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Using flowcharts to support students' mathematics problem-solving: Teachers' planning, monitoring, and evaluation strategies

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Abstract

This paper offers an autoethnographic account of mathematics teachers' use of flowcharts to support students' problem-solving. Over two school terms, two mathematics educators participated in fortnightly sessions to analyze surveys and interview data on flowchart integration collected from eight mathematics teachers. Using analytical autoethnography, the study explored the teachers' planning, monitoring, and evaluation strategies. The autoethnography reflective sessions incorporated member checking, professional practice considerations, literature, and curriculum documents. The paper highlights strategies for facilitating, tracking, and assessing problem-solving and concludes with insights and implications for mathematics teaching and learning.

Keywords: problem-solving, metacognition, planning, monitoring, evaluation

INTRODUCTION

Metacognitive strategies employed by teachers in planning, monitoring, and evaluating students' mathematics problem-solving are fundamental to effective mathematics education (Hornby & Greaves, 2022). Flavell (1979) defined metacognition as an individual's awareness and regulation of their own cognitive processes commonly described as thinking about thinking. Flavell (1979) proposed that metacognitive abilities can be consciously or automatically applied to guide cognitive activity before, during, and after problem-solving. These abilities encompass self-awareness, learning regulation, and cognitive organization (Patterson, 2011), and are recognized as teachable and essential 21st century skills for success in mathematics (Mevarech & Kramarski, 2014; Schoenfeld, 2013; Veenman & Elshout, 1999).

However, fostering students' metacognitive skills in mathematics requires teachers to first cultivate their own metacognitive competencies in instructional practice (Boaler et al., 2024). This is critical, as teachers' metacognitive practices enable them to identify and address barriers in students' problem-solving processes (NSW Department of Education, 2020). Despite its importance, there is a notable gap in the literature regarding practical integration of metacognitive strategies in mathematics classrooms (Boaler et al., 2024; Schneider & Artelt, 2010). Moreover, research indicates that many teachers overlook metacognitive approaches due to limited understanding of the concept (Dignath & Büttner, 2018). To address this, educators require stratagems with targeted support and professional development to effectively promote metacognitive strategies during teaching and learning (Depaepe et al., 2010; Dignath & Büttner, 2018). Teodorescu (2014) defines stratagems as structured and deliberately designed tactical approaches aimed at achieving specific instructional goals to enhance learning outcomes.

In classrooms, metacognition is enhanced when teachers scaffold and model the cognitive processes and then provide students with opportunities to practice, reflect and gain experience (Hornby & Greaves, 2022). A metacognitive stratagem to teaching practice is "about embedding a self-reflective style of thinking that enables students to be aware of and ultimately create learning goals, plan ways to achieve those goals, strategize how to deal with setbacks, and monitor their learning progress" (NSW Department of Education, 2020, p 1). Teachers are expected to monitor students' performances and create an environment where students can monitor each other (Boaler et al., 2024). Patterson (2011) argues that metacognition is central to mathematics problem-solving and is a higher-order

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Contribution to the literature

- Metacognition is central to mathematics problem-solving and is a higher-order cognitive process requiring both creative and critical thinking strategies and skills.
- The study explores mathematics teachers' metacognitive regulation strategies in the context of using flowcharts to support students' problem-solving in mathematics.
- There is a notable gap in the literature regarding practical integration of metacognitive strategies in mathematics classrooms.

cognitive process requiring both creative and critical thinking strategies and skills. Similarly, Schneider and Artelt (2010) argue that metacognition is central to mathematics problem-solving especially when students are exploring ways to solve the problem and determining the extent to which the proposed solution addresses the problem.

Contemporary research conceptualizes metacognition as comprising two interrelated components: metacognitive knowledge and metacognitive regulation. Metacognitive knowledge includes declarative (knowledge of facts), procedural (knowledge of how to perform tasks), and conditional (knowledge of when and why to apply strategies) dimensions. Metacognitive regulation refers to the self-directed management of cognitive processes through planning, monitoring, and evaluating one's practice (Hornby & Greaves, 2022; Mitsea & Drigas, 2019). This study examines mathematics teachers' metacognitive regulation strategies in the context of using flowcharts to support students' problem-solving in mathematics. Two mathematics educators analyzed data collected from eight high school mathematics teachers, aimed at exploring how flowcharts can be effectively integrated into instructional planning to enhance students' mathematical problem-solving capabilities.

Metacognitive Regulation

Metacognition, encompassing the organization of knowledge and strategic thinking, is fundamental to effectively planning, executing, and evaluating mathematics problem-solving instruction (Magno, 2010; Mevarech & Kramarski, 2014; Mitsea & Drigas, 2019; Soedjoko et al., 2019). According to Schraw (1998), metacognitive knowledge includes understanding oneself as a learner, being aware of various learning strategies, and knowing when and why to apply them. Metacognitive regulation, on the other hand, involves the ability to plan, monitor, and evaluate one's learning processes. Mitsea and Drigas (2019) similarly describe metacognitive regulation as a form of cognitive self-management through reflective self-appraisal, encompassing planning, monitoring, and evaluation. These components align with the three core phases of teaching mathematics problem-solving. Focusing on teachers' metacognitive regulation strategies during lesson planning, instructional delivery, and post-instruction reflection can significantly enhance the effectiveness of teaching and learning in mathematics problem-solving.

Mathematics teachers not only plan for student engagement in problem-solving but also support students in developing their own problem-solving strategies (Baumanns & Rott, 2022; Du Toit & Kotze, 2009). Planning involves organising a course of action, directing cognitive processes, and self-regulating learning (Mitsea & Drigas, 2019). In the classroom, this may include clarifying learning objectives, anticipating problem types, posing guiding questions, exploring content connections, and outlining general problem-solving procedures (Boaler et al., 2024). Effective problem-solving requires students to understand, analyze, and explore the problem before identifying flexible solution pathways (Goos et al., 2000). Goos et al. (2000) suggest that students can enhance their planning by rephrasing problems, identifying key terms and concepts, and outlining solution stages. Teachers can facilitate these processes through targeted instructional planning.

For students, metacognitive processes similarly involve deliberate reflection on how to approach a task (Boaler et al., 2024). Higher levels of metacognition are evident when students engage with tasks that challenge their problem-solving abilities (Du Toit & Du Toit, 2013), often leading them to experiment with multiple solution strategies. Thus, metacognitive regulation during planning may include revisiting the problem, thinking aloud, using diagrams to visualize the problem, and decomposing it into manageable parts (Boaler et al., 2024). Tools such as graphic organizers can support students in structuring their problem-solving plans (NSW Department of Education, 2020). This planning phase is typically followed by the monitoring stage, where students track and adjust their problem-solving approach.

Mathematics teachers organize knowledge and strategies to monitor students' progress in problem-solving and to support students in developing their own monitoring skills (Baumanns & Rott, 2022; Du Toit & Kotze, 2009). Monitoring involves implementing a planned approach and checking for errors or inconsistencies throughout the process (Mitsea & Drigas, 2019; Schraw, 1998). It provides direction toward achieving desired outcomes and helps students stay on track (Boaler et al., 2024). Boaler et al. (2024) emphasize that effective monitoring includes tracking progress and offering feedback to guide students toward success.

In mathematics instruction, monitoring should help students establish clear connections between the problem and the proposed solution. Tools such as graphic organizers or rubrics can clarify expectations and support students in assessing their progress. Thus, teachers play a critical role in guiding students to examine the mathematical relationships between concepts and procedures, evaluate the logic and coherence of their solutions, and determine whether their plans have been correctly implemented. Monitoring also enables teachers to identify students who are struggling and may require remediation or redirection (Goos et al., 2000). For students, monitoring can be understood as the process of assessing and evaluating their own learning, as described by Nelson and Narens (cited in Rhodes, 2019). Through intentional use of metacognitive strategies, students can acquire, share, and apply skills that enhance their mathematical thinking and performance (Radmehr & Drake, 2019; Veenman & Elshout, 1999). This phase naturally leads into the evaluation stage of problem-solving.

Mathematics teachers organize knowledge and strategies to evaluate students' progress in problem-solving and to guide students in evaluating their own problem-solving processes (Baumanns & Rott, 2022; Du Toit & Kotze, 2009). Evaluation involves verifying the outcome and assessing its reasonableness in relation to the problem and the process used to solve it (Pugalee, 2001; Queensland Curriculum and Assessment Authority [QCAA], 2018). Goos et al. (2000) highlight that evaluation includes determining whether different strategies yield consistent solutions or lead to reasonable conclusions. For both teachers and students, evaluation is about judging whether the solution effectively addresses the problem.

Teachers' metacognitive regulation strategies are essential in planning and implementing effective evaluation practices that enhance student learning. Comprehensive evaluation requires the application of higher-order metacognitive skills and decisions before, during, and after the learning process (Mitsea & Drigas, 2019). Patterson (2011) argues that problem-solving lies at the heart of metacognition, involving both creative and critical thinking skills that depend on the ability to organize one's thoughts. These metacognitive abilities can be explicitly taught, acquired, and refined. As such, both teachers and students can develop and apply these strategies to improve the teaching and learning of problem-solving in mathematics classrooms.

Problem-Solving

Enhancing problem-solving in mathematics education has long been a priority, yet research continues to grapple with articulating effective planning and instructional strategies for classroom practice (English & Gainsburg, 2016; Güner & Erbay, 2021). Effective problem-solving instruction requires that teachers possess the capacity to plan for student engagement, implement teaching strategies logically, and reflect critically on the outcomes (van der Stel & Veenman, 2014). Despite its importance, developing teacher knowledge, resources, and pedagogical strategies to support student problem-solving remains a significant challenge, underscoring the urgent need for clear, evidence-based guidelines (Ahmad et al., 2010; Güner & Erbay, 2021; Schoenfeld, 2016).

Central to this challenge is the need for teachers to apply metacognitive regulation strategies: planning, monitoring, and evaluating to support students effectively. However, many teachers struggle to identify which strategies to use, how to adapt them during instruction, and how to integrate them meaningfully into problem-solving contexts (Goos et al., 2000; Zawojewski et al., 2013). Research consistently shows that teacher strategies play a pivotal role in fostering student problem-solving capabilities (Schoenfeld, 2016). Therefore, greater emphasis must be placed on how teachers manage and apply their own metacognitive regulation strategies throughout the teaching process to enhance students' metacognitive regulation strategies and learning outcomes.

Problem-solving is a central and essential component of mathematics and mathematics education, reflected in curricula worldwide and prompting widespread advocacy for teaching mathematics through problem-solving approaches (Liljedahl et al., 2016; QCAA, 2018). Despite its prominence, there is no universally accepted definition of problem-solving (English & Gainsburg, 2016; Güner & Erbay, 2021). Some view problems as tasks requiring the application of new mathematical knowledge and techniques, while others define them as complex challenges that present obstacles to the solver (Schoenfeld, 2013).

Although Polya's (1971) problem-solving steps and strategies remain influential, their practical application in classroom settings has yielded limited success (English & Gainsburg, 2016; Mamona-Downs & Downs, 2013). This may partly explain why efforts by mathematics teachers to improve students' problem-solving skills have not consistently achieved the desired outcomes (Anderson, 2014; English & Gainsburg, 2016). A widely accepted perspective is that problem-solving involves a dynamic interaction between the solver's prior knowledge, a tentative solution plan, and relevant contextual information (Schoenfeld, 2016).

Effective problem-solving thus requires both mathematical knowledge and the integration of cognitive and metacognitive abilities. Cognitive skills support task comprehension and strategy application, while metacognitive skills enable regulation of the problem-solving process and informed decision-making (Goos et al., 2000; Güner & Erbay, 2021). Focusing on the metacognitive strategies essential for teaching and learning problem-solving can deepen understanding and offer valuable directions for future research.

Although widely endorsed in mathematics education, problem-solving presents persistent challenges for many teachers, particularly in helping students integrate mathematical concepts and processes into coherent, meaningful solutions (Artigue et al., 2020; Hacker, 1998). Effective problem-solving requires the organization of knowledge throughout the development and communication of a solution, which is closely tied to metacognitive regulation strategies employed by both teachers and students.

Students must engage in a structured process: understanding the problem, planning a solution, executing the plan, and reflecting on both the outcome and the process (Polya, 1971). This can also be framed through four instructional phases: discover, devise, develop, and defend, which guide students in constructing and justifying their solutions (Makar, 2012). To support this, teachers must cultivate their own metacognitive abilities to effectively guide students through each phase of problem-solving.

English and Gainsburg (2016) highlight the need to address how problem-solving competencies can be supported in classroom settings. Despite ongoing research in mathematics problem-solving, much of it remains theoretical and lacks practical application in teaching contexts (Dorier & Maass, 2020; Lester, 2013). Further investigation is needed to identify and develop metacognitive strategies that teachers can use to enhance students' problem-solving competencies in mathematics.

Enhancing teachers' metacognitive regulation strategies is crucial for effectively supporting students' metacognitive regulation strategies during mathematics problem-solving. Throughout the problem-solving process, teachers play a key role in guiding students to develop solution strategies, such as drawing diagrams, performing calculations, identifying relationships, and formulating conclusions, as well as interpreting, evaluating, and communicating their solutions (Artigue et al., 2020; Dorier & Maass, 2020). A teacher's ability to regulate their own metacognitive processes significantly influences their capacity to foster students' metacognitive regulation strategies and problem-solving skills.

This underscores the importance of focusing on teachers' metacognitive abilities. By applying metacognitive regulation strategies: planning, monitoring, and evaluating, teachers can actively promote students' metacognitive development and enhance their problem-solving competencies. In practice, developing students' problem-solving abilities begins with teachers' strategic planning, thoughtful execution of instructional approaches, and reflective evaluation of teaching effectiveness. Tools such as flowcharts can be particularly useful in supporting students' understanding and reasoning during mathematical problem-solving by visually organising steps and relationships within the problem. This aligns with the Australian curriculum: Mathematics 9.0, which incorporates tools like flowcharts to help students analyze and construct algorithms, and promote deeper engagement and understanding.

Using Flowcharts

Graphic organizers, such as flowcharts, are among the most effective tools for enhancing metacognitive awareness and regulation in learning (Aprilisa, 2019; Leny et al., 2020). The process of creating flowcharts inherently involves metacognitive activity, as learners must reflect on their understanding, organize information, and plan solution pathways, thereby fostering metacognitive learning (Huang & Tsapali, 2022). Flowcharts serve as visual representations that deconstruct complex information and procedures into manageable components, while also illustrating the relationships between these components (Chinofunga et al., 2024; Grosskinsky et al., 2019).

As diagrammatic sequences of strategies or steps, flowcharts have long been used across disciplines to support problem-solving, including in fields such as robotics and programming (Carlisle et al., 2005; Hooshyar et al., 2018). They help learners visualize procedural steps and anticipate how strategies will be applied (Chinofunga et al., 2024; Ledin & Machin, 2020; Reingewertz, 2013). In educational contexts, Norton et al. (2007) advocate for the active use and scaffolding of flowchart-based planning to help students recognize their potential in enhancing problem-solving capabilities. The effectiveness of flowcharts in problem-solving is largely attributed to their ability to promote deep cognitive engagement during the planning phase. As Jonassen (2012) notes, flowcharts can serve as mental models that represent a proposed approach to solving a task, thereby supporting both strategic thinking and reflective decision-making.

Creating flowcharts during problem-solving enhances students' ability to understand the problem, think critically, make sense of the task, explore solution pathways, and communicate their reasoning effectively (Leny et al., 2020; Norton et al., 2007). A key aspect of successful problem-solving is selecting the most appropriate strategy and making timely, informed decisions. One of the major advantages of using flowcharts is their capacity to visually represent complex processes, which supports comprehension of procedural flow, identification of inefficiencies or irrelevant steps, and ultimately leads to improved decision-making and problem resolution (Huang & Tsapali, 2022; McGowan & Boscia, 2016).

Teaching students to use flowcharts as part of their problem-solving toolkit helps them recognize new relationships between procedures, evaluate their solutions, and clearly articulate their thinking (Vale & Barbosa, 2018). Flowcharts also serve as a two-way communication tool between teacher and student or among student peers, facilitating collaborative learning and feedback (Aprilisa, 2019; Grosskinsky et al., 2019). Teachers can use flow charts to monitor students' progress, assess their understanding, and provide targeted guidance. Additionally, teachers can model the use of flowcharts to illustrate key processes and strategies they employ when planning, delivering, and reflecting on problem-solving instruction.

METHOD

This study employed an autoethnographic approach to investigate mathematics teachers' planning, monitoring, and evaluation strategies using flowcharts to support students' problem-solving. Two mathematics educators participated in fortnightly autoethnographic sessions to analyze data collected from eight high school mathematics teachers. Auto-ethnography, as defined by Adams et al. (2017), is a qualitative research method that draws on personal experience ("auto") to describe and interpret ("graphy") cultural practices, beliefs, and experiences ("ethno"). Chang et al. (2016) note that while some auto-ethnographers focus primarily on personal narrative, others adopt a more analytical stance aimed at interpreting broader educational or cultural phenomena.

This investigation followed an analytical autoethnographic approach that involved the mathematics teacher educator, the mathematics teacher and member checking with eight mathematics teachers. The study was guided by the central research question: What planning, monitoring, and evaluation strategies are required to support student problem-solving through the use of flowcharts? Across two school terms, the mathematics educators convened fortnightly for one-hour sessions to analyze survey and semi-structured interview data collected from eight high school mathematics teachers who investigated the use of flowcharts in supporting students' mathematical problem-solving. Employing an analytical autoethnographic approach, the inquiry examined teachers' planning, monitoring, and evaluation strategies associated with flowchart integration. These sessions functioned as retrospective reflections on the data sets, incorporating member checking, professional practice considerations, relevant literature, and curriculum documentation (Adams et al., 2017).

The autoethnographic analysis was structured around a three-phase metacognitive regulation framework (Mitsea & Drigas, 2019). Phase one focused on planning strategies to support student problem-solving through the use of flowcharts. Phase two examined monitoring strategies and how they facilitated students' progress in problem-solving. Phase three explored evaluation strategies that helped assess and enhance students' problem-solving outcomes. The following section presents a summary of the three-phase analysis and the educators' reflective insights.

REFLECTION AND ANALYSIS

This section presents the retrospective reflection and analysis of planning, monitoring, and evaluation strategies using flowcharts to support students' problem-solving. Our central position is that visually deconstructing these strategies into discrete, interconnected components through flowcharts is essential for clarity and effectiveness, as emphasized by Grosskinsky et al. (2019).

We propose the use of two distinct types of graphic organizers: one focused on teaching and the other on learning. These are referred to as teaching stratagems and learning stratagems, respectively. As previously noted, stratagems refer to carefully structured and intentional tactics designed to achieve instructional goals and enhance learning outcomes (Teodorescu, 2014).

> **Teaching stratagems** are flowcharts designed to visually represent the teacher's planning, monitoring, and evaluation strategies when facilitating problem-solving. They also incorporate differentiated strategies to support and remediate student learning.
>
> **Learning stratagems** are student-focused flowcharts that break down the steps students follow in planning, monitoring, and evaluating their own problem-solving processes. These stratagems can also serve as a guide for establishing success criteria.

Importantly, students can benefit from access to both types of stratagems. Teaching stratagems can scaffold student learning and peer collaboration, while learning stratagems empower students to take ownership of their problem-solving processes. The following discussion explores both stratagems across the three phases of metacognitive regulation: planning, monitoring, and evaluation.

Phase one focused on reflecting on and analyzing the planning strategies necessary to support student problem-solving through the use of flowcharts. As previously outlined, our approach involved developing two graphic organizers, the teaching stratagem and the learning stratagem to visually represent the planning strategies and procedures. We recommend beginning by

Table 1. Planning strategies to support student problem-solving

Teaching stratagem: Planning strategies to support teaching problem-solving	Learning stratagem: Planning strategies to support learning problem-solving
Demonstrate how to construct flowcharts to represent problem-solving steps.	I understand how to construct flowcharts to represent problem-solving steps.
Begin with problems involving a few steps to familiarize students with flowchart development.	I begin with problems involving a few steps to familiarize myself with flowchart development.
Select problems that allow for multiple solution paths to encourage strategic thinking.	I choose problems that allow for multiple solution paths to explore different strategies.
Identify stages that require reasoning or evaluation to guide solution direction.	I identify stages that require reasoning or evaluation to guide my solution approach.
Choose problems that prompt deeper thinking about solution strategies.	I engage with problems that prompt deeper thinking about how to solve them.
Provide opportunities for students to integrate multiple procedures to validate or optimize solutions.	I work on problems that require integrating multiple procedures to validate or optimize my solution.

explicitly teaching students how to construct procedural flowcharts. This should start with relatively simple problems that require only a few steps to solve, allowing students to become familiar with the structure and purpose of flowcharts. Ideally, these problems should offer opportunities for multiple solution paths, encouraging students to engage in metacognitive strategies such as revisiting the problem, thinking aloud, drawing diagrams, and breaking the problem into smaller, manageable parts.

Subsequent steps should involve identifying stages within the problem that require reasoning or decision-making to determine the most appropriate direction for the solution. Teachers should select problems that prompt deeper thinking and require students to integrate multiple procedures to validate or optimize their solutions. This process naturally leads to the exploration of alternative methods, which can be visually mapped using flowcharts. Importantly, students should have access to the teaching stratagem, which can serve as a scaffold for the processes outlined in their learning stratagem. This alignment supports coherence between instructional planning and student learning. **Table 1** summarizes the key planning strategies embedded in both the teaching and learning stratagems.

Phase two focused on reflecting on and analyzing the monitoring strategies necessary to support student problem-solving using flowcharts. As in phase one, we emphasize the use of both the teaching stratagem and learning stratagem to guide and scaffold the monitoring process.

Monitoring involves tracking progress, reflecting on the problem-solving steps, and providing feedback to ensure that the solution pathway is logical and mathematically sound. We recommend beginning with teacher-led scaffolding, where students are supported in developing single-procedure flowcharts before progressing to more complex, multi-solution representations.

A key component of monitoring is encouraging students to critically assess the flow of their solutions. This can be facilitated through targeted questioning, both teacher-generated and peer-to-peer, to prompt reflection and clarify reasoning. Students should be given opportunities to explain their flowchart steps to peers and the teacher, using class-developed or teacher-modelled flowcharts as reference points. Before students create individual flowcharts, we suggest beginning with collaborative group work. Group-based flowchart development fosters discussion, dialogue, and shared reasoning. Students should be encouraged to list and justify the steps involved in their group solutions before transitioning to individual flowchart construction. **Table 2** summarizes the key monitoring strategies embedded in both the teaching and learning stratagems.

Phase three focuses on reflecting on and analyzing evaluation strategies to enhance student problem-solving through the use of flowcharts. As in the previous phases, we emphasize the importance of both the teaching stratagem and learning stratagems in guiding evaluation practices. We recommend beginning with an assessment of the learning environment to determine whether it supports peer monitoring and constructive feedback. Effective evaluation occurs when both teachers and students are given opportunities to review each other's flowcharts to assess whether the visual representations lead to coherent and valid solutions.

Peer review followed by teacher evaluation, within an open-ended and dialogic space, is central to this phase. These interactions allow for critical reflection, clarification of reasoning, and refinement of strategies. The next step involves applying the flowcharts to authentic, real-world problems, thereby extending the dialogic space and deepening engagement with problem-solving. **Table 3** summarizes the key evaluation strategies embedded in both the teaching and learning stratagems.

Table 2. Monitoring strategies to support student problem-solving

Teaching stratagem: Monitoring strategies to support teaching problem-solving	Learning stratagem: Monitoring strategies to support learning problem-solving
Monitor how students explain the flow of their solutions–Are the steps logical and mathematically sound?	I can explain the flow of my solution, ensuring the steps are logical and mathematically sound.
Support students in first developing single-procedure flowcharts before progressing to multi-solution flowcharts.	I can logically develop single-procedure flowcharts before attempting multi-solution ones.
Use targeted questions to prompt students when steps are unclear.	I ask questions to clarify steps when they are unclear.
Encourage students to explain their flowchart steps to peers.	I can explain the steps in my flowchart to my peers.
Provide access to class- or teacher-developed flowcharts as reference tools.	I refer to class- or teacher-developed flowcharts to support my understanding.
Facilitate group-based flowchart development before individual work.	Before creating my individual flowchart, I worked collaboratively in a group.
Promote discussion and dialogue during flowchart development.	I actively discuss the process with peers while developing my flowchart.
Guide students to list solution steps before constructing flowcharts.	I list the steps involved in solving the problem before creating my flowchart.

Table 3. Evaluation strategies to support student problem-solving

Teaching stratagem: Evaluation strategies to maximize teaching problem-solving	Learning stratagem: Evaluation strategies to maximize learning problem-solving
Assess whether the learning environment enables students to monitor and provide feedback to one another.	I check whether the learning environment allows me and my peers to monitor and provide feedback to each other.
Evaluate whether student-developed flowcharts lead to coherent and valid solutions.	I check whether my flowchart leads to a coherent and valid solution.
Facilitate and assess peer review of student flowcharts.	I participate in peer review of flowcharts to improve my problem-solving approach.
Conduct teacher-led review of student flowcharts to provide targeted feedback.	I receive and reflect on teacher feedback on my flowchart.
Promote and evaluate the use of open dialogic spaces for collaborative reflection.	I engage in open dialogue with peers and teachers to reflect on my problem-solving process.
Apply flowcharts to authentic, real-world problems and evaluate their effectiveness.	I apply my flowchart to real-world problems and assess their effectiveness in solving them.

DISCUSSION

This section presents insights from our reflective analysis of planning, monitoring, and evaluation strategies using flowcharts to support students' problem-solving in mathematics. As previously discussed, effective problem-solving requires the organization and communication of knowledge throughout the solution process. To support this, we have proposed two types of graphic organizers, teaching stratagems and learning stratagems, which visually represent the metacognitive regulation strategies involved in planning, monitoring, and evaluating problem-solving activities. These stratagems align with Polya's (1971) foundational framework: understanding the problem, planning the solution, executing the plan, and reflecting on both the process and outcome. Through our reflective exchanges, we found that teaching and learning stratagems can stimulate metacognitive regulation by guiding decision-making and problem-solving processes (Goos et al., 2000; Güner & Erbay, 2021).

The integration of flowcharts into problem-solving instruction supports deeper understanding, critical thinking, and effective communication (Leny et al., 2020; Norton et al., 2007). Moreover, flowcharts function as two-way communication tools between teachers and students, and among students themselves, fostering dialogic engagement (Aprilisa, 2019; Grosskinsky et al., 2019). Within this framework, teachers can use teaching stratagems to scaffold student learning and monitor progress, while students can use learning stratagems to guide and track their own understanding and problem-solving processes. Both stratagems highlight essential strategies and stages that teachers and students can follow when planning, executing, and evaluating problem-solving tasks. Additionally, they can be used to inform and support the development of student success criteria, ensuring clarity and coherence in learning outcomes.

As previously highlighted, planning strategies that support the teaching and learning of problem-solving through flowcharts inherently involve metacognitive processes. Metacognitive learning is activated when learners construct flowcharts, as this requires them to reflect on their thinking, organize information, and plan solution pathways (Huang & Tsapali, 2022). Teaching students to use flowcharts, structured as learning stratagems and teaching stratagems can foster a dynamic exchange of ideas between teachers and students, and among peers. This two-way interaction supports the development of students' problem-solving and communication strategies, enabling them to articulate goals and execute specific procedures effectively (Boaler et al., 2024).

Flowcharts enhance students' ability to identify relationships among strategies and procedures, evaluate solutions, and think critically about how to approach problems (Vale & Barbosa, 2018). They also encourage deeper cognitive engagement by prompting students to integrate multiple steps and methods to reach optimal solutions. Norton et al. (2007) advocate for the active use and scaffolding of flowcharts to help students appreciate their value in facilitating problem-solving.

We recommend beginning with simple procedural flowcharts that involve only a few steps, allowing students to become familiar with the structure and logic of flowchart development. The use of teaching and learning stratagems can further scaffold key metacognitive strategies such as revisiting the problem, thinking aloud, drawing diagrams, and decomposing complex problems into manageable parts (Boaler et al., 2024). By engaging with these stratagem tools, students can develop higher levels of metacognitive awareness, especially when working with tasks that challenge their problem-solving skills (Du Toit & Du Toit, 2013). Importantly, students can plan and explore multiple solution approaches, supported by both teacher- and student-generated flowcharts. These stratagems provide essential scaffolding to strengthen students' problem-solving strategies and skills.

Monitoring progress and evaluating strategies during problem-solving is essential, particularly when using flowcharts supported by learning and teaching stratagem tools. These tools help students understand what is required to achieve accurate solutions and overall success. As noted by Güner and Erbay (2021), teachers play a critical role in guiding students to develop effective problem-solving approaches, which may include drawing diagrams, performing calculations, identifying relationships, making conclusions, and interpreting and communicating solutions (Artigue et al., 2020; Dorier & Maass, 2020). We recommend beginning the monitoring process by having both teachers and students track how the solution is unfolding, specifically, whether the steps are logical and mathematically sound. This is most effective when students first practice developing single-procedure flowcharts before progressing to more complex, multi-solution formats. Boaler et al. (2024) support this approach, emphasizing that monitoring should involve gradual progress tracking and timely feedback to guide students toward success.

The monitoring phase should therefore involve both teacher and student actively using stratagem tools to assess progress and identify what adjustments are needed. Before students create individual flowcharts, they should work collaboratively in groups, using class or teacher developed flowcharts as references. This scaffolding supports both learning and self-monitoring. Learning and teaching stratagems help identify incremental steps and clarify the connections between the problem, proposed strategies, and final solutions (Goos et al., 2000). We argue for the intentional use of these tools, alongside structured discussions of the flowcharts and problem-solving processes–as students develop their own solutions. This protocol not only supports metacognitive development but also helps students visualize and internalize the problem-solving process (Ledin & Machin, 2020), reinforcing both procedural understanding and strategic thinking.

We take the position that effective evaluation that enhances student problem-solving using flowcharts is best achieved when both teachers and students are actively engaged in reviewing each other's work through the use of learning and teaching stratagem tools, as well as class- or teacher-developed flowcharts (Hornby & Greaves, 2022). As previously discussed, teachers play a vital role in helping students develop strategies to solve mathematical problems, interpret results, evaluate reasoning, and communicate solutions clearly (Artigue et al., 2020; Dorier & Maass, 2020).

We recommend beginning the evaluation process by assessing whether the learning environment supports peer-to-peer monitoring and feedback. Students should be encouraged to review each other's flowcharts, assess the mathematical reasoning and solution pathways, and provide constructive feedback. This peer review should be followed by teacher evaluation of both the students' flowcharts and the peer review process itself. Applying these procedural flowcharts to authentic, real-world problems further strengthens the evaluation phase. This evaluation, conducted collaboratively by students and teachers, can support the development of higher-order metacognitive strategies, skills, and decision-making processes before, during, and after problem-solving activities (Mitsea & Drigas, 2019). These metacognitive abilities can be explicitly taught and cultivated through the structured use of flowcharts, enabling both teachers and students to refine their problem-solving practices and deepen their mathematical understanding.

Limitations of the Study

While this study offers valuable insights into the use of flowcharts to support students' mathematics problem-solving through metacognitive regulation strategies, several limitations should be acknowledged.

The study employed an autoethnographic methodology, which, although rich in reflective depth, is inherently subjective. The findings are based on the personal experiences and interpretations of key participants, which may introduce bias and limit the generalizability of the results to broader educational contexts.

The sample size was relatively small, involving data from only eight educators. This limited scope may not fully capture the diversity of teaching practices or student responses across different schools, regions, or educational systems. Additionally, the study did not incorporate quantitative data to measure student performance or learning outcomes, which restricts the ability to assess the effectiveness of flowcharts in enhancing problem-solving skills in measurable terms.

The study's context was closely tied to the Australian curriculum and specific school environments. As such, the applicability of its findings to other educational settings may be constrained. Moreover, while flowcharts were presented as beneficial tools, the study did not explore potential challenges students might face in using them, such as cognitive overload or misrepresentation of problem-solving steps. Finally, the study was conducted over two school terms, which may not be sufficient to observe long-term impacts on teaching practices or student learning. The absence of longitudinal data limits the understanding of how sustained use of flowcharts influences metacognitive development and problem-solving proficiency over time.

CONCLUSION

The autoethnographic process enabled a deeper reflection on the metacognitive strategies necessary for supporting student problem-solving through the use of flowcharts. This inquiry was structured around a three-phase metacognitive regulation framework: Phase one examined planning strategies to guide students in constructing procedural flowcharts. Phase two focused on monitoring progress and identifying strategies to support students as they developed and refined their problem-solving approaches. Phase three explored evaluation strategies aimed at maximizing student problem-solving outcomes.

An emerging framework was introduced, incorporating teaching stratagems and learning stratagems graphic organizers designed to scaffold the planning, monitoring, and evaluation processes for both teachers and students. These tools were discussed in relation to their practical implications for mathematics instruction and student learning. The autoethnographic approach highlighted the critical role of metacognitive strategies in the teaching and learning of mathematics problem-solving. It offered a reflective lens through which to examine instructional practices and student engagement. We hope this work encourages further research into the development and application of metacognitive strategies and skills that are essential for fostering high-quality mathematics teaching and learning.

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