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Towards onto-semiotic analysis of the university students of business sciences' mathematical activity when solving mathematics problems of a diagnostic

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Abstract

This article analyzes the mathematical activity of students entering a university mathematics course for the first time, based on their completion of a diagnostic questionnaire consisting of five problems in elementary arithmetic and algebra. The study is developed from an onto-semiotic approach, integrating the analysis of mathematical practices, cognitive configurations of primary objects, and the types of mathematical connections established by the students while solving the tasks. The methodology was qualitative, based on a content analysis of the mathematical activity of students in logical-mathematical reasoning. The results show that the first problems primarily activate procedural and instructional connections, associated with prior school knowledge, which is reflected in high success rates. However, as the onto-semiotic complexity of the tasks increases, especially in the linear function modeling problem, a significant decrease in performance is observed. This decrease is explained by the need to simultaneously coordinate multiple mathematical connections, particularly those related to the articulation of representations, the construction of meaning, and the activation of procedures. It is concluded that the difficulties identified do not stem from a lack of mathematical knowledge, but rather from a breakdown in the connections necessary to reinterpret school content within university contexts. These findings provide relevant elements for the design of teaching strategies aimed at facilitating the transition from school to university level.

Keywords: problem-solving, university students, mathematics education, mathematical connections

INTRODUCTION

Arithmetic and algebra are fundamental pillars of mathematical knowledge and play a decisive role in university education across multiple disciplines. Arithmetic, understood as the field encompassing basic operations (addition, subtraction, multiplication, and division), lays the foundation for subsequent learning and contributes to the development of essential cognitive skills such as logical reasoning, abstraction, and working memory (Morocho Guamán et al., 2025; Toainga et al., 2025). Algebra, for its part, introduces a form of thinking characterized by generalization and the use of symbols to represent relationships and structures, which is indispensable in university programs of a scientific, technological, economic, and administrative nature. In these contexts, arithmetic and algebra function as transversal tools that connect and support the learning of subjects such as calculus, statistics, physics and data analysis (Davis, 2024; Rodríguez-Nieto et al., 2024; Tong et al., 2024).

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Contribution to the literature

- The article conceives of mathematical activity as a practice, not merely a result, analyzing how students mobilize primary objects, mathematical practices (MPs), and semiotic functions (SFs) during problem-solving.
- It demonstrates that many errors stem not from a lack of knowledge, but from semiotic and epistemic conflicts, where students activate personal meanings that do not align with institutional meanings, particularly in the handling of signs, equality, and slope.
- It provides an integrated onto-semiotic approach (OSA)-extended theory of connections (ETC) analytical framework that allows for the identification of breaks in mathematical connections and provides a basis for diagnoses and didactic interventions to facilitate the transition to university-level mathematics.

Despite its importance, numerous studies have revealed persistent difficulties in mastering arithmetic from the earliest levels of education. These difficulties include errors in performing basic procedures and problems interpreting contextualized situations, which tend to persist and worsen over time (Chen, 2025; Rodríguez-Nieto et al., 2025a, 2025b). When students encounter algebra, these limitations intensify, as they need to transition from numerical to algebraic reasoning, a process that involves understanding symbols, establishing generalizations, and justifying transformations. Recent studies report frequent errors in interpreting statements and manipulating algebraic expressions, associated with the mechanical use of algorithms without conceptual support (Acosta et al., 2024; Ryals et al., 2025; Sánchez García, 2019; Santana et al., 2023).

The transition from secondary to higher education is a particularly critical stage in mathematics learning. Upon entering university, students face greater academic demands and curricula that presuppose a solid command of arithmetic and algebra. However, research conducted in various contexts shows that a significant proportion of first-year students exhibit substantial deficiencies in these areas, which affects their academic performance and hinders their learning of advanced courses (Chávez Roblero, 2018; Márquez et al., 2025; Parra Rodríguez et al., 2026). These findings are repeated in different regions, demonstrating that difficulties in solving algebraic problems and in translating between verbal and symbolic language constitute a widely documented phenomenon (Kenney & Ntow, 2024; Nti & Jupri, 2024).

Errors in learning algebra constitute an important source of information for understanding how students construct mathematical knowledge. Rather than being interpreted merely as procedural failures, these errors help identify weaknesses in conceptual understanding and in the mastery of symbolic rules that underpin algebraic thinking. Analyses of students' deficiencies and errors reveal that many difficulties are related to incomplete procedural knowledge and limited interpretations of algebraic structures (Dorner et al., 2025). Likewise, studies using cognitive maps have shown that many mistakes originate from persistent misconceptions about the meaning of variables, operations, and the equivalence of algebraic expressions (Fitria & Susanto, 2023). These difficulties indicate that learning algebra involves not only mastering procedures but also developing a solid conceptual understanding that enables students to correctly interpret and manipulate mathematical symbols.

From a pedagogical perspective, several approaches propose explicitly working with errors as part of the teaching and learning process. The use of incorrect examples, for instance, can help refine algebraic knowledge and strengthen the understanding of the structural features of equations, as it allows students to analyze and contrast correct and incorrect procedures (Barbieri & Booth, 2020). In the same direction, teaching techniques have been proposed to reduce students' mistakes when simplifying algebraic expressions by promoting more systematic reasoning processes (Rafiepour et al., 2023). Other approaches, such as the use of the bar model or the strengthening of pre-algebraic strategies, aim to improve the understanding of relationships between quantities and foster early generalization processes (Kuzu, 2022; Roos et al., 2024). Even in advanced areas of applied mathematics, such as cryptography based on algebraic structures and learning-with-errors problems, the analysis of errors plays a role in the development of robust mathematical models (Espitau et al., 2025).

In this context, problem-solving has been recognized as the central axis of mathematical activity and its teaching. Solving a problem involves understanding the situation, analyzing relationships, constructing models, and arguing for and validating solutions, going beyond the simple application of procedures (Cerón-Molina et al., 2025; Cervantes-Barraza et al., 2025; Rodríguez-Nieto & Font, 2025). Therefore, problem-solving and problem-posing occupy a central place in contemporary mathematics education (Cai & Rott, 2024; Geteregechi, 2025; Ministry of National Education, 2006; National Council of Teachers of Mathematics, 2000; Rodríguez-Nieto et al., 2025c; Santos-Trigo, 2024). Theoretical frameworks such as the OSA and the ETC allow us to analyze mathematical activity in terms of semiotic practices, objects, and processes (Godino et al., 2019), while APOS theory conceptualizes understanding as the coordination of actions, processes, objects, and schemas (Arnon et al., 2014). From these perspectives, the establishment of mathematical connections is key to deep and functional learning (Businskas, 2008; García-García & Dolores-Flores, 2021; León del Carmen et al., 2024; Rodríguez-Nieto et al., 2022b).

It is worth noting that problem-solving in mathematics education is fundamental to meeting the demands of Education 4.0. Through critical thinking, data analysis, and the use of digital tools, students develop skills to interpret real-world situations, innovate creatively, and make informed decisions in an increasingly technological and globalized society (Taja-on et al., 2025). Furthermore, it is important to analyze mathematics teaching strategies that integrate values from moral education and propose pedagogical approaches that promote critical thinking, social responsibility, and ethical decision-making, demonstrating how mathematics can contribute to students' holistic development through contextualized and reflective classroom activities (Harefa & Hulu, 2024).

Despite the various didactic proposals aimed at strengthening the learning of arithmetic and algebra, including visual models, creative problem-solving, problem-solving, and the use of digital technologies (Burgos & Tizón-Escamilla, 2025; Milanesio & Burgos, 2025a, 2025b; Morocho Guamán et al., 2025; Roos & Kempen, 2024; Wilkie, 2024), difficulties persist, especially at the beginning of university education, where factors such as the poor articulation between educational levels, the lack of contextualization of problems respecting semantic and syntactic aspects, and the low confidence of students to apply mathematics in problem-solving are influencing this situation in different university semesters (Hatisaru et al., 2025; Kusuma & Mariani, 2024; Sudirman et al., 2024). Given this important but also specific need, the objective of this article is to analyze the mathematical activity of students entering university for the first time by solving a diagnostic questionnaire composed of five problems of arithmetic and elementary algebra, in order to identify the mathematical connections they establish and the errors that emerge during problem-solving, providing elements to understand and strengthen the transition from secondary to higher education in mathematics.

THEORETICAL FRAMEWORK

Extended Theory of Connections

The ETC provides a conceptual framework for understanding mathematical learning through the relationships that learners establish among mathematical ideas, representations, procedures, and contexts. From the perspective of networking between ETC and the OSA, a mathematical connection can be conceived as the visible manifestation of a deeper structure of mathematical activity. Specifically, a connection represents the "*tip of the iceberg*" of a conglomerate composed of practices, processes, primary objects, and SFs activated by a subject when solving a task (Rodríguez-Nieto et al., 2022b). This metaphor highlights that mathematical connections are not isolated cognitive acts, but emergent phenomena resulting from complex interactions within mathematical activity (Rodríguez-Nieto & Font, 2025).

The ETC categorizes mathematical connections into three broad groups, depending on the nature of the task and the type of relationships involved. The first group corresponds to intra-mathematical connections, which involve relationships among mathematical concepts, representations, theorems, procedures, and meanings. These connections are fundamental for the internal coherence of mathematical knowledge and support conceptual development. The second group consists of extra-mathematical connections, which emerge when mathematical knowledge is applied to real-life situations or problems from other disciplines. In these cases, learners transfer information from a contextualized problem into a mathematical model, thereby activating modeling processes (Dolores-Flores & García-García, 2017). The third group encompasses ethnomathematical connections, understood as relationships between mathematics embedded in cultural practices and institutional mathematics, making visible the mathematical meanings constructed in everyday activities such as measuring, designing, or counting within specific cultural contexts (Rodríguez-Nieto, 2021).

A key contribution of the ETC lies in its detailed typology of mathematical connections, which makes the theory operational for empirical analysis. Instruction-oriented connections emphasize continuity with prior knowledge, highlighting how new concepts (C) are understood through previously learned concepts (A and B), either by association or by prerequisite relationships (Businskas, 2008). Modeling connections describe the relationship between mathematics and the real world or other scientific domains, where learners construct mathematical models using symbolic, algebraic, and graphical actions to address contextualized tasks (Campo-Meneses & García-García, 2023). Connections of different representations capture the ability to express mathematical objects through equivalent or alternate registers, supporting conceptual understanding through representational flexibility (Businskas, 2008).

Procedural connections involve the application of algorithms, rules, and formulas, emphasizing structured sequences of actions to operate with mathematical concepts. Part-whole connections include both generalization relationships and inclusion relationships, allowing learners to situate specific cases within broader conceptual structures. Implication connections refer to logical relationships in which one concept leads deductively to another. Feature-based connections emerge when learners identify and compare properties of mathematical concepts (Eli et al., 2013), while reversibility connections involve moving back and forth between related concepts or processes (García-García & Dolores-Flores, 2021). Meaning connections arise when learners attribute personal meaning to mathematical concepts, including self-constructed definitions. Metaphorical connections and mnemonic-based metaphorical connections highlight embodied and cognitive strategies that support understanding and recall (Rodríguez-Nieto et al., 2022a, 2024). Finally, idealizing connections capture the transition from concrete, ostensive objects to abstract mathematical entities (Ledezma et al., 2024). Recent developments extend the ETC toward neuro-mathematical perspectives, exploring brain activation patterns associated with mathematical connections (Cantillo-Rudas et al., 2024).

Onto-Semiotic Approach

The OSA emerged from the need to articulate and refine theoretical and methodological tools for analyzing the teaching and learning of mathematics. In this framework, mathematical activity whether personal or institutional is modeled in terms of practices and configurations of primary objects and processes activated during those practices (Drijvers et al., 2013). MP is broadly defined as any activity undertaken to solve problems, communicate solutions, validate results, or generalize ideas across contexts (Godino & Batanero, 1994).

OSA identifies six types of primary objects involved in MP: problematic situations, representations, definitions, propositions, procedures, and arguments (Godino et al., 2019). These objects do not appear in isolation; rather, they form interconnected systems known as configurations, which can be institutional (epistemic) or personal (cognitive). Furthermore, primary objects can be characterized according to several dualities, such as ostensive/non-ostensive, personal/institutional, expression/content, and unitary/systemic (Godino et al., 2007). These distinctions allow for a nuanced analysis of how mathematical meanings are constructed and transformed in practice.

Primary objects emerge through the activation of primary mathematical processes, including communication, problem posing, definition, enunciation, procedures, and argumentation. These processes are intertwined with broader cognitive processes derived from dualities such as materialization–idealization and representation–meaning (Font et al., 2013). A central construct within OSA is the notion of SF, defined as a triadic relationship between an antecedent (expression or object), a consequent (content or object), and a correspondence criterion established by a subject or institution. SFs provide an operational way to model mathematical understanding, competence, and meaning, framing knowledge as a network of relations between objects and practices (Godino, 2022; Godino et al., 2007).

Articulation Between ETC and OSA

Networking between theoretical frameworks enables researchers to analyze complex educational phenomena while preserving the conceptual integrity of each theory (Prediger et al., 2008). In this study, the articulation between ETC and OSA follows the work of Rodríguez-Nieto et al. (2022b), who examined how mathematical connections can be inferred and interpreted from both perspectives. Their analysis shows that while ETC provides a predefined typology of connections, OSA offers detailed tools to unpack these connections in terms of practices, primary objects, and SFs.

A key point of articulation is the recognition that mathematical connections in ETC can be interpreted as specific cases of SFs in OSA. From this perspective, a correct mathematical connection corresponds to an institutional SF, whereas an incorrect connection reflects a personal SF that does not align with institutional meaning. Thus, ETC allows for the identification and classification of connections, while OSA enables a deeper analysis of their nature, accuracy, and cognitive underpinnings.

The complementary use of both frameworks is particularly powerful for analyzing students' mathematical activity in problem-solving. ETC provides a structured lens to categorize the types of connections students establish, whereas OSA reveals how these connections emerge from configurations of objects, processes, and meanings. Together, these frameworks support a comprehensive understanding of mathematical understanding as a dynamic, semiotically mediated process, rather than a mere accumulation of procedures. This integrated perspective underpins the analytical approach adopted in the present study.

METHODOLOGY

This research was conducted using a qualitative, interpretive approach (Lichtman, 2023), with the aim of understanding the mathematical activity of students entering a university mathematics course for the first time. To this end, the resolution of a diagnostic questionnaire was analyzed through a thematic analysis grounded in the tools of the OSA and integrated with mathematical connections. This approach allowed for the interpretation not only of the results obtained, but also of the practices, objects, and connections employed by the students while solving the tasks.

Table 1. Problems proposed in the diagnostic questionnaire

No	Problem	Main mathematical content	Diagnostic intent
1	If $x = -2$ and $y = -3$, the value of $x - xy^2$ is: A. $x - xy^2 = -14$ B. $x - xy^2 = -8$ C. $x - xy^2 = 1$ D. $x - xy^2 = 16$	Algebraic substitution, exponent, and order of operations	Explore the control of the law of signs, the correct use of powers, and the order of operations in basic algebraic expressions.
2	The value of x that satisfies the equation $15x - 10 = 6x - (x + 2) + (-x + 3)$ is: A. $x = 2$ B. $x = 1$ C. $x = -2$ D. $x = -1$	Linear equations with parentheses	Analyze the ability to transform equations while preserving equality and coordinate elementary algebraic procedures.
3	The result of computing $\left\{\frac{81}{9} - \left(10 - \frac{36}{6}\right)\right\} - \frac{2}{3} + \frac{17}{3}$ is: A. 7 B. -10 C. 4 D. 10	Combined operations with fractions	Identify the handling of the order of operations and the coordination between whole numbers, fractions and parentheses.
4	Adding $x^3 - 2x^2 + 5x - 4$ to $-x^3 - 5x + 4$ the result is: A. $2x^3 + 2x^2 - 10x - 8$ B. $-2x^2 - 10x$ C. $-2x^3 + 2x^2 - 10x + 8$ D. $-2x^2$	Addition and simplification of polynomials	Explore the recognition of like terms, algebraic cancellation, and understanding of polynomial structure.
5	A company is analyzing the relationship between the number of units produced and the associated cost. The following data is recorded: When 2 units are produced, the cost is 5. When 6 units are produced, the cost is 17. Assuming that the relationship between the variables is linear: a. Find the equation of the line that passes through the points (2, 5) and (6, 17). b. Graph the line obtained, clearly indicating the given points. A. $y = 3x - 1$ B. $y = 3x - 11$ C. $y = -3x + 1$ D. $y = -3x + 11$	Linear function and mathematical modeling	Analyze the articulation between real context, algebraic expression and graphical representation (modeling and connections between records).

Diagnostic Questionnaire Design

The data collection instrument consisted of a diagnostic questionnaire composed of five mathematical problems, designed with an implicit onto-semiotic progression. The problems address elementary algebra content (algebraic substitution, linear equations, operations with fractions, addition of polynomials, and equation of a line), but were conceived not as routine exercises, but as tasks that generate MPs.

Each problem was designed to activate different primary objects (languages, concepts, propositions, procedures, and arguments) and encourage the emergence of various types of mathematical connections (procedural, different representations, part-whole, modeling, among others). In this way, the questionnaire (see **Table 1**) allowed us to explore the students' transition from predominantly algorithmic practices to functional and modeling practices.

Selection of Participants

The participants (P) were 66 students (20 women and 46 men) enrolled in the subject logical mathematical reasoning 1 at a university in northwestern Colombia. These are recent high school graduates, aged between 15 and 21 years, belonging to the programs of business administration (n = 44, 66.7%), international business (n = 1, 1.5%), and software development technician (n = 21, 31.8%), see **Figure 1**.

The selection was intentional, as the study focused on analyzing the mathematical activity of students encountering university mathematics for the first time. This disciplinary diversity allowed for a cross-disciplinary analysis of mathematical activity, beyond any specific academic program.

Figure 1. Evidence from research participants (Source: Authors' own elaboration)

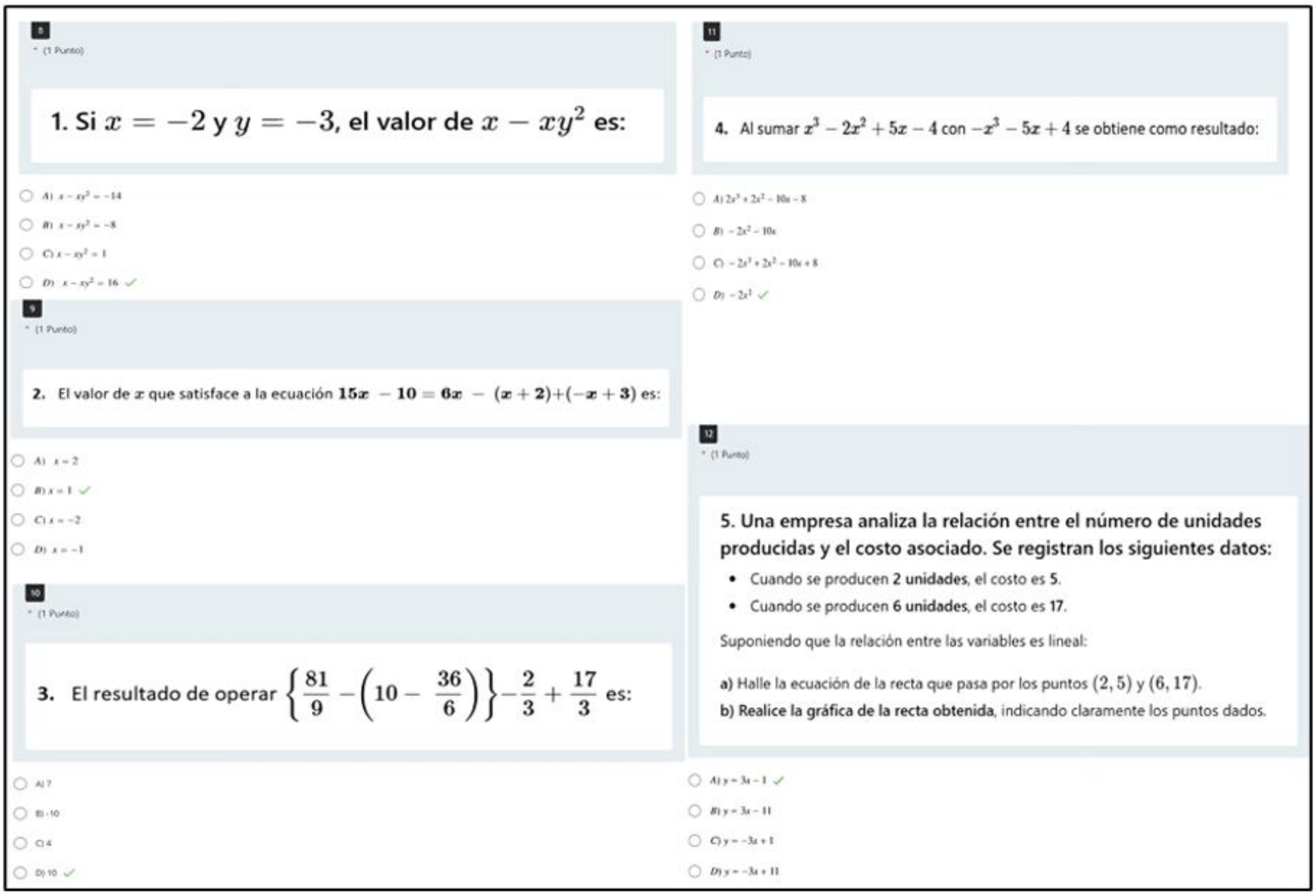
Application of the Questionnaire and Collection of Evidence

The questionnaire was administered during a one-hour face-to-face session, during which students solved the problems using pencil and paper, recording their final answers on a digital form (FORM), see **Figure 2**. This design allowed for the collection of both the final products and the problem-solving processes, as well as the researcher's observations while the students performed the mathematical activity (Chand, 2025; Rodríguez-Nieto & Font, 2025). Also, audiovisual evidence was collected through photographs and videos to document the intermediate procedures, drafts, graphical representations, and strategies employed by the students. This evidence complemented the written records and allowed for a more detailed analysis of the MPs employed during the activity.

Data Analysis

Data analysis was conducted using thematic analysis, following the phases proposed by Braun and Clarke (2006), and was theoretically informed by the OSA and the framework of mathematical connections. Instead of free inductive coding, interpretive deductive coding was employed, where codes corresponded to MPs, primary objects, and types of connections.

Based on this coding, cognitive configurations were constructed for each problem, recurring patterns of mathematical activity were identified, and these were correlated with the percentages of correct and incorrect answers. This process made it possible to explain the



8
* (1 Punto)
1. Si $x = -2$ y $y = -3$, el valor de $x - xy^2$ es:
A) $x - xy^2 = -14$
B) $x - xy^2 = -8$
C) $x - xy^2 = 1$
D) $x - xy^2 = 16$ ✓

9
* (1 Punto)
2. El valor de x que satisface a la ecuación $15x - 10 = 6x - (x+2) + (-x+3)$ es:
A) $x = 2$
B) $x = 1$ ✓
C) $x = -2$
D) $x = -1$

10
* (1 Punto)
3. El resultado de operar $\left\{\frac{81}{9} - \left(10 - \frac{36}{6}\right)\right\} - \frac{2}{3} + \frac{17}{3}$ es:
A) 7
B) -10
C) 4
D) 10 ✓

11
* (1 Punto)
4. Al sumar $x^3 - 2x^2 + 5x - 4$ con $-x^3 - 5x + 4$ se obtiene como resultado:
A) $2x^3 + 2x^2 - 10x - 8$
B) $-2x^2 - 10x$
C) $-2x^3 + 2x^2 - 10x + 8$
D) $-2x^2$ ✓

12
* (1 Punto)
5. Una empresa analiza la relación entre el número de unidades producidas y el costo asociado. Se registran los siguientes datos:
- Cuando se producen **2 unidades**, el costo es **5**.
- Cuando se producen **6 unidades**, el costo es **17**.

Suponiendo que la relación entre las variables es lineal:

a) Halle la ecuación de la recta que pasa por los puntos $(2, 5)$ y $(6, 17)$.
b) **Realice la gráfica de la recta obtenida**, indicando claramente los puntos dados.
A) $y = 3x - 1$ ✓
B) $y = 3x - 11$
C) $y = -3x + 1$
D) $y = -3x + 11$

Figure 2. Questionnaire administered to students, digitized in FORM (Source: Authors' own elaboration)

Table 2. Organization of codes, subthemes, and themes

Initial codes (examples)	Subthemes	Emerging theme
Direct application of algebraic rules; mechanical substitution; partial control of signs	Predominance of algorithmic practices	Theme 1. Centrality of the procedural at the beginning of university studies
Use of prior school knowledge; activation of memorized rules	Dependence on prior learning	
Incorrect cancellation of terms; sign errors	Underlying conceptual fragility	
Step-by-step resolution without justification	Procedures without argumentation	
Isolated handling of algebraic expressions	Fragmentation of primary objects	Theme 2. Breakdowns in the coordination of primary mathematical objects
Difficulty integrating symbolic and conceptual language	Semiotic disarticulation	
Equality treated as an operator rather than a relation	Weakness in algebraic propositions	
Success in computation, failure in interpretation	Calculation–meaning dissociation	
Transformations within the same register	Limited use of representations	Theme 3. Restricted use of mathematical representations
Difficulty transitioning from algebraic to graphical representations	Lack of connections between registers	
Graphs constructed as isolated drawings	Representation without functional meaning	
Correct calculation of slope	Partial activation of modeling	Theme 4. Obstacles in mathematical modeling
Difficulty interpreting the context	Break between reality and mathematics	
Use of formulas without contextual reference	Modeling reduced to a procedural activity	

Table 3. Review and validation of the themes

Theme	Empirical evidence	Analytical decision
Theme 1. Centrality of the procedural	High success rates in problems 1-3 (75-88%); predominance of procedures (Pc) over arguments	Retained as a central theme
Theme 2. Breakdowns in the coordination of objects	Errors in problems 3 and 4; incorrect cancellations; absence of arguments	Refined by incorporating the notion of onto-semiotic coordination
Theme 3. Restricted use of representations	Difficulties in problem 5; errors in graphs and equations	Retained and differentiated from theme 4
Theme 4. Obstacles in modeling	Low success rate (48%); failures to articulate context-model	Consolidated as the theme of greatest complexity

observed difficulties not as a lack of knowledge, but as disruptions in the coordination of mathematical objects and connections, typical of the transition from school to university level.

Phase 1. Familiarization with the data

Researchers become thoroughly familiar with the dataset. In the present study, this familiarization was achieved through a systematic review of 66 students' responses to a diagnostic questionnaire consisting of five mathematical problems. This stage included analyzing correct and incorrect solutions, identifying the procedures used by the students, and examining the distribution of scores in absolute and percentage terms. Examples of individual solutions (such as the case of student P4) were also analyzed, allowing for an initial understanding of the modes of mathematical reasoning activated by students upon entering university.

Phase 2. Generation of initial codes

In this phase, the data were systematically coded. The initial codes were not generated inductively, but rather through a theoretically informed coding process based on the OSA. The initial codes identified were MPs, primary objects (languages, concepts, propositions, procedures, and arguments), and types of mathematical connections (procedural, part-whole, representations, modeling, etc.). Each step in the problem-solving process was coded according to the objects and connections that were activated or absent.

Phase 3. Searching for themes

The initial codes were grouped to identify recurring patterns of mathematical activity. This process gave rise to provisional themes such as predominance of procedural and instructional connections in initial problems; breaks in the coordination of primary objects; difficulty in articulating representations; obstacles in modelling processes. The initial codes obtained in the theoretical coding were grouped interpretively, considering regularities in MPs, primary objects, and types of connections mobilized by students in the five problems in the questionnaire (**Table 2**).

Phase 4. Review of the topics

In this phase (**Table 3**), the emerging themes were contrasted with the complete dataset, including:

(a) percentages of correct and incorrect answers,

(b) analysis of written solutions, and

(c) cognitive configurations constructed for each problem.

During the review, redundant topics were discarded and subtopics were reorganized to ensure internal consistency, distinction between topics, and theoretical alignment with the OSA and the mathematical connections framework.

Phase 5. Definition and naming of the themes

The themes were defined with conceptual precision and named in a manner consistent with the theoretical framework. For example, themes such as the transition from procedural to functional thinking, ruptures in representational connections, and difficulties in idealization and modeling processes were established. Each theme was clearly delimited in terms of its primary objects and the mathematical connections that characterize it.

Theme 1. Predominance of procedural thinking inherited from the school level: The first theme reveals that students' initial mathematical activity relies primarily on algorithmic procedural practices linked to prior school knowledge. In the first problems of the questionnaire, students effectively activate procedures and symbolic languages, reflected in high success rates. However, these practices are developed with little explicit explanation of propositions and arguments, resulting in mathematical activity focused on "how to operate" rather than "why it works." This predominance is mainly associated with procedural and instructional connections, sufficient for routine tasks but limited for more complex mathematical demands.

Theme 2. Breakdowns in the coordination of primary mathematical objects: The second theme highlights that, as tasks require greater structural articulation, breakdowns emerge in the coordination between concepts, propositions, and procedures. Although students master isolated rules, they struggle to integrate them coherently, leading to systematic errors in operations with fractions and polynomials. From the perspective of the OSA (objective structure of mathematical analysis), these difficulties are explained by incomplete cognitive configurations, in which procedures are not supported by stable mathematical propositions. Implication and part-whole connections appear weakened, affecting the overall coherence of mathematical activity.

Theme 3. Limited articulation between mathematical representations: The third theme concerns the restricted use of representations, particularly in tasks requiring the coordination of algebraic and graphical representations. In the linear function problem, many students manage to obtain correct algebraic expressions but struggle to represent them graphically or interpret their functional meaning. This situation reveals MPs focused on executing graphical procedures without an integrated understanding of the mathematical object of function. The connections between representations and the connections of meaning appear fragmented, which limits the development of the functional thinking characteristic of the university level.

Theme 4. Difficulties in the construction of mathematical models: The fourth theme summarizes the main obstacles in mathematical modeling processes. Although the problem context is familiar, a significant number of students reduce modeling to the mechanical application of formulas, without articulating the context, the equation, and the graphical representation as a coherent system. This fragmentation leads to incomplete cognitive configurations, where the connections between modeling, idealization, and meaning are not fully activated. The low success rate on this problem does not indicate a lack of knowledge, but rather the difficulty in simultaneously coordinating multiple mathematical connections, which is a distinctive feature of transition to university-level mathematical thinking.

Phase 6. Report production

The defined themes were integrated into an analytical narrative that articulates MPs, cognitive configurations, and mathematical connections, supported by quantitative evidence (percentages of correct answers) and qualitative evidence (analysis of solutions). This phase culminates in the preparation of the report or article, where the thematic analysis explains the students' transition from school mathematics to university mathematics, highlighting didactic and curricular implications.

RESULTS

The results are organized through a theoretically informed thematic analysis, grounded in the OSA and within the framework of mathematical connections. Quantitative evidence, expressed as percentages of correct and incorrect answers, is articulated with qualitative evidence derived from the analysis of MPs, cognitive configurations of primary objects, and the types of connections mobilized by the students. This integration allows for the identification of recurring reasoning patterns, as well as the main breaks or errors and transitions in the observed mathematical activity, offering a characterization of the shift from school practices to forms of mathematical thinking typical of the university level. Below is an example of written work by students who focused on developing mathematical activity conducive to problem-solving (**Figure 3**). In addition, other productions were observed where students established connections between symbolic and graphical representations, measured by procedural connections, part-whole and modeling about the linear function associated with problem situation (**Figure 4**).

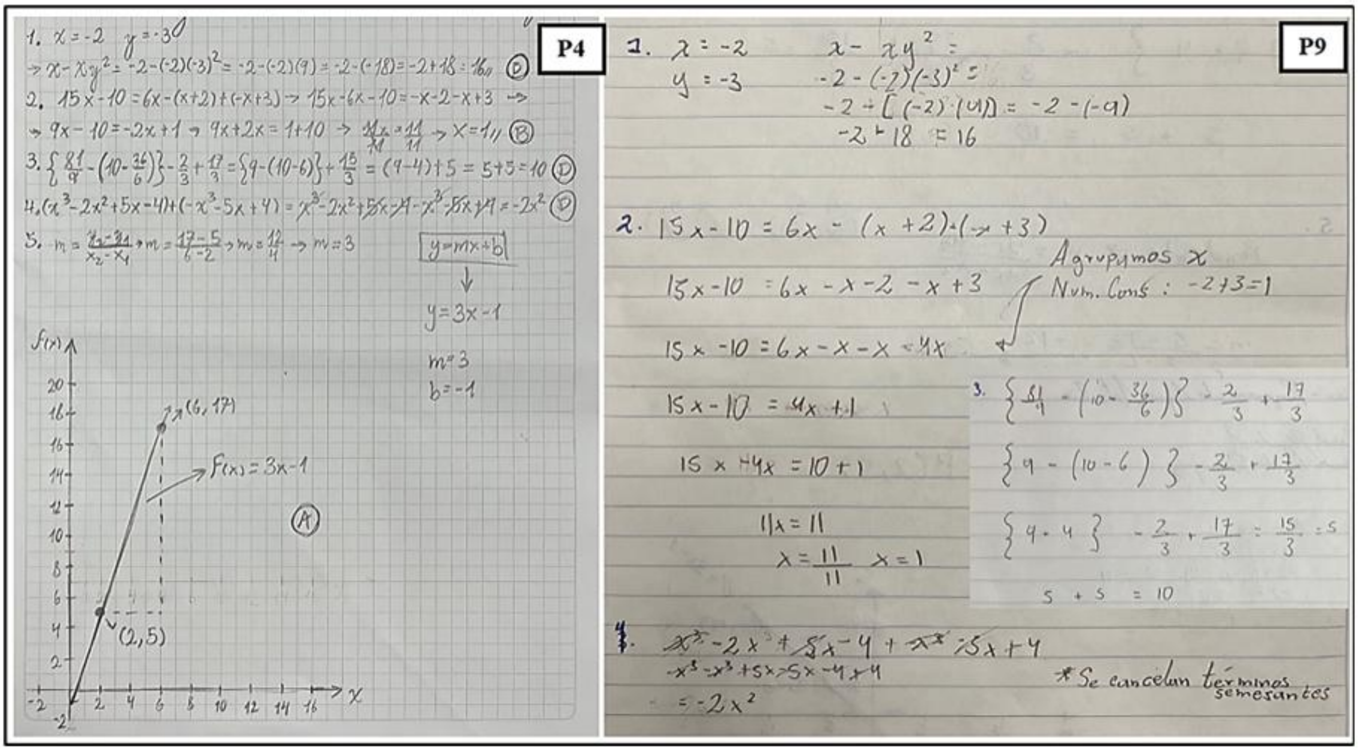


Figure 3. Evidence of problem-solving with mathematical connections (Source: Authors' own elaboration)

Problem 1. Algebraic Substitution and Control of the Order of Operations

This study analyzes the MPs activated in the evaluation of an algebraic expression using value substitution and order of operations. The percentages of correct and incorrect answers are presented, and the predominant cognitive configurations are interpreted, characterized by the activation of algorithmic procedures and procedural and instructional connections of school origin.

Algebraic substitution and order of operations:

Statement: If $x = -2$ and $y = -3$, find $x - xy^2$.

Mathematical narrative through mathematical practices (Mp)

1. Mp1. Reading and identifying data: the student identifies the numerical values assigned to the variables x and y.
2. Mp2. Analysis of the expression structure: recognizes that the expression includes exponentiation, multiplication and subtraction, which requires operational hierarchy.
3. Mp3. Power calculation: calculate $y^2 = (-3)^2$.
4. Mp4. Substitution and algebraic product: substitute the value of the power in the product xy^2.
5. Mp5. Final operation with sign control: performs the subtraction $x - xy^2$ and obtains the final numerical value.

Cognitive configuration of primary objects (problem 1)

Problem situation: Evaluate an algebraic expression by substituting numerical values.

Languages:

Verbal: variable, power, sign, order of operations.

Symbolic: $x = -2$, $y = -3$, $x - xy^2$, $-2 - (-2)(-3)^2$.

Concepts/definitions: Algebraic variable, power, substitution, order of operations.

Propositions:

1. Pr 1. The square of a negative number is a positive number.
2. Pr 2. In an algebraic expression, exponents are calculated before multiplications and differences.
3. Pr 3. Subtracting a negative number is equivalent to adding its absolute value.

Procedures:

1. Pc1. Calculate the power y^2.
2. Pc2. Substitute the obtained value into the product xy^2.
3. Pc3. Perform the subtraction operation, taking the signs into account.

Arguments:

Thesis: The value of the expression is 16.

Reasons: The order of operations and the correct treatment of signs were respected at each step.

Conclusion: The result obtained is mathematically valid.

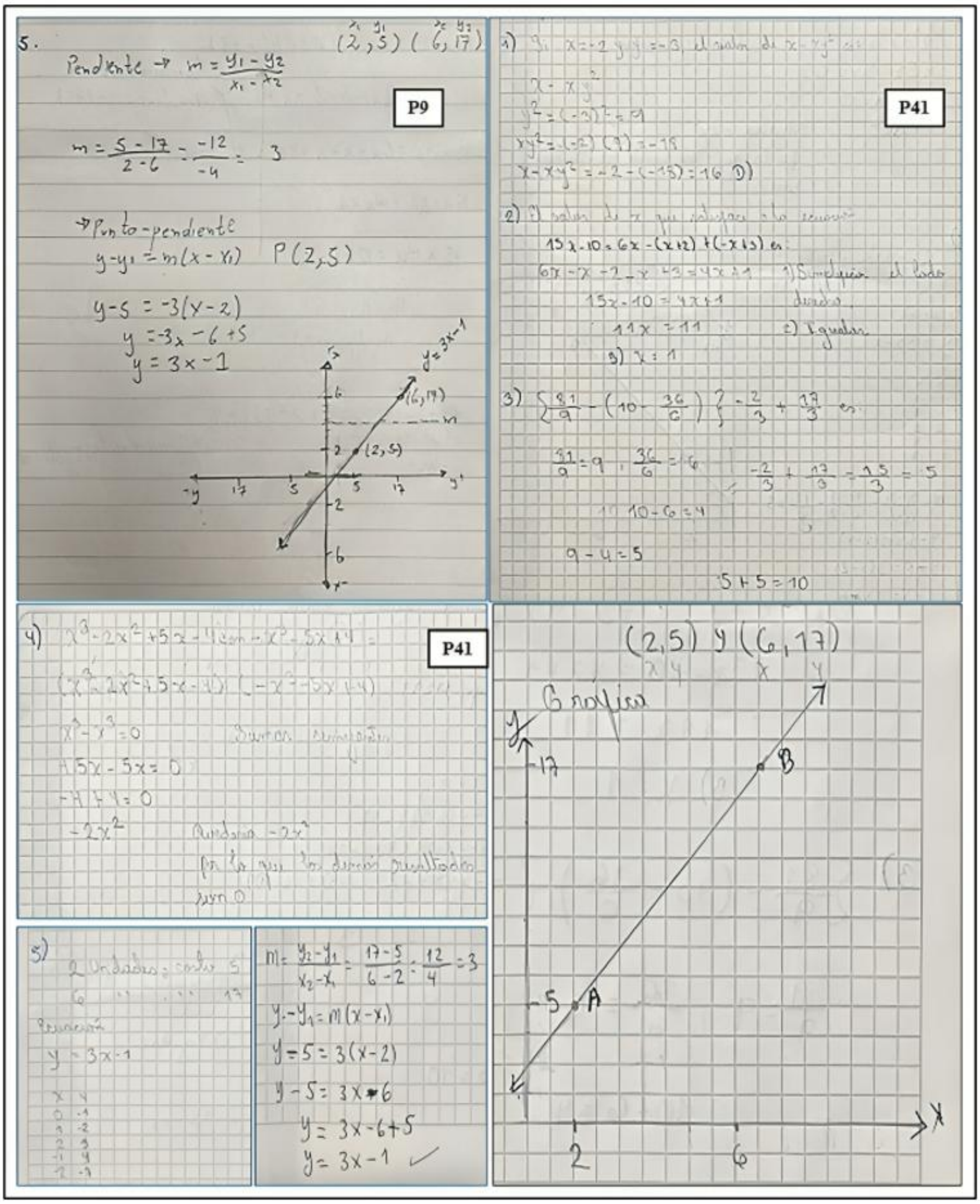


Figure 4. Written production that reflects connections between different representations (Source: Authors' own elaboration)

Global analysis of responses (problem 1)

The 88% success rate indicates that the majority of students correctly articulated Pm2-Pm5, activating the concepts of exponent, product, and sign. However, the remaining 12% showed errors in Pr1 or Pr3, revealing semiotic conflicts with the sign of the square or with the subtraction of negative numbers. The error is not an isolated procedural error, but rather a conceptual one (**Figure 5**).

Problem 2. Solving Linear Equations With Parentheses

This subsection addresses the mathematical activity associated with transforming a linear equation, highlighting simplification practices and the preservation of equality. It analyzes the cognitive frameworks constructed by students, as well as the breakdowns in coordination between algebraic procedures and propositions, based on observed error patterns.

Linear equation with parentheses:

Statement: $15x - 10 = 6x - (x + 2) + (-x + 3)$

Mathematical narrative through mathematical practices (Mp)

1. Mp1. Equation interpretation: the student recognizes a linear equation with parentheses and negative signs.

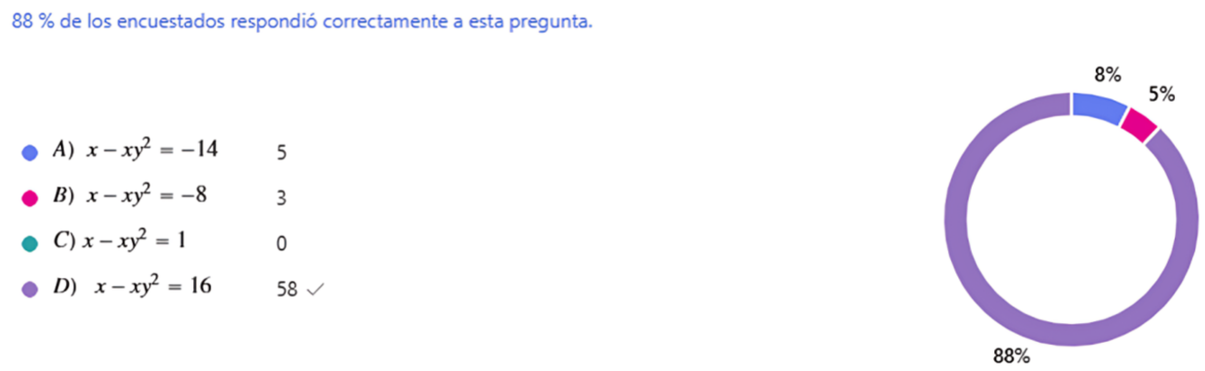


Figure 5. Participant responses to problem 1 (Source: Authors' own elaboration)

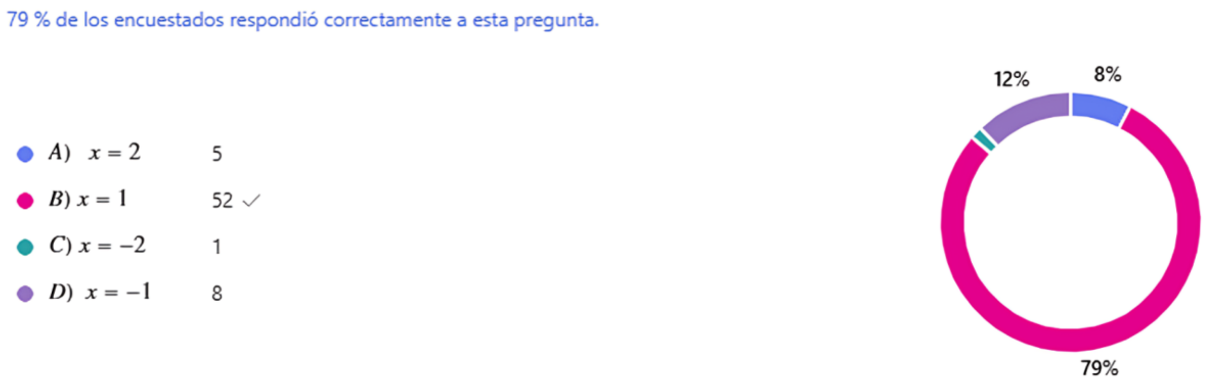


Figure 6. Participant responses to problem 2 (Source: Authors' own elaboration)

2. Mp2. Elimination of parentheses: the student correctly distributes the signs in each term.
3. Mp3. Algebraic reduction: the student groups like terms on each side of the equation.
4. Mp4. Isolation of the unknown: the student operates on both sides until the variable x is isolated.

Cognitive configuration of primary objects (problem 2)

Problem situation: Solve a linear equation with one unknown.

Languages

Verbal: equation, parentheses, clear or find the value of the unknown.

Symbolic: $15x - 10$, $4x + 1$, $11x = 11$.

Concepts/definitions: Linear equation, equality, like terms, unknown.

Propositions:

1. Pr1. Removing parentheses preceded by a negative sign reverses the internal signs.
2. Pr2. Equality is preserved by applying the same operation to both sides of the equation.

Procedures:

1. Pc1. Remove parentheses by distributing signs.
2. Pc2. Combine like terms in the equation.
3. Pc3. Solve for the unknown in the equation.

Arguments:

Thesis: The solution to the equation is $x = 1$.

Reasons: The equation was transformed into an equivalent one using valid operations.

Conclusion: The value found satisfies the original equation.

Global analysis of responses (problem 2)

The 79% success rate shows that most students correctly articulate Pm2-Pm4. The remaining 21% present errors associated with Pr1, that is, conflicts with the semiotic meaning of the negative sign in parentheses. The error is not in the clearing of the variable, but in the prior algebraic transformation (**Figure 6**).

Problem 3. Combined Operations With Fractions

This section presents the results related to solving a numerical expression involving fractions and multiple levels of grouping.

Mathematical narrative through mathematical practices (Mp)

1. Mp1. Simplifying fractions: the student reduces the given fractions.
2. Mp2. Solving parentheses: the student first performs operations on the expressions within the parentheses.

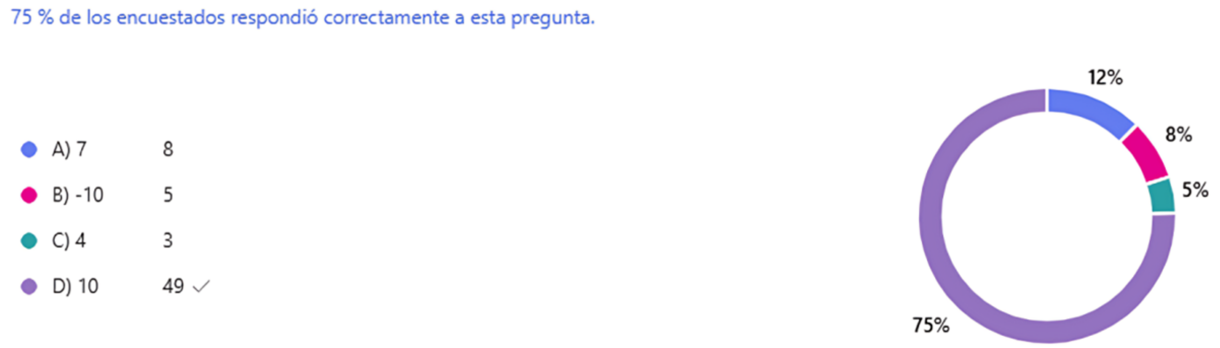


Figure 7. Participant responses to problem 3 (Source: Authors' own elaboration)

3. Mp3. Performing operations on the main expression: the student subtracts whole numbers.
4. Mp4. Operations with fractions with the same denominator: the student adds fractions with the same denominator.
5. Mp5. Integrating partial results: the student obtains the final value of the expression.

Cognitive configuration of primary objects (problem 3)

Problem situation: Evaluate a numerical expression with fractions and parentheses.

Languages

Verbal: fraction, common denominator, law of signs.

Symbolic: $\frac{81}{9}, \frac{17}{3}$.

Concepts/definitions: Rational numbers, fraction, parentheses.

Propositions:

1. Pr1. Fractions can be simplified by dividing the numerator and denominator by the same number.
2. Pr2. Fractions with the same denominator are added or subtracted by operating on the numerators.

Procedures:

1. Pc1. Simplify fractions using operations.
2. Pc2. Solving parentheses and using the law of signs.
3. Pc3. Adding homogeneous fractions.

Arguments:

Thesis: The value of the expression is 10.

Reasons: Each operation was performed respecting the arithmetic order and the rules of fractions with the law of signs.

Conclusion: The result is correct after performing the operations.

Global analysis of responses (problem 3)

The 75% success rate indicates partial mastery of Pm1-Pm4. However, the 25% error rate reveals difficulties in coordinating parentheses and fractions, especially in Pr2, where rules for fractions and integers are mixed without semiotic control (**Figure 7**).

Problem 4. Addition and Simplification of Polynomials

This section analyzes the mathematical activity deployed in the addition of polynomials, emphasizing the breaks in the identification of like terms and in algebraic cancellation.

Mathematical narrative through mathematical practices (Mp)

1. Mp1. Identifying like terms: the student recognizes terms of the same degree.
2. Mp2. Algebraic addition and cancellation: the student adds coefficients and cancels like terms with opposite signs.
3. Mp3. Recognizing the simplified result: the student identifies the term that remains after cancellation.

Cognitive configuration of primary objects (problem 4)

Problem situation: Add two polynomials and simplify the result.

Languages

Verbal: similar term, second- and third-degree expressions.

Symbolic: x^3, x^2, x.

Concepts/definitions: Polynomial, coefficient, Polynomial, coefficient, degree of the algebraic expression.

Propositions:

1. Pr1. Only similar terms can be added.
2. Pr2. Terms with opposite signs cancel each other out when added together.

Procedures:

1. Pc1. Identify similar terms.
2. Pc2. Add coefficients algebraically, taking characteristics into account.

Arguments:

Thesis: The result of adding the polynomials is $-2x^2$.

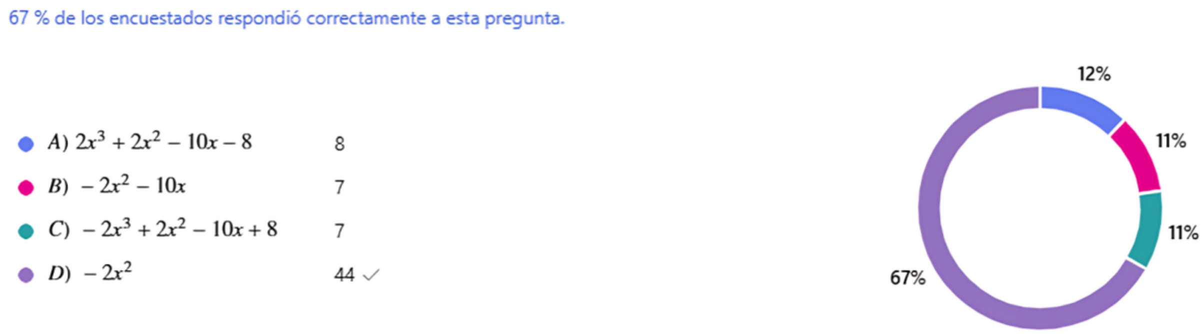


Figure 8. Participant responses to problem 4 (Source: Authors' own elaboration)

Reasons: All other terms are cancelled out when adding opposite coefficients.

Conclusion: The resulting polynomial is the correct simplified answer.

Global analysis of responses (problem 4)

The 67% success rate indicates that algebraic cancellation is not fully understood. Meanwhile, the 33% error rate is associated with mistakes in Pr1, where students add unlike terms, demonstrating an arithmetic rather than a structural understanding of the polynomial (**Figure 8**).

Problem 5. Linear Function and Mathematical Modeling

This section presents the results of the most complex onto-semiotic problem in the questionnaire. It analyzes the MPs associated with modeling, the articulation between context, equation, and graphical representation, and the connections between representation, meaning, and idealization mobilized by the students.

Equation of a line from two points:

Mathematical narrative through mathematical practices (Mp)

1. Mp1. Identifying the given points: the student recognizes the ordered pairs (2,5) and (6,17).
2. Mp2. Calculating the slope: the student applies the slope formula as the rate of change.
3. Mp3. Using an algebraic form of the line: the student uses the point-slope form.
4. Mp4. Algebraic transformation: the student obtains the explicit equation of the line.
5. Mp5. Graphical representation and validation: the student graphs the line and verifies that it passes through the given points.

Cognitive configuration of primary objects (problem 5)

Problem situation: Determine and represent the equation of a line from two points.

Languages:

Verbal: equation, point, Cartesian plane, slope, line, graph.

Symbolic: (2,5), (6,17), $m = \frac{y_2 - y_1}{x_2 - x_1}$, $y = mx + b$.

Graphic: Cartesian plane, line, point.

Concepts/definitions: Slope, linear function, point-slope equation, line.

Propositions:

1. Pr1. Two points determine a unique line.
2. Pr2. The slope represents the rate of change between the variables.
3. Pr3. A point satisfies the equation of a line if it fulfills the equality.

Procedures:

1. Pc1. Calculate the slope using two points.
2. Pc2. Substitute a point into the equation of the line.
3. Pc3. Graph the function in terms of $y = mx + b$.

Arguments:

Thesis: The equation of the line is $y = 3x - 1$.

Reasons: The calculated slope is correct, and the equation satisfies both given points.

Conclusion: The resulting line accurately models the stated relationship.

Global analysis of responses (problem 5)

The 48% success rate reveals a significant break in the chain of MPs. The 52% errors are concentrated in: failures in Pr2 (meaning of slope), mechanical use of formulas without validation (Pm5 missing), and a disconnect between algebraic and graphical language. This problem acts as a critical indicator of the transition to functional thinking (**Figure 9**).

However, student P11 stated in his written work that he can find the equation of a line, but he does not know how to graph the equation. Therefore, he does not establish the connection between different representations, moving from a symbolic register to a graphical one. These are epistemic and semiotic aspects that this student needs to develop in order to succeed in his "business administration" degree, which requires the

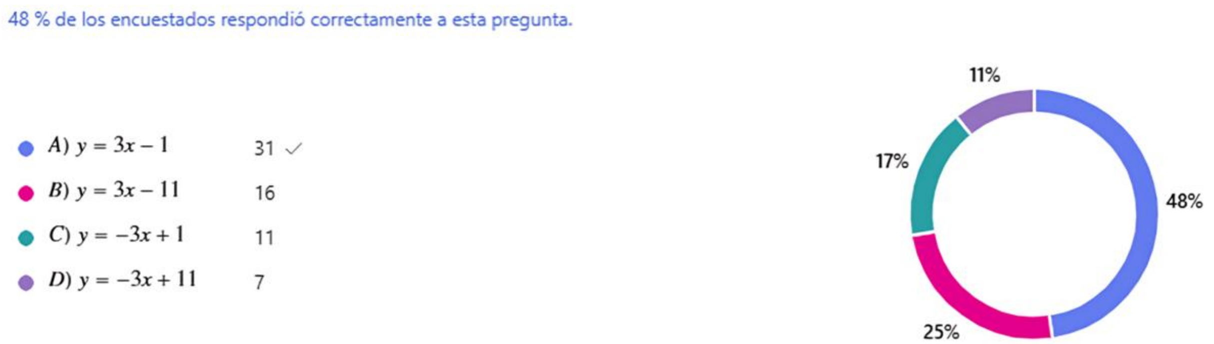


Figure 9. Participant responses to problem 5 (Source: Authors' own elaboration)

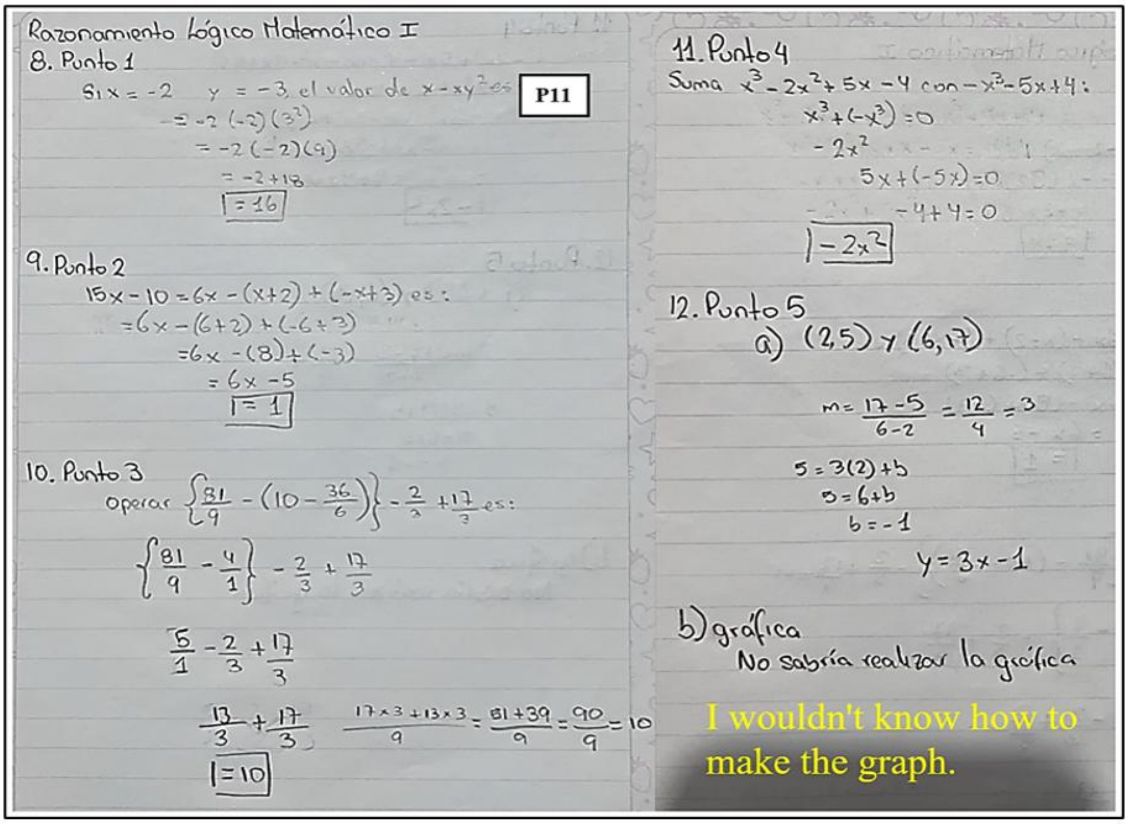


Figure 10. P11 completed the questionnaire, but without a graphical representation (Source: Authors' own elaboration)

analysis of functions based on linear functions (see **Figure 10**).

The analysis of the problems solved by the students allowed us to extract the following interpretations that should be considered in mathematical activity.

Mathematical Activity Is Conceived as a Practice, Not as a Result

From the perspective you have used (Mathematical Practices + Cognitive Configuration), the student's mathematical activity is not reduced to the correct answer but is understood as a system of intentional actions through which the student: interprets a situation; mobilizes mathematical objects; applies rules and procedures; and constructs arguments that justify their decisions. In this sense, each of the five problems activates specific MPs (Pm) that allow us to observe how the student thinks, even when the final result is incorrect.

The Analysis Is Conceived as a Diagnosis of the School-University Transition

The five problems are implicitly organized in an onto-semiotic progression: problem 1-problem 2: algorithmic practices inherited from the school level predominate. Problem 3-problem 4: the need to coordinate structures (fractions, like terms) arises. Problem 5: articulation between registers, concepts, and validation is required. The analysis shows that students do master routine school procedures (88%, 79%, 75%) but begin to have difficulties when mathematical activity requires: mathematical structure of the problem, semiotic control of signs, or articulation between representations (67%, 48%). Therefore, mathematical activity is conceived as a transitional process, where

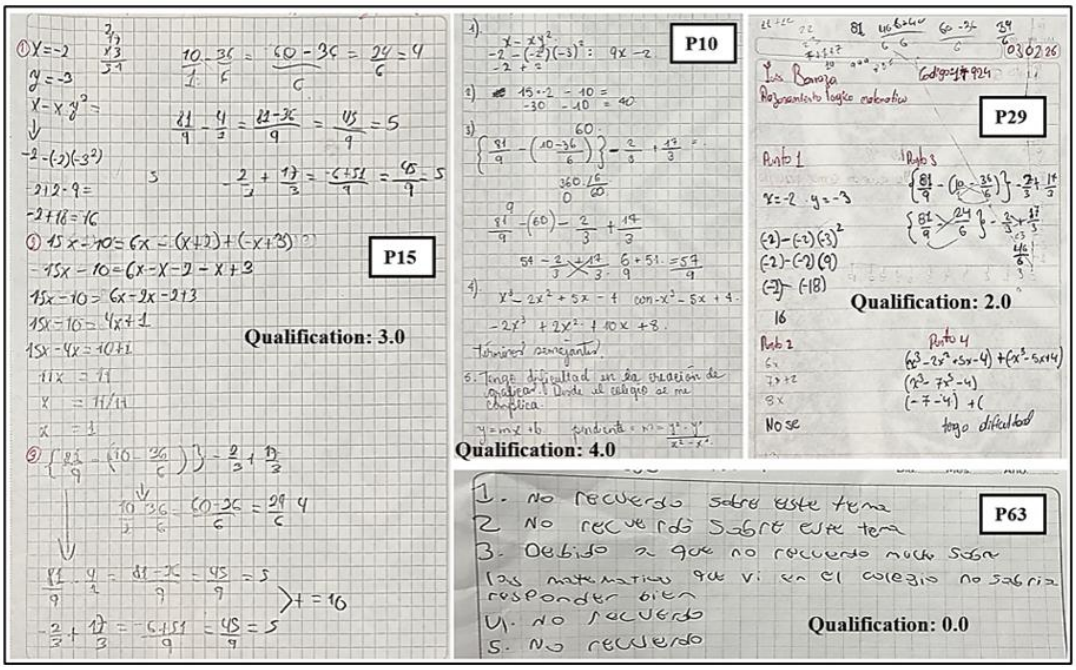


Figure 11. Examples of the different student grades (Source: Authors' own elaboration)

students still operate with school-based frameworks in the face of university-level demands.

The Focus Is on the Primary Objects That Students Manage (Or Fail) to Mobilize

Using the cognitive configuration of primary objects allows us to conceive of mathematical activity as a network of objects, not as a list of steps. In these five problems, it is observed that when students appropriately mobilize concepts (power, slope), propositions (Pr1 and Pr2), and procedures (Pc1-Pc3), the mathematical activity is consolidated and the success rate is high. However, when breakdowns occur, these are not random but localized, revealing conflicts in propositions (negative sign and slope), unjustified procedures, and a lack of argumentation (they do not validate results).

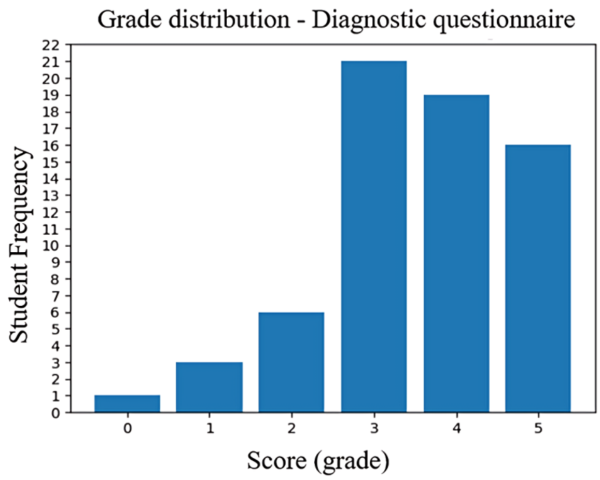


Figure 12. Student scores on the diagnostic (Source: Authors' own elaboration)

Error Is Conceived as a Semiotic and Epistemic Conflict, Not as Ignorance

In students beginning university mathematics studies, the observed errors are interpreted as:

(a) Semiotic conflicts: interpreting y^2 as negative or not inverting signs in parentheses

(b) Epistemic conflicts: using the formula for a straight line without understanding the meaning of slope.

This implies that the student: knows certain mathematical objects but has not yet integrated them into a stable MP.

The analysis reveals that the different levels of mathematical activity within the same group are inferred from the percentages of correct answers, showing that the group is not homogeneous, but rather that several levels coexist (**Figure 11**): students with stabilized MPs (e.g., P1, P4, P16, P27, P41, P58, and P66), students with partially activated practices (e.g., P2, P8, P15, P18, and P53), and students with fragmented practices (e.g., P3, P25, P26, P29, P38, P39, and P63). Furthermore, this analysis allows us to identify which practices are consolidated and consistent, which are emerging, and which are not yet activated and are characterized by errors.

Regarding the five problems, the mathematical activity of the beginning student is conceived as a transitional practice, characterized by advances, tensions, and productive conflicts that can be improved through the establishment of mathematical connections. OSA allows us to understand what students are actually doing when they "do mathematics," offering a solid foundation for designing relevant and progressive

Table 4. Student performance in the diagnosis

Score	Students	Percentage (%)	Performance level
0	1	1.5	Low (10 students)
1	3	4.5	
2	6	9.1	
3	21	31.8	Medium (21 students)
4	19	28.8	High (35 students)
5	16	24.2	
Total	66	100	

teaching interventions. **Figure 12** shows the group of students and their scores.

The bar chart with precise scales shows a discrete distribution of performance, with a greater concentration in the intermediate scores (3 and 4), indicating that most students are in a transitional phase toward consolidating their university-level MPs. The percentage analysis of the grades shows that 53.0% of the students show high performance (4-5), while 31.8% are at an intermediate level (3), showing MPs in transition. Only 15.1% of the group exhibit low performance (0-2), suggesting the need for targeted support strategies at the beginning of the course (**Table 4**).

However, the questionnaire's content is related to the students' field of study. For example, although the questionnaire covers seemingly "basic" content (elementary algebra, equations, operations, and lines), it explores not only techniques but also fundamental MPs: symbolic interpretation, control of signs and structures, algebraic reasoning, initial modeling, articulation between algebraic and graphical language, and basic argumentation. These practices constitute the core of university mathematics, regardless of degree program.

For students in business administration and international business programs, the questionnaire explores mathematical concepts useful for economic and administrative decision-making. Specifically, algebraic expressions (problem 1 and problem 3) help interpret costs, revenues, profits, margins, rates, and financial indicators. Linear equations (problem 2) aid in resolving equilibrium situations: break-even point, budget adjustments, and simple projections. The addition of polynomials (problem 4) influences the understanding of aggregate models: total costs and simplified supply and demand functions. The line and slope (problem 5) allow for the interpretation of trends, growth, price variations, and cost-benefit analysis.

For the students of software development technician, the mathematics of the questionnaire are important for structural elements linked to computational thinking and serve specifically as shown in **Figure 13**.

Finally, **Table 5** shows the connections activated in the students' mathematical activity to solve the diagnostic problems.

The analysis of the connections reveals that the first problems mainly activate procedural and instructional connections, while the final problem requires the simultaneous coordination of connections of different representations, modeling, meaning and idealization, which characterizes the beginning of university mathematical thinking.

DISCUSSION

The results obtained in this study confirm that the applied diagnostic tool serves an explanatory, rather than merely classificatory, function. Indeed, the high

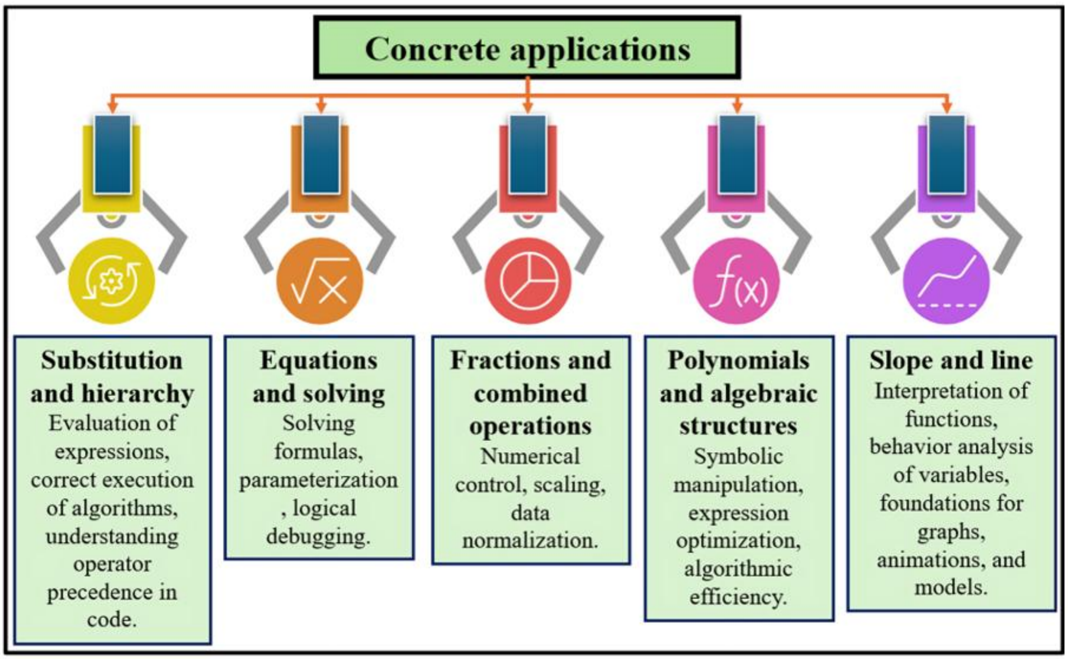


Figure 13. Applications for mathematics in the career of software development technician (Source: Authors' own elaboration)

Table 5. Connections between the five cognitive configurations are triggered by the problems

CP (Pb)	Type of connection	POI	Description of the connection
Pb1-Pb2	Meaning connection	Symbolic language, procedures (Pc), propositions (Pr)	The evaluation of algebraic expressions through substitution and sign control (Pb1) functions as necessary prior knowledge to solve linear equations with parentheses (Pb2). The student understands the equation by relying on previously activated practices.
Pb1-Pb2	Procedural connection	Pc1, Pc2, Pc3	Procedures of substitution, order of operations, and sign management are directly reused when eliminating parentheses and reducing terms in the equation.
Pb2-Pb3	Different representations	Symbolic language, propositions (Pr)	Control of the order of operations and algebraic equivalence in Pb2 is a prerequisite for operating numerical expressions with fractions in Pb3.
Pb2-Pb3	Procedural connection	Pc2, Pc3	Simplification and reduction rules and algorithms are applied, now in a context involving fractions and multiple levels of grouping.
Pb3-Pb4	Part-whole connection (generalization)	Concepts, propositions (Pr)	Numerical simplification (Pb3) is generalized to the algebraic simplification of polynomials (Pb4). Operating with numbers appears as a particular case of operating with algebraic expressions.
Pb3-Pb4	Procedural connection	Pc1, Pc2	Reduction and cancellation procedures are maintained but applied to structurally more complex objects (like terms).
Pb4-Pb5	Part-whole connection	Concepts (variable, expression), propositions (Pr)	The concept of algebraic expression and polynomial (Pb4) serves as a conceptual basis for understanding the equation of a linear function (Pb5).
Pb4-Pb5	Part-whole connection (inclusion)	Concepts	The algebraic expression $y = mx + b$ (Pb5) contains, as parts, the polynomials studied in Pb4.
Pb5	Connections of different representations	Symbolic and graphical language, arguments	The student moves from the algebraic register (equation) to the graphical one (line) and verifies given points, establishing equivalence between alternative representations.
Pb3-Pb5	Feature-based connection	Concepts, propositions (Pr)	The fraction as a ratio (Pb3) is recognized in Pb5 as slope, emphasizing the property of rate of change between quantities.
Pb2-Pb5	Implication connection	Propositions (Pr), arguments	From algebraic equality (Pb2), it is deduced that if a point satisfies the equation, then it belongs to the line (Pb5).
Pb1-Pb5	Reversibility connection	Procedures (Pc), arguments	The substitution used in P1 to compute a value is reversed in P5 to verify whether a point satisfies the equation of the line.
Pb5	Modeling connection	Concepts, languages, arguments	The units–cost relationship is translated into a linear mathematical model, connecting the real world with algebraic and graphical expressions.
Pb5	Meaning connection	Concepts, arguments	Students attribute meaning to the slope as "cost per unit" and to the intercept as "initial cost," constructing personal and contextual sense.
Pb5	Idealizing connection	Graphical language, concepts	Discrete data from the real context are idealized as a continuous line, transforming the concrete situation into an abstract mathematical object.

Note. CP: Connected problems & POI: Primary objects involved

success rates observed in the first problems of the questionnaire (problem 1-problem 3) demonstrate that students entering university for the first time possess an operational mastery of basic algebraic procedures, inherited from their schooling.

However, the OSA shows that this initial success is fundamentally based on cognitive configurations centered on procedures (Pc) and symbolic languages, with weak propositional explicitness and almost no argumentation. In this sense, the diagnostic tool does not reveal an absence of mathematical knowledge, but rather a specific form of predominantly algorithmic mathematical activity, functional for routine tasks, but fragile when faced with more complex structural demands.

This fragility becomes evident as the onto-semiotic complexity of the tasks increases. In problems 4 and, especially, problem 5, a progressive decrease in performance is observed (67% and 48% correct, respectively), associated with breakdowns in the coordination between primary objects and the incomplete activation of mathematical connections. The linear function modeling problem acts as a critical diagnostic point, since it requires the simultaneous articulation of procedures, concepts, propositions, representations, and arguments, in addition to establishing connections of modeling, meaning, and idealization. The identified errors are not random or isolated, but systematic, and reflect incomplete cognitive configurations characteristic of an unresolved transition between school mathematics and university mathematics.

Furthermore, these results are consistent with previous research documenting persistent difficulties in the transition from arithmetic to algebraic reasoning (Acosta et al., 2024; Sánchez García, 2019; Santana et al., 2023). Similar to the findings reported by Chávez Roblero (2018), Márquez et al. (2025), and Parra Rodríguez et al. (2026), the students in this study demonstrate difficulties interpreting statements, coordinating records, and attributing meaning to algebraic expressions, even when they have mastered calculation rules. Likewise, these findings align with international studies indicating that errors in algebra do not stem solely from procedural weaknesses, but also from semiotic and epistemic conflicts related to the understanding of mathematical objects (Kenney & Ntow, 2024; Nti & Jupri, 2024).

From the adopted theoretical perspective, the combined use of OSA and ETC allows for a deeper understanding of these results. While ETC makes visible the types of connections that students do or do not establish, OSA allows us to explain these connections in terms of practices, cognitive configurations, and SFs. Consistent with Rodríguez-Nieto et al. (2022b), the results confirm that many connections activated by students are personal rather than institutional, which explains why certain procedures are applied incorrectly without a stable conceptual understanding.

In response to the study's objective, it is conclusively concluded that the difficulties observed in first-year university students are not explained by a lack of basic mathematical knowledge, but rather by breakdowns in the articulation of mathematical connections necessary to reinterpret school content in university contexts. The diagnostic assessment, therefore, allows us to identify not only what students know, but also how they do mathematics, providing a solid foundation for designing teaching strategies aimed at strengthening integrated MPs, fostering the construction of meaning, and facilitating the transition to university-level mathematical thinking.

In this context, the present study provides a deeper interpretation of algebraic errors by analyzing them through the articulation of MPs, cognitive configurations of primary objects, and SFs. Unlike approaches that focus solely on identifying errors or proposing instructional strategies to correct them, this study shows that many of these errors emerge as breakdowns in the coordination of meanings, procedures, and representations within specific onto-semiotic configurations. From this perspective, the analysis of errors not only makes it possible to identify conceptual difficulties but also to understand how students mobilize or fail to mobilize the mathematical objects required to solve a task. Consequently, this approach contributes to explaining the structural nature of difficulties in algebra and highlights the importance of analyzing mathematical activity as a basis for designing more appropriate instructional interventions, particularly in the context of the transition toward university-level mathematical thinking.

The results of this study suggest *several implications for teaching practices* aimed at supporting students during the transition from secondary to higher education. First, it is important to incorporate structural visual representations that help students coordinate symbolic expressions with contextual meanings. Representations such as tables, diagrams, or graphical models can facilitate the interpretation of algebraic relationships and support the construction of meaning for concepts such as variables, slope, and functional relationships.

Second, instructional tasks should be designed to promote the simultaneous articulation of multiple representations, particularly verbal, algebraic, and graphical registers. The results of this study showed that many students were able to obtain correct algebraic expressions but experienced difficulties when interpreting or graphing them. Therefore, tasks that require students to move flexibly between different representations can strengthen the connections necessary for developing functional mathematical thinking.

Teaching strategies should emphasize the coordination of MPs, primary objects, and meanings, rather than focusing exclusively on procedural execution. Strengthening these aspects before students enter university can help consolidate arithmetic and algebraic knowledge and prevent these topics from becoming obstacles in subsequent mathematics courses.

CONCLUSION

The diagnostic assessment concludes that students entering a university mathematics course for the first time do not begin with a lack of knowledge, but rather with a predominantly procedural approach to mathematical activity, strongly rooted in prior school learning. The high success rates on the initial problems demonstrate that students have mastered basic algorithmic rules, such as algebraic substitution, simplification of expressions, and solving linear equations, relying primarily on procedural connections. However, as tasks demand greater onto-semiotic coordination, significant breakdowns emerge in mathematical activity. The diagnostic assessment shows that many students struggle to simultaneously articulate procedures, propositions, and concepts, resulting in incomplete cognitive configurations, particularly in operations with fractions, polynomials, and, most significantly, in the problem of modeling with linear functions. These difficulties are not explained by isolated calculation errors, but rather by failures in constructing mathematical connections of representation, implication, meaning, and idealization.

An important aspect of this diagnosis is that it allowed us to affirm that the main challenge in the transition to university mathematics lies not in teaching new procedures, but in reorienting mathematical activity toward the construction of meaning, the articulation between representations, and the modeling of situations. This result justifies the need for didactic proposals that promote the development of integrated MPs and the consolidation of fundamental mathematical connections from the beginning of the course. For future research, it is recommended to emphasize the need to apply mathematical concepts to real-life situations (Amo-Asante et al., 2025) and in the context of business administration, accounting, international business, and other engineering sciences to increase student performance and their problem-solving and quantitative reasoning skills.

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AI statement: The authors stated that no generative AI or AI-based tools were used in any part of the study, including data analysis, writing, or editing.

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