



The Effect of Preschool Mathematics Education in Development of Geometry Concepts in Children

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Received 19 May 2017 • Revised 15 July 2017 • Accepted 24 August 2017

ABSTRACT

Given the fact that early mathematics education is important for further learning of mathematics, authors of this paper examine the effects that preschool mathematics education on development of geometry concepts in children. On a sample of 290 children, through a one-on-one interview, we studied the development of geometry concepts in children at the end of preschool education in Serbia. Research results show that the preschool mathematics education achieves significant effects developing geometry concepts in children. Preschool mathematics education does not achieve equal effects on the development of all geometry concepts covered by the curriculum of preschool education, that there are no differences in the development of these concepts depending on the location of the preparatory preschool institution and the gender of the child, but that the education level of the child's parents has great impact on the development of these concepts.

Keywords: geometry concepts, mathematics, preschool mathematics education, preschool child

INTRODUCTION

In recent years, there has been a number of initiatives that emphasize the importance of preschool mathematics education. One initiative is especially important in this field, and that is the initiative from 2000 in which the *National Council of Teachers of Mathematics* (NCTM), while revising the standard for primary and secondary education from 1989, included the standard for preschool education for the first time. This initiative generated the common position of the *National Association for the Education of Young Children* (NAEYC) and the *National Council of Teachers of Mathematics* (NCTM) that "high quality, challenging, and accessible mathematics education for 3- to 6-year old children is a vital foundation for future mathematics learning" (2002: 1). The importance of these initiatives is confirmed by numerous documents and papers that promote preschool mathematics education and indicate that it should be more intensive and organized in this age (OECD, 2001; Penn, 2002; Dahlberg & Moss, 2005), because "the foundation for children's mathematical development is established in the early years" (Seefeldt & Wasik, 2006: 249) and basis for the later success in mathematics (Bodovski & Farkas, 2007; Denton & West, 2002; Jordan et al., 2009).

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State of the literature

- The foundation for children's mathematical development is established in the early years and basis for the later success in mathematics.
- Geometry teaching is more important in preschool period, when first critical geometrical observations are made, when instincts are developed, and concept and information are acquired, than in the other periods following it.
- Children form concepts of shapes even before they start school and that geometric concepts in children, although limited, are stabilized by the age of six.

Contribution of this paper to the literature

- The aim of this paper is to determine the role of preschool mathematics education in the development of geometric concepts in children.
- The research has shown that preschool mathematics education achieves significant effects developing geometry concepts in children, but does not achieve equal effects on the development of all geometry concepts.
- The research has shown that children have more success in identifying and naming geometric shapes on objects in their environment, than when they observe these same shapes as geometric images devoid of materiality.

When it comes to preschool mathematics education, two areas of mathematics have proven essential for this kind of education "1) numbers, operations and relations among them and 2) geometry, spatial thinking and measurement" (Cross, Woods, & Schweingruber, 2009: 2).

BACKGROUND OF THE STUDY

Various authors point out the importance of geometry in children's development. Dutch mathematician H. Freudenthal highlights: „Geometry is grasping space. And since it is about the education of children, it is grasping that space in which the child lives, breathes, and moves”(1973: 403). Results of some studies show that spatial thinking is an important prerequisite of mathematics achievement, and that it has impact on verbal and mathematics skills (Clements & Sarama, 2007; Stewart, Leeson, & Wright, 1997; Wheatley, 1990). The importance of geometric contents is also being contemplated in the context of the development of logical and mathematical thinking (Ćebić, 2006: 15; Prentović, 1998: 271; Dobrić, 1979: 121). Recognizing this fact, N. M. Beskin points out that no other school subject puts logical and mathematical thinking to the fore as much as geometry does, or, “that no other school subject possesses such possibilities for developing logical thinking as geometry does” (1948: 19).

Geometry teaching is more important in preschool period, when first critical geometrical observations are made, when instincts are developed, and concept and information are acquired, than in the other periods following it (Develi & Orbay, 2003; Kesicioğlu, 2013: 48). Despite the importance of geometric contents, and the fact that geometric objects in the form of models exist in the child's surroundings as different objects and phenomena, that the child is in the situation to manipulate their models on daily basis, observe them, and use them in games, geometric concepts are abstract and complex to adopt. For these reasons, planning and organizing activities aimed at children's adoption of geometric concepts is a challenging task for teachers. To achieve this task, one must be acquainted with psychological, pedagogical and methodological foundations of the development of these concepts. On the other hand “despite the growing attention given to the geometry skills in early grades, still numbers and operations are the first content areas to which children are usually introduced” (Yeşil Dağlı & Halat, 2016: 190).

Development of geometric concepts, in the opinion of J. Piaget and B. Inhelder (1967), occurs in developmental sequences, in which children first start to distinguish between objects based on the topological characteristics, and only later based on Euclid's axioms. A child of this age can adopt facts, recognize names and name shapes, but he/she is not capable of comprehending relations and connections between shapes, nor is he/she capable of deeper understanding of the very essence of geometric concepts. Also, the child's reasoning is global, undifferentiated, so the child identifies geometric shapes with objects of the same properties. A methodological

procedure for developing geometric concepts in preschool children is based on these findings, according to which children “are first introduced to 3D shapes, and then to geometric figures as parts of these 3D shapes” (Dejić, 2003: 84). This stage in the development of geometric concepts is the opposite to the system of Euclidian geometry, that is, axiomatic geometry where one starts from basic concepts and axioms, and it is also determined by the fact that children of this age are not capable of thinking and reasoning at the formal deductive level.

Developing geometrical reasoning, progressing from visual to descriptive and analytical reasoning may go hand in hand with developing the ability to form well-defined concepts in other domains as well (Levenson et al., 2011: vii). According to Van Hiele (1986), geometric concepts are developed through hierarchically organized levels. The first level of the Van Hiele’s theory is identified as the visual level. Main characteristic of this level is that a child gets acquainted with shapes through practical activities, observes them as a whole and does not make a difference between the parts that make up the whole. A child at this level is able to name the shape, but he/she cannot give any explanation for such naming, which is why this level marks the level of nonverbal thinking. At the second level, which Van Hiele labels as the descriptive level, the child is able to identify and describe relations and properties of certain shapes, but that knowledge is not logically organized, because children cannot recognize essential links between objects. At the third level, which is marked as the level of informal deduction, children express observed relations through words and begin to use mathematical language. Van Hiele (1986) points out that preschool children are not able to reach the level of formal logic and the level on which they could comprehend the laws of logic.

A successful development of geometric concepts in children indicates the regard for the level of development of the child’s mental structures and finding the most efficient way for their development. According to Van Hiele’s theory, geometrical reasoning levels can be improved if individual is supported by education (Van Hiele, 1986). The research reviewed to this point suggests that development of geometric knowledge is fueled by experience and education, not just maturity (Cross et al., 2009: 192). Also, we have to consider the fact that no geometric concept can be fully formed at this age, and that it cannot be built through a single activity. The process of developing these concepts should be integrated into overall development of the child, and into acquisition of knowledge in other subject areas. The most important thing in developing geometric concepts is to put the child in the position to observe objects of different shapes as often as possible, because only on the basis of sensory experience can the child transform perceptions into concepts, and ultimately express those concepts through words and images.

The results of research studies show that children form concepts of shapes even before they start school (Clements, 2001; Clements & Sarama, 2000a), and that geometric concepts in children, although limited, are stabilized by the age of six (Clements & Sarama, 2000b). Many mathematics education studies “have focused on number sense and counting skills of children in the elementary grades at the youngest and been conducted in western countries, leaving a gap about younger children’s conceptual understanding of geometric shapes in different cultural settings” (Yeşil Dağlı & Halat, 2016: 190).

For these reasons, we have chosen to examine the development of geometric concepts in children in our study, and thus to determine the effects of preschool mathematics education on their development. In addition to the importance of developing geometric concepts for the overall mental development of the child, development of thinking, abstraction and reasoning, their significance is also reflected later in the process of education, because established concepts represent a basis for further learning. The main objective of this research was to determine the development of geometric concepts in children at the beginning of their education, in order to comprehend the role of preschool mathematics education in that process.

Educational Context

Preschool education in Serbia is organized within preschool institutions, with children aged 3 to 6.5 and is not compulsory for all children. Namely, the General Bases of the Preschool Curriculum in the Republic of Serbia are divided into three units: 1) *Bases of the Curriculum Aimed At Children Aged 6 Months to 3 Years*, 2) *Bases of the*

Curriculum Aimed At Children from the Age of 3 to Their Introduction into the School Preparatory Program and 3) *Preparatory Preschool Program (Guidelines on the Basis of Preschool Curriculum, 2006: 1)*. Mathematics education becomes an integral part of activities with children in the *Curriculum Aimed at Children from the Age of 3 to Their Introduction into the School Preparatory Program* and lasts three years, but this program is not compulsory for all children. The law states that all children have to attend the *Preparatory Preschool Program* which lasts one year (*Law on Primary Education, 2014*). This program is organized in preschool institutions for all children who previously attended the *Curriculum Aimed at Children from the Age of 3 to Their Introduction into the School Preparatory Program*, whereas the program is organized within special classes in primary schools for children who didn't attend it. Both groups of children are in charge of preschool teachers who follow a unique program prescribed by the government (*Guidelines on the Basis of Preschool Curriculum, 2006*). Program content comprises the following thematic units: Observing and understanding the concept of space and spatial relations; Forming the concept of a set; Forming the concept of numbers (counting, forming the concepts of numbers up to 10, number sequence, position of the number in a number sequence); Developing the concept of plane shapes (circle, square, triangle, rectangle) and solid shapes (ball, cube, cuboid, cylinder); Measurements and units of measure (length, mass, volume, time) (*Guidelines on the Basis of Preschool Curriculum, 2006: 127*). Children who attend the preschool education program in preschool institutions from the age of three are introduced to and acquire logical and mathematical knowledge and adopt the aforementioned mathematical contents by age groups. Those who attend the *Preparatory Preschool Program* within special classes in primary schools were not able to acquire this knowledge in an organized way before, so they begin to adopt it now. For these reasons, we wanted to determine whether there are differences in the development level of geometry concepts with regard to the place where children attended and completed the preschool mathematics education program, because children who attended preschool institutions had the opportunity to adopt these contents during a three-year period, whereas children who started the program one year before primary school had significantly less time to do the same.

RESEARCH METHODOLOGY

Participants

The study sample was chosen from the population of preschool children from the Republic of Serbia and it consisted of 290 children (boys 141, girls 149, $M_{age} = 5.6$ years, age range: five to six years). The study sample consisted of children from socially homogenous middle-class population. The structure of the sample is presented in **Table 1**. Of the total number of children, 198 mastered the preschool curriculum in preparatory preschool institutions, and 92 children were encompassed by preschool mathematics education only for a year, as parts of special groups within their primary schools.

Table 1. Structure sample

Children	Gender		Place of preparatory preschool program		Fathers' level of education		Mother' level of education	
	Boys	Girls	Preschool institution	Primary school	primary/secondary school	college/university	primary/secondary school	college/university
f	141	149	198	92	204	86	195	95
%	48.6	51.4	68.3	31.7	70.3	29.7	67.2	32.8
Total	290		290		290		290	

Independent variable was represented by the following characteristics: the place where the preparatory preschool program was attended (a preschool institution, a school), the child's gender (boys, girls) and the parents' level of education (primary/secondary school, college/university degree).

Data Collection

The data necessary for the research were collected through oral examination of children. Each child has been tested individually. The test comprised three sections with four questions in each. In the first section, children were shown pictures of solid geometric shapes: cube, rectangular prism, ball and cylinder stripped of all materiality, and the children's task was to correctly recognize and name each shape (Figure 1). In the second section, children were shown pictures of plane shapes: square, circle, triangle and rectangle, with the same task (Figure 2). The third section involved real objects shaped like a cube, rectangular prism, ball and cylinder, and the children's task was to name the shape of the object (Figure 3). Testing was conducted by the authors of this paper in kindergarten classrooms and in the presence of their preschool teachers. Each child from the sample has accepted to participate in the research, and we also obtained the consent of their parents. All children are from a middle-class background and have been chosen according to the principle of random sampling. Children's responses are recorded on a record sheet containing information on each child (place where they attended the preparatory preschool program, gender, education level of the parents). Children's responses were expressed in the form of points. Each correct answer was worth 5 points. Since each of the three sections contained four questions, the maximum number of points a child could win was 60. We obtained data on the education level of parents from the class register.

A pilot study was conducted with 50 children. The pilot study revealed that questions were understandable. Two mathematics teachers graded the questions in this study independently, in order to provide their scoring reliability. The Cronbach's alpha coefficients calculated for each task ranged between .76 and .91. Discrimination of the test was determined by item analysis. The coefficient of discriminative value of task in the test varies from .14 to .25, which shows that the tasks were discriminative enough, because the coefficient is greater than .12 (Bandur et al., 2008: 137).

Data Analyses

The data obtained in the research were processed with the statistics software package IBM SPSS20. In the first part of the paper, data are expressed quantitatively in the form of frequencies and percents, to get a general insight into the number of children who possess a "developed" (are able to recognize and name a solid shape or plane shape) or "undeveloped" geometric concept (are unable to recognize or name a given solid or plane shape). The influence of independent variables on children's success on the test and the examination of differences among children with regard to their gender, place where they attended the preparatory preschool program and education level of their parents were tested with a single factor analysis of variance (ANOVA) for independent samples.

RESEARCH RESULTS AND DISCUSSION

The curriculum of preschool mathematics education anticipates that children should develop the concepts of geometric plane figures and geometric space figures. We determined the development of geometric concepts in children by examining their ability to:

- recognize and name geometric figures in space,
- recognize and name geometric figures in a plane and
- recognize and name geometric shapes on the objects in their surroundings.

First, we wanted to determine if the children had developed concepts of geometric space figures on a purely visual level, according to Van Hiele, and starting from the attitude of Piaget and Inhelder (1967) that the child can recognize and name figures, but not identify connections and relations between figures. Children were shown the following geometric figures in space: cube, rectangular prism, ball and cylinder (Figure 1).

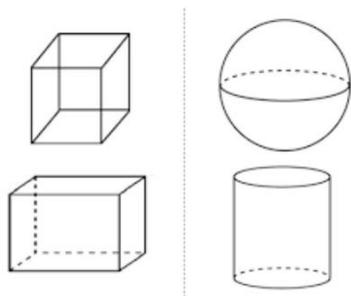


Figure 1. Task from the test

As it can be seen from Table 2, most children can recognize and name the cube (240 or 82.8%) and the sphere (219 or 75.5%). Then we have the cylinder (152 or 52.4%), and the smallest number of children (48 or 16.6%) have a developed concept of the rectangular prism.

Table 2. Development of the concept of geometric figures in space in preschool children

Geometric figures	Developed		Undeveloped		Total
	f	%	f	%	
Cube	240	82.8	50	17.2	290
Rectangular prism	48	16.6	242	83.4	290
Cylinder	152	52.4	138	47.6	290
Sphere	219	75.5	71	24.5	290

The result, according to which children’s most developed concepts at the beginning of schooling are those of the cube and the sphere, was to be expected. F. Fröbel emphasized that the most familiar shapes for a child of preschool age were the sphere and the cube, which, according to him, represented “mathematically pure forms with distinct contradictions” (according to: Prentović, 1998: 277–278). Also, geometry, in the opinion of Van Hiele “begins with play” (1999: 310), and children in the earliest period of their lives usually play with objects in the form of a sphere and a cube.

The next task was to examine children’s ability to recognize and name geometric figures in a plane (square, triangle, circle and rectangle). Children were shown images of a square, triangle, circle and rectangle, then asked to name them (Figure 2).



Figure 2. Task from the test

Research results show that most children from the study sample (281 or 96.9%) have a developed concept of the circle (they recognize and can correctly name the circle). Slightly lesser number of children (226 or 77.9%) has a developed concept of the triangle. The rectangle was recognized and correctly identified by 176 or 60.7% of children, and the smallest number of children (166 or 57.2%) has a developed concept of the square (see Table 3).

Table 3. The development of the concept of geometric figures in a plane in preschool children

Geometric figures	Developed		Undeveloped		Total
	f	%	f	%	
Square	166	57.2	124	42.8	290
Circle	281	96.9	9	3.1	290
Triangle	226	77.9	64	22.1	290
Rectangle	176	60.7	114	39.3	290

Other authors obtained similar results while studying these concepts. Twenty-five years ago, K.C. Fuson, and C. Murray (1978) determined that 85% of children aged five could recognize the circle, 80% the triangle, and 78% the square. In addition, these authors state that as much as 60% of children aged three and older can recognize and correctly identify the circle, the square and the triangle. Researching this problem, D.H. Clements and his associates present their conclusions, according to which 90% of children aged three years and five months to four years and four months can successfully recognize and identify the circle, whereas the same shape is successfully recognized and identified by 99% of children aged six (Clements et al., 1999: 200–201). Other authors came to the conclusion that children have misconceptions, especially about the rectangle and triangle shapes, and that they have the least misconception about the circle shape (Clements & Sarama, 2000a; Kesicioğlu, Alisinanoğlu & Tuncer, 2011). Studies of Clements and his associates show that 96% of the children could fully recognize circles. Although the children consider equilateral quadrangle as a square, 87 % could fully recognize squares. It is highlighted that little children are less successful in recognizing triangles (60%), this level is even lower for rectangles (54%) and children also have a tendency to define parallel edges as a rectangle (according to: Kesicioğlu, 2013, p. 48–49).

The level of development of geometric concepts is further illustrated by the children’s ability to spot and name geometric concepts in their surroundings. Children were shown objects shaped like a ball, cube, cuboid and cylinder (Figure 3).

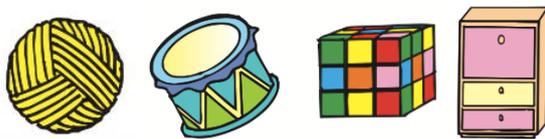


Figure 3. Task from the test

Most children recognize the shape of the cube (241 or 83.1%) and the sphere (240 or 82.8%) in objects in their surroundings (Table 4). Cylinder is recognized by only half of the children (147 or 50.7%), and the rectangular prism by only 52 or 17.9% of children.

Table 4. Development of the ability to spot and name geometry concepts on objects in the surroundings

Geometric figures	Developed		Undeveloped		Total
	f	%	f	%	
Sphere	240	82.8	50	17.2	290
Cube	241	83.1	49	16.9	290
Rectangular prism	52	17.9	238	82.1	290
Cylinder	147	50.7	143	49.3	290

If we compare these results with the results we obtained by testing the children’s ability to recognize and name geometric figures in space, in pictures devoid of any materiality, we can see they are almost identical. A slightly larger number of children recognized and correctly named the sphere on the objects of that shape, than in the picture which depicted a geometric image of the sphere.

If we view the results as a whole, children scored on average 37.71 points, out of the maximum 60, with a standard deviation 13.061 (Table 5) on the test for determining the development of geometric concepts.

Table 5. Development of geometry concepts in preschool children

N	Min	Max	Mean	Std. Error	Std. Deviation	Skewness	Kurtosis
290	5	60	37.71	.767	13.061	-.134	-.466

Therefore, preschool children scored on average slightly more than a half of the maximum number of points on the test which measures the development of geometric concepts. The concepts of the sphere, cube and circle are more developed than the concepts of the cylinder, rectangular prism, rectangle, triangle and square.

In **Figure 4**, we can see there is an approximately normal distribution of the number of students by the number of points scored on the test. The maximum 60 points was scored by 26 or 8.97% of children, and 15 or 5.17% scored 55 points. Most children from the sample (91 or 31.4%) scored between 35 and 40 points, whereas only six or 2.1% of children scored less than ten points. These results simultaneously tell us there is not a single child who does not have at least one developed geometric concept.

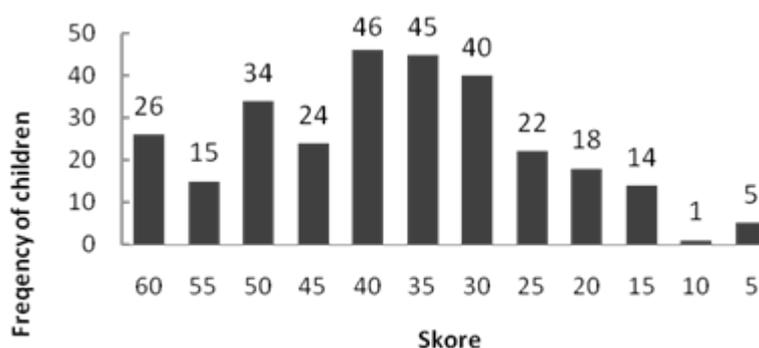


Figure 4. Frequency of children by the number of scored

As we can see, geometric concepts, viewed individually, are unevenly developed in preschool children. The reason for this lies in the very nature of geometric concepts, because some of them are intuitively more familiar to children, but a part of the responsibility must be placed on the organizers of preschool mathematics education.

We wanted to determine if there were any differences in the level of development of geometric concepts, dependent on the place where children attended the preparatory preschool program, primarily because of the fact that children who were educated in preschool institutions, had preschool mathematics education for four years, whereas children who were a part of special classes in primary schools, had it for only one year.

Research results show that children who attended the preparatory preschool program in a preschool institution scored on average 37.63 points on the test, which is approximately the same as children who attended the preparatory preschool program in special classes in primary schools, and scored on average 37.88 points (**Table 6**). By testing the statistical significance of the difference, we determined that it is not statistically significant ($F = .24$, $p = .878$).

Table 6. Development of geometry concepts in children, dependent on the place where they attended the preparatory preschool program

Place	N	M	SD	F	p
Preschool institution	92	37.88	13.056	.024	.878
Primary school	198	37.63	13.096		
Total	290	37.71	13.061		

We can conclude that the development of geometric concepts does not depend on the place where the preparatory preschool program was realized, and neither does it depend the duration of the program, i.e. whether children were a part of the preparatory preschool program since they were three, or just for one year before their primary education started. Our findings lead us to a conclusion that when it comes to the development of geometric concepts, same effects are achieved by preschool teachers who work with children for a year, as well as those who have a chance to work with children for four years in a preschool institution. These results stem from the fact that both groups of preschool teachers organize their work with children of approximately the same abilities, in approximately the same conditions, on the basis of the unified curriculum, which has clear objectives. Thus, children whose preschool education lasts longer, achieve the same results in developing geometric concepts as children who had preschool education for one year only. It should be also said that these results are an effect of preschool teachers' efforts to achieve designated objectives, which are defined and regulated in the *Guidelines on the Basis of Preschool Curriculum* (2006).

It is interesting to note that certain studies found that boys were more successful in spatial cognition (Levine et al., 1999), as well as in recognizing certain geometric shapes (rectangle and square) than girls, whereas there were no differences in recognizing the circle and the triangle (Kesicioğlu, 2013: 52). Explanation for these results, and the preschool boys' superiority in geometry are explained by the fact that boys spend more time playing with various models with which they construct and build space (Lego and other blocks), solve various spatial tasks, that they are often in the situation to copy and build three-dimensional models (Levine et al., 2005), compared to girls, who are more focused on social interaction. Therefore, we wanted to examine if there were any differences in the development of geometric concepts between boys and girls.

Boys scored on average 38.37 points on the test, and girls 37.08 points (Table 7). The difference between the obtained average results, which is 1.29 points in favor of the boys, is not statistically significant ($F = .704$, $p = .402$), which further implies that there are no significant differences in the development of geometry concepts between boys and girls.

Table 7. Development of geometry concepts by to the gender of children

Gender	N	M	SD	F	p
Boys	141	38.37	12.443	.704	.402
Girls	149	37.08	13.632		

Given the role of the child's family in this period of life, and the findings that "families can enhance the development of mathematical knowledge and skills as they set expectations and provide stimulating environments" (Cross, Woods, & Schweingruber, 2009: 343), we wanted to examine whether parents' level of education could influence the development of geometric concepts of children at the beginning of schooling (Table 8).

Table 8. Development of geometry concepts in preschool children compared to parents' level of education

Parent	Parents' level of education	N	M	SD	F	p
Father	primary/secondary school	204	36.50	12.936	6.025*	.015
	college/university	86	40.58	12.980		
Mother	primary/secondary school	195	36.56	12.992	4.614*	.033
	college/university	95	40.05	12.954		

* $p < .05$

In Table 8, we can see that children whose fathers have a college or a university degree were more successful on the tests of the development of geometric concepts, with an average 40.58 points, than children whose fathers had primary or secondary education only, who scored 36.5 points on average. By testing the statistical significance of the observed difference, which is 4.08 points in favor of children whose fathers have a higher level of education, we can say that it is statistically significant ($F = 6.025$, $p = .015$) at the level of .05. Thus, the father's level

of education has a significant impact on the development of geometric concepts of children at the beginning of schooling.

We get similar results when we compare the development of geometric concepts of children against the mother's level of education. Children whose mothers have a college or a university degree achieved better results (40.05 points), compared to children whose mothers only have primary or secondary education (36.56 points). Obtained variance between the two groups ($F=4.614$, $p = .033$) shows that the difference of 3.49 points in favor of children whose mothers have a higher level of education is at the level 0.05 of significance. Similar results were obtained while testing the level of development of the term natural number (Špijunović & Maričić, 2011: 507). However, the question is if this difference is a consequence of the greater ambition of parents with higher level of education, the fact that they have a greater mastery over mathematical contents, that their free time spent with their children is used for developing some geometric concepts, that their ambition concerning their children's education are greater, or some other reason. Either way, it is certain that family has an important role in children's mathematics education, because parents are the first teachers their children have.

CONCLUSION

Despite the fact that geometric concepts are materialized in reality, that they exist for real in the child's surroundings, that the child encounters them on daily basis during various activities, from studying to playing, they are still abstract, and complex to adopt. Studying the effects of preschool mathematics education on the development of geometric concepts in preschool children, we came to the following conclusions:

- when it comes to geometric figures in space, the most developed concepts are *the cube* (82,8%), *the sphere* (75,5%), whereas, only 16,6% of children can recognize and name *the rectangular prism*;
- when it comes to geometric figures in the plane, the most developed concepts are those of *the circle* (96,9%), then *the triangle* (77,8%), whereas, *the rectangle* is recognized and correctly named by 60,7% of children, and *the square* by 57,2% of children;
- children are more successful in recognizing and naming the geometric shapes of *the cube* and *the sphere*, than *the rectangular prism* and *the cylinder*;
- there are no significant differences in the development of geometric concepts between children who attended the preparatory preschool program in a preschool institution, and those who attended it in special classes within a primary school;
- there are no significant differences in the development of geometric concepts between boys and girls;
- parents' level of education has a significant impact on the development of geometric concepts in children.

Results obtained are consistent with the results from literature that refer to the development of geometry concepts and the fact that children of preschool age can form concepts of geometric shapes. We haven't been able to find similar research, organized with the same goal – to examine the effects of organized work with children in preschool mathematics education on the development of geometry concepts in similar or identical context.

The research has shown that preschool mathematics education achieves significant effects developing geometry concepts in children. However, we should bear in mind that preschool mathematics education does not achieve equal effects on the development of all geometry concepts covered by the curriculum of preschool education. Children have more knowledge about and more clear ideas on certain concepts (cube, sphere, circle), whereas some other concepts (cuboid, cylinder) are less developed. Also, the research has shown that children have more success in identifying and naming geometric shapes on objects in their environment, than when they observe these same shapes as geometric images devoid of materiality, which is a consequence of the child attachment to concreteness and his limited abilities for abstraction.

The results imply that the development of mathematical concepts should be placed in a realistic context familiar to the child, and that models should be taken from their surroundings. Special emphasis should be put on naming of the toys which represent models of solid geometric shapes, because many toys are associated with the cube, and some even have the word cube in their name (Lego, puzzle cubes, etc.), although they are not actually shaped like a cube, or even a cuboid.

In addition, the results show that children have more developed ideas about two-dimensional concepts, compared to three-dimensional concepts. This result is interesting, especially because of the fact that three-dimensional shapes are closer to children in the material sense, because children encounter them in everyday life through models, unlike geometric shape in a plane which are devoid of materiality. All this suggests that, despite the fact that geometry concepts are materialized in reality, that they really exist in the child's environment through their models, that the child encounters them daily in various activities, from games to learning, they are abstract and complex to adopt. Preschool teachers especially should keep this in mind, because they should plan the organization of activities the result of which should be the development of geometry concepts, integration of mathematics education with other forms of cognition, the need for further studying of this problem and improvement of preschool teachers' work in this field. In addition, the development of geometry concepts in preschool children should be organized through activities which require the child's maximum engagement, activities where learning is based on the experience of the child, in the context of real life and on sources of knowledge familiar to the child.

Given the importance of early mathematics education for later success in learning and the acquisition of mathematical contents, the results obtained in this research indicate the implications important for the practice. The research showed that preschool mathematics education can significantly contribute to the development of mathematical concepts in preschool children, but also that teachers who organize the work in this field should pay more attention to the development of certain mathematical concepts. We should bear in mind that boys and girls have equal abilities for developing geometry concepts, but also that the child's family has a significant role in preschool mathematics education. The result by which children from families with higher level of education have more developed geometry concepts shows that children's development of geometry concepts depends on adults and their environment in general, which further indicates that there are possibilities for improving work with children within preschool institutions. Activities aimed at developing geometry concepts should be undertaken on daily basis, using play and free activities, and should be integrated with all other areas of knowledge, instead of targeting this particular area exclusively.

The research also has certain limitations that can be used as guidelines for future research on the effects of preschool mathematics education on the development of mathematical concepts in children. The method of collecting data using a structured interview is also the cause of certain limitations. Observation of children would perhaps yield more reliable results. The results would be more complete if preschool teachers' opinions on the process of preschool mathematics education had been collected, and if we had examined their teaching methods.

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