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Using Reading Strategy Training to Foster Students' Mathematical Modelling Competencies: Results of a Quasi-Experimental Control Trial

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ABSTRACT

Ever since the national standards for teaching and learning mathematics in Germany were published, investigation of ways to support students' acquisition of mathematical competencies has increased. Results of these studies have been of special interest in empirical educational research. In this context, several recent studies have focused on the enhancement of students' reading comprehension skills as a means of supporting students' development of subject-specific competencies. Taking into account previous research, the empirical research project FaSaF investigated to what extent students' mathematical modelling competencies can be fostered using a 15-week training in reading strategy. Treatment effects have been investigated in three conditions: EC A, integrated reading strategy training; EC B, separate reading strategy training; and EC C, no reading strategy training. Data from German secondary school students ($N = 380$) who were about 13 years old were analyzed. The results indicate that students who have participated in reading strategy training experience an increase in mathematical modelling competencies but that the same increase can also be observed in students who have not participated in reading strategy training. Thus, the issue of fostering the acquisition of modelling competencies using reading strategy training is still open for debate.

Keywords: mathematical modelling, reading comprehension, intervention study, fostering mathematical modelling competencies, reading strategy training

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State of the literature

- Mathematics achievements, amongst others, are influenced by individual reading comprehension.
- Reading comprehension seems to be important for working on mathematical modelling problems. Although students' reading comprehension abilities play a prominent role in working on modelling problems, previous intervention studies have not focused on working on improving reading comprehension to support students in building up mathematical modelling competencies.
- Reading comprehension can be fostered using reading strategy training.

Contribution of this paper to the literature

- This study aimed at fostering selected mathematical modelling competencies with the help of reading strategy training. In this context, the impact of differences in the organization of reading strategy training was investigated.
- Our results indicate that students who have participated in the reading strategy training in our intervention experienced an increase (although low) in the selected mathematical modelling sub-competencies but that the same increase can also be observed in students who have not participated in our intervention.
- Our study points out that more research on mathematics-specific reading strategies focusing on the interplay between reading and finding mathematical relations is required.

INTRODUCTION

The issue of how to design learning-conducive, competency-oriented mathematics instruction is a key challenge in the research discourse and in educational policy. The *Fach-an-Sprache-an-Fach* (FaSaF¹) study has been addressing this research gap, taking as an example competency in mathematical modelling, which is a central component of the German Education Standards. In particular, it explores means of supporting students in developing this competency using reading strategy training, acknowledging that reading is an important facet of school learning. This article includes (1) a basic description of the interplay between language and mathematics, (2) key ideas in mathematical modelling, and (3) a discussion of fostering reading comprehension using reading strategies in general. Finally, we present (4) an intervention study in which we (5) investigate to what extent the acquisition of selected sub-competencies of mathematical modelling can be facilitated through targeted promotion of reading strategies. On the basis of the results, we (6) reflect on the intervention and the measurement instruments used in the study.

THE INTERPLAY BETWEEN LANGUAGE AND MATHEMATICS

For several years, "it [has been] widely acknowledged within the field of mathematics education that language plays an important (or even essential) role in the learning, teaching, and doing of mathematics" (Morgan, 2013, p. 50). In this context, various studies have demonstrated that mathematics achievements are influenced by individual language proficiency (Abedi & Lord, 2001; Baumert & Schümer, 2001; Heinze, Rudolph-Albert, Reiss, Herwartz-Emden, & Braun, 2009). Furthermore, language proficiency not only influences

multilingual students' mathematics achievement (Heinze et al., 2009) but also influences monolingual students' mathematics achievement, particularly those with low socioeconomic status (Prediger, Renk, Büchter, Gürsoy, & Benholz, 2013).

One aspect of language proficiency is individual reading comprehension: the active (re)construction of a text's meaning, a complex ability made up of various sub-processes (Lenhard, 2013). Empirical replication studies have determined that reading comprehension is an influential predictor for the successful completion of mathematics problems (Fuchs, Fuchs, & Prentice, 2004; Grimm, 2008). Since students who possess insufficient reading comprehension skills show deficits in dealing with mathematical test items (Leutner, Leopold, & Elzen-Rump, 2007), the process of extracting meaning from texts has been regarded as the precondition for understanding mathematical phenomena encountered in everyday life (Kaiser & Schwarz, 2003). However, despite an extensive body of research, an open question still remains: How can knowledge about the interplay between reading comprehension and mathematics achievement be used to develop adequate intervention programs in mathematics education? In order to explore answers to this question, we designed the present study to investigate empirically the possibility of fostering modelling competencies (as part of mathematics achievement) by fostering reading comprehension using reading strategy training.

REALISTIC PROBLEMS IN MATHEMATICS INSTRUCTION

As part of a changing problem-solving culture in mathematics instruction motivated by the German students' disappointing results in solving realistic problems in international school comparison studies, there have been increased efforts in the past years to integrate mathematical modelling problems into daily teaching practice (see, among others, Kultusministerkonferenz, 2003; National Council of Teachers of Mathematics, 2000). In contrast to the algorithmic mathematics problems long dominant in German mathematics instruction, mathematical modelling problems are realistic word problems involving the application of mathematics to situations outside of mathematics (Blum, 2011; Pollak, 2007). The goals in integrating mathematical modelling problems into daily teaching practice are to teach students the significance of mathematics for everyday life and to enable them to apply mathematics in a thoughtful way to present and future real-life problems (Niss, Blum, & Galbraith, 2007). It is thus a means for fostering "mathematical literacy": "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts" (OECD, 2013, p. 17).

Mathematical Modelling Process

The completion of mathematical modelling problems involves complex translation processes between reality and mathematics that may be illustrated by so-called modelling cycles (for an overview, see Borromeo Ferri, 2006). An example of an idealized modelling cycle describing the modelling process in seven cognitive steps is presented in Figure 1. The cognitive steps involved in solving the modelling problem "Annual Movie Theater Pass" (see Figure 2) are explained in Table 1.

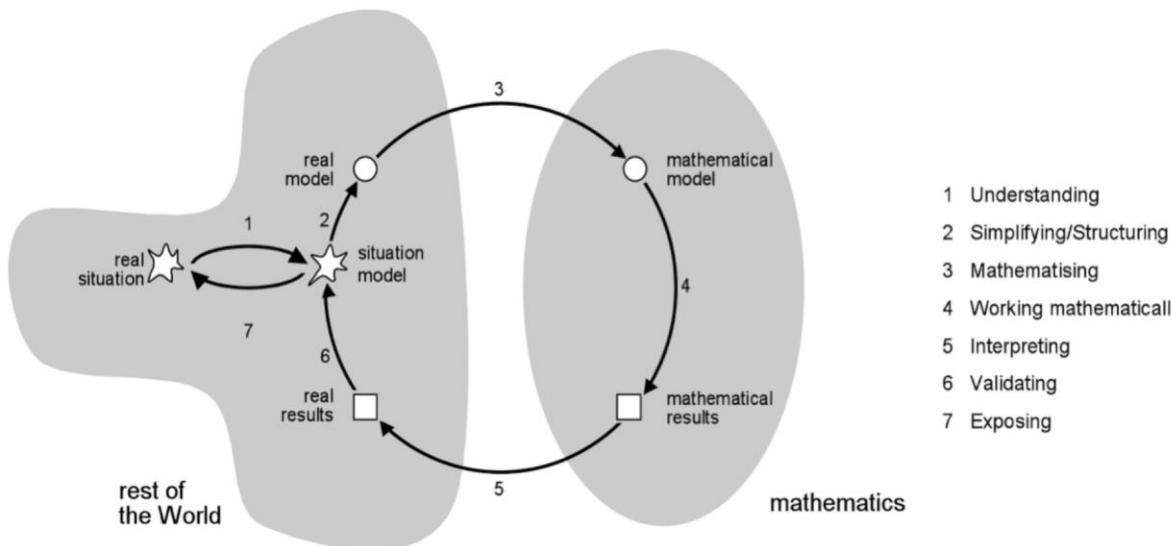


Figure 1. Modelling cycle according to Blum and Leiss (2007)

Annual Movie Theater Pass

Mr. Morgan comes across an interesting offer in the newspaper. The movie theater chain Kinomaxx is selling annual movie theater passes for 399 €. The pass allows one to go to the movies as often as one wants for an entire year. Mr. Morgan, a big movie fan and a regular moviegoer, is considering whether to buy an annual pass.



Decide whether it's worth it to buy an annual movie theater pass. Provide reasons for your decision.

Figure 2. Sample modelling problem "Annual Movie Theater Pass"

The performance of the steps presented in Table 1 describes the process of mathematical modelling in its entirety. In the context of this process, the ability and the willingness to perform a modelling process are understood as mathematical modelling competencies: "In short: modelling competency in our sense denotes the ability to perform the processes that are involved in the construction and investigation of mathematical models" (Niss et al., 2007, p. 12). The individual sub-competencies necessary for performing a modelling process in detail are defined with reference to the cognitive steps of the modelling cycle (see Table 1).² Since single cognitive steps of the modelling process can hardly be distinguished empirically (Borromeo Ferri, 2006), and since competencies in mathematizing are largely dependent on

Table 1. The seven modelling steps involved in solving the modelling problem “Annual Movie Theater Pass”

1. Understanding	The problem-solving process begins with reading the text and examining the accompanying picture. The reader must understand the circumstances in order to make the problem accessible (situation model): <i>“Is it less expensive to buy an annual pass for 399 € or to pay for admission each time one goes to the movie theater?”</i>
2. Simplifying/ Structuring	In order to formulate the problem in mathematical terms, the problem solver needs to make independent assumptions about the estimated costs of going to the movie theater and/or the estimated amount of times Mr. Morgan will go to the movie theater, information that is not provided in the formulation of the problem: <i>“Admission to the movie theater costs around 8 €.”</i>
3. Mathematizing	The problem solver needs to use mathematical concepts to treat these assumptions within the framework of a mathematical model: $399 \text{ €} : 8 \text{ €} = x$
4. Working mathematically	In the next step, the problem solver needs to perform the necessary mathematical operations in order to arrive at a mathematical result: $x = 49,875$
5. Interpreting	The mathematical result must then be translated into reality and rounded off in a meaningful way: <i>“Mr. Morgan would have to go to the movie theater at least 50 times in a year for the annual pass to be worth the cost.”</i>
6. Validating	Finally, the problem solver needs to reflect on the individual steps of the problem-solving process and the result: <i>“Does the movie theater even show that many films that Mr. Morgan really wants to see? Would he go to the movies every week even in the summer?”</i>
7. Exposing	The problem solver then formulates the final result in written or oral form: <i>“If Mr. Morgan went to the movies once a week, it would be worth it for him to buy the annual movie theater pass, but in that case he would really have to be a big movie fan.”</i>

the steps of understanding the task and simplifying the problem (Biccard & Wessels, 2011), different experimental studies concerning ways of fostering modelling competencies have used an adapted version of the modelling cycle. The adapted cycle has been reduced to three cognitive steps: (1) Understanding/mathematizing the task, which comprises the first three phases of the modelling cycle; (2) working mathematically; and (3) explaining the results, which includes interpreting and validating the result (see Djepaxhija, Vos, & Fuglestad, 2015; Schaap, Vos & Goedhart, 2011; Zöttl, 2010).

In the following we concentrate on the ability to simultaneously conduct the two cognitive steps (which we will refer to as sub-competencies) mentioned above—understanding and simplifying/structuring—because they cannot distinguished empirically (Borromeo Ferri, 2006, see above).

Fostering Mathematical Modelling Competencies

Because performing each individual step in the modelling process can cause problems for students (Galbraith & Stillman, 2006), a variety of studies have shown that modelling problems are difficult for students (Blum, 2011). More precisely, even the comprehension processes at the beginning of the modelling process, namely the translation from the real situation given in a written task to a mathematical model, can pose cognitive obstacles in the modelling process (Borromeo Ferri, 2006; Galbraith & Stillman, 2006; Reusser, 1994). However, it has to be stressed that “the translation of one’s understanding of a problem situation into a mathematical model constitutes a key step in the process of mathematical modelling” (Van Dooren, De Bock, & Verschaffel, 2013, p. 385). Based on this idea, it is not enough to merely extract the numbers included in the modelling problem and enter them into whatever mathematical algorithm seems to suggest itself—yet this is a strategy followed by many students (De Corte, Verschaffel & Op’t Eynde, 2000). According to these findings, the comprehension processes at the beginning of the modelling process influence mathematical modelling performances (Maaß, 2007; Voyer, 2010). Since students’ comprehension processes show themselves in the so-called situation model, which is “a representation of the content of a text, independent of how the text was formulated and integrated with other relevant experiences” (Kintsch & Greeno, 1985, p. 110), students’ understanding is related to problems with language (Maaß, 2007). Although Maaß did not describe in detail the meaning of “problems with language,” the main finding was that understanding the content of a text in a modelling task is crucial for starting to work on the problem. In other words (as pointed out by Biccard & Wessels, 2011; Leiss, Schukajlow, Blum, Messner, & Pekrun, 2010), students’ abilities in reading and reading comprehension play a prominent role for the construction of the situation model. At present, there is not enough empirical knowledge about fostering students’ mathematical modelling performances by promoting reading comprehension in mathematics education.

In addition, investigating ways of supporting students to successfully build up mathematical competencies has been promoted in the context of the development of national education standards for mathematics in several countries (Kultusministerkonferenz, 2003; National Council of Teachers of Mathematics, 2000). Empirical studies have also shown that students have an insufficient level of modelling competency by the end of lower secondary school (Blum, 2011; OECD, 2013). Therefore, researchers have conducted intervention studies on students at various grade levels to foster mathematical modelling competencies (for further discussions see, e.g., Blum, 2011). Some of these studies have demonstrated that support of students’ strategy use improves students’ mathematical modelling competencies (Schukajlow, Krug, & Rakoczy, 2015; Schukajlow, Kolter, & Blum, 2015; Stillman & Galbraith 1998; Stillman, 2011; Zöttl, Ufer, & Reiss, 2010). For this reason, Blum emphasized that an effective way of fostering students’ modelling competencies is “to teach learning strategies, cognitive strategies as well as metacognitive strategies such as planning, controlling, or

regulating" (Blum, 2015, p. 88). Unfortunately, only a few interventions have aimed to foster the acquisition of said strategies in mathematics classrooms (Leiss, 2007).

As argued above, reading comprehension plays a prominent role in working on modelling problems. Since an effective way of fostering students' modelling competencies is to support students' strategy use, we thus discuss the use of reading strategies for fostering general reading comprehension.

READING COMPREHENSION IN THE FOCUS OF SOCIAL AND SCIENTIFIC INTEREST

The starting point for a variety of studies in the past few decades that have focused on fostering students' reading comprehension has been the unsatisfactory reading competencies of German middle school students, as tested by the international comparative school achievement study, PISA 2000 (Artelt, Schiefele, Schneider, & Stanat, 2002; Kirsch et al., 2003). Since students' reading comprehension has proven to be a promising target dimension for interventions (Artelt et al., 2002), progress in students' reading comprehension has recently been made (Naumann, Artelt, Schneider, & Stanat, 2010). However, students still need further support to improve their reading comprehension. Since adequate reading comprehension is the complex result of an active examination of a text, it is influenced by a number of different factors that are related to, on the one hand, the text (the type of text, the complexity of the micro- and macrostructure of the text, and the amount of new information it includes) and, on the other hand, the individual (decoding skills, prior knowledge, lexicon, and affective factors such as motivation and self-perception) (De Corte, Verschaffel, & Van de Ven, 2001; following Hiebert & Raphael, 1996; Cromley & Azevedeo, 2007).

Whereas changing factors inherent to a text leads only to a short-term improvement in reading comprehension (namely only with regard to that particular text), promoting factors related to the individual can bring about long-term improvements. However, not all individual factors are equally suited for use as target dimensions of interventions designed to foster general reading comprehension. While it is difficult to support dimensions such as the capacity of working memory or basic cognitive skills, the carefully considered use of strategies has been a promising target dimension for supporting students' reading comprehension (Edmonds et al., 2009; Gersten, Fuchs, Williams, & Baker, 2001; Nordin, Rasihd, Zubir, & Sadjirin, 2013). Furthermore, findings of empirical research show that good and poor readers often differ with regard to their use of appropriate cognitive strategies as well as their metacognitive monitoring of comprehension (Mokhtari & Reichard, 2002; Paris, Lipson, & Wixson, 1983).

The Influence of Reading Strategies on the Development of Reading Comprehension

Reading strategies are related to learning strategies that support students in acquiring knowledge and in influencing and controlling their motivation (Friedrich & Mandl, 2006): "Strategic readers actively construct meaning as they read and interact with the text" (Nordin et al., 2013, p. 470). The term reading strategies is defined as any processes that

readers are conscious of executing in order to facilitate understanding from written texts (Artelt et al., 2002; Nordin et al., 2013).

In the PISA study, knowledge of reading strategies contributes substantially to the explanation of individual reading competence and is additionally even the second strongest predictor for general reading competence when controlling for basic cognitive abilities, verbal self-concept, and general decoding skills (Artelt et al., 2002). These findings reinforce the claim that reading strategies, which are still given only scant attention in school learning, should be integrated into daily teaching practice (Pressley, 1998; Lenhard, 2013). Therefore, “it is important to teach the strategies by naming the strategy and how it should be used” (Kükükoğlu, 2013, p. 710). Furthermore, teachers should give students opportunities to practice the strategies, either in pairs, small groups, or individually, and offer structured feedback to students (Kükükoğlu, 2013).

There are three general categories of reading strategies: cognitive strategies, which involve processes of extracting and processing information; metacognitive strategies, which focus on planning, controlling, and monitoring the learning process; and resource-based strategies, which are used to ensure a suitable learning environment (De Corte et al., 2001; Lenhard, 2013). Each of these three categories contains a substantial number of individual strategies (De Corte et al., 2001). Since readers use strategies to understand what they read before, during, and after reading (i.e., pre-reading, while reading, and post-reading) (Nordin et al., 2013), in the following, we present examples of several cognitive and metacognitive reading strategies of these stages that have already been useful (Gersten et al., 2001; Pressley, 1998):

Pre-reading: Research indicates that readers use strategies before they begin to read. In doing so, students are likely to make the texts more accessible during reading. While “pre-reading activities assist readers to activate what they know about a topic and foresee what they will read” (Nordin et al., 2013, p. 470), one major strategy before reading is *activating prior knowledge*. By using the title, table of contents, or pictures, readers are instructed to formulate their own prior knowledge before reading the text to be processed. After reading, the readers must see if their predictions are validated by the text. Research has shown that readers improve their individual understanding comprehension by making predictions (Duke & Pearson, 2002; Kintsch, 1994).

While reading: There are a variety of strategies effective readers use to build their understanding of the text and to become engaged in the reading process during reading. Most of these strategies are monitoring strategies to make sure that readers understand what they are reading. Since the relationship between reading comprehension and vocabulary knowledge is widely acknowledged, one monitoring strategy is *dealing with unclear text passages* by identifying and interpreting comprehension obstacles with the help of context or external aids (Gersten, 2001).

Post-reading: Since constructing meaning from text does not end with the termination of reading, readers have to identify and summarize major information of a text (Nordin et al., 2013). In order to do this, *dividing the text into thematic sections* and *highlighting keywords* can be very helpful. While dividing the text into thematic sections and giving each section a heading, the reader becomes sensitive to the structure of a text. By highlighting key words,

the reader identifies a text's necessary information. Afterwards, readers can make meaningful connections between pieces of information (Kükükoğlu, 2013). These types of meaningful connections can be done by creating a *concept map*. A concept map is "a type of graphic organizer that is distinguished by the use of labeled nodes denoting concepts and links denoting relationships among concepts" (Nesbit & Adesope, 2006, p. 415). Concept maps were developed as organizational tools to represent knowledge and are useful learning tools (Novak & Cañas, 2007).

(Reading) Comprehension Strategy Training in Mathematics

Even though in recent years the effectiveness of comprehension strategies for working on mathematical word problems has come under scrutiny (Capraro, Capraro, & Rupley, 2012; Kintsch & Greeno, 1985; Verschaffel, Greer, & De Corte, 2000), the literature on successful reading strategies—one kind of comprehension strategy—for working on mathematical word problem is limited. Because students' difficulties with mathematical word problems are often related to poor reading comprehension and because teachers normally tend to take students' reading competences for granted and focus only on teaching subject-specific skills, there have been calls for a more language-sensitive teaching in recent years (Thürmann, Vollmer, & Pieper, 2010). In this context, initial studies have been conducted to identify the interplay between reading and finding mathematics relations. One of these studies was conducted to investigate how to foster students' comprehension strategies (including reading comprehension strategies) for multi-step algebraic word problems. The findings of this study suggest that an interplay of six different strategies supports students' comprehension processes. Some of these six strategies focus on supporting students in finding relevant information and in making meaningful connections between pieces of information (Prediger & Krägerloh, 2015). However, further research is required to investigate the possibility of transferring these findings to other mathematical contexts, especially to mathematical modelling.

The underlying research gap

Since we do not know much about the influence of reading strategy training on students' modelling competencies, we created an intervention study based on the following ideas: (1) Students' comprehension processes play a prominent role in students' modelling performances. (2) The construction of the situation model is related to students' abilities in reading comprehension and difficulties can often be traced back to deficits in students' comprehension strategies. (3) An effective way of fostering students' modelling competencies is to support students' strategy use. In our intervention study, we investigate the effectiveness of reading strategy training on the comprehension processes at the beginning of the modelling process, namely the modelling sub-competencies associated with understanding and simplifying/structuring. Therefore, students received support in making use of the selected reading strategies presented here (activating prior knowledge, dealing with unclear text passages, dividing the word problem into thematic sections, highlighting key words, and creating a concept map). While the first three strategies are applied to ensure students' understanding of the text, the last two strategies focus on supporting students in finding information and in making meaningful connections between information.

Because a mathematical word problem is generally structured differently than a narrative or an expository text (Thürmann et al., 2010), we also analyzed whether it was more effective to foster reading strategies directly while working on modelling problems (integrated strategy training) or separately as an interdisciplinary aid (separate reading strategy training).

INTERVENTION STUDY IN THE FRAMEWORK OF THE RESEARCH PROJECT *FACH-AN-SPRACHE-AN-FACH*

The interdisciplinary research project FaSaF has been investigating the effectiveness of reading strategy training on the mathematical modelling competencies of seventh-grade students (about 13 years old) within the framework of a 15-week intervention study. The primary concern of the study described here was to foster mathematical modelling competencies by focusing on the comprehension-oriented sub-competencies of understanding and simplifying/structuring in the modelling process. In detail, the study pursued two research questions:

- To what extent is it possible to foster seventh-grade students' selected mathematical modelling sub-competencies with the help of reading strategy training?
- Are there differences in the efficacy of two different teaching approaches (integrated vs. separate reading strategy training) on the development of selected mathematical modelling sub-competencies?

Design of the Study

In the academic year 2014-2015, we conducted an intervention study (starting in November 2014 and ending in April 2015) undertaken in the interdisciplinary research project FaSaF. The study compared the effects of three different experimental conditions: Experimental Condition A (EC A), integrated reading strategy training; Experimental Condition B (EC B), separate reading strategy training; and Experimental Condition C (EC C), wait-list control group (see Figure 3).

In the course of ECs A and B, seventh-grade students participated in an optional reading strategy training for solving mathematical modelling problems. At seven different schools, we established an EC A and an EC B with a maximum of 16 students parallelized in accordance with basic mathematical skills and general language skills, including reading comprehension. The students received 90 minutes of reading strategy training from trained teachers one afternoon each week in addition to their regular lessons. Hence, the intervention covered a period of 4.5 months, during which the students received a maximum of 15 afternoons of weekly extra lessons, excluding vacations. On average, the students participated in extra lessons on 10.53 afternoons (standard deviation 3.84). In addition to ECs A and B, we established an EC C that did not receive any reading strategy training (wait-list control group). Therefore, in two additional schools our research instruments were administered to all seventh-grade students.

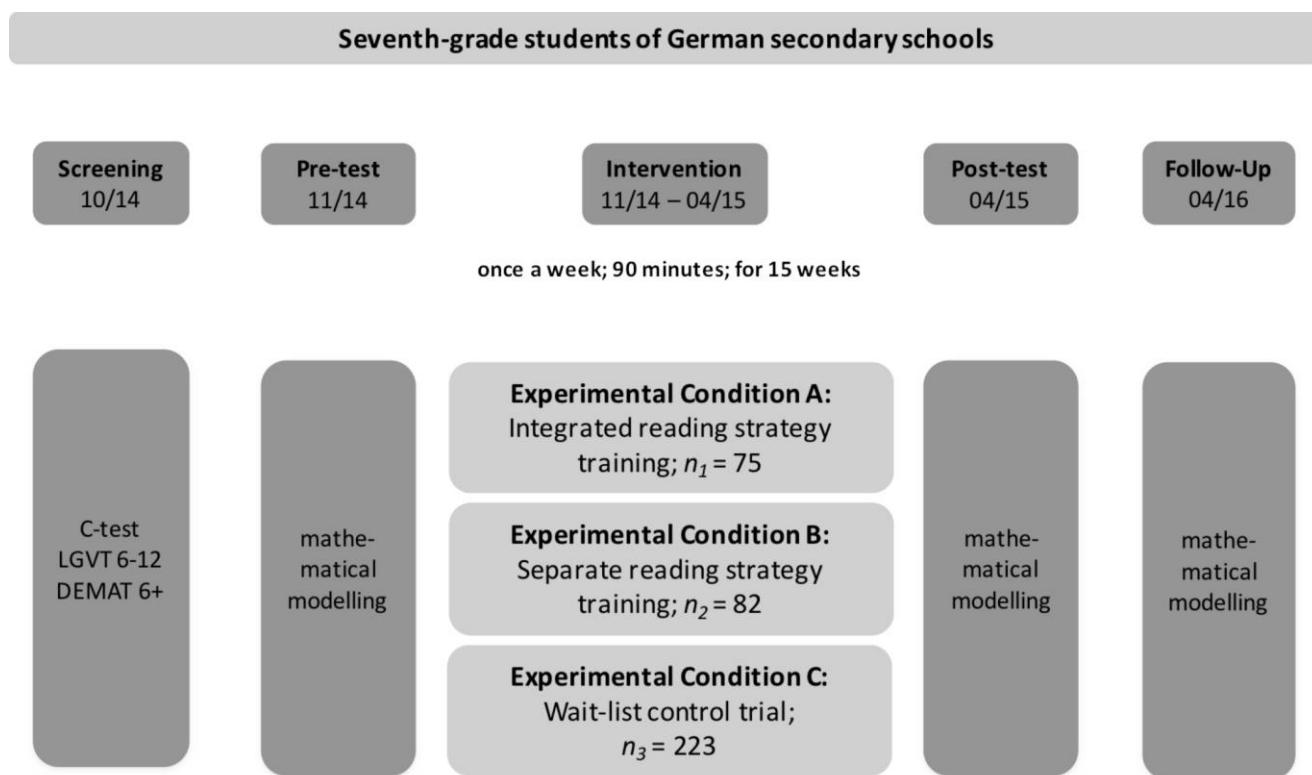


Figure 3. Design of the intervention study

Research Instruments

To investigate the results with regard to the research questions discussed above, we employed a variety of research instruments completed by the students. Before the beginning of the intervention study, the students underwent a 90-minute screening to determine their initial level of basic mathematical abilities, their general language skills, and their reading comprehension. As the instruments (C-Test, LGVT 6–12, and DEMAT 6+) were standardized research instruments, we evaluated them on the basis of the available evaluation forms and standardization tables. Since we took these results as a basis for establishing parallelized ECs A and B, students in EC C did not participate in the screening.

C-TEST

We used a C-test to measure the general language skills of the participating students. The C-test is a written test that is regarded as particularly valid for measuring general language proficiency (see Grotjahn, 2013).

LGVT 6–12

We used the LGVT 6–12 to measure the reading comprehension and the reading speed of the students participating in the study. It is a proven standardized reading speed and comprehension test developed for sixth- to twelfth-grade students that involves reading a

continuous narrative text. A validity test has confirmed that the LGVT 6–12 can serve as a valid measure for reading comprehension (see Schneider, Schlagmüller, & Ennemoser, 2007).

DEMAT 6+

We used part of DEMAT 6+ to measure the students' basic mathematical skills at the beginning of the intervention study (see Götz, Lingel, & Schneider, 2013).

To determine the students' subject-specific initial learning level with regard to mathematical modelling competencies, we assigned a pre-test to all three experimental conditions assessing mathematical modelling competencies before the start of the intervention (November 2014). Finally, upon completion of the intervention in April 2015, we again tested the students' mathematical modelling competencies in a post-test in order to measure possible increases in performance.

Mathematical Modelling

We used a research instrument designed specifically for the intervention study to measure the understanding and simplifying/structuring sub-competencies. Based on other empirical studies (see the Mathematical modelling process section), we did not try to distinguish these two cognitive steps of mathematical modelling empirically. Thus a total of 30 items were available for measuring these two sub-competencies. Each of these items was characterized by a moderately long informational text (10–16 lines and a picture) that included the relevant information for completing the item as well as information that was unnecessary for completion of the item (for a sample item, see Figure 4). Successfully completing the

Apple Juice

Apple juice is the classic fruit juice: Germans drink around 12 liters of apple juice per year. However, drinking apple juice is no innocent pleasure. After all, a glass of apple juice has more calories than a glass of cola: Apple juice contains almost 190 calories per 250 ml glass, while cola has 140 calories per 250 ml glass. But what happens to an apple on its way from the tree to the bottle? In 2014, 600,000 tons of apples were used to make 400 million liters of apple juice in Germany. After harvesting, rotten spots were first removed from these apples, and then they were carefully cleaned and chopped up into small pieces. The apple pieces were then put through the press. Finally, the pressed juice was heated to 80 °C to ensure that it will keep for at least two years. In the end, the apple juice was bottled. Thus, each 1 liter bottle of apple juice now available for sale contains approximately 1.5 kg of cleaned, chopped, pressed, and pasteurized apples.



Henry wants to calculate how many calories a German takes in on average each year through the consumption of apple juice.

Underline all of the numbers Henry really needs to calculate how many calories a German takes in on average each year through the consumption of apple juice.

Figure 4. Sample item "Apple Juice"

individual items involves, among other things, deciding which information provided in the text is relevant for performing the mathematical operations necessary for answering a given question. By selecting the necessary information to complete these items, the students provided the data for measuring the mathematical modelling understanding and simplifying/structuring sub-competencies (see the Mathematical modelling process section).

Twelve of the 30 items were used specifically at each of the two measurement time points. Another six items served as anchoring items (see Figure 5). This allowed us to use the probabilistic Rasch test model (OPL) to illustrate the students' competencies for the two measurement time points on a scale. Fifteen out of a total of 30 items were provided in multiple-choice format and coded dichotomously, while the other 15 items were provided in partial credit format (with 0, 1, and 2 points). For scaling purposes, the students' data was arranged to consider as pseudo-observations those students who had participated repeatedly in the testing due to the longitudinal design. Hence, a total of 883 observations were available for scaling, including 760 students who had participated in the study at both measurement time points. With the exception of one item (whose discrimination was somewhat too high [MNSQ = 0.79]), the scaling (performed with the program ConQuest) resulted in good item fits ($0.8 \leq \text{MNSQ} \leq 1.2$). Nevertheless, we left this item in the test instrument as it only exhibited a very minor deviation and all of the other characteristics of this item were rated very highly. The reliability of the scale for measuring the selected sub-competencies of mathematical modelling may also be described as good (EAP-Rel. = 0.810). In the end, we wrote out the person parameters as a WLE and then standardized them across all cases to a mean of 100 and a standard deviation of 20. For the following analyses, we adopted the students' WLEs as their new performance values.

Intervention

In the framework of ECs A and B (see Table 2), the students were given training in the five selected cognitive reading strategies, which have already been shown to be important aspects of fostering reading comprehension (see the Influence of reading strategies on the development of reading comprehension section). Furthermore, the participating students in ECs A and B completed five complex modelling problems developed especially for the project. The level of mathematical proficiency required to solve the modelling problems was controlled by only including mathematical concepts (various size ranges, functional relations, and direct and inverse proportionality) the students had dealt with previously in regular lessons in order to avoid making the modelling problems even more challenging for the students than they already were.

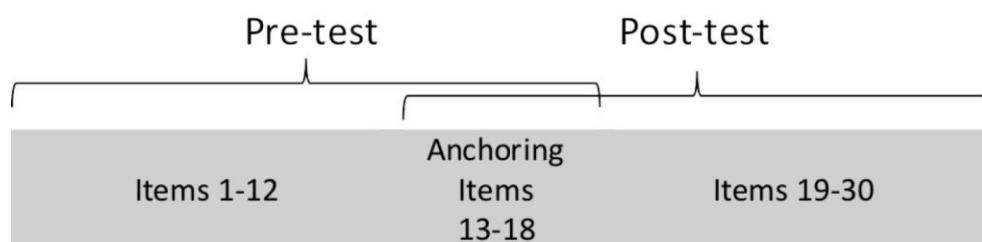


Figure 5. Distribution of test items across the measurement time points

Table 2. Differences in the treatments

EC A: Integrated reading strategy training

In interpreting the content of selected modelling problems, the students in EC A received explicit support in applying reading strategies designed to help them perform the modelling process independently and in a thoughtful way.

Reading strategy training	Within the integrated reading strategy training the students practiced the selected reading strategies while working on different modelling problems.
Completion of mathematical modelling problems	Although the intervention focused on supporting the students in performing the comprehensibility-enhancing processes of the modelling process, the students also completed the mathematical operations involved in solving each of the selected modelling problems. Leaving the mathematical operations out could have possibly had a negative effect on the students' motivation. After completing the modelling problems, the students discussed and reflected on their results in a verification phase.
Advantage	The integrated reading strategy training made it possible to take into account the specific structure of mathematical modelling problems.
Disadvantage	At the same time, however, the students were also regularly interrupted in the modelling process, as the focus of the support changed constantly between the reading strategies to be learned and the modelling problems to be completed.

EC B: Separate reading strategy training

We separated language support from subject-specific support by providing the students separate reading strategy training in the first 10 weeks of the intervention; the students then worked on selected mathematical modelling problems in the subsequent five weeks.

Reading strategy training	In the separate reading strategy training, the students were familiarized with selected reading strategies as interdisciplinary aids. While reading various factual texts the students practiced the selected reading strategies.
Completion of the mathematical modelling problems	To enable a comparison of how the students in the two experimental conditions dealt with mathematical modelling problems, we also gave the students in EC A the selected modelling problems that had formed the basis for the integrated reading strategy training in EC B. The students presented and discussed their results in a verification phase.
Advantage	The decision in favor of separate strategy training made it possible to focus first exclusively on the reading strategies and then on the completion of mathematical modelling problems. Thus, the students did not have to deal with a glut of new information.

EC C: Wait-list control group

Table 2. continued

Disadvantage	At the same time, the decision to separate language and subject learning came with the disadvantage that the mathematical modelling problems were completed without the explicit use of the previously trained reading strategies. Accordingly, the students were not explicitly instructed to use the reading strategies they had acquired on the mathematical modelling problems.
EC C: Wait-list control group	
	EC C did not include reading strategy training. The students in EC C thus received no additional language or modelling-specific support between the pre-test and the post-test. Other than regular lessons, there was no additional intervention.

While the mathematical modelling problems did not differ between ECs A and B, there were design-related differences in the material used to help the students acquire the reading strategies. The students in EC A acquired the various reading strategies with the help of mathematical modelling problems, while in EC B the strategies were introduced and applied with the help of selected factual texts.

Sample

The sample consisted of seventh-grade students (about 13 years old; N = 380) from nine secondary schools who answered both the tests on mathematical modelling competencies. Two groups of students (EC A = 75; EC B = 82) participated in the intervention, while another group of students (EC C = 223) only filled in the research instruments before and after the intervention.

RESULTS

We measured the selected mathematical modelling sub-competencies (understanding and simplifying/structuring) in pre- and post-tests. This performance data is presented in the following sections with reference to our two research questions. In the first section, we present descriptive data and correlations of all manifest/latent variables being used for answering the research questions. In sections two and three we present more in-depth analyses in the form of group comparisons (between the experimental conditions) and analyses of variance.

Descriptive Data and Correlations

Three hundred and eighty students answered both the tests at measurement points one and two. Descriptive data for these students is given in Table 3 (both separated for the experimental conditions as well as summed up for all participating students). Having a closer look at the correlations of this data, it becomes obvious (see also Table 4) that all variables correlate significantly; correlations range from $r = .30$ to $r = .73$.

Although the students in EC C did not take part in the screening, we assumed that their performances in these tests (DEMAT 6⁺, LGVT/C-test) would be comparable to those of the students in ECs A and B because of our random sample (see Bortz & Döring, 2009).

Table 3. Mean scores of general mathematical knowledge, language skills including reading comprehension, and mathematical modelling competencies

	Screening	Pre-test	Post-test
DEMAT 6⁺			
Whole sample (n = 133)	8.75 (3.45)	---	---
EC A (n = 64)	8.47 (3.32)	---	---
EC B (n = 69)	9.04 (3.57)	---	---
EC C	---	---	---
LGVT/C-test			
Whole sample (n = 133)	6.03 (2.41)	---	---
EC A (n = 64)	5.98 (2.37)	---	---
EC B (n = 69)	5.97 (2.52)	---	---
EC C	---	---	---
Mathematical modelling			
Whole sample (n = 380)	---	98.20 (18.88)	103.24 (19.96)
EC A (n = 75)	---	93.64 (17.65)	97.34 (19.31)
EC B (n = 82)	---	96.07 (19.81)	102.06 (21.98)
EC C (n = 223)	---	100.51 (18.63)	105.65 (19.02)

Table 4. Correlations of all research instruments (DEMAT 6+, LGVT/C-test, and pre- and post-test mathematical modelling) used in the study

	DEMAT 6 ⁺	General language skills and reading comprehension (LGVT/C-test)	Pre-test mathematical modelling	Post-test mathematical modelling
DEMAT 6 ⁺	1			
General language skills and reading comprehension (LGVT/C-test)	.298**	1		
Pre-test mathematical modelling	.559**	.422**	1	
Post-test mathematical modelling	.657**	.488**	.726**	1

*: Significant with $p < .05$ (two-tailed).

**: Significant with $p < .01$ (two-tailed)

Research Question 1: Is it possible to foster selected mathematical modelling sub-competencies (understanding and simplifying/structuring) of seventh-grade students with the help of reading strategy training?

The results of selected mathematical modelling sub-competencies from the pre- and post-test serve as a basis for answering this research question. As pointed out in Table 5, the students who participated in reading strategy training (EC A + EC B) in the context of this intervention study ($n = 157$) scored a mean of 94.91 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 99.81 points on the post-test. A t-test for paired samples confirmed that this increase was significant ($p < 0.001$).

However, the difference between the two measurement time points was minor, as shown by the effect size ($d = 0.25$; see Table 5).

Research Question 2: Are there differences in the influence of two different teaching approaches (integrated vs. separate) on the development of selected mathematical modelling sub-competencies (especially understanding and simplifying/structuring)?

Taking this (the combined results of EC A and EC B) as a basis, we discuss in the following what specific effect the two different teaching approaches (EC A and EC B) had on the students' performance in the area of the selected mathematical modelling sub-competencies.

As shown in Table 5, the students in EC A ($n = 75$) scored a mean of 93.64 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 97.34 points on the post-test. The students from EC B ($n = 82$) scored a mean of 96.07 points on the pre-test for measuring selected mathematical modelling sub-competencies and a mean of 102.06 points on the post-test (see Table 5). Furthermore, the minimum mean score increased from 25.58 to 50.52. The descriptive results indicate that both groups achieved increases in the selected mathematical modelling sub-competencies. Although these increases were significant in both experimental conditions (EC A, $p = 0.018$; EG B, $p = 0.001$), they must be rated as small on the basis of the effect sizes calculated for the two groups (EC A, $d = 0.20$; EC B, $d = 0.29$). A direct comparison does not show any significant differences between these two experimental conditions before ($p = 0.420$) or after the intervention ($p = 0.156$). However, at a descriptive level, there was a slightly larger effect for the students from EC B ($d = 0.29$; see Table 5). However, neither of the two reading strategy trainings proved to be more suitable for fostering the selected mathematical modelling sub-competencies. Both the students in EC A (integrated reading strategy training) and the students in EC B (separate reading strategy training) experienced a slight increase in their levels of the selected mathematical modelling sub-competencies.

Table 5. Differences in the mean scores between the measurement points (pre-test and post-test mathematical modelling)

	n	min.	max.	mean	sd		
Reading strategy training (EC A + EC B)							
Pre-test mathematical modelling	157	25.58	140.55	94.91	18.79	$t(156) = -4.29$	$d = 0.25$
Post-test mathematical modelling		50.52	161.56	99.81	20.81	$p < 0.001$	
Integrated reading strategy training (EC A)							
Pre-test mathematical modelling	75	48.40	140.55	93.64	17.65	$t(74) = -2.42$	$d = 0.20$
Post-test mathematical modelling		50.52	148.34	97.34	19.31	$p = 0.018$	
Separate reading strategy training (EC B)							
Pre-test mathematical modelling	82	25.58	140.55	96.07	19.81	$t(81) = -3.56$	$d = 0.29$
Post-test mathematical modelling		50.52	161.56	102.06	21.98	$p < 0.001$	
Wait-list control group (EC C)							
Pre-test mathematical modelling	223	25.58	147.65	100.51	18.63	$t(222) = -5.29$	$d = 0.27$
Post-test mathematical modelling		39.58	161.56	105.65	19.02	$p < 0.001$	

In order to ensure that the observable increases can be attributed to participation in the intervention, we examine the performance data of the students in EC C (no reading strategy training) in the following. The students in EC C ($n = 223$), who did not receive reading strategy training, scored a mean of 100.51 points on the pre-test for measuring the selected mathematical modelling sub-competencies and a mean of 105.65 points on the post-test (see Table 5). In a t-test for paired samples (see Table 5), the increases observed between the two measurement points in the students in EC C also turned out to be significant ($p < 0.001$). The difference between the two measurement time points is nominally lower here ($d = 0.27$) than in the students who participated in a separate reading strategy training (EC B). A single-factor analysis of variance with repeated measures did not reveal any significant differences in the increase between the three experimental conditions. This indicates that there are no differential courses of development in the various experimental conditions across all three conditions. Furthermore, a test of the levels of the selected mathematical modelling sub-competencies, conducted on the basis of a linear regression analysis controlling for the initial level of the selected mathematical modelling sub-competencies and the support conditions, did not reveal any differential effects either.

Consequently, the increases observed in the students in EC C do not differ from the increases confirmed in the students in EC A or EC B. With regard to the research questions under discussion in this study, we may thus conclude that while the students who participated in one of the two reading strategy trainings did experience minor increases in the selected mathematical modelling sub-competencies, the same increases can also be observed in students who did not participate in reading strategy training.

DISCUSSION

The students who participated in reading strategy training during the intervention (ECs A and B) achieved significant increases (with low effects; $d = 0.25$) in the selected mathematical modelling sub-competencies understanding and simplifying/structuring over time (see first research question). These increases may be observed for those students in the integrated reading strategy training (EC A, $d = 0.20$) as well as for those students in the separate reading strategy training (EC B, $d = 0.29$) (see second research question). Similar low effects can also be observed in other studies: Biccard and Wessel (2011) reported on an intervention study involving 12 seventh-grade students solving a variety of modelling problems over a period of 12 weeks. Their descriptive analyses showed that modelling competencies developed slowly and gradually. However, as the students in this study in the wait-list control trial (EC C, $d = 0.27$) also achieved significant increases, and as the courses of development did not differ across the three experimental conditions, the success of the intervention must be called into question. The low increases therefore cannot be attributed to a general slow development of modelling competencies, but must be explained differently.

In the following, we discuss which issues could be responsible for the lack of intervention-related increases in the context of our analyses. We therefore distinguish between issues associated with the choice of our specific method and issues related to the content of our intervention.

Discussion of the Method

Validity of the test instrument

In interpreting the results, it is necessary to consider the possibility that the test instrument we used in the intervention was not sensitive enough for the content. Although the instrument was shown to be suitable from the perspective of test theory (see the Research instruments section), this says nothing about its intervention-specific validity. The first question that needs to be addressed in this context is whether the linguistic complexity of the selected items requires the application of reading strategies to the extent provided by the test instruments, as the use of reading strategies has only proven to be effective for problems with a subjectively average level of difficulty (Hasselhorn, 2010). Secondly, it will be necessary to investigate whether the pre- and post-tests were set up to enable the students to apply elaborate reading strategies: The task of completing an extensive achievement test in a limited amount of time might have hindered the students from applying the reading strategies, which are already demanding per se (Lenhard, 2013): "Students who learned in ISL [informed strategies for learning] to think about the title, to skim before and after reading, to monitor comprehension, not to skip unknown words, and to reread text would be unable to use these strategies in the time-constrained testing procedure" (Paris & Oka, 1986, p. 52). Perhaps qualitative settings are needed to investigate whether students' comprehension processes at the beginning of the modelling process can be fostered by supporting students in using reading strategies.

Duration of the intervention

It also must be taken into consideration that the intervention lasted for only 15 lessons and that these 15 lessons were not only given separately from the regular school lessons but were also spread over a period of about 4.5 months: Students participating in the study took part in only 1 lesson per week. Therefore, many other variables may have influenced the effect of the intervention: "Studies of shorter duration were found to be more effective than long interventions" (Jacobse & Harskamp, 2011, p. 6; for some discussion about the influence of the duration of an intervention on the interventions' effect size, see, e.g., Hattie, Biggs, & Purdie, 1996).

Treatment control

It is also necessary to consider that the implementation of the reading strategy training involved various teachers due to the substantial number of groups receiving training. The training was taught by 14 different teachers, who received weekly instructions and were requested to design the lessons in accordance with a detailed handbook; however, we were not able to monitor each individual training session. As a means of providing treatment control, future interventions should involve another researcher attending at least a selection of the actual training sessions to take notes or film them.

As we did not measure the students' reading comprehension again in the post-test due to a lack of time, it is also unclear to what extent the intervention actually succeeded in fostering the students' reading comprehension skills and their knowledge of reading strategies.

Discussion of the Content of the Intervention

The significance of reading comprehension for performing the modelling process

Whereas the previous points addressed the intervention as such and the test instruments used in the intervention, we now turn to the significance of reading comprehension for the mathematical modelling process. The results of previous studies have demonstrated that reading comprehension skills are essential for the modelling process. Ultimately, however, it must be taken into consideration (also on the basis of the first point of discussion) that reading comprehension skills are a necessary yet by no means sufficient condition for the ability to adequately understand and simplify or structure mathematical modelling problems. Accordingly, it is conceivable that even these initial steps in the modelling process (understanding and simplifying/structuring) require an understanding of the basic principles of mathematics. Other studies (see Ludwig & Reit, 2013) have shown that "students simplified problems based on the mathematics they wanted to use on the problem" (Biccard & Wessels, 2011, p. 380). This impression is also borne out by the highly significant correlations between the individual test instruments used in the study (Table 4). It turns out that the mathematical modelling competency test instruments (especially the understanding and simplifying/structuring competencies) show highly significant correlations not only with the test instruments for measuring general language proficiency and reading comprehension skills (C-TEST and LGVT) but also with the test instrument for measuring basic mathematical skills (DEMAT 6⁺).

The key potential obstacles to understanding and simplifying/structuring a modelling problem might lie not only in students' poor reading comprehension but also in conceptual obstacles – semantic problem structures which are connected to students' access to different basic models (Prediger & Krägeloh, 2015). These so-called conceptual obstacles are often linked to comprehension obstacles. The comprehension of the underlying mathematics relations is necessary for understanding the given task (Prediger, Wilhelm, Büchter, Gürsoy, & Benholz, 2015).

Selection of reading strategies

We have already discussed students' reading comprehension skills being a necessary yet by no means sufficient condition for the ability to adequately understand and simplify or structure mathematical modelling problems. However, we have not discussed whether our selected reading strategies were suitable for working on mathematical modelling problems successfully.

The study presented here focuses on a general reading strategy training. Although the students of EC A were practicing the selected reading strategies while working on different modelling problems, the selected reading strategies were formulated in a very general manner. According to Prediger and Krägeloh (2015), instructional approaches for overcoming mathematics word-problem obstacles have to focus on mathematics-specific reading strategies. Students are in need of strategies that help them to focus on the interplay between reading and finding mathematical relations. For this purpose, in the intervention students received support in creating a concept map. However, students could not use this

strategy for working on the 18 post-test items due to a lack of time. Perhaps rather than supporting general comprehension strategies, more support on other mathematics-specific comprehension strategies focusing on relations connecting information would have been more useful.

Discussion of the Reasons for the Increases in Competencies across All Three Experimental Conditions

After discussing various reasons for the lack of intervention-related increases, we turn in the following to possible reasons for the significant and comparable increases in competencies across all three experimental conditions. How is it possible that students experience an (although low) increase in the selected mathematical modelling sub-competencies if the intervention was not successful?

The completion of test instruments as an opportunity for learning

According to Lipowsky (2015), the completion of test instruments can serve as an educational aid for fostering learning processes. In line with this description, empirical studies have confirmed that the completion of test instruments initiates learning-conducive effects (Roediger & Karpicke, 2006). It is thus possible that all the students profited from memory effects in completing the post-test due to having previously completed the pre-test.

Development of competencies

In addition, the students' increases in competencies can also be explained by their participation in regular lessons (Hofe, Pekrun, Kleine, & Götz, 2002). After all, the students attended their regular lessons for five months between the pre-test and the post-test. Unfortunately, we do not know anything about the content students were being taught in their regular lessons between the pre-test and the post-test. Thus, we cannot exclude the possibility that the students were practicing mathematical modelling in their regular mathematics lessons.

Our analyses did not succeed in confirming that reading strategy training has an influence on selected mathematical modelling sub-competencies (especially understanding and simplifying/structuring), that is, on the ability to understand and appropriately simplify or structure modelling problems. The aim of the remaining analyses will be to study whether there are any group-specific differences with regard to open modelling items the students had to work on in addition to the modelling tests presented here. Moreover, the extensive corpus process data analyzed during the intervention will also need to be analyzed at a qualitative level. These analyses will perhaps allow us to evaluate the success of the intervention in more specific terms. If the number of students per school allows for it, we also plan to conduct further quantitative analyses for specific types of schools. Finally, much research on the interplay of linguistic complexity and successful modelling processes is needed to be able to interpret the results being pointed out here: To what extent are difficulties in understanding and simplifying/structuring a modelling problem caused by difficulties in understanding singular words, difficulties in understanding singular sentences, or difficulties depending on the connections between the underlying

mathematical ideas and comprehension obstacles (e.g., the working group *Fach-und-Sprache* has been working on these basic problems in teaching and learning mathematics with regard to linguistic difficulties; see Leiss, Domenech, Ehmke, & Schwippert, submitted)?

What we can learn from our findings: As empirical research has demonstrated relationships between mathematics achievements and individual language proficiency, we also see in our results strong correlations ($r \geq .422$) between the mathematical modelling sub-competencies of understanding and simplifying/structuring and students' general language skills, including their reading comprehension. However, our findings did not succeed in confirming that our general reading strategy training had an influence on the selected mathematical modelling sub-competencies. This means that even if there is a strong connection between students' mathematical modelling sub-competencies of understanding and simplifying/structuring and students' general language skills, including their reading comprehension, it is not enough to merely foster students reading comprehension using general reading strategy training. Because obstacles to understanding and simplifying/structuring a modelling problem might lie not only in students' poor reading comprehension but also in conceptual obstacles, students are in need of mathematics-specific reading strategies. These strategies should help students to focus on the interplay between reading and finding mathematical relations. Further studies are needed to investigate this relationship.

SUMMARY

This article begins by examining possibilities for fostering mathematical modelling competencies described in the context of current empirical and educational policy discussions on designing competency-oriented mathematics instruction (see the Realistic problems in mathematics instruction section). In this context, we called attention to a research gap: Is it possible to foster the development of mathematical modelling competencies by providing students with reading strategies? As differences in reading comprehension have been explained by knowledge of reading strategies, among other factors (see the Reading comprehension in the focus of social and scientific interest section), we conducted a study to investigate whether it is possible to foster the mathematical modelling sub-competencies of understanding and simplifying/structuring in seventh-grade students by means of reading strategy training. Taking into account requests for language-sensitive teaching, we therefore developed two different teaching approaches for fostering the acquisition of reading strategies (integrated reading strategy training and separate reading strategy training) and tested them in the context of a 15-week intervention (see the Intervention study in the framework of the research project *Fach-an-Sprache-an-Fach* section). Our analysis of the intervention shows that while students who had participated in reading strategy trainings (ECs A and B) experienced a (low) increase in the selected mathematical modelling sub-competencies, the same increase could also be observed in students who had not participated in reading strategy training (EC C) (see the Results section). Based on these sobering results, we reflected on the intervention and the measurement instruments used in the study (see the Discussion section). We assume that solid reading comprehension skills are a necessary but probably not a sufficient condition for performing the first steps of the

modelling process (understanding and simplifying/structuring). Hence, the key potential obstacles to understanding and simplifying/structuring a modelling problem might lie not only in a lack of reading comprehension skills but also in the connection between the underlying mathematical ideas and comprehension obstacles. In order to support students' mathematical modelling competencies, more mathematics-specific reading comprehension strategies focusing on the interplay between reading and finding mathematical relations are needed. On the basis of the available process data as well as the open modelling items completed by the students, we aim at conducting further, more specific analyses.

NOTES

1 Fach-an-Sprache-an-Fach ("Subject to Language to Subject") is funded by the Mercator Institut für Sprachförderung und Deutsch als Zweitsprache [Mercator Institute for Language Acquisition and German as a Second Language].

2 Furthermore, researchers suggest that other competencies should be considered, e.g., metacognitive competencies (Stillman, 2011). The result is a complex combination of sub-competencies that serves as the necessary basis for mathematical modelling competencies in general (Niss et al., 2007).

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