitudes. *Research in Mathematics Education, 18*(3), 231-248. https://doi.org/10.1080/14794802.2016.1188139

Paas, F., & Sweller, J. (2012). An evolutionary upgrade of cognitive load theory: Using the human motor system and collaboration to support the learning of complex cognitive tasks. *Educational Psychology Review, 24*, 27-45. https://doi.org/10.1007/s10648-011-9179-2

Pagani, L. S., Fitzpatrick, C., Archambault, I., & Janosz, M. (2010). School readiness and later achievement: A French Canadian replication and extension. *Developmental Psychology, 46*(5), 984-994. https://doi.org/10.1037/a0018881

Palinkas, L. A., Horwitz, S. M., Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K. (2013). Purposeful sampling for qualitative data collection and analysis in mixed method implementation research. *Administration and Policy in Mental Health and Mental Health Services Research, 42*(5), 533-544. https://doi.org/10.1007/s10488-013-0528-y

QCAA. (2018). Mathematical methods. General senior syllabus. *Queensland Curriculum and Assessment Authority*. https://www.qcaa.qld.edu.au/downloads/senior-qce/syllabuses/snr_maths_methods_19_syll.pdf

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246. https://doi.org/10.3102/00346543075002211

Roche, A., Clarke, D. M., Clarke, D. J., & Sullivan, P. (2014). Primary teachers' written unit plans in mathematics and their perceptions of essential elements of these. *Mathematics Education Research Journal, 26*(4), 853-870. https://doi.org/10.1007/s13394-014-0130-y

Roseshine, B. V. (2009). The empirical support for instruction. In S. Tobias, & T. M. Duffy (Eds). *Constructivist instruction: Success or failure?* Routledge.

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 47*(6), 1525-1538. https://doi.org/10.1037/a0024997

Schuhl, S. (2020). *Mathematics unit planning in a PLC at work. Grades 3-5*. Solution Tree Press.

Schuhl, S., Kanold, T. D., Deinhart, J., Larson, M. R., & Toncheff, M. (2020). *Mathematics unit planning in a PLC at work®, grades 3-5: A guide to collaborative teaching and mathematics lesson planning to increase student understanding and expected learning outcomes*. Solution Tree.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145. https://doi.org/10.2307/749205

Smith, M. S., Sherin, M. G., & Steele, M. (2020). *The five practices in practice: Successfully orchestrating mathematics discussions in your high school classroom*. SAGE.

Stemhagen, K. (2016). Deweyan democratic agency and school math: Beyond constructivism and critique: Deweyan democratic agency and school math. *Educational Theory, 66*(1-2), 95-109. https://doi.org/10.1111/edth.12156

Stratton, S. J. (2018). Likert data. *Prehospital and Disaster Medicine, 33*(2), 117-118. https://doi.org/10.1017/S1049023X18000237

Sullivan, P., Clarke, D. M., Clarke, D., & Roche, A. (2013). Teachers' decisions about mathematics tasks when planning. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow*. MERGA.

Sweller, J. (2010). Cognitive load theory: Recent theoretical advances. In J. Plass, R. Moreno, & R. Brünken (Eds.), *Cognitive load theory* (pp. 29-47). Cambridge University Press. https://doi.org/10.1017/CBO9780511844744.004

Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive load theory*. Springer. https://doi.org/10.1007/978-1-4419-8126-4

Taber, K. S. (2019). Constructivism in education: Interpretations and criticisms from science education. In *Early childhood development: Concepts,* *methodologies, tools, and applications* (pp. 312-342). https://doi.org/10.4018/978-1-5225-7507-8.ch015

Tavakol, M., & Sandars, J. (2014). Quantitative and qualitative methods in medical education research: AMEE Guide No 90: Part I. *Medical Teacher, 36*(9), 746-756. https://doi.org/10.3109/0142159X.2014.915298

Truxaw, M. P., Gorgievski, N., & DeFranco, T. C. (2008). Measuring K-8 teachers' perceptions of discourse use in their mathematics classes. *School Science and Mathematics, 108*(2), 58-70. https://doi.org/10.1111/j.1949-8594.2008.tb17805.x

Wang, M. T. (2012). Educational and career interests in math: A longitudinal examination of the links between classroom environment, motivational beliefs, and interests. *Developmental Psychology, 48*(6), 1643-1657. https://doi.org/10.1037/a0027247

Watt, H. (2007). A trickle from the pipeline: Why girls under-participate in maths. *Professional Educator, 6*(3), 36-41.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*(7), 352-360. https://doi.org/10.3102/0013189X14553660

APPENDIX A

Open Ended Survey Questions

1. How can we enhance collaborative mathematics planning in schools?
2. How do you identify assumed knowledge critical to new knowledge?
3. How do you identify essential concepts (concepts that students must return in a topic)?
4. How do you sequence content in a unit?
5. Was the rationale for this framework realized?

Semi Structured Interview Questions

Planning framework (content sequencing)

1. How would you define/describe collaborative mathematics planning in your school?
2. What informs content sequencing as you go through planning?
3. How would/did the framework that is being proposed enhance content sequencing at your school?