


## The impact of mathematics learning environment supported by error-analysis activities on classroom interaction

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### Abstract

This study was designed to investigate the effect of mathematics learning environment supported by error-analysis activities on classroom interaction. To achieve this purpose, two classes of seventh grade students (aged 12-13 years) were randomly selected and were assigned into two groups; experimental (number of student=24) and control (number of students=24). The experimental group was exposed to error-analysis activities, whereas the control group studied the same mathematics content without any error-analysis activities. Moreover, two instruments were used to collect the data: an observation checklist including indicators of classroom interaction and a semi-structured interview, after ensuring their validity and reliability.

14 classes for each group were observed by two observers using an observation checklist. The findings of the study revealed that statistically significant differences were found between the rating means of classroom interaction of the two groups. Moreover, the qualitative analysis of the interviews revealed that the mathematics error-analysis activities contribute to improving the quality of teacher-student, student-student and student-content interaction. They enhance the quality of students' responses, help students be more engaged in mathematics learning through social interaction and more active in oral communication, improve their classroom predications and discussions and support student-content interaction through sustaining error-analysis to be a learning behavior. Based on these findings, it was recommended that mathematics learning environment supported by error-analysis activities could be adopted as a teaching-learning strategy to improve classroom interaction, which enhances students' mathematics learning in primary education.

**Keywords:** error-analysis, mathematics learning, classroom interaction, learning environment, primary education

### INTRODUCTION

Classroom interaction is regarded as one of the indispensable and influencing variables in mathematics learning environments. It is also a critical factor in determining the quality of students' mathematics learning (Bippus & Young, 2000; Davadas & Lay, 2018; Mccarthy et al., 2016). Because it is the focal point in teaching and learning settings, it is considered as one of the main factors that contribute to achieving high teaching quality and good learning outcomes among students.

Many researchers (Apriliyanto et al., 2018; Dallimore et al., 2004; Davadas & Lay, 2018; Kent, 2017; Mccarthy et al., 2016; Tatar, 2005) emphasized that in order for students to develop their abilities and establish an in-depth understanding of mathematical knowledge and skills, mathematics learning practices must concentrate on classroom teacher-students, students-teacher, and students-students interactions.

Therefore, educators emphasized that the best classroom interactions in mathematics learning are those that focus on the student to be the centered of learning

### **Contribution to the literature**

- The study contributes to the mathematics education by adopting the current learning theory that concentrates on supporting students to constitute their knowledge by creating an educational learning environment.
- This study encourages students to discover their mathematical errors and misunderstandings, search for their causes and solve them, noting that the activities of analyzing and solving mathematical errors correspond to the constructivist approach.
- This study opens new projects for educational researchers to conduct further studies in the field of pedagogy of the analysis of mathematical errors.

rather than the teacher. Based on the importance of this, the Jordanian mathematics curriculum reform movement identified the importance of activating the role of the student to be a centered in all classroom interactions.

Despite the importance of this trend, however, educational studies in the Jordanian environment showed that the teacher is still the centered of learning, with content-overloaded books, where this cannot offer effective classroom interaction. Moreover, the low performance in TIMSS and PISA among school students in Jordan might suggest that teaching methods and learning models are not aligned with the skills that should be learned by students, where classroom social interaction is essential for mathematics learning. In addition, mathematics teachers face challenges in their feedback moves in response to their students' errors, where students should learn from these moves in the learning process and increase the quantity and quality of classroom interaction.

In the context of mathematical errors, Ayuwanti and Dwiswoyo (2021) and Santagata (2005) claimed that no body denies the fact mathematical errors are part of the learning process, especially when the constructivist approach is used by students to connect the new information with their previous knowledge. Previous research by Heinze (2005) and Santagata (2005) revealed that mathematics teachers handle mathematical errors in a bothering way during classroom interactions, where they overlook the error-analysis process and do not give opportunities to students to deal with these errors, but rather mostly correct the errors by themselves without explanation. This was confirmed by Brodie (2008), where the focus of the teachers is to get the correct answers of the tasks presented during classroom interaction. Moreover, few studies at the international level have discussed the classroom interaction in relation to mathematical environment supported by error-analysis activities and relevant studies at the national level are of a diagnostic nature.

Based on the above, this study emerged from the fact that it reveals an aspect that serves the educational process, which is the improvement and increase of classroom interaction among students, which is an important universal standard for judging the quality of

the teaching-learning process (Gamlem, 2018). Moreover, this study lies in the fact that it adopts the pedagogy of mistake and the theory of negative experience, which considers errors as positive opportunities to learn mathematics. Therefore, mathematics teachers shall capitalize mathematical errors and employ them in the teaching-learning process in order to assist students in overcoming them (Gartmeier et al., 2008; Larrain & Kaiser, 2022; Parviainen, 2006; Wildgans-Lang et al., 2020).

Furthermore, the current study adopts the current learning theory that focuses on enabling students to constitute their knowledge by creating an educational learning environment that encourages students to discover their mathematical errors and misunderstandings, search for their causes and solve them, noting that the activities of analyzing and solving mathematical errors correspond to the constructivist approach as stated by Gedik et al. (2017) and Heinze (2005), in contrast to the behavioral approach that avoids errors and tries to emphasize successful students' activities only, which means that the positive knowledge matters only.

Hence, the importance of this study lies in providing activities of how to employ the analysis of mathematical errors and mathematical misunderstandings in facilitating and increasing classroom interaction, which may allow teachers and developers of teacher guides to be informed about mathematical errors and benefit from them. Therefore, this study provides a participatory learning environment dominated by effective discussions, acceptance and respect of the other, taking risks and not being afraid of failing to solve mathematical problems.

In the field of research, this study opens new ventures for researchers to conduct further studies in the field of pedagogy of mistake, the theory of negative experience and its effect on acquiring many mathematical concepts and various thinking skills, such as reflective thinking.

Based on the above, this study sought to investigate the effect of mathematical learning environment supported by error-analysis activities on improving classroom interaction. In other words, this study attempts to answer the following questions:

1. What is the effect of mathematics learning environment supported by error-analysis activities on classroom interaction?
2. How does mathematics learning environment supported by error-analysis activities improve classroom interaction?

Based on the above questions, the study aimed to investigate the effect of mathematical learning environment supported by error-analysis activities on improving classroom interaction among seventh grade students. Also, the study tried to find out the crucial role of mathematics learning environment supported by error-analysis activities on enhancing students' classroom interaction. In addition, it introduces a set of recommendations that could play an essential role in developing mathematical learning.

## THEORETICAL BACKGROUND

Effective learning of mathematics is directly associated to the teacher's ability to establish constructive and critical classroom interaction in mathematics learning environment. Classroom interaction is a fundamental component in the mathematics teaching and learning process. It reflects an in-depth understanding and vitality of acquiring mathematical knowledge, skills and experiences, as well as achieving a long-term effect in enabling students to apply knowledge in various life contexts (Al-Barakat et al., 2022b; Al-Hassan et al., 2022; Fraihat et al., 2022; Khasawneh et al., 2022). Bippus and Young (2000) mentioned that classroom interaction is the way in which students can participate and be engaged in various class discussions and refrain from negative behaviors.

Nura and Zubairu (2015) defined classroom interaction as all discussions, interactions and exchanges of ideas that dominate the classroom in an organized, purposeful and constructive manner, in order to help students develop a desire to learning and reinforce their motivation towards it.

From a constructivist learning viewpoint, classroom interaction is defined as the effective use of language in various real learning situations using interactions, sharing thoughts and working in groups to solve problems in authentic-life situation (Majid et al., 2010). On other hand, Jaber (2004) defined classroom interaction as a set of verbal and non-verbal behaviors of the teacher and students in a specific situation while achieving a balance between meeting their needs and achieving the educational goals.

More clearly, classroom interaction is defined as a type of student participation that is characterized by three characteristics: quantity, dependability and quality (Petress, 2006). More specifically, quantity was defined as the number of learning opportunities given to students to take part in all classrooms activities

constructively. Dependability relates to having learners who can contribute to different interactions "appropriately, obviously and respectfully" when asked to do so in the classroom, while quality of classroom interaction relates to students who show their cognitive abilities through giving evidence of personal awareness of conceptual and procedural knowledge discussed in the class and this involves some repetition of interactions (Murray & McConachy, 2018; Nura & Zubairu, 2015; Petress, 2006; Rocca, 2010; Theriault, 2019).

Many studies emphasized the significance of classroom interaction in the teaching-learning process; as it creates an effective learning environment, helps the teacher develops his/her teaching method, adjust the educational process, communicate with students, exchange ideas with them and helps students self-control and increase self-confidence. Additionally, classroom interaction assists students in developing multiple aspects of their cognitive and emotional personality (Henouda & Jaber, 2017).

There are two types of classroom interaction: verbal interaction, which means all types of speech used in the classroom, including providing instructions, guidance, encouragement expressions, asking questions by teacher, answering such questions by students and exchanging ideas between the teacher and students and between students themselves. The second type is non-verbal interaction, which refers to all types of movements and gestures used in the classroom, including head and hand movements, facial expressions, among others (Al-Khataiba et al., 2004).

Additionally, there are two models of classroom interaction; the first one is the teacher-centered classroom interaction in which the teacher is the knowledge owner and transmitter, and the role of the learner is a receiver of knowledge. The other model of classroom interaction is the student-centered classroom interaction, where the learner is active and involved in building his/her knowledge and the role of the teacher is determined through guidance and counseling (Apriliyanto et al., 2018; Qadri, 2012).

Qadri (2012) mentioned several factors affecting classroom interaction, some of which are related to school and classroom environments such as school location, size, number of classrooms and classroom capacity, whereas others are related to the teacher, such as teacher gender, his/her personality, attitude towards education and educational level, along with those related to the students, such as gender, individual differences and behavior.

In order to create an effective mathematics learning environment, interventions based on non-traditional (non-routine) mathematical tasks must be posed by teachers to facilitate and increase classroom interaction. Sánchez-Barbero et al. (2020) found that non-traditional mathematical tasks enhance classroom interaction, as

they are challenging for teacher and students while dealing with them in the classroom. Various studies (Kirlakidis & Johnson, 2011; Lichtenstein, 2005; Torok et al., 2004) reported that the classroom environment should be relaxing, playful, helpful and comfortable, use non-traditional mathematical tasks in a friendly-manner that allows students to explore activities together and provide several opportunities for them to develop informal communication methods during classroom interaction. Hegseth (2021) reported that when the teacher works on establishing a classroom environment of mutual respect and focuses on classroom interaction's standards in mathematics classes, students become more likely to engage effectively in class discussions and verbal interactions that improve their mathematical abilities.

The term learning environment refers to "the social, physical, psychological and pedagogical contexts in which learning occurs and which affect student achievement and attitudes, in addition to teacher support, involvement of students and personal relevance" (Afari, 2012, p. 34). In the same context, Fraser (2012) claimed that learning environment is an important support of student learning and classroom interaction, where students support each other, teachers behave as friends and help students participate, pay attention and link the taught content in the classroom with their life experiences. Moreover, Ayuwant and Dwiswoyo (2021) showed that teacher-student interaction has affected students' mathematics understanding, as was evident through students' interviews.

Apriliyanto et al. (2017) concluded that students' social interaction positively affects their achievement in learning mathematics. Smith et al. (2013) reported that active learning strategies are considered to have a positive effect on classroom interaction. Khuwailid and Seham (2018) concluded that the relation between classroom interaction and the competency approach is a complementary relation, meaning that each of them serves the other.

In view of the nature of mathematics, which requires special learning efforts by students to become at a high or at least an acceptable level of mathematical proficiency, teachers are required to select the best strategies and methods to help their students and invest any classroom setting in the teaching process to enable students to acquire mathematical concepts, master mathematics skills, be able to justify mathematical settings and reflect on their solutions to problems presented to them (Azevedo et al., 2012; Larrain & Kaiser, 2022; Wildgans-Lang et al., 2020; Yeh et al., 2019). Despite serious attempts to achieve the above, students face difficulties in learning mathematics and practice procedural and conceptual errors, misconceptions and misunderstandings of mathematical concepts during solving mathematical problems and participating in

class discussions (Ambasa & Tan, 2022; Legarde, 2022; Metcalfe, 2016; Murillo & Tan, 2019)

In the same context, behaviorism supports the previous views, as it assumes that learning is reinforced when the correct responses are rewarded (positive reinforcement), while incorrect responses are punished or ignored (withholding positive reinforcement). Many mathematics teachers who teach using traditional methods consider explicit attention to mathematical mistakes in the classroom to be critical, as it can interfere with reforming the correct result in the student's mind (Asparin & Tan, 2018; Mariano, 2019; Stonewater, 2005; Tarmizi & Bayat, 2012). Meanwhile, educators who adopt constructivism in education encourage the inclusion of error-analysis in curricula and teaching strategies as a strong learning opportunity (Bray, 2013; Kramarski & Zoldan, 2008; Mariano, 2019; Metcalfe, 2016; Rushton, 2018; Schommer-Aikins et al., 2005; Stonewater, 2005; Suleiman & Hamed, 2019; Tarmizi & Bayat, 2012).

In the context of error-analysis activities, Rach et al. (2012) suggested four practices that enhance learning from errors: error identification or error awareness, error-analysis in the sense of explaining the error, error correction and error prevention, in the sense of using strategies that avoid the repetition of the error in the future. Kyaruzi et al. (2020) argued that the process that involves the previous four practices is denoted by the analytic process-oriented approach to learning from errors.

Hansen (2005), Herholdt and Sapire (2014), Mallue (2015), and Rong and Mononen (2022) acknowledged that mathematical errors can be used as a starting point to learn mathematics, in addition to the positive role of error understanding in improving mathematics learning. Metcalfe (2016) stated that it is useful to allow students to make mistakes and even encourage them to commit them in the classroom instead of avoiding them to achieve optimal performance in high-risk situations, while Rushton (2018) supported the analysis of mathematical errors in learning mathematics to improve various thinking skills. In the same context, Monthienvichienchai and Melis (2006) indicated the benefits of including mathematical errors as opportunities to improve students' attitudes towards failure and their motivation toward understanding mathematical concepts, enhance the ability of mathematical reasoning and train self-regulation and self-interpretation for judging whether the solution steps are right or wrong.

Based on the above, it can be underlined that the inclusion of mathematical error analysis and misunderstanding of mathematical ideas in learning settings can motivate the teacher to reduce students' committing of these errors and facilitate and increase the interaction in the classroom, with the need to increase

the awareness of the teacher and students to deal positively with errors and misconceptions related to mathematical concepts, where the student reflects on the error and analyzes it in order to determine the reasons for falling into it and then correct it if possible. The teacher, in turn, performs his/her role in meditating on the student's errors, understanding what is going on in his/her mind, analyzing the procedural and conceptual errors and attempting to solve them in order to modify them with the student.

Lischka et al. (2018) used a broad definition of mathematical mistakes to include erroneous understanding, faulty procedures for solution, ineffective solution strategies in solving mathematical problems and incomplete mathematical arguments. Lischka et al. (2018) defined three standards for mathematical errors that deserve examination and analysis in the classroom, including the objective of the lesson, whether error-analysis stimulates student understanding and achieves goals; error prevalence and whether the error is common; and whether the learner has a fundamental misunderstanding of basic mathematical concepts. Furthermore, Priyani and Ekawati (2018) categorized mathematical errors into three categories; conceptual errors (misconceptions), procedural errors and the inability to complete solving a problem due to a previous error.

Students face many difficulties in understanding the various concepts and procedures listed under many mathematical topics. Also, there are common mathematical errors that appear among students during the class settings, homework and various assessments. Wagner (1981) claimed that learning usually includes making errors, while Rach et al. (2012) argued that many students do not use errors to enhance their mathematical learning, although they value the way in which their teachers deal with mathematics errors in the classroom and although they recorded low anxiety levels in error situations.

Moreover, classroom interaction is considered to be a significant criterion for the quality of teaching-learning process (Gamlem, 2018; Larrain & Kaiser, 2022; Wildgans-Lang et al., 2020). The success of this process in the classroom depends on creating an appropriate environment that encourages classroom interaction (Al-Khataiba et al., 2004; Wildgans-Lang et al., 2020). On top of that, classroom interaction is considered as one of the challenges that face mathematics teachers when planning to teach a specific mathematical subject. In addition, some students refrain from classroom interaction in mathematics due to several factors, some of which are related to the nature of mathematics, some are related to the teacher and others are related to their culture. Investing mathematical error-analysis to facilitate classroom interaction did not receive much research, which provides a justification for researchers to address and investigate it. Thus, the current study

attempts to investigate whether creating an environment that allows mathematical errors in the classroom provides opportunities to facilitate and increase classroom interaction.

## METHODOLOGY

### Sample of the Study

The study sample was selected using the convenient sampling method from seventh-grade students (aged 12-13 years) affiliated to a public school in Jordan, where one of the researchers teaches mathematics at this school. Two classes were randomly selected and were assigned into two groups: the experimental group consisted of 24 students and the control group consisted of 24 students. Both the experimental and the control groups were taught by the same teacher. The study sample in the experimental and control groups were randomly selected from five sessions available in the school. In addition, the study sample was selected from one area with similar conditions. All students are similar in academic, economic and social conditions. The treatment lasted four weeks (14 classes), each 45 minutes, and 14 classroom observations were conducted for each of the experimental and the control groups.

### Learning Environment Supported by Error-Analysis Activities

Teacher, students, and the mathematical content of ratio, proportion and proportional reasoning interacted to shape the learning environment. More specifically, the students learnt the following topics: ratio, proportion, direct proportion, inverse proportion, proportional division and drawing scale. The students were involved in learning activities that concentrate on error-analysis and they have to link their previous knowledge with the new one. The learning activities concentrate on the error-analysis process, which includes discovering, explaining and correcting errors by students themselves. The teacher assigned the learning activities (tasks with correct solutions and tasks with erroneous solutions) and motivated the students towards self-learning supported by activating feedback.

The adopted pedagogy of the learning environment in the current study is summarized by many principles aligned with the new view of teaching-learning process. These principles are errors are normal to occur in practicing and solving mathematics problems; errors support and enhance mathematical learning; errors play an important role in renewing the learner's mathematical knowledge; errors are a diagnostic assessment tool; and errors motivate the teacher to plan, implement and evaluate the teaching-learning process during problem solving. Regarding these principles, the learning environment was designed based on error-analysis, where many mathematical activities were

designed including real-life tasks with their erroneous solutions. Examples of these activities are:

1. **The fastest-car task:** "Car A travels a distance of 280 km in two hours, while car B travels 420 km in four hours. Which car is faster? Samer answered that car B is faster, because it travels 420 km while car A travels 280 km. Is Samer right? if not, what is his misconception? Explain it and correct that with reasoning" (comparison problem).

The reasoning of Samer is incorrect, he just compared the distances only, ignoring the time. He should use unit rate strategy, which means finding the distance traveled in one hour for both cars (car A travels 140 km/hr and car B travels 105 km/hr. So, A is faster). Or using the equivalent rates:  $2 \times 280 / 2 \times 2 = 560 / 4$ , where car A travels 560 km in four hours, and car B travels 420 km, so car A is faster.

2. **The fruit salad recipe task:** "To make fruit salad, we need two cups of banana for each three cups of apple. Manar has nine cups of apple and she used eight cups of banana to prepare the same recipe of fruit salad. What is Manar's error? Explain it and guide her to prepare the fruit salad with the right recipe" (missing-value problem).

The error in task 2 is using additive reasoning instead of multiplicative reasoning (where Manar added six to both two and three and did not realize that the situation is proportional), she should multiply  $3 \times 2$  and  $3 \times 3$  to get that she needs six cups of banana to prepare the salad.

3. **Bike task:** "A factory produces 50 bikes in five days. How many bikes does it produce in 30 days? Soha solved the problem, as follows: the number of bikes is directly proportional with the number of days; so the constant of proportionality  $= 5 \div 50 = 0.1$ . Thus, the number of produced bikes in 30 days  $= 0.1 \times 30 = 3$  bikes. Discover Soha's error, explain it and solve the problem correctly and check the correctness of the solution using different methods" (direct-proportion problem).

Soha realized that the situation is direct proportion, but she multiplied the constant of proportionality by 30 instead of dividing 30 by 0.1 to get 300 bikes. Also, she missed the multiplicative reasoning, where the number of days was doubled by six,  $5 \times 6 = 30$ , so the number of bikes should be doubled by six, so  $50 \times 6 = 300$  bikes.

4. **Fishing boat task:** "It takes nine men 14 days to build a fishing boat. How many days does it take 6 men to build it, where the rate of each man attainment is constant? Laila answered: The number

of men and the number of days to build the fishing boat are inversely proportional, because when the number of men increases, the number of days decreases. The number of men decreases by three, where  $9 - 6 = 3$ ; so the number of days will increase by three, and  $14 + 3 = 17$ ; i.e. six men need 17 days to build the boat. What is Laila's error? Explain it and correct the solution" (inverse-proportion problem).

Although Laila realized that the situation is inverse proportion, but she still use additive reasoning for proportion situations. She should find the proportion constant,  $9 \times 14 = 126$ , so number of days needed for six men to complete the work is  $126 \div 6 = 21$  days. Also she could use the parallel multiplication:  $9 \times 14 = y \times 6$ , so  $y = 126 \div 6 = 21$ .

5. **Mixed-colors task:** "Two colors; white and red, were mixed with a ratio of 1:2 in order to get 12 liters of a new color. Calculate the amount of liters used from each color. Sawsan answered: Six liters from the white color and six liters from the red one; so  $6 + 6 = 12$ . Is Sawsan's answer correct? Justify your answer and if the answer is incorrect, give the right solution" (proportional division problem).

It is clear that Sawsan did not understand the proportional division concept, just she looked for two numbers with sum 12 without considering that the ratio of the mixed colors is 1:2, and she had a misconception of the ratio part to whole. The correct solution is the whole  $= 2 + 1 = 3$ , the amount of one part  $= 12 \div 3 = 4$ , red liters  $= 2 \times 4 = 8$ , white liters  $= 1 \times 4 = 4$ , so  $8 + 4 = 12$ .

Mathematical errors in the current study refer to erroneous understanding or misunderstanding of mathematical concepts, procedural errors, ineffective solution strategies in solving mathematical problems or incomplete mathematical arguments. This is associated with three criteria for mathematical errors that are worth examining and analyzing in the classroom, motivating students' understanding, achieving the lesson's objectives and the prevalence of errors and reporting a fundamental misunderstanding among students of basic mathematical concepts. The common patterns of errors in ratio, proportion and proportional reasoning were the main concern, such as: errors in understanding ratio and proportion concepts, errors in forming a proportion from a proportional real-life situation, inability to differentiate between direct proportion and inverse proportion, inability to solve comparison problems, missing-value problems and proportional division.

Mathematical activities were designed based on the recommendations of the previous literature in teaching

proportionality and based on mathematical error-analysis activities, where the mathematical activities address examples containing correct solutions and others containing incorrect solutions (Cai & Sun, 2002; Hunter et al., 2013; Khabour et al., 2019; Khasawneh et al., 2022; Singh, 2000). These mathematical activities concentrated on encouraging students to detect mathematical errors, justify their reasons, correct and solve them again. Regarding the control group, the same content was used, and the focus was on activities with correct solutions only, without addressing erroneous examples and when a mistake was found in the homework, during class lessons or short tests, it is directly corrected without addressing its analysis, justification or reviewing. Moreover, the treatment took four weeks to be conducted.

One of the researchers himself taught the same content for both the experimental and the control groups, where the experimental group was taught through a mathematical environment supported by error-analysis activities. The learning environment for the experimental group encourages students' proper works, allowing mathematical errors in the classroom, reducing students' embarrassment when they make them and discussing expected mathematical mistakes that are worth to be examined and analyzed and that may appear during teaching the topic or in homework.

### Instruments of the Study

To answer the study questions, the mixed methods research was used. It combines and integrates elements of quantitative and qualitative methods. This refers to the ways in which qualitative and quantitative research procedures are used together to give the researcher greater insight. The integration process can take place during gathering and analyzing data, or during presenting findings. On this basis observation checklist and semi-structured interview were used in this study. These instruments can be presented, as follows.

#### *First: Observation checklist*

To achieve the objectives of the current study, an observation checklist was used, where its items were prepared and developed in light of the different procedural definitions of classroom interaction in different previous studies (Apriliyanto et al., 2018; Bippus & Young, 2000; Dallimore et al., 2004; Davadas & Lay, 2018; Madanat, 2019; Mccarthy et al., 2016; Nura & Zubairu, (2015); Tatar, 2005). The observation checklist, in its final form, included different indicators: Sympathy, respect and collaboration (items: 1, 3, 4, 6, & 8), raising questions (items: 9, 11, 15, & 20), interaction of students with content (items: 2, 5, 7, 13, 14, & 16) and teacher support (10, 12, 17, 18, & 19). All items in the observation checklist were formulated to be observable and measurable in classroom interactions.

Primarily, a 5-likert-scaled, 23-item checklist was prepared and verified for validity by a group of reviewers in the field of mathematics education and measurement and evaluation. Minor amendments were suggested by the reviewers related to deleting three items; namely, the checklist was reduced to consist in its final form of 20 items. Classroom observation was used to rate classroom interaction on a 1-to-5 response scale, where 1=very low practice, 2=low practice, 3=moderate practice, 4=high practice and 5=very high practice. Cronbach's alpha coefficient was calculated to measure the internal consistency of the observation checklist, which was established at 0.89, which is proper for the study purposes. Moreover, the validity of the internal consistency of the observation checklist was calculated, by checking the correlation between each item's rating and the total instrument's rating, where the correlation coefficients ranged from 0.35 to 0.77, which are acceptable for the purpose of the current study.

#### *Data collection & analysis of observation checklist*

After obtaining the personal approval of the respondents to conduct the classroom observation, twenty-eight classroom observations were made for the experimental and control groups. This means that 14 classroom observations were conducted for the experimental group and 14 classroom observations were conducted for the control group. All classroom observations were attended at a rate of 45 minutes per class for each classroom observation.

Each classroom observation was conducted by two observers, where each observer filled out one checklist for each classroom situation with complete independence. The number of observation checklists recorded for each learning class was two. This means that the total observations made by the two observers were 28 observations per group.

To establish reliability between the two observers in filling out the observation checklist, the intra-class correlation coefficient between observers' estimates of class observations was calculated at the level of each class observation (Yaffee, 1998). The consistency coefficient was 0.95 for their estimates for the classroom observations. In addition, Cooper's equation (Cooper, 1981) was used to find the coefficient of agreement between the observers' estimates on each classroom observation. The consistency coefficient was 0.97.

To analyze the data collected by the observation checklist, arithmetic means and standard deviations of the two observers' estimates of all classroom observations were calculated at the level of each item in the observation checklist. The maximum score for each item was five and the minimum score was one.

In order to explore the effect of mathematical error-analysis activities on classroom interactions, the mean ranks and the sum ranks of the 14 observations of each

of the experimental and the control groups were calculated to find out whether there were significant differences between the estimations of the classroom interaction of the two groups and Mann-Whitney U test was used.

### *Second: Semi-structured interview*

The purpose of the interview was to provide a clear vision in regard of the way in which mathematical error-analysis activities contributes to improving classroom interaction in math learning environments. In-depth semi-structured interviews with nine students of the experimental group were conducted. An interview schedule was prepared through a literature review (Apriliyanto et al., 2018; Bippus & Young, 2000; Dallimore et al., 2004; Davadas & Lay, 2018; Mccarthy et al., 2016; Nura & Zubairu, 2015; Tatar, 2005).

Semi-structured interview schedule was validated by a jury of mathematics education and measurement and evaluation specialists and piloted on 6 participants who were omitted from the sample of the study to gather data related to the aim of the interview. Semi-structured interview was chosen to achieve the aims of the study, because it gives the interviewer a superior depth of answers and freedom to probe more profoundly into and extend the replies of the respondents (Creswell, 2018; Creswell & Plano Clark, 2007; Makri & Neely, 2021; Patton, 2015).

### *Data collection & analysis of semi-structured interview*

Nine students from the experimental group were selected to participate in the interview. This selection was based on the students' level of academic achievement in mathematics according to their previous school grades. In order to ensure credibility in selecting the subjects of the interview sample, the students were chosen from three categories: three students from each of the high, the medium and the low levels of achievement.

Previous consent was obtained from all respondents to take part in the study and the interview appointments were fixed and organized. All respondents were informed of the purpose of the interview and guaranteed the confidentiality of the data they provide in the interviews, which were audio-taped and immediately transcribed.

The length of the interview ranged between 15 and 25 minutes per interviewee. All interviews were transcribed, and each transcript was shown to the respective respondent to assure whether or not it truthfully represented his/her responses. The interview sample was furthermore questioned whether they would like to add or omit anything. All respondents displayed their satisfaction with their primary answers. The interview transcripts were then carefully examined,

the data classified into main categories and sub-categories, percentages calculated for each category and excerpts selected to use in the presentation of the findings.

## **FINDINGS OF THE STUDY**

The findings of the study are presented as follows:

### **Part One: Findings of the Effect of Mathematical Error-Analysis Activities on Classroom Interaction**

The first question of the study aimed to find out the effect of mathematical error-analysis activities on classroom interaction among seventh-grade students. To achieve this aim, 28 teaching and learning situations were observed for the two groups (14 classes for the experimental group and 14 for the control group), where each lasted 45 minutes. In order to test the effect of error-analysis activities on classroom interaction among the experimental group's students who were taught using mathematical error-analysis activities, compared with the control group's students who were taught without using mathematical error-analysis activities, means and standard deviations of both groups' classroom interaction were calculated, as shown in **Table 1**.

**Table 1** shows differences in the rating means of classroom interactions between the experimental and the control groups. These differences confirm that there are clear differences in the rating means of each item of the observation checklist based on the learning method (mathematical error-analysis activities and non-mathematical error-analysis activities), where the total rating mean of the classroom interaction for the experimental group was 3.87, while the total rating mean for the control group was 3.24 out of 5.

To test the significance of the difference between the rank means of the classroom interaction of the two groups, Mann-Whitney U test was used, as **Table 2** shows. It is clear from **Table 2** that there are statistically significant differences ( $p < 0.05$ ) between the classroom interaction rank means revealed by the 14 observations of each of the experimental and the control groups. This means that the effect of the learning method on the classroom interaction is effective, and the statistically significant differences are in favor of the experimental group that was taught using mathematical learning environment supported by error-analysis activities.

In the context of the findings of the first part of the results, it is worth to illustrate the way the students comment on the errors they discover through error analysis process. First of all, the students were organized in small groups in order to find the errors, to explain it, to proceed to the correct solution and then to engage in the whole class discussion. In the meanwhile, each group reflects on the other groups' error analysis process, and the teacher asks questions and sometimes he presents hints.



**Table 1.** Means (Ms) & standard deviations (SDs) of ratings of experimental & control groups on observation items

No	Items	Experimental		Control	
		M	SD	M	SD
1	Maintaining order & being quiet in the classroom	4.71	0.469	4.00	0.134
2	Showing interest in the learning topic	4.71	0.469	4.00	0.235
3	Listening carefully to colleagues	4.71	0.469	4.00	0.167
4	Respecting & accepting the opinions of colleagues	4.50	0.650	3.71	0.445
5	Speaking in a correct mathematical language	4.43	0.756	3.71	0.469
6	Enriching group & whole-class discussions	4.43	0.756	3.71	0.479
7	Differentiating between main ideas & secondary ones regarding learning topic	4.43	0.756	3.71	0.726
8	Collaborating effectively with colleagues, especially when working in groups	4.14	0.663	4.00	0.239
9	Raising brief & clear questions	4.00	0.392	4.00	0.371
10	Connecting questions & notes related to the learning topic	3.86	0.363	3.86	0.363
11	Raising questions & notes in a timely manner	3.86	0.363	3.71	0.469
12	Presenting ideas in an organized method	3.79	0.426	2.86	0.363
13	Providing evidence that support mathematical argumentations	3.64	1.393	2.29	0.726
14	Offering appropriate results, conclusions & evaluations	3.50	0.941	2.36	0.842
15	Raising deep & important questions	3.36	0.497	2.57	0.514
16	Presenting creative ideas	3.21	1.311	2.36	0.842
17	Connecting the learning subject to related topics in other subjects	3.21	0.975	2.43	0.646
18	Connecting the learning topic to the needs of the students	3.21	0.893	2.86	0.363
19	Connecting the learning topics to real-life problems	3.21	0.699	2.79	0.426
20	Presenting ideas & questions to predict tasks' solution	2.50	0.855	1.86	0.363
Total		3.87	0.631	3.24	0.401

**Table 2.** Results of Mann-Whitney U test of differences between rank means of experimental & control groups on observation checklist

Group	No	Rank mean	Rank sum	Mann-Whitney U	Wilcoxon W	Z	Sig.
Experimental	14	18.93	265.00	36.000	141.000	-2.859	0.004
Control	14	10.07	141.00				

Also, the students were interested in the real life situations behind the erroneous examples, and although they faced difficulties in the beginning, they tried to read the tasks carefully, to ask questions to each other and to the teacher, to reflect on other’s error analysis process, to evaluate their work until they get the correct solution and to feel that they are responsible about their learning.

Regarding the previous comments, the error analysis process facilitated learning of the proportion topic through repeated trails and efforts, and it helped students to reconstruct knowledge, where conceptual change achieved through reasoning, self- regulation and self-judgment.

**Part Two: Findings of the Contribution of Error-Analysis Activities to Improving Classroom Interaction**

The second question aimed to find out the way in which the error-analysis activities contribute to improving students’ learning. To achieve this aim, a qualitative analysis of the responses of nine students of the experimental group who participated in the interviews was conducted. The results of the qualitative analysis focused on four main categories that relate to the contribution of mathematical error-analysis activities to improving classroom interactions. These categories were: improving the quality of students’ responses

through classroom interaction, improving the quality of students’ engagement, improving oral questions used by both teachers and students and improving student-content interaction. It is worth noting that the main categories were derived from the ideas encoded during data analysis. The results are presented, as follows.

*First category: Improving the quality of students’ responses in the classroom interaction*

This category relates to the quality of students’ learning in mathematics learning environment in terms of enabling them to improve the quality of response. Students’ responses show a strong distinction in class interaction. Some examples of student responses resulting from reflective thinking, deep thinking, discussion, sharing, criticism, etc.

Therefore, the results of the data analysis showed that all the respondents (100%) who participated in the interviews showed that mathematical errors-analysis activities contributed to improving the quality of their written or oral responses, since the effective classroom interaction during error analysis assisted students to make negotiations with the teacher and the students themselves and improved the responses of students either through their classroom discussions or through their reflecting on error-analysis processes, which need

an advanced quality of responses. Some participants' responses in the interviews are quoted, as follows:

"Certainly, I deal carefully with the erroneous examples, where my responses concentrate on evaluating the errors by discovering, reasoning and correcting them if possible. This means that my responses are not just to calculate or to give a superficial answer and at the same time, I do not correct the error directly until I find its source and explanation."

"I share in making comments in peer-to-peer dialogues that support my ability to analyze mathematical errors."

"When the teacher asks a question while discussing the errors, I think deeply and concentrate on the explanation of my response."

"Yes, I feel that qualities of my responses, either at the level of the whole-class discussions or through the collaborative groups, are sometimes reflective and other times evaluative."

### ***Second category: Improving the quality of students' engagement in classroom activities***

This category is about the quality of student engagement in a creative way. Examples include students being able to ask questions, persistence in tasks, enthusiasm and motivation to participate rather than passivity, and so on.

Data analysis revealed that most of the respondents in the interview sample (88.88%) showed that their roles are changed in the new mathematical learning environment, which is supported by error-analysis activities, where they became more active and shared making decisions. This is illustrated in the following quotes:

"I became more active in this learning environment and not just a listener."

"I try to construct the ratio and proportion concepts through error-analysis activities."

"Working in small groups encourages me to work hard in order to look for errors in the solutions of the problems."

"Error-analysis activities motivate me to ask questions in order to understand mathematical concepts."

"We spent most of the lesson engaged in mathematical error-analysis activities."

"When I face a difficulty in the error-analysis process, I persevere to think mentally in order to continue the task."

"Understanding ratio, proportion and proportional reasoning needs from me to be more engaged in solving real-life tasks."

### ***Third category: Enhancing oral questions by both teacher and students***

This category relates specifically to the role of the teacher and student in improving classroom interaction by asking verbal questions through error analysis activities. Examples of this include the students' desire to ask questions to deepen their understanding, to hold discussions during group work, to generate new ideas stemming from the teacher's questions, and so on.

The findings of data analysis showed that many interviewees (77.77%) who participated in the interviews stressed that they were encouraged to raise questions and that the teacher raises scaffolding questions while the students are involved in the learning activities. The questions raised by the teacher aimed to let students pay attention to whole-classroom discussions and motivate students to retrieve the previous knowledge in order to connect it with the new topic. Meanwhile, the questions raised by the students concentrate on asking for hints. This is illustrated in the following quotes:

"Often, the teacher uses oral questions to encourage us individually and through groups to look for the erroneous examples in order to increase classroom participation and understanding of the learning topics."

"The teacher stressed to ask questions, such as: What is the error in the solution? Can you explain it? Can you correct it? What concepts are involved in this task?"

"Usually, I ask questions, such as: How can I increase my understanding of ratio and proportion topic through the error-analysis process."

"The error-analysis process motivates me to ask questions orally, either to my teacher or to my colleagues in the collaborative group."

"Teacher's oral questions support our classroom interaction, because they concentrate on both low and high levels of mathematical thinking."

### ***Fourth category: Improving student-content interaction***

More than two thirds of the study sample confirmed the importance of this category. It relates to improving students' skills in interacting with content. Examples

include using mathematical language, analyzing tasks, reading the problem carefully, discussing the problem with students, and so on. This kind of interaction is essential in the mathematics learning environment, especially in mathematics error-analysis activities. The indicators of this interaction reflect how students express their ideas, how they reflect on their solutions and the others' solutions and how they substitute their satisfactions about certain idea. Also, they reflect the extent of reading the task in order to understand it and persevere to complete the task. Some responses are quoted, as follows:

"Usually, I read the task carefully in order to understand what it requests."

"Of course, when I read a certain task, I try to look for the key concepts in this task."

"When my collaborative group completes the task, we try to reflect on our final solutions and on the other groups' solutions."

"When I or my colleagues in the group complete the task successfully, I change my satisfaction about ratio and proportion ideas, which means that it can be understood, and it has wide real-life applications."

"I became more engaged in using mathematical language related to the ratio and proportion topic."

"I concentrate on the conceptual part rather than on the procedural one in solving the lesson tasks."

"I usually try to be precise in mathematical discussions and the teacher encourages us to communicate using mathematical language precisely."

"My talk is usually related to solving strategies that are aligned with proportional reasoning problems."

## DISCUSSION

The findings of this study related to classroom observation showed that classroom interaction in the learning environment supported by error-analysis activities was significantly better for the experimental group than for the control group, as indicated by the results of the Mann-Whitney U test. This result can be attributed to the effective role of mathematical error-analysis activities in classroom interaction, as a result of their positive features in classroom interaction that enabled students to enrich discussions, show interest in learning the mathematical content and listen well to each other and collaborate effectively with each other,

especially when working in groups, which in turn helped them link the content of ratio and proportion with real life-situations that need solutions.

This finding can be also ascribed to the role of mathematical learning environment supported by error-analysis activities in enabling students to raise questions in a timely manner, in order to come up with an appropriate understanding, conclusions and assessments based on providing evidence to support their comments. Moreover, providing opportunities for students to identify, explain and correct errors enabled them to connect the learning subject, especially proportional reasoning, to related topics in other subjects, to real-life situations and to the topics that were previously studied, in addition to linking this to the needs of the students and their expectations.

The previous finding is consistent with prior studies (Bray, 2013; Ingram et al., 2015; Willingham et al., 2018; Yarman et al., 2020), which emphasized that mathematical error-analysis contributes to providing students with opportunities to show interest in the learning topic, respect and accept students' opinions, speak in a correct language, develop useful questions, in addition to giving students an opportunity to express their thoughts, and criticize them along with others' thoughts.

In addition, this finding of the current study accords with previous studies (Bray, 2013; Ingram et al., 2015; Schleppebach et al., 2007), which confirmed that the use of mathematical errors increases interaction between teacher and students and among students themselves. This finding introduces empirical evidence regarding the effectiveness of error-analysis in providing an active social environment, encouraging students' classroom participation and discussions and providing students with comprehensive feedback.

Regarding the qualitative analysis of the participants' responses in the interviews, it was found that the quality of students' responses was more advanced, which is due to the cognitive requirements of mathematical error-analysis activities, such as inquiring, explaining, correcting and evaluating the errors. Also, it is due to the benefits of the learning environment supported by mathematical error-analysis activities in increasing classroom interaction and participation, and helping students reduce their anxiety of making errors in the classroom as well as overcoming some challenges that face them, through reducing embarrassment, reducing mathematical errors and encouraging students to keep on learning mathematics.

This result was reported by previous studies (Ayuwanti & Dwiswoyo, 2021; Brodie, 2008; Heinze, 2005; Ingram et al., 2015; Kent, 2017; Kyaruzi et al., 2020; MCCarthy et al., 2016; Palkki & Hästö, 2019; Qadri, 2012; Rach et al., 2012; Rushton, 2018; Willingham et al., 2018; Yarman et al., 2020), which indicated that mathematical

error-analysis enhances students' interaction and effective discussion, develops analytical skills, corrects misunderstandings and misconceptions and leads to creating a strong learning educational environment in the classroom, as it increases classroom interaction between teacher and students and among students themselves.

Error-analysis activities through mathematical learning motivated students to pay attention and to be serious in accepting mathematical topics presented by the teacher, where this increased classroom interactions represented in teacher-student, student-student and student-content interactions. Also, the actions of remediating mathematical errors create an environment that motivates questioning by both teachers and students. This means that the observation and the interview results indicated that one of the forms of interaction is raising questions and giving answers. For example, the teacher raised oral questions at the beginning and at the end of the lesson, such as: questions to check understanding the previous knowledge necessary for dealing with proportional reasoning, as well as asking students to give examples of proportional real-life situations against non-proportional ones, in addition to assessment questions at the end of the lesson. From the side of students, they raised explanatory questions in order to understand the different concepts related to proportional reasoning and sometimes asked for hints to be able to deal with the errors.

Moreover, collaborative group work while looking for the errors in the posed tasks, error-analysis activities and whole classroom discussions, which were part of the mathematical learning environment, encouraged students to negotiate, reflect on each other's solutions and interact with the mathematical content, particularly since the proportional reasoning topic and its related concepts represent a difficult issue. Since this topic needs rich previous knowledge and high levels of understanding and cognitive processes, beside connection of real-life situations, the quality of students' responses improved and were more advanced regarding the argumentations used to correct the errors within the tasks presented to students during the learning process, where students' engagement increased, and student-content interaction became more successful.

In addition to the above, the participants in the interviews from the experimental group confirmed that they benefited from mathematical error-analysis activities, since they helped students understand proportionality and the related topics. This result confirms that finding out errors in the classroom, along with interpreting and solving them, increase students' understanding of mathematical topics and enable them to avoid making mathematical mistakes again or at least reducing them, as well as increasing their interaction and encouraging them not to be afraid to participate. In addition, the learning environment supported by error-

analysis overcomes mathematics anxiety, where mathematics learning becomes fun for students and reduces awkwardness.

Furthermore, the environment of error-analysis activities helped students raise questions and notes in a timely manner in order to come up with an appropriate understanding, conclusions and assessments based on providing evidence to support their opinions, in addition to providing opportunities for students to analyze their mistakes, which enables them to connect the learning subject to related topics in other subjects and to topics previously studied, as well as linking this to the needs of the students and their teachers' expectations.

These findings are in line with the studies of (Alali & Al-Barakat, 2022; Bray, 2013; Ingram et al., 2015; Khasawneh et al., 2022; Willingham et al., 2018), which confirmed that the use of mathematical errors increases interaction between teacher and students and among students themselves. This provides an active social environment that enables students to show interest in the learning process, speak and criticize in an appropriate language, respect each other's thoughts and express their questions and conceptions, especially since mathematical error-analysis encourages students' classroom participation as they feel free when they make errors, thus encouraging class discussions. In general, this emphasizes that error-analysis activities enhance students' mathematics learning, since they provide students with comprehensive feedback.

Far from the effect of mathematical learning environment supported by error-analysis activities, Kurthen (2014), Lorenz (1980), Susak (2016), and Tiwari (2021) claimed that there are many factors that might affect the quantity and quality of classroom interactions in general, either teacher-student interaction, student-student interaction or student-content interaction. These factors are represented by "students' characteristics and behaviors", such as previous performance, previous knowledge, interest in interaction, school level, willingness to solve problems, dependency, among others, "teachers' skills and behaviors", such as questioning, positive feedback, perceptions and introducing real-life tasks and "class topic" (Al-Barakat et al., 2022a; Kurthen, 2014; Lorenz, 1980; Susak, 2016; Tiwari, 2021). In the current study, these factors might play a role in shaping the different aspects of classroom interaction, beside the role of error-analysis activities related to ratio, proportion and proportional reasoning. In this context, error-analysis activities played a role in motivating students to share, interact effectively, persist in discovering errors and correcting erroneous solutions and be independent. Furthermore, teachers' questioning, and feedback played a role in increasing the quality of students' responses and gave power for students to ask questions and interact with the mathematical content.

## LIMITATIONS, CONCLUSIONS, AND INSTRUCTIONAL IMPLICATIONS

One of the main limitations of this study is that the data were collected from the subjects of the study sample (seventh grade students) regardless of the difficulties and constraints they face in learning mathematics. Therefore, future research should focus on a sample of students who have difficulties and limitations in learning mathematics. In addition, the study was limited to a sample of seventh-grade students at one of the public schools in Jordan in the first semester of the school year 2021/2022, which may limit the generalization of its results to other non-similar samples. In addition, only 14 classroom observations were conducted for each group. Moreover, the error-analysis activities were limited to ratio, proportion and proportional reasoning topics, which may limit the generalization of the results to other mathematical topics. In addition, the results of the study were based on the psychometric properties of the data collection instruments developed for the purposes of this study.

Despite the limitations above, this study contributes to the mathematics education field by using the mathematical error-analysis activities in developing classroom interaction. One contribution of the current study is its demonstration of the role of error-analysis activities in enabling students to improve their responses, engaging in a creative way, asking verbal questions, and in interacting with content.

The study concluded that mathematical learning environment supported by error-analysis activities was effective and useful in classroom interaction during primary-mathematics teaching and learning. This main conclusion underlines that mathematical error-analysis activities contribute effectively to facilitate and increase classroom interactions between teacher and students, students with each other and students with the mathematics content in the mathematics lessons, as well as to construct positive attitudes and motivation to learn mathematics. Moreover, it was concluded that the error-analysis process implemented by the students themselves and guided by the teacher enhanced the quality of students' responses, the level of their engagement, oral questions raised by teacher and students and students' interaction with the mathematics content.

These conclusions are reported by the studies of (Bray, 2013; Gamlem, 2018; Gedik et al., 2017; Ingram et al., 2015; Kramarski & Zoldan, 2008; Makonye & Khanyile, 2015; Metcalfe, 2016; Murillo & Tan, 2019; Qadri, 2012), which showed that error-analysis not only directly influences students' cognitive skills and learning, but also affects the provision of a psychological atmosphere that contributes to motivating and arousing students to participate and be active learners in mathematics learning situations.

In light of the study results, the researchers recommend the need for future research that should address the effectiveness of mathematical learning environment supported by the error-analysis process in various mathematical topics, as well as in the development of other styles of mathematical justification and thinking, besides addressing other variables that may interact with mathematical error-analysis activities, such as students' demographic variables, students' behavior, teachers' behavior, metacognition habits and verbal and non-verbal classroom interactions.

In addition, the study recommends mathematics curriculum developers to include in mathematics textbooks a variety of correct and incorrect examples that focus on discovering errors, their sources, interpreting and solving them and designing mathematical tasks in the form of drills and assessments for students. In addition, the study recommends that mathematics teachers should introduce in learning settings erroneous solutions made by virtual students so that students can practice the error-analysis process. Further, the study recommends concentrating on mathematics teachers' professional development and training, in order to be aware of the pedagogical value of errors.

Finally, the study recommends mathematics teachers to develop a teaching-learning environment that allows mathematical errors in order to reduce students' embarrassment and mathematics anxiety when they commit mathematical errors, concentrating on conceptual knowledge in parallel with procedural knowledge and the use of various mathematical problems in the context of daily life.

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