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Young Children's Conceptual Understanding of Triangle

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This study explored 5-6 year-old children's conceptual understanding of one geometric shape, the triangle. It focused on whether children could draw a triangle from memory, and identify triangles of different types, sizes, and orientations. The data were collected from 82 children attending state preschool programs through a one-on-one interview, during which children completed five paper-pencil tasks. Findings showed that the majority of the children (93%-96%) successfully identified a prototyped triangle. Approximately half of the children experienced difficulties in identifying triangles of different sizes, types, and orientations. The most difficult area was identifying types of triangles, where an isosceles triangle and a right triangle were presented, followed by identifying triangles in different orientations, specifically flipped and rotated ones. Children appeared to identify and classify the triangle relying on the visual prototype. These findings provide support for the Prototyped Theory, van Hiele theory, and for works by Clement and colleagues.

Keywords: conceptual understanding of triangle; young children; Prototype theory; van Hiele.

INTRODUCTION

Research has shown that human babies are born with an innate ability of mathematics learning (Antell & Keating, 1983; Starkey, Spelke, & Gelman, 1990; Wynn, 1992), and that young children have a natural interest in mathematics (Cross, Woods, & Schweingruber, 2009; Geist, 2009). Studies have also shown that early mathematics skills are strongly associated with later school success (Bodovski & Farkas, 2007; Denton & West, 2002; Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009), and the early years are vital for establishing the foundations of mathematics learning and attitudes (Clements, 2001; Clements & Sarama, 2007; National Association for the Education of Young Children [NAEYC] / National Council of Teachers of Mathematics [NCTM], 2002). These findings collectively suggest that strengthening the early mathematics skills of children in preschool could be of a great importance to mathematics achievement as well as overall school success in the long run.

NCTM (2000) identifies five mathematics content areas, including numbers and operations, geometry and spatial sense, measurement, pattern/algebraic thinking, and displaying and analyzing data. As articulated in content standards, NCTM has

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identified geometry as one of three areas (the other two areas are numbers and measurement) that are particularly important for 3- to 6-year-olds. There is some evidence that children begin forming conceptual understanding of shapes in early childhood years, their understanding of shapes becomes pretty stable at the age of 6 (Gagatsis & Patronis, 1990), and "the ideal time to learn about shapes is between 3 and 6 years of age" (Clements, 1999, p.71). Based on the research, NCTM (2000) set expectations for children in grades preschool to 2, including

> ...recognize, name, build, draw, compare, and sort two- and three-dimensional shapes; ... describe attributes and parts of two- and three-dimensional shapes; ...investigate and predict the results of putting together and taking apart two- and three-dimensional shapes; ...recognize and apply slides, flips, and turns; ...recognize and create shapes that have symmetry; ...create mental images of geometric shapes using spatial memory and spatial visualization; ...recognize and represent shapes from different perspectives; ...recognize geometric shapes and structures in the environment and specify their location.

Mathematics education has recently begun to be part of early childhood education. Traditionally, many early childhood programs were play-oriented and focused on the development of social, emotional, and self-help skills; while formal mathematics education would not begin until the elementary grades (Hachey, 2013). Until recently, geometry was regarded as a topic which should be introduced to children in the late elementary school years. Despite the growing attention given to the geometry skills in early grades, still numbers and operations are the first content areas to which children are usually introduced. Teaching of geometric shapes is still less touched area in preschool years (Clements, 2004), and when it is addressed, the content is limited to recognizing and naming the geometric shapes (Casey et al., 2008; Oberdorf & Taylor-Cox, 1999).

Recently, the USA has set Common Core Standards (CCS) Initiative, a nationwide movement to establish core standards for several subjects, including mathematics. According to CCS, kindergarteners are expected to name and draw the shapes in different orientations and sizes, and describe, analyze, and compare properties and attributes of shapes (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). In Finland's educational system, children are expected to observe, recognize, explain, and name 2- and 3-dimensional shapes (Finnish National Board of Education, 2010). In the United Kingdom, the expectations for young children are to recognize and name common 2- and 3-dimensional shapes, including rectangles, squares, circles, triangles, and cubes in different orientations and sizes (United Kingdom Department for Education, 2013).

State of the literature

- Many mathematics education studies have focused on number sense and counting skills of children in the elementary grades at the youngest and been conducted in western countries, leaving a gap about younger children's conceptual understanding of geometric shapes in different cultural settings.
- Studies have shown that children use a visual prototype in identifying shapes and tend to make mistakes if shapes are in a non-prototypical form. However, it is unclear in which forms children experience difficulty.
- Many researchers have included multiple geometric shapes in a single study. Very few studies have focused on one geometric shape for a deeper understanding.

Contribution of this paper to the literature

- Children in this study which was conducted in Turkish state preschools showed similar patterns when identifying geometric shapes. That is, children appeared to identify and classify the triangle relying on the visual prototype.
- Findings shed light on forms of a non-prototyped examples children experience difficulties in identifying a triangle. Approximately half of children had difficulties in identifying triangles in different sizes, types, and orientations. The most difficult forms were isosceles and right triangles, followed by flipped and rotated triangles.
- Findings provide support for the Prototyped theory and van Hiele theory and are aligned with work by Clement and his colleagues.

The case in Turkey has not been much different than in other countries. Although some preliminary attempts were made in 1952 and 1989 (Alisinanoğlu & Bay, 2007), indeed, not until 1994, there was even a formal preschool education program, which would provide preschool teachers with a framework to use in planning a quality curriculum. In 1994, a formal preschool education program addressing development of children in the social, emotional, language, and cognitive domains and self-help skills was developed and revised in 2002, 2006 (Gelişli, & Yazıcı, 2012), and 2013. Specifically, the 2013 program identifies the skills needed to better prepare children for primary education and also provides preschool teachers with a framework to use in planning a quality curriculum. Among the many expectations in the cognitive domain, of special interest for this study are those for early mathematics, including topics such as numbers and operations, patterns, geometry, measurement, and spatial sense. Particularly focusing on the expectations for geometric shapes, three goals were set: [children] (1) name the geometric shapes (2) describe the attributes of the geometric shapes, and (3) recognize the objects that are similar to geometric shapes (National Ministry of Education [Milli Eğitim Bakanlığı; MEB], 2013). Basically, the program recommends teaching four geometric shapes: circle, square, triangle, and rectangle; and teaching polygons if children are ready.

Compared with the expectations in more developed countries (e.g. those in USA, Finland, and England) the expectations for Turkish 5-6 years old children in preschool classrooms appear to be similar or relatively lower. However, it is unknown what Turkish preschool children know about geometric shapes. Thus, this study was designed to explore 5- and 6- year-old children's conceptual understanding of geometric shapes in state preschool education programs in Turkey. Particularly, whether they were able to identify and differentiate triangles in various forms and contexts, and if they could create a mental image of the shape (i.e. draw from memory) were of interest.

Theoretical models of geometric understanding

In general, many researchers (e.g., Burger & Shaughnessy, 1986; Clements et al., 1999; Halat, 2006, 2007) have used van Hiele theory in explaining children's conceptual development of geometry. A Dutch mathematician, van Hiele (1959/1985) proposed that children go through hierarchical developmental stages of geometrical understanding. According to van Hiele, the stages are sequential and not discrete. That is, children go from one level to the next without skipping any level and they cannot be at the same level in all contexts; thus, children's conceptual level of geometry can be classified as being at one stage or another. In addition, children can move to the next level only by learning experience and teaching, being independent from age and maturation (van Hiele, 1986, 1999).

van Hiele (1986) initially identified five stages of geometrical understanding ranging from recognition of the shape by appearance to rigor. The first stage, Stage 0, also called visualization, is the entrance reasoning stage, in which children recognize and name the shapes based on their familiar appearance, usually on prototypes. Attributes of shapes (e.g., angles, corners, sides, lengths) are not attended, and a shape that does not match the prototype (e.g., a shape that is in a different form, size, or orientation) is not recognized. In the second stage, Stage 1, called analysis, children begin to recognize the attributes of shape, such as sides and corners. Studies explaining their findings through the lens of the van Hiele model have found that children at the preschool age are usually at the visualization or analysis stage (e.g., Burger & Shaughnessy, 1986; Clements et al., 1999; Halat, 2006, 2007).

Some researchers have disputed van Hiele's theory. For example, Clements and colleagues (Clements & Battista, 1992: Clements et al., 1999) argue that there is a pre-recognition level before van Hiele's visualization stage, called syncretic level, where children use declarative knowledge. In addition, they disagree with van Hiele's position which suggests that children operate within one level at a time. Instead, they argue that children can perform at more than one level simultaneously.

Another theory that has helped mathematics education researchers to explain children's reasoning behind the categorization of a shape is Prototype Theory (Rosch, 1973). The central tenet of the Prototype Theory is that concepts are formed by not primarily defined formal rules, features, or definitions, rather by the prototype(s), which is a typical and highly representative example(s) of concepts. Then, categorization occurs based on the similarity to the prototype(s). A prototype is a central member of a category and other members have a family resemblance based on the similarity. A central member of triangle concept, for example, is the equilateral triangle, and other types of triangles, such as an isosceles triangle, a scalene triangle, or a right triangle, have a family resemblance to the equilateral triangle.

Research has indicated that children identify and classify shapes based on a comparison to a visual prototype (Clement et al., 1999; Gal & Linchevski, 2010). Findings have also suggested that young children use a combination of visual prototypes and attributes that are imposed upon the visual prototype (Clements & Battista, 1992; Clements et al., 1999; Hershkowitz, 1989; Mason, 1989). Imposed attributes are those that are not formally determined as attributes such as sides, corners, or lengths, or are not necessarily formal attributes of a shape; instead they just match the prototype as how the shapes look (Hershkowitz, 1989). An example of an imposed attribute can be related to the orientation of the shapes on the page. For example an imposed attribute of a triangle might be that it has a horizontal base. A child at the visualization stage can recognize the square on the left in Figure 1 as it is a prototype, but not the one on the right side, because the figure on the right is rotated and the bottom of the shape is not parallel to the page.

Researchers have also attempted to understand children's reasoning behind categorization of a figure into a class of a shape. In a study with gifted fourth and eighth grade students, Mason (1989) found that there are three categories of reasoning in children's identification of geometric figures. The first one is based on the figures' appearance ("this look like a triangle"). The second one is based on the noncritical (called imposed by Hershkowitz) attributes of the figures, particularly for squares, rectangles, and triangles, which are usually acquired through the more frequent exposure to prototypical examples and limited or no exposure to other types. The last one is based on the critical attributes of the shapes, such as "a triangle has three sides and corners and the sides are closed". The author speculated a strong influence of the prototype figures on the identification of geometric figures. Similarly, in Hershkowitz's (1989) study with fifth, sixth, seventh, and eighth graders, children had difficulty in identifying a non-prototyped triangle, which was a

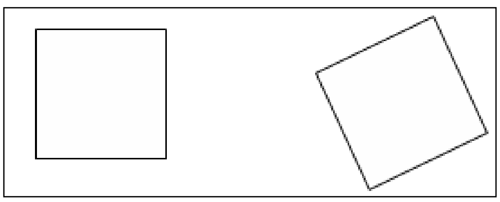

Figure 1. Two perspectives of a square
Note: Adapted from " In-Service Middle and High School Mathematics Teachers: Geometric Reasoning Stages and Gender," by E. Halat, 2008, The Mathematics Educator, 18, p.8. Copyright 2008 by Mathematics Education Student Association. Adapted with permission.

twisted equilateral triangle with the base that was not parallel to the bottom of the page. Studies with 3-6 years old children found a high rate of rejection of non-prototypical triangles (Hannibal, 1999) and rectangles (Clements & Battista, 1992).

Collectively, the findings of the existing research suggest that young children identify and classify the geometric shapes relying on visual prototypes of geometric shapes and this is independent from age or grade in school. Concept formation of geometric shapes involves the ability to use visual imagery. Children at early ages learn shapes by identifying visual representation of examples and non examples of shapes. As they gain more experience, they begin paying attention to common attributes of shapes with informal definitions, and then they combine the visual image and definitions (Walcott, Mohrb & Kastberg, 2009). Young children make incorrect identification of geometric shapes (Clements, 1999; Mason, 1989; Oberdorf & Taylor-Cox, 1999) due to the geometrical misconceptions, for which the possible reasons are the lack of exposure to vocabulary and to various forms of shapes (Oberdorf & Taylor-Cox, 1999).

Present study

Based on the given framework above, this study was designed to explore 5- and 6- year-old children's conceptual understanding of geometric shapes in state preschool education programs in Turkey. In particular, focusing on the geometric shape triangle, the study examined whether children were able to identify the triangles in different forms. An equilateral triangle with a horizontal base was considered as a prototyped triangle, the central member of the triangle concept, and any triangle that deviated visually from the prototyped triangle (any triangle in different size, orientation, or type) was considered as a non-prototyped example, or a resembling member of the triangle concept. It also investigated in what contexts children were or were not able to recognize or differentiate the triangle. The contexts were size, type, and orientation. Finally, it examined if they can draw the triangle from memory (i.e. mental representation).

This research is important for several reasons. First, an examination of existing literature suggests that majority of the studies with young children have been conducted in the western countries (Clements et al., 1999; Clements & Sarama, 2008; Hannibal, 1999; Mason, 1989; Walcott et al., 2009). It is important to study what children in different cultural settings can and cannot do. Second, limited numbers of studies (e.g., Clements et al., 1999; Hannibal, 1999) have focused on only one shape for a deeper understanding of children's conceptual knowledge. Next, studies on conception and misconception of geometrical shapes have usually focused on children in elementary grades at the youngest (e.g., Hershkowitz, 1987, 1989; Mason, 1989). Research exploring development of mathematical concepts of younger children has usually studied number sense and counting skills (e.g., Boonen, Kolkman, & Kroesbergen, 2011; Zur & Gelman, 2004) and paid less attention to young children's conceptual understanding of geometric shapes. Finally, the early childhood education field has rather based its practice on Piaget`s theory of logico-mathematical knowledge and development of spatial sense. Limited numbers of studies have focused on geometrical understanding of children in early childhood years and been inspired from different theoretical orientations (e.g., Clements et al., 1999; Hannibal, 1999; Walcott et al., 2009). This study aimed to explore preschool children's conceptual understanding of geometric shapes, with a theoretical framework rooted in Prototype Theory combined with van Hiele's geometrical concept development theory and with the addition of Clements and Hershkowitz's critiques.

METHOD

Participants

The study involved 82 preschool children (ages 5-6), 35 girls and 47 boys, recruited from three state-funded preschool education programs located in neighborhoods representing similar socioeconomic status. Children were grouped as 5-year-olds and 6-year-olds by their program when they were enrolled. The ages of children in months were not available to the researchers. One may argue that age in months might be an important factor for children's conceptual level of geometric shapes. However, the authors share the perspective with van Hiele (1999) that the conceptual development of geometric shapes are more about teaching and exposure than about age and maturation; and thus the ages in months may not be a critical factor as long as the age variation is reasonable. Because of the allowed age criterion to be enrolled in preschool education programs in Turkey, the age variation of children participated in the study was considered appropriate.

Data collection procedure, materials and analyses

The data were collected toward the end of the spring semester in 2014, through interviews in a one-on-one setting, where researchers administered a paper-pencil test to the preschoolers. The test, measuring children’s conceptual understanding of the geometric shape triangle, was developed by the researchers and included five tasks. According to Smith (2006), there are four difficulty levels of experiences with shapes for young children: Level 1 involves matching a form of shape to a similar one. Level 2 is about sorting the shapes by similarities. Level 3 is naming the shapes, and the Level 4 is drawing a shape from a model or memory.

The tasks were designed in different difficulty levels and presented to children in an order from simple to more complex, with the exception of the first task. The first task required the participants to draw a triangle, through which children were encouraged to represent mental image of the geometric shape, without providing an example or a reminder. This task is identified at the highest difficulty level by Smith (2006). It was given in the first place intentionally to identify if children had already formed an image of the triangle and if they could represent it before completing the subsequent tasks which include triangle figures in various forms. The second task asked the participants to differentiate the triangle from two other geometric shapes, a square and a star. All geometric shapes were prototyped examples of the shapes they represented and in regular sizes.

Tasks three through five were designed to measure children's abilities to identify the triangles in three different contexts, which were size, type, and orientation. The third task measured whether children were able to identify triangles in different sizes. It included figures of three equilateral triangles with horizontal bases in three different sizes among two others (a star-like shape and a square), for a total of five geometric shapes. All figures were printed on a line. In the fourth task, there were six geometric shapes: a circle, a pentagon, a square, an equilateral triangle with a horizontal base (prototyped), a right triangle, and an isosceles triangle. All figures were printed on the same line. In the fifth task, the pentagon was removed and it included seven geometric figures: a medium and a small size equilateral triangles with horizontal bases, an equilateral triangle with a flipped horizontal base (e.g. the base was at the top and pointy side was at the bottom), a rotated isosceles triangle (base was on the right and pointy side was on the left), a star, a square, and a circle in medium sizes. The figures of geometric shapes were not lined up, rather were placed randomly upper or lower positions relative to each other. The pictures and more detailed explanations of the tasks are included in the findings section.

During the interviews, the researchers read the questions to the participants, and children put their fingers on each shape that they thought it was a triangle. Researchers asked the preschool children to think loudly to unfold the reason behind their particular responses. Children took as long time as they needed to respond. The interview lasted approximately 6-15 minutes.

The researchers scored the participants' responses for correctness. They used descriptive statistics in the analyses of the data. Only several children explained their reasons for their responses, and the responses were limited to the statements such as "I know it is so", "I do not know", and "because". Thus, although initially we were interested in their reasoning behind their selections, we did not include those responses in the analyses as the quantity and quality of the responses were insufficient.

RESULTS

Task 1: Drawing a square

The first task included asking children to draw a triangle. Scoring of children's drawings showed that approximately 80.5 % (n=66) of the children correctly drew a triangle (see Figure 2 for examples). About 6.1% (n=5) did not draw any figure, and 13.4% (n=11) drew a geometric figure that cannot be defined as a triangle (see Figure 3 for examples).

Task 2: Distinguishing a triangle from two other geometric shapes, a square and a star

This task required children to recognize the triangle among the three geometric shapes, a square, a triangle, and a star, all of which were printed in the same size and were typical examples of the shapes they represent. That is, the square was printed parallel to the bottom of the page, and the triangle was an equilateral triangle with a horizontal page (see Figure 4).

Seventy-six children (92.7%) distinguished the triangle from the star and square. Three children identified another shape as triangle (one of them chose the star as a triangle, and two of them chose all three shapes as triangles), and three children did not respond.

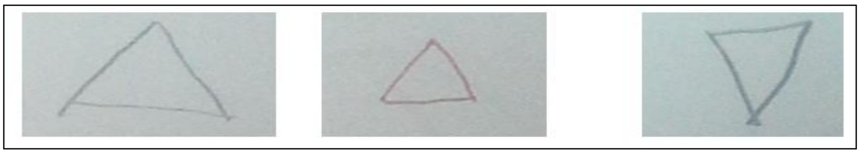

Figure 2. Examples for children's correct drawings of a triangle

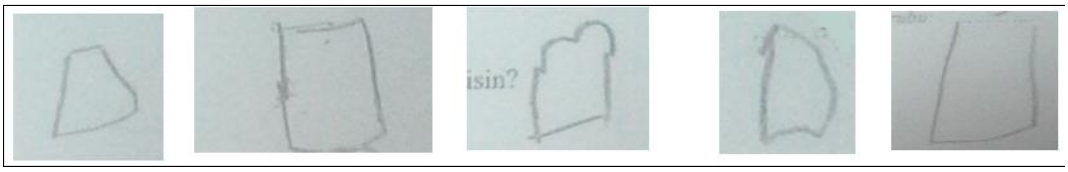

Figure 3. Examples for children's incorrect drawings of a triangle

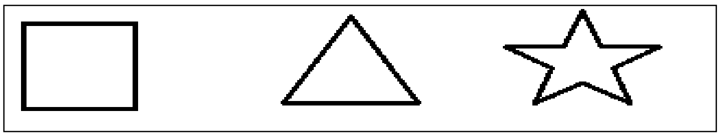

Figure 4. Task 2

Task 3: Identification of the three triangles in three different sizes

In this task, there were three triangles, a square, and a star-like shape printed on the page. The square with sides printed parallel to the page and a star-like shape were in regular sizes. The triangles were equilateral triangles with horizontal bases in small, medium, and large sizes (see Figure 5). Children were expected to identify all triangles in different sizes.

As shown on Table 1, about two-third of children (*n*=53, 64.6%) identified all three triangles in different sizes. One child left the question unanswered and the remaining 28 children (34%) missed at least one of the triangles. Six children (7.3%) identified only the largest one and did not consider the small and medium size triangles as triangles. Five children (6.1%) identified only the medium size triangle and did not identify either the small or large ones as a triangle. Ten children identified both the smallest and largest ones and missed the medium one, whereas seven children identified the medium and small ones, and missed the large one. Putting another way, 11 children (13.4%) did not identify the small triangle, 12 children (14.6%) did not identify the large triangle, and 16 children (19.5%) did not identify the medium triangle.

Task 4: Identification of an equilateral, an isosceles and a right triangle

In this task, children were shown a circle, a pentagon, a square, and three triangles. The three triangles were an isosceles triangle, a right triangle, and an equilateral triangle with horizontal bases (see Figure 6). Previous studies (e.g., Clements et al., 1999; Hannibal, 1999) used a variety of triangle types regardless of their classification by sides or angles, yet did not compare children's performances based on the triangle types. In this study we want to find out whether children's identification or classification of a triangle is related with being member of different types of triangles. To do so, an isosceles triangle, as a member of the triangle types categorized by side, a right triangle, as a member of the triangle types categorized by angle, and an equilateral triangle, as a member of both triangle types and also a prototyped triangle, upon which children build their conceptual understanding of the triangle were included. All shapes were in medium size. The pentagon and the

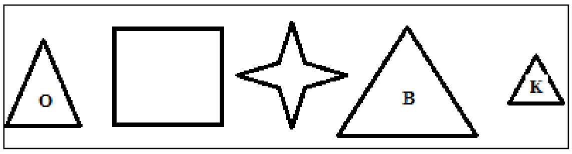

Figure 5. Task 3

Table 1. Descriptive statistics for identifying triangles in different sizes

Children's Responses	n	%
Unanswered	1	1.2
Identified triangles in all sizes (Small, Medium, and Large; K, O, B)	53	64.6
Identified only the largest triangle (B)	6	7.3
Identified only the largest and smallest triangles (B, K)	10	12.2
Identified only the medium and the smallest triangles (O, K)	7	8.5
Identified only the medium triangle (O)	5	6.1

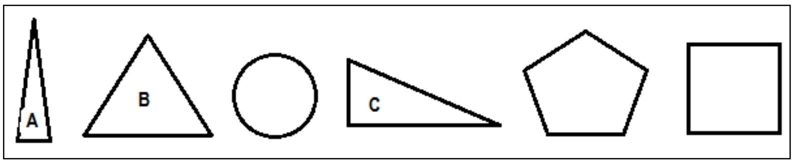

Figure 6. Task 4

square were printed parallel to the page. Children were expected to identify all three triangles.

Table 2 shows the frequencies and percentages for children's responses. The success rate for identifying all three triangles were 30.5% (n=25). On the other hand, 40.2% of children (n=33) identified only the equilateral triangle and did not recognize the isosceles or right triangle; 11% (n=9) identified both the equilateral and isosceles triangles, but the right triangle; and 8.5% (n=7) identified both the equilateral and right triangles but did not identify the isosceles triangle. Remaining children (n=8), 9.8% identified either the right triangle, or the isosceles triangle, or both. Overall, 91.2% of children identified the equilateral triangle. None of the children pointed to any other shapes for a triangle.

Task 5: Identification of rotated triangles and a small triangle

This task included a total of seven shapes (see Figure 7); four triangles, a circle, a star, and a square. There were a medium-sized equilateral triangle with a horizontal base (the prototyped one), a small-sized equilateral triangles with a horizontal base, a medium-sized flipped equilateral triangle (with a horizontal base at the top), and a 90-degree-rotated isosceles triangle (with horizontal base on the right). The circle and the star were medium-sized, and the square was small-sized with sides printed parallel to the page. Shapes were dispersed across the task box without any particular order.

As seen on Table 3, 42.7% of children (n=35) identified all four triangles; 19.5% (n=16) identified the medium- and small-sized equilateral triangles with horizontal

Table 2. Descriptive statistics for identifying triangles in different types

Children's Responses	n	%
Identified all types of triangles (Isosceles, equilateral, right; A, B, C)	25	30.5
Identified only equilateral triangle (B)	33	40.2
Identified equilateral and isosceles triangles (A, B)	9	11.0
Identified equilateral and right triangles (B, C)	7	8.5
Other	8	9.8

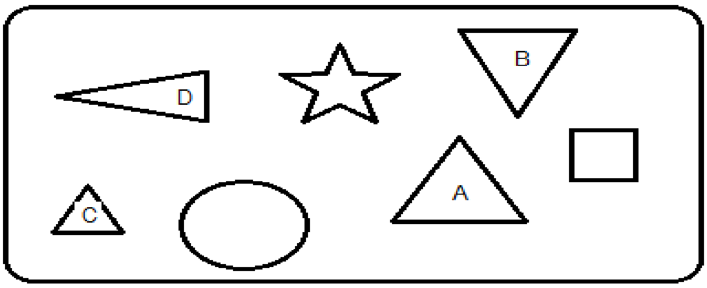

Figure 7. Task 5

Table 3. Descriptive statistics for identifying triangles in different types, sizes, and orientations

Children's Responses	n	%
Identified all triangles	35	42.7
Identified prototyped equilateral, flipped equilateral, and small size equilateral triangles (A, B, C)	16	19.5
Identified prototyped equilateral, flipped equilateral, and rotated isosceles triangles (A, B, D)	9	11
Identified prototyped equilateral and flipped equilateral triangle (A,B)	7	8.5
Identified only prototyped equilateral triangle (A)	5	6.1
Identified prototyped equilateral and small size equilateral triangle (A, C)	7	8.5
Other	3	3.7

bases, and the medium-sized flipped equilateral triangle, but missed the rotated isosceles triangle; 11% (n=9) identified an equilateral triangle with the horizontal base, a flipped equilateral triangle and a 90-degree rotated isosceles triangle, but missed the small-sized equilateral triangle; 8.5% (n=7) identified only the two medium-sized triangles one with the horizontal base and the other one is the flipped horizontal base; 8.5% (n=7) identified the both small- and medium-sized equilateral triangles with the horizontal bases; and 6.1% (n=5) identified only the typical, medium-sized equilateral triangle with the horizontal base. In other words, 19.5% missed the rotated isosceles triangle, 11% missed small equilateral triangle with the horizontal base, 8.5% missed the smaller size equilateral triangle with the horizontal base and the rotated isosceles triangle, 6.1% (n=5) missed a small equilateral triangle with the horizontal base, the rotated isosceles triangle, and the flipped equilateral triangle with a horizontal base, 8.5% (n=7) missed the flipped equilateral triangle and the rotated isosceles triangle. Approximately, 96.3% of children (n=79) identified the medium-sized equilateral triangle with the horizontal base.

DISCUSSION

Conducted with 82 preschool children between the ages of five and six, this study examined children's conceptual understanding of the triangle. It focused on whether children could draw a triangle from memory and identify triangles of different types, sizes, and orientations. The data were collected through a one-time one-on-one interview lasting about 6-15 minutes, during which children completed five paper-pencil tasks, all printed on the same page, in various difficulty levels ranging from differentiating a prototyped triangle (a medium size equilateral triangle with a horizontal base) from a square and a star to identification of triangles in different types, sizes, and orientations among other basic geometric shapes. The theoretical framework of the study took its roots from the Prototype Theory (Rosch, 1973) and Van Hiele's geometrical concept development theory (1959/1985, 1986, 1999), taking the Clements's and colleagues' works into consideration. In that sense, an equilateral triangle with a horizontal base was considered a prototyped triangle, the central member of the triangle concept, and any triangle that is deviated visually from an equilateral triangle with horizontal base was considered as a non-prototyped example, or a resembling member of the triangle category. The study tested no specific hypotheses, rather provided useful descriptive information. Central for this study was to determine in which areas children are confused when identifying the triangle. Three contexts were under investigation: sizes, types, and orientations.

The main findings of the study were that the majority of the children (93%-96%) identified a prototyped triangle, two-third of the children (64.6%) identified equilateral triangles with horizontal bases in different sizes, 30.5% recognized different types of triangles, and 42.7% recognized flipped or rotated triangles. These findings imply that children are more successful in identifying prototyped triangles, and they experience difficulties in identifying triangles in different sizes, types, and orientations. The most difficult area among the three contexts appeared to be types of triangles, where an isosceles triangle and a right triangle were presented, followed by the identification of triangles in different orientations (a flipped and a rotated triangle). Interestingly, although drawing a shape from memory (e.g. mental representation) is a higher level skill than identifying the pictures of shapes (Smith, 2006), children in this study were more successful in drawing a triangle (80.5%) than in identifying triangles in different contexts (see the percentages above).

The data of the study showed that 93%-96% of children were able to identify a prototyped triangle, the central member of the triangle family, across all tasks. On the other hand, the success rate for recognizing the resembling members of the triangle family, an isosceles triangle or a right triangle, was much lower. Among the different types of triangle, 48.5% of children missed the isosceles triangle, and 51.2% missed the right triangle. Putting in other ways, about 50% of children were confused when they were given different types of triangles. These findings suggest that majority of children in this study had formed the concept of triangle that a triangle has three sides, and three corners. Supporting the Prototyped Theory (Rosch, 1973), their concept formation was based on the prototyped example, as almost all children were able to recognize the prototyped triangle. On the other hand, for approximately half of the children, family membership for isosceles triangles and right triangle had not developed yet, as they had failed to identify both. These findings are also in agreement with existing research suggesting that children identify and classify shapes comparing a visual prototype (Clements, Swaminathan, Hannibal, & Sarama, 1999; Gal & Linchevski, 2010) and show higher rate of failure in recognizing non-prototypical triangles (Hannibal, 1999). The possible reason for missing non-prototyped examples may be, as suggested in the literature (e.g., Oberdorf & Taylor-Cox, 1999), the lack of exposure to triangles in various types, sizes, and orientations. In terms of practical implications, early childhood educators should provide children with opportunities to experience typical and atypical examples as well as non-examples of geometric shapes in various forms. In addition, early childhood teacher education programs should emphasize preschool children's misconceptions about geometric shapes in mathematics methods courses.

These findings imply that the majority of children in this study were at or above Van Hiele's visualization stage. Evidently, data showed that children's recognition of non-prototyped examples of triangle varied across the tasks. For example, children were more successful in recognizing a flipped equilateral triangle than in recognizing equilateral triangles of different sizes (approximately 82% vs. 64%, respectively). It is particularly important to note that approximately 15% of children failed to identify a prototyped triangle when its orientation was changed. One possible reason would be, supporting Clements's and colleagues' positions (Clements & Battista, 1992: Clements et al., 1999), some children in this study may be operating at more than one level simultaneously and some children may be transitioning to the next level, and experiencing conflict between prototype matching and property analyses (Clements et al., 1999). Moreover, this finding suggests that, congruent with Hershkowitz (1989), children may classify the shapes using imposed attributes.

This study comes with several limitations, which may give directions for future research. First, it was limited to data collected during a one-time, one-on-one interview, during which children were only involved in a paper-pencil task and were required to make selection(s). Observing children at different times and in different settings, as well as in different contexts may provide better information. In addition, future studies should include more non-typical examples of the shapes, including triangles with different types by angles, and sides, and in different rotations, for instance 45 degree and 75 degree rotations. Also, having the data about amount and quality of mathematics experience at home and in preschool may shed some light on children's success or failure to recognize the non-prototypical examples. Moreover, examining gender and age differences and their geometric skills in connection with other mathematics areas would also be useful. Finally, future research should also focus on exploring whether preschool teachers believe in expectations set for preschool children, whether they expose children to geometric shapes, and if they do, how frequent and in what quality they provide children with learning experiences of geometric shapes, particularly in different countries, like Turkey, would contribute to the literature.

In summary, children participated in the study which was conducted in Turkish state preschool programs showed similar patterns when identifying and classifying geometric shapes. Although approximately half of children's conceptual understanding is limited to the prototyped triangle, as they missed non-prototyped examples (different types, sizes, and orientations), almost all children participating in the study had formed a conceptual understanding that a triangle has three sides, and three corners, as they did not classify a square or a star in the triangle category. Moreover, even though children's verbal reasoning was not investigated, the data provides some evidence that children identified and classified the triangle relying on the visual prototype which is an equilateral triangle with a horizontal base. Collectively, these findings provide support for Prototyped theory and van Hiele theory, in addition to providing evidence aligned with work by Clement and his colleagues.

REFERENCES

Alisinanoğlu, F., & Bay, N. (2007, Eylül). *Okul Öncesi Eğitim Programlarının Tarihsel Gelişimi*. XVI. Ulusal Eğitim Bilimleri Kongresi. Tokat. Retrieved October 24, 2014 from http://www.pegem.net/akademi/kongrebildiri_detay.aspx?id=5773

Antell, S., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695-701.

Bodovski, K., & Farkas, G. (2007). Mathematics growth in early elementary school: The roles of beginning knowledge, student engagement, and instruction. *The Elementary School Journal, 108*(2), 115-130. http://dx.doi.org/10.1086/525550

Boonen, A. J. H., Kolkman, M. E., & Kroesbergen, E. H. (2011). The relation between teachers' math talk and the acquisition of number sense within kindergarten classrooms. *Journal of School Psychology, 49*, 281-299. doi:10.1016/j.jsp.2011.03.002

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*, 31-48.

Casey, M. B., Andrews, N., Schindler, H., Kersh, J. E., Samper, A., & Copley, J. (2008). The development of spatial skills through interventions involving block building activities. *Cognition and Instruction, 26*(3), 269-309. doi:10.1080/07370000802177177

Clements, D. H., &Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-64). New York, NY: Macmillan.

Clements, D. H. (1999). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 66-79). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H. (2001). Mathematics in the preschool. *Teaching Children Mathematics*, *7*, 270-275.

Clements, D. H. (2004). Geometric and spatial thinking in early childhood education. In D. H. Clements, Sarama, & A. M. Dibiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp.7-72). Mahwah, NJ: Erlbaum.

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. In J. F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 461-555). New York: Information Age Publishing.

Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal, 45*, 443-494.

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science, 333*(6045), 968-970. doi:10.1126/science.1204537.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young Children's Concepts of Shape. *Journal for Research in Mathematics Education, 30(2)*, 192-212.

Cross, T. C., Woods, T. A., & Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood: Paths towards excellence and equity*. Washington, DC: National Academies Press.

Denton, K., & West, J. (2002). *Children's reading and mathematics achievement in kindergarten and first grade*. Washington, DC: National Center for Education Statistics. NCES 2002-125. Retrieved from http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2002125

Duncan, G., Dowsett, C., Claessens, A., Magnuson, K., Huston, A., Klebanov, P., et al. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428-1446. http://dx.doi.org/10.1037/0012-1649.43.6.1428

Finnish National Board of Education (2010). *National Core Curriculum for Pre-primary Education*. Retrieved from http://www.oph.fi/download/153504_national_core_curriculum_for_pre-primary_education_2010.pdf

Gagatsis, A., & Patronis, T. (1990). Using geometrical models in a process of reflective thinking in learning and teaching mathematics. *Educational Studies in Mathematics, 21,* 29-54.

Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception*. Education Studies in Mathematics, 74,* 163–183.

Geist, E. (2009). *Children are born mathematicians: Supporting mathematical development, birth to age 8*. Upper Saddle River, NJ: Pearson.

Gelişli, Y. & Yazıcı, E. (2012).Türkiye`de Uygulanan Okul Öncesi Eğitim Programlarının Tarihsel Süreç İçerisinde Değerlendirilmesi. *Gazi Üniversitesi Endüstriyel Sanatlar Eğitim Fakültesi Dergisi, 29,* 85-93. from http://esefdergi.gazi.edu.tr/makaleler/290.pdf

Hachey, A. C. (2013). The early childhood mathematics education revolution. *Early Education & Development, 24*(4), 419-430. doi: 10.1080/10409289.2013.777286

Halat, E. (2006). Sex-related differences in the acquisition of the van Hiele levels and motivation in learning geometry. *Asia Pacific Education Review, 7*(*2),* 173-183.

Halat, E. (2007). Reform-based curriculum & acquisition of the levels. *Eurasia Journal of Mathematics, Science and Technology Education, 3(1),* 41-49.

Halat, E. (2008). In-service middle and high school mathematics teachers: Geometric reasoning stages and gender. *The Mathematics Educator, 18* (1), 8-14.

Hannibal, M. A. Z. (1999). Young children's developing understanding of geometric shapes. *Teaching Children Mathematics, 5*(6), 353-357.

Hershkowitz, R. (1987). The acquisition of concepts and misconceptions in basic geometry—Or when "a little learning is a dangerous thing.". In J. D. Novak (Ed.), *Proceedings of the second international seminar on Misconceptions and Educational Strategies in Science and Mathematics, vol. 3* (pp. 236–251). Ithaca, NY: Cornell University.

Hershkowitz, R. (1989). Visualization in geometry: Two sides of the coin. *Focus on Learning Problems in Mathematics*, *11*(1), 61-76.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45,* 850-867. doi:10.1016/j.lindif.2009.07.004

Mason, M. M. (1989, March). *Geometric understanding and misconceptions among gifted fourth-eighth graders*. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA. ED 310922

National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) [NAEYC/NCTM]. (2002). *Early childhood mathematics: Promoting good beginnings*. Washington, DC: National Association for the Education of Young Children

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and Standards for School Mathematics*. Math Standards and Expectations: Geometry Strand. Retrieved from http://www.nctm.org/standards/content.aspx?id=314

National Governors Association Center for Best Practices, Council of Chief State School Officers. *Common Core State Standards (Mathematics Content Standards: Geometry).* Washington D.C. Copyright Date: 2010. Retrieved from http://www.corestandards.org/Math/Content/K/G/

National Ministry of Education [Milli Egitim Bakanligi; MEB]. (2013).*Turkish Preschool Education Program (Okul Öncesi Eğitim Programı)*. Ankara. Retrieved from http://tegm.meb.gov.tr/dosya/okuloncesi/ooproram.pdf

Oberdorf, C. D., & Taylor-Cox, J. (1999). Shape up! *Teaching Children Mathematics, 5,* 340-345.

Rosch, E. H. (1973). Natural categories. *Cognitive Psychology, 4*, 328–350.

Smith, S. S. (2006). *Early childhood mathematics* (3rd Ed.), Boston, MA: Pearson Education.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36*, 97-128.

United Kingdom Department for Education (2013).*National curriculum in England Mathematics programmes of study: key stages 1 and 2.* Retrieved from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

van Hiele, P. M. (1959/1985). The child's thought and geometry. In D. Fuys, D. Geddes, & R. Tishchler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele* (pp. 243–252). ERIC/SMEAC.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. New York: Academic Press.

van Hiele, P. M. (1999). Developing geometrical thinking through activities that begin with play. *Teaching Children Mathematics, 6,* 310-316.

Walcott, C., Mohrb, D., & Kastberg, S. E. (2009). Making sense of shape: An analysis of children's written responses. *Journal of Mathematical Behavior, 28,* 30-40. doi:10.1016/j.jmathb.2009.04.001

Wynn, K. (1992). Evidence against empiricist accounts of origins of numerical knowledge. *Mind and Language, 7*, 315-332.

Zur, O., & Gelman, R. (2004). Young children can add and subtract by predicting and checking. *Early Childhood Quarterly Review*, *19*, 121-137. doi:10.1016/j.ecresq.2004.01.003

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