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# To what extent do mathematics textbooks offer opportunities for student reflection? A case study of two Chinese textbooks

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**Abstract**

Reflection plays a vital role in fostering students' metacognition, conceptual understanding, and problem-solving abilities. However, little is known about the extent to which mathematics textbooks create opportunities for students to engage in reflective thinking. This study investigates two widely used junior secondary mathematics textbook series in China to examine how and to what degree they promote reflection. Drawing on a multidimensional analytical framework, the study categorizes reflective opportunities along three dimensions: reflective content, reflective stage, and reflective behavior. Through systematic content analysis, 390 reflective prompts were identified and coded. Results reveal that knowledge-focused reflections are predominant (54.3%), with most prompts positioned during (69.3%) or after tasks (25.9%), and few encouraging pre-task reflection (4.8%). Behaviorally, statement-type prompts dominate (38.0%), while deeper analytical forms such as comparison, generalization, and explanation are relatively scarce. These findings suggest that while Chinese mathematics textbooks provide some opportunities for reflection, such opportunities remain unevenly distributed and limited in depth. The study underscores the need for textbook design to better support metacognitive and strategic reflection throughout the learning process.

**Keywords:** mathematics textbooks, student reflection, reflective opportunities, metacognition, content analysis

## INTRODUCTION

Reflection is widely recognized as a core component of meaningful and deep learning. As a higher-order cognitive process, it enables learners to monitor and evaluate their thinking, refine problem-solving strategies, and transfer knowledge to new contexts. In mathematics education, reflection has been closely associated with the development of metacognitive awareness, conceptual understanding, and adaptive expertise (Kramarski & Mevarech, 2003; Schoenfeld, 1992). Ausubel (1968) asserts that learning is deeply linked with cognitive structures, which is echoed by Bransford et al. (2000), emphasizing the importance of reflection in linking new knowledge to prior understanding.

In recent years, educational policies have increasingly emphasized the cultivation of reflective learning. Mathematics curriculum standards for compulsory education (Ministry of Education of the People's Republic of China, 2022) serve as the national curriculum framework guiding mathematics instruction for grade 1-grade 9 across mainland China. As a mandatory policy document, the standards define core competencies, learning objectives, and content requirements that textbook development and classroom teaching must follow. All officially published textbooks are developed in accordance with these national standards and undergo governmental review and approval. Similarly, the Hong Kong Education Bureau's (2016) guiding principles for quality textbooks advocate designing materials that engage students in analysis, evaluation, and self-regulation rather than rote memorization (OECD, 2005). Internationally, UNESCO's (2014) guidelines for textbook development recommend that textbooks promote learner-centered, participatory, and cognitively demanding content (UNESCO, 2014). These policy directions converge on a shared expectation


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**Contribution to the literature**

- An integrated framework is proposed for understanding reflection in mathematics textbooks through three dimensions: reflective content, reflective stage, and reflective behavior.
- In contrast to earlier studies that were mainly descriptive, this study offers a more systematic approach to examining how reflective opportunities are distributed and structured in mathematics textbooks.
- Empirical evidence is provided from Chinese junior secondary mathematics textbooks, a context that has received limited multidimensional attention in previous studies of textbook-based reflection.

that textbooks—often the most influential instructional resource—should actively create opportunities for students to reflect on their mathematical learning.

However, empirical research on student reflection, particularly within the Chinese mathematics education context, remains limited and fragmented. Existing studies often conflate reflection with general thinking skills or focus primarily on teachers' reflective practices (Rodgers, 2002), overlooking students' reflective engagement during learning. Large-scale surveys and classroom observations indicate that middle school students in China tend to concentrate on procedural execution rather than self-monitoring or strategic evaluation (Wu & Zhang, 2015; Zhao & Gao, 2014), suggesting a gap between curriculum expectations and classroom realities.

Given the pivotal role of textbooks as the main medium for translating curriculum standards into daily classroom practice, their capacity to promote reflection warrants close examination. When thoughtfully designed, textbooks can scaffold reflective thinking by embedding prompts that guide students to articulate reasoning, compare alternative strategies, and evaluate their learning. Yet previous studies show that reflective prompts in Chinese mathematics textbooks are often inconsistently distributed and typically concentrated at the end of units rather than integrated throughout the learning process (Fan & Zhu, 2007; X. Li et al., 2021). Such patterns may constrain students' sustained engagement in metacognitive reflection.

Despite policy emphasis on reflective learning, little is known about how textbooks actually support such reflection. This study therefore aims to evaluate the extent and nature of reflective opportunities embedded in two widely used Chinese junior secondary mathematics textbook series. Drawing on a multidimensional analytical framework, it examines how and to what degree these textbooks provide opportunities for students to engage in reflection across three dimensions—reflective content (what is reflected upon), reflective stage (when reflection occurs), and reflective behavior (how reflection is enacted).

**RQ1.** To what extent do Chinese junior secondary mathematics textbooks provide opportunities for student reflection?

**RQ2.** What types and patterns of reflective opportunities are most and least represented?

## LITERATURE REVIEW

Reflection has been widely recognized as a critical construct in mathematics education for fostering metacognition, problem solving, and conceptual understanding. As textbooks remain central to instructional practice, researchers have increasingly examined how reflection is structured and supported within textbook content. To understand how textbooks promote students' reflective thinking, it is necessary to consider not only whether reflection is present, but also how it is designed, situated, and enacted. This review therefore focuses on three interrelated dimensions commonly discussed in prior research: reflective content, reflective stage, and reflective behavior, followed by an overview of how reflective opportunities are represented in mathematics textbooks (Krathwohl, 2002).

### Reflective Content

Reflective content refers to prompts or questions that explicitly guide students to think about their own thinking—for example, justifying solutions, explaining reasoning, or comparing methods (Clarke et al., 2012; Fan & Zhu, 2007; Fan et al., 2013). Such prompts draw on metacognition theory (Flavell, 1979), self-regulated learning (Zimmerman, 2002), and the notion of "mathematical thinking" (Mason et al., 2010), all emphasizing that reflection helps students monitor and extend their understanding.

Empirical studies reveal significant variation in how reflective content is embedded across countries and textbook traditions. For instance, X. Li et al. (2021) found that reflective prompts in Chinese textbooks are often unevenly distributed and positioned after problem-solving tasks, while Rezat (2006) noted that German textbooks tend to place them in marginal or enrichment sections. Comparative analyses (Kaur, 2020; Pepin & Haggarty, 2001; Zhu & Fan, 2006) show that East Asian textbooks often emphasize procedural variation, whereas Western ones highlight self-explanation and multiple solution strategies. Despite increasing policy emphasis (e.g., Ministry of Education of the People's Republic of China, 2022; National Council of Teachers of Mathematics [NCTM], 2000), reflective content remains inconsistently integrated and limited in depth.

### Reflective Stage

Reflective stage concerns when reflection occurs within the learning sequence—before, during, or after a task. Theoretically, timely reflection supports goal setting, progress monitoring, and evaluation (Flavell, 1979; Paris & Winograd, 1990; Zimmerman, 2002). Drawing on Polya's (1945) four-step model and Clarke et al.'s (2014) reflective learning framework, researchers emphasize that pre-task reflection activates prior knowledge, mid-task reflection regulates strategy use, and post-task reflection consolidates learning.

However, empirical analyses indicate that reflection is rarely distributed evenly across these stages. In many mathematics textbooks, prompts are clustered at the end of units, emphasizing review rather than planning or regulation (Kang & Kilpatrick, 1992; Pepin & Haggarty, 2001). Even when curriculum standards explicitly encourage reflection (Ministry of Education of the People's Republic of China, 2022; NCTM, 2000), textbooks often fail to operationalize this across the learning process, leaving pre- and mid-task reflection underrepresented.

### Reflective Behavior

Reflective behavior refers to the observable cognitive or metacognitive actions through which students monitor, evaluate, and regulate their thinking. These include self-assessment, justification, comparison, error analysis, or generating examples (Schukajlow et al., 2012; Y. Li et al., 2021). Such behaviors bridge the gap between knowing what to reflect on and knowing how to reflect effectively (Flavell, 1979; Zimmerman, 2002).

Studies show that while reflective behaviors can promote metacognitive engagement and flexible strategy use (Mevarech & Kramarski, 2014), their inclusion in textbooks remains limited and often implicit. Without explicit scaffolding, students may treat reflective tasks superficially, reducing their potential impact (Fan & Zhu, 2007; Stylianides et al., 2018). Comparative research further suggests cross-cultural variation: for instance, English and Singaporean textbooks encourage justification and discussion, while Chinese textbooks favor brief statements or procedural summaries (Kaur, 2020; Pepin & Haggarty, 2001).

### Reflective Opportunities in Mathematics Textbooks

In recent years, an increasing number of studies have examined how textbooks provide opportunities for students to engage in reflection, yet the findings remain limited and fragmented. Textbook-based reflective opportunities refer to explicit or implicit prompts, tasks, or questions that invite learners to think about their reasoning, strategies, and understanding (Pepin & Haggarty, 2001; X. Li et al., 2021). When effectively designed, such opportunities can scaffold metacognitive engagement and deepen conceptual understanding by encouraging students to justify, compare, and evaluate their mathematical ideas (Kramarski & Mevarech, 2003; Schoenfeld, 1992).

However, empirical analyses show that reflective opportunities in mathematics textbooks are often unevenly distributed and limited in depth. Many prompts appear as review questions at the end of chapters rather than being embedded throughout the learning process (Fan & Zhu, 2007; Rezat, 2006). Studies also report that reflection tends to focus on factual or procedural aspects rather than on self-regulation, values, or the learning process itself (X. Li et al., 2021; Zhu & Fan, 2006). As a result, students are seldom guided to plan, monitor, or evaluate their learning strategically.

Overall, existing evidence suggests that while reflective prompts exist in mathematics textbooks, they are quantitatively limited, qualitatively shallow, and structurally unbalanced across content, stage, and behavior. These findings highlight the need for systematic analyses that quantify the extent and nature of reflective opportunities in textbook design, thereby linking theoretical frameworks of reflection with practical curriculum implementation.

Across the literature on reflective content, reflective stages, and reflective behavior, existing research provides valuable conceptual foundations for understanding how reflection is represented in mathematics textbooks Yet most studies remain descriptive or theoretical, focusing on defining dimensions rather than quantifying their manifestation in actual materials. Few studies have systematically examined the extent and distribution of reflective opportunities in Chinese mathematics textbooks using a multidimensional lens. Addressing this gap, the present study applies an established analytical framework to evaluate how and to what extent reflection is supported in practice across two widely used Chinese junior secondary mathematics textbook series.

## ANALYTICAL FRAMEWORK

Based on prior research on metacognition, self-regulated learning, and mathematics textbook analysis (e.g., Boud et al., 1985; Dewey, 1933; Flavell, 1979; Kramarski & Mevarech, 2003; Mezirow, 1990; Schön, 1983; X. Li et al., 2021; Zimmerman, 2002), this study adopts a multidimensional analytical framework to examine how reflection is represented in junior secondary mathematics textbooks. The framework codes textbook-based reflective opportunities along three interrelated dimensions: reflective content, reflective stage, and reflective behavior. These dimensions collectively capture what students are prompted to reflect on, when reflection occurs, and how reflection is enacted in mathematics learning.

**Table 1.** Dimensions of reflective content in mathematics learning

| Dimension | Description |
|---|---|
| Mathematical knowledge | Mathematics concepts, rules, theorems, formulas. Such as Pythagorean theorem. |
| Mathematical methods | Thinking strategies, or reasoning processes. Such as methods to solve equation. |
| Value of mathematical content | Reflection on the mathematical, scientific, or cultural value of the content or activity. For example, in analyzing public attitudes toward waste sorting, what role does mathematics play in collecting and analyzing data? |
| Cognitive process and the self | Reflection on one's exploration process, difficulties encountered, problem-solving path, collaboration, and self-regulation during learning. Such as what was your inquiry process? What challenges did you face, and how did you overcome them? |

**Table 2.** Dimensions of reflective stage in mathematics learning

| Dimension | Description |
|---|---|
| Before the task | At the beginning of a mathematical task, students identify goals, clarify tasks, and formulate a plan. Anticipation of outcomes is often emphasized. |
| During the task | Students monitor their progress, reflect on errors, and adjust strategies accordingly. Reflection may involve trying alternative approaches. For example, after using one solution strategy, a student may pause to consider a second, possibly more efficient method. Ongoing reflection on one's own learning process, strategy use, and resource allocation. Demonstrates awareness of purpose and progress during learning. |
| After the task | Students assess their performance using clear criteria. Objective judgment of outcomes is encouraged. For example, assessing one's solution accuracy, completeness, or alignment with expected standards.<br>Critical analysis of both learning process and outcomes. Students identify what worked, what didn't, and what could be improved. |

### Reflective Content

This dimension focuses on the substance of students' reflection—what they are encouraged to think about during or after their mathematical learning experience (**Table 1**). Prior research suggests that reflective content can be broadly categorized into two orientations: reflection directed toward mathematical learning objects (e.g., concepts, representations, and problem structures) and reflection directed toward the learning process itself (e.g., strategies, reasoning pathways, and errors) (Flavell, 1979; Schoenfeld, 1985). These reflections help consolidate conceptual understanding and foster metacognitive awareness and personal growth (Ausubel, 1968; Boaler, 2016; Bransford et al., 2000; Kramarski & Mevarech, 2003; Schoenfeld, 1992).

Prior studies emphasize that the presence of reflective prompts is crucial for promoting deep understanding and active learning, particularly in inquiry-based and problem-centered environments (Boaler & Greeno, 2000; Goos et al., 2007; Mason, 2000). Therefore, this dimension includes four key aspects:

(1) reflection on mathematical knowledge,

(2) reflection on mathematical methods,

(3) reflection on the value of mathematical content, and

(4) reflection on cognitive process and the self.

The first three aspects guide students to understand, apply, and appreciate mathematics beyond procedural knowledge, whereas the last one encourages students to monitor their learning strategies, cognitive processes, and self-regulation during tasks (Flavell, 1979; Zimmerman, 2002).

### Reflective Stage

Building on Polya's (1945) problem-solving model and Clarke et al.'s (2012) three-phase reflective learning framework, this dimension examines when reflection is prompted during the learning process. Specifically, reflection may occur before the task (planning), during the task (regulation and management), or after the task (evaluation and verification). These stages align with metacognitive cycles identified in Zimmerman's (2002) and Flavell's (1979) frameworks, which emphasize how timely reflection supports cognitive, emotional, and motivational regulation in problem-solving (Kuhn, 2000; Paris & Paris, 2001; Pintrich, 2004; Veenman et al., 2006). **Table 2** provides a detailed interpretation of each reflective stage in the context of mathematics learning.

### Reflective Behavior

Grounded in Flavell's (1979) theory of metacognition and Zimmerman's (2002) model of self-regulated learning, this dimension refers to the specific cognitive and metacognitive actions students take during reflection. These reflective behaviors encompass higher-order cognitive actions such as justification, analogy, and generalization (Leikin et al., 2017; Niss & Jensen, 2002), which deepen students' engagement with mathematical structures and reasoning (Boaler &

**Table 3.** Dimensions of reflective behavior in mathematics learning

| Dimension | Description |
|---|---|
| Comparison and connection | Analyzing the differences and similarities among mathematical concepts or solution strategies. Comparison is a specific type of connection. Reflecting by linking multiple mathematical contents or experiences without explicitly comparing. |
| Classification and analogy | Grouping a series of mathematical elements based on specific criteria. Drawing inferences based on shared attributes between two mathematical objects or strategies. |
| Explanation and example | Describing concrete situations or instances to support understanding of a concept or method. Translating learned content into personalized representations, enhancing comprehension. Explanation is a specific form of elaboration. |
| Generalization and discussion | Summarizing patterns, relationships, or the broader significance of mathematical content. Often involves abstraction. Engaging in peer conversations to share insights and receive feedback. Promotes collective reflection. |
| Statement and justification | Logically articulating meanings of mathematical concepts, solution strategies, or error analysis. Providing reasons for one's claims or strategies; includes supporting arguments or deriving conclusions through reasoning. |
| Others | Carefully examining features of materials or objects (e.g., observing diagrams, text, or manipulatives). Creating visual knowledge maps to express hierarchies and relationships among concepts. Generating original representations or alternative formulations of a concept or method. Demonstrates creative thinking. |

Greeno, 2000; National Research Council, 2001). Moreover, reflection through articulation, explanation, and dialogue contributes to identity formation and agency in mathematical learning (Mason, 2000; van Manen, 1995). **Table 3** presents the detailed dimensions and interpretations of reflective behavior:

These reflective behaviors align with the development of metacognitive skills, as students move beyond procedural engagement to construct meaningful connections and strategic thinking. Kramarski and Michalsky (2009) emphasize that explicitly teaching these behaviors can significantly improve students' reflective learning capabilities in mathematics.

Research on reflection in mathematics education has developed along several theoretically rich yet largely parallel strands. One line of inquiry, grounded in metacognition and conceptual learning (e.g., Flavell, 1979; Schoenfeld, 1985), emphasizes the content of reflection—what learners attend to when they reconsider mathematical ideas, strategies, or representations. A second tradition, informed by problem-solving and instructional cycle models (e.g., Clarke et al., 2014; Polya, 1945), highlights the temporal positioning of reflection—when reflective activity is prompted within phases of planning, regulation, and review. A third body of work focuses on the behavioral and cognitive manifestations of reflection, examining the specific metacognitive actions through which reflective thinking is enacted. Although each of these perspectives has advanced understanding of reflective learning, they are typically treated in isolation. Such fragmentation limits the capacity to analyze how reflection is systematically embedded within curricular materials. For example, focusing solely on reflective behaviors without considering their epistemic focus or instructional timing risks overlooking how reflective opportunities are structured and sequenced within textbooks. To address this gap, the present study synthesizes these complementary traditions into a three-dimensional framework encompassing reflective content (what learners reflect upon), reflective stages (when reflection occurs), and reflective behavior (how reflection is enacted). By integrating these dimensions into a unified analytical structure, the framework offers a theoretically coherent and methodologically robust approach for examining reflective opportunities in mathematics textbooks.

Within this integrative structure, each dimension serves a distinct analytical function. The reflective content dimension distinguishes between students' reflection on mathematical knowledge (e.g., concepts, rules, and methods) and their reflection on the learning process (e.g., cognitive strategies, self-regulation, and collaboration). The reflective stage dimension—drawing on models by Polya (1945) and Clarke et al. (2014)—captures when reflection is prompted: during the planning phase (before the task), the regulation phase (during the task), or the review phase (after the task). The reflective behavior dimension categorizes the specific metacognitive actions students engage in, including comparison, classification, exemplification, generalization, explanation, justification, and other behaviors such as observation and design. Together, these dimensions provide a comprehensive lens through which to analyze and support reflective learning in mathematics education.

## METHODOLOGY

### Sample

To examine how reflective practices are embedded in junior secondary mathematics education in China, this

**Table 4.** Overview of textbook sources and sampled reflective prompts

| Publisher | Publication year(s) | Grade level(s) | Total reflective prompts | Sampled prompts | Proportion (%) |
|---|---|---|---|---|---|
| BNU | 2013-2014 | 7-9 | 395 | 180 | 45.6 |
| PEP | 2012-2014 | 7-9 | 380 | 210 | 55.3 |

study selected representative textbooks from two major sources:

(1) Beijing Normal University (BNU) edition,

(2) People's Education Press (PEP) edition.

These editions were chosen due to their widespread adoption and curricular influence in different regions. All potential reflective prompts were first identified and coded from the six textbooks (grade 7-grade 9) in each series, yielding 395 prompts in the BNU edition and 380 in the PEP edition.

In this study, the term "reflective prompt" refers to any identifiable textbook element (including questions, statements, or task instructions) that invites reflective engagement. For consistency, all identified units of analysis are referred to as "reflective prompts" throughout the manuscript.

To ensure representativeness and reduce potential position-related bias, we employed a stratified random sampling strategy. Reflective prompts were grouped by instructional unit (e.g., chapters). Within each unit, prompts were randomly sampled proportionally to the total number of reflective prompts in that unit. This procedure ensured coverage across introductory sections, in-task components, and end-of-chapter features, rather than concentrating on early textbook sections. The final sample consisted of 180 prompts from BNU (45.6% of total) and 210 from PEP (55.3% of total), resulting in a total of 390 prompts analyzed. **Table 4** reports the publication years, grade levels, and the total number of reflective prompts in each textbook, as well as the proportion represented by the sampled prompts.

## Method

A content analysis was conducted on a sample of Chinese junior high school mathematics textbooks to investigate how reflection is represented in mathematics instruction. The analysis focused on identifying reflective prompts embedded within instructional materials and systematically characterizing them using a predefined analytical framework. This framework consisted of three dimensions: reflective content, reflective stage, and reflective behavior.

### *Coder training and preparation*

Two primary coders independently coded all identified reflective prompts. Both coders were graduate researchers in mathematics education who received prior training in qualitative content analysis. Before formal coding began, the coders participated in structured training sessions in which the analytical framework and coding manual were reviewed in detail. A pilot coding phase was then conducted using a subset of textbook materials to refine operational definitions and establish a shared understanding of the coding categories. During this stage, discrepancies were discussed and the coding guidelines were clarified.

### *Identification of reflective prompts*

The analysis first involved identifying reflective prompts in the textbook materials. Reflective prompts were defined as instructional prompts that explicitly encourage students to examine mathematical ideas, learning processes, or problem-solving strategies through reflective thinking. To ensure reliability in the identification process (i.e., locating reflective prompts and determining analytical unit boundaries), a separate inter-coder reliability test was conducted. The Cohen's kappa coefficient for the identification stage was 0.84, indicating substantial agreement between coders.

### *Coding of reflective prompts*

Following the identification stage, all reflective prompts were independently coded according to the three analytical dimensions of the framework: reflective content, reflective stage, and reflective behavior. Each reflective prompt could receive multiple codes across the three dimensions.

A total of 390 reflective prompts were identified across the two textbook series and were independently double-coded. When disagreements occurred during coding, they were resolved through discussion between the coders. If consensus could not be reached, a third senior researcher adjudicated the disagreement to determine the final coding decision.

### *Reliability analysis*

Inter-coder reliability for the coding stage was assessed using Cohen's kappa. The overall kappa coefficient for the coding stage was 0.87, indicating a high level of agreement and supporting the reliability of the analytical framework. Reliability was also calculated separately for each analytical dimension. The results indicated strong agreement across dimensions:

**Reflective content:** kappa = .931, 95% confidence interval [CI] [.873, .990], $p < .001$

**Reflective stage:** kappa = .944, 95% CI [.874, 1.014], $p < .001$

**Reflective behavior:** kappa = .866, 95% CI [.831, .900], $p < .001$

Observed percentage agreement was 95.4% for reflective content, 97.2% for reflective stage, and 90.0% for reflective behavior. Disagreements were resolved through discussion until consensus was reached, and the final dataset used for analysis was based on the consensus coding.

### Coding Procedure

Each reflective prompts could receive multiple codes across the three dimensions: reflective content, stage, and behavior. The research team comprised several trained professionals and followed a rigorous multi-step coding process, including coder training, initial coding, code verification, and a second round of coding to ensure accuracy and consistency.

Before formal coding began, the two coders participated in a structured training session in which the operational definitions of reflective prompts and the three-dimensional analytical framework were explained and discussed in detail. A pre-coding phase was conducted using a subset of 30 reflective prompts (approximately 8% of the dataset) to refine operational definitions and clarify coding rules.

Initially, each textbook was assigned to at least two independent coders. All team members underwent standardized training sessions to develop a shared understanding of the coding framework and ensure consistency in applying the codes. After the first round of coding, a collaborative review session was held to examine and verify the assigned codes. During this process, misclassifications were identified and corrected. A second round of coding was then conducted for refinement.

#### *Coding examples*

In the following example, the original coding for the reflective stage was "after the task (evaluation)." However, upon review, it was determined that the prompt focuses more on students' awareness of their actions and the rationale behind them. Therefore, it was reclassified under the "during the task (regulation)" category:

**Reflective prompt:** Give an example of an algebraic expression and an equation. What are their similarities and differences?

**Reflective content:** Reflection on mathematical knowledge

**Reflective stage:** After the task (evaluation) (revised to: during the task [regulation])

**Reflective behavior:** Comparison and connection, explanation and example

In the next example, the primary emphasis is on analogical reasoning between different types of knowledge. To better distinguish it from simple comparison, the behavior code was revised from "comparison" to "analogy":

**Reflective prompt:** Use analogies among three linear relationships to explore the connections between quadratic functions, quadratic equations, and quadratic expressions.

**Reflective content**: Reflection on mathematical methods

**Reflective stage:** Before the task (planning)

**Reflective behavior:** Comparison and connection (revised to: classification and analogy)

To enhance reliability in dimensions more prone to disagreement—namely, reflective content and reflective stage—a three-coder approach was used. For the reflective behavior dimension, dual coding was implemented. All coders worked independently, and inter-coder agreement was calculated for each dimension. Discrepant cases were subsequently discussed and resolved through consensus.

**Table 5.** Analytical framework for identifying reflective opportunities in mathematics textbooks

| Dimension | Sub-dimension |
|---|---|
| Reflective content | Mathematical knowledge |
| | Mathematical methods |
| | Value of mathematical content |
| | Cognitive process and the self |
| Reflective stage | Before the task |
| | During the task |
| | After the task |
| Reflective behavior | Comparison and connection |
| | Classification and analogy |
| | Explanation and example |
| | Generalization and discussion |
| | Statement and justification |
| | Others |

## RESULTS

### Extent and Dimensions of Reflective Opportunities in Mathematics Textbooks

This study applied a multidimensional analytical framework to systematically examine the extent and distribution of reflective opportunities in two Chinese junior secondary mathematics textbook series. The framework, summarized in Table 5, includes three first-order dimensions: reflective content, reflective stage, and reflective behavior.

Reflective content refers to what is reflected upon, including mathematical knowledge, mathematical methods, the value of mathematical content, and cognitive processes and the self. Reflective stage concerns when reflection occurs in the learning process—before, during, or after a task. Reflective behavior captures how reflection is enacted, including comparison, classification, explanation, generalization,

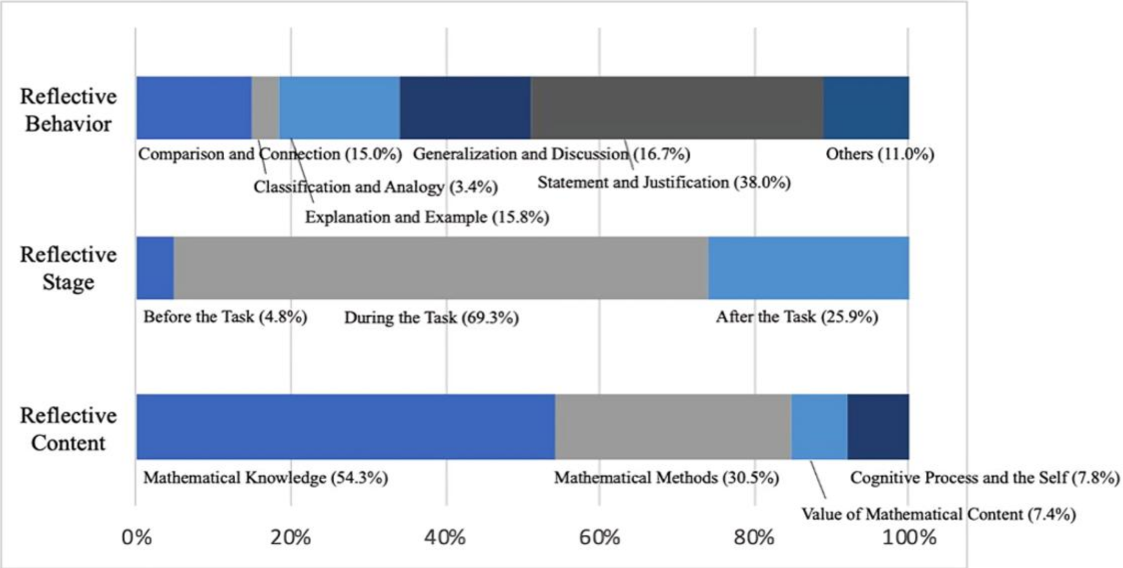


**Figure 1.** Distribution of reflective opportunities in two Chinese mathematics textbooks (Source: Authors' own elaboration)

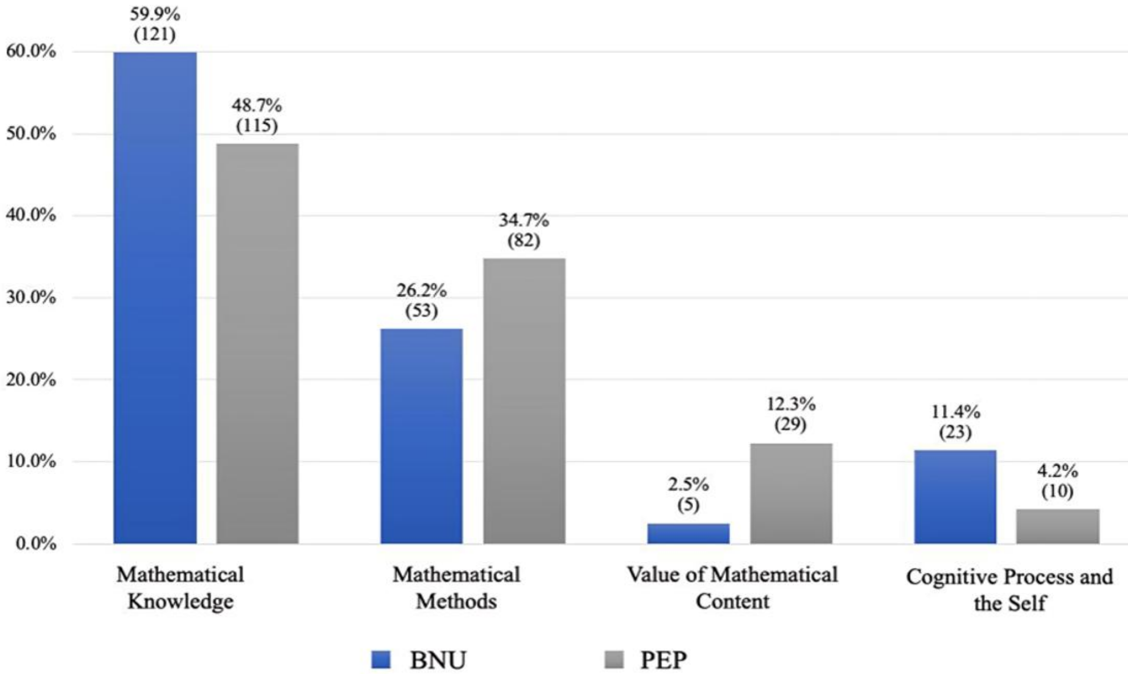


**Figure 2.** Distribution of reflective opportunities across content dimensions (Source: Authors' own elaboration)

justification, and other cognitive or metacognitive actions.

### Overview of Reflective Opportunities in Two Chinese Mathematics Textbooks

**Figure 1** shows the overall distribution of reflective opportunities across the three dimensions in the two textbook series. As shown in **Figure 1**, reflective opportunities were unevenly distributed across the three major dimensions. Among the four subcategories of reflective content, the majority of opportunities focused on mathematical knowledge and methods, while far fewer addressed the value of mathematics or students' cognitive and self-regulatory processes. Similarly, across the three subcategories of reflective stage, opportunities occurring before the task were notably scarce. For reflective behavior, statement and justification types predominated, while opportunities for classification or analogy were limited. The following sections present a detailed account of the extent and patterns of reflective opportunities within each dimension.

### Extent and Distribution of Reflective Opportunities by Content

The distribution of reflective content across the two textbook sources is shown in **Figure 2**.

As shown in **Figure 2**, reflective opportunities emphasizing mathematical knowledge accounted for the

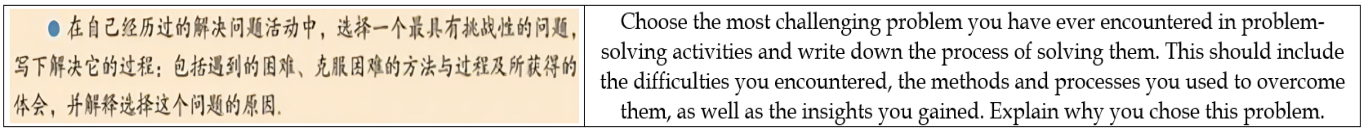


**Figure 3.** Example question-1 (Source: Authors' own elaboration)

largest proportion of identified instances. In the BNU edition, such instances represented 59.9% of reflective opportunities (121 out of 202), whereas in the PEP edition they accounted for 48.7% (115 out of 236). To provide a comparative overview across the two textbook series, the percentages reported in the text represent the mean proportion across editions; thus, reflection on mathematical knowledge averaged 54.3% across the two versions.

Reflection on mathematical methods and strategies followed a similar pattern, comprising 26.2% in the BNU edition and 34.7% in the PEP edition, with an average proportion of 30.5%. In contrast, opportunities addressing the value of mathematical learning (2.5% in BNU; 12.3% in PEP; average 7.4%) and those focusing on cognitive processes and self-awareness (11.4% in BNU; 4.2% in PEP; average 7.8%) were comparatively less frequent across both textbook series.

These findings indicate that both textbook series predominantly promote knowledge- and method-oriented reflection, while affective and metacognitive opportunities remain underrepresented. The BNU edition contained a slightly higher proportion of knowledge-based reflection, whereas the PEP edition included more method-focused prompts.

Reflection on mathematical knowledge and skills is particularly prominent. For example, the BNU edition (grade 7, volume 1) includes the prompt: "Use your own words to describe the similarities and differences between prisms and cylinders." Similarly, in the PEP edition (grade 7, volume 1), a problem in the main text presents two solution methods for the same task and asks students: "What are the differences in the order of operations between the two methods? Which law of operations is used in the second method? Which method requires fewer steps?" All textbook versions include similar reflective prompts in sections titled "review and summary."

On the other hand, reflection on cognitive processes and the self appears less frequently. One such example can be found in the PEP edition (grade 9, volume 2), where students are asked to recall how they used a special-to-general approach to study the properties of inverse proportional functions with positive coefficients. They are then prompted to consider: "Can you use a similar method to explore the graph and properties of inverse proportional functions with negative coefficients?"

In the comprehensive review section of the BNU edition (grade 7, volume 1), there is a question of this kind, as shown in **Figure 3**.

The explicit inclusion of such reflective prompts in the textbook is particularly valuable, as it serves as meaningful guidance for instruction.

The data reveal that reflective prompts in all textbook versions are predominantly concentrated on mathematical knowledge and mathematical methods, highlighting the textbooks' strong emphasis on the core content of the mathematics curriculum. Mathematical knowledge forms the foundation of students' learning, and reflecting on it helps consolidate their conceptual understanding. Mathematical methods, on the other hand, are essential tools for problem-solving, and emphasizing reflection on methods aligns well with the discipline's goal of cultivating logical thinking and problem-solving skills.

However, the proportion of reflective prompts targeting cognitive processes and the self is noticeably lower. This suggests a limitation in the current textbooks' ability to guide students in examining their own learning processes and recognizing their individual learning characteristics. Understanding how one learns is crucial for effectively adjusting learning strategies. Similarly, the low proportion of prompts prompting reflection on the value of mathematical learning objects indicates a gap between textbook content and curriculum standards, which emphasize helping students appreciate the value of mathematics.

## Distribution of Reflective Opportunities by Stage

The distribution of reflective opportunities across the three learning stages is presented in **Figure 4**. Most opportunities were embedded during tasks (approximately 69.3%), followed by those appearing after tasks (25.9%), while before-task reflection represented only 4.8% of all coded instances.

This indicates that both textbook series emphasize reflection during problem-solving but provide limited support for reflection during planning or goal-setting phases. For example, a PEP prompt designed to activate prior knowledge asks: "Now that we are introducing negative numbers, what new cases might arise in addition?" Such planning-oriented opportunities are rare. In contrast, many prompts focus on ongoing or evaluative reflection, such as: "What other ways might there be to construct a coordinate system?" (BNU edition, grade 7) "What have you gained from this

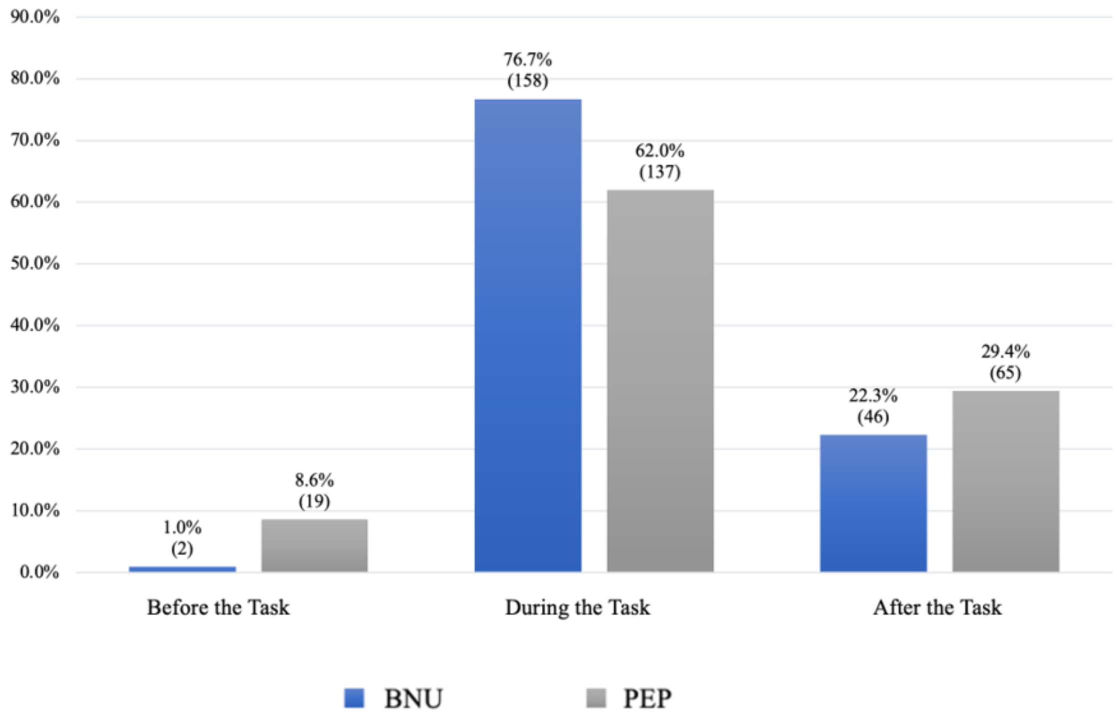


**Figure 4.** Distribution of reflective opportunities across stages (Source: Authors' own elaboration)

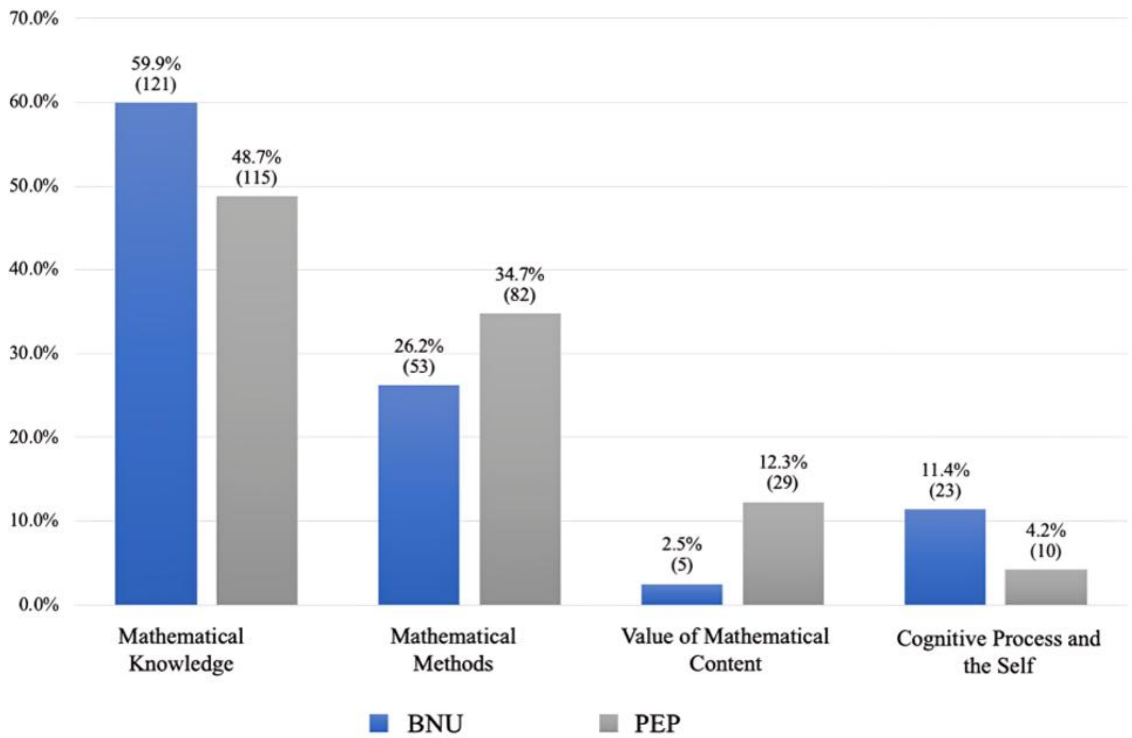


**Figure 5.** Distribution of reflective opportunities across behavioral dimensions (Source: Authors' own elaboration)

semester's mathematics study? What areas still need improvement?" (BNU edition, grade 9).

Overall, the findings suggest that textbooks offer limited opportunities for pre-task reflection, which restricts students' ability to anticipate and regulate their learning. Integrating reflection consistently before, during, and after tasks would more effectively support comprehensive metacognitive engagement.

## Distribution of Reflective Behavior

The behavioral distribution of reflective opportunities (Figure 5) shows that statement and Justification opportunities were most frequent (38.0%), followed by generalization and discussion (16.7%), explanation and example (15.8%), and comparison and connection (15.0%). In contrast, classification and

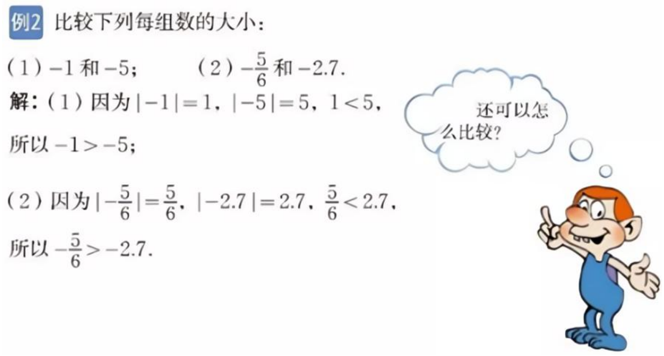


**Figure 6.** Example question-2 (Source: BNU edition, grade 7)

analogy appeared rarely (3.4%), while miscellaneous or creative forms (others) accounted for 11.0%.

This pattern reveals a strong emphasis on articulating and justifying reasoning—essential components of mathematical discourse—but fewer opportunities for analogy-making, categorization, or creative reasoning. Examples include: "Explore other ways to compare two numbers in size." (BNU edition, grade 7) (**Figure 6**).

Statement-based reflection promotes clarity and logical reasoning, while discussion and example-based reflection support collaborative and applied understanding. However, the low frequency of classification, analogy, and creative behaviors suggests that textbooks provide relatively narrow behavioral pathways for reflection. A more balanced inclusion of higher-order reflective behaviors could better promote students' critical and innovative thinking.

Overall, the analysis reveals that reflective opportunities in Chinese junior secondary mathematics textbooks are present but unevenly distributed. The majority target content-level reflection and occur during task execution, while opportunities addressing value, self-awareness, or pre-task planning are rare. Behaviorally, reflection is most often enacted through statements and justifications, with limited variety across analytical or creative forms. These findings underscore the need for more diverse, balanced, and strategically embedded reflective opportunities that support metacognitive and affective dimensions of mathematics learning.

## DISCUSSION

This study investigated the extent and distribution of reflective opportunities in two widely used Chinese junior secondary mathematics textbook series. By applying a multidimensional analytical framework, the study identified how reflection was represented across three dimensions—content, stage, and behavior—and revealed several consistent patterns.

### Uneven and Surface-Level Distribution of Reflective Opportunities

Findings indicate that reflective opportunities are present but unevenly distributed across dimensions. The majority of prompts focus on mathematical knowledge and methods, while relatively few engage students in reflecting on the value of mathematics, self-regulation, or learning strategies. Similarly, reflection is most frequently embedded during tasks, with limited prompts before or after the learning process. In terms of behavior, textbooks overwhelmingly favor statement- or justification-type prompts, providing fewer opportunities for deeper analytical or creative engagement such as comparison, analogy, or generalization.

This pattern is broadly align with prior international textbook research showing that mathematics materials often prioritize cognitive structure and procedural fluency over metacognitive development (Fan et al., 2013; Stylianides et al., 2018). However, the present findings extend this line of research by demonstrating not only the limited presence of reflective prompts, but also their structural concentration within specific stages and behaviors. In contrast to curriculum documents that emphasize holistic learner development (Ministry of Education of the People's Republic of China, 2022; NCTM, 2000), reflective engagement in textbooks remains predominantly task-bound and explanation-oriented. This structural imbalance suggests that reflection is treated more as a local instructional device than as an integrated pedagogical principle. Similar tensions between curricular ideals and textbook enactment have been reported in comparative studies (Niss & Jensen, 2002; OECD, 2005), indicating that the challenge of embedding sustained reflective practice may not be unique to the Chinese context.

### Interpreting the Patterns: Curriculum, Design, and Pedagogy

The predominance of knowledge- and method-oriented reflection can be attributed to long-standing curricular traditions emphasizing mastery of content and problem-solving procedures (Pepin & Haggarty, 2001; Zhu & Fan, 2006). This orientation may lead textbook authors to treat reflection as an auxiliary or evaluative task—often placed at the end of chapters—rather than as a core learning process integrated throughout instruction. Furthermore, reflective opportunities requiring self-monitoring or value-based judgment are cognitively demanding and may be perceived as less assessable within exam-driven contexts.

From a learning-theoretical perspective, the limited distribution of pre- and post-task reflection weakens the cyclical nature of self-regulation described in metacognitive models (Flavell, 1979; Zimmerman, 2002).

When reflection is primarily embedded during task execution, opportunities for strategic planning and evaluative synthesis are constrained. This finding resonates with broader critiques in the literature that textbooks tend to privilege immediate problem resolution over longitudinal reflective development.

Behaviorally, the dominance of explanation-based prompts suggests an epistemic emphasis on correctness and justification. While explanation supports conceptual articulation, it does not necessarily foster abstraction, comparison, or transfer. Previous studies on reflective scaffolding (Kramarski & Mevarech, 2003) indicate that higher-order reflective behaviors require explicit structuring. The relative absence of such prompts in the textbooks analyzed may therefore limit the development of adaptive expertise.

### Implications for Textbook Design and Classroom Practice

While the analytical framework provides a conceptual structure for examining reflection, the specific implications proposed here are derived from the empirical distribution patterns identified in the two textbook series. Although all indicators of the reflective framework are present in both textbook series, the empirical analysis revealed marked concentration within certain dimensions. The following implications therefore aim to address these structural imbalances rather than to compensate for complete absence.

First, diversify reflective content. Textbooks should include not only prompts on knowledge and procedures but also opportunities for students to reflect on mathematical values, real-world relevance, and personal learning strategies. This aligns with the mathematics curriculum standards for compulsory education (Ministry of Education of the People's Republic of China, 2022), which emphasize fostering appreciation of the utility and beauty of mathematics.

Second, distribute reflection across learning stages. Reflection should be intentionally embedded before, during, and after learning tasks. Pre-task prompts can guide planning and goal setting; mid-task prompts can support self-monitoring and adaptive regulation; and post-task prompts can facilitate synthesis and transfer. Such distribution aligns with cyclical models of metacognition and self-regulated learning.

Third, broaden reflective behaviors. Embedding reflective activities that engage students in justification, analogy, and self-evaluation resonates with the constructivist paradigm advanced by Bransford et al. (2000) and Boaler (2016). Integrating metacognitive scaffolds and reflection-based prompts can promote self-regulated learning (Paris & Paris, 2001; Pintrich, 2004), while connecting mathematical reasoning with personal meaning supports transformative learning (Mezirow, 1990; van Manen, 1995). These behaviors foster higher-order reasoning, collaborative inquiry, and creative problem-solving—core aspects of 21st century mathematical literacy. These design considerations highlight that the issue is not the absence of reflection, but its structural configuration within curricular materials.

In classroom practice, teachers also play a critical mediating role. Even when reflective prompts are present, their impact depends on how teachers integrate them into discussion, feedback, and formative assessment (Kaur, 2020; Stylianides et al., 2018). Professional development should therefore support teachers in recognizing and leveraging reflective opportunities to promote metacognitive discourse.

### Broader Educational Implications

The empirical patterns identified in this study, namely the dominance of knowledge-oriented prompts (54.3%), the scarcity of value- and self-awareness-related reflection (below 8%), the concentration of reflection during task execution, and the prevalence of statement-based behaviors, suggest that reflection in current textbooks is structurally constrained rather than absent.

When reflection is embedded primarily within procedural problem-solving, students may engage in explanation without systematically developing planning or evaluative capacities. The limited presence of pre- and post-task prompts weakens support for cyclical self-regulation. Moreover, the minimal proportion of value-oriented reflection indicates that textbooks rarely connect mathematical reasoning to personal meaning or broader contexts. The behavioral emphasis on explanation over comparison, abstraction, or generalization further narrows opportunities for higher-order reflective engagement. These findings suggest that future textbook revision should examine not only whether reflection is included, but how it is distributed across content, stages, and behaviors.

Beyond the Chinese context, the three-dimensional framework proposed in this study offers a transferable analytical tool for examining reflective opportunities in diverse curricular systems. Because the framework is grounded in internationally established theories of metacognition, problem-solving, and reflective practice, it provides a common conceptual language for cross-textbook and cross-cultural comparison. Future studies could apply this framework to textbooks from different educational systems to explore whether similar structural patterns emerge or whether variations reflect distinctive curricular traditions. In this sense, the present study contributes not only empirical evidence from China but also a methodological lens for advancing international textbook research.

## CONCLUSION

This study provides an empirical account of how Chinese junior secondary mathematics textbooks create opportunities for student reflection. By applying a multidimensional analytical framework encompassing reflective content, stage, and behavior, the analysis identified 390 reflective opportunities and revealed that while such opportunities exist, they are unevenly distributed and limited in depth. Reflection in these textbooks predominantly focuses on mathematical knowledge and methods, occurs mainly during task execution, and is enacted primarily through statements or justifications. Opportunities for students to engage in pre-task planning, post-task evaluation, self-regulation, or value-based reflection remain comparatively scarce. These findings contribute to the growing body of research on textbook analysis by shifting the focus from theoretical descriptions of reflection to a quantitative and systematic evaluation of its presence and structure within curriculum materials. Methodologically, the study demonstrates the applicability and reliability of a multidimensional coding framework for examining textbook-based reflection, offering a replicable tool for cross-cultural and longitudinal research.

Beyond its analytical contribution, the study carries important implications for curriculum design, textbook development, and classroom practice. To align more closely with competency-based education and the goals articulated in China's mathematics curriculum standards for compulsory education (Ministry of Education of the People's Republic of China, 2022), textbooks should intentionally integrate reflection across content domains, learning stages, and behavioral forms. Embedding reflection before, during, and after learning tasks can strengthen metacognitive regulation, while incorporating prompts that emphasize value, strategy, and self-awareness can promote holistic mathematical literacy. The purpose of this study was to provide a descriptive content analysis of reflective tasks rather than to test formal hypotheses. Given that the analyzed prompts represent an exhaustive or purposive sample of the selected textbooks rather than a random sample from a larger population, inferential statistical tests were not applied. Although a full census of all reflective prompts across the textbook series could provide additional precision, the present stratified sampling design—covering nearly half of all identified prompts—offers a robust and representative basis for the distributional patterns reported in this study. Future research may extend this line of inquiry by

(a) comparing reflective opportunities across educational systems and grade levels,

(b) examining how teachers and students actually engage with textbook-based reflective tasks in classrooms, and

(c) exploring how digital or adaptive textbooks can personalize reflection to support individual learners.

Future research could adopt a topic-based sampling approach to examine whether reflective patterns vary systematically across different mathematical domains (e.g., algebra, geometry, and data analysis). Because different areas of mathematics are characterized by distinct epistemological features, reflective opportunities embedded in textbooks may differ in both their frequency and pedagogical function. Such topic-focused analyses may therefore provide a more nuanced understanding of how reflective practices are structured within specific mathematical content areas. Through these efforts, reflection can move from a peripheral element to a central pedagogical mechanism that empowers students to think critically, learn autonomously, and appreciate mathematics as both a discipline and a way of reasoning about the world.

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**AI statement:** The authors stated that no generative artificial intelligence tools or AI-based software were used in the design, data collection, analysis, or writing of this study.

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