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University students' mathematical connections in problem-solving on frequency distribution tables: A new view of connections in statistics education

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Abstract

The article explores the mathematical connections activated by Colombian university students when solving a problem involving the construction of frequency distribution tables. This study arose because the literature and teaching experience reveal difficulties in statistics in finding the range and determining the number and size of classes, which are essential aspects for creating frequency tables with grouped data. Grounded in the networking between the extended theory of connections, the onto-semiotic approach, and levels of table reading, A qualitative methodology was implemented with 28 students who completed the task and actively participated in the class. The findings reveal that students activated mainly procedural, part-whole, implication, and different representation connections to construct grouped frequency tables, progressing from data organization to statistical interpretation. These connections supported the articulation of mathematical practices, primary objects, and semiotic functions, evidencing a structured but predominantly procedural understanding. However, students' comprehension was largely situated at level 1 and level 2 (reading the data and reading within the data), with limited evidence of higher-order reasoning such as inference or critical analysis. Only a small proportion of responses reached level 3, and no evidence of level 4 was identified. These results indicate that, although students successfully complete statistical procedures, their understanding remains incipient. This highlights the need for instructional approaches that foster deeper reasoning and critical statistical literacy through the activation of meaningful mathematical connections.

Keywords: mathematical connections, onto-semiotic approach, frequency distribution tables, statistics, body mass, university students

INTRODUCTION

Numerous studies indicate that the capacity to establish mathematical connections is a key component of effective learning and meaningful teaching in mathematics (Rodríguez-Nieto et al., 2025b). Such connections support students in developing a coherent, integrated, and usable understanding of mathematical concepts, enabling them to apply their knowledge in practical and context-specific situations. Moreover, promoting relationships among mathematical ideas enhances the development of critical thinking, analytical reasoning, and problem-solving abilities, which are essential both for academic achievement and for addressing challenges encountered in everyday life (Ministry of National Education [MNE], 2006; National Council of Teachers of Mathematics [NCTM], 2000; Rodríguez-Nieto & Font, 2025).

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Contribution to the literature

- This article highlights the central role of mathematical connections in statistics, showing that constructing frequency distribution tables is not a mechanical process, but a complex activity that articulates procedures, concepts, and meanings.
- It proposes a solid theoretical articulation between the extended theory of connections (ETC) and the onto-semiotic approach (OSA) offering a powerful analytical framework for understanding how students activate practices, objects, and semiotic functions when solving statistical problems.
- It demonstrates that university students activate multiple types of connections (procedural, part-whole, implication, and different representations), which allows them to progress toward deeper levels of understanding statistical tables and strengthens their statistical literacy.

Research on mathematical connections has revealed not only their importance in problem-solving, but also their ability to enrich communication, strengthen argumentation, and diversify procedures through the use of multiple representations (Berry & Nyman, 2003; Pino-Fan et al., 2018; Rodríguez-Nieto et al., 2023, 2025a). A revealing example is provided by Businskas (2008), who observed how future teachers established connections when addressing quadratic equations. Similarly, other studies have investigated these connections in geometric contexts (Eli et al., 2013; Rodríguez-Nieto, 2021), highlighting their versatility. However, Caviedes et al. (2024) note a worrying trend: many future teachers cling to numerical procedures and formulas, neglecting geometric and intuitive approaches that could enhance understanding. This preference demonstrates a disconnect between different ways of representing and understanding the same concept, thus limiting the richness of mathematical thinking.

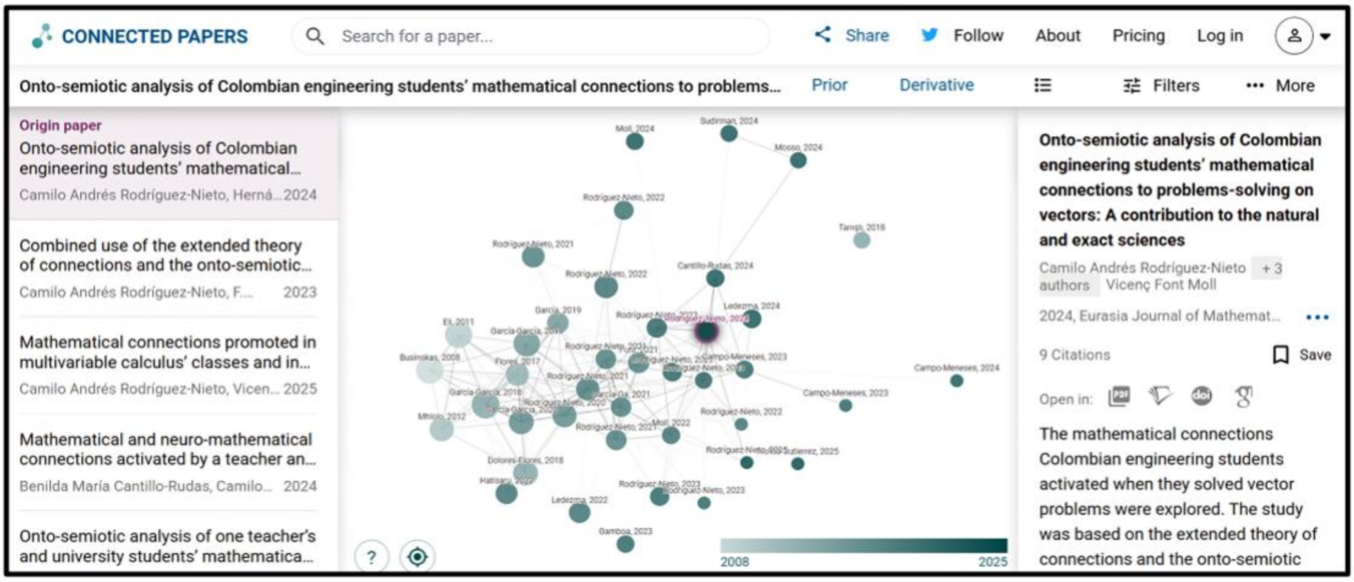
Just as mathematical connections are important in various studies of calculus, geometry, arithmetic, and others, they should also contribute to other contexts. For example, in statistics, studies on connections are under-recognized, even though they could be a fundamental resource for improving the understanding of students and teachers struggle with the problems and difficulties of interpreting graphs, constructing tables, performing operations with formulas, etc. (Pallauta & Batanero, 2024; Vásquez & Alsina, 2019; Verástegui-Gutiérrez et al., 2024). Recognizing this need, this study is motivated to explore connections in the construction of frequency distribution tables, evaluating the step-by-step procedures and their implications. Furthermore, the literature review presents the essential foundations of this research.

Establishing connections between mathematical concepts has been identified as a key aspect for meaningful learning, as evidenced by various studies developed from the ETC at different educational levels (Bahar et al., 2023; Businskas, 2008; Campo-Meneses & García-García, 2023; Rodríguez-Nieto, 2025). These investigations show that these connections not only allow a more functional understanding of the concepts, but also facilitate their application in everyday contexts, strengthening conceptual and procedural knowledge. Along these lines, Kenedi et al. (2019) highlight that the ability to establish mathematical relationships is an essential component of learning, since it contributes to a faster understanding and the development of solid mathematical skills.

Several studies have highlighted the importance of mathematical connections in teaching functions and calculus. Hatisaru (2023) addressed this topic by integrating teachers' specialized knowledge with different representations such as the Cartesian plane, diagrams, and ordered pairs. Campo-Meneses et al. (2021) analyzed the connections that students and teachers activate when working with exponential and logarithmic functions, highlighting processes such as reversibility and the use of multiple representations. In the field of calculus, research has shown that the lack of articulation between graphical, verbal, numerical, symbolic, and algebraic elements hinders a deep understanding of derivatives and integrals (García-García & Dolores-Flores, 2021; Rodríguez-Nieto et al., 2022a; Sahin et al., 2015).

The literature review reveals that establishing mathematical connections is a key aspect of effective learning and meaningful teaching of mathematics, enabling students to develop an integrated and functional understanding of mathematical concepts. For example, using AI connected papers, it was evident that no research focused on mathematical connections and statistics has been conducted on the teaching and learning that plagues this line of research. Instead, statistics is used to perform data analysis, which is more closely tied to methodological processes to address results quantitatively (**Figure 1**).

On the other hand, in an analysis carried out in Scopus, connections were identified as a highly researched topic, and authors such as García-García (2024) and Rodríguez-Nieto and Font (2025) are increasingly promoting analyzes to better understand mathematical concepts (see part a in **Figure 2**). Furthermore, these investigations on mathematical connections have been published in relevant journals in mathematics education and in other fields such as engineering, computing, physics, neuroscience, among



Origin paper
Onto-semiotic analysis of Colombian engineering students' mathematical connections to problems...
Onto-semiotic analysis of Colombian engineering students' mathematical...
Camilo Andrés Rodríguez-Nieto, Herná... 2024

Combined use of the extended theory of connections and the onto-semiotic...
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Mathematical connections promoted in multivariable calculus' classes and in...
Camilo Andrés Rodríguez-Nieto, Vicen... 2025

Mathematical and neuro-mathematical connections activated by a teacher an...
Benilda María Cantillo-Rudas, Camilo... 2024

Onto-semiotic analysis of one teacher's and university students' mathematica...

Onto-semiotic analysis of Colombian engineering students' mathematical connections to problems-solving on vectors: A contribution to the natural and exact sciences
Camilo Andrés Rodríguez-Nieto +3 authors Vicenç Font Moll
2024, Eurasia Journal of Mathemat...
9 Citations

The mathematical connections Colombian engineering students activated when they solved vector problems were explored. The study was based on the extended theory of connections and the onto-semiotic

Figure 1. Literature on connections found through connected papers (the authors' own elaboration)

others (see part b in **Figure 2**). Other searches found research on mathematical connections in Google Scholar in both Spanish and English (see part c in **Figure 2**).

The fact that research conducted in Connected Paper, Scopus, Google Scholar, and other databases show a broad literature on mathematical connections does not mean that there is research covering all mathematical, geometric, and statistical concepts, among others. Therefore, there is a special concern about statistics because students have difficulty solving problems involving variables, tables, graphs, functions, regressions, etc. (Su et al., 2024).

In this research, statistical tables occupy a central place, since in recent years statistics have gained great relevance in various areas of daily life such as science, society, and the humanities (Su et al., 2023). This importance is reflected in the constant presence of statistical data in media such as television, the internet, and the press, generally presented in tables, graphs, or summaries (Arteaga et al., 2011). Statistical tables are frequently used as a resource to structure and synthesize large amounts of data into compact formats, allowing for more accessible interpretation and analysis of the information (Beltrão, 2012; Eudave, 2009; Gea et al., 2022). Therefore, they are considered an essential component of statistical literacy, understood as the body of knowledge that every citizen must possess to navigate a data-saturated environment (Cazorla & Utsumi, 2010; Pallauta & Batanero, 2024). Furthermore, like graphs, tables represent a specific way of organizing information, with cognitive value for multiple purposes (Gabucio et al., 2010).

From a curricular perspective, different countries agree on the importance of introducing work with statistical tables and graphs, along with other statistics and probability content, from the first levels of primary education (e.g., Alsina & Vásquez, 2024; Bustamante-Valdés et al., 2024; de Alencar & Diaz-Levicoy, 2024; MNE, 2006; NCTM, 2000; Pallauta & Batanero, 2024; Vásquez & Alsina, 2019; Verástegui-Gutiérrez et al., 2024). In the case of Chile, the primary education curricular bases (Ministry of Education, 2012, 2013) detail the approach to statistical tables from the first grade. Furthermore, Pallauta and Batanero (2024) proposed a guide based on the theory of didactic suitability, which defines epistemic and cognitive indicators to evaluate the treatment of statistical tables in textbooks, thus contributing to teacher training.

In the first years of primary education in Chile, work with tally tables is linked to reading and data recording activities close to the student's context, while in second grade, these are related to recording the results of dice and coin rolls, highlighting the use of simple tables to organize information from these games. This type of representation continues to be used in later grades (Parra-Fica et al., 2024). However, research on the use of statistical tables at this educational level is still limited, possibly due to the non-mandatory nature of these contents in the early years of schooling. In this sense, studies such as that by Guimarães et al. (2008), which analyzed 2,080 activities in textbooks from 1st to 4th grade, show that, although tables have a significant presence, more than half are not directly linked to the organization of information, and that interpretation is prioritized over construction. Similarly, Amorim and Silva (2016) found that in fourth- and fifth-grade textbooks in Brazil, most activities related to tables refer to charts or databases, focusing primarily on interpretation. Furthermore, they highlight a higher frequency of situations based on real data, reflecting an intention to connect with students' everyday lives.

In a study conducted in Chile, Díaz-Levicoy et al. (2015) analyzed the use of statistical tables in first and second-year primary education textbooks. They

Mathematical connections in mathematics education

The potential of ethnomathematical and mathematical connections in the pre-service mathematics teachers' meaningful learning when problems-solving about brick ...
CA Rodríguez-Nieto, ML Pabón-Navarro... - Infinity ..., 2025 - e-journal.stkipsiliwangi.ac.id
... and mathematical connections in fostering meaningful learning through problem-solving in brick-making. Despite the importance of such connections in mathematics education, ...
Citado por 21 · Las 7 versiones

The impact of realistic problem-based learning on mathematical connection abilities: evidence from elementary schools in Indonesia
I Anugraheni, A Gufron, YW Purnomo - Cogent Education, 2025 - Taylor & Francis
... elementary mathematics education, aiming to develop students' mathematical connection skills in a ... The Synthesis Diagram of Mathematical Connections (MC) is illustrated in Figure 1 ...
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Mathematical connections promoted in multivariable calculus' classes and in problems-solving about vectors, partial and directional derivatives, and applications
CA Rodríguez-Nieto, VF Moll - Eurasia Journal of Mathematics ..., 2025 - ejmste.com
... • Also, the special role of mathematical connections used by the in-service ... of mathematical connections presented in this article is important because it shows details in mathematical ...

Mathematical connections in mathematics education

Aproximadamente 12.300 resultados (0,09 s)

Systematic Literature Review: Mathematical Connections in Solving Contextual Mathematics Problems
S Aulia, AF Jamil, AH Khuana - ... and Learning Mathematics, 2026 - ejournal.umm.ac.id
... connection skills are still an obstacle for junior high school students. This study aims to analyze junior high school students' mathematical connection ... ' mathematical connection skills ...

Mathematical Connection Research in Indonesia: A Bibliometric Study (2020–2025)
M Silvina, R Rohati, A Kurnalasari - Journal of General Education ..., 2026 - journal-gehu.com
... This study highlights mathematical connections as an important focus in mathematics education ... In general, the results of this study confirm that mathematical connections are one of the ...

... CIMATIK (Interactive Mathematics Canva) Media Based on Problem-Based Learning with a Deep Learning Approach to Enhance Students' Mathematical Connection ...
E Yunita, M Aminah, NT Rosita - Journal of Mathematics ..., 2026 - journal-gehu.com
... INTRODUCTION Mathematics education does not merely emphasize the mastery of ... ability to make mathematical connections. Mathematical connection ability enables students to link ...

Figure 2. Journals where works on connections have been published (the authors' own elaboration)

identified tally tables as the most common, although frequency tables, data tables, and double-entry tables also appear. When examining the reading levels proposed by Curcio (1989) and Friel et al. (2001), they found only the first two levels: literal reading of the data and reading within the data, the latter being the most frequent, especially in activities that involve comparing information or performing simple calculations. A variety of activities were also detected, including reading, completing, calculating, translating, asking questions, explaining, collecting data, and presenting examples. The most common tasks were calculation, completion, translation, exemplification, and reading. These findings served as the basis for designing and validating the questionnaire used in this research, the development of which is described in the methodology section.

Several studies in international contexts have highlighted the difficulties faced by primary and secondary school students in interpreting and working with statistical representations, both graphical and tabular (Rodríguez-Alveal et al., 2025). In Spain, Gabucio et al. (2010) administered a questionnaire to 205 primary and secondary school students, assessing their understanding of tabular structure, direct reading of data, and simple and global inferences. The results showed that literal reading tasks are the most accessible, while inferences, especially global ones, pose greater challenges regardless of the grade. In the Chilean context, Estrella and Olfos (2015) identified, through the analysis of the trans-numerative thinking of third-grade students, that only a few were able to construct basic frequency tables, despite demonstrating processes such as ordering, grouping, and calculating frequencies. Subsequently, Díaz-Levicoy et al. (2019) examined the reading levels of statistical graphs in Chilean students, detecting significant limitations at the interpretive and critical levels. These findings were reinforced by Arteaga et al. (2020), who also highlighted the difficulties in understanding statistical graphs among primary school students in Chile. More recently, Díaz-Levicoy et al. (2022) analyzed primary education textbooks in Peru, revealing a scarce presence of statistical tables and a predominantly literal approach. These studies coincide in pointing out the need to strengthen work with

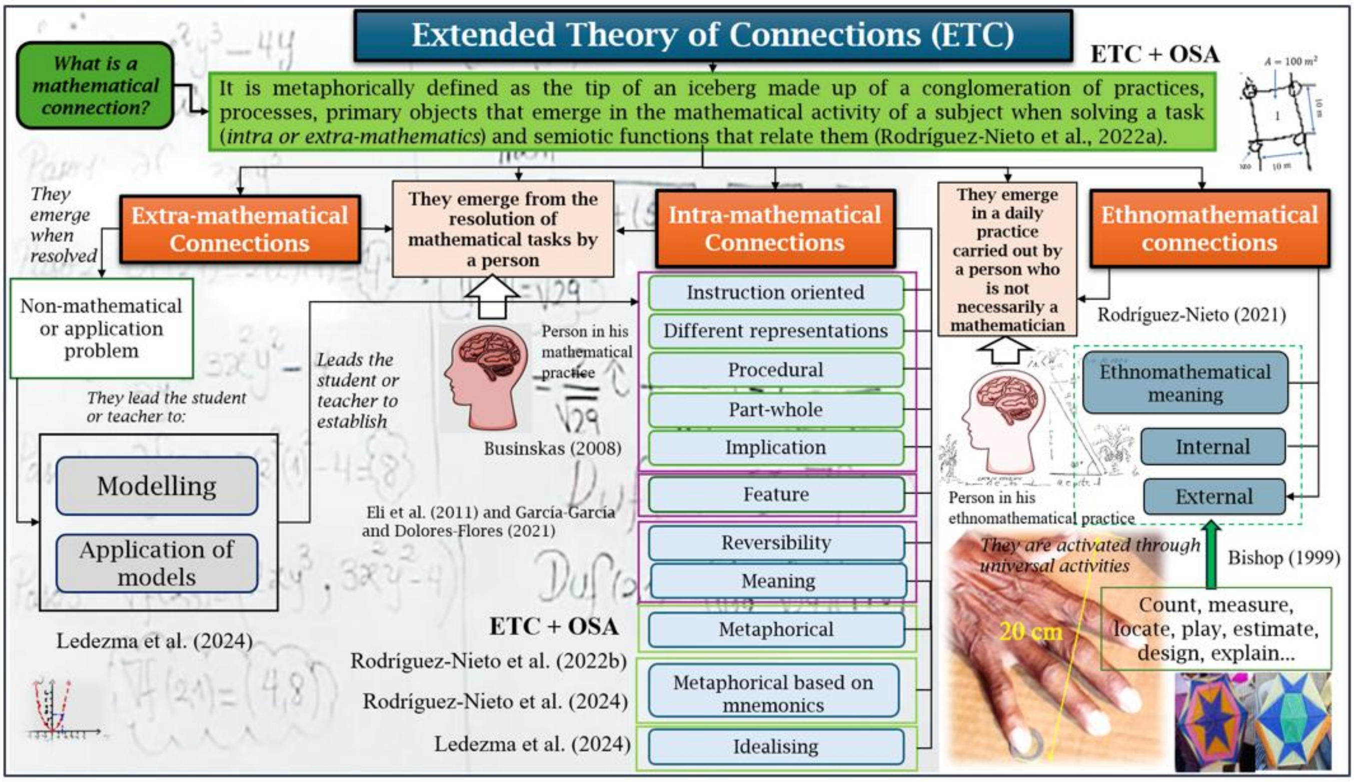


Figure 3. Synthesis of the ETC (Rodríguez-Nieto et al., 2024)

statistical tables and graphs from the early school years, incorporating tasks that promote data interpretation, comparison, and inference, in order to develop a deeper understanding and an authentic school statistical culture.

Although there is an abundant literature on mathematical connections, there remains a notable lack of research focused on working with frequency tables at different educational levels. This absence is relevant, since the ability to establish connections between different mathematical representations, such as transforming unorganized data into tables and, subsequently, into graphs or interpretations, is crucial for achieving a deep understanding of statistics (Alsina, 2020; Rodríguez-Nieto & Font, 2025). The reviewed studies show that many students approach tables, limiting themselves to literal readings or simple calculations, without relating them to broader concepts such as variability or probability. In this sense, fostering mathematical connections in the construction of frequency tables not only involves teaching students how to read and complete tables, but also promoting relationships between data, contexts, units of measurement, and visual representations, which strengthens statistical thinking and functional statistical literacy (Su et al., 2023). Therefore, the objective of this research is to explore the mathematical connections activated by university students when they solve a problem about the construction of frequency distribution tables.

THEORETICAL FRAMEWORK

Extended Theory of Connections in Mathematics Education

From the theoretical articulation between the ETC and the OSA to mathematical knowledge and instruction, a mathematical connection can be conceived as the visible part of an iceberg composed of a complex set of practices, processes, and primary objects identified in a subject's mathematical activity when solving a task, as well as the semiotic functions (SFs) that link them (Rodríguez-Nieto et al., 2022a). The Theory of Connections can be applied in three main groups, depending on the type of problems being addressed. The first group corresponds to intra-mathematical connections, which involve relationships among concepts, theorems, representations, and other mathematical ideas. The second group refers to extra-mathematical connections, which are activated when solving applied or non-mathematical problems through the transfer of information to a mathematical model (Dolores-Flores & García-García, 2017).

Finally, the third group concerns ethnomathematical connections, understood as the relationships between the mathematics practiced by people belonging to cultural groups (for instance, those who work in carpentry, masonry, or other trades) and the institutional or school mathematics found in curricular materials (Rodríguez-Nieto, 2021) (**Figure 3**).

The following section presents the typology of mathematical connections, providing a more practical and applicable dimension to the ETC. The typology of mathematical connections outlined below provides a more structured and pragmatic framework for applying the ETC. Each category highlights distinct ways in which learners relate mathematical concepts, representations, and experiences, enriching their understanding and cognitive flexibility.

Instruction-oriented connections: These occur when a learner understands a concept **C** through one or more prior concepts (**A** and **B**) that must already be comprehended. Such connections manifest in two forms:

(1) by linking new content to previous knowledge, and

(2) by treating mathematical ideas and procedures as prerequisites or foundational skills necessary for acquiring new concepts (Businskas, 2008).

This type of connection stems from the need to enhance the effectiveness of mathematics instruction by building continuity between prior experiences and new learning to foster conceptual development.

Modeling connections: These represent the relationships that individuals establish between the mathematical world and real-life contexts or between mathematics and other scientific disciplines. They occur when a mathematical concept is related to a real or applied situation, prompting the learner to construct a mathematical model to address it. In doing so, the individual draws on various forms of knowledge (both mathematical and non-mathematical) and engages in diverse actions (symbolic, algebraic, graphical, etc.) to produce a solution that is consistent with the problem's requirements (Campo-Meneses & García-García, 2023; Evitts, 2004).

Connections of different representations: These are identified when learners express mathematical objects using equivalent or alternative representations. Equivalent representations involve transformations within the same register, whereas alternative representations require a shift between registers (for example, from a graph to an algebraic equation) (Businskas, 2008). The use of multiple representations is fundamental for developing deep conceptual understanding, as it allows learners to connect different perspectives of the same idea.

Procedural connections: These arise when a learner applies rules, formulas, or algorithms to solve mathematical problems. They can be expressed as A is a procedure used to operate with concept B (Businskas, 2008). This category also includes step-by-step processes used to construct graphs, create geometric figures, or apply properties and theorems, as such sequences structure the actions needed to operate with specific mathematical concepts.

Part-whole connections: These occur when logical relationships are established between concepts in two ways:

(1) generalization, where A is a generalization of B and B is a particular case of A and

(2) inclusion, when one mathematical concept is contained within another (Businskas, 2008).

Implication connections: These are recognized when one concept (**P**) logically leads to another (**Q**) through a deductive relationship ($P \rightarrow Q$) (Businskas, 2008).

Feature-based connections: These appear when learners describe the attributes or properties of mathematical concepts, emphasizing similarities and differences that distinguish one concept from another (Eli et al., 2011).

Reversibility connections: These emerge when a learner can move from concept A to concept B and then reverse the process, returning from B to A (García-García & Dolores-Flores, 2021).

Meaning connections: These are established when a learner attributes personal meaning to a mathematical concept expressing what it represents to them. This includes cases in which students construct and articulate their own definitions for certain concepts (García-García & Dolores-Flores, 2021).

Metaphorical connections: These involve projecting properties or characteristics from a familiar, embodied domain to a more abstract or less familiar one, thus enabling the learner to structure and comprehend abstract mathematical ideas (Rodríguez-Nieto et al., 2022b).

Mnemonic-based metaphorical connections: These refer to relationships established between mnemonic devices (such as familiar phrases or memory aids) and mathematical objects, rules, or procedures, facilitating recall and strategic use (Rodríguez-Nieto et al., 2024). These connections can take three forms:

(1) keywords, which resemble the target term,

(2) acronyms, created by using the initial letters of words in a list to form another word, and

(3) acrostics, which involve constructing a sentence in which the initial letters correspond to the studied term (Rodríguez-Nieto et al., 2024).

Idealizing connections: These occur when a concrete (ostensive) representation is related to an abstract (non-ostensive) mathematical concept. Their function is to dematerialize the concrete object and transform it into an ideal mathematical entity for example, interpreting the rounded base of a tank as a circle or circumference (Ledezma et al., 2024).

The ETC continues to evolve and expand. Recent developments, such as those proposed by Cantillo-Rudas et al. (2024), introduce the notion of neuro-mathematical connections, which explore the cognitive

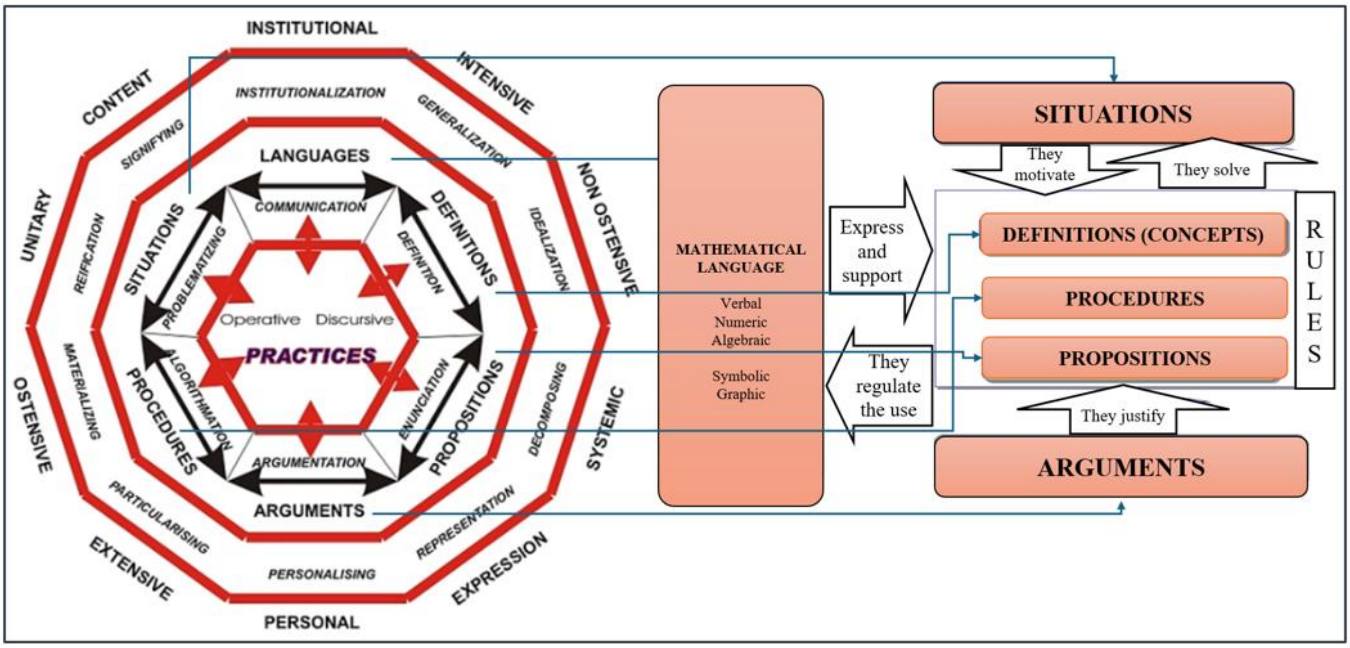


Figure 4. Schematization of mathematical knowledge from an onto-semiotic view (Font & Contreras, 2008; Font & Godino, 2006)

dimensions of the theory and examine the brain regions activated during mathematical activity.

Some Tools About Onto-Semiotic Approach

OSA emerged from the need to integrate and refine diverse theoretical and methodological tools in mathematics education and can itself be understood as a form of *networking* between theories aimed at addressing the inherent complexity of teaching and learning mathematics. Within this framework, mathematical activity if institutional or personal is systematically described and modeled through mathematical practices and through the configuration of primary objects and processes activated within those practices, enabling a deeper analysis of how mathematical meaning is constructed, negotiated, and communicated in educational contexts (Drijvers et al., 2013).

Mathematical practice is defined as "any situation or expression (…) carried out by someone to solve mathematical problems, communicate the solution obtained to others, validate it, or generalize it to other contexts and problems" (Godino & Batanero, 1994, p. 334). These practices involve the use of objects, understood broadly as any entity that participates in mathematical activity and can be identified as a distinct unit (Font et al., 2013). According to the OSA, these are six primary objects:

(1) problematic situations,

(2) representations,

(3) definitions,

(4) propositions,

(5) procedures, and

(6) arguments.

Together, these interrelated objects form what is known as the configuration of primary objects (Godino et al., 2019). During mathematical activity, primary objects emerge in various ways, reflecting different manners of perceiving, expressing, writing, and operating upon them. This diversity allows distinguishing between personal and institutional primary objects; ostensive or non-ostensive; unitary or systemic; intensive or extensive; and content or expression (Godino et al., 2007). Ultimately, these objects or systems of interrelated heterogeneous objects constitute configurations, which may be institutional (epistemic) or personal (cognitive), depending on the perspective and context in which they are analyzed (Godino et al., 2019).

Within mathematical activity, the emergence of primary objects occurs through the activation of a series of primary mathematical processes, including communication, problem posing, definition, enunciation, procedural execution (algorithms), and argumentation. These processes result from applying the process-product perspective to the primary objects. Additionally, they are complemented by other processes derived from the process-product duality applied to the five central dualities proposed in the OSA: personal/institutional, ostensive/non-ostensive, expression/content, extensive/intensive, and unitary/systemic. This interaction gives rise to corresponding dual processes such as personalization-institutionalization, materialization-idealization, representation-meaning, synthesis-analysis, and generalization-particularization (Font et al., 2013; Godino et al., 2007) (**Figure 4**).

Central to the OSA is the concept of SF, which serves as a bridge connecting mathematical practices with the processes and objects they activate. This notion provides a foundation for constructing an operational understanding of knowledge, mathematical comprehension, meaning, and competence (Godino et al., 2007). An SF is conceived as a triadic relationship established by an individual or institution between an antecedent (expression or object) and a consequent (content or object), following a specific criterion or correspondence code. These SFs become evident when mathematical activity is analyzed through the expression/content duality, revealing how meaning is generated and communicated in mathematical practice.

According to Rodríguez-Nieto et al. (2022a), the concept of SF within the OSA is broader and more comprehensive than the notion of mathematical connection proposed in the ETC, as connections can be understood as specific instances of SFs either of a personal or institutional nature. Within the ETC, a mathematical connection may or may not be accurate; from the OSA perspective, a correct connection aligns with the institutional one, whereas an incorrect connection corresponds to a personal interpretation. The articulation of both theoretical frameworks is valuable: the ETC provides a typology of mathematical connections, while the OSA enables a detailed analysis of those connections in terms of the practices, processes, objects, and SFs involved, thus offering a deeper insight into how individuals construct meaning and understanding around mathematical concepts.

Statistical Tables

In this research, statistical tables are understood as a trans-numeration tool (Wild & Pfannkuch, 1999), as well as a way of obtaining new information when changing from one representation system to another. For Campbell-Kelly et al. (2003) and Gabucio et al. (2010), they are a graphic organization format that uses a double axis to cross information concerning two sets of categories or variables, related and organized reciprocally; and where each cell represents quantitative data. Estrella (2014) defines statistical tables as rectangular structures formed by rows and columns that allow organizing, classifying and summarizing data of one or more variables, facilitating their analysis and understanding. The tables help visualize behavior of the data and extract relevant data. Among its components are title, which indicates the content and context of the represented information; the data body, formed by the intersection of rows and columns; the side header, which presents the categories of the variable; the top header, which shows the type of data in the columns; and the totals, which correspond to the sums by row or column.

From the perspective of Lahanier-Reuter (2003) there are different types of tables and he describes some that are used in mathematics education, among which the following stand out:

Data tables: These are simple tables that do not address the idea of frequency or distribution, but rather only the idea of variable and value. An example is recording the temperature over a week in a particular city.

Frequency tables: These represent the frequencies (obtained by grouping or counting identical data) associated with the values or categories of variables. An example of this situation is recording the ages of a company's employees.

Two-way table: A table in which two variables intersect; that is, a value is related to two variables at once. For example, representing the ages of a company's employees by gender.

In this research, frequency distribution tables are of interest, due to the difficulties students have in constructing them (Díaz-Levicoy et al., 2020).

Levels of Understanding of Statistical Tables

Some authors describe and characterize the different levels of comprehension for statistical tables and graphs. In this context, Curcio (1989) and Friel et al. (2001) propose levels for reading graphs, which have been adapted in research with statistical tables (e.g., Díaz-Levicoy et al., 2015, 2016; Mingorance, 2014). The four levels of reading graphs and tables proposed by Curcio (1989) and Friel et al. (2001) are presented below, ranging from the most basic (literal reading) to the most complex (critical analysis of information). These levels serve as a framework for accounting for students' understanding of these mathematical objects.

Level 1. Reading the data: This refers to a literal reading of the information presented in the table; it does not require in-depth interpretation of the data. For example, reading frequencies in classes.

Level 2. Reading within the data: This refers to determining a value through comparisons or simple arithmetic operations, including the interpretation of the information contained therein. For example, calculating the total number of students who participated in the survey, among others.

Level 3. Reading beyond the data: This involves determining information missing from the table through predictions or estimates. For example, inferring the maximum temperature from the maximum temperatures in a city shown in the table.

Level 4. Reading behind the data: This involves critically assessing the way in which the data was collected, as well as interpreting other people's criticisms of it, or questioning the quality of the data, which leads to a reflection on mathematical knowledge and context. For example, analyzing whether the question used to collect the data is appropriate or not.

Figure 5. Study participants (the authors' own elaboration)

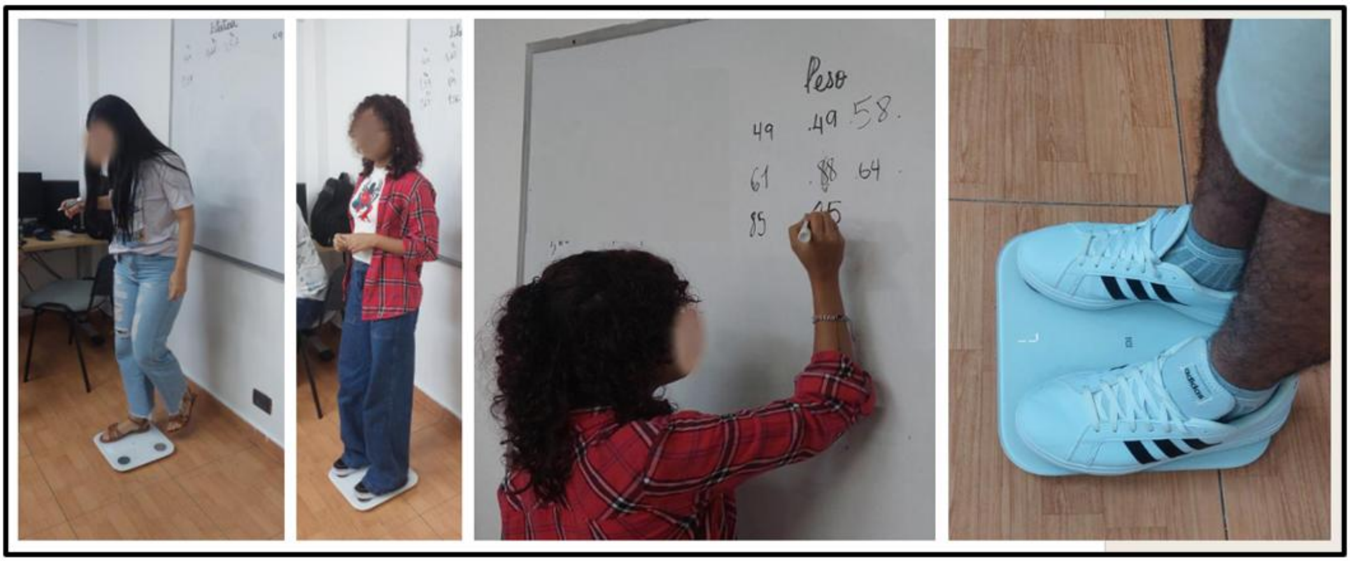


Figure 6. Using the scale to determine students' body mass (the authors' own elaboration)

METHODOLOGY

The methodology was qualitative (Maher & Dertadian, 2018) developed in four stages:

(1) design of a workshop that involves the creation of a problem based on the heights of the students,

(2) selection of university participants,

(3) implementation of the workshop and the problem on the construction of frequency distribution tables in the classroom to the students, and

(4) analysis of the data based on the theoretical foundation.

Participants and Context

This research involved twenty-eight university students with an average age of 18 (20 men and 8 women). They were enrolled in the first semester of a statistics for decision-making course in administrative science programs at a private university on the northern coast of Colombia's Caribbean Coast. These students came from the city of Barranquilla and other nearby municipalities (**Figure 5**). It is important to note that these students understood that the activities carried out and the evidence collected were part of an educational research project and not intended for financial gain; therefore, the students willingly agreed to participate and/or collaborate. Furthermore, the university where the researchers work suggests that the activities of the teaching staff contribute to the statistical training of students through everyday activities and be reported as high-level research.

Data Collection

Data collection was carried out in two phases during a statistics workshop using participant observation (Cohen et al., 2018): in the first, the professor spent an hour explaining to the students how to construct frequency distribution tables, considering key elements from organization to graphing. In the second phase, the professor encouraged a special dynamic where students went to the board in order of attendance to provide their body mass measurements taken on a scale (**Figure 6**).

This activity was videotaped, and evidence was collected on pencil, paper, and whiteboard in the form of written productions from the students and the teacher. This data was important for the teacher to formulate the problem situation presented below.

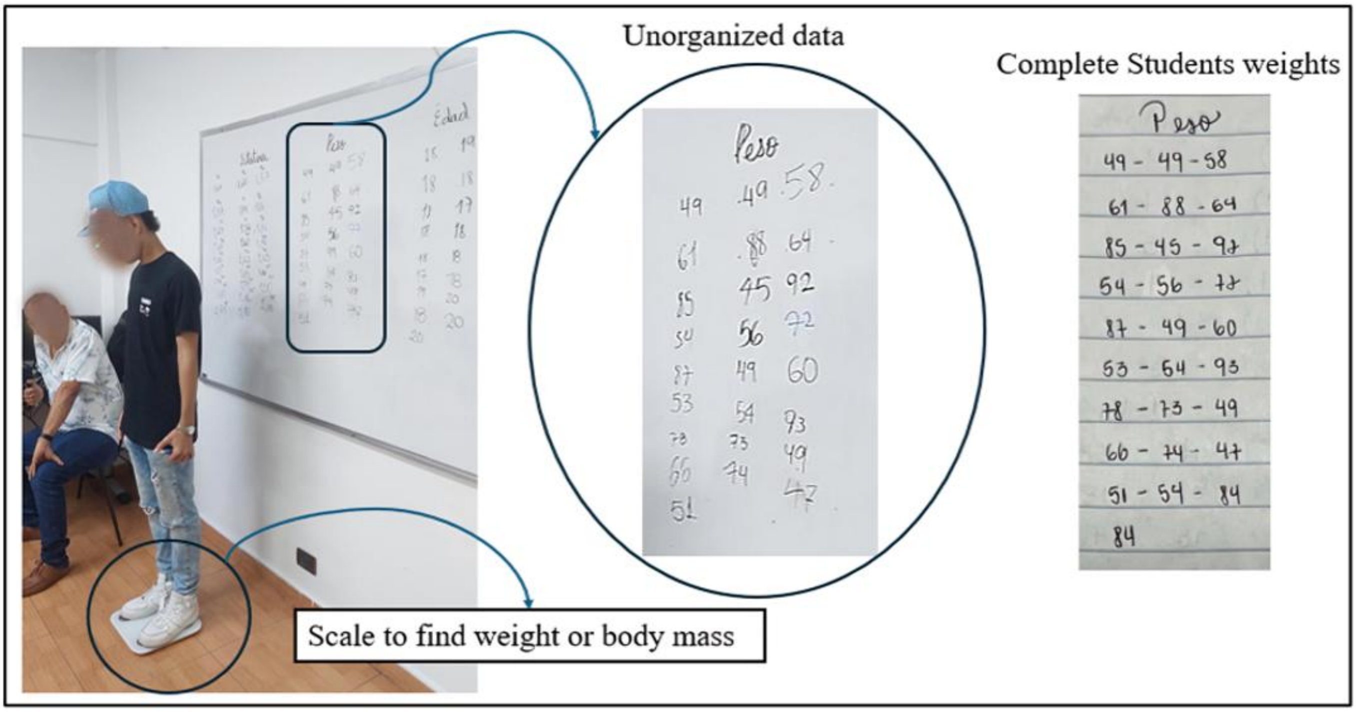


Figure 7. Measurement of the weight or body mass of the twenty-eight students (the authors' own elaboration)

Table 1. Phases for analyzing data based on ETC integrated with OSA (information adopted from Rodríguez-Nieto et al., 2024)

Phases	Description
1. Transcription of classroom observations, interviews, and data organization	The written productions of the teacher and students were organized in a way that allowed the researchers to become familiar with their responses. Moreover, this process is fundamental to ensure a thorough review, familiarization, reading, and detailed interpretation of the collected information.
2. Temporal narrative	The mathematical explanation of the problem situation's resolution is provided, including the most relevant mathematical objects used by the teacher and students.
3. Mathematical practice	These are sequenced actions that describe the students' mathematical activity, governed by rules established by institutions and useful for problem solving, in this case, in statistics. Within these practices, the foundation of each mathematical connection according to the ETC becomes evident.
4. Cognitive configuration	This refers to the system of primary mathematical objects that a subject mobilizes as part of the mathematical practices developed to construct the frequency distribution table. These primary objects are a fundamental part of the connection, as they generally constitute the beginning (antecedent) and the end (consequent) of its structure.
5. Semiotic functions	Relationships are established among the primary objects of the cognitive configuration. The mathematical connections suggested by the ETC are represented and consolidated.
6. Mathematical connections	The mathematical connections are reported based on the articulation between the ETC and the OSA.
7. Levels	Based on the students' responses and the mathematical connections identified step by step, the levels of understanding of frequency distribution tables are recognized.

Problem situation: From the list of data on people's weights (body mass) (**Figure 7**), you are asked to construct a frequency distribution table for grouped data, calculate class marks, frequencies, graph and interpret the results.

Data Analysis

To analyze the data, the mathematical connections analysis method was used based on the network work between the ETC and the OSA, considering the narrative, mathematical practices, cognitive configuration, SFs and mathematical connections (Rodríguez-Nieto et al., 2024, 2026), see **Table 1**. Subsequently, they identified the levels of understanding of frequency distribution tables.

The use of the mathematical connections analysis method, articulated between the ETC and the OSA, is justified by the need to understand the complexity of students' mathematical activity when constructing frequency distribution tables. This integrated approach

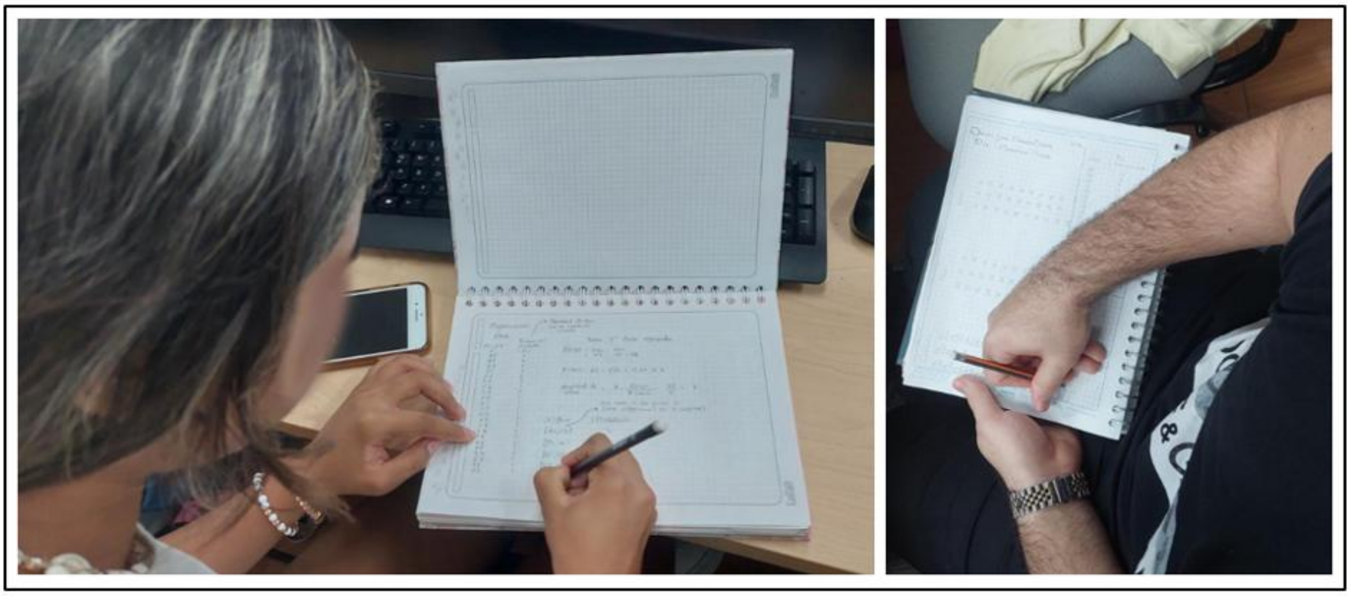


Figure 8. Evidence from students with their worksheet (the authors' own elaboration)

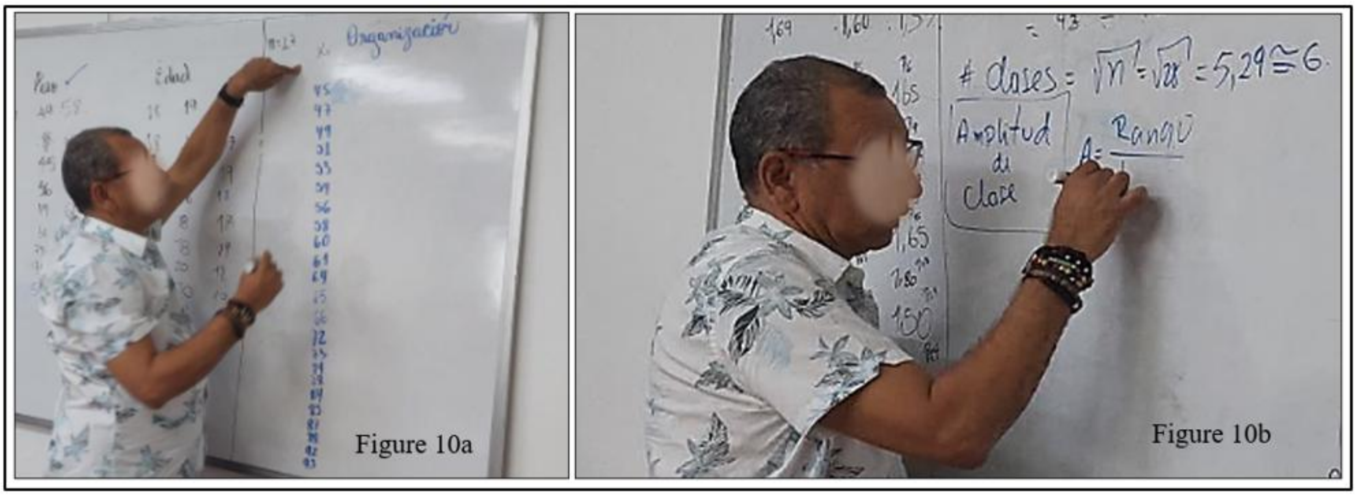


Figure 10a

Figure 10b

Figure 9. Teacher explaining the organization of data (the authors' own elaboration)

allows us to go beyond procedural correctness and analyze how mathematical practices, primary objects, representations, and meanings are mobilized during problem-solving. The phased structure ensures methodological rigor and transparency, facilitating the systematic reconstruction of reasoning and the identification of SFs. Furthermore, linking the connections to levels of understanding strengthens the interpretation of statistical learning.

RESULTS

In the data collection section, it was explained that each student went to the board in order of attendance to provide their weight or body mass measurement and were labeled as a participant (P1, P2, …, P28). To do so, the teacher used a scale to weigh in kilograms. Simultaneously, the students wrote their data on the board as a fundamental input for the workshop class and also wrote them on their worksheets (**Figure 8**).

To continue the analysis, the steps presented in **Table 1** were followed, but it is known that the researchers are already familiar with the data. Therefore, the analysis will be continued by integrating the temporal narrative with the mathematical practices, sequentially describing the mathematical activity of the teacher and the students to construct the frequency distribution table.

In this context, to construct a frequency distribution table, the teacher suggested that the students follow these steps for one hour:

Teacher's Narrative and Mathematical Practices

1. **TMp1.** Based on the information provided on the board, the student must order the data (weight) from lowest to highest: 45, 47, 49, 49, 49, 49, 51, 53, 54, 54, 54, 56, 58, 60, 61, 64, 66, 72, 73, 74, 78, 84, 84, 85, 87, 88, 92, 93 (see part a in **Figure 9**).
2. **TMp2.** Determine the absolute frequency of each value of the variable (weight).

3. **TMp3.** Find the range between the data for the variable weight, using the formula range = maximum value - minimum value.
4. **TMp4.** Find the number of classes or intervals in which the absolute frequencies are distributed, following the formula $\#of\ class = \sqrt[2]{n}$, where n is the number of data or sample size.
5. **TMp5.** Calculate the class width or interval width using the formula $A = \frac{Range}{\#of\ class}$ (see part b in **Figure 9**).
6. **TMp6.** Based on the previous steps, we proceed to construct the frequency table for grouped data. In this context, the first column lists the classes, and in the first class, the lower limit is the minimum value of the variable, to which the amplitude is added, obtaining the upper limit of the class. It should be noted that, in the classes, the lower limits are closed and the upper limits are open.
7. **TMp7.** The next class takes the upper limit of the previous class as the lower limit of the class being constructed, to which the amplitude is added, obtaining the upper limit. Following this process, the other classes are constructed until they cover the maximum value of the variable under study.
8. **TMp8.** In the second column, for each class the absolute frequencies are determined, which are obtained by counting the values that fall under the domain of each class.
9. **TMp9.** The third column contains the relative absolute frequencies, obtained by dividing the absolute frequency by the total number of data.
10. **TMp10.** The fourth column shows the accumulated absolute frequencies, obtained by adding the absolute frequencies from the first class.
11. **TMp11.** The fifth column shows the relative cumulative frequencies determined by dividing the absolute cumulative frequency by the total number of data.
12. **TMp12.** In the sixth column, the class marks or averages are determined by calculating the midpoint of each class, that is, the limits are added and divided by two.

Following the teacher's explanations, the students created their frequency distribution tables considering the weight variable. Below is P11's math activity, who followed six classes using her teacher's method, which is called method 1 for problem-solving.

Method 1. Student Mathematical Practices P13 for Constructing the Frequency Distribution Table

Similar to student P13, students P1, P2, P3, P4, P6, P7, P8, P9, P10, P11, P12, P14, P15, P17, P18, P19, P21, P22, P23, P4, P25, P26, P27 and P28 began solving the problem situation by activating the procedural connection and different representations (symbolic and tabular records) by organizing the data from lowest to highest: 45, 47, 49, 49, 49, 49, 51, 53, 54, 54, 54, 56, 58, 60, 61, 64, 66, 72, 73, 74, 78, 84, 84, 85, 87, 88, 92, 93 (TMp1). These students then calculated the range using procedural connection, considering the minimum value equal to 45 and the maximum value equal to 93, obtaining the range as 93-45 = 48 (TMp2). Based on the data number, the students determined the number of classes by applying the square root method and procedural connection using the formula $\#of\ clase = \sqrt[2]{n}$, when n the number of data or sample size: $\#of\ clase = \sqrt[2]{28} = 5{,}29 \approx 6$ classes (TMp3).

Table 2. Class width and absolute frequency

Weight interval (kg)	Absolute frequency (fa)
[45-53)	7
[53-61)	7
[61-69)	3
[69-77)	3
[77-85)	3
≥ 85	5
Total	28

Table 3. Relative absolute frequencies

Weight interval (kg)	Absolute frequency (fa)	Relative frequency (%)	Cumulative frequency (f)
[45-53)	7	25.0%	7
[53-61)	7	25.0%	14
[61-69)	3	10.7%	17
[69-77)	3	10.7%	20
[77-85)	3	10.7%	23
≥ 85	5	17.9%	28

To obtain the amplitude of each class, the students, through procedural connection, applied the formula $A = \frac{Range}{\#\ of\ class} = \frac{48}{6} = 8$ (TMp4). In this context, the amplitude is 8, and the first class starts at 45 with a closed interval. This information is an excellent input for constructing class intervals and identifying absolute frequencies (**Table 2**).

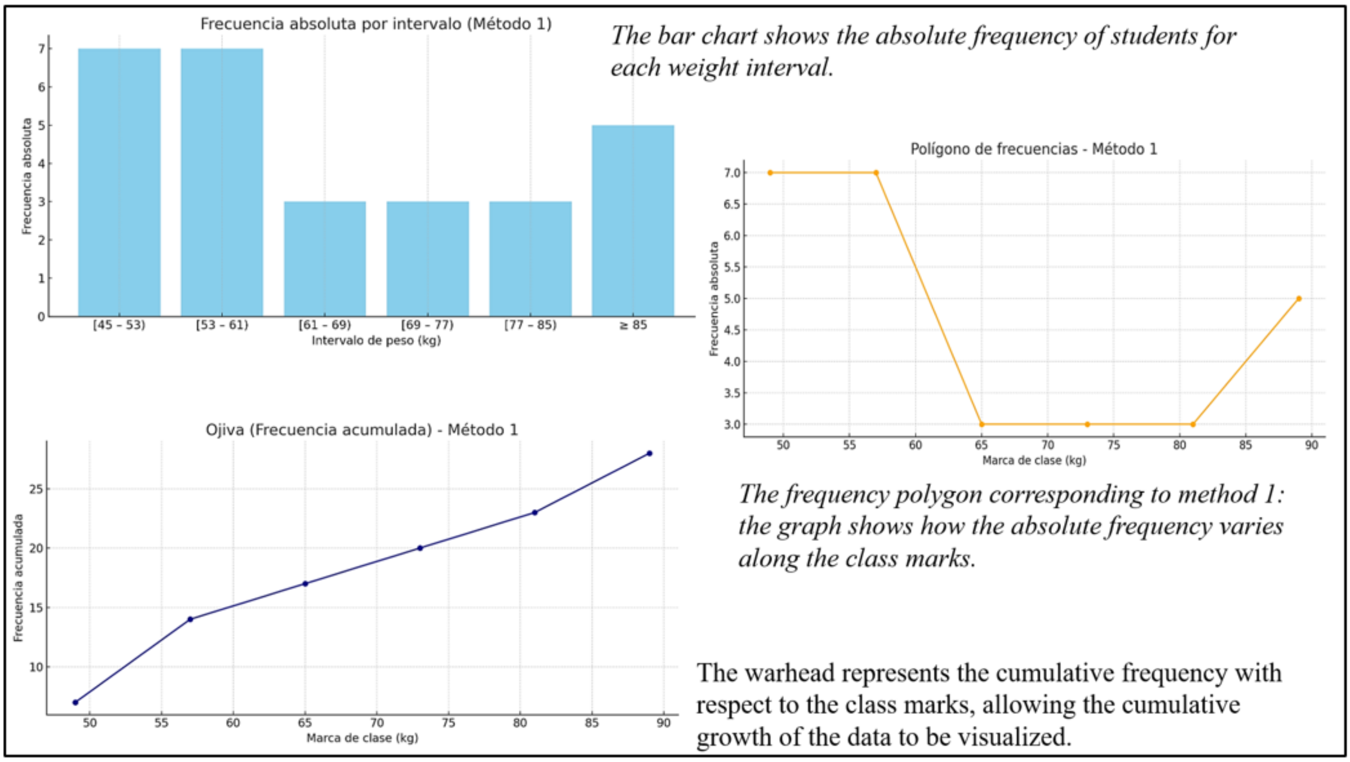
In the construction of **Table 2** it is observed that, to obtain each class, the students use the procedural connection by adding 8 to 45 obtaining 53, then adding 8 to 53 to obtain 61 (TMp5), but they also activate the part-whole connection by assuming a quantity or frequency of student weights in each interval (TMp6).

The students continued building the table by placing the relative absolute frequencies in the third column (TMp7), obtained through the procedural connection from the division between the absolute frequency and the total number of data, and in the fourth column they placed the accumulated frequency (**Table 3**).

Next, in the fifth and sixth columns of the table, the students displayed the relative cumulative frequencies (TMp8) and the class marks (TMp9) found through

Table 4. General frequency distribution table using method 1

Weight interval (kg)	Absolute frequency (fa)	Relative frequency (%)	Cumulative frequency (f)	Cumulative relative frequency (%)	Class mark (average)
[45-53)	7	25.0%	7	25.0%	49
[53-61)	7	25.0%	14	50.0%	57
[61-69)	3	10.7%	17	60.7%	65
[69-77)	3	10.7%	20	71.4%	73
[77-85)	3	10.7%	23	82.1%	81
≥ 85	5	17.9%	28	100%	89



The bar chart shows the absolute frequency of students for each weight interval.

The frequency polygon corresponding to method 1: the graph shows how the absolute frequency varies along the class marks.

The warhead represents the cumulative frequency with respect to the class marks, allowing the cumulative growth of the data to be visualized.

Figure 10. Graphics made by students for method 1 (the authors' own elaboration)

procedural connection. With this, they were able to construct the complete frequency distribution table (**Table 4**), which is actually a connection of different representations.

Onto-Semiotic Configuration of Primary Objects

Now, some of the primary objects related to each other are shown, for the resolution of the proposed problem situation:

Problem situation: The task proposed by the teacher, consisting of constructing a grouped frequency distribution table based on the students' weight data, constitutes the problem object that gives rise to the mathematical activity.

Representations: Several representations are identified:

(1) numerical (weights in kg),

(2) tabular (intervals, absolute and relative frequencies, cumulative frequencies, and class marks),

(3) percentages (cumulative relative frequencies in %), and

(4) graphical representations (later associated with data interpretation), thus, activating the multiple semiotics that characterizes statistical practices (**Figure 10**).

Definitions: Concepts such as "range," "class," "range," "frequency," "class marker," "cumulative frequency," among others, were explicitly or implicitly defined in the construction of the tables.

Propositions: Some propositions that guide the mathematical action include: "the range is obtained as the difference between the maximum and minimum values" or "relative frequency is the absolute frequency divided by the total data." These propositions guide the construction and validation of the procedures.

Procedures: A clear sequence is observed:

(1) data organization,

(2) range calculation,

(3) number of classes,

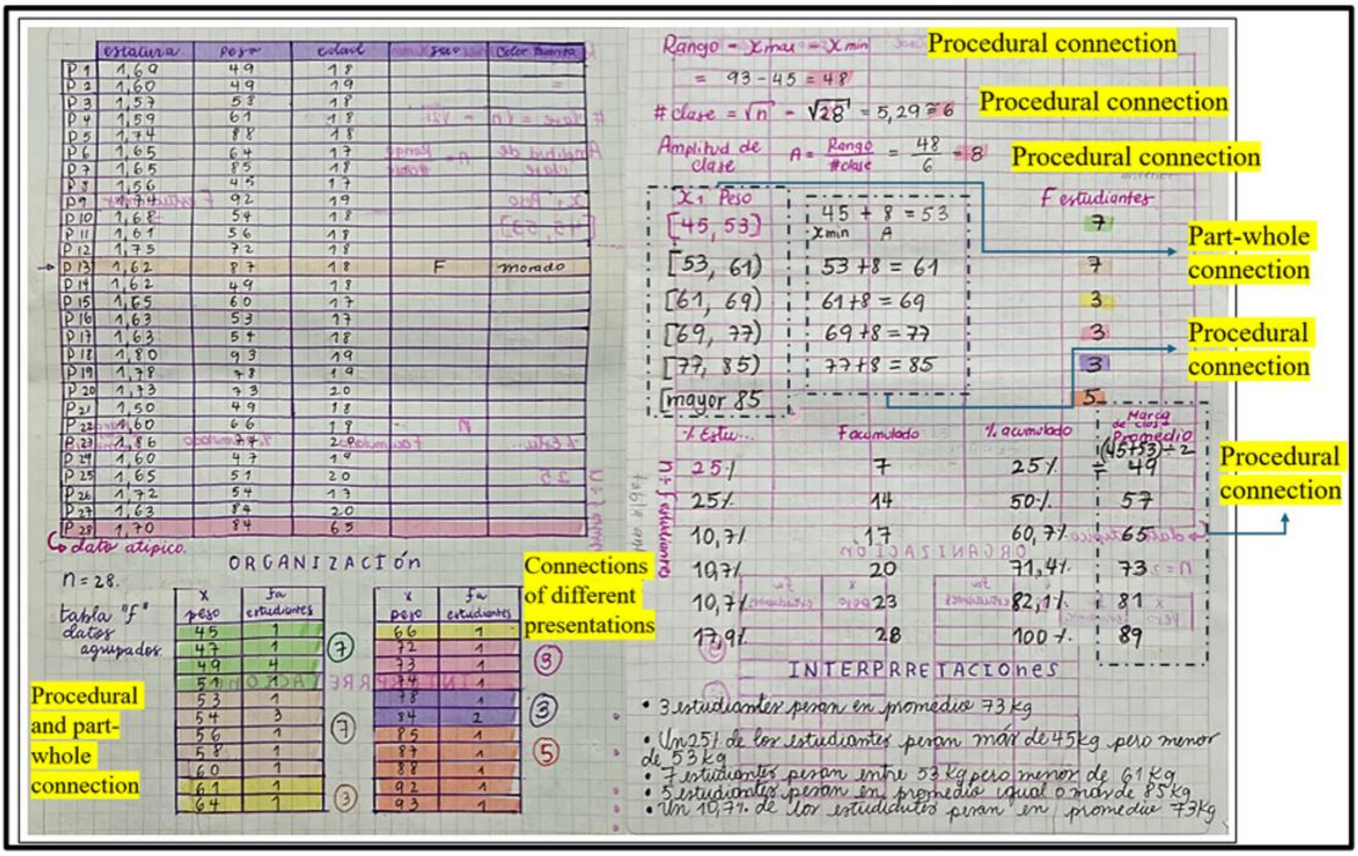
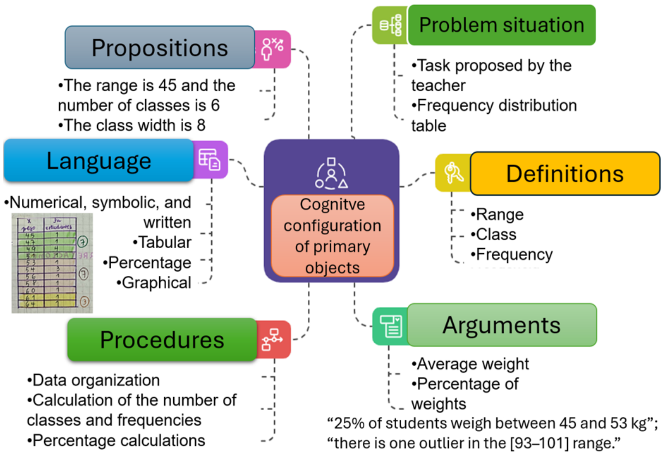


Figure 11. Procedures used by P13 (the authors' own elaboration)


Propositions
•The range is 45 and the number of classes is 6
•The class width is 8

Problem situation
•Task proposed by the teacher
•Frequency distribution table

Language
•Numerical, symbolic, and written
•Tabular
•Percentage
•Graphical

Cognitve configuration of primary objects

Definitions
•Range
•Class
•Frequency

Procedures
•Data organization
•Calculation of the number of classes and frequencies
•Percentage calculations

Arguments
•Average weight
•Percentage of weights
"25% of students weigh between 45 and 53 kg"; "there is one outlier in the [93–101] range."

Figure 12. Synthesis of the cognitive configuration of P13 (the authors' own elaboration)

(4) range,

(5) interval construction,

(6) frequency counting,

(7) percentage calculations, and

(8) class markers (**Figure 11**).

Arguments: 3 students weigh an average of 73 kg; 25% of the students weigh more than 45 kg but less than 53 kg; 7 students weigh between 53 kg but less than 61 kg; 5 students weigh an average of 85 kg or more; and 10.7% of the students weigh an average of 73 kg.

Figure 12 shows a summary of the optimal primary object configuration for solving the proposed task.

Mathematical Connections

The analysis shows how P13's mathematical activity goes beyond the simple application of rules, activating a complex network of mathematical connections that articulate institutional processes with personal meanings. This connection between the ETC and the OSA allows for a deeper understanding of the student's mathematical thinking, revealing how mathematical objects and practices interrelate to solve a statistical task (**Table 5**).

However, there are other students P5, P16 and P20 who built the frequency distribution table considering seven classes, for example, they used the class identified as [93, 101), which is statistically consistent.

Method 2. Student Mathematical Practices P16 for Constructing the Frequency Distribution Table

Students P5, P16, and P20 constructed the table differently, since they used Sturges' formula to find the class width and also the last class of the weights that fall in the interval [85, 93), but in this case 93 is not included in the class. Therefore, the students chose to add a seventh class [93, 101) to include this data, which is valid from a statistical point of view.

Table 5. Description of mathematical connections

Mathematical practice	Mathematical connections
Practices 1, 2, 3, 4, 5, 6, 7, 8, 9 and arguments (final): Interpretations such as "10.7% weigh 73 kg" or "25% weigh between 45 and 53 kg."	*Meaning:* Although implicit, it is identified when the student interprets that "5 students weigh equal to or more than 85 kg" or that "25% of the students weigh between 45 and 53 kg," showing that they attribute meaning to rank, class mark, and relative and cumulative frequencies as indicators of distribution and trend. Furthermore, by correctly solving the problem situation, it is evident that students possess institutional meanings for rank, class mark, frequency, arithmetic operations, etc.
Practice 1: Organizing data from lowest to highest. Practice 2: Calculating the range. Practice 3: Determining the number of classes using the square root of the sample size. Practice 4: Calculating the class width. Practice 5: Constructing class intervals by summing the widths. Practice 7: Calculating relative frequencies. Practice 8: Calculating relative cumulative frequencies. Practice 9: Calculating class scores.	*Procedural:* The student activated a structured sequence of algorithmic actions (data organization, range calculation, number of classes, amplitude, frequency counting, percentages, and class marks), which demonstrates a strong presence of this connection (ETC) supported by institutionalized procedures (OSA).
Practice 1: Converting raw numerical data to a tabular representation. Practice 7, 8, 9: Including percentages, cumulative data, and averages. Final product: Creating the general table (**Table 4**) with multiple records: numerical, percentage, tabular, and symbolic.	*Different representations:* Numeric representations (weights in kg), symbolic representations (formulas for range, number of classes, etc.), tabular representations (absolute and relative frequencies), percentage representations, and graphic representations were mobilized. This connection translates into semiotic functions within the OSA, establishing relationships between expressive registers and conceptual objects.
Practice 6: Assignment of quantities (frequency) within intervals.	*Part-whole:* By constructing intervals and assigning frequencies to each class, the student demonstrated an understanding of how individual values group together to form a statistical whole, reflecting the articulation between levels of generalization and particularization (OSA).
Practices 2, 3, 4: The student deduces values such as range, number of classes and amplitude from the initial data.	*Implication:* From certain values (e.g., total number of data, range), the student deduced other concepts such as the number of classes or the upper limit of the intervals, showing a logical relationship between concepts (P → Q).
Practices 2, 3, 4, 5 and 6: The student identifies the properties of the objects involved in the construction of the frequency distribution table.	*Feature:* In the analyzed activity, students: Calculate the range as the difference between the maximum and minimum. They determine the class width by considering the number of classes and the range. They identify intervals that are uniform. • They recognize the class mark as a representative value of the interval. • They relate the accumulated frequencies with the progressive sum of the absolute frequencies. • They interpret percentages as proportions of occurrence in relation to the total. That is, students are relating different properties of the data set (such as dispersion, centrality, frequency, and representativeness) that characterize the table as an organizational and synthesis tool. For example: "7 students weigh an average of 57 kg" associating the class mark with a measurement representative of all students in that range, recognizing a synthesis characteristic.

In this line, these students proceeded in a similar way to the students who applied method 1 until obtaining the 6 classes (**Figure 13**).

Table 6 shows the frequency distribution table obtained from the steps presented in **Figure 13**.

In addition, students P5, P16, and P20 established connections of different representations since they made

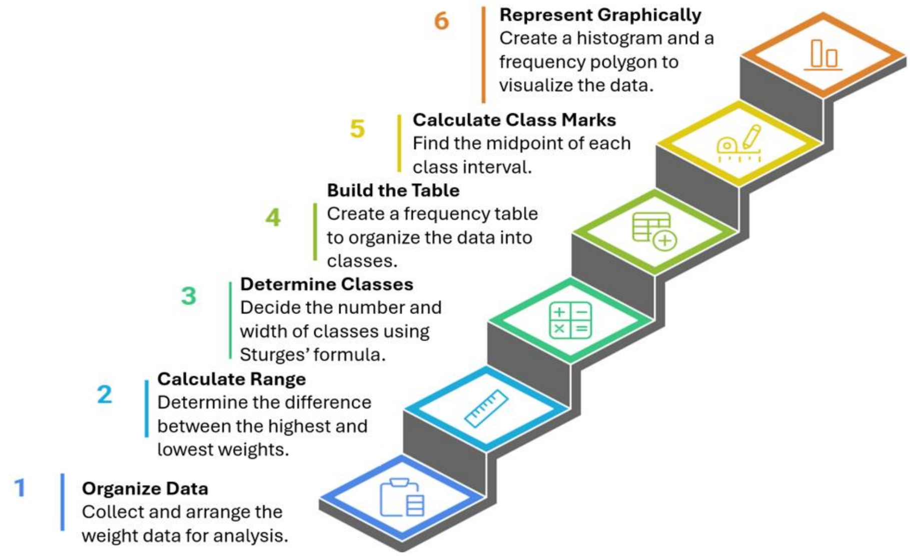
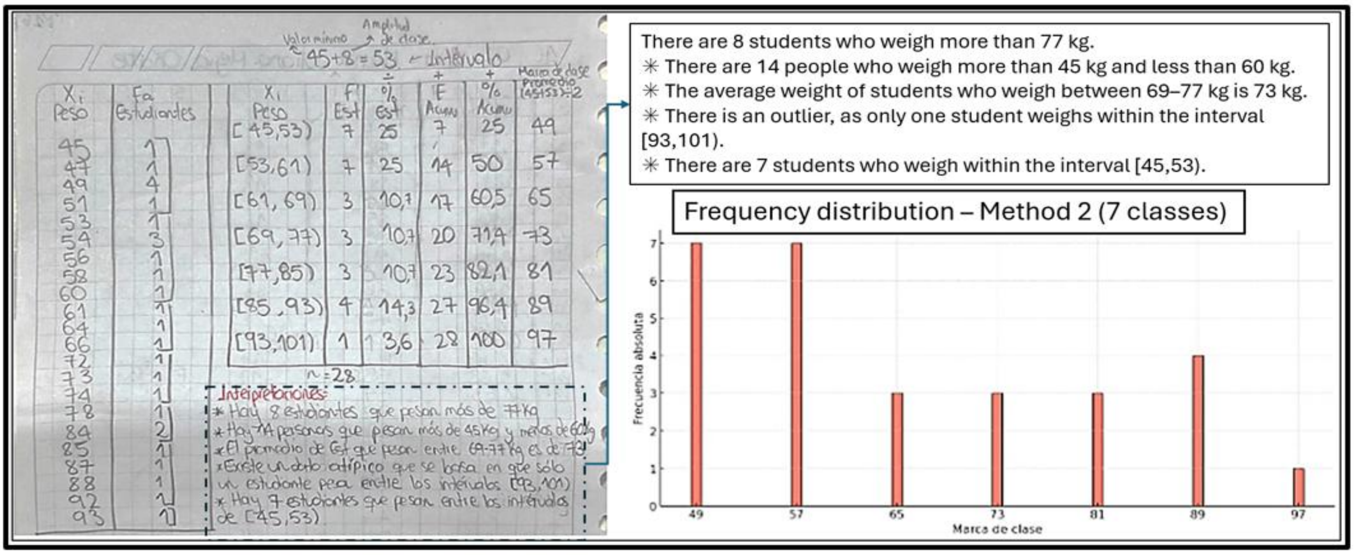


Figure 13. Logical steps or procedural connections to find the frequency distribution table (the authors' own elaboration)

Table 6. General frequency distribution table using method 2

Weight interval (kg)	Absolute frequency (fa)	Relative frequency (%)	Cumulative frequency (f)	Cumulative relative frequency (%)	Class mark (average)
[45-53)	7	25.0%	7	25.0%	49
[53-61)	7	25.0%	14	50.0%	57
[61-69)	3	10.7%	17	60.7%	65
[69-77)	3	10.7%	20	71.4%	73
[77-85)	3	10.7%	23	82.1%	81
[85-93)	4	14.3%	27	96.4%	89
[93-101)	1	3.6%	1	100%	97



X_i Peso	Fa Estudiantes
45	1
47	1
49	4
51	1
53	1
54	3
56	1
58	1
60	1
61	1
64	1
66	1
72	1
73	1
74	1
78	1
84	2
85	1
87	1
88	1
92	1
93	1

X_i Peso	fi Est	% Est	F Acum	% Acum	Marca de clase Promedio (45+53)÷2
[45,53)	7	25	7	25	49
[53,61)	7	25	14	50	57
[61,69)	3	10,7	17	60,5	65
[69,77)	3	10,7	20	71,4	73
[77,85)	3	10,7	23	82,1	81
[85,93)	4	14,3	27	96,4	89
[93,101)	1	3,6	28	100	97

Interpretaciones:
* Hay 8 estudiantes que pesan más de 77 kg
* Hay 14 personas que pesan más de 45 kg y menos de 60 kg
* El promedio de Est que pesan entre 69-77 kg es de 73
* Existe un dato atípico que se basa en que sólo un estudiante pesa entre los intervalos [93,101)
* Hay 7 estudiantes que pesan entre los intervalos de [45,53)

There are 8 students who weigh more than 77 kg.
✳ There are 14 people who weigh more than 45 kg and less than 60 kg.
✳ The average weight of students who weigh between 69–77 kg is 73 kg.
✳ There is an outlier, as only one student weighs within the interval [93,101).
✳ There are 7 students who weigh within the interval [45,53).

Figure 14. Procedures and graphs made by students for method 2 (the authors' own elaboration)

the graphs derived from the frequency distribution table using Excel tools and artificial intelligence (**Figure 14**).

Levels of Understanding of Statistical Tables

A thorough analysis of the students' interpretations and conclusions based on the statistical tables provided reveals a clear, hierarchical pattern of understanding, mostly concentrated at the most elementary levels of the Curcio (1989) and Friel et al. (2001) framework. The findings indicate consolidated competence in literal reading tasks, a functional ability to perform internal operations with the data, but a marked difficulty in moving toward deeper, more critical statistical reasoning.

Considering the information in **Table 7**, mastery of literal reading and basic operations was evident, corresponding to level 1 and level 2. The base of the comprehension pyramid is firmly established at level 1: reading the data, given that the majority of students (50.41%) demonstrated, without difficulty, the ability to make a direct and literal extraction of information. For example, statements such as "7 students weigh an average of 57 kg" or "3 students weigh between 69 kg and less than 77 kg" were recognized as recurrent, demonstrating that decoding explicit data in cells and rows is a fundamental skill acquired. This level, which does not require interpretation or manipulation of information, represents the essential starting point for any statistical analysis.

Table 7. Description of statistical table understanding levels

Student phrases that activate the levels	Frequency	Percentage
Level 1. Read the data	Total: 61	50.41%
#3 students on average weigh 89	1	0.83%
#7 students on average weigh 57 kg	1	0.83%
(1) 3 students on average weigh 73 kg	1	0.83%
(1) 3 students on average weigh 73 kg	1	0.83%
14 students weigh between 45 kg and less than 61 kg	1	0.83%
(2) 7 students weigh between 45 kg and less than 53 kg	1	0.83%
3 of the students weigh 83 kg	1	0.83%
3 students on average weigh 73 kg	1	0.83%
3 students weigh on average 65 kilos	1	0.83%
3 students weigh on average 73	1	0.83%
3 students weigh on average 73 kg	2	1.65%
3 students weigh on average 81 kg	3	2.48%
3 students weigh between 69 kg and less than 77 kg	1	0.83%
3 students weigh more than 61 kg and less than 69 kg	1	0.83%
3 students weigh more than 61 kg and less than 69 kg	1	0.83%
3 students weigh more than 61 kg and less than 69 kg	1	0.83%
3 students weigh more than 77 kg and less than 85 kg	1	0.83%
3 students weigh more than 77 and weigh less than 80 kg	1	0.83%
3 students weigh more than 77 and less than 85	1	0.83%
3 students weigh an average of 81 kg	2	1.65%
(3) 3 students weigh less than 85 kg but more than 77 kg	1	0.83%
5 students on average weigh 89 kg	2	1.65%
5 students on average weigh 89 kilos	1	0.83%
5 students weigh on average 89 kg	2	1.65%
5 students weigh more than 85 kg and less than 93 kg	1	0.83%
5 students weigh, on average, 89 kg	1	0.83%
(5) 5 students weigh on average 89 kg	1	0.83%
(5) Over 85 kg there are 5 students	1	0.83%
6 students weigh more than 69 and less than 85	1	0.83%
7 students weigh on average 49 kg	1	0.83%
7 students weigh on average 57 kg	2	1.65%
7 students weigh between 53 kg and less than 61 kg	1	0.83%
7 students weigh more than 53 kg and less than 61 kg	1	0.83%
7 students weigh more than 53 and less than 61	1	0.83%
7 students weigh less than 53 kg	1	0.83%
7 students weigh less than 53 and more than 45	1	0.83%
7 students weigh an average of 49 kg	1	0.83%
7 students weigh an average of 57 kg	2	1.65%
The [number] of students who are 18 years old are 13 people	1	0.83%
The average weight of students weighing between 69-77 kg is 73	1	0.83%
The average weight of 7 students is 49	1	0.83%
The average weight of people weighing 85 kg and 92 kg is 89	1	0.83%
There are 7 students who weigh on average 47 kilos	1	0.83%
There are 7 students who weigh between the intervals of [45, 53)	1	0.83%
Seven students on average 57 kg	1	0.83%
Seven students weigh on average 57 kg	1	0.83%
Three students weigh 65	1	0.83%
Three students weigh 73	1	0.83%

Table 7 (Continued).

Student phrases that activate the levels	Frequency	Percentage
Three students weigh on average 65 kg	1	0.83%
Three students weigh on average 73 kg	1	0.83%
Three students weigh on average 81 kg	1	0.83%
I interpret that 3 students weigh between 77-85 kg	1	0.83%
I interpret that 4 students weigh between 85-93 kg	1	0.83%
Level 2. Reading within the data	Total: 55	45.45%
(...) you can deduce 14 people in 45 kg and 60 kg	1	0.83%
11 of the 28 students weigh more than 69 kg	1	0.83%
14 students weigh between 45 and 61 kg	1	0.83%
16 students weigh 53 or more but less than 85 kg	1	0.83%
17 students weigh less than 69 kg	2	1.65%
(2) From 45 kg to 69 kg there are 14 students	1	0.83%
23 students weigh less than 85 kg	1	0.83%
25% of students weigh between 45-53 kg	1	0.83%
3 students make up 10% [of the sample]	1	0.83%
3 students represent 10.7% of the population ...	1	0.83%
(3) The percentage of students weighing between 61 kg and 69 kg is 10.7%	1	0.83%
(4) 23 students weigh less than 85 kg but more than 45 kg	1	0.83%
(4) There are 20 students who weigh between 45 kg and 77 kg	1	0.83%
5 students represent 17.9% of the entire population ...	1	0.83%
50% of students weigh between 53-61 kg	1	0.83%
9 students reflect a cumulative score of 82.7%	1	0.83%
Of the 21 students only 4 weigh 49	1	0.83%
The [percentage] of people who weigh more than 53 is 25%	1	0.83%
100% of students weigh more than 44 kg	1	0.83%
17% of students are around 17 years old	1	0.83%
17.9% of students weigh more than 85 kg	1	0.83%
18% of students weigh more than 85 kg	1	0.83%
25% of students weigh 45 kg or more and less than 53 kg	1	0.83%
25% of students weigh between 53 kg and 60 kg	1	0.83%
30% of students weigh between 61 kg and less than 85 kg	1	0.83%
45% of students weigh around 45 to 61 kg	1	0.83%
46% of students are 18 years old	1	0.83%
46.4% of students are 18 years old	1	0.83%
50% of students weigh more than 53 and less than 61	1	0.83%
50% of students weigh between 45 kg and less than 61 kg	1	0.83%
50% of students weigh between 53 and 67 kg	2	1.65%
50% of students weigh less than 61 kg	2	1.65%
50% of students weigh less than 81 kg	1	0.83%
60% of students weigh between 61 and 86 kg	1	0.83%
71.4% of the population ... weighs less than 77 kg	1	0.83%
71.4% of students weigh less than 77 kg	1	0.83%
82.1% of students are 19 years old or younger	1	0.83%
Among the 28 students, only 4 are 20 years old	1	0.83%
There are 14 people who weigh more than 45 kg and less than 60 kg	1	0.83%
There are 8 students who weigh more than 77 kg	1	0.83%
There are 9 people who are around the 61 kg and 84 kg range	1	0.83%
The largest number of students are 13 years old	2	1.65%
Most students are 18 years old.	1	0.83%
Most weigh between 45-61 kg	1	0.83%
The minority weighs 92-93 kg	1	0.83%
The minority of students is 17 years old	1	0.83%
10% of students weigh between 77-85 kg	2	1.65%
I interpret that there are more students under 30 years old	1	0.83%
I interpret that most of the students are of legal age	1	0.83%
I interpret that most students weigh less than 77 kg	1	0.83%

Table 7 (Continued).

Student phrases that activate the levels	Frequency	Percentage
Level 3. Reading beyond the data	Total 5	4.13%
(...) there is an atypical data in the intervals [93, 101)	1	0.83%
Of the 21 students we see an atypical fact	1	0.83%
There is an atypical fact, a student is 65 years old	1	0.83%
There is an atypical fact ... in which only one student weighs between [93, 101)	1	0.83%
There is a range of 7 people weighing around 50	1	0.83%

A significant number of students advanced to level 2: reading within the data (45.45%), which represents a cognitive leap from simple reading to interpreting and processing information. Within this level, three main types of skills were observed:

(1) **Data aggregation:** Students were able to combine information from multiple categories, such as in the statement "14 students weigh between 45 and 61 kg," which requires summing the frequencies of at least two contiguous intervals.

(2) **Calculating proportions:** Computing relative frequencies was a common task, reflected in conclusions such as "50% of students weigh less than 61 kg". This demonstrates the ability to relate one part of a data set to the whole. Comparison and ordering: Interpretations were identified that involved comparing frequencies to determine simple modes or trends, such as "most students are 18 years old."

These achievements indicate that students can actively operate with the data presented, reorganizing it and performing calculations to generate new information that is not explicitly visible in a single cell.

On the other hand, despite success at the first two levels, the results reveal a significant barrier to accessing higher levels of reasoning. Instances of level 3: reading beyond the data (4.13%), which involve inference or prediction, were extremely rare and limited to rudimentary observations. The statement "There is an outlier" was the clearest example, demonstrating a nascent ability to evaluate the distribution as a whole and identify unusual values. However, this skill was neither generalized nor developed further, suggesting that the ability to project, estimate, or make inferences from data is an area of conceptual weakness.

In this way, a complete absence of interpretations corresponding to level 4: reading behind the data was observed. In no case did the students question the nature of the information, the collection methodology, the appropriateness of the categories, or the context of the table. The data were perceived as a factual and unquestionable entity, and not as a construct susceptible to critical analysis. This omission is crucial, as it indicates that the most sophisticated dimension of statistical literacy is the ability to critically evaluate evidence and reflect on its meaning and limitations that is not being developed.

These findings outline a functional but superficial profile of statistical literacy. Students can read and calculate, but they fail to draw consistent inferences or critically evaluate. This suggests that current pedagogical practices may be prioritizing the mechanical and algorithmic aspects of statistics to the detriment of the development of genuine statistical thinking, which is essential for developing critical citizens capable of interpreting information in a data-saturated world.

DISCUSSION

The results obtained achieved the objective of this research (explore the mathematical connections activated by Colombian university students when solving a problem involving the construction of frequency distribution tables), since it was show that university students mobilized a diverse network of mathematical connections when constructing frequency distribution tables, including procedural, different representations, part-whole, implication, and feature, revealing a structured understanding with appropriate procedures. This activation of connections is directly associated with mathematical practices such as data organization, range and amplitude calculations, interval construction, determination of absolute and relative frequencies, and interpretation of class marks. From the perspective of the OSA, this mobilization translates into a rich cognitive configuration, where the following were articulated: the problem situation, representations, definitions, propositions, procedures, and arguments, especially visible in students who developed both method 1 and method 2.

The positive results obtained highlight the importance of teacher training in statistics, especially the need to move beyond approaches focused solely on algorithmic and mechanical procedures. While these are necessary, they are insufficient to foster a deep and meaningful understanding. In this respect, it is essential that teacher training programs integrate teaching strategies that link statistical content to real-world contexts, enabling students to interpret, analyze, and make data-driven decisions. This approach promotes the development of critical statistical literacy, in which students not only apply methods but also understand their meaning, evaluate information, and actively participate in an increasingly data-driven society.

In terms of the understanding of statistical concepts, the results indicate that most students were situated at level 1 and level 2 of the models proposed by Curcio (1989) and Friel et al. (2001), which correspond to literal reading of data and the execution of simple internal operations. This suggests that, although students are able to identify values, organize information, and apply basic procedures, their understanding remains predominantly procedural and limited to the data explicitly presented. In this sense, rather than evidencing a fully structured understanding, the findings point to a partial or incipient structuring of statistical knowledge, supported by appropriate procedures but lacking deeper levels of reasoning. Only a small group (4.13%) reached level 3 (reading beyond the data), demonstrating the ability to make inferences or identify outliers, and no responses were observed at level 4 (critical reading). This highlights the need to strengthen instructional approaches that promote higher-order thinking, critical analysis, and reflection on the nature, origin, and limitations of data.

In addition, these findings are consistent with previous research conducted in primary education. For example, Gabucio et al. (2010) found that students were more comfortable with literal and structural tasks but weaker in global inferences. Similarly, Díaz-Levicoy et al. (2015) reported that Chilean textbooks are dominated by activities associated with level 1 and level 2, while deeper reading tasks are rare or nonexistent. In line with this, studies such as those by Arteaga et al. (2020) and Díaz-Levicoy et al. (2019) indicate that even when analyzing statistical graphs, students tend to focus on superficial descriptions without reaching complex critical or interpretive levels.

Given this situation, this study offers a significant contribution by showing that, even at the university level, difficulties similar to those documented at earlier school levels persist. However, the diversity of connections activated in the construction of frequency tables represents an advance over more traditional approaches, which focus solely on calculation or mechanical reproduction. The inclusion of the feature connection, in particular, makes it possible to visualize how students articulate different mathematical attributes such as dispersion, centrality, or representativeness to make sense of the data, something that had not been clearly highlighted in previous studies focused on elementary school levels.

Furthermore, this research reinforces the need to integrate frameworks such as ETC and OSA, which allow not only to classify types of connections but also to analyze the mathematical practices and semiotic processes that underpin the understanding and production of mathematical and statistical knowledge, as shown in this research. This approach is especially useful for identifying differences between students who follow prescribed procedures and those who demonstrate a more meaningful understanding by establishing relationships between mathematical objects, representations, and meanings.

The contrast between the two methods applied (6 vs. 7 classes) reveals different ways of reasoning statistically in the same situation. Students who incorporated an additional class to include the extreme value (93 kg) revealed greater sensitivity to data variability and a contextualized application of concepts, which can be interpreted as an incipient manifestation of level 3 understanding, as well as a more sophisticated connection of implication and different representations. It is worth noting that these results also articulate with the recent concerns raised by Pallauta and Batanero (2024), who have pointed out that many curricular materials tend to prioritize structural aspects of statistical tables without promoting deeper understanding processes regarding the critical interpretation of data. In this article, this limitation becomes visible when observing that no student reached level 4 reading, that is, the critical assessment of the origin and use of data. This absence points to a teaching focused on algorithms and not on the development of critical statistical skills, a problem also noted in the curricular analyses of Alsina and Vásquez (2024) and de Alencar and Díaz-Levicoy (2024).

On the other hand, although this article is situated at the university level, the difficulties observed are related to those reported in early school stages. For example, Arteaga et al. (2020) documented how primary education students face significant challenges when attempting to overcome basic levels of data reading, and in the textbook review carried out by Díaz-Levicoy et al. (2022), they identified a limited focus on the processing of statistical tables, characterized by activities focused on literal reading and a scarce presence of tasks that encourage analysis and inference. This trend is also reflected in our results, where interpretations focused on the description of frequencies and percentages predominated, without a solid connection to the context or critical analysis. Despite this, a valuable contribution of this research is showing how students managed to activate connections that are rarely reported at school levels, such as the feature connection, which involves recognizing relevant attributes of mathematical objects (e.g., the use of the class mark as a representative value).

The absence of level 4 (critical reading) can be further interpreted from a didactical and epistemic perspective. From the viewpoint of the OSA, the tasks implemented privileged procedural practices and institutionalized algorithms but did not sufficiently promote argumentative processes or critical reflection on data production, validity, or context. Consequently, the SFs activated by students remained primarily operational rather than evaluative. From the perspective of the ETC, this suggests a limited activation of meaning-based and critical connections, restricting students' ability to question data sources or interpret their broader implications. This highlights the need to design tasks that explicitly incorporate uncertainty, data critique, and contextual analysis to foster higher levels of statistical literacy.

CONCLUSION

The results of this research show that, although university students are able to construct frequency distribution tables and activate various mathematical connections such as procedural, different representations, part-whole, implication, and feature, their level of understanding is mostly concentrated in the first levels of data reading (literal and basic interpretation). Some students are known to reach level 3 (simple inferences), but no manifestations of level 4, associated with critical data analysis, were observed. These findings show that, even at advanced levels of education, statistics teaching remains marked by an algorithmic orientation, with little development of reflective statistical thinking. The integration of OSA and ETC allowed for a more in-depth description of the cognitive and semiotic processes involved in statistical activity, which represents a significant contribution both to research in mathematics education and to the design of more comprehensive training proposals. Among the study's main limitations are the sample size, which restricts the generalization of the results, as well as the fact that the data are based on a single task, which may limit the diversity of responses observed.

Future research could expand the sample, include different educational levels, and design tasks that explicitly encourage argumentation, critique, and reflection on the data, incorporating real-life and diverse contexts. It would also be relevant to explore how teacher training can strengthen mathematical connections in statistics teaching, especially with regard to understanding the characteristics and functions of tables. Finally, the research present delved deeper into the interaction between activated connections and levels of understanding, which would allow us to advance toward a more meaningful statistical education connected to students' realities considering other extra-mathematical problem situations highlighted in the measurement situations carried out, and it is also important for future research with everyday examples.

The lack of responses at level 4 (critical reading) can be explained, firstly, by the predominance of teaching practices focused on the application of procedures and algorithms, as evidenced in the activity, where students followed well-defined sequences to construct tables, but without questioning the origin, quality, or limitations of the data. Furthermore, the proposed tasks did not explicitly require critical argumentation or contextual analysis, which limits the development of this level of understanding.

Further on, based on this research, it will be possible to explore the didactic suitability of mathematical connections activated in statistics classrooms, with particular attention to the design and implementation of teaching sequences aimed at fostering advanced levels of statistical understanding, especially critical and reflective data reading. In this regard, it would be valuable to examine how the intentional activation of more complex connections such as reversibility, implication, multiple representations, and meaning supports students' progression in understanding frequency distribution tables. Expanding the scope of research to different educational levels (primary and secondary) and to teacher education contexts would also make it possible to analyze how the networking between the OSA and the ETC can inform pedagogical practices that go beyond a predominantly algorithmic focus. Finally, future studies should promote the design of comparative and context-based learning experiences that encourage reasoning beyond literal data reading, as well as the development of assessment tools that integrate both the identification of mathematical connections and levels of understanding, contributing to a more meaningful, contextualized, and critical statistics education.

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