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The impact of integrating visual and illustrative models in learning trigonometry

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Abstract

This article analyzes the methodological potential of a multiple-representation approach based on visual and illustrative modeling in teaching the fundamental concepts of trigonometry. During the learning process, students frequently experience difficulties in understanding the intrinsic relationships among the unit circle, right triangles, and the graphs of trigonometric functions. To address these challenges, the study proposes an alternative model for constructing the trigonometric circle. The proposed model aims to reveal the logical origins of trigonometric concepts, support their spatial and intuitive perception, and establish a holistic connection among the unit circle, graphical representations, and geometric models. The article examines the solution of trigonometric inequalities as an instructional example by employing various visual and analytical methods and provides a rationale for their didactic effectiveness. The impact of this approach on the efficiency of learning trigonometry was evaluated with 94 final-year high school students from two selected public schools in Uzbekistan (45 in the control group and 49 in the experimental group). A mixed-methods research design was adopted, integrating quantitative data obtained from student assessments with qualitative data collected through interviews. The findings indicate that students' abilities to analyze problems, justify solutions, and make generalizations were significantly improved.

Keywords: achievement, mathematics education, mixed method, multi-representational approach, trigonometric inequalities, trigonometric functions, unit circle, visual and illustrative modeling

INTRODUCTION

Galileo stated that

> "… although the universe is open to our observations at every moment, it cannot be understood unless one learns and comprehends the language in which it is written and the letters of that language. The universe is written in the language of mathematics, with measure and harmony. The letters of this language consist of triangles, circles, and other geometric forms. Without these, not even a single word of the universe can be understood; those who lack these tools merely wander in a dark labyrinth" (Pappas, 2007, p. 9).

Like any language, the language of mathematics possesses universal symbols. When these symbols are associated with concepts, they gain meaning, thereby making mathematics more understandable and enjoyable (Baki, 2006). Therefore, it is extremely important that every student becomes familiar with the fundamental concepts of trigonometry—its language and the symbols (letters) in which it is written—and forms a correct initial understanding. Thorough learning of these topics from the very first lessons will provide an effective foundation for the successful continuation of trigonometry education (Keser, 2017). Expanding the possibilities for presenting the fundamental concepts of

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Contribution to the literature

- This study enriches mathematics education literature by showing that a multiple-representation approach strengthens students' understanding of fundamental trigonometric concepts.
- It demonstrates that integrating visual, illustrative, and analytical representations fosters improved problem-solving, reasoning, and conceptual connections, particularly through the use of trigonometric inequalities.
- Using a mixed-method design, the study offers evidence-based implications for curriculum and classroom practice, highlighting the value of visual and dynamic modelling tools in enhancing secondary students' conceptual learning.

trigonometry through visual models significantly reduces the difficulties that arise in the learning process. In the traditional approach, these concepts are generally introduced on the basis of the unit (trigonometric) circle. However, incorporating alternative methods for constructing the trigonometric circle into the educational process reveals multidimensional visual representation opportunities. This enables trigonometric concepts to be understood consciously and logically rather than through mechanical memorization and contributes to the development of students' conceptual thinking skills.

Despite this critical importance, trigonometry is consistently described as a domain in which students experience conceptual difficulties, particularly in establishing connections among geometric, algebraic, and graphical representations (Delice, 2003; Hatt, 2024; Mumcu & Aktürk, 2020). These difficulties generally stem from instructional approaches that prioritize procedural skills over the development of deep conceptual understanding, thereby limiting students' ways of thinking. To overcome these challenges, various instructional techniques and technologies are employed (İbili, 2019; Smith & Cekiso, 2020).

Baki (2000) emphasizes that many mathematical concepts are abstract in nature and require high-level cognitive activities. In the concretization of abstract concepts, the role of visualization is undeniable. Thus, visualization can be described as the use of geometric structures through which abstract concepts can be explained in a concrete manner. Borba and Villarreal (2005) define visualization as a cognitive process that follows a dual pathway between the learner's understanding and an external tool, while Arcavi (2003) states that students who are flexible in moving between these pathways are more successful. Boz (2005) emphasizes the importance of technology-supported visualization for a better understanding of mathematical concepts.

Özdemir et al. (2005) and Hegarty and Kozhevnikov (1999) have emphasized that using different techniques (solution methods) together facilitates mathematical understanding, and that visual strategies, in particular, play a significant role in both conceptual learning and long-term retention. In trigonometry education, the use of graph-based visual strategies (Doğan & Abdildaeva, 2013) alongside analytical thinking not only facilitates students' multidimensional thinking and knowledge transfer but also supports the formation of conceptual knowledge and accurate concept images. In mathematics, a concept becomes meaningful when it is related to other mathematical concepts.

Recent research emphasizes the importance of aligning trigonometry instruction with constructivist and cognitive learning theories that promote visualization and conceptual reasoning (Canonigo, 2025; Laudano, 2021; Moore, 2013). Likewise, recent studies show that allowing students to explore trigonometric relationships through visually supported tasks enhances conceptual understanding and flexibility in reasoning (Marufi et al., 2022; Nanmumpuni & Retnawati, 2021).

Developing students' deep conceptual understanding, reasoning skills, ability to interpret multiple representations, and capacity to apply knowledge in different contexts has become increasingly important in contemporary mathematics education. In response to this instructional demand, the present study introduces an innovative approach designed to support the integration of visual and illustrative models in trigonometry studies. It contributes to the field of mathematics education by empirically evaluating a visual and explanatory modeling approach designed to integrate unit circle, right triangle, and function graph representations into a coherent conceptual framework. Beyond its direct classroom applications, the proposed approach also aims to serve as a pedagogical resource that enhances instructional clarity in trigonometry and contributes to mathematics teacher education by addressing abstract concepts through alternative visual pathways.

As an example, let us consider the following model (part a in **Figure 1**). Let the rectangular coordinate system xoy be given. Let there be another coordinate line (denoted by a) that is perpendicular to the xoy plane and intersects it at the point with coordinates (1; 0). The origin of the coordinate system on the line a coincides with this point. The positive direction of numbers is chosen upward, and the length of the unit segment is taken to be equal to the unit segments on the ox and oy coordinate axes. Now, imagine a right circular cylinder with base radius $R = 1$, whose axis of symmetry passes

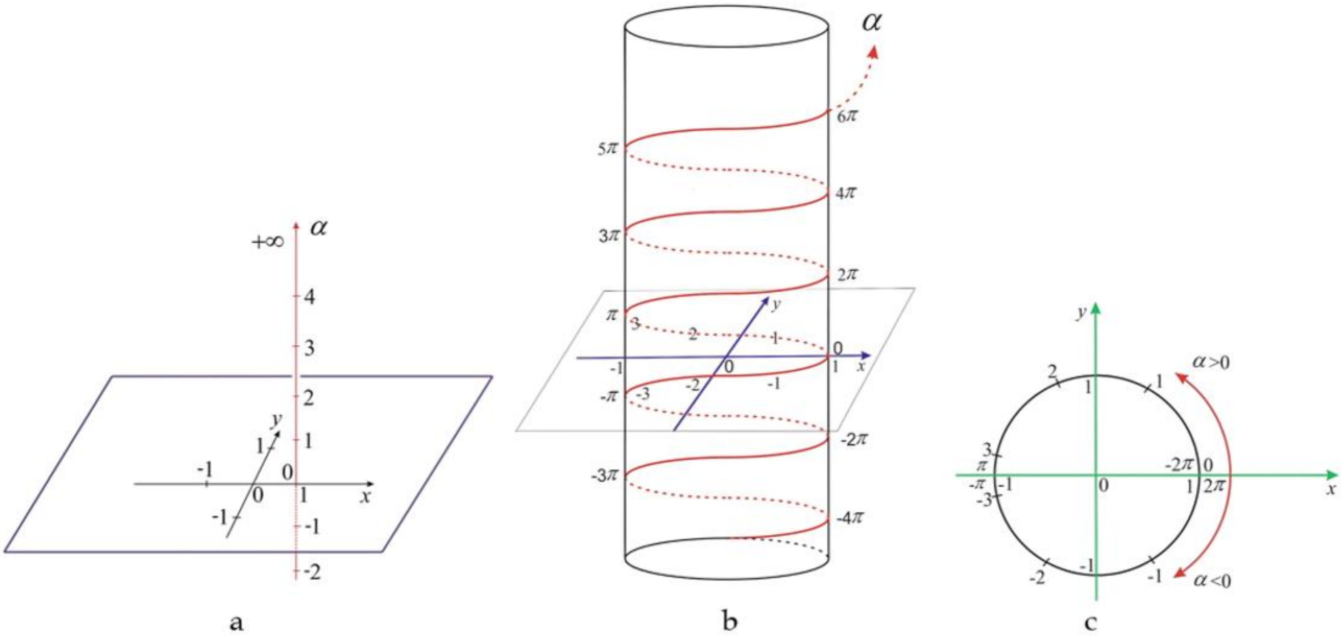

Figure 1. Model (Source: Authors' own elaboration)

through the origin of coordinates and which intersects the xoy coordinate plane. The ray of the coordinate line a corresponding to positive numbers is tightly wound around the surface of this cylinder in a counterclockwise direction. In order to maintain the continuity of the spiral, the ray corresponding to negative numbers is wound around the surface of the cylinder in the clockwise direction (part b in **Figure 1**).

All turns of the resulting "coordinate spiral" are laid onto the xoy plane from above and below. When the obtained result is viewed from above, we observe that a trigonometric circle with center at the origin and radius equal to one is formed in the xoy coordinate plane (part c in **Figure 1**). Thus, to construct the point corresponding to a positive number on the trigonometric circle, one should start with the origin of coordinates (that is, from the point where the trigonometric circle intersects the positive direction of the ox axis) and move along the trigonometric circle in a counterclockwise direction. For the graphical representation of negative numbers, the movement along the trigonometric circle is carried out in the clockwise direction. When constructing any arc on the trigonometric circle, the movement from the initial point of the arc to its terminal point must always be performed in a counterclockwise direction, that is, from a smaller numerical value to a larger one. In constructing graphs of trigonometric functions, the "flattened" a coordinate axis can be regarded as the abscissa axis.

This study identifies students' difficulties in trigonometry as challenges in

(a) interpreting the unit circle beyond mere procedural angle-value associations,

(b) converting between geometric, graphical, and algebraic representations, and

(c) justifying solutions to trigonometric problems, especially inequalities.

These aspects informed the design of the visual and illustrative model as well as the creation of evaluation and interview tools.

The proposed model exhibits certain conceptual and pedagogical differences from unit circle-based and dynamic visualization approaches such as GeoGebra. First, instead of treating the unit circle as a completed object, the model adopts a coordinate-spiral-based structure in which angular progression is represented as continuous spatial development. This structure facilitates the emergence of periodicity and helps students perceive trigonometric functions as accumulated rotations. Second, unlike common GeoGebra-based representations (Yorganci, 2018; Yorganci & Subasi, 2025), this model is presented as a didactically structured visual sequence. Thus, each stage of the structure functions as a conceptual bridge, explicitly connecting geometric motion, coordinate representation, and graphical interpretation. Finally, from a pedagogical perspective, the model prioritizes explanatory transparency over technological interactivity. While dynamic software can enhance interaction, if not carefully structured, it may obscure the underlying mathematical reasoning. Therefore, our model is positioned not as an alternative to the existing unit circle or digital tools, but as a complementary approach emphasizing conceptual continuity, spatial intuition, and multiple representations. This distinction constitutes the core methodological and pedagogical innovation of the current study.

One of the aspects that most strongly attracts students' interest is the possibility of solving problems not only by traditional "standard" methods, but also through aesthetically refined visual or illustrative solution techniques. Therefore, during instruction, the teacher should not overlook these opportunities and should aim to develop students' visual thinking and creative approaches. Detailed mathematical examples illustrating the application of the visual model to trigonometric inequalities are provided in **Appendix A**.

Accordingly, the purpose of this study is to examine the perceptions of final-year high school students regarding an approach based on visual and explanatory modeling, with a focus on identifying the internal structure and functional relationships of mathematical objects and understanding their essence at a spatial and intuitive level. In line with this purpose, the study addresses the following research questions:

Research Questions

RQ1. What difficulties do final-year high school students in Uzbekistan encounter in trigonometry?

RQ2. What is the effect of learning with visual and illustrative models on the trigonometry achievement of final-year high school students?

RQ3. How do final-year high school students perceive the effectiveness of the visual and illustrative model approach in their trigonometry lessons?

Research Hypotheses

H_0. There is no significant difference in the post-test mean scores between students in the experimental group and those in the control group.

H_1. There is a significant difference in the post-test mean scores between students in the experimental group and those in the control group.

LITERATURE REVIEW

Research in mathematics education has consistently emphasized the use of multiple representations—such as the unit circle, right triangles, and trigonometric functions—to strengthen students' understanding of trigonometry, highlighting their importance. However, previous studies indicate that presenting these multiple representations together does not automatically lead students to make independent inferences. Many studies argue that meaningful learning occurs when instructional approaches explicitly establish connections between representations and provide the necessary guidance. This issue is of central importance in addressing persistent conceptual difficulties in trigonometry and directly informs the design of the present study (Daher, 2020; Lyublinskaya, 2006; Mailizar et al., 2025).

In recent years, numerous studies have been conducted with the aim of reducing students' difficulties in learning trigonometry. These studies have primarily examined the effectiveness of integrating multi-representational approaches, digital technologies, and didactic models in the teaching process. For example, Mailizar et al. (2025) found that visual explanations delivered through short video lessons in trigonometry have a positive impact on students' ability to recall and consolidate knowledge. Their findings indicate that educational resources rich in visual components facilitate the comprehension of complex trigonometric concepts. In the study by Hamzić et al. (2024), test tasks on trigonometry based on triangles and the unit circle were analyzed, revealing that students tend to make more errors specifically on questions related to graphical and geometric interpretations. This observation confirms the need for consciously forming connections among different models in trigonometry.

Research conducted by Singh et al. (2023) demonstrated that the use of extended (augmented) technologies significantly improves spatial reasoning and the acquisition of trigonometric concepts. These results substantiate the role of visual-spatial modeling as an important methodological tool in trigonometry instruction. Spangenberg (2021) emphasizes that a teacher's subject-specific pedagogical knowledge is directly related to how and when visual and graphical models are employed in teaching trigonometry. According to the author, the teacher's ability to integrate multiple representations is considered one of the key factors determining students' conceptual understanding. An analysis of epistemological obstacles encountered in trigonometry shows that gaps in students' understanding between concepts (triangle-circle-graph) seriously hinder the formation of deep knowledge. This, in turn, confirms the relevance of the multi-representational approach proposed in this article. In the study by Mosese and Ogbonnaya (2021), it was proven that constructing and interpreting graphs of trigonometric functions using the GeoGebra software improves students' academic achievement. This work highlights the didactic significance of integrating graphical and algebraic approaches.

Overall, the literature review indicates that a multi-representational approach based on the integration of visual, graphical, and geometric models in teaching trigonometry is an effective means of reducing students' learning difficulties. Instructional materials developed by Hashrin and Lestyanto (2024), based on interactive tools, demonstrate the effectiveness of multimodal presentations (text, images, animations, and interactive tasks) in learning trigonometric ratios. The authors emphasize that the use of flipbooks increases students' independent learning activity and levels of understanding. This approach allows visual and illustrative models to be presented in a sequential and logical manner.

In the study by Asfyra et al. (2024), the topic of area calculation in trigonometry was taught through a website-based approach integrated with a professional (vocational) context. The results show that visual materials grounded in real-life contexts enhance students' motivation and facilitate understanding of the practical meaning of trigonometric formulas. This approach underscores the importance of combining visual models with contextual learning.

An electronic module based on the flipped classroom model proposed by Machmud et al. (2025) aims to position students as active agents in learning trigonometry. In this study, visual materials (videos, graphs, and animations) are studied prior to class, while problem-based and analytical activities are conducted in the classroom. This model promotes deep learning by integrating visual tools with analytical and logical activities. From a didactic design perspective, one of the notable studies is the work of Maknun et al. (2020).

The way the trigonometric circle is constructed plays a critical role in shaping students' conceptual understanding. While traditional approaches to the unit circle may be effective in promoting procedural fluency, they might not fully reveal the underlying geometric logic connecting angular motion, coordinate representation, and periodic behavior. Students are more likely to perceive trigonometric functions as dynamically generated quantities when they interact with multiple representations (Maknun et al., 2020). The authors substantiate the advantages of the unit circle approach in drawing and analyzing graphs of trigonometric functions. This approach reveals the relationships among the trigonometric circle, graphical, and algebraic models, and contributes to the formation of a holistic understanding of trigonometry in students. Bornstein (2020), in turn, examines the issue of teaching transformations of trigonometric functions (shifts, stretches, and reflections) through technology-based instruction. The study shows that observing, in real time, how functions change with respect to parameters using interactive graphical tools reduces students' errors.

Beyond algebraically procedural performance, learning trigonometry increasingly requires students to engage in higher-order reasoning, particularly in tasks such as solving trigonometric inequalities, interpreting their graphical behavior, and constructing solution sets. Previous studies have shown that students' reasoning across representations is often limited, leading to errors in interpretation and generalization; however, alternative visual-representational models have been found to support students in coordinating multiple forms of representation simultaneously. In complex tasks like trigonometric inequalities, visual models can serve as cognitive tools that bridge geometric intuition and analytical reasoning. Nevertheless, relatively few studies have addressed this area (Arhin & Hokor, 2021; Dündar, 2015; Fahrudin et al., 2019; Ngu & Phan, 2020; Owusu et al., 2025; Tunzana et al., 2025). This work confirms the didactic effectiveness of linking visual models with analytical analysis. However, existing studies have not sufficiently addressed the systematic methodological use of alternative visual models for constructing the trigonometric circle, nor the multi-aspect visual-representational possibilities in solving trigonometric problems. Addressing this gap, the present study investigates how a visual and illustrative model supports students' reasoning and problem-solving processes beyond mere procedural accuracy. Our literature study, which provides significant insights into trigonometry learning, demonstrates that students encounter conceptual challenges while navigating the interrelations among geometric, graphical, and algebraic representations of trigonometric concepts. Research also shows that students' conceptual understanding and spatial reasoning can be enhanced through visual and technology-supported instructional approaches. While most of these studies examine visualization through digital tools or isolated instructional interventions, only a limited number of studies investigate systematically designed visual models that integrate multiple representations within a coherent conceptual framework. This study addresses the identified gap by investigating students' challenges in learning trigonometry (**RQ1**), assessing the impact of a visual and explanatory modelling approach on students' performance (**RQ2**), and reviewing students' perceptions of this instructional method (**RQ3**). This research seeks to furnish empirical data on the efficacy of organized visual modelling in enhancing conceptual understanding in trigonometry, by correlating the research questions with the deficiencies revealed in prior studies.

METHODOLOGY

Research Design

The research was conducted using both quantitative and qualitative data. A quasi-experimental design was used, and mixed techniques were applied to gather the data. A mixed-methods approach was adopted to address the overall objectives of the research, to gather complementary data, and to integrate and interpret these data effectively (Creswell & Plano Clark, 2017). To gain a more complete understanding of the results and to use the strengths of both types of data, the researchers combined quantitative and qualitative data. Through interviews, the researchers gained a complete knowledge of high school students' comprehension of trigonometry as they learned it. They also explored how visual and illustrative models helped students learn from their perspectives.

Table 1. Gender frequency distribution

Gender	Frequency	Percentage (%)
Female	55	58.5
Male	39	41.5
Total	94	100

Sample and Participants

This study was conducted with voluntary participants from two distinguished public high schools in the Fergana region of Uzbekistan, recognized for admitting students with strong academic performance and high achievement in secondary education. These schools were selected because their students possessed sufficient mathematical background and cognitive readiness to meaningfully engage with the content addressed in the study.

The sample consisted of ninety-four final-year high school students enrolled during the 2024-2025 academic year. The experimental group at Fergana City General Secondary Education School No: B had forty-nine students. In contrast, the control group at Fergana City General Secondary Education School No: An included forty-five students. A purposive sampling method was employed, as the study required participants who had been previously exposed to trigonometry topics included in the national curriculum. Throughout the study, all participating students received formal mathematics instruction at their respective schools. This sampling technique also prioritized these schools due to the researchers' convenient access to both locations.

Participation in the study was voluntary, and informed consent was obtained in accordance with ethical research guidelines. Participants were assured that any assessments collected during the study would not appear on official records such as transcripts or report cards, and that they could withdraw from the study at any time without any consequences. To ensure confidentiality, no personally identifiable information about the participants or the schools was disclosed. The gender distribution of the students included in the study is presented in **Table 1**.

As can be seen, the majority of the participants are female, and all participants are final-year high school students. The reason for their selection is that they had previously received and were continuing to receive instruction in trigonometry at the time of the study.

Research Instruments

Data collection involved pre- and post-tests, including a trigonometry achievement test (TAT), a survey, and a semi-structured interview. The test items were carefully selected from high school basic mathematics textbooks and previous exam questions, using the basic mathematics curriculum objectives as a guide (Matematika Fanidan O'quv Dasturi, n.d.).

To obtain student responses from both groups, a survey using a 3-point Likert scale was given. This survey included six closed-ended questions. Additionally, open-ended semi-structured interviews were conducted to collect more detailed input. The use of open-ended questions in semi-structured interviews allows participants and the interviewer to engage in comprehensive discussion, providing researchers with the advantage of collecting rich qualitative data (McLeod, 2024).

The interviews aimed to assess students' perceptions of how well the visual and illustrative model approach helped them learn trigonometry. Furthermore, the study examined whether this approach enhanced students' self-confidence, research skills, and conceptual understanding of the subject.

Validity and Reliability

The validity of the TAT was ensured through the development of test items derived from the high school mathematics curriculum and textbooks sanctioned by the Ministry of Education of Uzbekistan. Furthermore, we incorporated trigonometry questions from prior high school graduation and university entrance examinations, specifically the Davlat Test Markazi (DTM), into the test. Content validity was established through expert review; mathematics education specialists meticulously examined the survey items and subject-specific questions within the interview guides, offering essential revisions and professional insights.

To evaluate the reliability of the TAT and the survey used in this study, the researchers used the test-retest method. Creswell (2012) explains that this method assesses reliability by giving the same test to the same group of people twice, with a significant time gap between the two tests. The test-retest dependability of research tools rises when the outcomes from two separate uses are increasingly similar. In this study, the researchers administered the TAT and the survey to students at a pilot school and then re-administered the same instruments to the same students one month later. The results were used to revise the instruments. After this procedure, the tools were updated and modified. Using SPSS, the correlation coefficients were generated to assess the test-retest reliability of TAT and the survey, comparing the two sets of replies. TAT was used as both a pre-test and a post-test to assess participants' achievement in trigonometry. The test consisted of 30 items, all scored using a dichotomous scoring scheme (1 point for a correct answer, 0 points for an incorrect or unanswered item), with a maximum possible score of 30. As seen from the item-level analysis of the pre-test, the test items targeted students' abilities in interpreting diagrams, applying basic trigonometric equations, performing algebraic calculations, and solving real-life problems. The same instrument was administered before and after instruction to measure changes in achievement. The correlation coefficients, which indicate the reliability of the trigonometric achievement test and the survey, were found to be .82 and .89, respectively. Reliability values of .70 or higher are frequently considered suitable for research instruments in educational measurement. Since both coefficients exceed this commonly accepted threshold, the instruments can be considered sufficiently reliable for the purposes of this study.

Treatment

This study used a quasi-experimental design. The researchers created instructional models based on visual and illustrative methods to teach trigonometry to the experimental group. At the same time, the control group received new lessons on the same trigonometry topics, but these lessons used traditional teaching methods. Lesson plans for both groups included specific goals, content, teaching and learning activities, and assessment tools, all taken from the high school basic mathematics curriculum for trigonometry in the Fergana region.

The intervention, which followed a pre-test given in March 2025, lasted eight weeks, ending in May 2025. Each group participated in two 40-minute lessons each week. The experimental group's initial lesson focused on introducing the visual and illustrative models approach and how it applied to trigonometry, helping students understand the methodology. In addition, students were divided into smaller groups of five during the teaching and learning activities. Following the high school basic mathematics curriculum, the lessons from the first to the eighth week were designed to help students understand the basic concepts of trigonometry.

The instructional content covered the following topics: angle concepts and measurement systems; the unit circle and the representation of angles on the unit circle; definitions of trigonometric ratios; trigonometric values of special angles; fundamental properties of trigonometric functions; graphing trigonometric functions; basic trigonometric identities; and problem solving and applications (Matematika Fanidan O'quv Dasturi, n.d.). While the control group received regular instruction using the conventional teaching methods typically employed by the researchers, the experimental group received regular instruction using the visual and illustrative models methodology.

The lessons designed for the experimental group were grounded in the integration of visual and illustrative models in trigonometry learning, drawing on the theoretical and empirical perspectives of researchers such as Arcavi (2003), Canonigo (2025), Laudano (2021), and Mosese and Ogbonnaya (2021). The instructional cycle involved presenting trigonometric problems through visual representations; analyzing these problems using diagrams and graphs; finding solutions through visual reasoning and GeoGebra (and similar applications) simulations; reporting and justifying results using visual arguments; and reflecting on the problem-solving process. Finally, students' solutions and reasoning were evaluated, with emphasis placed on both conceptual understanding and the effective use of visual and illustrative tools to support trigonometric learning.

In contrast, the control group was instructed using a traditional approach that emphasized procedural fluency and the memorization of step-by-step solution methods. After presenting example problems, finding answers, and describing the methods used, the researcher ended each class by highlighting the procedural patterns needed for each problem type. This was done to help students remember the material. Throughout the instructional period, students in the control group maintained continuous dialogue with the researchers regarding difficulties they encountered, allowing the researchers to support students by facilitating the memorization of multi-step concepts. In addition, elements of the visual and illustrative models approach were briefly introduced to the control group to help them understand the new methodology, and problems drawn from high school graduation examinations and university entrance examinations administered by the Uzbekistan State Testing Center (DTM) were also used. The traditional instruction in the control group was predominantly procedural and formula-based, relying on symbolic representations supplemented by teacher explanations. Any static visuals used were occasional and intended for illustrative purposes; they were not systematically integrated into instruction nor employed as conceptual models. In contrast, the experimental group used visual and illustrative models as the primary tools for concept development and reasoning. Therefore, the limited use of visuals in the control group does not constitute treatment contamination. Due to the mixed-methods approach of the study, hypotheses were developed just for research questions pertaining to quantifiable accomplishment results. The research topics concerning students' perceptions and representational comprehension were exploratory, and hence they were analyzed qualitatively instead of through hypothesis testing.

FINDINGS

RQ1. What Difficulties Do Final-Year High School Students Encounter in Trigonometry?

Following a descriptive analysis of the pre-test responses from the entire sample of 94 students, which aimed to pinpoint the challenges they faced in solving trigonometric problems, the students' understandings of trigonometric concepts were then assessed. To further explore these perceptions, a closed-ended questionnaire

Table 2. Students' difficulties in the pre-test

Item	Challenges	Frequency (%)		
		C	I	NA
1	Challenges in constructing a right triangle from diagram analysis	39 (41.5)	36 (38.3)	19 (20.2)
	Challenges related to the use of basic trigonometric equations	38 (40.5)	37 (39.4)	19 (20.2)
	Challenges in accurate algebraic computation	27 (28.7)	48 (51.1)	19 (20.2)
2	Challenges in interpreting trigonometric information during diagram construction	33 (35.1)	37 (39.4)	24 (25.5)
	Challenges in performing correct algebraic calculations	9 (9.6)	61 (64.9)	24 (25.5)
3	Challenges in applying trigonometry to real-life problems	3 (3.2)	57 (60.6)	34 (36.2)

Note. C: Correct; I: Incorrect; & NA: No attempt

Table 3. Students' understanding of trigonometric principles

Understanding	Frequency (%)		
	A	D	U
The instructor helped me learn trigonometric principles by using a step-by-step teaching method	9 (9.6)	72 (76.6)	13 (13.8)
I have difficulty understanding trigonometric terms	78 (83.0)	11 (11.7)	5 (5.3)
Learning trigonometry involves memorizing formulas without understanding	64 (68.1)	22 (23.4)	8 (8.5)
I find it difficult to learn trigonometric concepts because they are abstract	53 (56.4)	35 (37.2)	6 (6.4)
I lack confidence in learning and solving trigonometric problems	69 (73.4)	19 (20.2)	6 (6.4)
I have difficulty understanding real-life trigonometry problems	81 (86.2)	10 (10.6)	3 (3.2)

Note. A: Agree; D: Disagree; & U: Undecided

was subsequently administered, requiring students to choose from a set of provided options.

Table 2 shows that in the pre-test, item 1, a more detailed look at the three difficulties in this area reveals that 39 of the 94 students (41.5%) could easily analyze the diagram and create a right triangle. In contrast, 36 students (38.3%) struggled with this and were unable to do it.

In addition, 38 students (40.5%) were able to use the basic trigonometric equation correctly, while 37 students (39.4%) experienced difficulty and failed to apply the basic trigonometric equation. Of the 94 students, 27 (28.7%) were able to get the final solution correctly, showing they used the right algebraic methods. However, 48 students (51.1%) failed to obtain the correct answer, and 19 students (20.2%) were unable to solve item 1 at all.

In the pre-test's third item, students struggled to apply trigonometric concepts to real-world problems and solve them. Only 3 students, or 3.2% of the sample, could solve the real-world trigonometry problem correctly. In contrast, 57 of the 94 students, which is 60.6%, found the problem difficult and could not solve it completely. Moreover, 34 students (36.2%) did not attempt to solve the task at all.

The findings indicate that students encountered difficulties in comprehending trigonometric diagrams and conducting substantive analyses, thereby hindering their ability to employ suitable algebraic computations to fulfill the assigned tasks. Moreover, the majority of students resorted to rote memorization of procedures, a consequence of inadequate conceptual grasp and insufficient problem-solving experience. This, in turn, led to challenges in addressing real-world trigonometric problems. Consequently, students' struggles in solving trigonometric problems stemmed from fundamental arithmetic errors and the application of inappropriate procedures.

Table 3 shows that 9 students (9.6%) in the sample agreed that the teacher used a step-by-step presentation to help them understand trigonometric concepts, whereas 72 students (76.6%) disagreed and 13 students (13.8%) remained undecided. Additionally, 78 students (83%) agreed that the terminology used in trigonometry topics was difficult to understand, while 11 students (11.7%) disagreed and 5 students (5.3%) were undecided. Moreover, 64 students (68.1%) agreed that trigonometry requires memorizing formulas rather than understanding concepts, whereas 22 students (23.4%) disagreed and 8 students (8.5%) were undecided. Furthermore, 53 students (56.4%) stated that they experienced difficulty learning trigonometry due to the abstract nature of the concepts, while 35 students (37.2%) disagreed and 6 students (6.4%) were undecided. Regarding a lack of self-confidence in learning and solving trigonometric problems, 69 students (73.4%) agreed, while 19 students (20.2%) disagreed, and 6 students (6.4%) were undecided. Additionally, 81 students (86.2%) agreed that real-world trigonometric problems were difficult to understand, whereas 10 students (10.6%) disagreed and 3 students (3.2%) were undecided.

The data indicated that a significant number of students perceived limitations in commonly used instructional approaches. Furthermore, the results suggested that the abstract character of trigonometry itself presented a comprehension challenge, prompting students to resort to rote memorization of formulas,

Table 4. Descriptive statistics of pre-test scores in groups

Group	N	M	SD	Maximum	Minimum
Control	45	4.67	4.6	19	0
Experimental	49	4.10	4.8	15	0

Table 5. Independent sample t-test of pre-test scores in groups

Group	N	M	SD	t-value	df	p
Control	45	4.67	4.6	0.585	92	0.560
Experimental	49	4.10	4.8			

often without a clear understanding of their underlying principles. Finally, as shown in **Table 3**, these results confirm that students experienced difficulties in solving real-life trigonometric problems as a consequence of their lack of understanding of trigonometric concepts.

RQ2. What Is the Effect of Learning With Visual and Illustrative Models on the Trigonometry Achievement of Final-Year High School Students?

Pre-test scores

The pre-test scores of both the control and experimental groups were assessed and compared to see whether there were any significant differences between them before the intervention.

The data in **Table 4** show that the control group had a mean (M) score of 4.67, while the experimental group's M was 4.10. Both groups had a minimum score of p < .001. The maximum scores were 19 for the control group and 15 for the experimental group. Looking at the M scores, the control group (M = 4.67) performed slightly better than the experimental group (M = 4.10). To determine if the difference between the two groups' M scores was statistically significant, an independent samples t-test was done at a 95% confidence level. The results are shown in **Table 5**.

The data presented in **Table 5** indicate that the control group (M = 4.67, standard deviation [SD] = 4.6) and the experimental group (M = 4.10 , SD = 4.8) did not exhibit a statistically significant disparity; t (92) = 0.585, p = .560 > 0.05. Consequently, this observation suggests that the participants in both groups possessed comparable levels of conceptual comprehension of trigonometry before the intervention was implemented.

Post-test scores

After the intervention, a post-test was given to each group to measure any improvements in their achievement. The next section summarizes the results of the post-test scores for both groups.

According to **Table 6**, the results show that the M post-test score was 10.71 for the control group and 15.84 for the experimental group. The minimum scores were 0.2 for the control group and 0.4 for the experimental group, respectively. Furthermore, both groups attained

Table 6. Descriptive statistics of post-test scores in groups

Group	N	M	SD	Maximum	Minimum
Control	45	10.71	8.5	30	2
Experimental	49	15.84	7.7	30	4

Table 7. Paired sample t-test scores on the pre- and post-test

Group	N	M	MD	SD	t-value	df	p
Control	45	4.67	6.04	7.3	5.54	44	< 0.001
Pre-/post-test		10.71					
Experimental	49	4.10	11.74	6.6	12.49	48	< 0.001
Pre-/post-test		15.84					

Note. MD: Mean difference

a maximum score of 30. Upon comparing the M scores of the control group in **Table 5** (M = 4.67) and **Table 6** (M = 10.71), alongside the M scores of the experimental group in **Table 6** (M = 4.10 and M = 15.84), it is apparent that individuals in both groups exhibited enhancement in their post-test performance. This provides evidence that participants' performance improved following the intervention. To determine whether the differences between pre-test and post-test scores within each group were statistically significant, paired-samples t-tests were conducted. The results are presented in **Table 7**.

The data in **Table 7** show that students in the control group had higher trigonometry scores on the post-test compared to the pre-test; t (44) = 5.54, p = .001 < 0.05. This suggests that the traditional teaching method and the teacher's influence helped improve students' understanding of trigonometric concepts. In addition, the experimental group's pre-test and post-test scores showed a significant improvement in their performance in trigonometric concepts after using visual and illustrative models; t (48) = 12.49, p = .001 < 0.05. Therefore, the activities based on visual and illustrative models significantly improved students' performance and understanding of the subject.

This research topic focuses on assessing the effectiveness of the visual and illustrative model method in acquiring trigonometric ideas by comparing its influence on students' accomplishments to that of standard training. To address this research question, the following hypotheses were tested:

H_0. There is no significant difference in the post-test mean scores between students in the experimental group and those in the control group.

H_1. There is a significant difference in the post-test mean scores between students in the experimental group and those in the control group.

To establish the significance of this impact and the extent of change, an independent samples t-test was performed on both the control and experimental groups' post-test scores. **Table 8** summarizes the findings from the independent samples t-test.

Table 8. Independent sample t-test of post-test scores in groups

Group	N	M	SD	t-value	df	p	Eta squared
Control	45	10.71	8.5	3.06	92	0.003	0.144
Experimental	49	15.84	7.7				

The independent samples t-test results, as shown in **Table 8**, revealed a statistically significant difference between the control group (M = 10.71, SD = 8.5) and the experimental group (M = 15.84, SD = 7.7); t (92) = 3.06, p = .003< 0.05. This indicates that the experimental group, which used visual and illustrative models to teach trigonometric concepts, performed better than the control group. Moreover, the eta squared value of 0.144 suggests a large effect size, meaning that 14.4% of the differences in post-test TAT scores (the dependent variable) are related to the instructional approach (the independent variable).

According to Cohen's general guidelines for interpreting eta squared values (Field, 2024), an eta squared value of 1% represents a small effect size, 6% represents a medium effect size, and 14% represents a large effect size. The findings reveal a marked disparity in post-test mean achievement scores when comparing students exposed to the visual and illustrative models approach with those who received instruction via the traditional method. Consequently, the implementation of the visual and illustrative modeling approach in trigonometry instruction yielded a statistically significant enhancement in student achievement relative to the conventional approach.

RQ3. How Do Final-Year High School Students Perceive the Effectiveness of the Visual and Illustrative Model Approach in Their Trigonometry Lessons?

In this section, the findings obtained from semi-structured interviews conducted with five students randomly selected from the experimental group are presented. To protect their identities during the interviews, the students were assigned codes: *P1, P2, P3, P4*, and *P5*. The topics outlined in the interview guide were utilized to assist in transcribing and analyzing the interview material. Because thematic saturation was reached and no substantially new themes emerged in subsequent interviews, this sample size was considered sufficient for the exploratory and explanatory purpose of the qualitative component. To reduce interviewer bias and enhance reliability, a semi-structured interview protocol was used to ensure consistency across interviews. In addition, the initial coding of the interview transcripts was reviewed by a second researcher experienced in qualitative analysis, and any discrepancies in coding were discussed until consensus was reached. This process is important for supporting the trustworthiness and accuracy of the qualitative findings. The next section presents the findings from the interviews:

Question: What was your opinion about trigonometry when it was first introduced in mathematics classes? P1 responded as follows:

> "Trigonometry, honestly, is a tough nut to crack, and I'm not a fan. I struggled to grasp the ideas because our math teacher rushed through the material. He seemed to think we'd already mastered geometry, which we hadn't, and that left me lost. It was a real struggle to keep up."

P2 responded to the same question with:

> "When it comes to trigonometry, our teacher only talked about the concepts when he came to class; he did not involve us. He then solved the examples he wrote on the board by himself. Therefore, I had difficulty learning and understanding the topic."

P5 provided the following answer to the same question:

> "At first, I thought trigonometry was tough. However, my perspective has changed because I now understand it."

Question: How would you describe your overall experience of learning trigonometry? P3 responded as follows:

> "At the beginning, learning and solving trigonometric problems was not easy. In contrast, the current trigonometry learning activities have been the most beneficial experiences I have had, mainly because the instructor uses good teaching methods that keep us engaged and make learning enjoyable. This approach has helped me understand trigonometry better. Moreover, the occasional jokes used in class also had a positive effect on us."

The answer to the same question, P4 responded,

> "I consider the trigonometric activities very interesting, especially because they include many unusual questions and operations. I learn a lot from these activities; they encourage me to think about and analyze problems before starting to solve them. Now, I can say that my understanding has improved."

Similarly, P2 stated:

> "Now I understand trigonometric concepts and can approach them without fear. Therefore, I believe I will perform well in this subject."

Question: In your opinion, is the new approach to teaching and learning trigonometry beneficial? If so, why? P1 responded:

> "Of course! We can now communicate our thoughts and offer answers with the use of visual and illustrative models, thanks to this innovative approach. We may actively participate in the teaching process using this."

P3's response to the same question was:

> "Yes! Being able to analyze questions more comfortably and present our solutions through the diagrams and graphs we use in class helps us gain confidence in learning and understanding the topic."

P4 responded similarly:

> "Indeed, simulations and visual arguments offer a way for everyone to engage in learning, helping us to identify answers and seek out knowledge."

Question: How did the use of visual and illustrative models change the way you learned trigonometry, relative to the teaching techniques used before? P3 responded:

> "When I actively participate in a task, I find it simpler to recall how to do it compared to merely being told how to do it. Therefore, being part of the activities in the Visual and Illustrative Models approach helped me understand better than simply listening to concepts."

P5 stated:

> "The visual and illustrative models method is practical, which means it includes everyone in the teaching and learning process."

P1 added:

> "It improved my understanding of trigonometric concepts much more than the previous method."

Question: What specific teaching and learning activities, while using visual and illustrative models to learn trigonometry, resonated with you, and what made them effective? P1 responded:

> "The use of visuals, the excellent support of simulations, and the presentations. These experiences helped me enhance my communication abilities and build my self-confidence."

In response to the same question, P2 responded,

> "Using diagrams and graphs to locate information encourages students to understand and learn about the subject."

Question: Do you think mathematics teachers should always utilize visual and illustrative approaches while teaching math? Why? P5 nodded and stated:

> "Absolutely! Applying this method across all areas of mathematics would significantly aid our comprehension."

P2 replied,

> "Absolutely! This approach will motivate students to engage fully in every aspect of the teaching and learning experience, and it will also facilitate a more seamless learning process."

Based on the students' remarks, they originally found trigonometry challenging. The majority of students asserted that their math teachers' teaching methods contributed to this difficulty during their lessons. In addition, students expressed satisfaction that the visual and illustrative models approach broadened their perspectives and eliminated boredom in mathematics classes. Furthermore, students believed that this approach played a significant role in improving communication skills, enhancing conceptual understanding, and increasing self-confidence. The authors also advocated using the visual and illustrative method for other math courses. They believed the approach would help students learn mathematics better and grasp it more deeply.

DISCUSSION

Preventing gaps in students' understanding of trigonometric definitions and facts is critically important. In our study, this issue manifested itself in three main areas that made it difficult for students to solve problems successfully: diagram analysis, the use of basic trigonometric equations, and the required algebraic computations. Because of gaps in their existing knowledge, several students were unable to even begin tackling the problems. First, although in the traditional approach students acquire trigonometric concepts as relative magnitudes within the context of right triangles, the connection of these concepts to the unit circle and functional graphs is generally not sufficiently understood. This lack of understanding leads to serious difficulties in solving trigonometric equations and inequalities, including selecting appropriate roots, taking periodicity into account, and generalizing solutions. To address this problem, the proposed visual model—constructing a trigonometric circle based on a coordinate spiral—was confirmed by our findings.

Through the use of this model, the concept of angle is interpreted not only as a geometric object but also as a dynamic process intrinsically connected to the set of real numbers. As a result, concepts such as positive and negative angles, the direction of arcs, and periodicity become more intuitive and spatially comprehensible for students. This finding supports studies in the literature (Doğan & Abdildaeva, 2013; Mumcu & Aktürk, 2020; Spangenberg, 2021).

It was determined that students had difficulty recalling both prior and newly learned trigonometric concepts, which hindered their ability to cope with trigonometric problems and resulted in insufficient procedural skills. This finding is strongly supported by previous research (Delice, 2003; Hatt, 2024), which revealed that students exhibit conceptual misunderstandings and technical errors stemming from memorizing solution procedures rather than understanding them. Participants also acknowledged that learning trigonometry is difficult because it involves abstract concepts and requires memorization of formulas without sufficient understanding.

Based on the interview data, participants identified trigonometry as a particularly difficult subject in high school. Students believed this difficulty arose from the subject's abstract nature and the teaching methods used by their math teachers. Furthermore, the interview responses showed that students in the experimental group were willing to learn trigonometry and understand its concepts.

The paired sample t-tests, which compared the scores of the experimental and control groups, showed that the experimental group, which used the visual and illustrative model approach, improved more from the pre-test to the post-test than the control group. In addition, the independent-samples t-test results showed a significant difference in post-test performance between those who used the visual and illustrative model technique and those in the control group, who were taught using traditional methods. This difference in achievement between the two groups suggests that the experimental group performed better on the post-test. The eta-squared value of 14.4% indicates a notable effect.

The results show a statistically significant difference in average achievement scores between students who used the visual and illustrative model strategy and those who used the traditional method in the post-test. Compared to the traditional approach, the visual and illustrative model strategy in trigonometry led to a significant improvement in student performance. Therefore, using visual and illustrative models during the treatment period clearly improved students' performance and achievement in trigonometry. This result is consistent with the findings of researchers (Maknun et al., 2020; Marufi et al., 2022) who have shown that the visual and illustrative model approach enhances students' mathematics learning, conceptual understanding, and academic achievement.

The findings indicate that students' improved performance and reasoning abilities can be attributed not merely to increased visual exposure, but to the explicit structuring of relationships among geometric, graphical, and analytical representations. In contrast to traditional instruction, where these representations are often presented together without systematic coordination, the proposed visual-illustrative model supported students in understanding how trigonometric functions emerge from rotational and spatial processes. This finding is consistent with prior research emphasizing the coordination of representations as a fundamental mechanism for conceptual learning in mathematics (Bornstein, 2020; Machmud et al., 2025).

The visual-illustrative approach examined in this study demonstrates that it supports not only students' algebraic procedural accuracy but also their reasoning processes. In contrast to instruction focused primarily on symbolic manipulation, the proposed model enabled students to interpret their solutions both visually and analytically, thereby fostering deeper conceptual engagement with trigonometric structures.

Students' reflections in the collected feedback highlighted the role of the visual models in making abstract relationships more transparent and cognitively accessible. Many students reported that the new approach helped them "see" the connections among previously fragmented representations. These perceptions provide important insight into how visual-illustrative modeling can function as a cognitive scaffold in advanced studies of trigonometry.

Students believed that using visual and illustrative models created an environment where content could be presented visually, analyzed with diagrams and graphs, and solutions found through visual reasoning and simulations. This approach increased student interest in the subject, improved their understanding of the material, and helped them become more confident learners. Consequently, they recommended that the visual and illustrative model approach be used in other branches of mathematics as well. This study's findings corroborate the conclusions of much previous research (Borba & Villarreal, 2005; Hamzić et al., 2024; Hashrin & Lestyanto, 2024; Mailizar et al., 2025) that shows how visual and illustrative models help students learn actively and grasp mathematical concepts. Therefore, from the very beginning of trigonometry instruction, continuous attention should be given to how the same properties appear and are visually represented in triangles, the unit circle, and the graphs of trigonometric functions. By the way, while **Appendix A** includes extended mathematical illustrations, these are intentionally separated from the empirical findings to maintain a clear distinction between didactic exposition and research reporting.

Limitations

Despite the significant contributions of this study to the literature, certain limitations should be considered. A quasi-experimental design was employed with a limited sample drawn from academically strong schools in a specific context, and the instructional intervention was relatively short in duration. Longer-term implementations are needed to examine the sustainability of learning gains and the transfer of conceptual understanding to other areas of mathematics. Although this study was conducted in a single geographic region, the sampling method—implemented in two academically strong high schools to examine the instructional potential of the proposed model—was appropriate, yet it may have implications for the generalizability of the findings. Students in these schools generally demonstrate higher academic levels, prior mathematical content knowledge, and familiarity with abstract reasoning, which may have influenced their responses to the intervention. Consequently, the observed gains in conceptual understanding and achievement may not be directly transferable to students of varying academic levels in schools with more heterogeneous profiles or limited instructional resources. Therefore, future research is needed to investigate the applicability and impact of this approach across diverse educational settings, including schools with different achievement levels, curriculum structures, and broader socio-educational contexts. Finally, the qualitative findings are based on students' self-reported perceptions, which may be subject to response bias. Although measures were taken to enhance reliability—including a structured interview protocol and peer-reviewed coding—future studies could triangulate interview data with classroom observations or teacher reflections.

CONCLUSION

During the study, the didactic effectiveness of a multiple-representation approach based on visual and illustrative modeling in teaching the fundamental concepts of trigonometry was analyzed. The focus of the analysis was on the relationships among the unit circle, right triangles, the graphs of trigonometric functions, and an alternative model proposed in this paper for constructing a trigonometric circle. An example involving the solution of a trigonometric inequality using different methods demonstrated that problem solving based on visual models significantly enhances students' understanding of the solution process. In the unit-circle-based solution, the trigonometric inequality was interpreted geometrically, making the process of determining solution intervals logically transparent. The graphical method allowed for a visual evaluation of the relationships between functions. The analytical solution obtained by introducing an auxiliary angle was integrated with the preceding visual representations, leading to a deeper understanding of the results. The use of dynamic mathematics software such as GeoGebra further increased the effectiveness of visual modeling. Dynamic changes in graphs, visualization of the effects of parameters, and real-time representation of solution intervals increased student engagement and supported the development of independent analytical skills.

This study examined the instructional efficacy of visual and illustrative modelling techniques in trigonometry training, emphasizing representational comprehension, reasoning, and academic performance. The results demonstrate that deliberately designed visual models—particularly alternate structures of the trigonometric circle—can significantly enhance students' capacity to integrate geometric, pictorial, and analytical representations. Theoretically, the study enhances previous frameworks on multiple representations by prioritizing structure above mere co-presentation. The suggested model illustrates that visual representations may serve both as explanatory instruments and as generative frameworks that facilitate conceptual advancement and reasoning in trigonometry. In practice, the findings indicate that teaching trigonometry can be improved by using visual models as main teaching tools instead of just extra resources. This approach provides teachers with a structured way to support students' understanding of complex topics, such as trigonometric inequalities and functional behavior.

Consequently, the number of errors in solving trigonometric concepts (equations and inequalities) decreased, while the quality of solution justification improved. Overall, the research findings indicate that the systematic implementation of a visual and illustrative modeling approach from the early stages of trigonometry instruction is appropriate. This approach not only ensures the solid acquisition of knowledge but also supports the development of students' mathematical thinking, spatial reasoning, and logical-analytical skills.

In conclusion, examining a problem through multiple visual representations consistently stimulates students' interest, which in turn facilitates retention of the material. This approach not only enriches the external presentation of the content but also enables a deeper understanding of its internal logical structure. Through visual-illustrative representations, relationships among concepts become clearly visible, and the transition from abstract reasoning to concrete understanding is facilitated. As a result, students' abilities to analyze problem statements, justify solution methods, and generalize results are enhanced. Most importantly, this visual-illustrative approach promotes conscious knowledge acquisition rather than mechanical memorization.

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APPENDIX A: ILLUSTRATIVE TEACHING ARTEFACTS

The mathematical explanations and worked examples presented in this appendix are provided as illustrated teaching artefacts intended to clarify the structure and pedagogical logic of the proposed visual and illustrative model. These materials are not part of the empirical data analysis but are included to support transparency, replicability, and instructional transferability of the intervention.

Example 1. Solve the inequality $sin2a - 3cos2a \geq 0$.

Method 1. Let us introduce a substitution: $z = 2a$. Then the inequality becomes $sinz - 3cosz \geq 0$, or $\frac{sinz}{cosz} \geq 3$. Considering that on the unit circle $sinz = y$ and $cosz = x$, this inequality can be written as $y \geq 3x$. We now look for the points $(x; y)$ on the unit circle that satisfy $y \geq 3x$ (**Figure A1**). To do this, we draw the line $y = 3x$. Its slope is 3 ($tgz = 3$), so $z = arctg3$. From the intersection of the line $y = 3x$ with the unit circle, we obtain the solution $arctg3 + 2\pi k \leq z \leq \pi + arctg3 + 2\pi k, k\epsilon Z$. Returning to the variable a ($z = 2a$), we divide the inequalities by 2: $\frac{1}{2}arctg3 + \pi k \leq a \leq \frac{1}{2}arctg3 + \frac{\pi}{2}(1 + 2k)$, $k\epsilon Z$. This method is practically convenient and allows for a quick solution of the inequality.

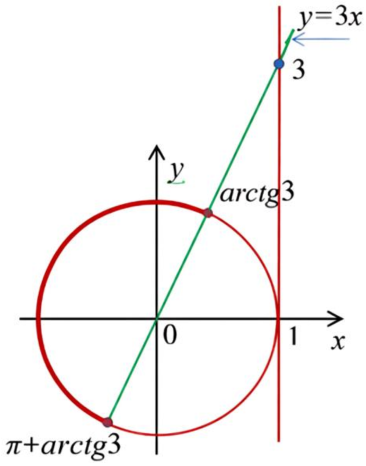

Figure A1. Method 1 for example 1 (Source: Authors' own elaboration)

Method 2. We plot the graphs of the following functions: $f(x) = sin2x$ and $g(x) = 3cos2x$. Next, we determine the intervals where the inequality $f(x) \geq g(x)$ holds true. That is, we identify the intervals in which the graph of $g(x)$ lies below the graph of $f(x)$ at each point (**Figure A2**).

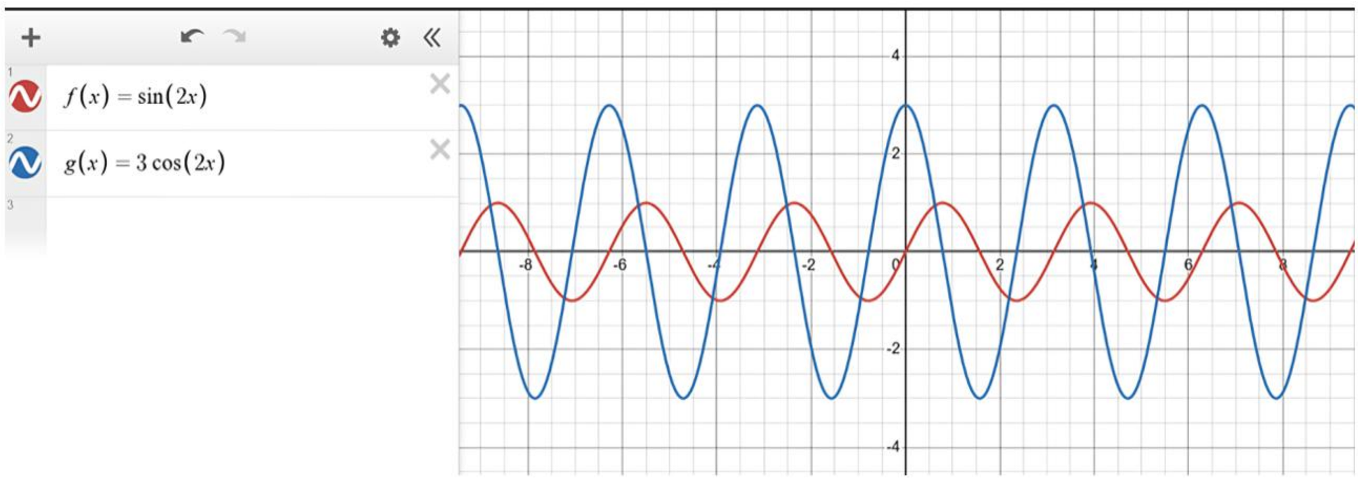

Figure A2. Method 2 for example 1 (Source: Authors' own elaboration)

Method 3. By introducing an auxiliary angle, we transform the inequality into the following form: $\sqrt{10}\sin(2x - arctg3) \geq 0$, or equivalently $\sin(2x - arctg3) \geq 0$. Next, we plot the graph of the function $f(x) = \sin(2x - arctg3)$ and identify the intervals where it is zero or takes positive values (see **Figure A3**).

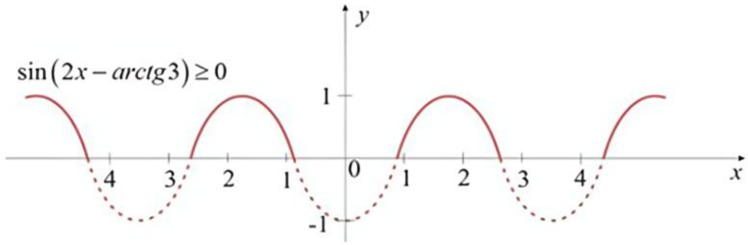

Figure A3. Method 3 for example 1 (Source: Authors' own elaboration)

Note. In the GeoGebra program, the $arctg$ function is written as tg^{-1}. In conclusion, such methodological ideas have previously been proposed by several authors, and in our opinion, their implementation in teaching practice is appropriate and beneficial.

Method 4. We restrict ourselves to the case where $z = 2a$ is an acute angle in a right-angled triangle. It is known that the ratio $sinz = 3cosz$ is satisfied in a right-angled triangle where one leg is three times the length of the other, i.e., $z = artctg3$. If we keep the shorter leg fixed and increase the longer leg indefinitely, the value of $sinz$ becomes progressively greater than $3cosz$ (**Figure A4**). Therefore, the solution can be expressed by the interval: $arctg3 \leq z \leq 90^0$ or equivalently $\frac{1}{2}arctg3 \leq a \leq 45^0$.

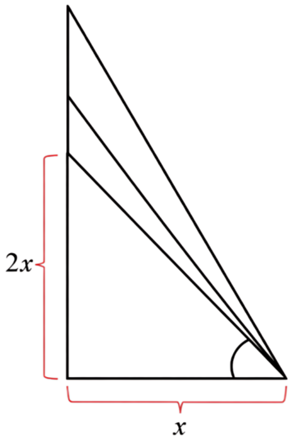

Figure A4. Method 4 for example 1 (Source: Authors' own elaboration)

Example 2. Calculate, $arctg1 + arctag\frac{1}{2} + arctg\frac{1}{3}$.

Method 1. In the Cartesian coordinate system, we take the points $A(0;0)$, $B(2;1)$, $C(5;0)$ and $D(2;0)$ and connect them (**Figure A5**). As a result, triangle ABC is formed with height $BD = 2$. In this configuration, $\angle BAD = arctg\frac{1}{2}$, $\angle BCD = arctg\frac{1}{3}$. Using the law of cosines in triangle ABC: $25 = 15 - 2\sqrt{50}\,cos\angle ABC \Rightarrow cos\angle ABC = -\frac{\sqrt{2}}{2}$, $cos\angle ABC = \frac{3\pi}{4}$. Thus, $\angle BAD + \angle BCD = arctg\frac{1}{2} + arctg\frac{1}{3} = \pi$. Taking into account that $arctg1 = \frac{\pi}{4}$, we obtain: $arctg1 + arctag\frac{1}{2} + arctg\frac{1}{3} = \frac{\pi}{2}$.

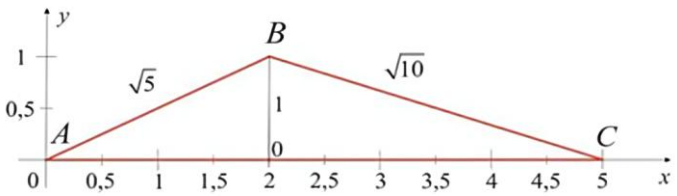

Figure A5. Method 1 for example 2 (Source: Authors' own elaboration)

Method 2. Since $arctg1 = \frac{\pi}{4}$, it follows that $arctg\frac{1}{2} + arctg\frac{1}{3} \epsilon \left(0; \frac{\pi}{2}\right)$. Using the tangent addition formula: $tg\left(arctg\frac{1}{2} + arctg\frac{1}{3}\right) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = 1$. Hence, $arctag\frac{1}{2} + arctg\frac{1}{3} = \frac{\pi}{4}$. Therefore, $arctg1 + arctag\frac{1}{2} + arctg\frac{1}{3} = \frac{\pi}{2}$.

Method 3. Consider a rectangle with side lengths 2 and 3, and construct the required tangents from point A (**Figure A6**). Let $\angle EAB = arctg\frac{1}{3}$, $\angle CAF = arctg\frac{1}{2}$. Then, $\angle FAE = \frac{\pi}{4} = arctg1$ and $\angle AFE = \frac{\pi}{2}$. Thus, triangle AFE is a right isosceles triangle, from which it follows that $arctg1 + arctag\frac{1}{2} + arctg\frac{1}{3} = \frac{\pi}{2}$.

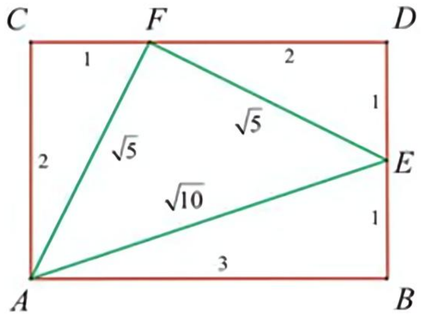

Figure A6. Method 2 for example 2 (Source: Authors' own elaboration)