



The effects of differentiation of content in problem-solving in learning geometry in secondary school

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The aim of the study was to examine the effects of problem-based learning which was established on differentiation of content at three levels of complexity in the processing of the content of Analytical geometry in the plane. In this context, an experimental research was conducted, on a sample of secondary school students (N = 165) in order to examine whether methodical approach designed on the principles of problem-based learning based on differentiation of content gives better effects in learning compared to the traditional mode. The results of the final measurement shown that the experimental group achieved better overall success than control group. The results suggest that the proposed methodical approach contributes to better student performance in teaching geometry and that the most significant progress is achieved in the group of students who are average in terms of success and with ones below the average.

Keywords: analytical geometry, differentiation content, geometry, mathematics, problem-based learning

INTRODUCTION

Teaching mathematics, nowadays, is characterized by an increasing focus on the acquisition of knowledge that is applied to solving problems in everyday life, students develop independence in learning, creating conditions that will allow „the desire and need to learn strategies for resolving and exploring different mathematical problems in the framework of problem situations” (Cotić & Felda, 2011: 163), training for „exploring, problem solving, creative thinking, data processing, logical reasoning and evaluation of results” (Felda & Cotić, 2012: 51). In order to succeed in that, “teachers must get a grasp and a grip on the knowledge

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society in which their pupils live and will work” (Hargreaves, 2003). This implies that teachers in a learning process need to be „teachers who understand learning as well as teaching who can address students’ needs as well as the demands of their disciplines, and who can create bridges between students’ experiences and curriculum goals” (Darling-Hammond, 2005). On the other hand, a lot of criticism is directed towards teaching that encourages memorizing, listening and repeating of what has already been learned (Paul, 1992), teaching that does not insist on developing problem-solving skills, which does not contribute to developing the ability to think.

Today there is general consensus that the process of learning in mathematics is seen as an active process of acquiring knowledge, „a process in which the role of the teacher is to help students in acquiring new and restructuring the old knowledge and so on, rather than as a process in which students passively adopt certain mathematical content and acquire readymade knowledge” (Maričić & Špijunović, 2015: 285). Numerous teaching strategies can contribute to creation of an environment in which a student in mathematics class can be an active constructor of knowledge, which will develop his ability to think and to reason, and also to acquire quality knowledge and apply it to solve problems in practice. In this study, we want to draw attention to the problem –based learning in the context of creating such an environment for learning in mathematics classes.

Background of the study

The basics of the problem-based learning can be found in American Project-Method, whose founders are J. Dewey, W. H. Killpatrick and E. Collings. At the beginning of the 20th century J. Dewey advocated for teaching that was based on the discussion and research work of the students drawn from realistic textual problems.

The problem-based learning ability enables the students to find appropriate solutions to problems that confront them (Hmelo-Silver, 2004: 258). The primary purpose of mathematical problem-solving instruction „is not to equip students with a collection of skills and processes, but rather to enable them to think for themselves” (Lester, 1985: 66). In this kind of teaching system, a research activity of the student is dominant, during the learning process which is conducted through problem-solving, and the teaching itself „supports mathematical learning” (Alemu, 2010: 50). Therefore, problem solving is the foundation of various mathematical activities (Reis, 2004). Teaching the student is based on the active problem solving and in accordance with constructive views upon studying. The student is in the center of learning. In the process of problem solving, students constantly connect gained knowledge with a new one that arises during the process of discovery, and connect existing experience with a new one, connect theory and practice, while learning through an active process of constructing knowledge and gaining new ones that have a greater transfer value in

State of the literature

- Problem-based learning puts the student into an active role through creation of problem-based situation which represents the natural context of learning.
- Differentiation of the content in problem-based learning, acknowledges the differences between students, therefore they can progress in accordance with their abilities.
- In the process of problem-solving, student connects gained knowledge with a new one that is created in the process of discovery, connects existing experience with a new one, connects theory and practice, and learns through an active process of constructing knowledge while acquiring knowledge that has a greater transfer value in further learning.

Contribution of this paper to the literature

- The aim of this paper is to experimentally check the value of problem-based learning which is based on the differentiation of content in learning geometry.
- Differentiating the learning content during problem-based learning takes into account the differences between students, so students can progress in accordance with their capabilities.
- Experimental verification of problem-solving learning contributes to improved achievement of students in mathematics class.

the future (Remillard & Kaye, 2002: 27). That way the student develops the ability to think, and above all, the ability to analyze, synthesize, generalize, to obtain the notions of abstraction, analogy as well as the ability to draw conclusions and to enable him to set and verify certain hypothesis. Critical and creative thinking of students is developed, especially the ability of solving problems. Even research shows that if students are actively involved and talk about what they are doing, they will preserve about 90% of the material that they have been learning (Ainsworth & Loizou, 2003: 679-681).

In addition to the cognitive dimension, learning within problem oriented teaching, according to the researchers, is developing more positive student attitudes, fosters deeper approach to learning and helps students to retain knowledge longer than traditional teaching (Achike & Nain, 2005; Peterson, 2004; Remillard & Kaye, 2002).

That is why problem-based learning „is one of the most important active learning/student centered approaches that promote students' problem solving abilities" (Alemu, 2010: 57). Of course, the whole process of establishing this type of teaching and participation of students in that process, involves a great deal of support of the teacher who plays the role of the instructor, leader and facilitator, someone who directs the process and if necessary – helps.

The practice of mathematical classes is usually not based on mentioned principles. Many students view mathematics as a „string of procedures to be memorized, where right answers count more than right thinking" (Mierson & Parikh, 2000:12-18). Teaching mathematics is based on the dominant activity of the teacher in which he sets-up problems but also solves them, and the students' task is to memorize the procedure of the teacher and repeats the process on similar tasks. That type of learning does not create quality knowledge nor does it develop the ability to solve problems which represents the basis of mathematical literacy. In that type of class „students often come to view problem solving as that of delving into a mysterious bag of tricks to which only a select few are privy" (Wilson et al., 2005: 93). The result is that students demonstrate an inability to consistently monitor their progress, and have varying degrees of success in recognizing that a solution attempt is not progressing toward the desired goal (Salman, 2005: 25-26).

According to NCTM principles and standards „mathematics teaching at university asks for constructivist-based instruction using problem-based learning method in which the students' own productions and constructions play a central role (2000: 31). That means that the students have to be active constructors of the base of their knowledge in the learning process at all levels of mathematical education (Remillard & Kaye, 2002), so they could successfully continue that process at the university. Learning within the framework of problem-based teaching ensures that the student will be a subject in the process of gaining knowledge, and not become a passive learner. That can be achieved if the student himself participates in determining the goal of his work and thereby „actively, creatively and naturally includes his intellectual, cognitive and mental strength in the process of learning" (Kadum, 2005).

The challenge of teaching mathematics from such a constructivist perspective is to create experiences that engage students and encourage them to discover new knowledge in mathematics education settings (Zan & Martino, 2007: 158-160). Besides that, students who learn to apply active learning approaches are also expected to acquire more useful and transferable knowledge (Remillard & Kaye, 2002:27). It is important to emphasize that „students' abilities to solve mathematical word problems not only rely on finding the right answer but also involve understanding and mastering more complex strategies such as the ability to plan, monitor, and evaluate" (Abdullah et all., 2014: 166).

The sole organization of the classes that should be based on the principle of problem-based learning is not a simple process, because it requires preparation of the

student and introduction in the working mode based on these principles. It is possible to notice next phases in the structure of the class:

1. Formulate a problem situation,
2. Defining the problem,
3. Set up the hypothesis,
4. Problem solving,
5. Analysis of results and conclusions.

Problem-based learning should be established on the initial problem-based situation which is connected to the previous knowledge of the student and from which a new knowledge should be constructed. Along with the problem-based situation, the goal is also clearly defined. The key issue is that the student has to find a way to achieve the goal. However, learning based on the principle of problem-solving strategy implies the existence of the fundamental knowledge and abilities of the student that will enable him to define the problem, set the hypothesis, solve the problem and test the hypothesis. The situation in practice regarding teaching mathematics is characterized by the existence of large individual differences among students, both in terms of knowledge they possess, and in terms of mathematical abilities, opportunities, interest to acquire and learn mathematical content. That is why we consider that better results in teaching mathematics would be achieved in problem-based learning which is differentiated according to the current level of knowledge, ability and possibility of the student to learn. In that manner, the teaching process is optimally adapted to individual differences of students (Ilić, 2002). The problem-based situation and requirements are differentiated according to students to match their level of capabilities and skills. This makes differentiation of teaching in problem-based learning reasonable, because if we give students the requirements that are higher than their current possibilities they will, in the view of Vygotsky, have real value, but will also awake curiosity, encourage them to think, search and research and lead to the development of their capabilities. Therefore, the problem-based task should be optimally reasonable, otherwise too difficult tasks do not motivate students for the appropriate activities, i.e. do not encourage students to learn.

The essence of differentiated teaching is based on the principle of respecting the differences among students and organizing teaching that follows the students' abilities and interests, and its main goal is the advancement of students according to their abilities. Differentiation, according to D. George, is the „process in which the objectives of the curriculum, teaching methods, assessment methods, resources and learning activities are planned to meet the needs of individual students“ (2003: 106). That way, the teaching becomes directed towards the student (Roeders, 2003). Our attention regarding the implementation of the problem-based learning which is based on the principle of three-level content differentiation is directed towards the geometry content in secondary schools, because the research shows that „most pupils think that geometry is very difficult to learn“ (Soedjadi, 1991; Kerans, 1994; Fauzan, 1998).

Using the concrete example, we will show the phases in the teaching activity that is based on stated principles (Figure 1).

After performing a segmental form of the equation of an ellipse, students along with the help of the teacher reach the equation of an ellipse c :

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \quad (1)$$

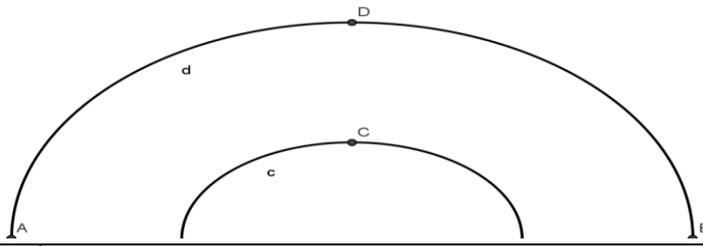
Differentiation of problem-based task at three levels	
<p>The road passes through a tunnel of semi elliptical shape, it is 4 m high, and 10 meters wide. To extend the road, engineers must design a tunnel that is 1,5 times wider and 5 meters higher than the original tunnel. What are the equations that describe the designs of both tunnels?</p> 	
Group A (below average)	What is the equation that describes the design of the second tunnel? Write the coordinates of the focus of the ellipse <i>c</i> and the ellipse <i>d</i> and write how many times the linear eccentricity of the ellipse <i>d</i> is greater than the linear eccentricity of the ellipse <i>c</i> ?
Group B (average)	What is the equation of an ellipse that describes the design of a larger tunnel? Can a truck with width of 4 meters and a height of 3.5 meters pass through a smaller tunnel? Explain the answer.
Group C (above average)	If the truck is 3,5 meters high, how many meters can the truck be wide (at most) so it could pass through a smaller tunnel? If the truck is 4,5 meters wide, how many meters can the truck be high (at most) so it could pass through a bigger tunnel?

Figure 1. The phases of in the teaching activity

Students from the group A should write the solution of their problem – based task on the blackboard. Equation of the ellipse that describes the design of a bigger tunnel is

$$\frac{x^2}{100} + \frac{y^2}{81} = 1 \tag{2}$$

and the length of the linear eccentricity of smaller and larger ellipse are respectively $e_1 = 3$ $e_2 = \sqrt{19}$. By dividing their length they conclude that the linear eccentricity of the ellipse *d* is 1.45 times larger than the linear eccentricity of the ellipse *c*.

Students of the group B interpret their own solution. When the value of width of the truck $a=4$ is inserted in the equation of the ellipse (1) they will get $\frac{16}{25} + \frac{y^2}{16} = 1$, from which the $y = 3.2$ m. As the height of the truck is $3.5\text{ m} > 3.2\text{ m}$, students conclude that the truck with the given dimensions cannot pass through the tunnel.

Students of the group C are explaining their answers. If the height of the truck is 3.5 m, by inserting that value into the equation (1), they will get $\frac{x^2}{25} + \frac{3.5^2}{16} = 1 \Rightarrow x \approx 2.42$. If we connect this task with the problem – based task from the group B, we can see that the truck from the task of the group B has to be 2.42 m wide at most, so it could pass through the smaller tunnel.

Analogously, assuming that the width of the truck is $x = 4.5$ m and incorporating that value into the equation (2) we come to the conclusion that the maximum height of the truck must not be greater than 8.04 meters.

In this study we wanted to exam whether the methodical approach that is established on problem-based teaching of mathematics differentiated at three levels, increases the efficiency of learning the specified content.

RESEARCH METHODOLOGY

Research sample (N = 165) was selected from the population of high school students in Bosnia and Herzegovina. The sample has the characteristics of a convenient sample, because the sample consists of the students with technical orientation; geometry content is highly present in their education and represents a significant basis in their future education. Students come from approximately uniform social backgrounds. Two groups of students have been formed: experimental (N=88, age 17.2 to 18.1) and control group (N=79, age 16.9 to 18.0). Students of the experimental group and control group were not the students from the same school. Balancing the experimental and control groups was not done in an artificial manner, by transferring students from one class to another due to the working conditions at school, but we used the statistical method of analysis of covariance (ANCOVA) in order to statistically control the dependent variable, because the "adjusted" variance corresponds to the variance that would be obtained in experimentally homogenous groups. The procedure of analysis of the covariance is based on getting the reduced calculation of the experimental error, taking into consideration the regression of the final measure (Y) in regards to the initial measure (H). Students in both groups belonged to the socially homogeneous middle social class.

The research is based on application of the experimental method – experiment with parallel groups. The base of the experimental program was the *Mathematics curriculum for the third grade of technical and similar schools* within the teaching unit Analytical geometry in the plane. The experimental program lasted for 16 teaching lessons which besides learning the new content, also included lessons of repeating and exercising. In the control group, teaching units were processed in the traditional manner. The experimental program was designed on the basis of problem-based learning, which was differentiated on three-level achievement for the following teaching content:

1. Condition of parallelism and verticality of two plains,
2. Equation of plain through one point,
3. Equation of plain through two points,
4. Equation of a circle,
5. The condition of a contact of plain and circle,
6. Equation of the tangent and normal of the circle,
7. Equation of the ellipse,
8. The condition of a contact of plain and ellipse,
9. Equation of the tangent and normal of the ellipse,
10. Equation of the hyperbole,
11. The condition of contact of plain and hyperbole. Tangent and normal of the hyperbole,
12. Equation of the parabola,
13. The condition of contact of plain and parabola. Tangent and normal of the parabola.

The experimental program was realized by the teachers of mathematics who work in these classes, based on the clear instructions and obtained activity contents that were based on the three-level complexity with problem-based approach to teaching. Based on the initial testing, students of the experimental group were divided into three groups according to their level of knowledge or achievement (A, B, C). Group A „below average“: consists of students whose success on the initial test was below 40%. Group B „average“: consists of students whose success on the initial test was above 40% and below 75%. Group C „above average“: consists of students whose success on the initial test was above 75%.

The implementation of the experimental program and the initial testing was preceded by a pilot study that was conducted on a sample of 49 students, based on which the experimental program was verified and the initial forms of the instruments were made.

Accordingly, in order to examine whether the methodical access has positive effects on learning and success of the students in geometry we constructed two tests:

- Initial test (IT) – test of the initial knowledge from the field of solving tasks in the rectangular Descartes coordinate system,
- Final test (FT) – final test from the teaching unit of Equation of a straight line and Second-order curve.

Each test contained 10 tasks and every student could achieve maximum 4, 6 or 8 points, depending on the difficulty of the task, and the total number of points on the test was 60.

Two weeks before the beginning of the experimental program, students were subjected to initial test (IT), after which the experimental and control groups were formed, as well as the groups of level A, B and C in each of the groups. Final test (FT) is applied after the conduction of the experimental program. The authors of this study have independently designed both tests, and in order to ensure their liability, the tests were individually scored by four mathematics professors. The objectivity of the test is ensured by placing each student in approximately similar testing situation, by making sure that the independent examiners are following unique instructions and by evaluating the assignments based on the same key principle. The correlation (Pearson correlation coefficient) between different evaluators of the test is very high: IT ($r = .98, p < .01$) and FT ($r = .99, p < .01$). After the discriminant analysis for each of the 10 tasks per test individually, the reliability of instruments is determined by calculating the Cronbach's coefficient for IT ($\alpha = .81$) and FT ($\alpha = .85$).

The data obtained through research were processed by using the statistical software package IBM Statistics SPSS20, where single-factor analysis of variance was used (ANOVA) and the analysis of covariance (ANCOVA) for statistical equalization of experimental and control group as well as the longitudinal monitoring of the effects of the experimental program.

RESEARCH RESULTS AND DISSCUSION

The first goal that we wanted to examine was to determine whether the chosen methodological approach based on the principles of problem-based learning along with differentiating content on the three levels in the processing of the equation of the straight line and second-order curve gives a positive effect in solving test in the final examination. After completing the initial testing, both groups achieved approximately similar results: experimental group ($M=20.35; Sd=14.59$), and the control group ($M=22.40; Sd=14.35$), therefore we can conclude that the students of the control group achieved better results. The calculated variance of the initial measurement ($F(1,163) = .823; p = .366$) indicates that there is no statistically significant difference in the level of knowledge between the students of the experimental and control group regarding in the initial testing.

After conducting the experiment, a measurement was performed by using the final test which showed an improvement of results of the students who belong to the experimental group. When we observe the average number of points per student achieved at the final measurement in Table 1, we can notice an improvement regarding the students from the experimental group ($M = 22.11; Sd = 13.301$), while there is a slight decrease in the average number of points in the control group of students ($M = 17.30; Sd = 12.021$) in relation to the initial testing. The analysis of

the variance ($F(1,163)=5.884$; $p= .016$) indicates the existence of the statistically significant differences between students of experimental and control group after the conducted experimental program (Table 2).

In order to prove that the statistical difference between the experimental and control group on the final test is a consequence of the applied methodical approach, and not the consequence of the unevenness of the experimental and control group, we have calculated the covariance (ANCOVA). The result of the initial testing for both groups was taken as a covariate. That is confirmed by the calculated covariance between groups ($F(1,162) = 14.309$; $p < .001$) (Table 3).

Obtained results suggest that learning by solving tasks at three levels of complexity, through the problem-oriented approach in teaching Analytical geometry, had a significant impact in resolving the tasks at the final test, which means that this type of approach improves the knowledge of the student. These findings are in accordance with other researches which show that problem-solving was proven to enhance achievement in trigonometry (Nfon, 2013: 52; Mandaci & Kendir 2013; Mwelese & Wanjala, 2014). Akor (2005) in his research proves that Polya’s problem-solving strategy in the teaching of geometry on secondary school enhances students’ achievement in geometry in relation to the traditional teaching. Besides that, research show that problem-solving question promptly leads to improved knowledge acquisition (Bulu & Pedersen, 2010; Huang et al., 2015: 160; Perveen, 2010; Raes et al., 2012), and that problem-solving is – to some extent – a context of independent

Table 1. Descriptive indicators of the success of experimental and control group at the initial and final testing

		N	Mean	Std. Dev	Std. Error	95% Confidence Interval for Mean		Min	Max
						Lower Bound	Upper Bound		
						IT	E group		
	C group	77	22.40	14.353	1.635	19.14	25.66	0	54
	Total	165	21.30	14.473	1.126	19.08	23.53	0	60
FT	E group	88	22.11	13.301	1.418	19.30	24.93	0	60
	C group	77	17.30	12.021	1.370	14.57	20.03	0	59
	Total	165	19.87	12.908	1.005	17.88	21.85	0	60

Table 2. ANOVA analysis

		Sum of Squares	df	Mean Square	F	Sig.
Initial test	Between Groups	172.637	1	172.637	.823	.366
	Within Groups	34182.599	163	209.709		
	Total	34355.236	164			
Final test	Between Groups	952.073	1	952.073	5.884*	.016
	Within Groups	26374.994	163	161.810		
	Total	27327.067	164			

*The difference is significant at the $p < .05$

Table 3. ANCOVA analysis

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	11060.194 ^a	2	5530.097	55.074	.000	.405
Intercept	3356.515	1	3356.515	33.427	.000	.171
IT	10108.121	1	10108.121	100.666	.000	.383
Group	1436.793	1	1436.793	14.309*	.000	.081
Error	16266.873	162	100.413			
Total	92450.000	165				
Corrected Total	27327.067	164				

*The difference is significant at the $p < .001$

skill, which is both possible to train and transfer (Hattie, 2009; Marcucci, 1980, according to: Trincherro & Sala, 2016: 663). Also, research show that the mathematical abstraction levels and forms of the students can be improved by exposure to mathematical abstraction through a problem solving learning approach using ill structured problems, and that solving problems with characteristics of authenticity, complexity and openness can improve students' high-level mathematical thinking and strengthen their problem-solving skills in real life (Hong, 2016: 267-269).

We also observed the success of the experimental and control groups according to the level A, B and C (Table 4) during measurements. The analysis of the variance on the initial measuring between the subgroups of the experimental and control group (A, B and C) shows that the statistically significant difference in average number of achieved points does not exist: ($F_{xA}(1,72)=.599$; $p=.441$), ($F_{xB}(1,63)=.032$; $p=.858$) i ($F_{xC}(1,24)=1.224$; $p=.280$).

If we compare the achieved results within groups of levels after the applied final test, we will notice certain differences in achieved average number of points per student for all groups. We are curious about whether these differences are statistically significant, and if they are, which of the groups of levels contain the greater difference. This way we can provide an answer to the question to which groups of levels did the applied methodical approach brought the best results. If we compare the obtained variance between subgroups of the experimental and control group (A, B and C): ($F_{yA}(1,72)=9.906$; $p=.002$), ($F_{yB}(1,63)=11.723$; $p=.001$) i ($F_{yC}(1,24)=1.022$; $p=.322$) we can conclude that the statistically significant difference regarding the success on the final test exist between among the students of the experimental and control group within groups A and B, while the students of the group C did not make a statistically significant difference (Table 5). The conducted experimental program had the most influence on the students from the average group.

Table 4. Descriptive indicators of the groups of levels A, B and C on the tests

Group	N	Mean IT	Std. Dev.	Mean FT	Std. Dev	Std. error	Min	Max
Group A (experimental)	42	9.85	0.72	14.88	6.954	1.073	0	26
Group A (control)	32	10.75	0.926	9.97	6.229	1.101	0	23
Total	74	10.24	0.569	12.76	7.045	.819	0	26
Group B (experimental)	34	23.26	1.475	24.35	10.462	1.794	4	51
Group B (control)	31	23.64	1.521	16.81	6.710	1.205	5	32
Total	65	23.44	1.051	20.75	9.590	1.190	4	51
Group C (experimental)	12	48.83	2.029	41.08	16.822	4.856	10	60
Group C (control)	14	46.28	1.238	35.14	13.137	3.511	18	59
Total	26	47.46	1.153	37.88	14.946	2.931	10	60

Table 5. The analysis of variance for groups A, B and C after the final test

Group	Source of Variation	df	Mean Square	F _x	p	Mean Square	F _y	p
Between Groups A	Between groups	1	14.479	.599	.441	438.248	9.906*	.002
Within Groups	Within groups	72	24.155			44.241		
Total	Total	73						
Between Groups B	Between groups	1	2.347	.032	.858	923.458	11.723**	.001
Within Groups	Within groups	63	72.916			78.771		
Total	Total	64						
Between Groups C	Between groups	1	41.938	1.224	.280	228.023	1.022	.322
Within Groups	Within groups	24	34.272			223.193		
Total	Total	25						

**The difference is significant at the $p < .001$

*The difference is significant at the $p < .05$

We can see that the applied methodical approach did not have any statistical influence on the success of the students from geometry who belonged to the group of students with the best results on the initial test (C group). These students also achieved the best results on the final test. The reason can be found in a fact that this group of students has a greater knowledge of geometry, that their abilities are greater in relation to other students, and the manner in which they acquire knowledge is not crucial for their success, because even students from the control group achieved approximately the same results even though they were working in different conditions. On the other hand, students from the B group belong to a group of average students, and at their group the applied methodical approach gave the best result, which guides us to conclusion that this type of teaching, especially the differentiation of the content, has a great value in working with students in teaching mathematics. The fact that the students of the experimental group A (the lowest level) made a statistically significant difference in the average number of points in relation to the students of the same level in the control group. Research findings that are in accordance with findings of other research, show better adjustment of teaching students by differentiation which gains better results with students of lower knowledge level and have a smaller influence on students with higher level of knowledge. (Flores et al., 2012, Bokosmaty et al., 2015). It is especially important to develop learning environments that assess levels of learner' prior knowledge and accordingly alter instructional support levels (Bokosmaty et al, 2015) and that differentiated geometry teaching affects the spatial ability level of students (Kok & Davasligil, 2014). By building these "problem-solving competencies, students will strengthen their conceptual understanding, procedural fluency, strategic competence, productive disposition, and adaptive reasoning abilities" (Abdullah et al., 2014:166).

CONCLUSION

Classes of mathematics are constantly being innovated by introducing new methodical solutions, with an aim of achieving the best results within it, so that students could acquire the highest quality of knowledge and to train themselves in applying that knowledge in solving practical problems in life. In this study we tried to create a methodical approach based on the problem -based learning along with differentiating the content on three levels of complexity and examine its impact on the quality of knowledge and achieving the best results in secondary school mathematical classes within the field of geometry. On the basis of the results gained in the framework of the experimental research, we can conclude that:

- the applied methodical approach contributes to the better success of the student in learning mathematics,
- the greatest progress of the applied methodical approach is realized with group of students that are average regarding the success, and with the ones below the average, whereas in the group of the best students there is no statistically significant difference.

The research has shown that the choice of the methodical approach is very important in the designing the mathematics classes. By applying this methodic approach, students achieved a greater quality of knowledge and were more successful in solving tasks from students who did not learn under this approach. It should be noted that the progress of students was achieved through solving concrete problem tasks. In addition to that, differentiating of the content for students according to their knowledge and with problem-based approach to learning, achieved substantial results regarding the students' success, because the content is adjusted to students' knowledge and in accordance with that - student advances. Of course, in the process

of learning the student can shift to a higher level or, in case of experiencing the difficulties with the content – return to a lower level.

Teaching that is based on the problem-solving can contribute to the greater thinking activity of the students, who in turn exhibit greater activity during the class, versatile approach to mathematical contents, rationality, creativity and criticism. Learning in which the students are faced with problem-based situation that needs to be solved, represents a natural context of learning during classes, which cannot be said for classes organized in classical manner. In addition to that, the differentiation of the content for students in accordance with their current level of knowledge and possibilities creates conditions for adjusting their manner of learning and gaining knowledge.

Methodical contribution of the research is visible from the analysis of the contemporary teaching practice in the field of problem-based learning in teaching mathematics, based on which the models for the application in classes are constructed. Students who follow previously memorized paths in a traditional approach do not have the opportunity to create their own approaches (Hines, 2008). This approach of organizing teaching classes can also be applied not just in Analytical geometry but also in other areas in the field of mathematics, such as algebraic or geometric content. The greatest contribution of this paper will be if its results and suggestions become a part of everyday teaching practice and stimulus for writing new papers in mathematics teaching methodology.

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