

# Technological, Pedagogical, and Content Knowledge (TPACK) and Beliefs of Preservice Secondary Mathematics Teachers: Examining the Relationships

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Received 5 January 2017 • Revised 8 October 2017 • Accepted 12 December 2017

## ABSTRACT

The purpose of this paper is to examine the relationships between preservice secondary mathematics teachers' beliefs and knowledge regarding teaching mathematics with technology. By conducting three semi-structured interviews, I investigated four preservice secondary mathematics teachers' technological, pedagogical, and content knowledge (TPACK) and their beliefs about the nature of mathematics, learning and teaching mathematics, and technology use in the mathematics classroom. The findings of this study suggest that preservice teachers with constructivist-oriented or student-centered beliefs about the nature of mathematics, learning mathematics, and technology use displayed higher levels of mathematical knowledge, pedagogical content knowledge, and technological content knowledge, respectively, than preservice teachers with traditional or teacher-centered beliefs about mathematics, learning, and technology use.

**Keywords:** dynamic geometry, preservice secondary mathematics teachers, teacher beliefs, technological pedagogical content knowledge

## INTRODUCTION

With the increased need for technology integration in mathematics classroom, the role of teachers has become more important. According to the National Council of Teachers of Mathematics (2000, p. 26), "The teacher plays several important roles in a technology-rich classroom, making decisions that affect students' learning in important ways. Initially, the teacher must decide if, when, and how technology will be used." Mathematics teachers are expected to make prudent decisions when integrating technology into their teaching. Many teacher education and professional development programs offer technology courses for preservice and current teachers. These programs encourage student-centered technology uses that "support inquiry, collaboration, or re-configured relationships among students and teachers" (Culp et al., 2005, p. 302) and technology uses that enable students to engage in higher levels of thinking with less cognitive load by providing visualization and representation of problems (Jonassen, 2003). However, teachers have tended to use technology to display lesson content or support their existing practices rather than to implement inquiry-based, collaborative, or problem-solving activities and projects (Culp et al., 2005; Ottenbreit-Leftwich et al., 2010). According to the "Speak Up 2007" survey of Project Tomorrow (2008), 51% of the responding teachers primarily assign homework or drill-and-practice work using computers as a way to facilitate student learning. Project Tomorrow (2011) also compared the results from two "Speak Up" surveys (2008, 2010) to show how technology use in the classroom has changed over time. Although some relatively sophisticated uses of technology (e.g., conducting investigations and creating graphic organizers) were significantly higher in 2010 than in 2008, the majority of teachers' technology uses were still limited to providing homework and practice. Moreover, the percentage of assigning homework and practice work increased from 2008.

According to Ertmer (2005), teachers' beliefs are one of the major barriers to the integration of technology into their teaching. Since technology integration has been encouraged and funded, barriers caused by external constraints (e.g., access and support) have been resolved in most schools (Ertmer et al., 2012). Thus, preservice or current teachers' beliefs may constitute a more deeply ingrained barrier to student-centered technology use than

### Contribution of this paper to the literature

- Mathematics teacher educators should provide learning and teaching experiences with student-centered approaches and positive experiences with technology and training to integrate technology into mathematics instruction.
- To develop preservice teachers' technological pedagogical content knowledge, mathematics teacher educators should focus on developing preservice teachers' content knowledge and pedagogical content knowledge even if neither type of knowledge includes technology.
- This study provides a foundation for further investigation of how preservice mathematics teachers' beliefs and technological pedagogical content knowledge are related and how this relationship impacts their integration of technology into their teaching.

external limitations (Ertmer, 2005; Ottenbreit-Leftwich et al., 2010). Although the goals of mathematics teacher education include aligning preservice teachers' beliefs with student-centered learning and developing their knowledge for the effective incorporation of technology into mathematics teaching, little research has examined both preservice mathematics teachers' beliefs and their technological pedagogical content knowledge (TPACK). The purpose of this study is to investigate preservice secondary mathematics teachers' beliefs and TPACK and how they relate to each other.

## LITERATURE REVIEW

In the literature, there are diverse terms that many researchers have used to define types of teaching or learning. In general, there are two contrasting sets of adjectival terms: constructivist/student-centered and traditional/teacher-centered. First, "constructivist" is a derivative of "constructivism," which refers to a learning theory. In this perspective, learners are viewed as creators of their own understanding by combining what they already believe to be true based on past experiences with what they learn from new experiences (Richardson, 1997). In addition, knowledge is viewed as a product of an individual's construction of the experiential world. Thus, mathematics is viewed as a human creation that is continually expanding. "Student-centered" aligns with "constructivist" in that students are the main agents of their own learning. Student-centered approaches tend to emphasize interactive activities in which students can address unique learning interests and needs to deepen their understanding (Hannafin & Land, 1997).

"Traditional" beliefs or approaches are based on the idea that teaching is mainly the transmission of knowledge from teacher to student and that learning is the passive reception of transmitted knowledge. In traditional classrooms, teachers have authority and can control students' learning activities. From this perspective, mathematics is viewed as a collection of facts, rules, and skills that is fixed, absolute, certain, and applicable (Raymond, 1997). "Teacher-centered" approaches are closely related to "traditional" approaches in that knowledge is primarily transmitted by the teacher through telling. Teacher-centered approaches tend to focus more on content knowledge than on student thinking or processing and place "control for learning in the hands of the teacher" (Brown, 2003, p. 50).

### Beliefs and Knowledge about Teaching with Technology

Although teachers' beliefs have been considered a crucial research topic in mathematics education since the 1980s (Pehkonen & Pietilä, 2003), there are various definitions of "beliefs" in the literature. Furinghetti and Pehkonen (2002) derived two types of knowledge from the definitions of beliefs proposed by 18 mathematics educators: objective and subjective. Objective knowledge is accepted by the mathematics community such that individuals are able to approach this type of knowledge and construct "their own conceptions of mathematical concepts and procedures, i.e. they construct some pieces of their subjective knowledge" (p. 53), whereas subjective knowledge is informal, personal, and private knowledge that is not necessarily made public and evaluated by other people. Pehkonen and Pietilä (2003) considered beliefs as subjective knowledge that is personal, experience based, and tacit. In this study, I view knowledge as objective knowledge (i.e., formal, verifiable, or justifiable) and beliefs as subjective knowledge (i.e., individual psychological understanding about the world based on personal experience).

**Belief systems.** Green (1971) suggested three dimensions of belief systems: the relationships between beliefs, the degree of strength of beliefs, and the characteristics of clustering beliefs. The first dimension concerns a "quasi-logical" structure of belief systems. According to Green, belief systems have a particular order: *primary* and then *derivative*. This order cannot be considered logical, however, because beliefs are arranged according to the logic in individuals' belief systems. The second dimension, the "psychological strength" of beliefs, is viewed as either *central* or *peripheral*. Central beliefs are the most strongly held, whereas peripheral beliefs can be more easily challenged

and changed. The third dimension is related to the claim that “beliefs are held in clusters, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs” (p. 48). This dimension implies that it is possible to simultaneously hold conflicting core beliefs that reside within different belief clusters.

In another perspective on belief systems as sensible systems, Leatham (2006) argued that teachers develop beliefs in ways that make sense to them. This perspective is informed by Thagard’s (2000) coherence theory of justification: “To justify a belief ... we do not have to build up from an indubitable foundation; rather we merely have to adjust our whole set of beliefs ... until we reach a coherent state” (p. 5). Beliefs are viable within a belief system when they make sense in the context of an individual’s other beliefs. Thus, when contradictory beliefs in different clusters are revealed, the individual must address and rectify the conflict because sensible belief systems do not allow for overt contradictions (Singletary, 2012).

Nevertheless, teachers can hold conflicting beliefs at the same time because their beliefs “are connected to different contexts, certainty and consciousness” (Drageset, 2010, p. 32). Such concurrence of contradictory beliefs can be explained by Green’s (1971) third dimension of belief systems. Drageset noted that a belief can be simultaneously derivative and psychologically central or simultaneously primary and psychologically peripheral. For example, beliefs lacking psychological strength may not influence teachers’ decisions or actions even if they are primary beliefs. In this study, my view of preservice teachers’ belief systems is conceptualized based on Green’s (1971) structure of belief systems. This provided insight into how preservice teachers’ beliefs are organized and related to their other beliefs; it also explains conflicting beliefs and inconsistencies between beliefs and behavior.

*Preservice teachers’ beliefs about technology use.* Preservice teachers bring “highly idealistic, loosely formulated, deeply seated, and traditional” entering beliefs about teaching and learning into their teacher education programs (Richardson, 2003, p. 6). These beliefs strongly affect how they interpret courses and classroom practices, how they translate and use their knowledge, and how they determine their later practices as teachers (Pajares, 1992, p. 310). Preservice teachers’ beliefs and attitudes are also crucial factors in the integration of technology into their future teaching (Kay & Knaack, 2005). Some researchers have reported that preservice teachers tend to have limited or teacher-centered beliefs about technology use. Messina and Tabone (2015) indicated that most preservice teachers viewed technology as devices assisting teaching (e.g., providing teaching aids or creating materials to teach) rather than enhancing student collaboration, creativity, and active involvement. In Turner and Chauvot’s (1995) study, preservice secondary mathematics teachers believed that successfully exploring a mathematics topic with technology requires that students already have knowledge about the topic; the teachers stated they would use technology with their students after they had taught the students how to perform or calculate by hand.

Other researchers have indicated that preservice teachers have shown confidence and intention to use technology for student-centered learning. Preservice elementary teachers in Choy et al.’s (2009) study had positive intentions to incorporate technology into their future teaching to facilitate student-centered learning. During their student teaching, however, they tended to use technology to prepare handouts, record grades and attendance, convey information, or gain students’ attention rather than to facilitate student-centered learning. Preservice teachers may also choose not to use technology in their student teaching even if they are competent in its use (Crompton, 2015). This decision can be due to their beliefs or previous experiences with technology in learning or teaching. If they have few or negative experiences with technology, they may not use it in their teaching. External barriers such as lack of time or support from the teacher community and the schools (Amado & Carreira, 2006) as well as preservice teachers’ lack of knowledge on how to teach mathematics (Choy et al., 2009) can also be major barriers to technology integration.

**Technology, pedagogy, and content knowledge (TPACK).** Many studies have provided evidence that preservice teachers’ knowledge about teaching with technology is another factor that strongly influences their integration of technology into mathematics instruction (e.g., Choy et al., 2009; Pamuk, 2012). The effective integration of technology into instruction requires teachers’ intertwined and specialized knowledge. To identify and understand what knowledge teachers need to incorporate technology into their teaching, Mishra and Koehler (2006) developed the technological pedagogical content knowledge (originally TPCK) framework which is now known as technology, pedagogy, and content knowledge (TPACK) framework, building on Shulman’s (1986) construct of pedagogical content knowledge (PCK). The TPACK framework consists of three main knowledge domains—content, pedagogical, and technological knowledge (CK, PK, and TK)—and the intersections between and among them: PCK, technological content knowledge (TCK), technological pedagogical knowledge (TPK), and TPACK (also called TPCK; see [Figure 1](#) and [Table 1](#)). In this study “TPACK framework” refers to the whole Technology, Pedagogy, and Content Knowledge framework, “TPACK components” refers to the knowledge components that comprise the TPACK framework, and “TPACK” or “TPCK” refers to a specific type of knowledge that intersects with all three: content, pedagogical, and technological knowledge. [Table 1](#) provides summarized descriptions of the TPACK framework components introduced by Mishra and Koehler (2006) and discussed by Abbitt (2011b).

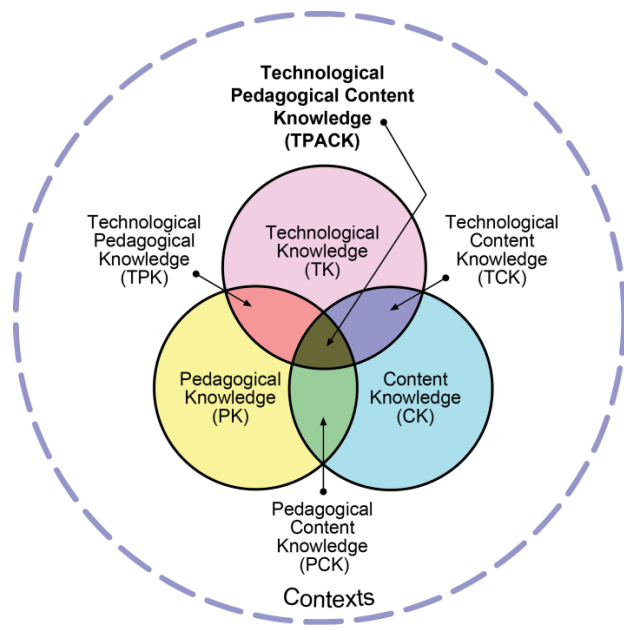


Figure 1. The TPACK framework and its knowledge components (<http://tpack.org>)

Table 1. Descriptions of the TPACK Framework Components (Mishra & Koehler, 2006; Abbitt, 2011b)

Components of TPACK	Description
Content Knowledge (CK)	Knowledge of the actual subject matter to be learned or taught, including central concepts, theories, and organizing or connecting ideas.
Pedagogical Knowledge (PK)	Knowledge of the processes and practices or methods of teaching and learning, including classroom management, development and implementation of lesson plans, and student assessment.
Technology Knowledge (TK)	Knowledge of the standard and advanced technologies, including the skills to install, remove, and operate particular technologies.
Pedagogical Content Knowledge (PCK)	Knowledge of pedagogy that is applicable and appropriate to teaching specific content.
Technological Content Knowledge (TCK)	Knowledge of the manner in which technology and content relate to, influence, and constrain each other.
Technological Pedagogical Knowledge (TPK)	Knowledge of the capability of various technologies including affordances and constraints that influence pedagogical designs and strategies in a teaching and learning setting.
Technological Pedagogical Content Knowledge (TPACK or TPCK)	Knowledge of the interaction among content, pedagogical, and technological knowledge that requires an interweaving of specialized knowledge for teaching with technology.

Niess (2005) also used the term *TPACK*, but viewed it as extended PCK, referring to technology-enhanced PCK. Niess (2015, p. 22) developed four different aspects that consist of teachers' TPACK, with five levels of development for each aspect: (1) an overarching conception of what it means to teach with technology, (2) knowledge of students' thinking and understanding of specific topics with technologies, (3) knowledge of curricular materials that incorporate technologies, and (4) knowledge of instructional strategies and representations for teaching subject matter with technologies. In her description of aspects and levels of teachers' TPACK, teachers' beliefs play an important role. Teachers develop their TPACK through "a constructive and iterative process" of confronting, reflecting on, and carefully revising their various experiences for teaching mathematics with appropriate technologies based on their beliefs and knowledge (Niess, 2015, p. 24). Therefore, teachers' beliefs and TPACK are closely related, and TPACK should be considered with beliefs.

**Preservice teachers' TPACK.** Preservice teachers in Choy et al.'s (2009) study were aware of the benefits of technology use and were not reluctant to integrate technology into their teaching. However, they lacked pedagogical knowledge and skills in planning to integrate technology into their lessons, which may have influenced their use of technology in teaching. Similarly, in Pamuk's (2012) study, preservice middle school or high school teachers demonstrated a lack of TPACK and had a difficult time developing intertwined knowledge such as TPK or PCK. Although they had well-grounded technology backgrounds, they displayed limited TPK. Pamuk suggested that the preservice teachers' lack of pedagogical experience may cause their limited TPK and argued that developing their PCK is important and should be a priority in technology integration.

## Relationship between Beliefs and Knowledge about Teaching with Technology

Crompton (2015, p. 243) stated that “TPACK cannot be considered as a body of knowledge that exists independently of teachers’ beliefs.” In Abbitt’s (2011a) study on preservice teachers in early childhood education, preservice teachers’ beliefs and TPACK are positively correlated, and their TPACK may be predictive of their self-efficacy beliefs about technology integration. Abbitt suggested that efforts to improve teachers’ TPACK, especially technology-related knowledge (TPK, TCK, and TPCK), may result in enhanced self-efficacy beliefs. Similarly, Mudzimiri (2010) found a progression in preservice secondary mathematics teachers’ beliefs about technology use while their TPK, TCK, and TPACK were improved through a technology-intensive mathematical modeling and methods courses. Smith et al. (2016) investigated the relationships between preservice middle school mathematics teachers’ beliefs and their TPACK. Preservice teachers’ TPCK levels were the lowest among the TPACK components, and preservice teachers with sophisticated and student-centered beliefs about mathematics, learning and teaching, and technology use displayed higher CK, PCK, and TPCK, respectively, than preservice teachers with traditional or teacher-centered beliefs. Thus, preservice middle school mathematics teachers’ beliefs about mathematics, pedagogy, and technology use are aligned with their levels of TPACK components.

Other researchers have found a discrepancy between teachers’ beliefs and TPACK. So and Kim (2009) indicated that preservice elementary and secondary teachers recognized the advantages of student-centered learning approaches and saw the benefits of integrating technology into teaching and learning. Preservice teachers, however, faced difficulties in applying their pedagogical beliefs or understanding about problem-based learning in creating tasks and problems, integrating information and communications technology tools, and identifying their role in lesson design artifacts. Chai et al. (2013) stated that teachers displayed low levels of technology-related knowledge (TK, TPK, TCK, and TPACK) and needed robust knowledge regarding technology integration even though they possessed highly constructivist-oriented pedagogical beliefs.

Given the mixed results and few studies that have considered both beliefs and TPACK in the context of mathematics, more research on the relationships between preservice or current mathematics teachers’ beliefs and TPACK is needed to clarify and understand teachers’ beliefs and TPACK for their integration of technology into mathematics teaching. Most researchers have focused only on preservice teachers’ pedagogical or technology-related beliefs even though many researchers acknowledged the importance of beliefs regarding the nature of mathematics (e.g., Ernest, 1989b; Raymond, 1997). This study is guided by the following research question: How do preservice secondary mathematics teachers’ beliefs (i.e., beliefs about the nature of mathematics, learning and teaching mathematics, and technology use in the mathematics classroom) relate to their levels of TPACK components?

## METHODOLOGY

The review of the literature revealed that many studies used self-report surveys to measure preservice or inservice teachers’ pedagogical beliefs (e.g., Abbitt, 2011a; Chai et al., 2013). However, these and Likert-type questionnaires have been well documented as inadequate to accurately capture participants’ responses (e.g., Best & Kahn, 2006; Gure, 2015; Kothari, 2004). Because individual items may be open to interpretation and numerical responses do not provide detailed information about beliefs, it is difficult to accurately measure participants’ beliefs using self-report surveys or Likert-type questionnaires. In addition to beliefs, self-report surveys may not accurately reveal participants’ knowledge. Among the research reviewed, many studies used self-report surveys to measure participants’ knowledge (e.g., Abbitt, 2011a; Chai et al., 2013; Choy et al., 2009). However, this self-report measure, one of the frequently-used methods, has limitations. Survey items do not provide fundamental questions or statements related to specific content knowledge. Instruments of measurement need to be customized to certain content knowledge. Moreover, we cannot say the self-reporting system assesses or measures teachers’ actual TPACK (Abbitt, 2011b) because a self-report is based on teachers’ subjective, not objective, thoughts or self-judgments. Thus, for the current study, a qualitative research methodology, specifically a multiple-case study, was selected in order to accurately identify, describe, and examine individual preservice secondary mathematics teacher’s beliefs and TPACK components (CK, PCK, TCK, and TPCK), and to understand how they are related to each other. Yin (1984) defined a case study as “an empirical inquiry that investigates a contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence are used” (p. 23). In addition, a multiple-case study methodology allows a researcher to construct contextualized experiences and systemic analysis processes (Stake, 2006). Therefore, a multiple-case study methodology was the most appropriate method to identify, describe, and examine individual participants’ beliefs and TPACK, and to find possible relationships between their beliefs and TPACK from across-case analysis (Creswell, 2013). For a multiple-case study, moreover, Stake (2006) stated that 4 to 10 cases are enough to provide substantial information on the interaction between the cases without overwhelming amounts of differences, thereby restricting comparisons.

## Participants and Context

The participants who volunteered for this study were four undergraduate preservice teachers enrolled in a mathematics teacher education program at a university in the southern region of the United States. The participants, Hank, Grace, Abby, and Kate (all names are pseudonyms), were enrolled in a secondary mathematics pedagogy course focused on learning and teaching geometry, probability, and sequences and series. One of the course goals was to develop preservice teachers' knowledge about technology in mathematics teaching and learning and how technology influences student thinking and conceptual understanding. The course instructor regularly provided activities facilitating the preservice teachers' use of technology to explore mathematical concepts, including dynamic geometry environments (DGEs) such as *Geometer's Sketchpad 5* (GSP; Jackiw, 2009). The participants were familiar with using technology, such as computers, calculators, the Smart Board, or GSP and with solving and explaining geometry problems. I collected data after the course was over so that (a) participants had been exposed to relevant geometric topics, and (b) there was no implied connection between participating in the study and the course grade.

## Data Collection

I collected various types of data from three semi-structured interviews (beliefs, task-based, and performance interviews). Each of all three interviews lasted approximately one hour and were video- and audio-recorded.

**A beliefs interview.** First, to infer participants' beliefs, I used Smith et al.'s (2016) semi-structured beliefs interview protocol consisting of four categories: the nature of mathematics, learning mathematics, teaching mathematics, and technology use in mathematics class. Based on the works of Raymond (1997) and Zakaria and Musiran (2010), which investigated preservice or current teachers' teaching practices and beliefs about mathematics and pedagogy, Smith et al. (2016) developed the interview questions for the first three categories. The questions for beliefs about technology use in mathematics class were based on Landry's (2010) study on the instrument for measuring middle school mathematics teachers' TPACK. The beliefs interview protocol has 74 questions in total, including the follow-up questions (see [Appendix I](#)).

**A task-based interview.** I used Hollebrands and Smith's (2010) task-based interview protocol, which contains four tasks designed to assess participants' TPACK components within geometry topics (see Appendix II). In the task-based interview, students' DGE use was addressed, and the participants were asked to create activities or tasks using GSP to help the students acquire a deeper understanding of the concepts or to remedy the students' difficulties or misconceptions. I examined the participants' geometric concepts and their understanding of what the students understand and how the students think in specific technological pedagogical mathematical contexts. During this interview, each participant was given a laptop with GSP, the pedagogy course textbook, a compass, a protractor, a ruler, blank paper, pencils, and markers.

**A performance interview.** Finally, I designed a performance interview in which participants were asked to describe and demonstrate how they would teach the polygon exterior angle sum theorem (the exterior angle theorem: The sum of the measures of the exterior angles of a convex polygon is 360) using GSP. This interview aimed to measure participants' knowledge not covered in the task-based interview and its rubric, to find evidence to support the task-based interview results, and to investigate participants' instructional design and planning process or decision-making based on their pedagogical reasoning and their ability to teach mathematics using GSP (Harris et al., 2010). The theorem was chosen based on the pedagogy course instructor's recommendation. In the pedagogy course, the participants learned various geometric topics. The exterior angle theorem was one topic the participants did not experience in the pedagogy course. In addition, there were diverse possible strategies or ways to teach the theorem. Thus, the exterior angle theorem was appropriate to differentiate the participants' levels of TPACK components. Before the performance interview, I provided information about what participants would do in the interview. The participants were allowed to prepare teaching materials (e.g., pre-constructed GSP materials or worksheets) in advance or could develop such materials during the interview. During this interview, each participant was given a laptop with GSP, blank paper, and pencils. In the task-based and performance interviews, videos of the participants' work on their computers were recorded using a screen capture software program. I also collected any electronic files and artifacts created by the participants during the interviews.

## Data Analysis

To analyze the beliefs interview data and assign codes, I used Ernest's (1989a) classification of beliefs and Goos et al.'s (2003) perspectives of technology use (see [Table 2](#)). Ernest's belief classification provided insights into mathematics teachers' views and a useful category to conceptualize teachers' beliefs about the nature of mathematics, learning, and teaching mathematics (Smith et al., 2016; Kaiser & Vollstedt, 2007). Goos et al.'s categories regarding how to use technology in mathematics class have been used to investigate how students or

**Table 2.** Classifications of Mathematics Teachers' Beliefs

Beliefs about	Classification of beliefs	Description
Nature of Mathematics (Ernest, 1989a)	Instrumentalist	Mathematics is a set of facts and rules
	Platonist	Mathematics as a unified body of certain knowledge that does not change
	Problem Solving	Mathematics as a human creation that is continually changing
Learning (Ernest, 1989a)	Passive Reception of Knowledge	Child exhibits compliant behavior and masters skills. Child passively receives knowledge from the teacher
	Active Construction of Knowledge	Child actively constructs understanding. Child autonomously explores self interests
Teacher's Role (Ernest, 1989a)	Instructor	Goal of instruction is for students to master skills and perform correctly
	Explainer	Goal of instruction is for students to develop conceptual understanding of a unified body of knowledge
	Facilitator	Goal of instruction is for students to become confident problem solvers
Using Technology in the classroom (Goos et al., 2003)	Master	Dependence on technology, not capable of evaluating the accuracy of the output generated by technology
	Servant	Fast, reliable replacement for mental or pen and paper calculations
	Partner	Cognitive reorganization, use technology to facilitate understanding, to explore different perspectives
	Extension of Self	Incorporate technological expertise as a natural part of mathematical and/or pedagogical repertoire

**Table 3.** Preservice Teachers' Levels of TPACK Components in the Task-based Interview

Name	Task	CK	PCK	TCK	TPCK
Grace	1	Advanced	Advanced	Intermediate	Beginner
	2	Intermediate	Advanced	Intermediate	Beginner
	3	Intermediate	Beginner	Intermediate	Beginner
	4	Advanced	Beginner	Beginner	Intermediate
	Overall	Intermediate	Intermediate	Intermediate	Beginner
Hank	1	Intermediate	Advanced	Advanced	Intermediate
	2	Intermediate	Advanced	Advanced	Intermediate
	3	Beginner	Intermediate	Beginner	Intermediate
	4	Intermediate	Intermediate	Advanced	Intermediate
	Overall	Intermediate	Intermediate	Advanced	Intermediate
Abby	1	Advanced	Advanced	Advanced	Advanced
	2	Beginner	Intermediate	Advanced	Beginner
	3	Advanced	Intermediate	Advanced	Intermediate
	4	Advanced	Advanced	Intermediate	Intermediate
	Overall	Advanced	Intermediate	Advanced	Intermediate
Kate	1	Advanced	Advanced	Advanced	Advanced
	2	Intermediate	Advanced	Advanced	Intermediate
	3	Advanced	Advanced	Advanced	Advanced
	4	Advanced	Advanced	Advanced	Advanced
	Overall	Advanced	Advanced	Advanced	Advanced

teachers interact with technology in their classrooms (e.g., Morton, 2013; Nzuki, 2010) and what preservice teachers think about technology use in mathematics classrooms (e.g., Smith et al., 2016). After coding, I wrote each participant's narrative about his/her beliefs. Then, by providing each participant with their narrative, I performed a member check (Creswell, 2013). All participants agreed that I accurately captured and described their beliefs.

I analyzed the task-based interview using Hollebrands and Smith's (2010) scoring rubric. The rubric was designed to assess participants' knowledge about the geometry, pedagogy, and technology required to complete the tasks (see Appendix III). The rubric only scores content-based TPACK components: CK, PCK, TCK, and TPCK. Based on the participants' work in each of the four tasks, I assigned one of four levels (emergent, beginner, intermediate, advanced) for each of the four TPACK components (see Table 3).

To assign an overall level of knowledge to each of the TPACK components, I converted the four levels, emergent, beginner, intermediate, and advanced to numerical values, 1, 2, 3, and 4, respectively; the average was then computed. When the average was a whole number, I assigned the level of the corresponding whole number. If the average was a decimal, I assigned a whole number that is close to the decimal value. For example, when the average was 2.75, I considered it as a 3, and assigned the Intermediate level. Whenever the average was exactly in between two whole numbers, such as 2.5, I examined the participant's work across the four tasks of the TPACK

**Table 4.** Classification of Participants' Beliefs

Name	Nature of Mathematics Ernest (1989a)	Learning Ernest (1989a)	Teaching Ernest (1989a)	Technology Goos et al. (2003)
Grace	Instrumentalist & Platonist	Passive	Explainer	Servant
Hank	Instrumentalist & Platonist	Passive & Active	Facilitator	Partner
Abby	Instrumentalist	Passive & Active	Facilitator	Partner
Kate	Problem Solving	Active	Facilitator	Partner

**Table 5.** Participants' Levels of TPACK Components in the Task-based Interview

Name	CK	PCK	TCK	TPCK
Grace	Intermediate	Intermediate	Intermediate	Beginner
Hank	Intermediate	Intermediate	Advanced	Intermediate
Abby	Advanced	Intermediate	Advanced	Intermediate
Kate	Advanced	Advanced	Advanced	Advanced

components and assigned the level that most consistently represented their knowledge. For instance, when determining Grace's overall level of CK, the average was 3.5. I assigned the intermediate level because Grace lacked basic mathematical knowledge across four tasks.

Finally, I analyzed the performance interview based on three categories – content, pedagogy, and technology – with four levels (emergent, beginner, intermediate, advanced). In the content category, I focused on whether the participants had knowledge about mathematical concepts related to the theorem, a deductive or inductive proof, or connections between mathematical ideas or concepts (e.g., definitions of mathematical figures, the sum of interior angles of a triangle, the sum of interior angles of an  $n$ -sided polygon, and parallel line postulates). In the pedagogy category, I focused on what strategies the participants used to teach the theorem to their imaginary students and whether they could anticipate their imaginary students' thinking or potential difficulties. I considered whether the participants allowed their imaginary students to explore many examples to find the theorem themselves or directly provided the theorem, how they led their imaginary students to come up with theorem proofs, or what type of learning environment they provided (e.g., individual learning or collaborative learning). In the technology category, I attended to the participants' knowledge about technology, especially GSP. I observed whether the participants knew how to use GSP to implement certain tasks (e.g., constructing polygons, measuring and marking angles, and using diverse tools of GSP). I also focused on how they used technology to support their pedagogical strategies. I considered whether they used the dragging feature of GSP to have their imaginary students explore many cases and make conjectures based on those cases, or whether they used the benefits of GSP to provide their imaginary students with diverse ways to explore the theorem or helped the imaginary students intuitively understand the theorem (e.g., using parallel line postulates and a circle to show that the sum of exterior angles is 360 degrees).

For the participants' final levels of TPACK components, the results of both the task-based and performance interviews were integrated. While there are four TPACK components (CK, PCK, TCK, and TPCK) that I focused on in this study, the data from the performance interview were analyzed based on three categories (Content, Pedagogy, and Technology). Basically, I used the performance interview data to support results of the task-based interview and to determine participants' final levels of TPACK components. Thus, I considered a participant's content category level in the performance interview when assigning the participant's final level of CK. A participant's pedagogy category level in the performance interview was taken into account when assigning the participant's final level of PCK and TPCK, and the technology category level in the performance interview was considered when assigning final level of TCK and TPCK.

## FINDINGS

After classifying each participant's beliefs and examining their levels of TPACK components and performance, I found that each participant's belief classification (see [Table 4](#)) and set of TPACK components (see [Table 5](#)) and performance levels (see [Table 6](#)) were unique. [Table 7](#) shows that participants' beliefs classification and the final levels of TPACK components by combining the results from the task-based and performance interviews. In addition, there seemed to be possible relationships between certain belief categories and levels of TPACK components across data from all three interviews. In this paper, I focused my analysis on the potential relationships between participants' beliefs and TPACK components.



**Table 6.** Participants' Levels of the Performance Interview

Name	Content	Pedagogy	Technology
Grace	Intermediate	Beginner	Beginner
Hank	Beginner	Intermediate	Intermediate
Abby	Beginner	Intermediate	Intermediate
Kate	Advanced	Advanced	Advanced

**Table 7.** Participants' Beliefs Classification and Final Levels of TPACK Components

Name	Beliefs				TPACK			
	Mathematics	Learning	Teaching	Technology	CK	PCK	TCK	TPCK
Grace	Instrumentalist & Platonist	Passive	Explainer	Servant	Intermediate	Intermediate	Intermediate	Beginner
Hank	Instrumentalist & Platonist	Passive & Active	Facilitator	Partner	Intermediate	Intermediate	Advanced	Intermediate
Abby	Instrumentalist	Passive & Active	Facilitator	Partner	Intermediate	Intermediate	Advanced	Intermediate
Kate	Problem Solving	Active	Facilitator	Partner	Advanced	Advanced	Advanced	Advanced

In the beliefs interview, some participants seemed to have different beliefs simultaneously, so I was unable to assign a single code. For instance, Grace had both *instrumentalist* and *Platonist* views of the nature of mathematics. She said that the more she learned math, the more she realized that she needed to conceptually understand and find the connection between concepts rather than just calculating and following procedures. She explained that students should understand how they solve a problem and why they solve it that way: "Once students understand the reason of learning, they can figure out the relationship between mathematical concepts." She viewed mathematics as connected knowledge or concepts, which is aligned with a *Platonist* view of mathematics. But she simultaneously believed that mathematics is a set of formulas and calculations and that quick problem solving is important, which is consistent with an *instrumentalist* view. She said, "Physics is most like math because it requires critical thinking ... and diagrams for visual learners. Physics also has many formulas, rules, and calculations." When I asked how teachers can develop students' critical thinking, Grace said, "Use specific examples ... I can say [use] a quick way to solve problems or like mental math". She considered critical thinking as a way to solve problems or calculate quickly.

### Content Knowledge and Beliefs about the Nature of Mathematics

The participants who held *instrumentalist* and/or *Platonist* views of mathematics displayed lower levels of CK, while the participant who held a *problem-solving* view displayed a higher level of CK. Hank, like Grace, had both *instrumentalist* and *Platonist* views of mathematics. He said, "I define mathematics as a manipulation of numbers. ... There are formulas ... but also an understanding of how to use these numbers." He likened mathematics mastery to building a house: "With math, you need a really good foundation ... then you can build up." Abby held an *instrumentalist* view. She described mathematics as a puzzle that seems to be mere a set of mathematical problems:

*I think geometry for one ... When you're finding missing angles ... like finding a missing puzzle piece, that's one aspect of geometry. But then algebra, you probably don't see it as much, but you're solving for x. X is the missing puzzle piece you're always trying to solve for.*

When I asked what subject is most like mathematics, Abby said, "Science and math always go hand in hand with solving problems." She did not consider the process of problem solving nor did she focus on how students construct their own solutions but instead emphasized finding the answer to the problem. Grace, Hank, and Abby displayed an *intermediate* level of CK across the task-based and performance interviews. For example, Grace was able to explain the properties of reflection and rotation and the process of how to perform reflections and rotations, but she seemed to have a misconception about the center of rotation (i.e., only the vertices of a figure can be the center of rotation). Although Hank found that the location of the intersection of perpendicular bisectors (circumcenter) changes, he believed that it "depends on the length of the sides rather than the angles." Abby already knew the Pythagorean Theorem and that all angles of a rectangle and square are 90 degrees, but she was unable to use this knowledge to prove why the diagonals of a rectangle and square are congruent. She displayed an *advanced* level of CK in the task-based interview but was unable to demonstrate her knowledge about the exterior angle theorem and its proof in the performance interview. Given evidence from both the task-based and performance interviews, Abby seemed to have an *intermediate* level of CK overall.

Kate held a *problem-solving* view of mathematics. She said mathematics "can be described as a language because of the way it can explain things." When asked what other subjects are most like mathematics, she answered, "I

think writing proofs and all that stuff, it's more like a logical flow, so I'd say it's most like English class." She believed that doing mathematics is similar to creating something that explains the world with logical reasoning. Kate displayed *advanced* CK in both task-based and performance interviews. She correctly stated not only the properties of different quadrilaterals, rotation, reflection, and circumcenter, but also the inclusion relation among quadrilaterals. She was able to prove that rectangles and squares have congruent diagonals using the Pythagorean theorem and correctly demonstrate two different proofs of the exterior angle theorem (i.e., algebraic and geometric proofs) in the task-based and performance interviews. Therefore, participants with traditional beliefs about mathematics seemed to have fragmentary mathematical knowledge and were unable to connect or use what they already knew to develop theorem proofs. On the other hand, the participant with constructivist-oriented beliefs about mathematics had strong mathematical knowledge and was able to use her knowledge and reasoning to prove the theorems and fulfill various tasks in both task-based and performance interviews.

### **Pedagogical Content Knowledge and Beliefs about Learning Mathematics**

The participants who had *passive* and/or *active* views of learning mathematics displayed lower PCK, while the participant who held only an *active* view displayed higher PCK. Grace, Hank, and Abby held *passive* and/or *active* views of learning mathematics. For example, describing how she learned about mod in a mathematics content course in college, Grace said she had to practice doing mod to be able to calculate and solve mod problems. When I asked how students learn mathematics, Grace described the procedures of problem solving: "You see the problem ... you look at what is given and what is asked. Find a connection between the hypothesis ... then you work until [you find] the solution." She viewed learning mathematics as mastering skills or procedures by repeating problem solving, which is a *passive* view. Hank held both *passive* and *active* views. He said that students learn through "doing again and again, repetition." At the same time, he explained that mathematics can be taught conceptually through diverse approaches including the use of technology. He added,

*Let the kids struggle a little bit. That's always when I learn the most. ... I would just struggle and have no clue what to do, and then all of a sudden I would get it and I would learn it.*

Abby said that students "learn math through their own ways," adding that learning "is your own thing. It's not something that someone can make you do. Or they can help you, but they can't make it come to you." She recognized that learning is an individual process, not a passive reception of knowledge from teachers. She also believed that students learn mathematics through "a trial and error process" saying, "The [learning] process for them [kids] is just trying and trying and trying and until they get it." She emphasized the need for repetition to get the right answer. The participants' *passive* view of learning mathematics seemed to be aligned with their *intermediate* PCK levels. For instance, Grace could show whether students' claims were correct, but she did not know what questions to ask or what activities to use to help students develop their mathematical understanding beyond correcting their misconceptions. She also focused more on conveying the proof of the exterior theorem than on providing opportunities to explore examples. Hank was sometimes unable to identify students' geometric thinking levels, and some of his tasks did not lead students to discover properties or mathematical concepts. Without any hints or leading questions about the sum of exterior angles, Hank would directly ask students whether the sum of a triangle's exterior angles was 360 degrees, which is the core idea that students need to discover themselves. Abby did not correctly interpret students' thinking because of her misconceptions about rotation. Although she knew about the distance property of the circumcenter, she did not ask meaningful questions or provide tasks to help students find it. She was also unable to lead students to develop the deductive proof of the exterior angle theorem.

Kate held an *active* view of learning mathematics. She emphasized students' active learning and believed that students can learn mathematics through problem solving, sharing, and discussing their ideas with peers. She explained the learning process as follows:

*The students should get a sort of problem, they should try to think about it and figure out the best that they can and if possible, that should be a social thing ... They could come together and talk about what they found and say, "Okay, this is what we found, this is why it's true" ... So, I think it's important for them to do the figuring out and then come together to define something or say why something works.*

In the task-based and performance interviews, Kate displayed *advanced* PCK. She was able to develop appropriate tasks or activities to help students explore diverse cases, discover properties, and fully understand mathematical concepts. In the performance interview, besides the algebraic calculations, she provided a geometric way using parallel line postulates to show that the sum of exterior angles of a pentagon is 360 degrees. Kate's beliefs about learning mathematics were aligned with what she would do in her teaching. She said that the students would

participate in individual exploration and then, in a class discussion, share and examine their mathematical thinking and ways to prove the exterior angle theorem based on their individual exploration. Thus, participants who believed that students learn mathematics by mastering skills and repeating the same procedures seemed to have limited pedagogical knowledge. On the other hand, the participant who viewed learning mathematics as an active construction of knowledge could provide meaningful questions or tasks and diverse ways to deepen the students' understanding of the content.

### Technological Content Knowledge and Beliefs about Technology Use

The participant with a *servant* view displayed a lower TCK level, while the participants with a *partner* view displayed higher TCK levels. I categorized Grace's beliefs about technology as a *servant* view in which technologies are quick and accurate tools to replace mental calculations or pen and paper works. Grace believed that using calculators hinders students' mental math and did not want students to be dependent on technology. She also described a teacher-centered view of technology use:

*Sometimes students can use the Smart Board but with the teacher observing them. I don't think students are allowed to write on the Smart Board by themselves. ... I use the Smart Board as a saving tool, for saving my lecture and lesson.*

Grace viewed technology as a tool to save works or communicate with students or their parents rather than as a tool for developing students' mathematical understanding. Across the task-based and performance interviews, Grace displayed a *beginner* level of TCK. She seemed to feel more comfortable with writing on paper than using GSP when thinking about mathematical concepts or demonstrating her mathematical knowledge. She correctly drew a reflection line on the paper when she performed reflection of a triangle, but when using GSP, she did not know that a reflection line was needed to reflect the figure. She initially did not know how to construct the reflection line and the extended line of the side of the figure using GSP. Because she believed that technology enables students to work quickly, she said she would not use GSP for the exterior angle theorem when dealing with polygons that had many sides, since she thought it would be time-consuming work.

Hank, Abby, and Kate all expressed a *partner* view of technology. Hank believed that technology allows students' mathematical thinking to "grow exponentially" and that students can deepen their conceptual understanding by manipulating and "playing around" with figures using technology, especially GSP. Abby believed that technology could change the main agent of learning in the classroom from the teacher to the students, as technology allows students "to explore and do what they want and talk to each other about it." Kate stated that the different aspects provided by technology, including the visual aspect, were an advantage of using technology in learning and teaching mathematics. She added:

*It [technology] also helps them to figure out what's going on so they can look at it. "I can change this one parameter. What happens to the rest of it?" That's important because it kind of gives them the background for why something is true or what something does, and that kind of thinking is what goes into writing proofs.*

Hank, Abby, and Kate displayed *advanced* TCK levels and were willing to use GSP during the task-based and performance interviews. They were able to construct various mathematical figures using GSP (e.g., quadrilaterals, pentagons, and extended lines and exterior angles of figures) and use diverse affordances of GSP (e.g., dragging, labeling, measuring, calculating, and rotating features) to create tasks or activities in which the students could explore examples and make their own mathematical conclusions. Therefore, the participant who viewed technology as a fast and accurate tool to amplify what one can do by hand had limited knowledge about how to use GSP to construct or represent mathematical figures or concepts. On the other hand, the participants who believed that technology could facilitate students' mathematical understanding by providing opportunities to explore different perspectives were familiar with GSP and knew basic skills to operate GSP and its diverse affordances and constraints.

## DISCUSSION

In this study, the preservice secondary mathematics teachers had some overall features. Traditional views were more predominant in their descriptions of the nature of mathematics (*instrumentalist* and *Platonist* views) and learning mathematics (*passive* view) than in teaching mathematics (*facilitator* view) and technology use (*partner* view). Second, the preservice teachers' levels of CK and PCK were lower (*intermediate* level) than their levels of TCK (*advanced* level). Third, similar to the results of Smith et al.'s (2016) study, preservice teachers displayed the lowest levels of TPCK among the TPACK components.

My analysis of preservice secondary mathematics teachers suggests possible relationships between preservice teachers' beliefs and TPACK components. This study found that the more sophisticated the beliefs about mathematics, learning, and technology, the higher the levels of CK, PCK, and TCK, respectively. These results are similar to the findings of Smith et al.'s (2016) study. Both Smith et al.'s (2016) and my study indicate that preservice mathematics teachers with constructivist-oriented beliefs about mathematics and learning displayed higher CK and PCK than preservice teachers with traditional beliefs about mathematics and learning. However, in terms of the relationship between preservice teachers' beliefs and knowledge about technology use, the results from this study and that of Smith et al. (2016) were different. In Smith et al.'s study, the preservice middle-school teachers' beliefs about technology use were more closely related to their TPACK levels than their TCK levels.

### **Beliefs and Knowledge about Technology**

In this study, preservice teachers' beliefs and knowledge about technology use seem to have a strong influence on their TPACK level. The preservice teacher with a limited view of technology use and lack of TCK (Grace) demonstrated lower TPACK than the other participants (Hank and Abby), who had more student-centered views of technology use and higher TCK, even though they had similar beliefs and knowledge about mathematics and pedagogy. Moreover, a limited view of technology use and low TCK seem to be associated with preservice teachers' limited experiences with technology in mathematics classes, as discussed by Crompton (2015). Unlike the other participants, Grace rarely used technology in her middle school and high school mathematics classes. In her college mathematics classes, she used technology only for displaying content and communicating with professors, not for exploring mathematics concepts. Her lack of robust knowledge and experiences with technology in student-centered learning approaches may influence her level of TPACK. This result is consistent with the finding of Chai et al. (2013) that a lack of technology-related knowledge is associated with low TPACK.

### **Beliefs and Knowledge about Mathematics and Pedagogy**

Preservice teachers' beliefs and knowledge about mathematics and pedagogy seem to affect their TPACK levels. Although preservice teachers (Hank and Abby) held student-centered views of technology use and displayed advanced TCK, their TPACK might not be high if they had more traditional beliefs about mathematics and learning and low CK and PCK. The acquisition of technology-related knowledge does not always ensure successful technology integration (Polly et al., 2010). Consistent with the results of Choy et al. (2009), preservice teachers had positive attitudes toward technology, expressed a willingness to use technology in their future teaching, and showed good technological knowledge, but they did not have appropriate knowledge to ask meaningful questions or create tasks that facilitate students' conceptual learning. Kim et al. (2013) had similar findings: Teachers with more student-centered pedagogical beliefs tended to integrate technology more seamlessly into their teaching than those with more teacher-centered pedagogical beliefs.

### **Importance of Beliefs and TPACK**

As indicated by both other studies and the current study's findings, preservice teachers with similar beliefs and knowledge about mathematics and pedagogy may use technology differently to teach mathematics due to limited beliefs and knowledge about technology use. Or, although preservice teachers have strong technical knowledge and positive attitudes toward technology use in mathematics classes, they may not know how to use technology effectively to teach mathematics. Preservice teachers need to improve their knowledge of mathematics, how students think about and learn mathematics with/without technology, and how to use technology to teach mathematics. Just having knowledge, however, would not be enough. Preservice teachers also need to view mathematics as a continually expanding field in which students can construct their own mathematics through active engagement, teachers can facilitate students' conceptual learning, and technology can support student-centered approaches. Thus, preservice teachers should develop all areas of content, pedagogy, and technology in their beliefs and knowledge to be able to use technology effectively to teach mathematics.

## **IMPLICATIONS**

This study provides insights into what aspects mathematics teacher educators (MTEs) should consider to cultivate teachers who effectively teach mathematics using technology. In this study, preservice teachers' beliefs about the nature of mathematics and learning mathematics were more traditional and inflexible than their beliefs about teaching mathematics and technology use. Their beliefs about teaching and technology use may be more amenable to change because they have had less experience with teaching and technology. Thus, MTEs should place more emphasis on developing preservice teachers' student-centered beliefs about teaching and technology.

In addition, the preservice teachers displayed lower CK and PCK than TCK. At times, they could not provide appropriate questions or tasks for students due to their lack of CK, which may have caused their low PCK. They knew how to use GSP only for themselves, not for mathematics instruction. To develop preservice teachers' TPCK, MTEs should focus on developing preservice teachers' CK and PCK even if neither type of knowledge includes technology. MTEs should provide learning and teaching experiences with student-centered approaches and positive experiences with technology and training to integrate technology into mathematics instruction. Having preservice teachers work in a constructivist or student-centered environment could affect their beliefs as well (Liljedahl et al., 2007).

## STUDY LIMITATIONS

It could be argued that the small sample of four preservice teachers limits the generalizability of the findings of this study. However, although different results could be obtained from a larger sample, the current study focused on detailed analysis of the preservice teachers' interviews and discussion of their beliefs and use of technology. This analysis and observation of how preservice teachers utilized technology to perform various tasks and to teach a specific mathematical concept enabled the researcher to more deeply understand the relationships between preservice teachers' beliefs and TPACK.

Another limitation is that the task-based interview's rubric was founded on student-centered principles. Thus, the rubric is likely biased toward participants who hold student-centered beliefs, and those participants may achieve higher levels of TPACK components than participants with teacher-centered beliefs. Some participants in this study displayed low levels of some TPACK components even though they held student-centered beliefs. Thus, the relationships between the participants' beliefs and TPACK components were not predetermined.

More studies with larger samples and in more diverse mathematical contexts are needed to extend this line of research. However, the case studies discussed in this paper can provide an important foundation for further investigation of how preservice mathematics teachers' beliefs and TPACK are related and how this impacts their integration of technology into their teaching.

## REFERENCES

- Abbitt, J. T. (2011a). An investigation of the relationship between self-efficacy beliefs about technology integration and technological pedagogical content knowledge (TPACK) among Preservice Teachers. *Journal of Digital Learning in Teacher Education*, 27(4), 134-143. <https://doi.org/10.1080/21532974.2011.10784670>
- Abbitt, J. T. (2011b). Measuring technological pedagogical content knowledge in preservice teacher education: A review of current methods and instruments. *Journal of Research on Technology in Education*, 43(4), 281-300. <https://doi.org/10.1080/15391523.2011.10782573>
- Amado, N., & Carreira, S. (2006). The student teacher and the others: Multi membership of the process of introducing technology in the classroom. In H. Moraová, J. Novotná, M. Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 33-40). Prague, Czech Republic: PME.
- Best, J. W., & Kahn, J. V. (2006). *Research in education*. London: Pearson Education Inc.
- Chai, C. S., Chin, C. K., Koh, Ling, J. H. & Tan, C.L. (2013). Exploring Singaporean Chinese language teachers' technological pedagogical content knowledge and its relationship to the teachers' pedagogical beliefs. *The Asia-Pacific Education Researcher*, 22(4), 657-666. <https://doi.org/10.1007/s40299-013-0071-3>
- Choy, D., Wong, A. F., & Gao, P. (2009). Student teachers' intentions and actions on integrating technology into their classrooms during student teachings: A Singapore study. *Journal of Research on Technology in Education*, 42(2), 175-195. <https://doi.org/10.1080/15391523.2009.10782546>
- Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: SAGE.
- Crompton, H. (2015). Pre-service teachers' developing technological pedagogical content knowledge (TPACK) and beliefs on the use of technology in the K-12 mathematics classroom: A review of the literature. In C. Angeli & N. Valanides (Eds.), *Technological Pedagogical Content Knowledge* (pp. 239-250). New York, NY: Springer. [https://doi.org/10.1007/978-1-4899-8080-9\\_12](https://doi.org/10.1007/978-1-4899-8080-9_12)
- Culp, K. M., Honey, M., & Mandinach, E. (2005). A retrospective on twenty years of educational technology policy. *Journal of Educational Computing Research*, 32(3), 279-307. <https://doi.org/10.2190/7W71-QVT2-PAP2-UDX7>
- Drageset, O. G. (2010). The interplay between the beliefs and the knowledge of mathematics teachers. *Mathematics Teacher Education and Development*, 12(1), 30-49.

- Ernest, P. (1989a). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249-254). London, The United Kingdom: Falmer Press.
- Ernest, P. (1989b). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, 15(1), 13-33. <https://doi.org/10.1080/0260747890150102>
- Ertmer, P. A. (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? *Educational technology research and development*, 53(4), 25-39. <https://doi.org/10.1007/BF02504683>
- Ertmer, P. A., Ottenbreit-Leftwich, A. T., Sadik, O., Sendurur, E., & Sendurur, P. (2012). Teacher beliefs and technology integration practices: A critical relationship. *Computers & Education*, 59(2), 423-435. <https://doi.org/10.1016/j.compedu.2012.02.001>
- Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (vol. 31, pp. 39-57). Dordrecht, The Netherlands: Springer. [https://doi.org/10.1007/0-306-47958-3\\_3](https://doi.org/10.1007/0-306-47958-3_3)
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *The Journal of Mathematical Behavior*, 22(1), 73-89. [https://doi.org/10.1016/S0732-3123\(03\)00005-1](https://doi.org/10.1016/S0732-3123(03)00005-1)
- Green, T. F. (1971). *The activities of teaching*. New York, NY: McGraw-Hill.
- Gure, G. S. (2015). Different scale construction approaches used to attitude measurement in social science research. *International Journal of Research in Economics and Social Sciences*, 5(1), 26-44.
- Harris, J., Grandgenett, N., & Hofer, M. (2010). Testing a TPACK-based technology integration assessment rubric. In C. D. Maddux, D. Gibson, & B. Dodge (Eds.), *Research highlights in technology and teacher education 2010* (pp. 323-331). Chesapeake, VA: Society for Information Technology and Teacher Education.
- Hollebrands, K. F. & Smith, R. C. (2010, January). Assessing prospective secondary teachers' knowledge of geometry, technology, and pedagogy. In K.F. Hollebrands (Chair), *Methods and purposes for assessing high school teachers' knowledge of geometry*. Symposium conducted at the annual conference of the Association of Mathematics Teacher Educators. Irvine, CA.
- Jackiw, N. (2009). The Geometer's Sketchpad (Version 5) [Software]. Retrieved from <http://www.keypress.com/x24795.xml>
- Jonassen, D. (2003). Using cognitive tools to represent problems. *Journal of Research on Technology in Education*, 35(3), 362-382. <https://doi.org/10.1080/15391523.2003.10782391>
- Kaiser, G., & Vollstedt, M. (2007). Teachers' views on effective mathematics teaching: Commentaries from a European perspective. *ZDM*, 39(4), 341-348. <https://doi.org/10.1007/s11858-007-0036-1>
- Kay, R. H., & Knaack, L. (2005). A case for ubiquitous, integrated computing in teacher education. *Technology, Pedagogy and Education*, 14(3), 391-412. <https://doi.org/10.1080/14759390500200213>
- Kim, C. M., Kim, M. K., Lee, C. J., Spector, J. M., & DeMeester, K. (2013). Teacher beliefs and technology integration. *Teaching and Teacher Education*, 29, 76-85. <https://doi.org/10.1016/j.tate.2012.08.005>
- Kothari, C. R. (2004). *Research methodology: Methods and techniques*. New Delhi, India: New Age International Publishers.
- Landry, G. A. (2010). *Creating and validating an instrument to measure middle school mathematics teachers' technological pedagogical content knowledge (TPACK)* (Doctoral dissertation). University of Tennessee, 2010. Retrieved from [http://trace.tennessee.edu/utk\\_graddiss/720](http://trace.tennessee.edu/utk_graddiss/720)
- Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education*, 9(1), 91-102. <https://doi.org/10.1007/s10857-006-9006-8>
- Liljedahl, P., Rolka, K., & Rösken, B. (2007). Affecting affect: The reeducation of preservice teachers' beliefs about mathematics and mathematics teaching and learning. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The learning of mathematics. Sixty-ninth yearbook of the National Council of Teachers of Mathematics* (pp. 319-330). Reston, VA: National Council of Teachers of Mathematics.
- Messina, L., & Tabone, S. (2015). Technology proficiency, TPACK and beliefs about technology: A survey with primary school student teachers. *Research on education and media*, 5(1), 11-30.
- Mishra, P. & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *The Teachers College Record*, 108(6), 1017-1054. <https://doi.org/10.1111/j.1467-9620.2006.00684.x>
- Morton, B. L. (2013). *Tracing growth of teachers' classroom interactions with representations of functions in the connected classroom* (Doctoral dissertation). Retrieved from [http://rave.ohiolink.edu/etdc/view?acc\\_num=osu1373903545](http://rave.ohiolink.edu/etdc/view?acc_num=osu1373903545)

- Mudzimiri, R. (2010). Developing TPACK in pre-service secondary mathematics teachers through integration of methods and modeling courses: Results of a pilot study. In D. Gibson & B. Dodge (Eds.), *Proceedings of society for information technology & teacher education international conference* (pp. 3485–3490). Chesapeake, VA: AACE.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and teacher education*, 21(5), 509-523. <https://doi.org/10.1016/j.tate.2005.03.006>
- Niess, M. L. (2015). Transforming teachers' knowledge: Learning trajectories for advancing teacher education for teaching with technology. In C. Angeli & N. Valanides (Eds.), *Technological pedagogical content knowledge* (pp. 19-37). New York, NY: Springer. [https://doi.org/10.1007/978-1-4899-8080-9\\_2](https://doi.org/10.1007/978-1-4899-8080-9_2)
- Nzuki, F. M. (2010). Exploring the nexus of African American students' identity and mathematics achievement. *Journal of Urban Mathematics Education*, 3(2), 77-115.
- Ottenbreit-Leftwich, A. T., Glazewski, K. D., Newby, T. J., & Ertmer, P. A. (2010). Teacher value beliefs associated with using technology: Addressing professional and student needs. *Computers & Education*, 55(3), 1321-1335. <https://doi.org/10.1016/j.compedu.2010.06.002>
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of educational research*, 62(3), 307-332. <https://doi.org/10.3102/00346543062003307>
- Pamuk, S. (2012). Understanding preservice teachers' technology use through TPACK framework. *Journal of Computer Assisted Learning*, 28(5), 425-439. <https://doi.org/10.1111/j.1365-2729.2011.00447.x>
- Pehkonen, E., & Pietilä, A. (2003). On relationships between beliefs and knowledge in mathematics education. In M. A. Mariotti (Ed.), *Proceedings of the CERME-3 (Bellaria) meeting* (pp. 1-8). Retrieved from [http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings\\_draft/TG2\\_draft](http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG2_draft)
- Polly, D., Mims, C., Shepherd, C. E., & Inan, F. (2010). Evidence of impact: Transforming teacher education with preparing tomorrow's teachers to teach with technology (PT3) grants. *Teaching and Teacher Education*, 26(4), 863-870. <https://doi.org/10.1016/j.tate.2009.10.024>
- Project Tomorrow. (2008). *21st century learners deserve a 21st century education: Selected national findings of the Speak up 2007 survey*. Retrieved from <http://www.tomorrow.org/speakup/pdfs/National%20Findings%20Speak%20Up%202007.pdf>
- Project Tomorrow. (2011). *The new 3 E's of education: Enabled, engaged, empowered-how today's educators are advancing a new vision for teaching and learning*. Speak Up 2010 National Findings. Retrieved from [http://www.tomorrow.org/speakup/pdfs/SU10\\_3EofEducation\\_Educators.pdf](http://www.tomorrow.org/speakup/pdfs/SU10_3EofEducation_Educators.pdf)
- Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, 28(5), 550-576. <https://doi.org/10.2307/749691>
- Richardson, V. (2003). Preservice teachers' beliefs. *Teacher beliefs and classroom performance: The impact of teacher education*, 6, 1-22.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 15(2), 4-14. <https://doi.org/10.3102/0013189X015002004>
- Singletary, L. M. (2012). *Mathematical connections made in practice: An examination of teachers' beliefs and practices* (Unpublished doctoral dissertation). University of Georgia, Athens, GA.
- Smith, R. C., Kim, S., & McIntyre, L. (2016). Relationships between prospective middle grades mathematics teachers' beliefs and TPACK. *Canadian Journal of Science, Mathematics and Technology Education*, 16(4), 359-373. <https://doi.org/10.1080/14926156.2016.1189624>
- So, H. J., & Kim, B. (2009). Learning about problem based learning: Student teachers integrating technology, pedagogy and content knowledge. *Australasian Journal of Educational Technology*, 25(1), 101-116. <https://doi.org/10.14742/ajet.1183>
- Stake, R. E. (2006). *Multiple case study analysis*. New York: Guilford.
- Thagard, P. (2000). *Coherence in thought and action*. Cambridge, MA: MIT Press.
- Turner, P., & Chauvot, J. (1995). Teaching with technology: Two preservice teachers' beliefs. In G. M. Millsaps (Ed.), *Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 115-121). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Yin, R. (1984). *Case Study Research*. Beverly Hills, CA, U.S.A.: Sage Publications.

Zakaria, E., & Musiran, N. (2010). Beliefs about the nature of mathematics, mathematics teaching and learning among trainee teachers. *The Social Sciences*, 5(4), 346-351. <https://doi.org/10.3923/sscience.2010.346.351>

## APPENDIX I

### Beliefs Interview

#### *Interview questions on beliefs about the nature of mathematics:*

- a. **When you hear the term mathematics, what do you think of? In other words, how do you define mathematics?**

Possible questions to pose:

- Why do you think you view mathematics in this way?
- What other subject is mathematics most like? Least like?

- b. **Why do we need to learn mathematics?**

Possible questions to pose:

- Could you describe how you are thinking about the need of mathematics in your everyday life?
- How can mathematics be useful in your everyday life?
- Could you give me some examples?

#### *Interview questions on beliefs about mathematics learning:*

- a. **How do you think students learn mathematics?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

- b. **How do you remember feeling about your mathematics experiences in middle school?**

Possible questions to pose:

- How do you think about the way you have learned mathematics?
- What do you think was the hardest part about learning mathematics?
- Can you remember when you enjoyed learning mathematics?

- c. **What do you think is the most important aspect of mathematics that students should learn? In other words, what part of mathematics do you want students to be really good at?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

#### *Interview questions on beliefs about mathematics teaching:*

- a. **What do you think the role of mathematics teacher should be? You can give more than one role.**

Possible questions to pose:

- Which one is the most important?
- Could you tell me why you think that way?
- Could you give me an example?



**b. Could you describe your thoughts on your mathematics teachers in middle school and the instructional strategies they used to teach mathematics?**

Possible questions to pose:

- Why do you think your mathematics teachers taught this way?
- Could you tell me why you think that way?
- Could you give me an example?

**c. In order to be a good mathematics teacher, what do you think are the most important things for a teacher to do?**

I will make a list of what you say.

Possible questions to pose:

- Could you rank these things most important to least important?
- Could you tell me why you think that way?
- Could you give me an example?

**d. What do mathematics teachers need to know in order to be successful?**

I will make a list of what you say.

Possible questions to pose:

- Could you rank these things most important to least important?
- Could you tell me why you think that way?
- Could you give me an example?

*Interview questions on beliefs about the use of technology for learning and teaching:*

**a. In your mathematics classes in middle school, how often did you use technology?**

Possible questions to pose:

- Could you give me an example of the way how you have used technology?
- What kinds of technology did your mathematics teachers use?
- How often did your mathematics teachers use it?

**b. In your mathematics classes in high school, how often did you use technology?**

Possible questions to pose:

- Could you give me an example of the way how you have used technology?
- What kinds of technology did your mathematics teachers use?
- How often did your mathematics teachers use it?

**c. In your mathematics classes in college, how often did you use technology?**

Possible questions to pose:

- Could you give me an example of the way how you have used technology?
- What kinds of technology did your mathematics teachers use?
- How often did your mathematics teachers use it?

**d. How do you think the use of technology affects students' mathematical thinking?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

**e. Are there any advantages or disadvantages in using technology instead of pen and paper?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example to illustrate how it helps or not?

**f. How do you think the use of technology to teach mathematics? Does using technology change the teacher's role in the classroom?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you describe the role of teacher when teaching mathematics using technology?

*Interview questions on beliefs about the use of technology for their own teaching:*

**a. Describe your confidence in your ability to use technologies for mathematics instruction.**

Possible questions to pose:

- Could you tell me which term of a scale indicates how you feel about your confidence among Very Confident, Confident, Not Confident, and Very Not Confident?
- Could you tell me why you think that way?
- Could you give me an example?

**b. What technology has been available for you to use to teach mathematics?**

Possible questions to pose:

- How do you use technology for the purpose of effective mathematics instruction?
- How do you think technology could be used for the purpose of assessment? Please provide examples.
- How do you think you could use technology for the purpose of communication? Please provide examples (colleagues, parents, etc).

**c. When preparing lessons that incorporate technology, what do you take into account?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

**d. What kinds of support would be most helpful in order to use technology more often in the mathematics classroom?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

**e. What types of technology do you think you will need to better meet the needs of students when you become a teacher?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

**f. What types of technology do you think you will need to better meet your needs as a teacher?**

Possible questions to pose:

- Could you tell me why you think that way?
- Could you give me an example?

## APPENDIX II

## Task-based Interview

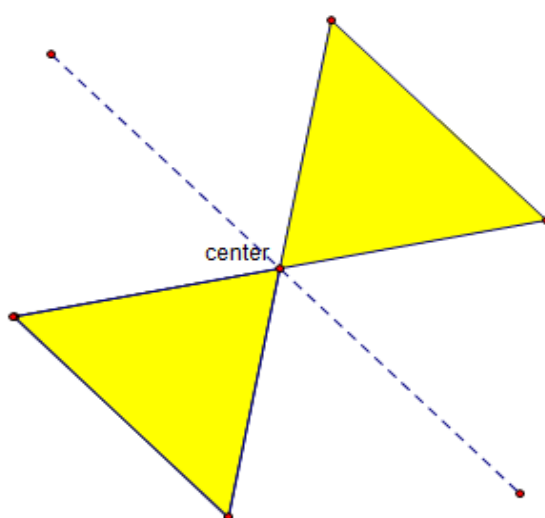
*Task 1*

Suppose students in your middle or high school mathematics class are studying rectangles and squares. They open a dynamic geometry sketch that contains a rectangle and a square, each of which have been constructed. Students are asked to consider properties of rectangles and squares, based on their exploration of the sketch. One pair of students has measured the diagonals and they have noticed they are always congruent. They claim, "quadrilaterals have congruent diagonals."

- Is this claim always true, sometimes true, or never true? Explain.
- How would you characterize their current level of geometric understanding?
- Create a sketch using a dynamic geometry environment that you would like students to use to explore diagonals of quadrilaterals. Be sure to include directions and/or questions you would provide to students as they use this sketch.

*Task 2*

After studying rotations, reflections, and translations using a dynamic geometry tool a student is playing around with rotations through an angle of 180 degrees and reflections. After some time the student claims: "A rotation through 180 degrees is the same as a reflection!" The student includes a screen capture that looks similar to the picture below. They explain, "when I reflect the triangle on the right and when I rotate the triangle on the right, I get the same thing."



- a. Is the statement "A rotation through 180 degrees is the same as a reflection" true? Explain how you arrived at that conclusion.
- b. What does the student understand about rotations and reflections?
- c. What question or task using technology would you pose to the student to learn more about how they are thinking about rotations and reflections? Explain.

### **Task 3**

Next week you are teaching a lesson on triangle centers and you are considering the following task.

*Draw a large acute triangle on a sheet of paper. Fold the paper to form creases representing the perpendicular bisectors of each side of the triangle. What conclusions can you reach regarding the three perpendicular bisectors of the sides of the triangles?*

- a. Use the blank sheet of paper to complete the task. Describe what you notice.
- b. Explore the same task using GSP. Describe what you do with the technology.
- c. How would you extend the original task to take into consideration what you learned in part b?
- d. How would you modify the original task to use technology with students? Give a restatement of the task. What pedagogical decisions and technological decisions did you make when redesigning this task?

### **Task 4**

When using the sketch of a constructed rectangle in a dynamic geometry program a student, Mary, drags a vertex of the rectangle so that it becomes a square. Mary claims that "a rectangle is a square."

- a. How would you characterize the Mary's current mathematical understanding? How might Mary have developed this understanding?
- b. What important mathematical ideas does a student need to understand to know about relationships between rectangles and squares?
- c. What instructional strategies and/or tasks would you use next with Mary? Why?

APPENDIX III

Rubric for Task-based Interview

Task 1

Task 1	Content Knowledge	Pedagogical Content Knowledge	Technological Content Knowledge	Technological Pedagogical Content Knowledge
	(A) Responds that the claim is sometimes true.	(A) Identifies that the student is able to notice that for a square and a rectangle that the diagonals are always congruent based on their measures.	(A) Accurately constructs or draws a quad using a DGE that is a counter-example.	(A) Uses the DGE technology to focus students on properties of different quadrilaterals and their relationships to the diagonals in the task.
	(B) Knowledge that there exists at least one quadrilateral for which the diagonals are not always congruent.	(B) Identifies that the student is at level 2 (descriptive) but probably not at level 3.	(B) Uses measures to find the lengths of the diagonals.	(B) Creates more than a single example using DGE technology to show the student that they are incorrect in the task.
	(C) States that for at least the rectangle and square the diagonals are always congruent.	(C) Has students consider at least one counterexample of a quadrilateral that has congruent diagonals.	(C) Drags to create multiple examples in a DGE.	
	(D) Provides a correct mathematical justification for why the statement is sometimes true using proofs that involve triangles or other properties.	(D) Asks students to consider at least one example of a quadrilateral that has congruent diagonals.	(D) Accurate constructions of the 2 of the following quad: <ul style="list-style-type: none"> <li>• Square</li> <li>• Rectangle</li> <li>• Parallelogram</li> <li>• Rhombus</li> </ul>	(C) Designs an exploration for students by creating accurate constructions and utilizing the measurement and dragging features
	Emergent: 0 or no response.	Emergent: 0 or no response.	Emergent: 0-1 of A - D or no response.	Emergent: 0 of A - C or no response.
	Beginner: 1 of A - D	Beginner: 1 of A - D	Beginner: 2 of A - D	Beginner: 1 of A - C
	Intermediate: 2 of A - D	Intermediate: 2 of A - D	Intermediate: 3 of A - D	Intermediate: 2 of A - C
	Advanced: 3 of A - D	Advanced: 3 of A - D	Advanced: All of A - D	Advanced: All of A - C

Task 2	Content Knowledge	Pedagogical Content Knowledge	Technological Content Knowledge	Technological Pedagogical Content Knowledge
(A)	Knowledge that a 180 degree rotation is never the same as a reflection when the domain and range are defined as all points in the plane.	(A) Displays knowledge about why students might think a rotation and reflection are the same.	(A) Understands how to perform a rotation using the technology by marking a center of rotation, indicating an angle of rotation, selecting the preimage polygon and labeling the preimage and image.	(A) Creates a task using an appropriate figure to highlight the differences between rotations and reflections (non-regular polygon).
(B)	Uses reasoning about orientation, such as a rotation preserves orientation and a reflection reverses orientation to explain why a rotation and reflection are different.	(B) Designs task that helps students see differences between rotation and reflections (uses labels for points, non-symmetric figure, matrices, etc)	(B) Understands how to perform a reflection using the technology by marking the mirror line, selecting the preimage and labeling the preimage and image polygon.	(B) Considers lines of reflection that are not parallel to a side of the preimage in the task. (dragging)
(C)	Understands that the images of symmetric polygons under a reflection and rotation of 180 degree may appear to look the same.	(C) Task or questions leads students to discover properties of reflections and rotations	(C) Demonstrates a knowledge of how to label points	(C) Focuses on the labeling of points to illustrate differences in orientation in the task.
(D)	Understands images will align only when line of reflection is perpendicular to a line of symmetry and when the center of rotation is strategically placed on the line of reflection.	(D) Describes what students know about reflections and rotations	(D) Uses dragging	(D) Considers other locations of the point of rotation that are not on the line of reflection in the task (dragging point).
(E)	Knowledge of properties of reflection or rotation or how to perform a reflection and rotation	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D
	Emergent: 0 or no response. Beginner: 1 of A - E Intermediate: 2 of A - E Advanced: 3 of A - E			

Task 3	Content Knowledge	Pedagogical Content Knowledge	Technological Content Knowledge	Technological Pedagogical Content Knowledge
(A)	Knowledge that the circumcenter is equidistant from the vertices of the triangle	(A) Asks students to consider the distance the circumcenter is from each of the vertices	(A) Constructs the perpendicular bisectors to locate the circumcenter.	(A) Gives an equivalent restatement of the task using technology so students are still considering circumcenters.
(B)	Knowledge that the perpendicular bisectors are concurrent - that there is a point of intersection	(B) Considers what students may have already done in class when modifying the tasks	(B) Uses the measurement tool in an appropriate manner.	(B) Creates more than a single example to show that the relationships hold for all triangles
(C)	Knowledge that the circumcenter of a triangle is the center of a circle the circumscribes the triangle (names circumcenter)	(C) Has students consider different types of triangles (D) Asks students to create a circle using the circumcenter and a vertex of the triangle.	(C) Uses dragging to modify the original triangle and examine different locations of the circumcenter (D) Uses the circle tool to create a circumcircle	(C) Constructs a figure that will enable students to discover relationships of a circumcenter. (D) Makes appropriate use of multiple features of the tool such as dragging, measures, constructing, etc.
(D)	Demonstrates knowledge about the location of the circumcenter (Inside for acute, on for right, and outside for obtuse).			
	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 of 4 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D

Task 4	Content Knowledge	Pedagogical Content Knowledge	Technological Content Knowledge	Technological Pedagogical Content Knowledge
	(A) Recognizes that a rectangle is not a square	(A) State's student misconception		(A) Describes a technological sketch that can help with student's misconceptions and justifies its appropriate use (Does not focus necessarily on properties, focuses on figures)
	(B) Recognizes that a square is a rectangle	(B) Understands where the student's misconceptions may have come from and relate them to technology or van Hiele levels	(A) Understands the drag feature in DGE and how it maintains the properties of the original construction	
	(C) Uses knowledge of differences between a rectangle and square to justify why a rectangle is not a square (which includes the following properties of a square) <ul style="list-style-type: none"> <li>• 4 congruent sides</li> <li>• Perpendicular diagonals</li> <li>• Diagonals are angle bisectors</li> <li>• Diagonals create 4 congruent right triangles</li> </ul>	(C) Uses knowledge of properties of squares and rectangles and differences between these two figures to pose questions to the students	(B) Uses measures to show that a rectangle is not a square since all sides are not congruent	(B) Designs an appropriate activity with the technology that assists students in learning the relationships between squares and rectangles by focusing students on the properties of each figure.
	(D) Uses knowledge of rectangles and squares to justify why a square is a rectangle (includes the following properties common to both) <ul style="list-style-type: none"> <li>• 4 right angles</li> <li>• Opposite sides congruent</li> <li>• Congruent diagonals</li> </ul>	(D) Task or questions leads students to understand that squares are always rectangles, but rectangles are not always squares	(C) Constructs a square and drags it to show that a square can never be a rectangle	(C) Makes appropriate use of multiple features of the tool such as dragging, measures, constructing, etc.
	Emergent: 0 or no response Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - D Intermediate: 2 of A - D Advanced: 3 of A - D	Emergent: 0 or no response. Beginner: 1 of A - C Intermediate: 2 of A - C Advanced: 3 of A - C

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