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Transformative teaching strategies for algebraic thinking: A systematic review of cognitive, pedagogical, and curricular advances

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Abstract

This systematic literature review (SLR) investigates transformative teaching strategies for enhancing students' algebraic thinking, a foundational competency in mathematics education. Recognizing the long-term significance of algebraic reasoning, this review synthesizes findings from 25 peer-reviewed articles published between 2022 and 2024, identified through comprehensive searches in Scopus and Web of Science, and selected using the preferred reporting items for systematic reviews and meta-analyses framework. The sample size aligns with accepted standards for SLRs in emerging educational domains. Drawing from recent scholarship, this study constructs a structured framework across four interrelated themes: (1) cognitive transitions in algebra learning, illuminating how students shift from arithmetic to abstract reasoning; (2) innovative pedagogical strategies that promote active, student-centered learning; (3) representational fluency, highlighting the role of visual, symbolic, and contextual tools in bridging conceptual gaps; and (4) developmental alignment in curriculum and assessment design, advocating for instructional sequencing tailored to learners' cognitive growth. The synthesis reveals that integrating cognitive, pedagogical, and curricular dimensions significantly strengthens algebraic reasoning. Despite its methodological rigor, the study is limited to English-language journal articles and excludes grey literature, which may constrain the comprehensiveness of findings. Moreover, the literature reflects developments only up to 2024, and more recent innovations may not be captured. Nonetheless, this review contributes a timely, evidence-based model for guiding instructional reform in algebra education and underscores the need for targeted, flexible strategies that support students' conceptual progression toward higher-level mathematics.

Keywords: algebraic thinking, teaching algebra, learning algebra, algebra education, teaching and learning strategies

INTRODUCTION

Algebraic thinking is widely regarded as a cornerstone of mathematical literacy and a prerequisite for advanced STEM learning (NCTM, 2023). However, it is essential to distinguish between two fundamental conceptions of algebra: algebra as the generalization of arithmetic, which typically characterizes early school algebra, and algebra as the formal study of abstract structures, which is more common in advanced mathematics. Conflating these two interpretations risks oversimplifying the pedagogical challenges and the developmental appropriateness of instructional strategies. International assessments such as TIMSS and PISA continue to report stagnant student performance in algebraic domains (Mullis et al., 2023; OECD, 2022), signaling persistent shortcomings in how early algebra is taught and prompting a critical re-evaluation of current instructional models.

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Contribution to the literature

- This study strengthens algebraic thinking, which is critical for preparing students for advanced mathematical learning, as current teaching practices often remain fragmented across cognitive, pedagogical, and curricular domains.
- This systematic review integrates representation theory and developmental mechanisms such as Vygotsky's ZPD to explain how scaffolding, differentiated instruction, and representational fluency collectively support the transition from arithmetic to algebra.
- The findings of this review show that coordinated, developmentally aligned and representation-rich strategies significantly enhance students' algebraic reasoning, offering both theoretical clarity for future research and actionable guidance for curriculum designers, educators, and policymakers.

A growing body of research supports the early introduction of algebraic thinking; however, this position is not without contention. While early exposure may enhance students' capacity for generalization and structural reasoning (Moretti et al., 2021; Pincheira & Alsina, 2021), others caution against prematurely shifting cognitive demands onto learners unprepared for abstraction (Wettergren, 2022). This debate raises essential questions about the timing, developmental readiness, and pedagogical framing of algebra in primary education–questions this review aims to address.

In addition to cognitive concerns, sociocultural, curricular, and systemic factors significantly influence the success of algebraic instruction. What works in one educational context may falter in another due to variations in curriculum expectations, classroom norms, or resource availability. For instance, Kwaffo's (2023) study in Ghana highlighted gains through differentiated instruction (DI), yet such findings cannot be universally applied without examining the cultural and curricular constraints that shape teaching practice.

This review centers on three recurring strategies in the literature: DI, early algebraic activities, and teacher noticing. While often studied independently, these strategies share a theoretical interdependence rooted in cognitive development and responsive teaching. DI enables tailored support, early algebraic tasks cultivate generalization from a young age, and teacher noticing enhances educators' ability to respond to students' thinking. A synthesized understanding of these elements as components of a cohesive pedagogical framework is largely absent from current literature, resulting in fragmented approaches to fostering algebraic thinking.

Critically, the existing literature lacks a comprehensive synthesis that examines how these three strategies intersect across cognitive, pedagogical, and curricular dimensions. Most prior studies focus on isolated interventions or short-term gains, offering limited insight into long-term learning progression. There is also insufficient attention to how these strategies can be developmentally aligned and contextually adapted. This systematic review addresses this gap by integrating diverse research strands into a unified model, justified through the urgent need for scalable, evidence-based practices in algebra education.

METHODOLOGY

The Preferred Reporting Items for Systematic Reviews and Meta-Analyses

The preferred reporting items for systematic reviews and meta-analyses (PRISMA) framework, established by (Page et al., 2021), is a widely endorsed standard for conducting systematic literature reviews (SLRs), promoting transparency, thoroughness, and consistency throughout the process. Following PRISMA guidelines enables researchers to increase the accuracy and rigor of their analyses by systematically identifying, screening, and including studies in their review. PRISMA also emphasizes the value of randomized studies, recognizing their role in reducing bias and contributing strong evidence to the review. For this analysis, Web of Science (WoS) and Scopus were selected as primary databases due to their extensive coverage and reliability.

The PRISMA approach encompasses four essential phases: identification, screening, eligibility, and data abstraction. During identification, relevant studies are located through a comprehensive search of databases. Meanwhile, screening involves applying predefined criteria to exclude irrelevant or low-quality studies. In the eligibility phase, the remaining studies undergo further evaluation to meet inclusion requirements. Finally, data abstraction focuses on extracting and synthesizing information from the included studies, which is essential for drawing meaningful, reliable conclusions. Other than that, this systematic approach supports a rigorous review process, yielding dependable results that can inform future research and practice.

Research Questions

Research questions (RQs) are fundamental to an SLR as they establish the review's foundation and guide its entire process. By setting the scope and focus of the SLR, they assist in defining inclusion and exclusion criteria, ensuring the review remains pertinent and aligned with the research topic. Notably, well-defined questions lead

Table 1. The search string

Database	Search string
Scopus	TITLE-ABS-KEY (algebra* AND ("algebra* think*" OR "algebra* reason*" OR "math* think*") AND (student* OR learn*)) AND (LIMIT-TO (DOCTYPE, "ar")) AND (LIMIT-TO (PUBSTAGE, "final")) AND (LIMIT-TO (SRCTYPE, "j")) AND (LIMIT-TO (LANGUAGE, "English")) AND (LIMIT-TO (PUBYEAR, 2022) OR LIMIT-TO (PUBYEAR, 2023) OR LIMIT-TO (PUBYEAR, 2024)) (Date of access: November 2024)
WoS	algebra* AND ("algebra* think*" OR "algebra* reason*" OR "math* think*") AND (student* OR learn*) (Topic) and 2024 or 2023 or 2022 (Publication Years) and Article (Document Types) and English (Languages) (Date of access: November 2024)

to a thorough and systematic literature search, covering relevant studies comprehensively, which reduces bias and ensures a complete examination of the existing evidence. Additionally, RQs aid in organizing and categorizing data from included studies, creating a framework for analyzing results and synthesizing insights into meaningful conclusions. They bring clarity and precision to the review, concentrating on specific issues and producing more relevant and actionable findings. Furthermore, well-formulated questions enhance the transparency and reproducibility of the review, allowing others to replicate the process, verify findings, or extend the review's scope. Ultimately, RQs ensure that the review aligns with its intended objectives, whether identifying knowledge gaps, assessing intervention effectiveness, or analyzing trends, making them essential to the rigor and focus of a high-quality SLR.

Formulating the RQs is the most critical step in the planning phase and serves as the foundation for any SLR, as it guides the overall review methodology (Kitchenham & Charters, 2007). Considering that this SLR aims to identify and analyze the state of the art in the field, the study applied the PICO framework–a mnemonic strategy particularly suited for developing RQs in qualitative studies, as proposed by Lockwood et al. (2015). The PICO framework assists in organizing RQs clearly and systematically by breaking down the study's key components, ensuring that the research remains focused, and the questions are precisely formulated. This structured approach facilitates a targeted literature search and efficient study design. Accordingly, this study arrived at four RQs as outlined below:

1. How do elementary students develop algebraic thinking skills through different stages of cognitive growth?
2. What teaching strategies are most effective for fostering algebraic thinking among middle school students in varied classroom settings?
3. How do high school students use visual and symbolic representations to enhance their understanding of algebraic concepts in problem-solving tasks?
4. How does the alignment of curriculum design and assessment practices impact the development of algebraic thinking among students in diverse educational systems?

Table 2. The selection criterion is searching

Criterion	Inclusion	Exclusion
Language	English	Non-English
Timeline	2022-2024	< 2022
Literature type	Journal (article)	Conference, book, review
Publication stage	Final	In press

Systematic Searching Strategies

Identification

This study implemented key stages of the systematic review process to compile a comprehensive collection of pertinent literature. The initial phase involved the careful selection of keywords, followed by the identification of related terms through the use of dictionaries, thesauri, encyclopedias, and existing research. Once all relevant terms were established, the researchers formulated search strings tailored for the Scopus and WoS databases, as detailed in **Table 1**. This foundational step yielded a total of 1,186 publications from both databases that were closely aligned with the research focus of the study. Therefore, including diverse literature ensured a thorough exploration of the topic, enhancing the robustness of the review. By utilizing systematic methods for literature collection, the authors aimed to minimize bias and maximize the relevance of the selected studies. This approach laid a solid groundwork for subsequent phases of the review process. Ultimately, the breadth of literature gathered will contribute significantly to the analysis and synthesis of findings related to the research objectives.

Screening

In the screening phase, each potentially relevant research item was carefully assessed to confirm its alignment with the study's RQs. During this step, duplicate documents were removed, resulting in the initial exclusion of 1,065 publications and leaving 121 for further analysis. Consequently, these remaining articles were evaluated using specific inclusion and exclusion criteria to retain only the most relevant sources (see **Table 2**). Publications included book series, reviews, meta-syntheses, meta-analyses, conference proceedings, and recent book chapters not previously analyzed,

ensuring a range of perspectives from authoritative sources. Note that only English-language studies from 2022 to 2024 were included to capture the latest research. Three additional publications were excluded due to duplication, yielding a refined and high-quality selection of literature for the study's in-depth analysis.

Eligibility

In the eligibility phase, the third step of the systematic review process, a total of 119 articles were identified and prepared for further examination. At this stage, each article's title, abstract, and main content were rigorously reviewed to ensure alignment with the study's predefined inclusion criteria and research objectives. This examination aimed to filter out studies that did not meet the necessary standards due to relevance, scope, or accessibility. Note that articles were excluded if they fell outside the field of interest, had titles deemed unrelated, contained abstracts misaligned with the study's aims or lacked access to full-text data. Through this careful vetting process, 94 articles were ultimately excluded as they did not provide sufficient empirical evidence or applicability to the research topic. These exclusions refined the pool to those studies most relevant to the study's objectives. Following this selection, 25 articles met the eligibility criteria and were retained for the final review. This curated set of articles will serve as the foundation for the subsequent analysis and synthesis phases.

Quality of appraisal

Following the systematic review protocol by Kitchenham and Charters (2007), once primary studies are selected, quality assessment (QA) becomes a critical step. Primary studies–original research articles, papers, or documents–serve as the foundational sources of evidence in systematic reviews and must be analyzed for validity and comparability. The QA process allows for thoroughly examining these studies, facilitating a quantitative or qualitative comparison directly addressing the review's RQs. This study applied QA criteria established by Abouzahra et al. (2020), consisting of six key QA measures tailored for SLRs. Correspondingly, each criterion is evaluated on a three-tier scoring system. A "yes" (Y) rating, worth 1 point, indicates the criterion is fully satisfied by the study; "partly" (P), worth 0.5 points, signifies partial fulfilment with some limitations; and "no" (N), worth 0 points, is assigned if the criterion is entirely unmet. This structured scoring approach standardizes QA across studies, supporting an objective and reliable comparative analysis.

QA1. Is the purpose of the study clearly stated?

QA2. Is the interest and the usefulness of the work clearly presented?

QA3. Is the study methodology clearly established?

QA4. Are the concepts of the approach clearly defined?

QA5. Is the work compared and measured with other similar work?

QA6. Are the limitations of the work clearly mentioned?

Table 2 outlines a QA process used to evaluate a study based on specific criteria. Three experts assess the study using the criteria listed, and each criterion is scored as "yes" (Y), "partly" (P), or "no" (N). Here is a detailed explanation:

1. Is the purpose of the study clearly stated?

   This criterion verifies whether the study's objectives are clearly defined and articulated. A clear purpose helps set the direction and scope of the research.

2. Is the interest and usefulness of the work clearly presented?

   This criterion evaluates whether the study's significance and potential contributions are well-explained. It measures the relevance and impact of the research.

3. Is the study methodology clearly established?

   This assesses whether the research methodology is well-defined and appropriate for achieving the study's objectives. Clarity in methodology is crucial for the study's validity and reproducibility.

4. Are the concepts of the approach clearly defined?

   This criterion examines whether the theoretical framework and key concepts are clearly articulated. Clear definitions are essential for understanding the study's approach.

5. Is the work compared and measured with other similar work?

   This evaluates whether the study has been benchmarked against existing research. Comparing with other studies helps position the work within the broader academic context and highlights its contributions.

6. Are the limitations of the work clearly mentioned?

   This criterion investigates whether the article identifies the limitations of the work clearly mentioned. Clear limitations are essential for understanding the study's approach.

Each expert independently assesses the study according to these criteria, and the scores are then totaled across all experts to determine the overall mark. For a study to be accepted for the following process, the total mark, derived from summing the scores from all three experts, must exceed 3.0. This threshold ensures that only studies meeting a certain quality standard proceed further.

Table 3. Number and details of primary studies database

No	Reference	Title	Journal	Scopus	Wos
1	Sun et al. (2023)	The developmental progression of early algebraic thinking of elementary school students	Journal of Intelligence	/	/
2	Ding et al. (2023)	Multiple pathways for developing functional thinking in elementary mathematics textbooks: A case study in China	Educational Studies in Mathematics	/	/
3	Erbilgin and Gningue (2023)	Using the onto-semiotic approach to analyze novice algebra learners' meaning-making processes with different representations	Educational Studies in Mathematics	/	/
4	Chimoni et al. (2023b)	Unfolding algebraic thinking from a cognitive perspective	Educational Studies in Mathematics	/	/
5	Ferreira et al. (2023)	Professional learning opportunities for teachers of the early years in algebra teaching: A study on the practice of a teacher educator	Acta Scientiae	/	/
6	Johansson and Kilhamn (2024)	From process to object in teachers' introductory algebra discourse	International Journal of Mathematical Education in Science and Technology	/	/
7	Acosta et al. (2024)	Computational thinking and repetition patterns in early childhood education: Longitudinal analysis of representation and justification	Education and Information Technologies	/	/
8	Ennassiri et al. (2023)	An activity based on figurative patterns: Moroccan students' reasoning	Academic Journal of Interdisciplinary Studies	/	/
9	Wilkie (2024)	Coordinating visual and algebraic reasoning with quadratic functions	Mathematics Education Research Journal	/	/
10	Ellis and Özgür (2024)	Trends, insights, and developments in research on the teaching and learning of algebra	ZDM Mathematics Education	/	/
11	da Silva Melo and Bisognin (2024)	Development of algebraic thinking in elementary school: An analysis from design-based research	Acta Scientiae	/	/
12	Hiltrimartin et al. (2024)	Analyzing students' thinking in mathematical problem solving using Vygotskian sociocultural theory	Revista de Gestao Social e Ambiental	/	/
13	Chimoni et al. (2023a)	Two different types of technologically enhanced intervention modules to support early algebraic thinking	Education and Information Technologies	/	/
14	Torres et al. (2024)	The evolution from "I think it plus three" towards "I think it is always plus three." Transition from arithmetic generalization to algebraic generalization	International Journal of Science and Mathematics Education	/	/
15	Anaya et al. (2024)	Curricular proposal to address diversity in mathematics class: A design on sequences and patterns	Eurasia Journal of Mathematics, Science and Technology Education	/	/
16	Ureña et al. (2022)	Generalization strategies and representations used by final-year elementary school students	International Journal of Mathematical Education in Science and Technology	/	/
17	da Silva et al. (2023)	Learning reported by three teachers and the teaching of algebra in the first grades	PNA	/	/
18	MacKay et al. (2024)	Mathematics content in early childhood classroom libraries: Alignment with common core mathematics standards	Early Childhood Education Journal	/	/
19	Gilmore (2023)	Understanding the complexities of mathematical cognition: A multi-level framework	Quarterly Journal of Experimental Psychology	/	/
20	Žakelj et al. (2024)	Demetriou's tests and levels of algebraic abilities and proportional reasoning in seventh, eighth, and ninth grades	European Journal of Science and Mathematics Education	/	/
21	Wilkie and Hopkins (2024)	Generalizing actions with the subtraction-compensation property: Primary students' algebraic thinking with tasks involving vertical towers of blocks	Educational Studies in Mathematics	/	/
22	Ferrara and Pozio (2023)	Entanglements of mathematics education research and large-scale assessment: Rethinking formulas as relational	Cognition and Instruction	/	/
23	Lopes (2023)	Advanced Algebraic Thinking Processes in Students' Modelling Activities	Teaching Mathematics and Its Applications	/	/
24	Dessbesel et al. (2023)	Mediation in mathematics teaching and learning in deaf education: Algebraic thinking	Acta Scientiae	/	/
25	Wilkie and Hopkins (2024)	Primary students' relational thinking and computation strategies with concrete-to-symbolic representations of subtraction as difference	Journal of Mathematical Behavior	/	/

Data Extraction and Analysis

An integrative analysis was implemented as a key evaluative strategy to systematically examine and synthesize the diverse research designs, primarily quantitative, included in this study. This analysis aimed to uncover and structure pertinent themes and subthemes associated with the research focus. The initial phase involved data collection, during which the authors gathered 25 publications for in-depth thematic analysis, as outlined in **Table 3**. Within this set of studies, the authors meticulously reviewed each publication to identify relevant assertions and content directly related

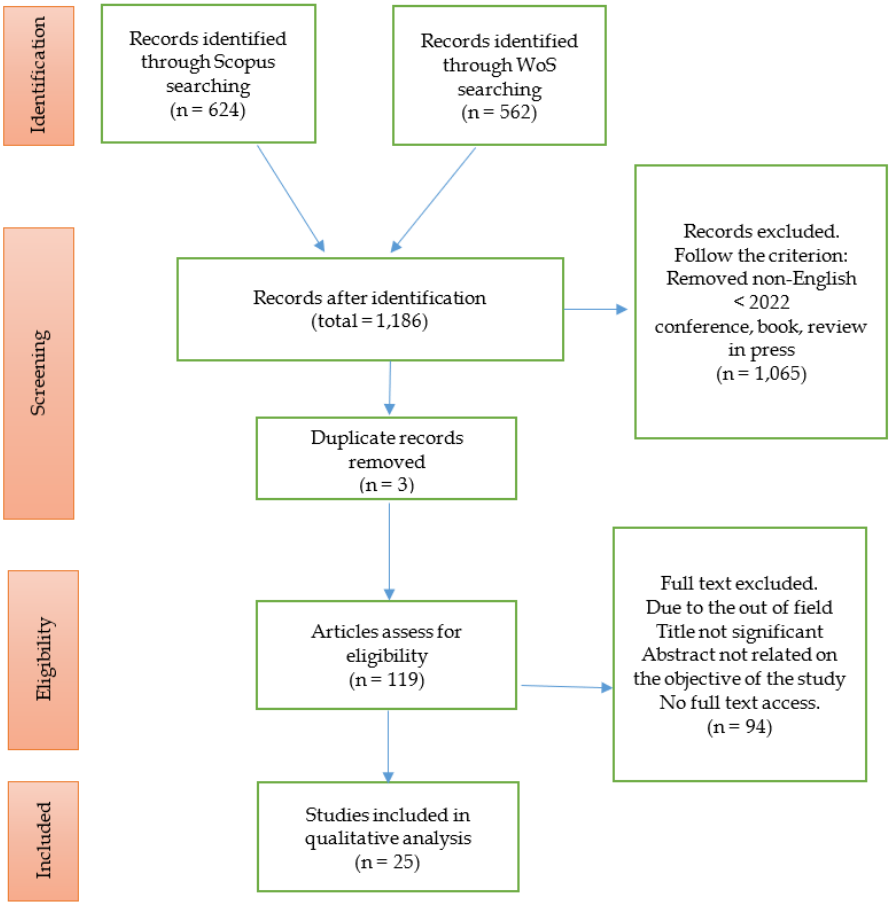

Figure 1. Flow diagram of the proposed search study (Adapted from Page et al., 2021)

to the themes under investigation. Subsequently, a focused examination of studies concerning algebraic thinking was conducted, assessing both the methodologies employed and the resulting findings. Following this, the authors collaborated to construct themes based on the evidence, working with co-authors to ensure accuracy and thematic coherence. Throughout the process, a log was maintained to document observations, analytical reflections, and emerging interpretations, supporting transparency in data analysis. Consequently, comparisons were made to detect any inconsistencies in the themes developed, allowing the authors to address and resolve discrepancies through discussion. This collaborative approach ensured that any differing perspectives on the concepts were fully examined and agreed upon. **Figure 1** shows the flow diagram of the proposed search study.

RESULTS AND FINDINGS

Quality Assessment

Table 4 summarizes the QA scores for the 25 included studies, showing notable variation in methodological robustness. While 48% (12 studies) achieved ≥ 90% scores, most excelled in theoretical framing (QA2-QA3: 92%) but only 68% explicitly addressed limitations (QA6), indicating insufficient reflexivity.

Summary

Highest score: The paper by Sun S., Sun D., Xu T., Sun S., Sun D., Xu T., da Silva Melo C. B., Bisognin E., Torres M.D., Moreno A., Vergel R., Cañadas M.C., Hiltrimartin C., Afifah A., Scristia, Pratiwi W. D., Handrianto C., Rahman M.A. and Chimoni M., Pitta-Pantazi D., and Christou C., achieved the highest score with 100% due to clear articulation of purpose, usefulness, methodology, defined concepts, comparison with other work, and mention of limitations.

Lowest score: The paper by Ferreira M.C.N., da Ponte J.P., Ribeiro A.J. and da Silva Dessbesel R., da Silva S.D.C.R., and Shimazaki E.M. scored the lowest (66.67%), as it partly met the criteria for the concepts of approach and comparison with other work, and did not mention the limitations.

Integrated Results and Findings

The synthesis of 25 selected studies, following QA, revealed that only 48% achieved excellence (≥ 90%), with notable variation in methodological robustness. Most demonstrated strong theoretical framing (QA2-QA3: 92%), but only 68% explicitly acknowledged limitations

Table 4. Assessment performance for selected primary studies

Data	QA1	QA2	QA3	QA4	QA5	QA6	TM	P (%)
PS 1	1	1	1	1	1	1	6	100
PS 2	1	1	1	1	0.5	0.5	5	83.30
PS 3	1	1	1	1	1	1	6	100
PS 4	1	1	1	1	1	1	6	100
PS 5	1	1	0.5	1	0.5	1	5	83.30
PS 6	1	1	1	1	1	0.5	5.5	91.70
PS 7	1	1	1	1	0.5	1	5.5	91.67
PS 8	1	1	0.5	1	0.5	0	4	66.67
PS 9	1	1	1	1	1	1	6	100
PS 10	1	1	1	1	1	1	6	100
PS 11	1	1	1	1	0.5	0.5	5	83.33
PS 12	1	1	1	1	0.5	1	5.5	91.67
PS 13	1	1	1	1	1	0.5	5.5	91.67
PS 14	1	1	1	1	0.5	0.5	5.5	91.67
PS 15	1	1	0.5	1	0.5	0	4.5	75
PS 16	1	1	1	0.5	1	0.5	5.5	91.67
PS 17	1	0.5	1	1	0.5	1	5	83.33
PS 18	1	0.5	1	1	0	0	4	66.67
PS 19	1	1	1	1	0.5	0	5	83.33
PS 20	1	1	1	1	0.5	0.5	5.5	91.67
PS 21	1	1	1	1	0.5	1	5.5	91.67
PS 22	1	1	1	1	1	1	6	100
PS 23	1	0.5	1	1	0.5	0.5	4.5	75
PS 24	1	1	1	1	0.5	1	5.5	91.67
PS 25	1	1	1	1	0.5	0.5	5.5	91.67

Note. TM: Total mark & P: Percentage

(QA6), signaling a lack of reflexivity. Integrating the four identified themes–developmental perspectives, teaching strategies, cognitive processes/representations, and curriculum/assessment design–showed these dimensions are interdependent. Students' cognitive development is shaped by the simultaneous implementation of aligned pedagogical strategies and curriculum structures, rather than by isolated interventions.

Developmental findings indicated a cognitive shift to abstract algebraic reasoning around age 14 (Žakelj et al., 2024), but mechanisms driving this change were often under-theorized. Few studies applied frameworks like Vygotsky's zone of proximal development (ZPD) to explain how scaffolding mediates transitions from arithmetic to algebraic abstraction. Those that did (e.g., Torres et al., 2024) found staged scaffolds, paired with representational bridging, accelerated progression. Individual differences–working memory, arithmetic fluency, and language proficiency–moderated development but were rarely addressed in planning.

Teaching strategies such as DI and early algebraic exposure improved relational thinking and representational fluency (Kwaffo, 2023; Wilkie & Hopkins, 2024). However, effectiveness depended on teacher expertise, class size, and resources. In large or resource-limited classrooms, DI's adaptability was constrained, requiring simplified tiering or peer-assisted scaffolding. Technology-enhanced approaches (Chimoni et al., 2023a) produced higher gains when contextualized in authentic problems (effect size ≈ 0.38) compared to abstract digital drills (≈ 0.21).

Cognitive processes and representation strategies revealed that visual, symbolic, and verbal forms work synergistically in algebra learning, consistent with Duval's (2006) representation theory. High-impact studies (e.g., Erbilgin & Gningue, 2023) showed that transitions between representations–especially from figural patterns to symbolic notation–require explicit mediation. Yet, most interventions lacked structured protocols, leading to uneven outcomes (**Table 5**).

Curriculum and assessment alignment was critical but under-addressed. Only 40% of high-scoring studies embedded formative assessments within developmental sequences (Anaya et al., 2024). Where formative and summative assessments were deliberately aligned, students displayed greater retention and transfer of skills. Much of the literature treated alignment as an ideal without analyzing its application. UDL-based designs enhanced engagement but their scalability in exam-driven systems remains under-examined.

This review moves beyond descriptive aggregation by critically analyzing how developmental, pedagogical, cognitive, and curricular dimensions converge to shape algebraic thinking. Key conclusions are:

(1) scaffolding and representation transitions are central yet under-utilized;

(2) contextual constraints must be considered before generalizing DI and technology strategies; and

(3) curricular and assessment designs should align with developmental stages and embed formative assessments (**Figure 2**).

Limitations include overrepresentation of quasi-experimental designs, scarce longitudinal data, and possible publication bias. Nonetheless, this synthesis

Table 5. Summary of high-impact strategies and reported effects

Strategy/focus area	Frequency (n = 25)	Average effect size/impact	Common limitations
DI	11 (44%)	Medium-high (0.30-0.45)	Resource constraints, training gaps
Early algebraic exposure	9 (36%)	Medium (0.25-0.38)	Limited follow-up
Functional thinking tasks	14 (56%)	High (> 0.40)	Inconsistent design
Representation bridging (Duval theory)	13 (52%)	High (> 0.40)	Lack of protocols
Technology-enhanced situated learning	10 (40%)	Medium-High (0.38 vs. 0.21)	Access inequality
Curriculum–assessment alignment	8 (32%)	High (0.40-0.50)	Rarely operationalized

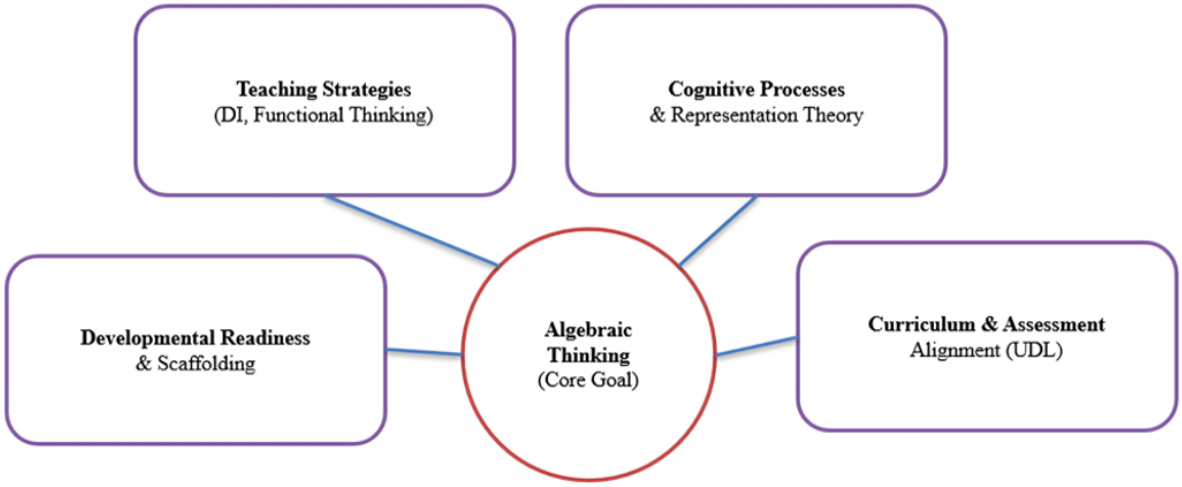

Figure 2. Integrated framework for fostering algebraic thinking (Source: Authors' own elaboration)

offers a theoretically grounded and empirically supported framework advocating a developmentally aligned, representation-rich, and context-sensitive approach to teaching algebra, providing conceptual clarity for theory-building and actionable guidance for educators and policymakers.

DISCUSSION

The findings of this review respond directly to the central RQ: how can algebra learning strategies be structured to strengthen students' algebraic thinking from the early to middle grades? The evidence suggests that meaningful progress occurs when pedagogical methods, curriculum design, and cognitive development considerations are implemented in a coordinated manner. For instance, the developmental patterns identified–such as the shift toward formal abstraction around age fourteen–cannot be fully understood without situating them in the broader context of scaffolding, Vygotsky's ZPD, and the deliberate sequencing of representational tasks. This integrated perspective demonstrates that isolated strategies, however effective in controlled studies, are unlikely to sustain deep learning unless supported by curriculum structures and formative assessment cycles.

Importantly, the synthesis underscores the interdependence of developmental readiness, DI, and representation-rich teaching. While early algebra exposure can accelerate conceptual generalization, its success depends on the teacher's ability to adapt strategies to varied working memory capacities, language backgrounds, and prior mathematical experiences. Cultural context also emerged as a moderating factor; for example, the collaborative problem-solving approaches documented in East Asian studies may not transfer seamlessly to classrooms with different sociocultural dynamics. Likewise, strategies such as DI face practical challenges in resource-limited settings, where class sizes, material constraints, and teacher training gaps can limit their reach. These observations suggest that policy and professional development programs must attend not only to *what* strategies are promoted, but also to *how* and *where* they are implemented.

The methodological scope of this review–restricted to studies published in English between 2022 and 2024–ensures an up-to-date synthesis but also narrows historical and linguistic diversity. Consequently, the representation of long-term or non-English studies is limited. The reliance on a high proportion of quasi-experimental designs also has implications for internal validity, as causal claims should be interpreted with caution. Furthermore, the concentration of studies in certain regions, particularly high-income countries, raises questions about external validity and the generalizability of findings. Publication bias remains a possibility given the prevalence of positive results in the reviewed literature, and selection bias may have occurred despite multi-phase screening. These limitations invite careful interpretation of the findings and emphasize the need for replication in varied contexts.

From a theoretical standpoint, this review contributes an integrated conceptual model that positions developmental, pedagogical, cognitive, and curricular dimensions as mutually reinforcing pillars for algebra instruction. This model advances the literature by combining representation theory, developmental psychology, and curriculum alignment into a single, practical framework for classroom application. It also fills a conceptual gap in earlier reviews by explicitly mapping how transitions between arithmetic and algebra are mediated through scaffolding and formative assessment. This process aligns with Vygotsky's ZPD, where scaffolding enables students to progress from their current arithmetic competence toward more advanced algebraic reasoning.

CONCLUSION

This systematic review concludes that strengthening algebraic thinking requires deliberate alignment between cognitive development stages, instructional design, and curriculum–assessment structures. Introducing algebraic concepts early, when accompanied by rich representational tasks and developmental scaffolds, enables students to transition more smoothly from concrete reasoning to abstract generalization. The evidence supports a shift away from viewing algebra as a discrete, secondary-level topic, towards embedding it as a core strand in mathematics education from the earliest years.

Practically, the findings call for teacher training that equips educators to diagnose and respond to students' representational and cognitive needs, especially in culturally and resource-diverse contexts. Policymakers and curriculum designers should integrate flexible structures that allow for DI while ensuring assessment practices promote–not inhibit–conceptual growth. At the theoretical level, the integrated framework proposed here enriches existing models by linking cognitive mechanisms with concrete pedagogical actions, offering a roadmap for both future research and classroom practice.

Future studies should empirically test this framework in longitudinal, cross-cultural settings, explore its adaptability to resource-limited schools, and further investigate how individual learner differences shape algebraic thinking development. By grounding future work in both theory and practical constraints, the field can move closer to universally effective, equitable, and enduring approaches to algebra education.

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