

## Technology for Teaching Students to Solve Practice-Oriented Optimization Problems in Mathematics

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### ABSTRACT

The transition to new educational standards puts forward the applied orientation of training school education. The universality of mathematical methods allows to reflect the connection of theoretical material of various fields of knowledge with practice on the level of general scientific methodology. Practice-oriented activity, as a manifestation of the content of the mathematics course of the secondary school, determines the importance of mathematics in preparing students for continuing education in the process of professional development. But at the same time there arises the need to stop understanding the educational activity only as a process of obtaining ready-made knowledge. The relevance of the problem is caused by the insufficiently developed method of teaching to solve practice-oriented optimization problems in the school course of mathematics as a means of strengthening its applied orientation. The purpose of the article is to develop and substantiate the technology of teaching students to solve practice-oriented optimization problems in the course of implementing the applied orientation of the mathematics course. The article discusses methodological aspects of organizing pupils' training in mathematics, substantiates the need to use practice-oriented optimization problems in mathematics in secondary schools, suggests a set of tasks and considers various methods for solving practice-oriented optimization problems, and also reveals the specificity of solving problems of this type.

**Keywords:** mathematical education, applied orientation of training, practice-oriented training, teaching methods, optimization problem

## INTRODUCTION

### The Relevance of the Research

The most important requirement of the society for school leavers' training is forming a broad scientific worldview based on solid knowledge and life experience, readiness to apply the acquired knowledge and skills in their life (Kvon et al., 2017). Implementing this requirement provides for the education system oriented on the development of students' qualities necessary for life in modern society and practical interaction with the objects of nature, production, daily life. It increases the need to strengthen the applied orientation of education, which implements both teaching and educational goals of training. Therefore, a graduate of the modern school needs practice-oriented knowledge in order to socialize and adapt successfully in society (Romanovskaya, 2006).

One of the ways to solve the problem of the socialization of school graduates is to introduce practical tasks into the content of the mathematics course: economic, vocational, social and other types of tasks. The use of tasks with

### **Contribution of this paper to the literature**

- Determining the principles of implementing the applied orientation of the school course of mathematics, as well as the functions and stages of teaching to solve practice-oriented optimization problems as a means of ensuring the applied orientation of school mathematics.
- Introducing didactic technologies to make it possible to realize a great didactic potential of practice-oriented optimization problems in the process of teaching mathematics.
- Developing didactic material for teaching schoolchildren to solve practice-oriented optimization mathematical problems.

practical content contributes to the provision of a more conscious mastering of mathematical theory and practice, creates the conditions for linking mathematics to life, developing intersubject connections, and contributes to the more successful socialization of graduates in modern society.

Practice-oriented training involves studying disciplines, traditional for the education fundamental, in combination with applied disciplines of technological or social orientation. To do this, it is necessary to restructure the educational system - without losing its fundamental nature; it has to acquire new, practice-oriented content (Yaburova, 2006). The updated content should play a key role in preserving fundamental science, developing applied sciences required for the sustainable development of Russian society (Yalalov, 2008).

An important role in the system of preparing students for applying the acquired knowledge for practical purposes belongs to the study of the school course of mathematics, since mathematics has the means of developing universal educational activities that will allow a person to realize his/her potential both as an individual and as a specialist (Federal state educational standard of basic General education, 2013). In addition, the universality of mathematical methods makes it possible to reflect the connection of theoretical material with practice on the level of general scientific methodology. This determines the importance of mathematics in the formation of students' ability to solve problems arising in the process of practical human activity.

One of the means of strengthening the applied orientation of teaching mathematics is optimization problems. Their purpose is to find the best (optimal) variant of utilizing available resources (material, time) from the point of view of some criterion or criteria. Optimization problems have great didactic possibilities for realizing goals of practice-oriented training (Zhmurova & Generalova, 2016). However, the use of such problems as a means of implementing practice-oriented instruction in mathematics is still insufficient. This is due to the rapid development of science and technology and slow updating of educational materials (Kolyagin & Pikan, 1985; Polisadov, 2014).

According to the conducted polls, teachers of mathematics rarely use optimization tasks in their educational activities. The class of optimization problems, which teachers of mathematics consider at their lessons, is rather narrow: mainly, problems are solved for finding the extremum of a function on some interval and the largest (least) value of a function. In the last two years, problems with economic content have been added to the optimization problems solved at the lessons of mathematics. Methods for solving optimization problems also do not differ in variety. Basically, these are methods of differential calculus, occasionally various graphic illustrations are used. Thus, it can be concluded that the teaching methodology is not sufficiently developed for solving practice-oriented optimization problems in the school course of mathematics, which actualizes the search for new forms and methods in this field.

## **Goals and Objectives of the Study**

The aim of the work is to develop and substantiate the technology of teaching students to solve practice-oriented optimization problems in the process of implementing the applied orientation of the profile school mathematics course.

To achieve this goal, the following tasks were identified: to determine the functions and stages of solving practice-oriented optimization problems as a means of implementing the applied orientation of the school mathematics course; to identify the features of teaching and to develop a technology teaching to solve these problems, the use of which will develop students' ability to formulate and solve these practice-oriented optimization problems on operational, technological and generalized levels; and also to develop didactic support for teaching to solve practice-oriented optimization problems in the secondary school mathematics course.

## **LITERATURE REVIEW**

Ideas and experience of implementing practice-oriented training in the context of ensuring the applied orientation of school curricula is covered by many scientists.

The main means of implementing the applied orientation of teaching mathematics are problems. The works of Apanasov (1975), Ashurov (1990), Balk and Petrov (1986), Vozniak (1990), Tereshin (2014), Shapiro (1990), Bondarentko (2013), and others are devoted to the research of didactic possibilities of applied problems.

Dalinger (1996) identifies ways of implementing the applied orientation of teaching mathematics: using applied problems in the teaching process (problems set out of mathematics and solved by mathematical means); involving in the content of the training material practical problems (problems from the real surroundings connected with the formation of practical skills required in everyday life), including the use of local history materials, elements of production processes; converging methods for solving training problems with methods used in practice.

The theoretical foundations of the methodology for implementing the applied orientation in the process of teaching mathematics are developed by Erentraut (2005), Solovieva (2012), Dalinger (2013) and other researchers.

The studies have found that implementing the provisions on applied orientation formulated in the normative documents determines the need for the development of: the content of the material (Pinsky, 2003; Polyakov & Kuznetsov, 2004), the content of schoolchildren's educational activities in assimilating the social experience to form individual skills on this basis to be ready to solve professional problems (Bublikov, 2000; Guzeev & Bershadsky, 2003).

The works of Kalugina (2010), Kolyagin and Pikan (1985), etc. are devoted to the ideas of strengthening the practical aspect of schoolchildren's training due to integrating the processes of forming theoretical knowledge and developing practical skills, which, of course, should increase the efficiency of the knowledge acquired.

Shtepa (2008) in his works gives an interesting experience in teaching how to solve forecasting and optimization problems in the school computer science course.

In order to identify the role and place of optimization problems in school mathematics education we have conducted a survey of students in grades 5-11 (188 respondents) and teachers of mathematics (308 respondents). The diagnostics of the data obtained during the survey indicates the importance of optimization problems in school mathematics education - 77.6% of teachers and 68.1% of students consider it necessary to study this kind of problems (Zhurova & Generalova, 2016).

Yaremko (2013) notes the need of a professional in daily practice, to solve incorrect problems along with correct ones, to work with incorrect objects, which demands forming an individual who can act in a variety of ambiguous conditions. The author asserts the need to introduce in the educational practice issues that will equip schoolchildren with the methodology for actions in incorrect conditions, actions to prove the correctness, to recognize the incorrectness and to transform it into the correctness. The means of achieving this are practice-oriented mathematical problems.

The National Council of Teachers of Mathematics (NCTM, 2000) has formulated principles and standards for school mathematics in which one of the main roles belongs to practice-oriented training.

Anthony and Biggs (1996) emphasizes that mathematics is indispensable in modeling economics, finance, business and management, and other areas. The content of teaching mathematics in school follows from one specific concept of general education. This concept does not create content for learning, but only assumes that mathematics is a cultural achievement, a fact of social life, an educational subject and a field of knowledge that can be delivered to students (Heymann, 2003).

At present, there is a consensus that one of the central components of the mathematics course in school is applying mathematical knowledge to the "real world" (Geretschläger, 2017). Furner and Kumar (2007) substantiate the need for a wide application of mathematics in natural sciences, giving it an important role in understanding the relationship between scientific concepts in different fields. They emphasize that the success of each student in these areas of knowledge depends on the extent to which they are integrated with mathematics, because it can attract students to in-depth motivation study of these subjects.

The work of Bock and Bracke (2015) is devoted to the problems of strengthening the applied orientation of the school mathematics course through mathematical modeling and practice-oriented problems. The authors have studied the role of mathematical modeling in the process of teaching mathematics and noted that active solution of practice-oriented problems increases the effectiveness of teaching mathematics and promotes the development of students' interests.

The issues of practice-oriented learning are discussed in the work of Xia, Lv, Wang, and Song (2016).

Kim and Cho (2015), based on their work experience, conclude that schoolchildren learn more meaningfully within the framework of integrated education, as it helps them to find a link between school education and their real life.

Special attention is paid to STEM education (S - science, T - technology, E - engineering, M - mathematics). Shahali (2017) has presented the results of the study, which prove high effectiveness of educational activities in the

form of an educational program, when separate subjects are not singled out, and work is carried out in the framework of an integrated study on “topics”.

Smith (2001), Pardhan and Mohammad (2005) examine issues of practice-oriented professional development.

The book (Collins et al., 1999) contains a program for motivating high school students, which allows students to demonstrate the value of mathematics in the world around them, improve their fluency in the mathematical language to solve problems of real life.

## MATERIALS AND METHODS

### Theoretical Basis

Applied orientation determines the target orientation of teaching on forming students’ interdisciplinary knowledge, skills, conceptual thinking, scientific worldview. That is why there arises an issue not only about the prospects for interaction of various subjects in school, which contributes to the development of a system of knowledge, schoolchildren’s clear vision of ideas common to different subjects (Khutorskoy, 2012; Vinokurova & Episeeva, 1999), but also an issue about the means of developing universal educational activities, which will allow a person to realize himself both as a person and as a specialist.

While analyzing this problem, we relied on the integrative approach involving interrelation of mathematical and natural-science knowledge in schoolchildren’s educational activity, on the technologies of developmental training (Davydov, 2004; El’konin, 1989), the technologies of personality-oriented teaching (Yakimanskaya, 2000) and the technology of practice-oriented learning. The essence of the practice-oriented learning is to build the educational process on the basis of acquiring new knowledge and developing practical experience of their use in solving vital tasks and problems. The principles of organizing practice-oriented training are: motivational support of the educational process; connection of training with practice; conscious and active training, activity approach.

Practical means of implementing the applied orientation of teaching mathematics in school is using applied or practice-oriented problems (Ivanova, 1998; Sarantsev, 2002), in particular, optimization problems.

Practically-oriented problem is a kind of plot tasks, which requires implementing all stages of the mathematical modeling method (external mathematical, not intramathematical) in their solution. They are constructed by selecting situations in which mathematical knowledge is a means of solving practical problems. Such problems are not problems in the traditional sense of the word, but represent a “vital-imitative” situation in which students see the value of scientific knowledge for the reality surrounding them. In the process of solving such problems, they become aware of how they can use mathematics in practical, future professional activity, in society, in specific psychologically significant situations.

Practice-oriented mathematical problems can provide different degrees of integration and contribute to the strengthening of interdisciplinary links of mathematics with other educational subjects. In addition, they can be widely used both directly at the lesson, and in additional education. They can also be used in project activities (Guzeev, 1995; Rohlov, 2006).

### Methods of Research

The following methods were used to obtain statistical data on the research problem: interview, questioning of schoolchildren and teachers, analysis of psychological-pedagogical and mathematical-methodical literature on the topic of the research, analysis and generalization of the experience of teachers and the author’s own experience in the system of basic and additional mathematical education, modeling the technology for solving problems, analysis of the results of the educational activity, the method of thought experiment, systematization and generalization of facts and concepts, modeling, design, method of expert assessment, analysis of learning activities, the development of training materials, diagnostic tools, pedagogical experiment.

### Experimental Research Base

Approbation, generalization and implementation of the research results are carried out:

- by experimental teaching the elective course “Optimization problems and methods to solve them” for pupils of Grade 10; the course has been conducted since 2014 on the basis of an educational organization - school # 14 of the city of Kirov (over 50 students annually);
- in the process of implementing the project “Bridging gaps between educational levels”, carried out as part of the Development Program of the Federal State Educational Institution of Higher Education “Vyatka State University” for 2016-2020;

- conducting distance and full-time courses for schoolchildren preparing for the profile State exam in mathematics (84 hours), in which over 300 pupils from schools of the city of Kirov and the Kirov region have taken part since 2014;
- in the form of reports and presentations at scientific conferences and seminars of various levels, including international ones, publications in collections of scientific articles and scientific and methodical periodicals.

### Stages of Research

The study is conducted in three stages. The first stage revealed the state of the problem studied in theory and practice of teaching mathematics to pupils in the middle school. For this purpose, we completed research and analysis of psychological, pedagogical and methodological literature on the problem studied, questionnaires of the educational process participants, observation and analysis of the experience of teachers of mathematics to explore possible ways of implementing the applied orientation of training and organizational forms of educational activity to prepare students for future professional tasks effectively.

The second stage was devoted to developing methodical approaches to introducing the proposed technology to solve applied problems into the educational process using practice-oriented optimization problems as an example. Their implementation has been discussed and continues to be discussed at conferences and seminars of various levels, which leads to a consistent improvement of the methodology of work on interdisciplinary projects in teaching mathematics.

The third stage was being implemented simultaneously with the second one when the experimental teaching was conducted on the proposed methodological aspects in schools of the city of Kirov and the Vyatka State University.

## RESULTS

### Practice-Oriented Problems as a Means of Implementing the Applied Orientation of The School Mathematics Course

The formation of thinking can occur both directly through the applied nature of the mathematics course, and indirectly through teaching processes of mathematical modeling and mathematization of arbitrary life situations. Using practice-oriented teaching technologies turns a student from a passive object of pedagogical influence into an active subject of educational and cognitive activity. The main means of implementing the applied orientation of the mathematics course is a specially selected system of practice-oriented problems.

In the process of solving such problems, students develop cognitive skills, the ability to design their knowledge independently, the ability to navigate in a vast information space, to analyze the information obtained, the ability to put forward hypotheses independently, to make decisions (finding directions and methods to solve the problem); critical thinking, the ability to carry out research and creative activities.

It is known that the process of solving a practice-oriented problem consists of three stages (Samarskiy & Mikhailov, 1997):

- 1) the stage of formalization, transfer of the proposed problem from a natural language into the language of mathematical terms, i.e. construction of a mathematical model;
- 2) the stage of solving the problem inside the model;
- 3) the stage of interpretation of the solution obtained, i.e. translation of the obtained result (mathematical solution) into the language in which the original problem was formulated.

It should be noted that the school mainly pays attention to work on the second stage of solving the problem within the already constructed model, while formalization and interpretation remain insufficiently disclosed.

One of the means of strengthening the applied orientation of teaching mathematics is the practice-oriented optimization problems. The spectrum of such problems is quite large (Zhmurova & Generalova, 2016). These are problems to compose and solve equations, and problems to compare numbers and/or quantities, and problems with missing or redundant data, geometric problems, including using various dynamic models. Optimization problems characterizing various economic processes and phenomena are widespread, for example, resource allocation, rational cutting, transportation, enterprises consolidation, network planning, the problem to achieve the destination as soon as possible, the problem to organize production in order to obtain maximum profit at a given resource cost, the problem of managing the system of hydropower stations and reservoirs in order to obtain the maximum amount of electricity, the problem of the fastest heating or cooling of the metal to a given temperature regime, the problem of best damping vibrations and many other problems. All optimization problems have a common property - the goal is known, which often requires dealing with complex systems, where it is not so much

about solving optimization problems as it is about researching and forecasting the states depending on the elected control strategies.

The analysis of school textbooks in mathematics shows that for the first time students meet optimization problems in the 8th grade when studying the topic "Quadratic function", later in the 11th grade the problems of this type are given in the topic "Application of the derivative to the study of a function". But in school textbooks, practice-oriented tasks are very few (Dalinger & Simonzhenkov, 2007).

It is not so easy to choose problems that form elementary skills of applying mathematics. Many of the textbook problems are unnatural from the applied positions. The search and systematization of instructive and at the same time fairly simple tasks of this kind is a very pressing problem.

It is advisable to use any possibility to show that an abstract problem can be connected with practice in order to eliminate possible false idea of schoolchildren that the tasks may be applied, and may not be practical, not useful in life. For example, "The yard has the shape of a triangle. Where is it necessary to dig a pole for the lamp suspension so that to illuminate best the points of the triangle sides nearest to the pole?" or "The forest glade has the form of a triangle. In what point is it safer to build a fire?"

### **Methods to Solve Optimization Problems**

When solving any specific optimization problem, it is necessary to complete the following sequence:

- 1) To set the optimization problem conceptually. To do this, it is necessary to define (classify) the object of optimization as fully as possible, i.e., the process or the system; if the system, then dynamic or static; if the object is dynamic, then continuous or discrete, deterministic or stochastic. It is also necessary to formulate an optimization criterion, that is, to formulate the optimization goal (what is necessary to obtain from the process or system). And, finally, it is necessary to define the possibilities of optimization fully, that is, to formulate constraints on the variables and parameters of the optimization object.
- 2) To define the unchangeable part of the optimization object fully, that is, to select those characteristics and parameters that are specified as requirements to the process or system, or exist objectively. To determine the variable part of the optimization object, that is, to isolate characteristics and parameters, the change of which can affect the optimization criterion.
- 3) To select the mathematical method and construct a mathematical model of the optimization object, that is, to describe the optimization object in the language of the chosen mathematical method.
- 4) In accordance with point 3, to determine the type and nature of restrictions imposed in this problem on the variable characteristics and parameters of the process or system.
- 5) In accordance with points 3, 4 to formulate and formalize the criterion, to choose the method of solution and formally put the problem of optimization.
- 6) To develop an algorithm to solve the optimization problem.
- 7) To work out this algorithm or develop a program to solve the optimization problem on a computer.
- 8) To formulate the answer.

### **Examples of Practice-Oriented Optimization Problems and Methods for Their Solution**

Mathematical and science disciplines (especially in their intersubject connections) give a wide scope for strengthening the applied orientation of training, and this, in turn, helps students to achieve substantive results. Lessons in solving problems, built on the basis of an integrative approach, develop students' potential, stimulate their knowledge of the surrounding reality, develop their logic of thinking, and provide training for a competitive specialist in the integrated information space of modern society (Gubanova, 2001).

Schoolchildren often think that problems may be applied, i.e. useful in life, and not practical, which are not useful in life. To eliminate such errors, it is advisable to use any possibility to show that an abstract problem can be connected with applied problems. For example, "The yard has the shape of a triangle. Where is it necessary to dig a pole for the lamp suspension so that to illuminate best the points of the triangle sides nearest to the pole?" or "The forest glade has the form of a triangle. In what point is it safer to build a fire? "

It is not easy to select problems that will form the elementary skills of applying mathematics. Many of the textbook problems are unnatural from the applied positions. The search and systematization of instructive and at the same time fairly simple tasks of this kind is a very pressing problem. It is also possible to compile practice-oriented problems using the following algorithm.

- 1) To determine the purpose of the task, its place in the lesson, in the topic, in the course.
- 2) To determine the focus of the problem (professional, intersubject).

- 3) To identify the types of information to compile the problem.
- 4) To determine the degree of students' independence in receiving and processing information.
- 5) To select the structure of the task.
- 6) To determine the form of the answer to the question of the problem (single-valued, multivariate, non-standard, no response, response in the form of a graph).

The topics of optimization problems can be quite diverse. For example, economical use of financial, natural and other resources in conditions of their limitation; measures against traffic jams in big cities; the development of an optimal route for transporting goods from beyond the Urals to the European regions of our country; load distribution in the electrical network.

On the one hand, such problems require accurate formulation from the point of view of physics and technology, on the other hand, rather complex mathematical methods.

Here is a fragment of a developed bank of practice-oriented optimization problems. Let us consider various ways of their solution.

### Economic Problems

**Task 1.** Aluminum and nickel are mined in two mines. In the first mine there are 100 workers, each is ready to work 5 hours a day. In this case, one worker produces 1 kg of aluminum or 3 kg of nickel per hour. In the second mine there are 300 workers, each is willing to work 5 hours a day. In this case, one worker produces 3 kg of aluminum or 1 kg of nickel per hour. Both mines deliver the extracted metal to the plant, where an aluminum and nickel alloy is produced for the needs of industry, in which 2 kg of aluminum accounts for 1 kg of nickel. In this case, the mines agree to conduct the extraction of metals so that the plant can produce the greatest amount of the alloy. How many kilograms of alloy under such conditions can the plant produce daily?

#### Solution

*Way 1 - by compiling a linear reference function*

Let  $x$  workers mine aluminum the first mine, then  $100 - x$  workers mine nickel. The amount of extracted aluminum is  $5 \cdot x$  kg, the amount of extracted nickel is  $15 \cdot (100 - x)$  kg.

Let  $y$  workers get aluminum daily in the second mine, then  $300 - y$  workers get nickel. Then the amount of extracted aluminum is equal to  $15y$  kg, the amount of extracted nickel is  $5 \cdot (300 - y)$  kg.

The total amount of aluminum mined is  $5 \cdot x + 15 \cdot y$  in kg, the total amount of nickel extracted is:

$$15 \cdot (100 - x) + 5 \cdot (300 - y) = 1500 - 15 \cdot x + 1500 - 5 \cdot y = 3000 - 15 \cdot x - 5 \cdot y \text{ in kg.}$$

The function of the alloy is:  $F(x) = (5 \cdot x + 15 \cdot y) + (3000 - 15 \cdot x - 5 \cdot y)$ ;  $F(x) = -10 \cdot x + 10 \cdot y + 3000$ ;

We take into account the condition under which the alloy of aluminum and nickel is produced: 2 kg of aluminum and 1 kg of nickel. Then  $5 \cdot x + 15 \cdot y = 2 \cdot (3000 - 15 \cdot x - 5 \cdot y)$ . Hence  $y = -1.4 \cdot x + 600$ . We substitute this expression in the alloy function:

$$F(x) = -10 \cdot x + 10 \cdot (-1.4 \cdot x + 600) + 3000;$$

$$F(x) = -24 \cdot x + 5400.$$

This linear function is decreasing. The greatest value it takes at  $x = 0$ .

Hence,  $F(0) = 5400$ .

Answer: 5400 kilograms is the largest amount of alloy that the plant can produce every day.

*Way 2 - with the help of logical reasoning and the equation*

Since more nickel is mined in the first mine, it is logical that all workers in this mine produce nickel for the greatest benefit. Then 1500 kg of nickel will be produced in the first mine. In the second mine, more aluminum is mined. Let all 300 workers extract aluminum. Then they will extract 4,500 kg of aluminum. The alloy requires aluminum twice as much as nickel. Hence, 1500 kg of nickel needs 3000 kg of aluminum. By the condition of the problem, the amount of aluminum is larger. So, workers of the second mine need to be redistributed to extract not only aluminum but also to extract nickel, taking into account the proportion of the alloy. Let  $x$  workers of the second mine extract aluminum, then  $300 - x$  workers get nickel. Write the equation:

$$5 \cdot 3 \cdot x = 2 \cdot (5 \cdot (300 - x) + 1500);$$

$$15 \cdot x = 6000 - 10 \cdot x;$$

$$x = 240.$$

Let's find  $y$ :  $y = 300 - 240 = 60$ . Hence, 240 workers must produce aluminum, 60 workers extract nickel. Then aluminum will be extracted  $240 \cdot 5 \cdot 3 = 3600$  kg, nickel  $1500 + 60 \cdot 5 = 1800$  kg. Total is  $3600 + 1800 = 5400$  kg.

Answer: 5400 kilograms is the largest amount of alloy that the plant can produce every day.

Way 3 - by brute force search

Since more nickel is mined in the first mine, let all workers extract nickel. Then, the first mine will extract 1500 kg of nickel. More aluminum is mined in the second mine. Let all 300 workers extract aluminum. Then they will extract 4,500 kg of aluminum. The alloy requires aluminum twice as much as nickel. Hence, 1500 kg of nickel requires 3000 kg of aluminum. And we have more aluminum. What to do? Hence, the workers of the second mine are to be redistributed to extract not only aluminum but also to extract nickel. We apply the method of brute force search.

Suppose 10 workers of the second mine extract nickel, and 290 workers extract aluminum. Then, they will extract  $290 \cdot 5 \cdot 3 = 4350$  (kg) of aluminum, and  $1500 + 10 \cdot 5 = 1550$  (kg) of nickel. We note that the data do not satisfy the proportion 1: 2. Hence, it is necessary to increase the number of workers mining nickel.

Suppose 20 workers of the second mine extract nickel, and 280 workers - aluminum. Then aluminum produced will be  $280 \cdot 5 \cdot 3 = 4200$  (kg), and nickel -  $1500 + 20 \cdot 5 = 1600$  (kg). We note that the data do not satisfy the proportion 1: 2. Hence, it is necessary to increase the number of workers producing nickel again.

Let's say 40 workers of the second mine extract nickel, and 260 workers - aluminum. Then,  $260 \cdot 5 \cdot 3 = 3900$  (kg) of aluminum will be extracted, and  $1500 + 40 \cdot 5 = 1700$  (kg) of nickel. We note that the data do not satisfy the proportion 1: 2. Hence, it is necessary to increase the number of workers producing nickel again.

Suppose 60 workers of the second mine extract nickel, and 240 workers - aluminum. Then aluminum produced will be  $240 \cdot 5 \cdot 3 = 3600$  (kg), and nickel -  $1500 + 60 \cdot 5 = 1800$  (kg). We note that the data satisfy the proportions 1: 2, that is, 1 part of nickel accounts for 2 parts of aluminum: 1800: 3600. So,  $3600 + 1800 = 5400$  (kg) of aluminum and nickel will be extracted. And the quantity of products from the alloy then will be equal 1800 pieces.

Answer: 5400 kilograms is the largest amount of alloy that the plant can produce every day.

*Problem 2. The entrepreneur bought a building and is going to open a hotel in it. The hotel is to have standard rooms of 21 square meters and luxury rooms of 49 square meters. The total area that can be assigned to the rooms is 1099 square meters. The entrepreneur can divide this area into rooms of different types as he wants. The usual room will bring the hotel 2000 rubles per day, and the "luxury" room - 4500 rubles per day. What is the largest amount of money the businessman can earn per day at his hotel?*

*Solution*

Way 4 - with the help of logical reasoning and arithmetic operations

Let's find the cost of 1 m<sup>2</sup> of the standard room:  $2000: 21 = 95\frac{2}{7}$  (rubles)

Find the cost of 1 m<sup>2</sup> of the "luxury" room:  $4500: 49 = 91\frac{41}{49}$  (rubles)

Conclusion: Since the cost of 1 m<sup>2</sup> of the standard room is higher, it is more profitable to place more standard rooms on this area, and as few as possible "luxury" rooms. Let's start searching the number of luxury rooms from the smallest possible number. Let number the luxury rooms be 0. Then the 1099 is not divisible evenly by 21. Assume that there will be 1 luxury room. Then:  $1099 - 49 = 1050$  m<sup>2</sup>

$1050: 21 = 50$  (standard numbers). So, on an area of 1050 m<sup>2</sup> it is possible to place 50 standard numbers. Then the hotel can earn per day:  $50 \cdot 2000 + 1 \cdot 4500 = 104500$  rub.

Answer: 104500 rubles is the largest amount that an entrepreneur can earn per day.

*Problem 3. The distance between two farms A and B along the highway is 60 km. Farm A produces 200 tons of milk daily, Farm B - 100 tons per day. Where is it necessary to build a milk processing plant, so that transportation number of ton-kilometers is the smallest?*

*Task 4. A 167-meter-long running water pipe should be installed in the villa site. There are pipes 5 and 7 meters long. How many pipes should be used to make the smallest number of joints (pipes cannot be cut)?*

*Problem 5. It is known that 1 kg of oranges contains 150 mg of vitamin "C", and 1 kg of apples - 75 mg of vitamin "C". How many oranges and how many apples should be included in the daily ration, so that at minimum cost it turned out to be 75 mg of vitamin "C", not less than 0.25 kg of oranges and not less than 0.25 kg of apples, if 1 kg of oranges costs 60 rubles, and 1 kg of apples - 40 rub?*

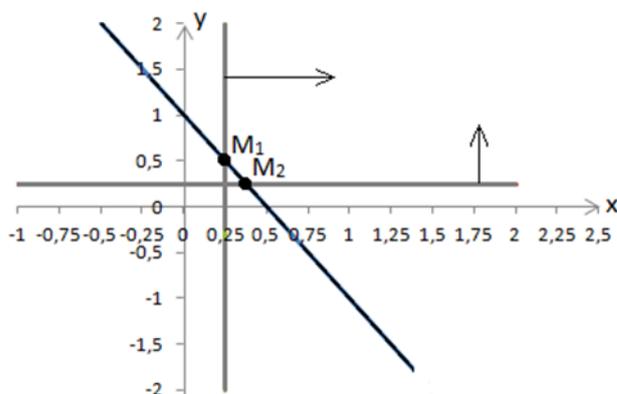
*Solution*

Way 5 - using the graphical method

Let's put the data in the **Table 1** and **Figure 1**:

**Table 1.** Fruits and vitamins

Fruits	Daily ration	Vitamin C(in 1 kg)	Cost of 1 kg
Oranges	$X$ kg	150 mg	60 r
Apples	$y$ kg	75 mg	40 r



**Figure 1.** Graphic method

**Table 2.** Production

Cost per product		A	B	Resources
Material	Steel (kg)	10	70	320
Material	Non-ferrous metals (kg)	20	50	420
Equipment	Lathe machines (machine-hr)	300	400	6200
Equipment	Milling machines (machine-hr)	200	100	3400
Profit per one item (thousand rubls)		3	8	

The constraints are:  $x \geq 0.25$ ;  $y \geq 0.25$ ;  $150x + 75y = 75$ .

Target function:  $F(x, y) = 60x + 40y$ .

It is necessary to find such values of the variables  $x$  and  $y$  for which **the target function takes a minimum value**.

Let's construct the domain of admissible solutions of the problem:

Let  $60x + 40y = 0$ ; hence  $y = -6 / 4x$

We construct the graph of the function  $y = -6/4x$  and carry out a parallel transfer along the axis  $Oy$  upwards, which is equivalent to an increase in the values of the expression  $60x + 40y$ .

In order for the target function to take a minimum value, its graph must intersect the segment  $M_1M_2$  at the point  $M_2$ . It is the intersection point of the straight lines  $y = 0.25$  and  $y = -2x + 1$ . Hence,  $y = 0.25$ ,  $x = 0.375$ . Further we find:  $F(x, y) = 60 \cdot 0.375 + 40 \cdot 0.25 = 16.25r$ .

Answer: for the daily ration to contain 75 mg of vitamin "C" under the condition of minimal costs, a person should eat 0.375 kg of oranges and 0.25 kg of apples every day.

**Problem 6.** For the production of two types of goods A and B, the plant uses steel and non-ferrous metals as raw materials, the stock of which is limited. The lathe and milling machines are employed in the production of these goods in the amount indicated in the **Table 2**.

It is necessary to determine the output plan, at which the maximum profit will be achieved if the operating time of the milling machines is fully used.

**Problem 7.** The sewing workshop has 164 m of fabric. It takes 4 m to sew one gown, and 3 m to sew one pajama. How many gowns and pajamas should be made to obtain the greatest profit from the sale of products, if the gown costs 7 rubles, and pajama 6 rubles? It is known that at least 14 gowns are required to be made.

### Geometric Problems

**Problem 1.** It is necessary to fence a rectangular section with a fence of 200 m long. What are the dimensions of this rectangle so that its area is the largest?

**Problem 2.** A rectangular area adjoining the building wall was set up for the parking lot. The site was fenced on three sides with 200 m metal gauze, and its area was at the same time the largest. What are the dimensions of the site?

**Task 3.** The window has the form of a rectangle, completed in a semicircle. The perimeter of the figure is 6 m. What are the dimensions of the window so that it passes the greatest amount of light?

**Problem 4.** It is necessary to cut out a beam of a rectangular section from a round log with thickness  $d$  sm. The strength of the beam is proportional to  $ab^2$  ( $a$ ,  $b$  are the beam cross section in sm). At what values of  $a$  and  $b$  will the strength of the beam be the greatest?

**Problem 5.** Three cities  $A$ ,  $B$ ,  $C$  are not on one line, and  $\angle ABC = 60^\circ$ . At the same time, a car leaves town  $A$  and a train leaves from point  $B$ . The car moves towards point  $B$  at a speed of 80 km/h, the train - to city  $C$  at a speed of 50 km/h. At what point in time (from the beginning of the movement) is the distance between the train and the car the smallest, if  $AB = 200$  km?

**Problem 6.** There is building material from which a 12 m long fence can be built. We want to fence a rectangular site of the largest area adjoining the house. What are its dimensions?

### Physical Problems

**Problem 1.** On the training range an anti-aircraft gun fires a shot in a vertical direction with an unburstable packet. It is required to determine the maximum height of the packet, if its initial velocity is  $v_0 = 300$  m/s. Resistance to air is neglected.

**Problem 2. On the optimal transportation (the fastest arrival to the destination).** A brigade of workers in the field has to evacuate defective equipment as soon as possible to the city. You can get into the city in three ways:

- 1) along the shortest path from point  $B$  (brigade) to point  $C$  (city), with the evacuator speed equal to the speed of movement along the field,  $V_{\text{field}} = 25$  km/h;
- 2) first, along the shortest path to the road from point  $B$  to point  $R$  (road), with a speed  $V_{\text{field}} = 25$  km/h, then along the road from point  $R$  to point  $C$  with speed  $V_{\text{road}} = 80$  km/h;
- 3) first to a certain point  $E$  on the road, with a speed  $V_{\text{field}}$ , and then along the road to the point  $C$  with the speed  $V_{\text{road}}$ .

Find the optimal way in which the time taken to deliver the defective equipment will be the least.

### Research Projects

**Project 1. "Credit purchase"**. It is necessary to investigate the possibility of making a purchase, for which there is no money. What is more profitable - to earn and save, keeping money in the "bank", earn and save by opening an account with a savings bank; make a purchase in credit, which is to be paid back from the money earned? What types of loans are more profitable?

**Project 2. "Repairs"**. The Ivanovs family decided to repair the floors in their apartment, it was also decided that their expenses for floor repairs are not to be over 50,000 rubles. Using the proposed sources, make the necessary calculations, draw a conclusion and give practical recommendations to the Ivanovs family, supported by mathematical calculations and explanations why this recommendation should be used.

**Project 3. "Bus routes"**. Our city is divided into two parts by the river. As in many big cities, there are road jams at the entrances to the bridges every morning and evening. One way to solve the problem is to introduce high-speed bus routes along bus only lanes. However, for such routes to be popular, their laying requires serious assessments of the volume of passenger traffic. It is required to offer some of the most promising routes that could unclough the transport system of this city, indicate their length and the optimal interval of bus traffic.

Suppose that the fuel costs, the driver's salary and the bus wear are respectively  $a$ ,  $b$  and  $c$  rubles/km. (These data can be found from the public specifications of popular models used in public transport.) If the length of the route is  $l$  km and the total number of runs during the peak hour is  $K$ , then the cost of the route will be  $X = K \cdot l \cdot (a + b + c)$ . The number of runs can be estimated through the interval  $m$  as follows:  $K = 120 / m$ .

Let's now estimate the revenues from the route. With the number of passengers equal to  $n$  and the fare value equal to  $q$  rubles, one bus will bring  $\beta = n \cdot q$  rubles. The proportion of  $\gamma$  passengers that take advantage of the route can be estimated as follows: let the traffic interval of existing routes be equal to  $M$ , and fast route -  $m$ . Then we assume that  $\gamma = (M-m) / M$ . If the total number of passengers is  $P$ , then the collection from the route for the peak hour is  $Y = \gamma \cdot q \cdot P$ .

To find the optimal parameters, we get the problem of maximizing the function  $U = Y - X$ . It is possible to introduce a dependence on several parameters and find the minimum of the corresponding function using the

program written by students. An example of the dependence on one of the variables - interval  $m$  - can be demonstrated by plotting a graph.

Based on the analysis of the results of the written program, the students offer the required routes, their length and the optimal interval of bus traffic.

Carrying out such projects requires students to know mathematics within the course of a general education school, to master universal computer technologies, to be able to program in the course of informatics course of 8-9 classes, as well as the ability to search and use the necessary reference literature.

### Teaching Technology to Solve Practice-Oriented Optimization Problems

Teaching to solve the problems under consideration occurs in stages, and the problem can be the same, but on different stages it will be offered in different ways. Let's show this on examples.

I stage: forming the ability to solve problems on the algorithmic level and the ability to formulate applied problems on the operational level. The level of mathematical associations is local. Supporting mental actions and educational and cognitive skills: analysis, synthesis, comparison, specification, and systematization.

*Task 1: It is necessary to lay a water pipe from house A to house B, located on different sides of the asphalt road. The cost of laying the pipe under the asphalt road is 6000 rubles, and in any other place - 3600 rubles per meter. How is it more economical to lay a pipe? (All the necessary data are reflected in the figure proposed by the teacher).*

1. Solve the problem by the following algorithm:
2. Answer the question: "How else can the pipe be laid?"
3. Determine the cost of laying the pipe in each of the proposed options.
4. Formulate the problem of optimization.
5. Compose a function that reflects the cost of laying the pipe.
6. Examine the function obtained for an extremum.
7. Compare the minimum costs with the costs obtained when performing step 3, draw a conclusion.

II stage: forming the ability to solve problems on a heuristic level and the ability to formulate these problems on a technological level. The level of mathematical associations is intrasubject. Supporting mental actions and educational and cognitive skills: analysis, synthesis, comparison, concretization, generalization, systematization, development, deepening, forecasting, choice of rational way (mode) of the activity.

*Task 2: It is necessary to lay a water pipe from house A to house B (see the figure suggested by the teacher). The cost of laying the pipe under the asphalt road is 6000 rubles, and in any other place is 3600 rubles per meter (the unit segment is 10 m). How is it more economical to lay the pipe?*

1. Solve the problem by having determined preliminary the data necessary to solve the problem.
2. How else can the pipe be laid?
3. Formulate a practical problem for this case.

III stage: forming the ability to solve applied and practical problems on the creative level and the ability to formulate applied problems at a generalized level. Supporting mental actions and educational and cognitive skills: analysis, synthesis, comparison, concretization, generalization, development, deepening, expansion, systematization, forecasting, the choice of rational way of the activity, the ability to put forward hypotheses, the ability to make value judgments.

*Task 3: It is necessary to lay a water pipe from house A to house B (see the figure suggested by the teacher). 341,000 rubles are granted for the pipe laying. Is this enough?*

1. Solve the problem.
2. Formulate the inverse problem and solve it.
3. Formulate a practical conclusion.

From the problems considered, it is seen that the problem can be the same, but on different stages it will be proposed in different ways.

The teaching technology to solve practice-oriented problems is aimed at moving schoolchildren from the algorithmic level of the solution and the operational level of formulation to the heuristic level of the solution and the technological level of formulation and further to the creative level of the solution and the general level of formulation of practice-oriented tasks.

**Table 3.** The results of questioning of technical directions educators of Vyatka State University from the point of view of the assessment of first-year students' knowledge in mathematics in percentage

Year	Knowledge level (%)					
	Very low	Low	Below the average	Average	Above average	High
2016	7	15	25	30	12	11
2017	12	19	34	24	4	7

### Specificity of the Solution of Practice-Oriented Optimization Problems

- 1) Choosing a model is an important question that requires a lot of time. If an ineffective model is chosen, then this is a waste of time and disappointment in optimization methods. The basic requirements that a model must meet:
  - there must be at least two variants of parameter values that satisfy constraints and boundary conditions, because if there are no solutions, then there is nothing to choose from;
  - we must clearly know in what sense the desired solution should be the best, otherwise neither mathematical methods nor PC will help. The choice of the model is completed by its conceptual statement.
- 2) Conceptual setting. The elements of the mathematical model should be formulated clearly:
  - initial data (deterministic and random);
  - required variables (continuous and discrete);
  - limits in which the values of the unknown quantities can lie in the optimal solution;
  - dependence between variables (linear or non-linear);
  - criteria by which to find the optimal solution.
- 3) In a practice-oriented optimization problem, unknown data and conditions form a more compound complex and are less clearly defined. In addition, there should be presented data relevant to the condition (and accessible), and it is also possible to have background data and conditions that can be neglected, which will simplify the solution.
- 4) Collection of input data is a necessary stage of work when searching for the optimal solution. It is advisable to begin solving problems of large dimension with a control example. This will require collecting a small amount of initial data at the initial stage of the work in order to evaluate quickly the correctness of the model.
- 5) In a practice-oriented optimization problem, concepts and knowledge necessary to solve it have an intersubjective integrative character and are less clearly defined.
- 6) Analysis of the solution is the most important tool for making optimal decisions.
- 7) Taking the optimal solution is the final stage of the work. The decision is made not by the computer, but by the person who should be responsible for the results of the decision.
- 8) Graphical presentation of the solution result and analysis is a powerful factor in information visibility necessary for taking a decision.

Thus, on the one hand, practice-oriented optimization problems are the goal of education, on the other hand they serve as a means of implementing the applied orientation of the school mathematics course. And in the second case they are one of the important means of forming mathematical knowledge.

### Results of the experimental part of the study

To test the effectiveness of the developed methodology, we have conducted an experiment during the practical course of mathematics.

The following results were obtained as a result of the questionnaire survey conducted by the educators of the Vyatka State University and the Kazan Federal University for two years, in which they were asked to assess the knowledge of their students in school mathematics (Table 3).

Diagnostic results show that students come to the university with a rather weak level in mathematics and large gaps in knowledge for the course of the basic school.

In addition, the results of diagnosing the level of motivation to study mathematics show that the motivation of students is mainly external, the level of internal motivation is medium or below average. Namely, 37.5% of schoolchildren are dominated by internal motivation, while 62.5% of schoolchildren are dominated by external motives. That is, more than 60% of schoolchildren do not have sufficient internal motivation, which is the main

**Table 4.** Results of the experiment

	Average score	Quality of knowledge, (%)	Level of training, (%)	Degree of training, (%)
Traditional training	3,2	38,4	69	41
Training with using practice-oriented tasks	3,8	50	94	63
<b>Indicator changes</b>	<b>+0,6</b>	<b>+11,6</b>	<b>+25</b>	<b>+22</b>

force pushing a person to active cognitive activity. It means that most students are not aimed at gaining knowledge because they are interested in the subject, but because there is an external stimulus.

In the opinion of teachers, the reason for the reduced internal motivation is often the unjustified convergence of tasks, the functions of science and the academic subject. Lessons at school rarely discuss practical problems and analyze situations from everyday life, as a result of which the learning process becomes unnecessarily complicated and detached from the real life of the learner, which leads to a loss of interest in learning the subject, and as a consequence, the quality of knowledge decreases.

To conduct the experiment, traditional and practical-oriented tasks on the topics “Actions with fractions”, “Proportion”, “Interest” were selected. At each of the training sessions, verification work was carried out. The results of the experiment are presented in [Table 4](#).

An analysis of the level of motivation to study mathematics shows that the introduction of the described methodology into the practice of teaching allowed to increase the level of motivation of schoolchildren to study this subject - the level of external motivation (52.2%) became higher than the level of internal motivation (47,8%).

Thus, the experiment has showed that teaching mathematics using practice-oriented tasks is more effective than traditional training, which is confirmed by the growth of each academic indicator.

## DISCUSSIONS

At present, the applied nature of teaching mathematics acquires social significance. Strengthening the applied orientation of education is possible through introducing new teaching technologies based on a problem approach (Tumaikina, 2000).

The research defined the components of mathematical content: a logical-forming unit (a set of knowledge of formal logic and control-assessment skills), a language block (the mathematical language of describing objects) and a narrow-subject block (the general educational core of subject knowledge) and an emotional-valuable block (personally significant interdisciplinary, historical-mathematical and similar knowledge). As a result of the experiment it was established that a positive effect for strengthening the applied orientation of mathematics can be achieved on the basis of taking into account all the components of the methodology developed by us for its use.

In the course of the research, the following results were achieved: there were determined the principles of implementing the applied orientation of the school mathematics course, as well as the functions and methods of teaching to solve practice-oriented optimization problems as a means of providing applied orientation of school mathematics.

The study has shown that practice-oriented optimization problems have a great didactic potential in the process of teaching mathematics and allow to demonstrate clearly the relationship of mathematics with the real life. In addition, they contribute greatly to the formation of pupils’ skills to observe, generalize, and reason by analogy, that is, they help to achieve a high level of development of students’ thinking and speech so that they can further expand and deepen their knowledge independently, apply it in other areas, and find practical solving problems in different new situations.

An important component of the technology of teaching students to solve practice-oriented problems is composing and formulating the condition of the problem, since the formation of these skills makes it possible to determine the degree of students’ readiness to set the tasks of a professional and life plan independently. The solution of various practice-oriented optimization problems allows not only to find the correct solution of a specific problem, but also to apply it to a number of similar tasks from the real practical human activity.

The given examples of problems and a technique of work with them show that students’ mastering the method of solving problems occurs on certain levels. On the first level there is an algorithmic solution, on the second level the required and given values are to be obtained independently, on the third level subject knowledge is necessary since the problem contains a lot of unknown data.

The technology of teaching students to solve practice-oriented optimization problems is aimed at moving schoolchildren from the algorithmic level of the solution and the operational level of formulation to the heuristic

level of the solution and the technological level of formulation and further to the creative level of the solution and the general level of formulation of practice-oriented tasks.

## CONCLUSION

The results of the analysis of the main theoretical provisions of the existing educational technologies in the context of the development of practice-oriented skills has shown that their active interpenetration and introduction into the educational process make it possible to increase students' motivation for studying mathematics and natural science disciplines. In addition, it contributes to the formation of schoolchildren's key competencies which they will use in future professional activities. Training with using practice-oriented problems also leads to a more solid uptaking of information, since associations with specific actions and events arise.

Using optimization problems in mathematics studies is justified by the fact that with sufficient completeness they lay the understanding of how a person seeks, constantly achieves the solution of life problems, so that the results of his activities are as good as possible. Solving the problems of this type, students see, on the one hand, the abstract nature of mathematical concepts, and on the other, their great effective applicability to the solution of vital practical problems. These tasks help to get acquainted with some ideas and applied methods of the school mathematics course, which are often used in labor activity, in learning the surrounding reality.

The significance of the study is that its results are brought to the level of practical application, namely, the principles for ensuring the applied orientation of the school mathematics course have been formulated; practice-oriented optimization problems have been chosen as a means of implementing the applied orientation; the functions and stages of implementing their learning solutions have been considered.

The developed didactic material and suggested didactic technologies to teach schoolchildren to solve practice-oriented optimization mathematical problems make it possible to realize their great didactic potential for preparing students for future social and professional life.

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