

Using Dynamic Mathematics Software to Teach One-Variable Inequalities by the View of Semiotic Registers

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Using Dynamic Mathematics Software to Teach One-Variable Inequalities by the View of Semiotic Registers

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Solution set of any inequality or compound inequality, which has one-variable, lies in the real line which is one dimensional. So a difficulty appears when computer assisted graphical representation is intended to use for teaching these topics. Sketching a one-dimensional graph by using computer software is not a straightforward work. In this paper, an innovative approach is suggested by using GeoGebra, the free dynamic mathematics software. This approach was theoretically based on Sackur's suggestion, which is created by semiotic register theorem of Duval. The dynamic application, described in this paper, also provides an opportunity to observe more complex, even impossible to solve analytically, inequalities' solution sets visually. At the end of the paper, it is also explained that how this approach also has the potential of allowing the students to make detailed reasoning towards finding solutions of inequalities and compound inequalities.

Keywords: teaching inequalities, graphical representation, GeoGebra, compound inequalities, semiotic registers

INTRODUCTION

Graphical representation is rarely used by students while solving an inequality. Algebraic approach is valid and easy to apply for most of the routine inequality questions like $2 - 3x > x + 1$. On the other hand, trying to solve some inequalities like $3/x > 2+x$ same as fishing in a blurry lake. Students need to know and apply algebraic rules about inequalities. In this metaphor, Algebraic rules are represented by fishing tools and algebraic representation of the inequality is represented by the blurry lake. Sackur (2004) asked the question above to her students and she reported that all the students who used an algebraic method to solve it made the expected error. They multiplied by x whatever the sign of x could be, thus giving an incorrect answer

as $x \in (-3,1)$. Quite a few students used a graphical solution (Sackur, 2004). More generally, it was also determined that students live difficulties in relationship between graphical representation and algebraic representation of linear functions (Birgin, Gurbuz & Catiloglu, 2012). Besides, there are also explicit evidences that even pre-university students give no semantic content to the inequality and usually they find no conceptual differences between equation and inequality (Blanco & Garrote, 2007).

This study aimed to design and propose an innovative teaching environment of any inequality by focusing on graphical representation of the inequalities. Sackur applied the theorem of semiotic registers of Duval to the solving inequalities graphically (Sackur, 2004). She identified 4 registers as in the table 1.

Duval emphasizes the fact that comprehension in mathematics assumes the coordination of at least two registers (Duval, 2000). Sackur defined this coordination as conversion from I to III and from II to IV by simplifying the structure below (Sackur, 2004).

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State of the literature

- Algebraic approaches may not be sufficient to provide enough understanding on inequalities and different representations, especially visual ones, are needed.
- Graphical representations are rarely used by students.
- Dynamic mathematics software usually show the solution set of one-variable inequalities on the plane directly not on the line.
- Semiotic registers theorem says that a concept can be coordinated by simplified registers.

Contribution of this paper to the literature

- A one-variable inequality’s solution set needs to be visualized on the line not on the plane. This work defines how a one-variable inequality’s solution set can be visualized on the line by using Dynamic Mathematics Software.
- A one-variable inequality was coordinated in terms of Semiotic Registers. This approach provided us to understand more complicated one-variable inequalities also.
- The approach, defined in this paper, also is an example of using correct graphical representation of one-variable inequalities.

Learners can reach the ability of using multiple representations for reasoning thanks to technology (Demir, 2011). Sanchez and Garcia (2012) also declared that understanding also implies that the student has to be able to express the understood idea through different representations. Regarding the graphical representation of inequality, the dynamic mathematics software may be the better choice of technology. GeoGebra is one of the free dynamic mathematics software which is started to develop within a master’s thesis in computer science and mathematics education in 2002 and started to use all over the world after 2006 (Hohenwarter & Preiner, 2007). GeoGebra allows observing both graphical and algebraic representations on its multiple views. There exists considerable evidence, which shows GeoGebra is software that can be used simply and efficiently in all level of mathematics teaching by its friendly user interface (Edwards & Jones, 2006; Kabaca et.al, 2010). Besides, there are considerably enough evidences on positive effect of dynamic software in the literature (Ozgun Koca, 2004; Abramovic & Ehrlich 2007).

While using computer software to help students on observing graphical approach to a one-variable inequality, a difficulty is encountered. Any computer software, including GeoGebra, sketches two-dimensional graphs directly. If we want to sketch a one-dimensional graph representing the solution set of a one-variable inequality, an indirect procedure needs to be proceeded. Yushau used MathCad to sketch one-dimensional graphs of some specific inequalities and stated that he applied a trick to create the application (Yushau, 2004).

The learning environment, described in this study, emphasizes making students to consider on the registers of solution for the inequalities (Table 1) and proposes the use of dynamic software accordingly. First, the solution set of a basic inequality is obtained. Then the application is improved to obtain any inequality.

Sketching the Solution Set of One Variable Inequality

Although a one variable inequality’s solution set is a one dimensional graph, GeoGebra provides a two dimensional drawing platform as in every mathematics software. Any one variable function can be represented on two dimensional Euclidean space easily. This opportunity was used by the idea of Sackur’s register suggestion. First, the case of basic inequalities was handled and the visualization of semiotic registers was described. Second, the dynamic application was converted to more functional version. Third, the application was improved as finding the solution set of compound inequalities also. Last, it was also illustrated that how the solution set of more complex inequalities can be visualized.

The Case of Basic Inequalities

To obtain the graph of solution set of $2 - 3x > x + 1$ follow the following steps;

(a) Write the expressions $2 - 3x$ and $x + 1$ in the input bar to obtain the graph of functions $f(x) = 2 - 3x$ and $g(x) = x + 1$ respectively. Simply write the expressions on input bar and execute. You will obtain the window as in figure 1.

Remark 1: Your own graphs will be black. You can edit the colours by using “object properties” dialog box, which is opened by right click. This editing makes easier to see the algebraic and graphical representations from algebra view and graphical view.

Table 1. Registers of graphical solution for inequalities

I	II	III	IV
Algebraic	<i>Functional</i>	Graphical bi-dimensional	Graphical mono-dimensional
$3/x > 2+x$	$f(x) = 3/x$ $g(x) = 2 + x$	$y = 3/x$ $y = 2+x$	$x \in [...]$

(b) Ask yourself or your students “for which values of x , $f(x)$ is greater than $g(x)$?” you should get the answer “We need to determine the region, where $f(x)$ line is above the $g(x)$ line, on the x -axis.”

Now, we need to create a point which can be moved on x -axis. First create a slider- t changing in the interval which has enough size like $[-20, 20]$ (see figure 2). Then, create a point $(t, 0)$.

(c) You can rename this point as “solution”. We will use this point to produce the solution set. Make the solution point trace on and let the point makes a trace while animating.

(d) At this time, the solution point’s trace covers whole x -axis. We need to edit this point’s condition to be shown (see figure 3) as $f(t) > g(t)$ where t is the x -component of the point solution. This means “show the point when $f(t) > g(t)$ ”

(e) After defining the condition to show the point solution, animate the slider- t . You will see the graph of solution set on x -axis (see figure 4).

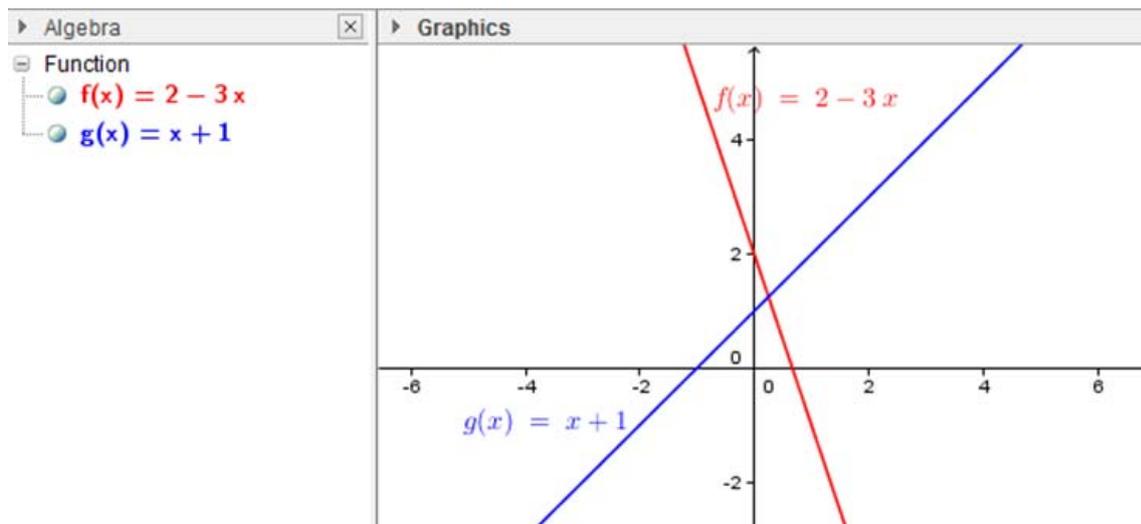


Figure 1. Graphs of the functions constituting the inequality

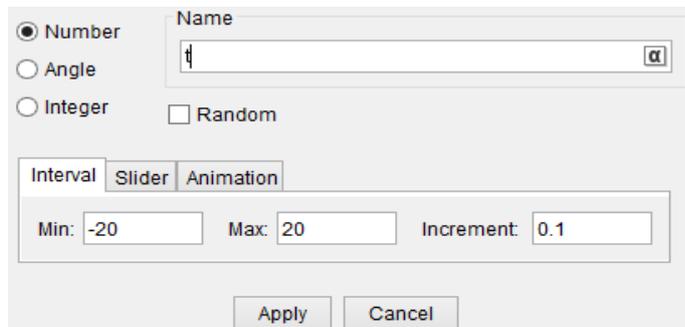


Figure 2. GeoGebra dialog box to create a slider

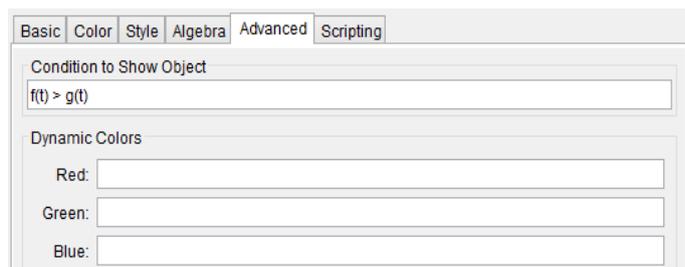


Figure 3. GeoGebra dialog box for editing the “condition to show object” of solution point

Even if the slider-t is animated under its complete interval, the solution point will trace the interval as satisfying the inequality. This is the real solution rather than filling the whole area in the plane and allows students to make reasoning on the condition. It is also enough to change the condition to obtain the graph of the inequality $2 - 3x < x + 1$.

Improving the Application

The above application is valid to find any inequality's solution set. On the other hand, it is possible to make this application more functional. Let's find a solution to avoid re-editing the condition to show the point solution when direction of inequality is wanted to be changed by following steps;

(a) Construct a Boolean Values with name of a (a check box). Then construct two dynamic texts as $(f > g)$ and $(f < g)$ by inserting a text. The names f and g, which represent the functions, must be inserted by selecting from objects box, otherwise they will appear just as ordinary text.

(b) Make the dynamic text $(f > g)$ is shown when the Boolean value a is true (the check box is checked)

and the dynamic text $(f < g)$ is shown when the Boolean value a is false (the check box is unchecked). This will help us to control the direction of inequality.

(c) Now the condition to show the point solution should be redefined as " $f(t) > g(t) \wedge a = \text{true}$ " which means show the point when the inequality $f(x) > g(x)$ is active and the Boolean value a is true. By this way when you click on check box, the point solution will appear for the condition of $f(x) > g(x)$

(d) Last, create a second solution point $(t, 0)$ and define its condition to be shown as " $f(t) < g(t) \wedge a = \text{false}$ ". So, it is enough to unclick the check box to show the graph of the inequality $f(x) < g(x)$.

So far, an application that allows every inequality's graph can be drawn was created. Some of the examples were given below.

Remark 2: Once you animated the slider-t a play-pause button symbol will appear at the left bottom of the graphics view. You can play or pause the animation by using that symbol. After playing the animation you must wait until the slider-t completes its cycle. You should press Ctrl+F to clean trace of the point that draws the solution set before every new question.

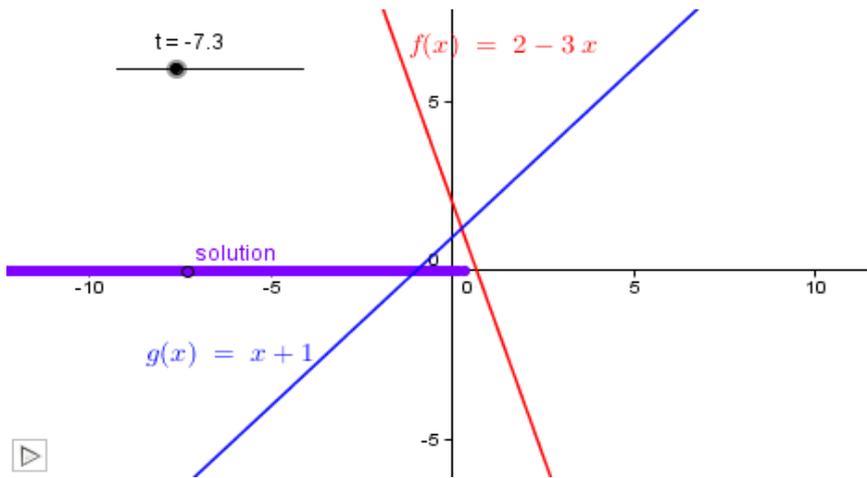


Figure 4. Trace of the solution point $(t, 0)$ indicating the exact solution set on the x-axis

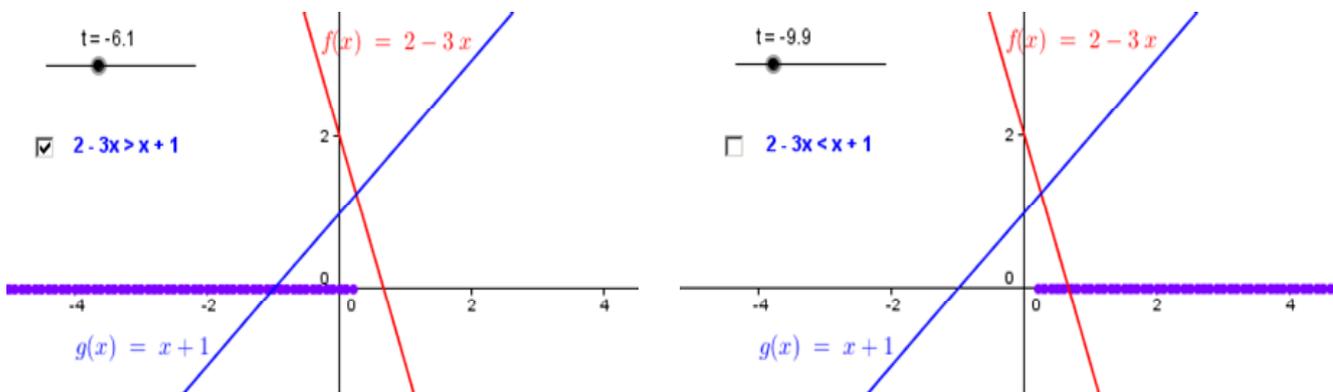


Figure 5. Two alternative solutions for two different direction of inequality

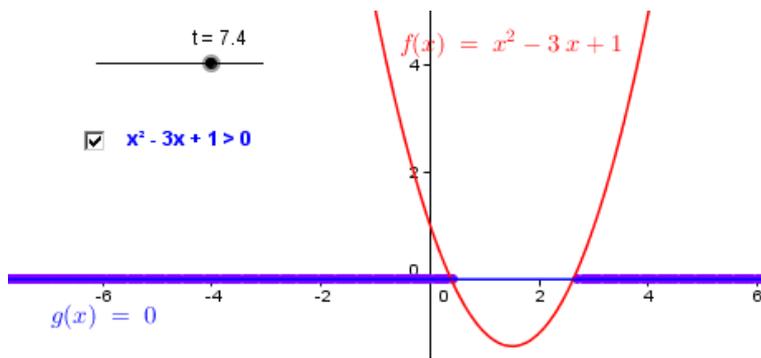


Figure 6. Graphical solution for the Example-1

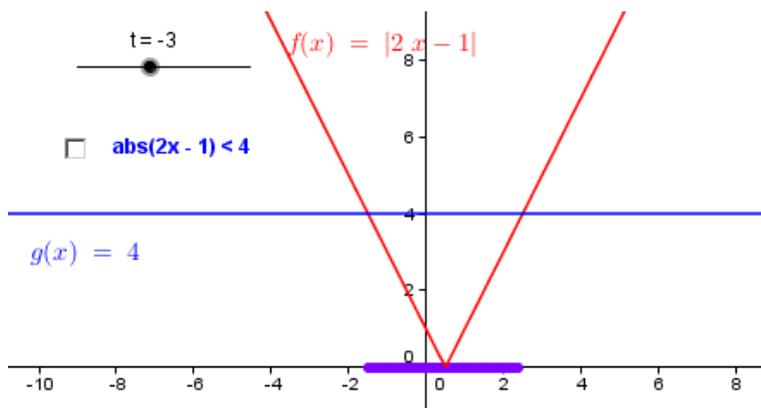


Figure 7. Graphical solution for the Example-2

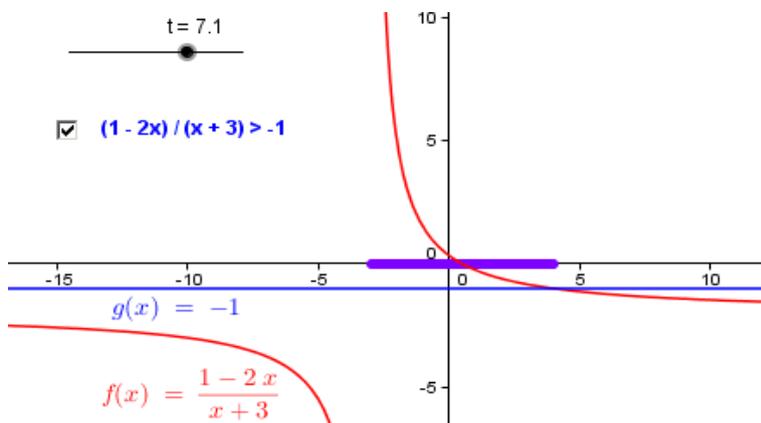


Figure 8. Graphical solution for the Example-3

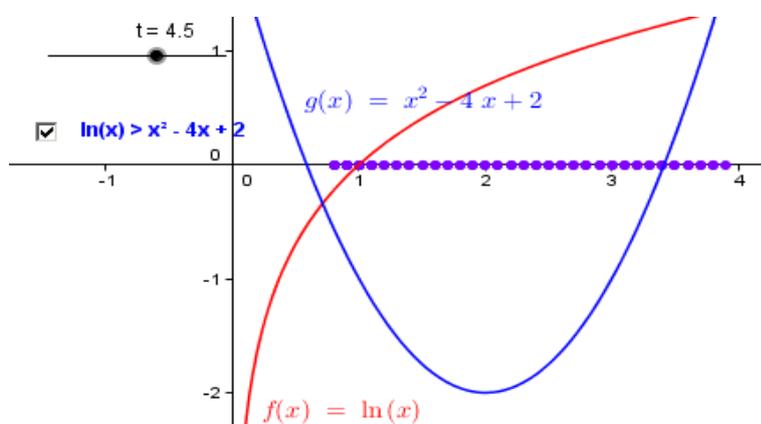


Figure 9. Graphical solution for the Example-4

Examples

Let's examine graphical solution of different inequality situations. The functions can be redefined by double clicking and writing its new definition simply.

Example 1: Find the solution set of $x^2 - 3x + 1 > 0$
 $f(x) = x^2 - 3x + 1$ and $g(x) = 0$ (see figure 6)

Example 2: Find the solution set of $|2x - 1| f(x) = |2x - 1|$ and $g(x) = 4$ (see figure 7)

Example 3: Find the solution set of $(1-2x)/(x+3) f(x) = (1-2x)/(x+3)$ and $g(x) = -1$ (see figure 8)

Example 4: Find the solution set of $\ln(x) > (x - 2)^2 - 2$
 $f(x) = \ln(x)$ and $g(x) = (x - 2)^2 - 2$ (see figure 9)

Remark 3: If a solution set could not be found on GeoGebra graphical view, there may be two reasons. The solution set is empty or the solution set is not in the interval of the slider-t. In this case, you should enlarge the interval.

THE CASE OF COMPOUND INEQUALITIES

In this case we have at least one more inequality. Construct two functions $h(x)$ and $k(x)$ as both sides of the new inequality and use a new Boolean value to control $h(x) > k(x)$ or $h(x) < k(x)$ as defined the previous sections.

Two inequalities may be compounded by one of the two logical operators which are "and" and "or". To determine that which operator compounds the inequalities construct a new Boolean value, named b and two texts as "and" and "or". Make these two texts to be shown when the Boolean value b is true and false respectively.

We have two inequalities and these inequalities may be compounded by one out of two logical operators. So, there exist 8 possibilities for the solution set. In this case, we need 8 points to produce the solution set with

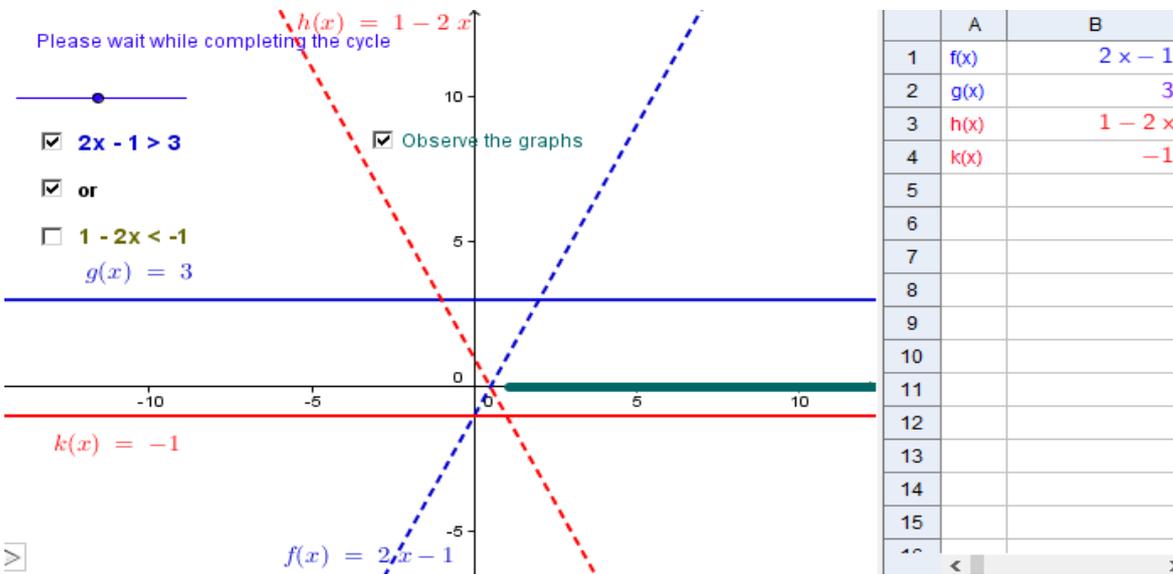


Figure 10. Examining the solution of an inequality system

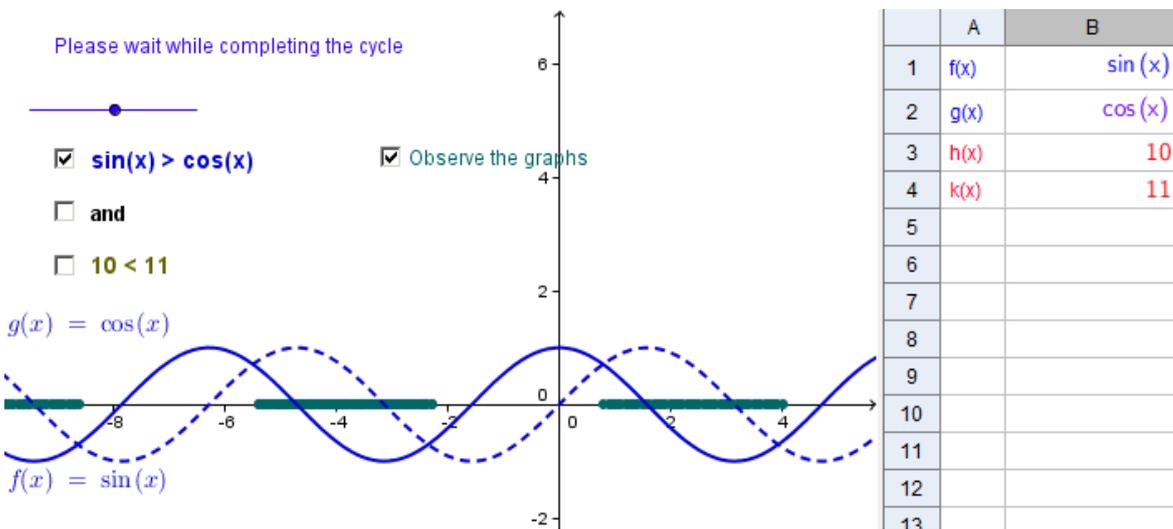


Figure 11. Solving a trigonometric inequality

the trace of these points. Since we have two points previously, it is enough to construct 6 new points (t, 0). Rename these points as solution1, solution2, . . . , solution 8.

Conditions to be shown of every solution point (t, 0), which plots the solution set by its trace, must be defined as in the table 2. Here a, b and c are the names of Boolean values where a controls the first inequality's direction as > for "true" and < for false, b controls the second inequality's direction in the same way and c controls the logical operators and for false and or for true (See table 2).

So, an integrated application, solving any compound inequality with two inequalities, was produced. Functions may be associated by spreadsheet view to make easier to change. In the sample screen view in figure 10, solution set of the inequality system of $2x - 1 > 3$ and $1 - 2x < -1$ can be seen. A check box is also added to hide or show the graphs. By this way user can show the graphs of functions representing the left and right hand sides of the inequalities. By this way, users make a reasoning why that region is the solution or this

reasoning can be made before finding the solution, and then the reasoning can be checked.

This system allows user to solve any inequality and making graphical reasoning. Let's think on the inequality $\sin(x) = \cos(x)$. To examine the example by this application, it is just enough to make second inequality as true for all real numbers and choosing the conjunction as "and" (see figure 11).

Users can observe the solution set is the region where $\sin(x)$ curve is under the $\cos(x)$ curve. This application can be considered as a program, solving the inequalities numerically and allowing users to observe the result graphically. Previous example can also be solved algebraically and the solution set can be stated as $(\pi/4 + 2k\pi, 5\pi/4 + 2k\pi)$ where k is any integer. Even an inequality, which is impossible or hard to solve algebraically, can be examined.

Consider on the inequality $\sin(x) < \ln(x)$. Although this inequality is impossible to solve by an analytical way, GeoGebra can present the solution as $x \cong 2.2$ by the suggested application on GeoGebra worksheet (see figure 12).

Table 2. Editing the conditions of the solution points to be shown

Solution point	Inequality system	Condition to show the point
solution1	$f(x) > g(x)$ and $h(x) > k(x)$	$f(t) > g(t) \wedge h(t) > k(t) \wedge a = \text{true} \wedge b = \text{true} \wedge c = \text{false}$
solution2	$f(x) > g(x)$ and $h(x) < k(x)$	$f(t) > g(t) \wedge h(t) < k(t) \wedge a = \text{true} \wedge b = \text{false} \wedge c = \text{false}$
solution3	$f(x) < g(x)$ and $h(x) > k(x)$	$f(t) < g(t) \wedge h(t) > k(t) \wedge a = \text{false} \wedge b = \text{true} \wedge c = \text{false}$
solution4	$f(x) < g(x)$ and $h(x) < k(x)$	$f(t) < g(t) \wedge h(t) < k(t) \wedge a = \text{false} \wedge b = \text{false} \wedge c = \text{false}$
solution5	$f(x) > g(x)$ or $h(x) > k(x)$	$(f(t) > g(t) \vee h(t) > k(t)) \wedge a = \text{true} \wedge b = \text{true} \wedge c = \text{true}$
solution6	$f(x) > g(x)$ or $h(x) < k(x)$	$(f(t) > g(t) \vee h(t) < k(t)) \wedge a = \text{true} \wedge b = \text{false} \wedge c = \text{true}$
solution7	$f(x) < g(x)$ or $h(x) > k(x)$	$(f(t) < g(t) \vee h(t) > k(t)) \wedge a = \text{false} \wedge b = \text{true} \wedge c = \text{true}$
solution8	$f(x) < g(x)$ or $h(x) < k(x)$	$(f(t) < g(t) \vee h(t) < k(t)) \wedge a = \text{false} \wedge b = \text{false} \wedge c = \text{true}$

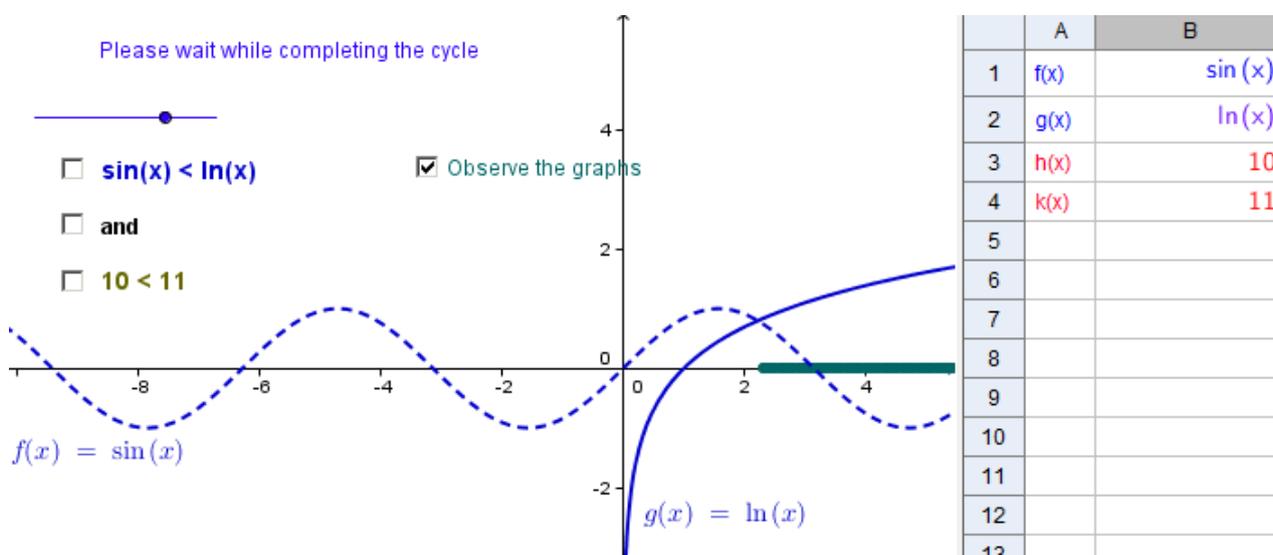


Figure 12. Examining the solution by graphical approach

DISCUSSION AND FURTHER CONSIDERATIONS

In this study, the process of creating a dynamic worksheet for solving two compound inequalities was described. This dynamic worksheet may be useful in mathematics teaching and learning environments in two main ways. First and basic way is using the worksheet as an integrated application for solving inequalities and the second is focusing on its procedure of construction.

Let's start by describing the second one. By a holistic approach in this worksheet, the x-axis is being scanned in an interval by checking for which x-values $f(x)$ curve is under (upper) the $g(x)$ curve for the inequality of $f(x) < (>) g(x)$. To implement this approach for solving the inequality, the algebraic form of the inequality is converted from its algebraic form into a functional form consists of two separate functions. Then, one-dimensional solution graph on the x-axis of the inequality found from the two-dimensional graph of these functions. Two conversions can be observed in the construction process of the worksheet. By the view of semiotic registers theorem, the comprehension occurs by conversion between at least two registers (Duval, 2000). By regarding the algebraic form and functional form of the inequality, bi-dimensional graph and one-dimensional graph as four different registers (Sackur, 2004), the construction process of suggested worksheet presents an environment for comprehension.

Last version of the worksheet provides a possibility to solve compound inequalities by simply writing and choosing some check boxes like Yushau's suggestion created by using MathCad (Yushau, 2004). On the other hand, GeoGebra is free and any worksheet can be exported as an html file. So any computer can run this application without installing GeoGebra. While teaching the inequalities students may start to study on this GeoGebra worksheet. Then, they may be asked "estimate the reason for scanning a part of x-axis". After enough reasoning, students can reach the result of "necessity for finding the solution of the equation of $f(x) = g(x)$ ". This is more meaningful than saying "assume the lower or greater signs as equal sign". By this approach, students can also reach the result of "solution set can be found at least approximately, even if solving the equation $f(x) = g(x)$ is difficult or impossible to solve analytically".

Consequently, the dynamic application described in this paper is a good example of how dynamic mathematics software can be worthwhile tool with a suitable theoretical basis and can help students to understand the relationship between graphical and algebraic representation of linear functions (Birgin, Gurbuz & Catlioglu, 2012). Furthermore, the learning environment has the potential of making students to explore the reason of finding solution set of an equation

$f(x) = g(x)$, which is obtained by assuming the inequality $f(x) = g(x)$ as an equation, unlike just telling them to assume. This opportunity has the potential of understanding the difference of inequality and the equation as expediency for the Blanco and Garrote's finding (2007). At last, regarding to make learners to use multiple representation in technology enriched environment (Demir, 2011) the positive effects of the usage of dynamic software in teaching mathematics (Ozgun Koca, 2004; Abramovic & Ehrlich 2007) may be increased. Furthermore, even if a student able to understand how to find a solution set of a one variable inequality, he/she needs to express the idea by different representations (Sanchez & Garcia, 2012) this paper suggested an innovative graphical approach which also help to develop extended understanding for further grades.

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