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Teaching Euclidean geometry with GeoGebra: Perceptions for in-service mathematics teachers

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Abstract

The teaching of Euclidean geometry is characterized by ineffective instructional methods used by in-service teachers as well as the low proficiency levels by learners. The purpose of this study was to survey in-service mathematics teachers' GeoGebra integrative skills in the teaching of geometry. This study was quantitative and pre- and post-questionnaires were used to collect data. Descriptive and inferential statistics were used to perform statistical analysis of quantitative data. 12 schools were randomly selected, and purposive sampling was employed to select 29 in-service mathematics teachers. The study revealed that the intervention impacted positively on in-service teachers instructional strategies with a high statistical significance and a gain of medium to large effect size on both the pre- and post-intervention. After the training, participants felt that integrating GeoGebra in geometry teaching affords mathematics teachers the opportunity to use learner-centered approaches, teach geometry with confidence and maintain learners' attention and alertness in class.

Keywords: Euclidean geometry, GeoGebra, in-service teachers, training, TPACK, perceptions

INTRODUCTION

The teaching profession is dynamic and responsive to changes in learner composition and instructional approaches. Teachers' responsiveness to the changing educational landscape is hinged on continual teacher development. The teacher professional development (TPD), also termed in-service training "the instruction provided to teachers to promote their development in a certain area such as technology or subject mastery" (Gaible & Burns, 2005, p. 31). Hewson and Luft (2014) indicate that it is important that teachers engage in teacher education activities that focuses in improving teaching practice and learner attainment. Thus, "in-service training is not only desirable but also an activity to which each school system must commit human and fiscal resources if it is to maintain a skilled and knowledgeable staff" (Ronald, 2004, p. 170). Teachers who embrace and integrate technology in geometry teaching have the power to improve learners' performance by further engaging learners in their lessons (Marange et al., 2021). A study by Mthethwa et al. (2020) highlights some recent increases in the enhancement of geometry learning and teaching with the aid of educational technologies such GeoGebra. GeoGebra is a highly popular software in the field of learning and teaching of mathematics. It is an interactive and free dynamic geometry software (DGS), which is used for creation and manipulation of geometric constructions (Akgul, 2014). DGSs such as GeoGebra affords both teachers and learners the opportunity to visualize geometric objects. GeoGebra application facilitates the process of geometry learning and teaching (Haciomeroglu et al., 2009). The research question for this study was: "How do in-service mathematics teachers perceive training to integrate GeoGebra in Euclidean geometry teaching?"

The introduction of Euclidean geometry in further education and training school curriculum in South Africa in the revised curriculum and assessment policy statement in 2012 brought many pedagogical challenges to many secondary mathematics teachers. One of the

This article is an extract from the first author's doctoral thesis.

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Contribution to the literature

- This study revealed a greater need for TPD, which incorporates educational technologies in the teaching and learning of selected topics in mathematics.
- A training of in-service teachers in the integration of GeoGebra was conducted by the first author, which resulted in good perceptions by the teachers.
- After the training, participants felt that integrating GeoGebra in geometry teaching affords mathematics teachers the opportunity to use learner-centered approaches, teach geometry with confidence, and maintain learners' attention and alertness in class.

challenges the first author observed was teachers' inadequate use of the instructional approaches in teaching of geometry. The teaching of geometry predominantly reflects teacher-centered approaches to learning. The use of teacher-centered instructional approaches was mostly observed in most secondary schools. It was evident that mathematics teachers encouraged passive learning and made learners feel that they had nothing to contribute because they (teachers) were dominant throughout the lesson. The approach most teachers used led to boredom in class. The teachers had no resources other than textbooks. The first author observed that challenges experienced by learners in understanding geometry were not different from those experienced by teachers in teaching it. This study sought to develop in-service teachers on the use of GeoGebra when teaching geometry and explore their perceptions. Using GeoGebra can transform the teacher's approach to teaching from a teacher-centric to a learner-centric approach; from direct instruction in which the teacher imparts the knowledge, to research-based or cooperative learning.

LITERATURE REVIEW & THEORETICAL FRAMEWORK

Professional development (PD) is described as an "ongoing in-service development program that focuses on the whole range of knowledge, skills and attitudes required to educate learners effectively" (Steyn & van Niekerk, 2002, p. 250). Exposing teachers to ongoing TPD promotes effective teaching. Such TPD afford teachers the opportunity to improve their own knowledge and skills through workshops and training. Whitworth and Chiu (2015) point out that the goals of any TPD are to improve teacher education and ultimately learner-learning. Through TPD, in-service teachers are introduced to modern and advanced teaching strategies to improve the quality of teaching approaches. This allows teachers to incorporate innovative teaching methods into the classroom and encourages them to embrace new methods. Desimone et al. (2002) claim that TPD, which places more emphasis on subject matter and focuses on the way learners learn, is seen as encouraging, particularly for a teaching approach that seeks to improve learners' conceptual understanding. Cohen and Hill (2000) and Kennedy (1998) emphasize that a TPD program is considered ineffective in changing teacher practice if it lacks a strong content component. In other words, positive changes in teacher practice are the result of effective TPD. Therefore, according to Desimone (2009), an effective teacher education program leads to increased teacher knowledge and positive changes in the teachers' attitudes and beliefs. Some countries in Africa are lagging in the global trend of TPD (Ajani, 2018). In Kenya, for instance, 350,000 teachers in public and private schools were requested to undergo mandatory two-year in-service professional training course to enhance their capacities (Adika & Mung'ala, 2018). The Nigeria education ministry emphasized the importance of capacitating teachers in both private and public schools (Alabi & Ige, 2014). The imbalance in in-service teacher training in southern and northern Nigeria has raised concerns about educational gaps. Hence, Alabi and Ige (2014) emphasize the need for TPD programs to fill the identified educational gap that exists among working teachers. Furthermore, Schacter (2001) asserts that geometric processes performed using chalkboard or pencil and paper can be confirmed using the visualization aspects of GeoGebra.

The problems of teaching Euclidean geometry concepts due to the topic's nature of requiring illustration and demonstration can be solved by involving teachers in technology integration training. So, this study covers one of the emerging mathematics didactics in mathematics education (de Oliveira & Alves, 2019). This is because this study intends to "cover aspects related to the teaching of mathematics, emphasizing the difficulties arisen in the learning process of the mathematical concepts" (Pais, 2002, p. 9). Bueno et al. (2021) attest that technological pedagogical and content knowledge (TPACK) becomes a powerful way to enable teachers to understand the connections between content, pedagogy and technology. Wood et al. (2005) noted that training for TPACK helped teachers to understand pedagogical strategies and reduce their anxiety. Teaching professional survey conducted by Turkish Education Association (2009) stated that TPACK is important in teachers' professional success. Saralar et al. (2018) recommend incorporating technology-integrated courses into teacher training. This is because of the fact that the present technology that is used is so closely related to 21st century generation of today,

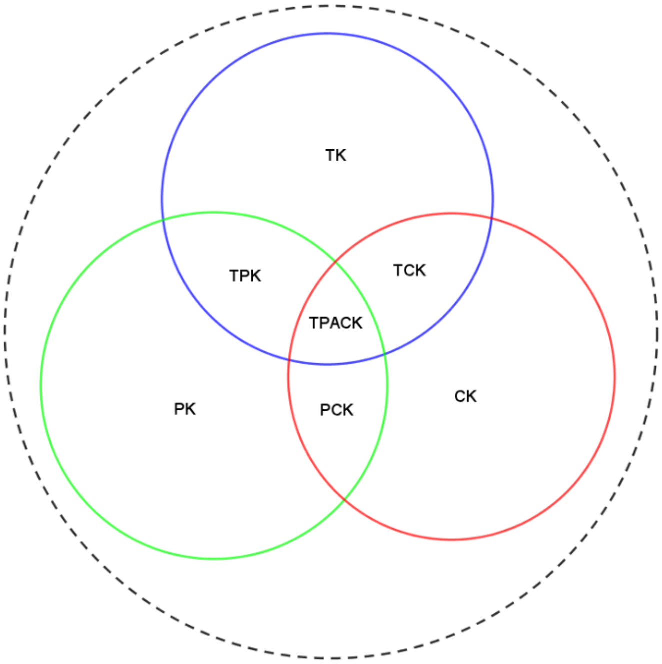

Figure 1. Technology, pedagogical, & content knowledge domains (Koehler & Mishra, 2009, p. 63)

therefore, teaching and learning environments cannot be considered without technology. Mudzimiri (2012) warned that teachers' TPACK will not develop immediately but it takes time. Mudzimiri (2012) emphasize that teachers' TPACK development can be influenced by factors such as technological experience, content knowledge (CK) and beliefs about technology. According to Verhoef et al. (2015), teachers in the Netherlands designed their lessons in ways that helped learners to understand derivatives of functions using GeoGebra application. In Nepal also, teachers were observed to be confident in the use GeoGebra in their teaching (Uwurukundo et al., 2020). In the same article, Uwurukundo et al. (2020) further indicate that Taiwanese teachers used GeoGebra as a learning object for learners to visualize mathematics concepts that were given in textbooks as activities. Hence, Bos (2009) asserts that "if mathematics is seen as problem solving and thoughtfully teamed with technology, deep conceptual learning can be a reality" (p. 527). Allison Lu (2008) investigated Taiwanese teachers' use GeoGebra and how it impacts their teaching practices in mathematics learning and teaching. The study revealed that teachers used different strategies to integrate GeoGebra into their teaching practices, which include preparing teaching material, presentation of geometry content and concepts and learners' classwork and investigation activities of geometry.

This study was based on TPACK theory. According to Mishra and Koehler (2006), TPACK was developed to serve two purposes. First, it aims to help teachers plan their PD by an indication of what teachers ought to know concerning pedagogy, technology, and content and how they relate to each other. Secondly, it may be helpful for teachers who are designing learning activities to help them see how technology intersects (or does not) with content, pedagogy and content-specific pedagogy. TPACK is a framework for teacher knowledge that affirms the link between technology, content and specific pedagogical approaches. It shows how teachers' understanding of pedagogy, technology and content can interact to enable effective teaching using digital educational technologies (Mishra & Koehler, 2006). Using TPACK as a framework for identifying and exploring knowledge of technology might potentially have implications for PD experiences and the type of education designed for qualified teachers (Schmidt et al., 2009). It is important to note that the principles of TPACK are guided under the professional activities of mathematics teachers, which originated from the French branch of mathematics didactics (Alves & Catarino, 2019). In this context, teachers interacted with the researcher, and they were part of the research design. The aim of this teacher-researcher relationship and teacher involvement in the design processes was purely for the understanding of teaching and learning purposes. This principle was based on the use of didactic engineering and situation theory. Moreover, TPACK theoretical framework emphasizes that technology integration for pedagogy for specific content requires the dynamic and transactional relationship between three the fundamental knowledge domains and their points of intersection intersections, as shown in **Figure 1**. These knowledge domains are CK, technological knowledge (TK), and pedagogical knowledge (PK). The interaction of the three main components of TPACK must be the foundation of any technology that teachers want to use in the classroom to enhance learning (Mishra & Kohler, 2006).

The role that teachers play in ensuring that they acquire TPACK skills is based on the theory of situations and professional didactics. The theory of situations is a driving force towards TPACK skills acquisition. It requires teachers' action and the interest in their teaching profession. Alves (2018) claim that theory of situations has a strong element of pragmatism. Professional didactics is an action conceptualization theory, which is grounded in cognitivist learning theory (Alves & Catarino, 2019). In this study, professional didactics involved adult learning and the gradual acquisition of both technical and professional skills of TPACK by in-service teachers.

METHODOLOGY

The study employed quantitative research methodology to achieve the research goal and the survey research design was followed. The survey research design consists of a scientific sampling method with designed questionnaires to measure the characteristics of a given population by means of statistical methods (Sukamolson, 2007).

Table 1. Mean score interpretation table

Mean range	Interpretation
4.3 to 5.0	Strongly agree
3.5 to 4.2	Agree
2.7 to 3.4	Neutral
1.9 to 2.6	Disagree
1.0 to 1.8	Strongly disagree

The study employed targeted sampling and the sample consisted of professional mathematics teachers working in secondary schools in an education district in South Africa. A total of twelve schools were randomly selected, seven of which were city schools and five rural schools. A total of 29 employed mathematics teachers (eight women and 21 men) from 12 schools were purposively selected and considered to participate in the study. Cohen et al. (2011) pointed out that targeted sampling must be used to access knowledgeable individuals for a specific purpose. Therefore, targeted sampling was used to identify study cases, where the greatest benefit could be derived from the phenomenon under study (Merriam, 2009). This study used pre- and post-TPACK questionnaires as instruments to gather quantitative data. Quantitative data was collected on TPACK components, which are TK, technology pedagogical knowledge (TPK), technology content knowledge (TCK), and the central core TPACK. TPACK questionnaires used the Likert rating scales. Descriptive and inferential statistics were performed in order to analysis and interpret the collected data. The descriptive statistics consisted of standard deviation (SD), mean (M), frequency tables, and histograms. The t-test and the z-score test of independence were among the inferential statistics that was calculated on GeoGebra classic 5. The researchers interpreted the calculated mean of each item TPACK using the mean score interpretation score interval in **Table 1** (Nunnary & Berstein, 1994). For instance, if the mean of TK 4 (*frequently played around with technology)* is 3.97, then it means the respondents generally *agreed* on the fact that they *frequently played around with technology* (see **Table 1**). Comparison of the mean and interpretation of pre- and post-questionnaires were then used to make decisions and reach conclusion.

Furthermore, in order to analyze the differences between pre- and post-intervention perceived knowledge and skills of integrating GeoGebra in geometry teaching, the researcher employed the t-test and in some cases the z-score test of the mean and effect size, Cohen's d. Thus, in inferential statistics the study used different test of independence calculated with SPSS and GeoGebra classic 5.

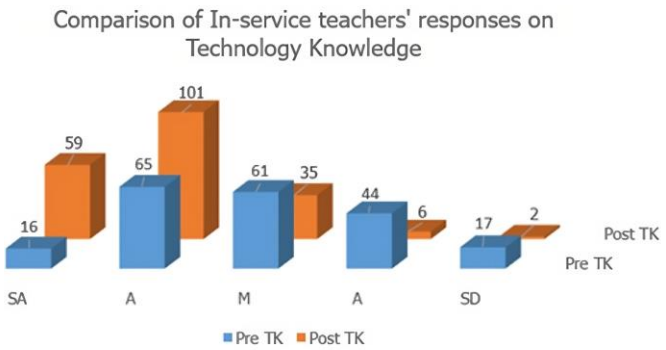

Figure 2. Comparison of in-service teachers' responses on TK (Source: Authors' own illustration)

FINDINGS

In-Service Teachers' Technological Knowledge Before & After Training

A total of 203 responses were registered for the in-service teachers' technology knowledge component of TPACK questionnaire. As seen, **Figure 2** and **Table 2** show the comparison results of the in-service teachers' technology knowledge (TK) before and after training.

The results show that in-service teachers' TK improved. For instance, **Figure 2** shows a greater improvement of strongly agree responses of in-service teachers' TK from 16 (8%) before the training to 59 (29%) after the training. A total of 101 responses indicated agree on post-training, which is 36 more than the agree responses registered prior to the training. At most 61 responses shows at least a disagreement before the training compared to eight disagree responses registered after the training. This means that 53 (26%) who registered at least a disagree before the training, registered at least an agree after the training. Further, a total of 61 responses was recorded as neutral (maybe) prior to the intervention and post-intervention responses declined by 26 (13%). The mean, SD, and effect size *(d)* of the seven items of in-service teachers' TK was calculated, as shown in shown in **Table 2**.

Table 2. In-services' teachers' TK before & after integration training

TK item description & code	Before training		After training		Effect size *(d)*
	M	SD	M	SD	
I know how to solve technical problems (TK1).	2.93	2.59	3.97	3.55	0.33
I can learn GeoGebra easily (TK2).	3.55	3.22	4.45	3.96	0.25
I keep up with important new technologies (TK3).	3.31	2.96	4.07	3.60	0.23
I frequently play around with GeoGebra (TK4).	3.07	2.77	3.97	3.52	0.28
I know about a lot of different technologies (TK5).	3.03	2.65	3.76	3.52	0.24
I have the technical skills to use GeoGebra (TK6).	3.17	2.80	4.24	3.78	0.32
I have had opportunities to work with tech (TK7).	2.59	2.29	3.76	3.34	0.41

Table 3. Pre- & post-in-service teachers' technology knowledge (TK)

	M	SD	n	Standard error	*z*-score	*p*-value	Cohen's *d*	Cronbach's alpha
Pre-TK	3.09	2.77	203	0.3183	-2.9536	0.0031	0.29	0.98
Post-TK	4.03	3.59	203					

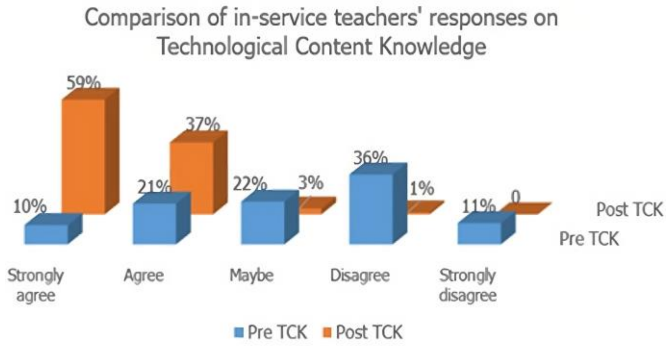

Figure 3. Comparison of in-service teachers' TCK responses (Source: Authors' own illustration)

The results show that five (71%) items registered a neutral before the training. In-service teachers' TK improved impressively with at least six (86%) items registering at least an agree after the training. Furthermore, the greatest change occurred in the item "I have had sufficient opportunities to work with different technologies (TK7)" with a gain of 1.17. The item with the least reported change with gain of 0.23 was "I keep up with important new technologies (TK3)". Results in **Table 3** showed a high significant increase in in-service teachers' TK when comparing pre- and post-training results.

As seen in **Table 3**, according to the *z*-test, the mean difference of 0.94 in favor of the post-in-service teachers' TK was observed. The overall views (before: M=3.09, SD=2.77; after: M=4.03, SD=3.59, *p*-value=0.0031) also showed highly statistically significant with a medium effect size (*d*=0.29) before and after the training. In-service teachers' responses on TK also shows that GeoGebra integration training helped teachers with the understanding of educational technologies in the teaching of geometry.

Teachers' Technology Content Knowledge Before & After Training

There were 174 in-service teachers' responses recorded when participants registered the extent of their agreement or disappointment on pre- and post-TPACK questionnaire. The comparison of these responses is shown in **Figure 3** expressed as percentages. For instance, pre-intervention results showed that in-service teachers self-rated their TCK of pre-intervention as at least an agree with a total response of 21% compared to 37% at least an agree responses recorded post-intervention.

At most 47% responses registered at least a disagree before the training. However, post-intervention registered only 1% (four) responses for a disagree. This means that 46% responses, which registered at least a disagree in pre-intervention registered at least an agree post-intervention. Also, out of 22% of responses recorded as neutral prior to the training, 19% then registered at least an agree post-training. The mean of TCK2 increased by 0.86 compared with pre-intervention of the same measuring item.

Table 4 further shows in-service teachers self-rated their TCK as agreed before the training with TCK1, TCK3, and TCK6 rating as disagree.

The situation improved significantly after the training with all the six items rated strongly agree (see **Table 1**). Apparently, the largest effect size occurred in the item with largest mean average TCK6 with gain of 0.71, whilst the item TCK2 with the least mean had the least reported effect size with gain of 0.22. As shown in **Table 4**, the mean and SD of in-service teachers' TCK perceptions (before and after the training) significantly increased for all the items. The largest average change occurring in the item "I can use GeoGebra software to conduct geometric-related inquiry activities (TCK6)".

Table 4. In-service teachers' TCK prior to GeoGebra training

TCK item description & code	Before training		After training		Effect size *(d)*
	M	SD	M	SD	
I know about technologies that I can use for understanding & doing geometry (TCK1).	2.66	2.33	4.34	3.88	0.53
Using GeoGebra can fundamentally change way teachers & learners understand geometry concepts (TCK2).	3.93	3.54	4.79	4.29	0.22
I can use GeoGebra software tools to demonstrate geometric subject content (TCK3).	2.59	2.29	4.45	3.97	0.57
I know what technologies can be applied to teach geometry (TCK4).	2.93	2.65	4.45	3.98	0.45
I can use proper technologies to represent geometry subject content (TCK5).	2.69	2.35	4.41	3.95	0.53
I can use GeoGebra software to conduct geometric-related inquiry activities (TCK6).	2.24	1.91	4.48	4.00	0.71

Table 5. In-service teachers' technological content knowledge

	M	SD	n	Standard error	*z*-score	*p*-value	Cohen's *d*	Cronbach's alpha
Pre-TCK	2.84	2.56	174	0.3607	-4.5749	0.000	0.49	0.94
Post-TCK	4.49	4.01	174					

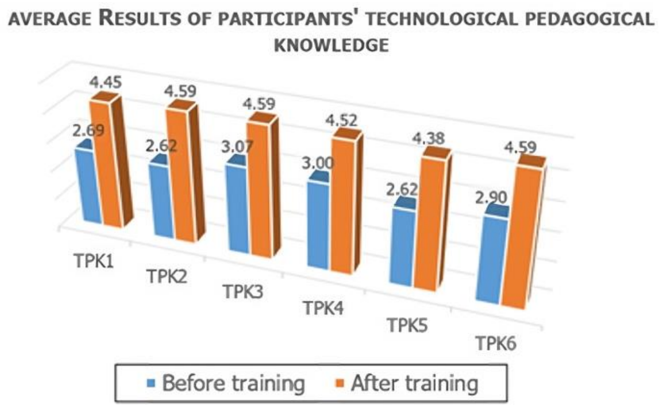

Figure 4. Comparison of in-service teachers' TPK pre- & post-intervention (Source: Authors' own illustration)

This item increased by 2.24 compared to the mean before the training. The least mean increase was reported in the item "Using GeoGebra can fundamentally change the way teachers and learners understand geometry concepts (TCK2)".

Findings in **Table 5** showed a significant increase in the overall in-service teachers' responses on their TCK. The overall views post-technology integration training show the mean of 4.49 and SD of 4.01, which was 1.65 and 1.45 increase in mean and SD, respectively prior the training. Furthermore, in **Table 5**, the *p*-value<0.01 showed highly statistically significant with a medium effect size (*d*=0.49) before and after the training.

In-Service Teachers' Technology Pedagogical Knowledge Before & After Training

A total of 174 responses were recorded on in-service teachers' TPK responses. Mean and SD were computed of the data from in-service teachers' TPK construct. This construct contained six items and the results are shown in **Figure 4**.

None of the six items rated at least an agree prior to the intervention. In fact, on average, the pre-intervention results showed that in-service teacher's self-rate their TPK as neutral (maybe) in four items and two items as disagree (see **Table 1**). Items such as "I can use GeoGebra technologies to enhance learners' enthusiasm for learning (TPK2)" and "I can select appropriate technologies to optimize geometric teaching (TPK5)" recorded the least mean of 2.62 before the training. A significant increase in teachers' TPK was observed in all six items of construct. Thus, post-intervention results showed that in-service teachers self-rated their TPK as strongly agree in all items of the construct. On average each item of TPK construct recorded a mean above 4.00 (see **Table 6**) and **Figure 3** with the largest mean of 4.59 registered in items such as TKP2, TPK3, and TPK6.

The effect sizes for these six items used to measure in-service teachers' TPK were between 0.4 and 0.6 Cohen's *d*, which implies medium to large effect sizes. Thus, item "I can use GeoGebra technologies to enhance learners' enthusiasm for learning (TPK2)" recorded the largest area change with a gain of 0.60 whilst item "I can adaptively use GeoGebra technology in various teaching activities (TPK4)" recorded the least area change with a gain of 0.45. The overall results in **Table 7** show that in-service teachers' TPK on GeoGebra integration improved significantly (before: M=2.82, SD=2.46; after: M=4.52, SD=4.03, *p*-value=0.000). The results also showed significant difference with a large effect size (Cohen's *d*=0.51) before and after the training.

Teachers' Technological Pedagogical & Content Knowledge Before & After Training

The results in **Table 8** and **Figure 5** are based on 174 in-service teachers' TPACK responses registered before and after the training.

Table 6. In-service teachers' TPK pre- & post-GeoGebra integration training

TPK item description & code	Before training		After training		Effect size (*d*)
	M	SD	M	SD	
I can choose appropriate technologies to improve geometry teaching (TPK1).	2.69	2.42	4.45	3.96	0.54
I can use GeoGebra technologies to enhance learners' enthusiasm for learning (TPK2).	2.62	2.24	4.59	4.09	0.60
I see the use of GeoGebra technology in classroom from a critical perspective (TPK3).	3.07	2.65	4.59	4.09	0.44
I can adaptively use GeoGebra technology in various teaching activities (TPK4).	3.00	2.61	4.52	4.04	0.45
I can select appropriate technologies to optimize geometric teaching (TPK5).	2.62	2.24	4.38	3.90	0.55
I can utilize GeoGebra technology to improve classroom interaction (TPK6).	2.90	2.55	4.59	4.09	0.50

Table 7. In-service teachers' pre- & post-TPK

	M	SD	n	Standard error	*z*-score	*p*-value	Cohen's *d*	Cronbach's alpha
Pre-TPK	2.82	2.46	174	0.3579	-4.7495	0.000	0.51	0.97
Post-TPK	4.52	4.03	174					

Table 8. In-service teachers' TPACK pre- & post-GeoGebra training

TPACK item description & code	Before training		After training		Effect size (*d*)
	M	SD	M	SD	
Integrating GeoGebra in teaching geometry content will be easy & straightforward for me (TPCK1).	3.10	2.78	4.55	4.05	0.42
I can teach lessons that combine geometry, GeoGebra, & teaching approaches (TPCK2).	2.48	2.13	4.52	4.02	0.63
I can use strategies that combine content, GeoGebra, & teaching approaches that I learnt (TPCK3).	2.79	2.52	4.38	3.90	0.48
I can provide leadership to coordinate the use of content, GeoGebra, & teaching approaches (TPCK4).	2.72	2.46	4.38	3.91	0.51
I can select technologies that enhance what I teach, how I teach and what learners learn (TPCK5).	2.97	2.67	4.38	3.90	0.42
I can choose technologies that enhance the content for a lesson (TPCK6).	2.90	2.59	4.45	3.97	0.46

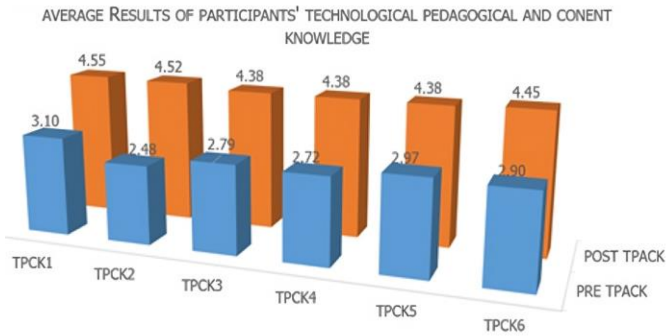

Figure 5. Comparison of in-service teachers' TPCK before & after intervention (Source: Authors' own illustration)

Findings in **Table 8** and **Figure 5** showed a significant mean and SD difference between pre- and post-intervention results of the items used to measure participants' TPACK.

In fact, before the training results showed that, on average, participants self-rated their TPACK as neutral (maybe) on five (83%) items and as disagree on one (17%) item. These results changed significantly after the training with participants self-rating their TPACK as strongly agree with a mean above 4.00 in all the items of TPACK. Thus, items such as 'I can teach lessons that appropriately combine geometry, GeoGebra technology and teaching approaches. (TPCK2)' recording the largest mean difference of 2.03. The least mean difference of 1.41 was reported in item "I can select technologies to use in my classroom that enhance what I teach, how I teach and what learners learn (TPCK5)".

Results in **Table 8** further show the effect sizes for these six TPACK items ranging between 0.42 and 0.63 Cohen's *d*, which implies medium to large effect sizes. Thus, item "I can teach lessons that appropriately combine geometry, GeoGebra technology and teaching approaches (TPCK2)" with the most gain of 0.63. On the other hand, item "Integrating GeoGebra in teaching geometry content will be easy and straightforward for me (TPCK1)" and item "I can select technologies to use in my classroom that enhance what I teach, how I teach and what learners learn (TPCK5)" reported the least gain of 0.42. A *z*-score test, difference of means to compare in-service teachers' TPACK perceptions before and after the GeoGebra integration training was highly statistically significant for all items. Thus, overall perceptions (before: M=2.83, SD=2.53; after: M=4.44, SD=3.96, *p*-value<0.01) showed reasonable difference with medium effect size of 0.49 pre- and post-intervention (**Table 9**). Participants' responses on TPACK shows that intervention assisted in-service teachers with understanding of technologies in geometry teaching.

In spite of the opportunities offered by TPACK framework, the study observed little significant changes in some of the items in the component of TPACK. This includes items such as "I know about a lot of different technologies (TK5)". This item was least rated in in-service teachers' TK with a mean difference of 0.72 and with a medium effect size of 0.24. The item "Using GeoGebra can fundamentally change the way teachers and learners understand geometry concepts (TCK2)" did not receive popular attention compared to the items in TCK construct. TCK2 reported a mean difference of 0.68 with a least area change of 0.22, which implies the medium effect size. Also, items "I see the use of GeoGebra technology in classroom from a critical perspective (TPK3)" and "I can adaptively use GeoGebra technology in various teaching activities (TPK4)" report same mean difference of 1.52 and medium effect sizes of 0.44 and 0.45, respectively. Lastly, the item "I can select technologies to use in my classroom that enhance what I teach, how I teach and what learners learn (TPCK5)" registered the least mean differences of 1.41 with the lowest effect size of 0.42 in TPACK construct.

Table 9. In-service teachers' TPACK

	M	SD	n	Standard error	*z*-score	*p*-value	Cohen's *d*	Cronbach's alpha
Pre-TPACK	2.83	2.53	174	0.3262	-4.5194	0.000	0.49	0.95
Post-TPACK	4.44	3.96	174					

DISCUSSION

Through in-service teachers' positive perceptions, they developed their knowledge domain of TPACK framework after training. This is due to significant difference found in pre- and post-intervention results. Highly statistically significant results for TCK, TPK, TK, and TPACK with medium to large effect sizes agree with a study conducted by Bueno et al. (2021). They found that TPACK has become an effective method to provide teachers with opportunities to understand relationships between pedagogy, content and technology. Compared results for pre- and post-intervention in all aspects of TPACK shows that in-service teachers changed their perspectives about technologies in geometry teaching. These findings are supported by Mishra and Koehler (2006) who found that TPACK changes the technological perspective of teachers and nature of classrooms. Also, a significant difference of the pre- and post-questionnaire results noticed in **Table 9** clearly shows that in-service teachers significantly increased their TPACK due to training. Wood et al. (2005) confirm that appropriate training for TPACK instructional programs helped teachers to increase their knowledge and reduce their anxiety. Specifically, results of pre- and post-training observed in items of TCK like TCK1, TCK3, TCK4, and TCK6 shows that participants fundamentally increased their TCK in geometry teaching. This finding is further supported by Wood et al. (2005) who found that engaging in technology integrating training increases teachers' familiarity to technology and decreases their anxiety with its use in classroom.

Participants' lacked awareness of the potential of the GeoGebra application as a tool in geometry teaching. But they managed to show their knowledge and skills in integrating GeoGebra in geometry teaching. Significant gain in their perceived developed competencies as reported in post-intervention study results was evidence of this. Thus, in-service teachers' integration of GeoGebra in teaching of geometry fosters teachers' ability to use educational technologies. This supports Koehler and Mishra's (2006) observation that participants found a significant increase of teacher perceptions of TPACK. Despite the fact that in-service teachers had very low level of awareness of GeoGebra integration in geometry teaching before intervention, findings show a greater improvement. For instance, findings observed from items like TK3, TCK1, TPK3, and TPCK1 showed that training increase teachers' attempt and willingness to use GeoGebra technologies in geometry classes and improve their TPACK (Saralar et al., 2018). Thus, mean difference of 0.76, 1.69, 1.52, and 1.45 of TK3, TCK1, TPK3, and TPCK1, respectively in favor of post-intervention results was observed. This has led to participants' realization of the importance of TPACK training as observed in item TPCK4 when participants showed their interest in taking lead in training other mathematics teachers in district. The item stated that "I can provide leadership in helping others to coordinate the use of content, GeoGebra technologies and teaching approaches at my school and/or district (TPCK4)". **Table 9** shows positive change of mean of 1.66 with largest effect size (gain=0.51). This finding concurs with studies conducted by Saralar et al. (2018) and Turkish Education Association (2009), which finds that TPACK is important in teachers' professional success. Saralar et al. (2018) recommend involvement of technology-integrated training in teacher education programs. These findings show that in-service teachers needed more time to be mentored and to practice this new technological approach to geometry teaching. This is similar to findings from Alabi and Ige (2014), who indicate the need to regularly train and retrain Nigerian teachers. The need for more time to familiarize with GeoGebra integration was also supported by Mudzimiri (2012) who found that teachers' TPACK will not develop immediately due to factors like technological experience, CK, and beliefs about technology. Limited technological experience has resulted in TK5 being ranked the least in TK component of TPACK. Inadequate geometry CK of some of in-service teachers affected TCK2, hence reported the least effect size in TCK construct. It can be concluded that as teachers gain more teaching experience, they continue to expand their knowledge base, which in turn strengthens the connection between pedagogy, content, and technology.

CONCLUSIONS

The study was a first attempt to engage in-service mathematics teachers in an intensive PD training in technology integration in geometry teaching in secondary schools as an intervention. Evidence from this study revealed that in-service mathematics teachers can develop to teach geometry by combining technology with content and teaching approaches. Introduction of instructional approaches that are likely to lead to teachers' improvements of their practices as professionals made this TPD to be more than just a training. This study was envisioned to suggest ways in which in-service mathematics teachers can enhance, refine, or reconstruct their instructional practice in teaching geometry in powerful and purposeful ways. Integrating GeoGebra in geometry teaching allows teachers not only to use learner-centered approach, but to show abstract geometric concepts, to set geometry questions and also to maintain order in classrooms due to its ability to keep the audience attentive. Although the study mainly focused on secondary school Euclidean geometry, the findings presented in **Table 8** and **Table 9** showed that GeoGebra software is capable of being used to demonstrate other abstract mathematical concepts in other topics. That is to say, the overall TPACK results in **Table 8** and **Table 9** mean that the study's findings can be generalized if in-service teachers can be trained to integrate GeoGebra in other mathematics topics such as functions and trigonometry. Hence, what this study sought was to bring to the attention of in-service teachers a dynamic and visual ability of GeoGebra and check their perceptions based on their GeoGebra integrative skills when teaching geometry. Based on the positive impact derived from the findings of this study, department of basic education teacher development officials together with mathematics specialist should consider including *Train of the Trainers Workshop* to bridge the gap that exists between the intended pedagogical practices as described in curriculum and assessment policy statement and what teachers practice in the classroom. Apparently, there is a significant inadequacy of technological-pedagogical skills in mathematics in-service teachers and in the teaching profession at large. Practicing teachers use more of a teacher-centered approach in teaching geometry, while curriculum and assessment policy statement places more emphasis on a learner-centered approach.

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