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Chinese High School Students' Mathematics-Related Beliefs and Their Perceived Mathematics Achievement: A Focus on Teachers' Praise

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ABSTRACT

This study examined the relationships between mathematics teachers' praise and students' mathematics-related beliefs and their perceived mathematics achievement rank in their respective classes. This study provided reliability and validity information about an instrument to assess high school students' mathematics beliefs. Students' mathematics-related beliefs are positively related to teachers' praise, as well as students' perceptions of their own achievement rank in their respective classes. While it was found that the proportion of students who reported that their teachers praised them was relatively low (about 28%), the more praise teachers provide, the more desirable mathematics-related beliefs students have. In addition, the more praise teachers provide, the higher students perceived achievement. The findings of this study suggest that teachers' praise may foster more desirable mathematics-related beliefs and achievement.

Keywords: China, mathematics, students' mathematics-related beliefs, teacher praise

INTRODUCTION

Students' beliefs related to mathematics have long been an important topic of study in mathematics education, both because of their relationship to affect and motivation (McLeod, 1992; Middleton, Jansen, & Goldin, 2017) and because of their potential influence on students' mathematical thinking and activity (Schoenfeld, 1989, 1992). Indeed, the teaching and learning of mathematics necessarily involves affective and noncognitive considerations including beliefs, emotions, values, and attitudes (Cai et al., 2017; Clarkson, Bishop, & Seah, 2010; Leder, 1993; Moyer, Robinson, & Cai, 2018; Pepin & Roesken-Winter, 2015). Op't Eynde, De Corte, and Verschaffel (2002) developed a comprehensive categorization of mathematics beliefs: beliefs about mathematics education, beliefs about the self, and beliefs about social context. Each of these categories of mathematics-related beliefs is potentially related to students' mathematical behavior.

Moreover, mathematics-related beliefs have been studied as predictors of students' mathematical achievement (House, 2009; Lay, Ng, & Chong, 2015). Given the potential ways in which mathematics-related beliefs both influence and are influenced by factors, such as mathematics achievement and interactions with parents and teachers, it is hardly surprising that students' mathematics-related beliefs are increasingly emerging as an important topic in the international mathematics education research community. Studies have shown that teachers' behaviors have an effect on educational aspirations, cognitive ability, mathematics achievement, mathematics learning, motivational beliefs about mathematics, mathematical self-efficacy, and so forth (e.g., Levpušček & Zupančič, 2009; Phillipson, 2010; Zijlstra, Wubbels, Brekelmans & Koomen, 2013). However, there has not been a systematic body of research focusing on the influence of factors such as teachers on students' mathematics-related beliefs.

Students in East Asian countries and regions have excelled in large-scale international academic assessments such as TIMSS and PISA. Studies have shown that this has a certain relationship with the performance of teachers

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Contribution of this paper to the literature

- Provides a valid and reliable instrument to assess Chinese students' mathematics-related beliefs.
- Students' mathematics-related beliefs are positively related to teachers' praise, as well as students' perceptions of their own achievement rank in their respective classes.
- While it was found that the proportion of students who reported that their teachers praised them was relatively low (about 28%), the more praise teachers provide, the more desirable mathematics-related beliefs students have.
- The more praise teachers provide, the higher students perceived achievement.
- The findings of this study suggest that teachers' praise may foster more desirable mathematics-related beliefs and achievement.

(Phillipson & Phillipson, 2007). China is a typical representative of the Confucian Heritage Culture (CHC) countries (Fan, Wong, Cai, & Li, 2004) which exhibit these culturally situated factors. The research reported here was designed to investigate the current status of mathematics-related beliefs among Chinese senior high school students, to analyze the influence of Chinese teachers on the beliefs of high school students in mathematics, and to provide a reference point for education in other countries and regions.

In particular, this study is designed to investigate the following two research questions:

1) How are teachers' appraisals related to students' mathematics-related beliefs and their perceived mathematics achievement?
2) How are students' mathematics-related beliefs related to their perceived achievement?

THEORETICAL BASES

Teacher Interpersonal Behavior

Early research showed that when students feel that their teachers value, care for, and support them, they are more likely to have higher motivation for academic achievement (Goodnow, 1993; Wentzel, 1997). Patrick, Ryan, and Kaplan (2007) pointed out that such support includes the teacher caring about students' learning, trying to help them learn, and encouraging them to do their best. In addition, some studies have shown that teacher care (teacher support) has a significant effect on students' academic performance after controlling for family factors (Marchant, Paulson & Rothlisberg, 2001). Indeed, Levpušček and Zupančič (2009) found that the support and encouragement of mathematics teachers had an even greater impact on students' mathematics achievement than parental education, although they noted that "the combined effect of both student-rated family and classroom context on eighth graders' math performance is stronger than the effect of either context alone" (p. 563).

Based on the existing research, Lapointe, Legault, and Batiste (2005) further explored the influence of teacher interpersonal behavior on students' mathematics learning motivation and found that mathematics teachers' interpersonal relationship behaviors have a significant effect on both regular students and students identified as talented. Zijlstra, Wubbels, Brekelmans, and Koomen (2013) investigated the relationship between 6-9 year-old students' perceptions of teacher interpersonal behavior and achievement in mathematics and found that students' views on mathematics teachers' control and connection were important predictors of student achievement in mathematics.

In summary, the existing studies mostly examine the effects of the behavior of teachers on overall mathematics academic achievement and mathematics learning motivation but have not paid detailed attention to the influence of mathematics teacher behavior on students' mathematics-related beliefs (although see the discussion of Levpušček & Zupančič, 2009, about the mediating role of self-efficacy between parental and teacher influences and mathematics achievement). The present study focuses in particular on the encouragement of mathematics teachers' evaluative speech, specifically the influence of mathematics teachers' praise on students' mathematics-related beliefs. Based on the existing research, it is assumed that positive recognition by the mathematics teacher has a positive influence on students' mathematics-related beliefs, and that its influence may be greater than parental education expectations.

Mathematics Achievement and Self-Efficacy

A large number of studies have shown that there is a significant correlation between student mathematics-related beliefs and student mathematics achievement (e.g., House, 2009; Lay, Ng, & Chong, 2015). These studies typically use students' mathematics achievement as a dependent variable and some particular student mathematics-related beliefs, including beliefs about mathematics self-efficacy, as predictors. For example, Skaalvik

Table 1. Participants' Background Information

	Number of participants	%
Gender		
Male	250	50.4
Female	246	49.6
Grade		
10	265	53.4
11	136	27.4
12	95	19.2

and Skaalvik (2011) explored the relationship between mathematics self-concept and self-efficacy and mathematics achievement. They found that students' self-efficacy was a strong predictor of achievement even beyond prior achievement. Pantziara (2016) noted, however, that few of these studies "provide causal ordering between the two constructs" (p. 8) of mathematics self-efficacy and mathematics achievement. Nevertheless, House (2009) concluded after a series of studies that students with better mathematics performance tend to have more positive mathematics-related beliefs, and those with poor mathematics performance have more negative mathematics-related beliefs. In other words, students' academic achievement in mathematics may have a positive effect on students' mathematics-related beliefs and vice versa.

In this study, the students' perception of their relative mathematics achievement ranking in their class is used as the measure of student achievement, and we assume that the students' perceived class mathematics achievement ranking may have a positive influence on their mathematics-related beliefs (and vice versa). In a study of mathematics students in Finland, Hannula, Bofah, Tuohilampi, and Metsamuuronen (2014) found that mathematics achievement and self-efficacy had a reciprocal relation, where the dominant effect was from achievement to self-efficacy. They also found a weaker unidirectional effect from achievement to emotion. Cai and Merlino (2011) also found that students' success in mathematical achievement was linked to positive mathematical dispositions. Students who succeeded in the process of learning mathematics through concerted effort in overcoming challenges gained a sense of satisfaction that bolstered their motivation to continue challenging themselves in mathematics. Cai and Merlino (2011) posited that this could produce a positive disposition-performance cycle.

METHOD

Participants

A survey was administered nationwide using a combination of online sampling and regional cluster sampling. The survey began with regional cluster sampling, and this was followed by an online sampling to ensure a high response rate. In total, 496 valid questionnaires were collected. The participants came from the following parts of China: Central China, North China, East China, South China, Northwest China, Northeast China and Southwest China. The background information of the participants is shown in **Table 1**.

Instrument

The survey instrument was adapted from a questionnaire originally developed by Op't Eynde and De Corte (2003). Because PISA and TIMSS are international assessment projects that include items related to mathematics beliefs, we also drew on those assessments in the development of the questionnaire. However, although we included items drawn from PISA and TIMSS, the instrument developed by Op't Eynde and De Corte had useful theoretical properties with respect to beliefs subscales. Thus, we made use of this theoretical structure by adopting the first three of the four subscale categories established by Op't Eynde and De Corte. We focused our fourth subscale on mathematics anxiety, using items from TIMSS.

The survey instrument underwent three stages of revision and refinement before being used in this study. In each stage, a group of mathematics education researchers have served as expert to ensure the content validity. In this section, we discuss the process of developing the Chinese version of the instrument.

The preliminary edition and its piloting

The Students' Mathematics-Related Beliefs Questionnaire (SMRBQ) compiled by Op't Eynde and De Corte (2003) was revised for use with Chinese high school students. We called the revised instrument the High School Student Mathematics Belief Questionnaire. The English version of the SMRBQ was translated into Chinese and then translated back into English by graduate students with good English language skills. We revised the Chinese version by comparing the result with the original English version. Taking into account applicability under the background of Chinese mathematics education and the accuracy of the terms in the questionnaire, combined with the characteristics of mathematics studying of Chinese high school students, and drawing on the PISA-2012 background questionnaires and the TIMSS-2015 background questionnaires on student mathematics-related belief items, we added or deleted some items, eventually forming the preliminary draft of a mathematics-related belief questionnaire with 52 items for high school students. The preliminary draft of the questionnaire retained three of the four subquestionnaires included in the original questionnaire. These were 1) beliefs about the role and the functioning of their own teacher, 2) beliefs about the significance of and competence in mathematics, and 3) mathematics as a social activity (Op't Eynde & De Corte, 2003). The preliminary draft also included items, primarily from TIMSS-2015, comprising a fourth subquestionnaire that focused on mathematics anxiety. All 52 items were randomly arranged. Nine of the items were scored in reverse.

The preliminary edition of the questionnaire was piloted with 190 students in two high schools of Tianjin City. In total, 182 responses were collected, and 174 valid responses were examined with item analyses. The participants in the pilot were 80 boys and 94 girls; 61 students were in 10th grade and 113 students were in 11th grade. The pilot data were used to examine the clarity of the language used in the items. The questionnaire was revised based on the pilot data. In particular, based on the analysis of results from piloting, the feedback from the subjects, and the suggestions of the in-service secondary mathematics teachers, the following changes were made: two items were deleted because they did not achieve significant differences between the high and low groups; one item was deleted because the Critical Ratio was low; and regarding the item, "I need mathematics to help me learn other subjects," we added examples such as "Physics, Geography, and so forth" to the text of the item in order to make the questionnaire easier for high school students to understand. Thus, the second edition of the questionnaire contained 49 items and had the same structure as the initial draft questionnaire.

The second edition and its validation

The second edition of the questionnaire was administered in 13 secondary schools from nine cities in China. In total, 960 questionnaires were distributed, 936 were returned, and 902 were valid. Through item analysis and exploratory factor analysis, we deleted those items which did not meet the psychometric criteria (with low loading). The participants in this second round were 460 boys and 448 girls; 402 were in 10th grade, 356 were in 11th grade, and 150 were in 12th grade.

Based on the total scores of students' mathematics-related beliefs, the top 27% were considered the high score group and the bottom 27% the low score group. The independent samples t test was conducted for the scores on each item for the high and low score groups, and a correlation analysis was also implemented between all the items and the total scores of each subquestionnaire. The deletion criteria for the correlation coefficient between all items and the subquestionnaire was set at 0.40. One item with insignificant t value and 3 items with correlation coefficients less than 0.40 were deleted, and 45 items were retained. The correlation coefficients between the scores from the 45 items and the total score of the subquestionnaires was distributed between 0.45 and 0.85 (all $p < 0.001$).

An exploratory factor analysis was conducted on the remaining 45 items. The results indicated that the KMO values of each subquestionnaire in the exploratory factor analysis ranged from 0.82 to 0.95, and Bartlett's sphere test reached a significant level ($p < 0.001$). The results revealed that each subquestionnaire was suitable for the factor analysis.

According to the research objectives and theoretical conceptions, after the first factor analysis of each subscale, we first deleted the items with the highest load on the other extracted factors and then conducted factor analysis again, so that the item-by-item deletion was performed successively for a series of factor analyses until the final extracted factors were in line with the theoretical conceptions. In the end, the entire questionnaire retained 40 items, and the load of factors for each item on the subscales ranged from 0.57 to 0.89. The explained variation ranged from 54.83% to 61.49%.

The Cronbach's α coefficients of the subscales and the α coefficients of the 40 items were examined. The results showed that the α coefficient after the two items in the subscale of Beliefs about the significance of and competence in mathematics had been deleted was 0.934, and the α coefficient after an item in the subscale of Mathematics as a social activity was deleted was 0.882, both of which were greater than the Cronbach's α coefficients of the respective subscale. The α coefficient of the Beliefs about the role and the functioning of their own teacher is equal to the Cronbach's α coefficient of the subscale, which indicates that after the deletion of the item, the reliability of the subscale was not changed. Finally, 36 items were retained.

The scores of students' mathematics-related beliefs were ranked. The top 27% were considered the high score group and the bottom 27% the low score group. The scores of high and low groups on each item were tested by independent samples t test. The results showed that the high score group's scores on the 36 items were significantly different from the low score group's scores, and all items had good discrimination.

Table 2. Goodness of Fit of Four-Factor Model for the High School Students Mathematics Belief Questionnaire

χ^2	df	NC	RMSEA	GFI	AGFI	PGFI	CN
1200.66***	629	1.91	0.05	0.82	0.80	0.73	209

Note: ***p < .001

Table 3. Correlation Coefficients of the High School Students Mathematics Belief Questionnaire, the IMBS Subscale of Time-consuming Mathematics Problem Solving, and FSMAS

	Subscale 1	Subscale 2	Subscale 3	Subscale 4	Overall scale	IMBS	FSMAS
Subscale 1	1						
Subscale 2	0.58***	1					
Subscale 3	0.61***	0.53***	1				
Subscale 4	0.36***	0.34***	0.31***	1			
Overall	0.82***	0.91***	0.81***	0.76***	1		
IMBS	0.28***	0.55***	0.33***	0.46***	0.52***	1	
FSMAS	0.44***	0.40***	0.53***	0.11***	0.49***	0.29***	1

Note: ***p < .001

The third edition and its reliability and validity

The third edition of the questionnaire was administered to students from five secondary schools in four cities. These schools did not administer the two earlier versions of the questionnaire. There were 370 questionnaires distributed; 362 were returned, and 351 of those were valid. We reconfirmed the factors through confirmatory factor analysis and obtained the reliability and validity indices. The participants in this round were 179 boys and 167 girls, with 5 participants missing a gender record; 109 were in 10th grade, 137 were in 11th grade, and 105 were in 12th grade.

The reliability analysis of the high school students' belief in mathematics questionnaire showed that the Cronbach's α coefficient of each scale was between 0.84 and 0.95 (see **Table 4**), the retest reliability was between 0.79 and 0.87, which are reasonably high.

With respect to the construct validity, a maximum likelihood method was used to perform a confirmatory factor analysis on the officially tested sample (n = 351). The four subscales were used to constitute a four-factor model. We selected χ2, NC, RMSEA, GFI, AGFI, PGFI, and CN as evaluation indicators for the fit of the model. The results are shown in **Table 2**.

For the fit of the hypothesis model, χ2 = 1200.66, df = 629, p < 0.001, and χ2 reached a significant level, which does not meet the standard. However, χ2 is susceptible to sample size and sample distribution, so this value is for reference only. NC (χ2/df) = 1.91, which is between 1.00 and 3.00, indicating that the model's fit is high. Both PGFI = 0.73 > 0.50 and CN = 209 > 200 indicate that the four-factor model meets the fit criteria. Neither GFI nor AGFI reached the standard of 0.90, but Anderson and Gerbing (1984) advocated using 0.80 as the critical value for goodness of fit, so both GFI = 0.82 and AGFI = 0.80 are acceptable. RMSEA, as an indicator of noncentralized Chi-square indicators in the third category, which is not affected by sample distribution and sample size, is very suitable for estimating the power of statistical tests. The smaller the RMSEA value is, the better the fit of the model is. The RMSEA value of the four-factor model was 0.05, which means a good fit. In summary, the four-factor model fit index reached the basic standard.

For the criterion validity, the subscale of time-consuming mathematics problem solving and the mathematical usefulness attitude scale (FSMAS) in the Indiana Mathematical Belief Scale (IMBS) (Fennema & Sherman, 1976) were treated as criteria. There was a significant positive correlation between the scales (see **Table 3**). The correlation coefficients between the four subscales were between 0.31 and 0.61. The correlation coefficients between the four subscales and the overall scale were between 0.76 and 0.91. This shows that the four subscales are relatively independent and can make a greater contribution to the entire questionnaire. The overall questionnaire has good construct validity.

Measures

The revised High School Students Mathematics Belief Questionnaire was used to measure the levels of the students' mathematics-related beliefs. Four additional questions were included in the Questionnaire on High School Students' Belief in Mathematics to investigate mathematics teachers' praise, and mathematics achievement ranking in the class. **Table 4** lists the questionnaire items used for these four topics.

Table 4. Four Additional Items from the Questionnaire

Topic	Questionnaire Item
Mathematics teachers' praise	Math teachers often praise me. 1. Strongly disagree 2. Disagree 3. Neutral 4. Agree 5. Strongly agree
Students' perceived math achievement ranking in the class	What is your math achievement ranking in the class? 1. High (top 10%) 2. Upper middle (10% to 35%) 3. Middle (35% to 65%) 4. Lower middle (65% to 85%) 5. Low (bottom 15%)

RESULTS

We present the results in three sections. The first section deals with relationships between students' mathematics-related beliefs and the measured factors regarding mathematics teachers' praise. The second section examines relationships between students' perceived mathematics achievement rankings in the class and mathematics teachers' praise. The third section deals with the relationship between students' perceived mathematics achievement rankings in the class and students' mathematics-related beliefs.

Mathematics-Related Beliefs and Teacher Praises

In order to examine the relationships between the mathematics-related beliefs scores (total and for each subscale) and teachers' praise, we conducted an ANOVA analysis for each factor and each belief score (total or subscale). **Table 5** shows the results for teachers' praise and students' mathematics-related beliefs. As indicated in **Table 5**, only about 28% (139 out of 496) of the students agreed or strongly agreed that their teachers gave them praise. In contrast, over one third of the students disagreed or strongly disagreed that their teachers gave them praise. Overall, the higher the degree of teachers' praise, the more positive the students' mathematics-related beliefs were. For example, the mean overall mathematics-related beliefs score for those students who strongly agreed that their teachers praised them was 147. However, the mean score for those students who strongly disagreed that their teachers praised them was 102. These findings for overall mathematics-related beliefs remained consistent across the first three subscales of mathematics-related beliefs. For the fourth subscale, the ANOVA analysis did not show a significant difference.

Table 5. Teachers' Praise and Students' Mathematics-Related Beliefs

	Teachers' Praise	Mean	*SD*	# of Students	*F*	η^2
Mathematics-Related Beliefs (total)	Strongly Disagree	102.2910	24.24269	61	51.008***	.294
	Disagree	114.6824	17.19791	110		
	Neutral	127.1509	18.01370	186		
	Agree	134.7883	15.92462	94		
	Strongly Agree	147.0222	25.59163	45		
Beliefs about the role and the functioning of their own teacher instruction	Strongly Disagree	39.9180	11.97956	61	45.216***	.269
	Disagree	45.1578	8.06569	110		
	Neutral	49.9856	8.05507	186		
	Agree	53.2522	6.70143	94		
	Strongly Agree	59.1556	9.37588	45		
Beliefs about the significance of and competence in mathematics	Strongly Disagree	27.8648	10.68291	61	33.072***	.212
	Disagree	31.6791	7.79893	110		
	Neutral	36.6707	7.85458	186		
	Agree	39.7275	7.84698	94		
	Strongly Agree	43.2667	11.43440	45		
Mathematics as a social activity	Strongly Disagree	20.0984	7.11502	61	29.369***	.193
	Disagree	23.0364	5.01544	110		
	Neutral	25.5538	4.52857	186		
	Agree	27.2447	4.35936	94		
	Strongly Agree	29.0889	6.60173	45		
Mathematics anxiety	Strongly Disagree	14.4098	6.14377	61	0.459n.s.	.004
	Disagree	14.8091	4.04900	110		
	Neutral	14.9409	4.26700	186		
	Agree	14.5638	4.08659	94		
	Strongly Agree	15.5111	6.55959	45		

Note: ***p < .001, n.s. p > .05

Table 6. Students' perceived class rank

Class Rank	Number of participants	%
High (top 10%)	41	8.3
Upper middle (10% - 35%)	119	24.0
Middle (35% - 65%)	158	32.1
Lower middle (65% -85%)	108	21.8
Low (bottom 15%)	69	13.9

Perceived Mathematics Achievement and Teacher Praise

In addition to examining the relationships between the mathematics-related belief subscale scores and teacher praise, we also examined the relationships between the students' perceived mathematics achievement (reported as their perceived ranking in class) and teacher praise. **Table 6** shows the distribution of students' perceived mathematics achievement rankings in the class. A High class rank indicates that the subject's mathematics grades were generally in the top 10% of the class, Upper Middle represents a ranking from 10% to 35%, Middle means 35% to 65%, Lower Middle means a ranking of 65% to 85%, and Low indicates that the math grades are in the bottom 15%.

Chi-square tests were conducted to probe the relationships between students' perceived mathematics achievement and teacher praise. As **Table 7** shows, students' perceptions of their own class ranking were significantly associated with their reporting of receiving their teachers' praise. That is, there was a clear relationship between teacher praise and students' perceived achievement ranking in class.

Table 7. Teachers' Praise and Students' Perceived Achievement

		Mathematics teachers' praise					
		Strongly Disagree	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**	**Total**
Students' Perceived Ranking in Their Respective Classes	Low	19	26	18	1	5	69
	Lower Middle	15	27	33	21	12	108
	Middle	12	36	70	36	5	159
	Upper Middle	12	16	50	30	11	119
	High	3	5	15	6	12	41
		χ^2 = 78.047***					

Note : ****p* < .001

Table 8. Students' Perceived Achievement and Students' Math-Related Beliefs

	Perceived Achievement	**Mean**	***SD***	**# of Students**	***F***	**η^2**
Mathematics-Related Beliefs (total)	Low	108.3862	21.94739	69	29.507***	0.194
	Lower Middle	118.0581	22.48730	108		
	Middle	123.5675	20.06813	159		
	Upper Middle	134.9147	17.89734	119		
	High	142.9258	20.64717	41		
Beliefs about the role and the functioning of their own teacher instruction	Low	45.1978	10.65033	69	8.254***	0.063
	Lower Middle	47.6970	10.49410	108		
	Middle	48.7280	9.66997	159		
	Upper Middle	51.4177	8.61173	119		
	High	54.4146	8.38742	41		
Beliefs about the significance of and competence in mathematics	Low	27.2319	8.38266	69	47.681***	0.280
	Lower Middle	32.3148	9.13619	108		
	Middle	35.2420	8.06983	159		
	Upper Middle	41.0096	7.77585	119		
	High	44.7307	7.27783	41		
Mathematics as a social activity	Low	22.1159	5.24835	69	13.425***	0.099
	Lower Middle	23.7037	6.32346	108		
	Middle	24.7107	5.22330	159		
	Upper Middle	27.1933	5.01401	119		
	High	27.6098	5.87741	41		
Mathematics anxiety	Low	13.8406	5.59049	69	2.212n.s.	0.018
	Lower Middle	14.3426	4.61238	108		
	Middle	14.8868	3.88111	159		
	Upper Middle	15.2941	4.75564	119		
	High	16.1707	5.62095	41		

Note: ****p* < .001, n.s. *p* > .05

Perceived Achievement and Mathematics-Related Beliefs

Finally, we examined the relationships between students' perceived mathematics achievement rankings and their mathematics-related beliefs. We carried out an ANOVA for perceived achievement and each of the belief subscale scores as well as the overall belief score (see **Table 8**). A statistically significant relationship was found for each of the belief scores except for the Mathematics anxiety subscale.

DISCUSSION

The goal of this study was to examine, in the context of Chinese high school students, the relationships between students' mathematics-related beliefs and teacher praise as well as students' perceptions of their own mathematics achievement in their classes. Using a framework derived from Op't Eynde and De Corte (2003) as well as relevant items adapted from TIMSS and PISA, we developed the High School Student Mathematics Belief Questionnaire to assess Chinese students' mathematics-related beliefs. The results of this study provide support in the context of Chinese secondary education for several findings from the literature on mathematics-related beliefs.

First, the results show that teachers' praise is positively related to Chinese high school students' mathematics-related beliefs. In other words, students who reported that their teachers praised them were more likely to possess more desirable mathematics-related beliefs. This supports the finding of Levpušček and Zupančič (2009) regarding the importance for mathematics achievement of the support and encouragement that mathematics teachers can offer to students. It is notable, however, that in this Chinese sample, only a relatively small percentage (about 28%) of students reported that their teachers gave them praise. This is a surprisingly small percentage that leaves room to explore how increasing teacher praise might influence students' mathematical beliefs and achievement.

Second, students' reported mathematics achievement level in their classes was positively related to teacher praise. This is consistent with the findings of some studies (e.g., Phillipson, 2010; Rodríguez et al., 2017). The uniqueness of this study is that students' perceived mathematics achievement rank is quite consistent with the data.

Finally, math anxiety is something of an outlier among the four mathematics-related beliefs subscales in the sense that the relationship between this subscale is not correlated in the same way as the other subscales with the other variables. This finding could be interpreted as reflecting the possibility that a certain degree of math anxiety is a good thing for math learning. However, too low or too high a degree of math anxiety may not be healthy for students' math learning. Prior research on math anxiety has revealed that the relationships between math anxiety and achievement are exceedingly complex (Ma, 1999). Based on our findings in this study, we might hypothesize that neither too little anxiety nor too much anxiety produces a strong correlation with achievement. This suggests that a future direction for exploration would be to understand how much math anxiety is beneficial for supporting desirable math beliefs and achievement.

The findings of this study have implications for pedagogical practice. In order to help students develop desirable mathematics-related beliefs, teachers may be able to strategically use praise. In this study, over 70% of the students viewed that their teacher did not give them praise. This implies that there is quite a bit of room to increase praise to encourage more desirable beliefs. This study confirms the positive effects of mathematics teachers' praise on students' mathematics-related beliefs and achievement. This suggests that good communication between mathematics teachers and students could greatly benefit students' mathematics-related beliefs. However, blindly pursuing the delivery of children to prestigious classes in an effort to pressure them to academically perform in mathematics may result in comparatively poor mathematics grades, resulting in less desirable mathematics-related beliefs and further affecting students' mathematics self-efficacy and performance.

A limitation of this study is that it is based purely on survey data collected from students. Thus, information on teachers' praise is channeled entirely through the reporting of the students. A future study is needed to go beyond examining only students' perceptions about teachers' praise to also collect data directly from teachers, as well as to collect actual student mathematics achievement data.

Endnote

1. A series of multiple linear regression analyses were also explored for these data, with the total belief score and each subscale score as the dependent variable and the teacher praise as predictor. Given the categorial/ordinal nature of the teacher praise variable, these analyses did not illuminate the data beyond the ANOVA analyses (neither did they contradict the ANOVA findings). Thus, those results are not reported here.

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APPENDIX

High School Student Mathematics Belief Questionnaire

The questionnaire items listed below are grouped by subscale. The items' positions in the questionnaire as administered were randomized.

Subscale 1: Beliefs About the Role and Functioning of Their Own Teacher

I am interested in what my teacher says in mathematics lessons

My mathematics teacher is easy to understand

The mathematics teacher asks me or my classmates to specify our thinking or reasoning

The mathematics teacher asks questions to check whether we have understood what was taught

The mathematics teacher gives students an opportunity to express opinions

The mathematics teacher sets clear goals for our learning

The teacher tells me what I need to do to become better in mathematics

My mathematics teacher has clear answers to my questions

At the beginning of a mathematics lesson, the teacher presents a short summary of the previous lesson

Our mathematics teacher listens carefully when we ask or say something

Our teacher explains why mathematics is important

Our mathematics teacher appreciates it when we have tried hard, even if our results are not so good

Our mathematics teacher gives us time to really explore new problems and to try out possible solution strategies

Subscale 2: Beliefs about the significance of and competence in mathematics

I can understand even the most difficult material presented in a mathematics course

Mathematics is one of my favorite subjects

If I wanted to I could do better in mathematics

I am good at working out difficult mathematics problems

I enjoy learning mathematics

I learn high school mathematics quickly
I'm very interested in mathematics
I look forward to my mathematics lessons
It is important to learn mathematics
I need mathematics to learn other school subjects such as physics, geography, etc.
I like doing mathematics

Subscale 3: Mathematics as a social activity
Mathematics is benefit for human development
Mathematics is used by a lot of people in their daily life
Development of many disciplines depends on mathematics
I learn many interesting things in mathematics
I think I will be able to use what I learn in mathematics also in other courses
Mathematics enables men to better understand the world he lives in
I am willing to pursue a major in mathematics at university

Subscale 4: Mathematics anxiety
Mathematics is harder for me than any other subject
Mathematics is more difficult for me than for many of my classmates
I have no confidence in myself before learning new mathematics
High school mathematics makes me anxious
Mathematics is not one of my strengths

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Confronting Mathematical Pseudo-Community: A Domain-specific Analysis of a Middle School Mathematics Department

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ABSTRACT
Teacher professional community refers to the extent to which teachers collaborate to further their own and their students' learning. Peck (1995) drew distinctions between pseudo-community and professional community, where pseudo-communities rely on suppression of conflict and the tacit understandings that it is "against the rules" to challenge others' beliefs and ideas. The teacher education literature is predominantly composed of domain-general characterizations of teacher community. This study investigates the pseudo-communities of five middle school mathematics teachers' through a domain-specific characterization of criteria of professional community. These characterizations are made using teacher responses to surveys and interaction analysis during a meeting between middle school mathematics teachers and researchers. These analyses allow for the investigation of why domain-general characterizations of pseudo-community may be limited.

Keywords: mathematics education, in-service teacher education

INTRODUCTION

In defining communities of practice Lave and Wenger (1991; Wenger, 1998) provided criteria by which to distinguish true communities from mere groups of individuals involved in similar work: joint enterprise, mutual engagement, and a shared repertoire. Wenger argued that a group has achieved the status of "community" if and only if it is working collaboratively on a project in accordance with a set of norms of practice and with a language and set of tools that are specific to the community. Little (1982; 2012) presented four "critical practices of adaptability" that focus specifically on teacher communities of practice and add to the work of Wenger. Little suggested that these teaching communities should "engage in frequent, continuous, and increasingly concrete and precise talk about teaching practice; be frequently observed and provided with useful critiques of their teaching; plan, design, research, evaluate, and prepare teaching materials together; and teach each other the practice of teaching." (p. 331) Analysis by Borko (2004), Shulman and Wilson (2004), Franke, Carpenter, Levi, and Fennema (2001), and the Center for Research on the Context of Secondary School Teaching (CRC) (McLaughlin, 1993; McLaughlin & Talbert, 2006) further suggests that a focus on students' understanding is necessary for a professional community of teachers. In addition, Avalos (2011), DuFour (2004), Louis, Kruse, and Marks (1996), described activities that are critical to a teaching community. Louis, Kruse, and Marks suggested that "peer coaching relationships, teamed teaching structures, and structured classroom observations are methods used to improve both classroom practice and collegial relationships" (p. 760). Adding to this conceptualization of professional teaching communities, researchers (e.g., Horn & Kane, 2015; Stoll & Louis, 2007; Tharp & Gallimore, 1988) have highlighted trust as requisite. Similarly, in their studies of urban elementary school teachers, Bryk, Camburn, and Louis (1999) and Bryk and Schneider (2003) found the strongest predictor of professional community was social trust among faculty. Likewise, Eaker, DuFour, and DuFour (2002) cited trust as one of three foundational aspects (time and mutual commitment being the other two) for creating professional communities of teachers. Newmann and Associates (1996) and Gamoran, et al. (2003) presented five key aspects of a teachers' professional community that support the findings from the above research: (a) a shared sense of purpose; (b) a collective focus on student

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Contribution of this paper to the literature

- Provides domain-specific principles and criteria with which to describe a teacher professional community.
- Provides a domain-specific characterization of a pseudo-community within a middle-school mathematics department based upon the professional teaching community literature.
- Provides a means to consider how to develop from a pseudo-community to a professional community of mathematics teachers.

learning; (c) collaborating to improve students' understanding of mathematics; (d) engaging in reflective dialogue about their instructional practices; and (e) making their own teaching practices public.

Grossman, Wineburg, and Woolworth (2000; 2001) noted that professional teaching communities do not emerge spontaneously. As a professional teaching community begins to develop, there is a natural tendency by individuals to "play community" to act as if they are already a community that shares values and beliefs, focuses on student learning, collaborates to improve students' understanding of mathematics, engages in reflective dialogue about their instructional practices, and has sufficient trust to make their own teaching practices public. Grossman et al. (2000; 2001) described how the maintenance of pseudo-community pivots on the suppression of conflict where face-to-face interactions are regulated by a tacit understanding that it is "against the rules" to challenge others or press them too hard for clarification. Pseudo-community supports discourse that is casual, superficial, and at times explicitly and intentionally fails to address many (if not all) of those elements that make up professional teaching community.

Much work around the development of pseudo-community (e.g., Grossman, Wineburg, & Woolworth, 2000; 2001; Lieberman, Miller, Wiedrick, & von Frank, 2011; Little, 2012) has focused on the development of domain-general criteria. Although these domain-general criteria of pseudo-community are useful towards generic understandings of how teacher development advances towards professional community, they are limited in their applicability to how these domain-general principles apply within particular domains (e.g., secondary mathematics). Domain-general models of learning focus on general cognitive structures for knowledge development, while domain-specific models focus on cognitive structures that are highly dependent on context. Some recent work in mathematics education has focused on domain-specific models of teacher learning. These domain-specific models have focused exclusively on how pre- and in-service teachers learn to consider and implement instruction within specific mathematical contexts. For example, Hill et al. (2008) and Hill (2010) described the domain-specific development of mathematical knowledge for teaching (MKT), comprised of mathematical disciplinary knowledge and mathematical knowledge for instruction. The current study improves upon previous domain-general characterizations of professional learning by focusing on domain-specific principles present (or absent) within a pseudo-community. Investigating the domain-specific practices of a group of teachers allows for a more adequate differentiation between professional and pseudo-community of mathematics teachers.

THEORETICAL FRAMEWORK

The definition of professional community that is referenced in this study is that enumerated by Newmann and Associates (1996) and Gamoran and colleagues (2003). However, because these authors framed these categories as domain-general, I have modified them for domain-specific contexts within this study. I have altered four of the five presented categories in order to describe domain-specific pseudo-communities (i.e., of mathematics teachers). There is one category that was described by the authors as having a domain-specific focus: collaborating to improve students' understanding of mathematics. This category is included in the study in its original domain-specific form. Domain-specific elements that have been added to the remaining four categories are shown parenthetically: a shared sense of purpose (around mathematics teaching), collective focus on student learning (about mathematics), engaging in reflective dialogue about their (mathematical) instructional practices, and making their own (mathematical) teaching practices public.

The term "pseudo-community" has been used in various ways. Peck (1995), Grossman, Wineburg, and Woolworth (2000; 2001), Lieberman, Miller, Wiedrick, and von Frank (2011), and Little (2012) have used the term to focus on face-to-face relations and professional collaborations amongst groups of teachers that appear congenial because the expression of conflict and dissent is squelched. These interactions and collaborations rarely focus on central issues of teaching and learning. In this study I develop an understanding of pseudo-communities of mathematics.

Table 1. Participants

Teacher	Subject	Grade
John	Mathematics	8th
Melissa	Mathematics	5th
Amanda	Mathematics	5th
Erika	Mathematics	6th
Jessica	Mathematics	6th
Amy	Mathematics	7th

Research Questions

This study focuses on the following research questions: (a) To what extent do middle school mathematics teachers represent a professional or pseudo-community and (b) in what ways do a focus on domain-specific practices affect the characterization of professional or pseudo-community?

METHODS

This is a mixed methods study that includes quantitative analysis of survey data and qualitative interaction analysis of video data.

Participants

The sample is shown in **Table 1**. The participants were all middle school mathematics teachers (n = 6) from a public middle school in the southeastern United States.

The teachers participating in this study were predominantly female (male = 1) and White. The collaboration between teachers and researchers came about when the school principal reached out to the author to lead a professional development. This professional development included a series of participant-observations, where the mathematics department chair (Erika) took on the role of a student in her colleagues' classes.

Procedure

Prior to a meeting between the mathematics teachers and the researchers (i.e., the teacher-researcher meeting), the Teacher Pre-Study Survey (Appendix A) was distributed to the teachers. These surveys were composed of nineteen questions that targeted elements of professional community. The teachers returned the completed surveys and submitted them to the researchers during the teacher-researcher meeting. This meeting was video recorded.

Teacher Pre-Study Survey. The teacher survey measures teachers' perceptions of the support structures for developing instructional practices in mathematics (Appendix A). It includes items concerning (a) the level of teachers' participation in mathematical teacher networks, their propensity to offer instructional advice, and who they offer advice to; (b) the degree to which the interactions between teachers in these networks focus on mathematical concepts and how to relate them to students' reasoning; (c) the degree to which teachers have a shared vision for mathematics instruction and student learning; and (d) the professional development that teachers have received in support of improved instructional practices in mathematics.

Teacher-researcher Meeting. All teachers in this study consented to participate in a professional development that would implement a novel model of peer observation. The researchers met with the six middle school mathematics teachers at one time, for approximately two hours to plan for the professional development. This teacher-researcher meeting was video-recorded and rendered for further analysis.

Analysis

Teacher Pre-Study Survey. The survey results were categorized according to five criteria domain-specific categories, informed by the work of Newmann and Associates (1996) and Gamoran and colleagues (2003): a shared sense of purpose (around mathematics teaching), collective focus on student learning (about mathematics), engaging in reflective dialogue about their (mathematical) instructional practices, and making their own (mathematical) teaching practices public. Two researchers coded the responses into all five categories. The inter-rater agreement was 92%. Because the data are ordinal, Mann-Whitney U tests were conducted to compare the mean teacher Likert scores with those scores corresponding with not engaging in the activity the survey questions targeted.

Teacher-researcher Meeting. Transcripts were created from the video data collected from the teacher-researcher meeting. Using interaction analysis (Jordan & Henderson, 1995), the researchers characterized participant talk (from the transcripts) into the five categories above (e.g., shared sense of purpose around mathematics teaching). The analysis of the transcription is triangulated with results from the Teacher Pre-Study Survey.

RESULTS

Data from the survey and meeting analyses indicated that the teachers did not have a shared sense of purpose (around mathematics teaching), a collective focus on student (mathematics) learning, engage in collaboration to improve students' understanding of mathematics, engage in reflective dialogue about their (mathematical) instructional practices, or make their own (mathematical) teaching practices public.

Shared Sense of Purpose (around Mathematics Teaching)

These teachers indicated the absence of a shared sense of purpose around mathematics teaching. For example, no two teachers had ever observed one another's mathematics teaching (1b, $Mdn = 0$; 4a, $Mdn = 0$; 4b, $Mdn = 0$) (Appendix B). In addition, teachers indicated that they did not jointly plan mathematics instruction (6e, $Mdn = 0.5$), share materials related to mathematics instruction with colleagues (6f, $Mdn = 1$), all use mandated state mathematics standards (6j, $Mdn = 0.5$), or jointly analyze student work in mathematics (6k, $Mdn = 0.5$). Moreover, teachers disagreed with colleagues about how to teach mathematics (3c, $p < 0.05, \text{Mann} - \text{Whitney U test}$) and were unaware of how their colleagues taught it (3f, $p < 0.05, \text{Mann} - \text{Whitney U test}$).

During the teacher-researcher meeting, CM (one of the researchers) tried to focus a discussion on Erika's upcoming observations of her colleagues (the central component of the upcoming professional development). CM tried to support Erika to think about mathematics teaching and learning. However, the group of teachers had no shared practices to support Erika to discuss issues central to mathematics instruction (Interview Transcript 1).

Interview Transcript 1

CM: What are you coming into this [observation] expecting?

Erika: I'm hoping if there are certain things I see with students that are more effective, like certain lines of questioning. Or, things that aren't as effective like certain days when we think we're on it, but the students miss the boat. We think that we've taught it; I know for a fact there are things that I think I've taught the fire out of and [students] take the test and it's as if you never taught it before (pause) as if they've never seen it before…Maybe if I see a student light bulb somehow [I will make a note in my journal], maybe a comment about the process that got them [students] to it.

Erika's response suggested that there was a shared sense of purpose, but not around *mathematics teaching*. She had nebulous, superficial, and conflicting ideas about what areas in mathematics teaching and learning were worth investigating. Her ideas were not sufficiently clarified to support a common language or understanding of mathematics teaching among her colleagues. For instance, Erika noted that she was interested in investigating "certain lines of questioning" or "things that aren't as effective [for students]," when teachers believe they've "taught it." However, these topics were too broad to be productively engaged with her colleagues.

Frequently, teachers indicated that the element of their mathematics instruction that they wanted Erika to focus on were those that did not involve mathematics (i.e., a domain-general shared sense of purpose). Accordingly, it is difficult to have a shared sense of purpose around mathematics teaching absent any discussion of mathematics teaching (Interview Transcript 2).

Interview Transcript 2

Amanda: And I told [Erika] I wanted to know if I have some strange tick. Or if I say "Okay" [too frequently].

Amanda was concerned about the number of times she said, "okay," "alright," and "be sure you're listening" within a teaching lesson. These topics show a general shared sense of purpose. However, mathematics teachers must indicate a shared sense of purpose around mathematical pedagogical and disciplinary practices.

Collective Focus on Student Learning (of Mathematics)

The survey data did not show a collective focus on student learning of mathematics. Teachers indicated that their colleagues did not support them to facilitate mathematical classroom discussions (12d, $Mdn = 1.0$) (Appendix B)or mathematical group work (12e, $Mdn = 1.0$), where students were encouraged to share their thinking. In addition, administrators did not provide teachers with feedback to improve their mathematics instruction (10b, $Mdn = 1.5$).

Two concepts were raised by these mathematics faculty that could be categorized as peripherally focusing on student learning of mathematics: procedural fluency and higher order thinking (Interview Transcript 3).

Interview Transcript 3

Amanda: I [often] see a child who is caught up in the [procedures] and they're not thinking past just the [procedures]. [I am concerned with student] (pause) fear of the [procedures because I want to] go to the next level with them [(i.e., focus on conceptual understanding)] …With the [gifted and talented] children, I want to know if I'm asking questions that promote the higher order thinking. Am I making them think, or am I really going after the [procedures]? I don't want to not do [procedures], but I want to make sure we go beyond it [to concepts]. Are my questions leading to that or are they not?

The discussion of procedural fluency and higher order thinking indicated a collective focus on student learning, but not a collective focus on student learning of *mathematics*. For example, procedural fluency and higher order thinking are described in vague and non-specific terms; accordingly, there is little possibility for a collective focus on student learning of *mathematics*. Amanda raised procedural fluency in an attempt to discuss student thinking. Procedural fluency is defined as knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently; accordingly, conceptual understanding refers to an integrated and functional grasp of mathematical ideas (Mathematics Learning Study Committee, 2001). Amanda discussed how "higher order thinking" was impeded by an exclusive focus on procedures. Resnick (1987) defines higher order thinking as requiring non-algorithmic, complex, multiple possible solutions, nuanced judgment and interpretation, the application of multiple criteria, uncertain solutions to posed problems, self-regulation, imposing meaning, and effortful thinking.

Although teachers were attentive to difficulties that occur when there is an exclusive focus on mathematical procedures, without a clear conception of the relation between mathematical concepts and procedures, it is unlikely that the teachers would be able to effectively communicate about these aspects of student learning of mathematics. When further reflecting on what about student thinking in mathematics she plans to attend to during her observation, Erika raised the construct of "engagement." She considered how engaged she would be when she participated as a student in her colleagues' classes (Interview Transcript 4).

Interview Transcript 4

Erika: [When thinking about what I will focus on during my observations, I will think about] how actively engaged was I with the activity, how much time on task? Was there some time when I wasn't spending doing things?

Erika discussed the construct of engagement with no connection to student mathematical learning.

Collaborating to Improve Students' Understanding of Mathematics

Teachers indicated that while they had addressed mathematical concepts in professional development sessions ($13\mathrm{d}_1, p < 0.05$, Mann − Whitney U test; $13\mathrm{f}_1, p < 0.05$, Mann − Whitney U test) (**Appendix B**), either such work did not impact their instruction or they were unable to describe how it had ($13\mathrm{d}_2, Mdn = 2.0$; $13\mathrm{f}_2, Mdn = 1.5$). Teachers also indicated that the administration had not structured sufficient time for collaboration around mathematics teaching (7, $Mdn = 0.5$). In addition, the administration had not supported professional development or collaboration around mathematics teaching and learning, beyond that required by the school district (12j, $Mdn = 2.0$). For example, the administration had never supported teachers to engage students with challenging non-routine mathematical tasks (12h, $Mdn = 2.0$). Although some professional developments focused on leading classroom discussions where students were to mathematically justify their answers ($13\mathrm{e}_1, Mdn = 2.5, p < 0.05$), support high level mathematical tasks ($13\mathrm{g}_1, Mdn = 2.0, p < 0.05$), and engage students in high level mathematics ($13\mathrm{i}_1, Mdn = 2.5, p < 0.05$), teachers were not supported to incorporate these practices into their teaching ($13\mathrm{e}_2, Mdn = 2.0$, $13\mathrm{g}_2, Mdn = 2.0$, $13\mathrm{i}_2, Mdn = 3.0$). Moreover, through professional development teachers were neither supported to provide students with high level mathematics solutions ($13\mathrm{h}_1, Mdn = 1.0$) nor able to incorporate these practices into their teaching ($13\mathrm{h}_2, Mdn = 2.0$). Thus, there was insufficient and ineffective collaboration around central *mathematical* teaching practices.

John, a first-year teacher, evidenced collaboration to improve students' understanding, but not collaboration to improve students' understanding of *mathematics*. John began a discussion with Erika that showed how collaboration around mathematics teaching, within a pseudo-community, breaks down. John began a discussion about mathematical content. He raised his concern about student difficulties with operations on fractional numbers with Erika (Interview Transcript 5).

Interview Transcript 5

John: Fractions – you know I told you a while back that like half my kids couldn't add and subtract fractions…They just don't want to do the work.

Erika: They don't want to think.

John: They don't want to do anything…

Erika: Well, I don't know where to help you.

This discussion indicated the presence of a mathematical pseudo-community. John raised a concern he had about his students understanding of an important Common Core State Standard (National Governors Association, 2010) (e.g., adding and subtracting fractions). This created an opportunity for these teachers to engage in collaboration around student understanding of mathematics around this content. However, the topic focus changed from content to student indolence: "They just don't want to do the work." This derailed the conversation to one where Erika could exit with a simple superficial: "Well, I don't know where to help you." Next, Melissa presented another opportunity to engage in collaboration around student understanding of mathematics, which quickly devolved into superficial and general discussion (Interview Transcript 6).

Interview Transcript 6

Melissa: [Erika's observation of my instruction will begin] at a part of the curriculum that is review…[I]f you [would observe me] during fractions, I had a million fun things to do, lots of manipulatives; lots of fun things to do. If you came during lots of geometry we did a lot of cool activities. Now, we're doing multiplying, multiplying 3-digits by 2 [digits], whatever. And it's not a very exciting part of our curriculum. So, when you're sitting there in my class I want you to just (pause) if a good idea pops into your head, I just want you to [tell me].

Although this collaboration was about mathematical content, Melissa presented a view that did not address specific issues within the content. Her view seemed to be that mathematics content is either exciting or not. When she described her unit on fractions, she indicated that she "had a million fun things to do, lots of manipulatives." This characterization of fractions as fun neglected to focus on the complexity of the big ideas within the content (e.g., part-whole relations, representing fractions using linear measure, equivalence of fraction).

Melissa next described the importance of engaging in mathematics with real world applications (Interview Transcript 7). Again, the focus deviated from student understanding of mathematics.

Interview Transcript 7

Melissa: So, maybe something that helps them see [how this content is important] (pause) I do use this all the time (pause) I talk about when you play football you don't just go out to practice and start playing football. You exercise and this is like exercise for your brain. I use that a lot for things when they can't really (pause) when it's hard to apply it to a real-life activity. You know, and the boys fall for that one. But um, if you can just find some way to make it so they'll just think, ah; that's why I need to know that or this is how I can apply that (pause). Geometry's easy to make relevant. I mean, my husband's an architect; I can always pull out the old (pause) this is a job where you use this. Or, if you're going to be an interior decorator or if you're hanging curtains or anything like that.

Here, Melissa presented a conception of what types of mathematical ideas were important to teach. Melissa made an argument that mathematical ideas that are "relevant" are more compelling to students than those that are not. However, when probed it was not clear what she meant by relevant. To Melissa, mathematics was relevant if it helped students see how they could use mathematical ideas outside of the classroom and made clear that professions exist that use these mathematical concepts. In addition, when it was difficult to see how mathematical ideas applied outside of the classroom, Melissa used a "football" metaphor. She forced students (especially male students) to see that their mathematics was practice for what would one day be relevant.

With one exception, this group of teachers did not collaborate to improve students' understanding of mathematics. During that one exception, Erika described one big idea within number, the role of place value. This was a qualitatively different way of engaging with this content than Melissa's (or the other mathematics teachers) (Interview Transcript 8).

Interview Transcript 8

Erika: [With multiplication of 3 and 2 digit numbers,] (pause) if you had 324 times 25, [it's important] for [students] to understand that it's 324 times 20 and 324 times 5; that's why you add the extra zero.

Here, Erika initiated an important discussion that focused on mathematical ideas within the content. She explained how place-value operates in a base-10 number system, over multiplication. Unfortunately, Melissa responded by dismissing this important mathematical idea and restating how she believed Erika's observation could best benefit her; she asked Erika to let her know if she could come up with anything that might make the multiplication of 3 and 2-digit numbers fun, by letting Melissa know if she has any great activities. She was unaware that Erika's discussion of content was such a recommendation. Thus, Erika's turn-of-talk described student understanding of mathematics; unfortunately, her colleagues were unable to productively take up this topic. Accordingly, there was no collaboration.

Engaging in Reflective Dialogue about their (Mathematical) Instructional Practices

These teachers did not discuss mathematical instructional practices. For example, teachers did not discuss state mathematics standards (6j, $Mdn = 0.5$) (**Appendix B**) or analyze student work (6k, $Mdn = 0.5$). No one (teachers, administrators, or coaches) supported the development of new mathematical instructional practices (12n, $Mdn = 1.0$).

Amanda discussed one instructional practice that she employed (Interview Transcript 9). This indicates her engagement in a reflective dialogue about instructional practices, but not about *mathematical* instructional practices.

Interview Transcript 9

Amanda: When I question children, there are two roles that I play … One is: I am the teacher. The other is: you are telling me, as a person who does not know…how to do something, how to do it. And I will go to that role when I see a child who is caught up in the [procedure] and they're not thinking past just the [procedure]. So, they're going to talk me past the [procedure]…

Amanda attempted to engage in reflective dialogue about a pedagogical practice she employed. Within this practice, when she determined that a student did not understand the concept behind a procedure she required them to explain the procedure. Explanation and justification have been found to be a valuable mathematics disciplinary practices (Yakel & Cobb, 1996). However, neither Amanda nor her colleagues could engage in a reflective dialogue about mathematical instructional practices because (a) Amanda could not clarify the mathematics involved in this practice and (b) her colleagues did not indicate that they engaged in similar practices.

Making their own (Mathematical) Teaching Practices Public

No two teachers had observed each other's mathematical teaching (4a, $Mdn = 0.0$; 4b, $Mdn = 0.0$; 12k, $Mdn = 1.0$) (Appendix B). In addition, all teachers indicated they possessed little knowledge of their mathematics colleagues' teaching methods (5a $p < 0.05$, Mann − Whitney U test) or the content they covered (5b, $p < 0.05$, Mann − Whitney U test).

The teacher-researcher meeting further indicated that this group of teachers had spent little time making their mathematics teaching practices public. In fact, they had spent little time making any of their teaching public. Amanda noted that during her twenty-year tenure as a teacher, she had never been observed by a colleague (Interview Transcript 10).

Interview Transcript 10

Amanda: I mean, I've taught for 20 something years…[and] I've never had this opportunity [to be observed]. So, I look forward to it.

Erika was asked what she expected to learn from the observations she was to conduct. Even though she was the mathematics department chair, it was clear she had done little (if any) observation of her mathematics teacher colleagues (Interview Transcript 11).

Interview Transcript 11

Erika: I'm not sure. I think when (pause) I'm going to have to reflect after that first day of thinking [and observing], "what happened that first day." After that first day, I'm going to have to sit back and look again because after that first day I'm not 100% sure of what I [will go] in looking for. I know it's the role of the student. I know, maybe, how actively engaged was I with the activity, how much time on task? Was there some time when I wasn't spending doing things? Um, so I'm not sure. I'm not 100%; I think I'll just have to see the first day and how that works out.

Erika and her colleagues had only a vague sense of what the observer should be attending to in their mathematics classrooms because they did not have a clear sense of the role of classroom observation. The teachers implicitly knew that it was important to attend to mathematical teaching practices. However, because observation had not been a common practice, they were unaware of what specific practices were worth attending to when their colleagues make their mathematics teaching public.

DISCUSSION

The majority of literature on teachers' professional communities take a domain-general focus (e.g., Little, 2012). For instance, analyses by Borko (2004), Shulman and Wilson (2004), and the Center for Research on the Context of Secondary School Teaching (CRC) (McLaughlin, 1993; McLaughlin & Talbert, 2006) take a domain-general focus on teachers' knowledge of student understanding. In addition, Avalos (2011), DuFour (2004), and Louis, Kruse, and Marks (1996), described domain-general activities that are critical to the building of teachers' professional community. Further, Stoll and Louis (2007), Bryk, Camburn, and Louis (1999), Bryk and Schneider (2003), and Tharp and Gallimore (1988) highlighted trust as requisite to teachers' professional community; however, they do not address whether and how community trust may appear and operate in diverse ways across different disciplines. Finally, Newmann and Associates (1996) and Gamoran, Anderson, Quiroz, Secada, Williams, and Ashman (2003) presented key aspects of a teachers' professional community that are similarly domain-general. While some researchers (e.g., Horn & Kane, 2015) discuss pseudo-community and professional community in the context of mathematics instruction, this does not indicate that this research is not domain-general. In Horn and Kane's article, mathematics is a background for a discussion of domain-general principles. For example, Horn and Kane describe the relationship between teacher communities and the nature of teacher talk without a domain-specific focus on teachers' mathematical talk.

The middle school mathematics teachers described in this study represent a pseudo-community (Grossman et al., 2001) because analysis of survey responses and teacher-research meeting discussion transcripts indicate that they do not have a (a) shared sense of purpose around mathematics teaching, (b) collective focus on student learning about mathematics, (c) collaborate to improve students' understanding of mathematics, (d) engage in reflective dialogue about their mathematical instructional practices, and (e) make their own mathematical teaching practices public.

The teacher responses to the teacher surveys substantiated that these teachers are members of a mathematics pseudo-community. For example, no two teachers had ever observed each other's mathematics lessons (1b, $Mdn = 0$; 4a, $Mdn = 0$; 4b, $Mdn = 0$; Appendix) (Appendix B). In addition, teachers indicated that their colleagues did not support them to facilitate mathematical classroom discussions (12d, $Mdn = 1.0$) or mathematical group work (12e, $Mdn = 1.0$). Moreover, teachers indicated that the administration had not structured sufficient time for mathematics department collaboration (7, $Mdn = 0.5$); teachers did not discuss state mathematics standards (6j, $Mdn = 0.5$) or analyze student work in mathematics (6k, $Mdn = 0.5$).

This study highlights the importance of using domain-specific principles to characterize a pseudo-community. In addition, this work demonstrates the differences between domain-general and domain-specific characterizations of pseudo-community. For instance, there is minimal utility in teachers "attending to a collective focus on student learning" without a specific focus on *mathematical* learning. In this study Amanda discussed student tendencies to focus on mathematical procedures. Such a discussion suggests a collective focus on student learning, but not on mathematical learning. These teachers were not yet ready to collaboratively consider the relations between mathematical procedures and concepts; accordingly, they cannot be considered a professional community of mathematics teachers. In another example, Melissa collaborates to improve students' understanding. However, she does not collaborate to improve students' understanding of *mathematics*. This is seen when she notes, "So, maybe something that helps them see [how this content is important]…I do use this all the time…I talk about when you play football you don't just go out to practice and start playing football. You exercise and this is like exercise for your brain…" Collaboration to improve students' understanding without a focus on mathematics, does not help students develop mathematical understandings. In addition, it does not develop a professional mathematics teacher community.

CONCLUSION AND IMPLICATIONS

This mixed methods case study characterizes a group of secondary mathematics teachers according to their survey responses and interactions during a two-hour faculty meeting. The study shows that classifications of pseudo-community and professional community should attend to domain-specific properties. This work takes domain-general aspects of professional community (Gamoran, Anderson, Quiroz, Secada, Williams, & Ashman, 2003; Newmann & Associates, 1996) and focuses them on practices specific to *teachers' professional community of mathematics*.

While some research on in- and pre-service teachers' learning is orienting towards a domain-specific focus (Hill et al., 2008; Hill, 2010), there is much more work to be done. For example, Weinberg (in press) further characterizes the development of teachers' professional communities of mathematics (from teachers' pseudo-community of mathematics) through the analysis of an in-service professional development focused on leading a group discussion within a model of peer observation and de-privatization of practice. Future work should look at similar professional development to support teachers' professional communities of mathematics within both affluent and high-poverty schools and districts.

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APPENDIX

Appendix A: Teacher Pre-Study Survey

		No	Yes
1.	In the past year, did you do any of the following?	No	Yes
a.	Participate in regularly scheduled collaborations with other teachers on issues of instruction	1	2
b.	Observe, or be observed, by other teachers in your classroom (for at least 10 minutes)	1	2

		No Teachers	Some Teachers	Most Teachers	All Teachers	Don't Know
2.	This question concerns how <u>teachers</u> interact in your school. Please indicate about how many teachers in your school do each of the following:	No Teachers	Some Teachers	Most Teachers	All Teachers	Don't Know
a.	Work together to develop curriculum and instructional materials	1	2	3	4	5
b.	Observe each other teaching	1	2	3	4	5
c.	Offer advice or help to each other	1	2	3	4	5
d.	Share ideas on teaching	1	2	3	4	5
e.	Promote innovative teaching practices	1	2	3	4	5

		Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
3.	Now consider conditions of <u>mathematics teaching</u>. How well does each of the following statements describe conditions in your school?	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
a.	The school administration promotes innovations in mathematics education	1	2	3	4	5
b.	Teachers in this school regularly share ideas about mathematics instruction	1	2	3	4	5
c.	There is a lot of disagreement among teachers about how to teach mathematics	1	2	3	4	5
d.	I work regularly with other teacher(s) on mathematics curriculum and instruction	1	2	3	4	5
e.	I feel supported by other teachers to try out new ideas in teaching mathematics	1	2	3	4	5
f.	I don't know how other teachers in this school teach mathematics	1	2	3	4	5

Questions 4 through 6 pertain to your interactions with other mathematics teachers.

		Never	1-2 Times	3-5 Times	6-10 Times	More than 10 Times
4.	In the past year, how often have the following events occurred?	Never	1-2 Times	3-5 Times	6-10 Times	More than 10 Times
a.	A mathematics teacher (*other* than a mathematics coach in my school) observed my teaching (for at least 10 minutes)	0	1	2	3	4
b.	I observed a mathematics teacher teach in a classroom (for at least 10 minutes)	0	1	2	3	4

		None	Some	All
5.	Indicate the number of teachers about whom the following statements are true:	None	Some	All
a.	I have detailed knowledge of the *instructional methods* used by other middle school mathematics teachers at my school	0	1	2
b.	I have detailed knowledge of the *mathematics content* covered by other middle school mathematics teachers at my school	0	1	2

		Never	1-2 Times	Quarterly	Monthly	At Least Weekly
6.	In the past year, how often have you done the following with another mathematics teacher?	Never	1-2 Times	Quarterly	Monthly	At Least Weekly
a.	Discussed administrative tasks and how to fulfill them	0	1	2	3	4
b.	Discussed/clarified the key mathematical ideas in a particular lesson or unit	0	1	2	3	4
c.	Discussed mathematical ideas that are usually difficult for students to understand	0	1	2	3	4
d.	Discussed different ways in which students solve a particular problem	0	1	2	3	4
e.	Jointly planned for instruction	0	1	2	3	4
f.	Shared materials related to mathematics instruction	0	1	2	3	4
g.	Discussed how to manage classroom routines and procedures (e.g., collecting homework)	0	1	2	3	4
h.	Discussed how to organize the classroom for instruction (e.g., small groups, whole class, etc.)	0	1	2	3	4
i.	Discussed the behavior of specific students	0	1	2	3	4
j.	Clarified our understanding of the state standards in math	0	1	2	3	4
k.	Discussed students' work	0	1	2	3	4

Questions 7 through 18 pertain to your interactions with your school principal (or assistant principals).

		No	Yes
7.	Did your principal structure this school year's schedule to provide time for collaboration with your mathematics department or your cross-disciplinary team?	0	1

		Daily	Weekly	Monthly	Quarterly
8.	How often does the schedule your principal created this school year provide common time for mathematics teachers' collaboration?	1	2	3	4

		Never	1-2 Times	Quarterly	Monthly	At Least Weekly
9.	In the past year, how often have the following events occurred?	Never	1-2 Times	Quarterly	Monthly	At Least Weekly
a.	I discussed my teaching with a school principal or an assistant principal	0	1	2	3	4
b.	A school principal or an assistant principal observed my teaching (for at least 10 minutes)	0	1	2	3	4
c.	A school principal or an assistant principal reviewed my students' work	0	1	2	3	4

		Not at All	To a Small Extent	To a Moderate Extent	To a Great Extent
10.	In the past year, to what extent has your principal (or assistant principal) done the following?	Not at All	To a Small Extent	To a Moderate Extent	To a Great Extent
a.	Worked with me to resolve student behavioral problems in my classroom	1	2	3	4
b.	Provided me with feedback to improve my instruction after observing my teaching	1	2	3	4
c.	Enabled me to purchase additional instructional materials that I needed	1	2	3	4
d.	Appreciated challenges in using the curriculum effectively	1	2	3	4

		Strongly Disagree	Disagree	Agree	Strongly Agree
11.	To what extent do you agree or disagree with the following statements?	Strongly Disagree	Disagree	Agree	Strongly Agree
a.	The purpose of my school principal (or assistant principal) visiting my classroom is to directly assist me in improving my teaching	1	2	3	4
b.	The purpose of my school principal (or assistant principal) visiting my classroom is to evaluate my teaching in terms of job performance	1	2	3	4

		No One	Other Teachers	Principals	Math Coach
12.	In the past year, who has expected you to do the following activities: <u>Mark all that apply</u>.	No One	Other Teachers	Principals	Math Coach
a.	Adhering to a prescribed pacing in my instruction	1	2	3	4
b.	Making sure that my students' test scores are high	1	2	3	4
c.	Addressing the state/district objectives and standards	1	2	3	4
d.	Having whole classroom discussion in which students explain how they solved tasks	1	2	3	4
e.	Having small-group discussion in which students explain how they solved tasks	1	2	3	4
f.	Using the adopted curriculum as a basis for my classroom instruction	1	2	3	4
g.	Keeping my students quiet and disciplined during classroom instruction	1	2	3	4
h.	Using challenging, non-routine tasks with my students	1	2	3	4
i.	Collaborating with other mathematics teachers	1	2	3	4
j.	Participating in professional development beyond district or school requirements	1	2	3	4
k.	Observing other mathematics teachers' instructional practices	1	2	3	4
l.	Using him/her/them as a resource when instructional problems arose	1	2	3	4
m.	Making my lesson plans available for inspection	1	2	3	4
n.	Trying new instructional approaches in my classroom	1	2	3	4

Questions 13 through 17 pertain to the school or district professional development you have received in the past year (including last summer).

	Topic was Addressed				Impacted my Instruction			
13. To what extent were the following topics addressed in professional development sessions, and, if they were addressed, to what extent have they impacted your instruction? (Mark one box for each: If the topic was not addressed, you can leave the second part blank.)	Not at All			To a Great Extent	Not at All			To a Great Extent
a. Meeting state standards or assessment requirements	1	2	3	4	1	2	3	4
b. Managing the classroom and/or student discipline	1	2	3	4	1	2	3	4
c. Analyzing students' mathematics work	1	2	3	4	1	2	3	4
d. Deepening my knowledge of mathematics	1	2	3	4	1	2	3	4
e. Leading discussions where students have to justify their mathematics solutions	1	2	3	4	1	2	3	4
f. Understanding the central mathematical ideas in the curriculum	1	2	3	4	1	2	3	4
g. Using challenging, high-thinking tasks	1	2	3	4	1	2	3	4
h. Holding students accountable for producing high-level solutions	1	2	3	4	1	2	3	4
i. Using strategies to engage all students in learning high-level mathematics	1	2	3	4	1	2	3	4

14. To what extent do you agree or disagree with the following statements about school and district professional development sessions during the past year (including last summer)?	Strongly Disagree	Disagree	Agree	Strongly Agree
The professional development sessions…				
a. Included opportunities to work productively with other teachers	1	2	3	4
b. Advocated practices I do not believe in	1	2	3	4
c. Led me to try new instructional approaches with my students	1	2	3	4
d. Made me question my beliefs and assumptions about which teaching methods work best with students	1	2	3	4
e. Were effectively related to previous sessions	1	2	3	4

15. To what extent have school and district professional development sessions in the past year (including last summer) been consistent with each of the following?	Not at All	To a Small Extent	To a Moderate Extent	To a Great Extent
a. The way your teaching performance is evaluated	1	2	3	4
b. Your own goals for instruction	1	2	3	4

If you answered 'No' to question 16, skip to question 17.

16. Have you made efforts to change your teaching based on your experience in the professional development sessions in the past year (including last summer)?	No	Yes
	0	1

17. What has been the response of the following people to your efforts to change your teaching based on your experience in the professional development sessions?	Resistance				Support
a. School administrators	1	2	3	4	5
b. Other teachers	1	2	3	4	5

18. To what extent is the mathematics curriculum at your school consistent with each of the following?	Not at All	To a Small Extent	To a Moderate Extent	To a Great Extent
a. Your personal philosophy of teaching	1	2	3	4
b. Ways of teaching mathematics promoted in professional development sessions	1	2	3	4
c. Mission of your school	1	2	3	4

19. In the past year to what extent have the following people influenced your instructional practices?	Not at All	To a Small Extent	To a Moderate Extent	To a Great Extent
a. Your principal (or assistant principals)	1	2	3	4
b. Your mathematics coach	1	2	3	4
c. Other mathematics teachers at your school	1	2	3	4
d. Other mathematics teachers not at your school	1	2	3	4
e. Your district mathematics leader(s)	1	2	3	4

Appendix B: Survey Items, Teacher Median Scores, and P-values

Item	Median	p-value
1a	1.00	$p < 0.05$
1b	0.00	$p < 0.01$
2a	2.00	$p < 0.05$
2b	2.50	$p < 0.01$
2c	3.00	$p < 0.01$
2d	3.00	$p < 0.01$
2e	2.50	$p < 0.01$
3a	4.50	$p < 0.01$
3b	3.50	$p < 0.05$
3c	2.00	$p < 0.05$
3d	2.50	$p < 0.05$
3e	4.00	$p < 0.05$
3f	3.00	$p < 0.05$
4a	0.00	$p = 0.92$
4b	0.00	$p = 0.92$
5a	1.00	$p < 0.05$
5b	1.00	$p < 0.05$
6a	1.50	$p < 0.05$
6b	1.00	$p < 0.05$
6c	1.50	$p < 0.05$
6d	1.50	$p < 0.05$
6e	0.50	$p = 0.07$
6f	1.00	$p = 0.14$
6g	1.00	$p < 0.05$
6h	1.00	$p < 0.05$
6i	1.50	$p < 0.05$
6j	0.50	$p = 0.07$
6k	0.50	$p = 0.07$
7	0.50	$p = 0.07$
8	4.00	$p = 0.14$
9a	1.50	$p < 0.05$
9b	1.00	$p < 0.01$
9c	1.00	$p = 0.07$

10a	1.50	$p = 0.07$
10b	1.50	$p < 0.05$
10c	2.00	$p < 0.05$
10d	3.00	$p = 0.10$
11a	3.00	$p < 0.01$
11b	3.00	$p < 0.01$
12a	2.00	$p = 0.09$
12b	3.00	$p < 0.01$
12c	2.00	$p < 0.05$
12d	1.00	$p = 0.14$
12e	1.00	$p = 0.92$
12f	3.00	$p < 0.05$
12g	1.50	$p = 0.09$
12h	2.00	$p = 0.09$
12i	3.00	$p < 0.01$
12j	2.00	$p = 0.09$
12k	1.00	$p = 0.46$
12l	2.00	$p < 0.05$
12m	3.00	$p < 0.05$
12n	1.00	$p = 0.19$
$13a_1$	3.50	$p < 0.05$
$13a_2$	3.00	$p < 0.05$
$13b_1$	1.50	$p = 0.07$
$13b_2$	3.50	$p = 0.92$
$13c_1$	2.00	$p < 0.05$
$13c_2$	2.00	$p < 0.05$
$13d_1$	2.50	$p < 0.05$
$13d_2$	2.00	$p = 0.10$
$13e_1$	2.50	$p < 0.05$
$13e_2$	2.00	$p = 0.10$
$13f_1$	2.00	$p < 0.05$
$13f_2$	1.33	$p = 0.14$
$13g_1$	2.00	$p < 0.05$
$13g_2$	2.00	$p = 0.14$
$13h_1$	1.00	$p = 0.17$
$13h_2$	1.00	$p = 0.14$
$13i_1$	2.00	$p < 0.05$
$13i_2$	2.00	$p = 0.14$
14a	3.00	$p < 0.01$
14b	1.00	$p = 0.19$
14c	3.00	$p < 0.01$
14d	2.00	$p < 0.05$
14e	3.00	$p < 0.01$
15a	1.00	$p = 0.19$
15b	3.50	$p < 0.01$
16	1.00	$p < 0.05$
17a	3.50	$p < 0.01$
17b	3.50	$p < 0.01$
18a	4.00	$p < 0.01$
18b	4.00	$p < 0.01$
18c	2.00	$p < 0.01$
19a	2.00	$p < 0.05$
19b	2.00	$p = 0.14$
19c	2.50	$p < 0.05$
19d	2.00	$p < 0.05$
19e	2.00	$p < 0.05$

Note. P-values indicate Mann-Whitney U Test difference between teacher endorsed survey values and the lowest possible survey value. See survey (**Appendix A**)

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Representations of Mathematicians in Lower Secondary Mathematics Textbooks

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ABSTRACT

We report on a study focused on identifying and describing the representations of mathematicians contained in Mexican textbooks of lower secondary level. We considered *representations* not only in the text but also in drawings, photographs, and illustrations in general. The term *mathematician* was understood in a broad sense: any person (or group of people) that in the textbook either (1) was explicitly referred to as a mathematician, (2) was credited with the development of a mathematical concept or tool, or (3) was displayed performing some sort of mathematical activity (such as counting, modelling, etc.). The results show that the representations that most frequently appear in the textbooks are white male mathematicians (mainly Europeans), who lived in ancient times; the representations of female mathematicians are almost nil. At the end of the paper the implications of these results are discussed, and some directions for future research are suggested.

Keywords: images of mathematicians, lower secondary education, mathematics textbooks, representations of mathematicians

INTRODUCTION

It is well known that mathematics students and the general public hold several perceptions or images—usually negative—associated with mathematics and mathematicians. Such images have been identified as factors that can be decisive for the students to feel (un)identified with, (un)attracted to, and (dis)interested in the study of mathematics (Picker & Berry, 2000; Rock & Shaw, 2000). Moreover, some authors suggest that the negative images of mathematics and mathematicians that some students possess are a factor that inhibits them, especially women, from pursuing an advanced degree in mathematics (Mendick, 2005; Piatek-Jimenez, 2008). But, where do these images come from?

The images that students have about mathematics and mathematicians are constituted over time and are shaped by experiences and elements coming from different social contexts. Rensaa (2006) states that these experiences and elements come from two different spheres: the *public society*, constituted for example by parents' and friends' images as well as representations of mathematics and mathematicians in the mass media and popular culture, and the *school society*, composed of teachers' and peers' images and the existing representations in school materials and content.

There are several studies focused on studying representations of mathematics and mathematicians in the public society. For instance, Furinghetti (1993) explores the representations of mathematics and mathematicians in different films and literary works, Rensaa (2006) investigates adults' images of mathematicians; Mendick (2007) analyses students' representations of mathematics and mathematicians in popular culture, and, more recently, Evans et al. (2014) investigate the representations of mathematics in advertisements included in several daily

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Contribution of this paper to the literature

- With this research we contribute to the study of the school resources (particularly lower secondary mathematics textbooks) that can shape students' images of mathematics and mathematicians.
- The research reported contributes to the study of representations of scientists and mathematicians in Latin American societies, which have been scarcely investigated.
- The study problematizes the role of mathematics textbooks in the constitution of a dominant discourse around "the mathematician", and examines some of the consequences of such discourse in students' subjectivities.

newspapers from the United Kingdom. Although there are also studies focused on the school society, some components of this society remain under-researched.

One of the most studied components of the school society are the images that students have about mathematics and mathematicians (e.g. Martin & Gourley-Delaney, 2014; Picker & Berry, 2000); however, other elements such as teachers' images of mathematics and mathematicians, or the representations of mathematics and mathematicians in school materials, have been little studied (see for example Beswick, 2012). With this research we want to contribute to the study of the sources in school society—as defined by Rensaa (2006)—that can shape students' images of mathematics and mathematicians. In particular, our study has a double purpose: on the one hand, it focuses on identifying, classifying and describing the representations of mathematicians in Mexican lower secondary mathematics textbooks; on the other hand, our study problematizes the role of mathematics textbooks in the constitution of a dominant discourse around "the mathematician", and examines some of the consequences of such discourse in students' subjectivities.

Since the nineties, Levin and Mayer (1993) pointed out that the studies related to illustrations in textbooks, have focused mainly on the study of mathematical representations such as graphs and figures, but have left in the background the study of pictorial representations such as photos and illustrations. Although the interest in studying textbooks' illustrations, their functions and effects in the conceptualization has continued since then (see for example Liu and Qi, 2014), the representation of mathematicians in textbooks remains an unexplored area. Thus, a contribution of our study is to expand our knowledge about the potential effects that the representations of mathematicians that are included in textbooks may have on the students.

To develop our research, we analysed the bestselling Mexican mathematics textbooks for secondary level, which reach hundreds of thousands of children. Our analysis focused on identifying the representations of mathematicians in these textbooks. We believe that our research not only contributes to the study of sources in the school context that can shape the images students have of mathematicians, but also contributes to the study of representations of scientists and mathematicians in Latin American societies, which have been scarcely investigated (see for example De Gómezgil, 1975; Medina-Jerez, Middleton, & Orihuela-Rabaza, 2011; Aguilar, Rosas, Zavaleta, & Romo-Vázquez, 2016).

After this introduction we will continue with a review of the literature related to this study, namely studies on representations of mathematicians and studies on representations of scientists in textbooks; next we will describe the method we used to select the textbooks and guide their analysis; we then continue with a discussion of the representations of mathematicians identified in the textbooks, ending with a discussion of the results and their implications.

A BRIEF REVIEW OF RESEARCH ON REPRESENTATIONS OF MATHEMATICIANS

The research on representations of mathematicians in particular, and scientists in general, has its origin in the seminal work of Mead and Metraux (1957) about the image of the scientist among high school students. This was the first research study that revealed the existence of a stereotypical image of a scientist among students: a middle-aged man, who may be unshaven and unkempt, working in a laboratory surrounded by instruments and conducting experiments. As the years passed, a research area specialised in the study of representations of scientists in different contexts and populations was developed; the results produced within this research area showed the persistence and ubiquity of these stereotypical perceptions over time and among different populations (see for example Chambers, 1983; Finson, 2002). Over time the studies on representations of scientists became more specialised, for example by discipline, resulting in the emergence of studies focused on representations of engineers, mathematicians and practitioners in other scientific careers.

One of the first collective efforts to investigate the popular image of mathematicians—and mathematics—can be traced back to the late eighties in the call for papers for the fifth ICMI study. There, Howson, Kahane and Pollak (1988) posed the following questions: "What is the popular view of a mathematician? To what extent does that view influence both the wish to study mathematics, or, should the possibility arise, to support mathematicians in their work?" (p. 208). From this moment, several research studies emerged addressing these questions from different perspectives.

People's Images of Mathematicians

The studies by Rock and Shaw (2000) and Picker and Berry (2000) were the first to reveal how students imagined mathematicians and the kind of work they carry out. Rock and Shaw (2000) ran a survey among 215 children from kindergarten through eighth grade (14–15 years old) where they were asked about the work of mathematicians (What do mathematicians do? What types of problems do mathematicians solve? What tools do mathematicians use?), and were also asked to draw mathematicians at work. They found that children tend to think that mathematicians do the same kind of mathematics that they do in the classroom, and that there is a lack of knowledge among the students about the types of work that mathematicians develop in the real world. With regard to their pictorial representations, they report that: "Most children's drawings showed smiling figures and more female than male figures; more than half indicated no definite racial characteristics" (pp. 553–554). On the other hand, Picker and Berry (2000) studied pictorial representations of mathematicians produced by 476 lower secondary school pupils (12–13 years old) from the USA, United Kingdom, Finland, Sweden, and Romania. They found that often mathematicians are depicted as teachers of mathematics—often using violence and intimidating students—but that they are also represented as a foolish person lacking common and fashion sense, or possessing special powers that may include magic potions and wizardry.

The presence of these stereotypical and negative images of mathematicians among people was subsequently confirmed in other studies with different populations. For example, Rensaa (2006) interviewed 31 adults about how they imagined mathematicians; she found that more than 40% of respondents have an image of a mathematician as being "a middle aged man with glasses, old-fashioned dressed and middle fitted, unsocial and boring" (p. 3). Similarly, Piatek-Jimenez (2008) explored the images that five female undergraduate mathematics students had about mathematicians; she found that the female students perceived mathematicians as exceptionally intelligent, obsessed with mathematics, and socially inept. It is important to note that there are studies suggesting that people who have a positive relationship with mathematics (e.g. they like mathematics or they are good mathematics students) have different images of mathematics and mathematicians, that is, less negative and more realistic images (Aguilar et al., 2014; Mendick, 2007).

Representations of Mathematicians in Mass Media

In one of the first studies of representations of mathematicians in mass media, Furinghetti (1993) explored the representations of mathematics and mathematicians in different films and literary works. She found that the mathematician is represented as a "meek man (until he rebels against vexation and injustice), absent-minded, naive and, generally speaking, biddable, but also endowed with clear (and sometimes strict) ideas about ethic and morals" (p. 36). Subsequent studies focused on identifying representations of mathematicians in popular culture found similar results; for example, by exploring how mathematicians are represented in movies, books, and songs, Wilson and Latterel (2001) find that mathematicians are often represented as troubled, mentally ill, or socially maladjusted individuals. In another example, Moreau et al. (2010) analyse the discourse in popular cultural texts, and conclude that mathematicians are generally represented as white, heterosexual, middle-class men.

Although several scholars suggest that the representations of mathematics and mathematicians that are present in mass media and popular culture can shape people's images, these representations are still poorly investigated. Particularly unexplored are the representations contained in school materials that are extensively distributed among pupils, such as textbooks, and which are the focus of this research. In the next section we review some studies on representations of scientists in textbooks, which are a precursor and inspiration for our study.

REPRESENTATIONS OF SCIENTISTS IN TEXTBOOKS

During recent years, research on mathematics textbooks has gained momentum because of the great interest that this research area has aroused within the international community of mathematics educators. The development of this research area manifests itself in a latent way through specialized conferences and publications. For instance, the *International Conference on Mathematics Textbook Research and Development*, the special issue of the journal *ZDM* focused on textbook research in mathematics education (Volume 45, Issue 5, September 2013), or the recently published *ICME-13 Monograph* (Fan, Trouche, Qi, Rezat, & Visnovska, 2018). However, despite the apparent growth and importance of research on mathematics textbooks, the studies on representations of scientists in general and of mathematicians in particular, remain scarce. We do not know much about how these individuals are represented in mathematics textbooks.

There are different arguments about the relevance of the study of representations of scientists and science itself in textbooks. For example, it is known that the representations contained in science textbooks influence the images of the nature of science that students and teachers possess (McComas, Almazroa, & Clough, 1998); there is also evidence of the existence of similarities between students' images of science and scientists and the representations contained in the textbooks they use (She, 1995). It has been further argued that the representations of scientists in textbooks are a key factor in the lack of diversification in science, that is, a dearth of diversity—in terms of race, gender, cultural background, etc.—of the individuals that are depicted as scientists. Despite these arguments, not many studies have been conducted to understand how scientists are represented in textbooks around the world; even more rare are studies focused on studying representations of mathematicians in school materials.

Yacoubian, Al-Khatib and Mardirossian (2017) conducted a study of Lebanese textbooks and found stereotyped images of scientists that predominantly depict white men of European descent; the textbooks do not represent scientists from other regions of the world. In addition, scientists are presented as individuals working alone, who perform experiments in their laboratories following the scientific method and operating within Eurocentric paradigms.

In a pair of studies that analysed the representations of a select group of scientists in a sample of Canadian textbooks, van Eijck and Roth (2008, 2013) found that the representations of scientists in biology textbooks are inconsistent with the past and current practices of scientists; for example, the authors claim that the representations of scientists "largely ignores the process of scientific practice as mediated by a scientist's community and instead focuses on other scientists separated from the scientist in space and time" (2013, p. 24).

There are other studies focused on analysing diversity in the representations of science and scientists in textbooks; one of the reasons that it is important to have a great diversity in curriculum materials such as textbooks is that it encourages minority students to identify themselves with science and scientists. Brooks (2008) analysed five American physical science textbooks in which she identified a lack of diversity in the representations of scientists; for example, she found that the kind of scientists most often depicted in the textbooks were white men. According to the author, these representations perpetuate the stereotype that science is for white men. In a more recent study, Ceglie and Olivares (2012) analysed two American biology books focusing on the diversity of their representations; the researchers confirmed the results of Brooks (2008) in that the dominant representation of a scientist in these textbooks is that of a white man, while there is a glaring under-representation of black and Hispanic individuals.

The study reported in this paper will help to fill an evident gap in the research literature in mathematics education: the lack of studies on the representations of mathematicians contained in the textbooks used by mathematics students and teachers around the world. In the next section we explain the method we follow to develop the study.

METHOD

One of the purposes of this study is to identify and describe the representations of mathematicians contained in Mexican textbooks at lower secondary level; in this section we describe the method followed to achieve this aim. In particular we describe the process of selection of textbooks, as well as the concepts and questions that guided the analysis of the selected textbooks. Regarding the second purpose of the study, the problematization of the role of mathematics textbooks in the constitution of a dominant discourse on "the mathematician", such problematization is based on this analysis of the textbooks, and presented in the discussion section at the end of the paper.

Selection of Textbooks

Lower secondary education in Mexico is typically followed by students between 11 and 15 years old; it is divided into three grades, each with a duration of one year. For this study we selected the five bestselling textbooks for each grade level; thus, the total sample used in the study was fifteen textbooks. We decided to select textbooks of this educational level for our study because some authors point to this period as a sensitive one for the determination of students' attitudes towards mathematics (Aiken, 1970; Picker & Berry, 2000).

To determine which textbooks are bestsellers at the lower secondary level in Mexico, we turned to the *Catalogue of Free Textbooks*, which is a report published by the Ministry of Education of Mexico (2015). This report indicates the circulation and distribution of each of the textbooks approved by the Ministry for its use in classrooms. The sample of selected textbooks is significant because, according to the official data, the selected titles reach a total of 4,000,321 children, and represent more than the 54% of the total number of textbooks distributed at the lower secondary level in Mexico. **Table 1** indicates the fifteen textbooks selected for this study, their positions in the sales

Table 1. Bestselling mathematics textbooks at lower secondary level in Mexico

	Distribution ranking position	Textbook	Number of items distributed
First grade	1	Block, S.D. & García, P.S. (2015)	271,649
	2	Arriaga, R.A. & Benítez, C.M.M. (2015a)	202,852
	3	Hernández, Z.P., Hernández, H.M. & Magallanes, G.G. (2014)	111,216
	4	Sánchez, S.F. (2015a)	101,316
	5	Escareño, F. & López, E.O.L. (2015a)	67,639
Second grade	1	García, P.S. & Block, S.D. (2015)	282,342
	2	Arriaga, R.A. & Benítez, C.M.M. (2015b)	175,325
	3	Sánchez, S.F. (2015b)	109,524
	4	Ramírez, C.M., Castillo, C.R., Vergara, R.D., Flores, O.M.E. & Azpeitia, V.J.G. (2015a)	95,129
	5	Xique, A.J.C. (2015)	74,274
Third grade	1	Block, S.D., Mendoza, V.D.B., García, P.S. & García, Z.J.C. (2015)	256,356
	2	Arriaga, R.A. & Benítez, C.M.M. (2015c)	178,965
	3	Escareño, F. & López, E.O.L. (2015b)	102,194
	4	De Icaza, P.A. (2015)	68,677
	5	Ramírez, C.M., Castillo, C.R., Vergara, R.D., Flores, O.M.E. & Azpeitia, V.J.G. (2015b)	74,998

ranking for each grade, and the number of items distributed. In the **appendix** the complete references of the selected textbooks are included.

What Counted as a Representation in this Study?

According to Goldin (2014), the term "representation" refers to: visible or tangible productions such as a graph or a formula (an external representation), mental or cognitive constructs of people (an internal representation), or even act or process of inventing or producing representations. During the analysis of the textbooks it was noticed that the format of the representations of mathematicians varied. For instance, there were images depicting well-known mathematicians or representing groups of people performing some mathematical activity; there were also texts describing, for example, the work or contribution of a particular mathematician. Thus, with the aim of using a unit of analysis covering the range of representations that could be found in the selected textbooks, we decided to consider as a *representation* not only text but also drawings, photographs, and illustrations in general.

What was Considered as a Mathematician?

In the study the term *mathematician* was understood in a broad sense; that is, a mathematician was considered to be any person (or group of people) that in the textbook either (1) was explicitly referred to as a mathematician, (2) was credited with the development of a mathematical concept or tool, or (3) was displayed performing some sort of mathematical activity (such as counting, modelling, etc.).

After the concepts of representation and mathematician were defined, we proceeded to analyse each book, page by page, to identify all the representations of mathematicians; if there was discrepancy or doubt about the way a representation should be interpreted, an explicit discussion was organised to try to reach a consensus on its interpretation. Once a representation of a mathematician was located, we focused our attention on six aspects of the representation. Such aspects served as a basis for the coding of the representations of mathematicians. They were constructed taking into consideration some of the research results reported in the literature on representations of scientists in textbooks. The six aspects and their rationale are presented next:

1. *Individuals or groups of persons represented*. There are studies showing that scientists are often represented working alone, thus denying the social and collective nature of the production of knowledge (Al-Khatib & Mardirossian, 2017; van Eijck & Roth, 2013). Thus, we were interested in knowing if mathematicians were represented individually or collectively, in order to identify the kind of perspective on mathematical activity that was favoured in the textbooks. In addition, it was decided if the individual or group represented was identifiable, that is, if it was the representation of a "known mathematician" (such as Pythagoras or the Maya). This identification was a basic element in the analysis, since it allowed to count the frequency of the mathematicians represented, as well as the application of the subsequent codes.

2. *Period of time when the represented mathematician lived*. Another aspect of interest was determining whether the textbooks promoted an image of the mathematician as someone who lived in ancient times, or as a contemporary individual. This is an aspect that we think may contribute to the constitution of a poor conception among students about mathematicians, namely, to perceive them as people who lived in antiquity, without a

Table 2. Gender, geographical origin, and time period of the most frequently represented mathematicians in the mathematics textbooks of first grade

		Frequency	Period of time in which they lived	Geographical origin	Gender
Mathematicians most frequently represented	Thales of Miletus	9	c. 620 B.C.E.–546 B.C.E.	Western Europe	Male
	Pythagoras	6	c. 570 B.C.E.–495 B.C.E.	Western Europe	Male
	Eratosthenes	4	c. 276 B.C.E.–195 B.C.E.	Western Europe	Male
	Euclid	3	c. 330 B.C.E.–260 B.C.E	Western Europe	Male
	Isaac Newton	3	1643–1727	Western Europe	Male
	Babylonians	3	c. 1792 B.C.E.–539 B.C.E	Near East	Group
	Ancient Egyptians	3	c. 3150 B.C.E.–31 B.C.E	Mediterranean Africa	Group

connection nor presence in modern societies. Thus, we tried to determine—by consulting external sources of information such as encyclopedias—the time when the mathematicians depicted were alive in order to determine whether the representations included in the textbooks were of ancient or contemporary mathematicians.

3. *Geographic origin of the represented mathematician*. Considering the studies of Brooks (2008); Ceglie and Olivares (2012) that highlight the importance of diversity in the representations of scientists in textbooks, we identify the geographical origin of mathematicians represented in textbooks, taking as reference "The cultural regions of the world" (Ministry of Education of Mexico 2010, p. 17) North America, Latin America, Western Europe, Eastern Europe, Mediterranean Africa, Africa, Near East, Middle East, Far East, Indochina and Oceania.

4. *Gender of the mathematician represented.* Since there is research pointing out the under-representation of women in science textbooks (Pienta & Smith, 2012), there was an interest in identifying the proportion of female mathematicians represented in the textbooks considered in the study. Thus, when possible, the representations of mathematicians were classified as female (woman), male (man), or group (when there was a representation of a group of people). There was not much room for differing interpretations of the masculine and feminine categories since, as we shall see in the results section, the representations of female mathematicians were almost nil.

5. *Role of the representation*. It is known that the textbooks illustrations that are related to their accompanying text, are more effective in promoting understanding than those illustrations that only have a "decorative" role (Levin & Mayer, 1993, pp. 106-107). For this reason, in addition to identifying the representations of mathematicians, it was sought to determine the role that these representations play in the lesson where they appeared. To organize the results, the following two categories were used: (1) the representation is used to illustrate a concept or to introduce a mathematical activity, or (2) the representation is not used to introduce an activity nor a concept, but only has ornamental or anecdotal value.

6. *Format of the representation*. Finally, the representations were classified according to their format. It is important to identify the format in which the illustrations appear, since different research studies indicate that when pictures are incorporated into text material, the memory for that material can be improved (see Mandl & Levin, 1989; Willows & Houghton, 1987). Thus, the representations of mathematicians in this study were classified as (1) textual descriptions, or (2) illustrations—such as photographs or drawings.

The coding of the representations of mathematicians was performed independently, we constituted three teams of two members each, to perform the coding separately. Subsequently, the teams met to try to achieve a consensus on the coding. As in the previous phase where the representations of mathematicians were identified, discrepancies over the coding were solved through collective dialogue, however, these discrepancies were not significant nor frequent. Once consensus was reached, the information associated with each of the six aspects of the representations of mathematicians, was concentrated in spreadsheets to facilitate its further interpretation.

Next we present the results obtained after identifying and coding the representations of mathematicians contained in the mathematics textbooks included in this study.

RESULTS

In total, 158 representations of mathematicians were found in the fifteen textbooks analysed: 75 in the first grade textbooks, 32 in the second grade textbooks, and 51 in the textbooks for third grade. Below we present the results concerning the mathematicians most frequently represented in the textbooks, organized by school grade.

Representations of Mathematicians in the First Grade Textbooks

Among the most frequently represented mathematicians in the first grade mathematics textbooks are: Thales of Miletus (represented nine times), Pythagoras (represented six times), and Eratosthenes (represented four times). **Table 2** shows more information about the most frequently represented mathematicians.

Figure 1. Representation of Thales of Miletus with an anecdotic value, included in the textbook by Arriaga and Benítez (2015a)

As for the roles played by the representations of mathematicians in the first grade textbooks, most (56) are representations that do not serve to introduce a mathematical activity or a specific topic, but only have anecdotal value; nineteen representations are used to introduce some mathematical activity or to illustrate a concept. An example of a representation with an anecdotic value is the representation of Thales of Miletus in the textbook by Arriaga and Benítez (2015a). This representation consists of a photograph of a sculpture of Thales of Miletus, which is used as part of a timeline that appears as a front page for one of the units of the textbook (see **Figure 1**); the picture of the sculpture representing Thales of Miletus is located in the timeline about the year 600 B.C.E. and is accompanied by the following caption: "In his traveling around the world, Thales of Miletus contributed to the development of geometry" (our translation, p. 67).

An example of a representation that is used to introduce a mathematical activity is the following extract from Sánchez (2015a) in which Eratosthenes is mentioned as the person who found a method to locate prime numbers, the Sieve of Eratosthenes, and the reader is asked to determine certain prime numbers using this algorithm:

> *"Hector and Yolanda discovered that Eratosthenes found a method to locate the prime numbers that appear in a sequence, this procedure is called the Sieve of Eratosthenes.*

Determine the prime numbers between 1 and 150, following the method of Eratosthenes [Next the algorithm is illustrated step by step]" (p. 74, our translation).

Regarding the format of the identified representations, the majority (72) are textual descriptions, while three are illustrations. An example of an illustration of a mathematician is the picture of Thales of Miletus described above; an example of a textual description of a mathematician is the following text referring to Pythagoras taken from Escareño and Lopez (2015a):

> *"Pythagoras was a Greek mathematician who taught around 580 B.C.E. He considered sacred the number 10, written in a triangular shape, and he called it Trianon" (p. 25, our translation).*

Representations of Mathematicians in the Second Grade Textbooks

In second grade textbooks the ancient Egyptians were the most frequently represented mathematicians (four times). Other mathematicians frequently represented are reported in **Table 3**.

Table 3. Gender, geographical origin, and time period of time of the most frequently represented mathematicians in the mathematics textbooks of second grade

		Frequency	Period of time in which they lived	Geographical origin	Gender
Mathematicians most frequently represented	Ancient Egyptians	4	c. 3150 B.C.E.–31 B.C.E	Mediterranean Africa	Group
	Archimedes	2	c. 287 B.C.E.–212 B.C.E	Western Europe	Male
	Diophantus	2	c. 201 C.E.–285 C.E	Western Europe	Male
	Al-Khawarizmi	2	c. 780 C.E.–850 C.E	Near East	Male
	Robert Boyle	2	1627–1691	Western Europe	Male
	Blaise Pascal	2	1623–1662	Western Europe	Group

Table 4. Gender, geographical origin, and time period of time of the most frequently represented mathematicians in the mathematics textbooks of third grade

		Frequency	Period of time in which they lived	Geographical origin	Gender
Mathematicians most frequently represented	Ancient Egyptians	4	c. 3150 B.C.E.–31 B.C.E	Mediterranean Africa	Group
	Archimedes	4	c. 287 B.C.E.–212 B.C.E	Western Europe	Male
	Fibonacci	3	1170–1250	Western Europe	Male
	Eratosthenes	2	c. 276 B.C.E.–c. 195 B.C.E.	Western Europe	Male
	Leonardo da Vinci	2	1452–1519	Western Europe	Male
	Ancient Indians	2	c. 7000 B.C.E.–500 C.E	Middle East	Group

Regarding the role of the representations of mathematicians in the second grade textbooks, eighteen are representations with an anecdotic value, while fourteen serve to introduce some mathematical activity or to illustrate a concept.

Concerning the format of the representations, 28 are textual descriptions of mathematicians, and four are illustrations.

Representations of Mathematicians in the Third Grade Textbooks

As in the textbooks for second grade, the ancient Egyptians and Archimedes are among the most frequently represented mathematicians in the third grade textbooks. Other mathematicians frequently represented are shown in **Table 4**.

The representations of mathematicians in the selected third grade textbooks appear most often as isolated elements without an explicit connection to the topic addressed; out of a total of 51 representations, 35 appear as anecdotal elements, while sixteen serve to introduce mathematical activities or to illustrate concepts.

Regarding the format of the representations, 44 are textual descriptions of mathematicians while seven are illustrations of mathematicians.

DISCUSSION

The results of this research show that the analysed textbooks contain representations of mathematicians that can favour the construction of stereotypical images of mathematicians among students, such as those that have been previously reported in the literature (Picker & Berry, 2000; Rock & Shaw, 2000).

Observe for instance the individuals or groups of persons most often represented (with a frequency equal to or greater than four) and the period of time when they lived: Thales of Miletus, Pythagoras, Eratosthenes, the ancient Egyptians, and Archimedes. This is only a sample of a more general phenomenon, namely, that these textbooks expose students mainly to representations of mathematicians who lived several centuries ago. We believe that the predominance of representations of ancient mathematicians does not favour the construction of an image of a mathematician who has a role in modern societies; in our analysis we found few representations of contemporary mathematicians and their work; in particular, in all the textbooks analysed only eleven representations of mathematicians who lived in the 20th century were found—for example Andrew Wiles, George Stibitz, and Wolfgang Haken. It could be argued that the prevalence of representations of ancient mathematicians in these textbooks is due to the fact that most of the mathematical topics addressed at this level were developed precisely by ancient mathematicians—as in the case of the Pythagorean theorem; however, as we have shown in the results of this study, most of the time the representations of these ancient mathematicians do not serve to introduce a mathematical concept or activity. Rather, the representations have an anecdotic or ornamental role. As will be discussed later in this section, this kind representation with an anecdotic role represents an area of opportunity to broaden the diversity of representations of mathematicians and promote a richer image of them, not limited to representations of ancient mathematicians.

With regard to the geographical origin of the mathematicians represented in the textbook sample, it is remarkable that the representations are predominantly from Western Europe. Despite the fact that these are textbooks aimed at Mexican students, none includes representations of Mexican mathematicians nor representations of regional ethnic groups that have been prominent in the application and development of mathematics, such as the Mayan for instance; an exception is the textbook by Ramírez, Castillo, Vergara, Flores and Azpeitia, (2015b) which mentions Mesoamerican people and the Nariño culture from Colombia. We believe that the prevalence of representations of white European mathematicians encourages the grounding of a stereotype present in several students, namely that scientists and mathematicians possess such particular racial origin (Chambers, 1983; Mendick, Moreau, & Hollingworth, 2008; Picker & Berry, 2000); on the other hand, as has been argued in studies on diversity in representations of scientists in textbooks, the lack of racial diversity in representations of scientists in textbooks can hinder students from minority racial groups in identifying themselves as possible or potential scientists; in the same way we believe that the absence of representations of mathematicians of American Indian origin does not encourage Mexican students, at whom these textbooks are aimed, to identify themselves as people who could become mathematicians in the future.

In their study on the gendering of representations of mathematics and mathematicians in popular culture, Mendick et al. (2008) declare that the popular culture texts strongly support the association of mathematics with masculinity through three mechanisms: "the dominant representations of mathematicians being men, the disappearing of women's mathematical contributions and the ways that women doing mathematics are subordinated in a range of ways including their youth and their positioning as appendages to 'greater' male mathematicians" (p. ii). Our study shows that the analysed textbooks have a gendered representation, an eminently male representation of mathematicians, that is achieved through the first of the mechanisms mentioned by Mendick et al. (2008), the dominant representations of mathematicians being men: out of a total of 158 representations of mathematicians identified in the textbooks, we located only one representation of a woman, Hypatia the Greek mathematician. This dominant representation of mathematicians as men in mathematics textbooks could encourage women to produce masculinized images of mathematicians, which in turn could hinder their identification of themselves as potential members of the community of mathematicians and discourage them from pursuing further studies in mathematics (Piatek-Jimenez, 2008).

Concerning the role of the representations of mathematicians in the lessons of the textbooks, the results show that out of the 158 representations identified, 109 are representations that do not serve to introduce any concept or mathematical activity. They are mostly representations of ancient mathematicians who have an anecdotal role in the lesson; these representations are used in a small grain size illumination approach, as discussed by Jankvist (2009). The role of the remaining 49 representations is to introduce some activity or mathematical concept. Our point here is that if the majority of the representations are not necessarily related to the topic or to the mathematical activities in the lesson, then this is an area of opportunity that could be used to enhance and diversify the representations of mathematicians included in the textbooks, in order to foster a richer, more diverse and updated image of a mathematician among students. This universe of representations of mathematicians with an anecdotal or ornamental role could be enriched by including representations of contemporary mathematicians, female mathematicians, mathematicians coming from racial, cultural, and social minority groups, children and youth participating in mathematical competences, etc. The opportunities to enrich the representations of mathematicians in textbooks are vast.

A finding about the format of the representations of mathematicians in the textbooks analysed is that most (144) are textual descriptions of mathematicians, while the rest (14) are graphic illustrations of mathematicians. Both types of representations have different qualities; for instance, the graphic illustrations of mathematicians allow students to associate with the mathematician represented a face, a style of hair or clothing, etc. It is not clear to us—and it is beyond of the scope of this study to investigate—if the nature of the textual descriptions of mathematicians affect the constitution of students' images of mathematicians in the same way or with the same intensity as the illustrations do. For instance, Picker and Berry (2000) analysed the images that students from five countries had of mathematicians, through drawings that they produced; the researchers found that in all countries there was at least one drawing of Albert Einstein, which, according to the researchers, indicates that these images come from sources such as the media, comic books, and cartoons. As can be noticed, those are sources filled with illustrations and graphical components. However, since the descriptive and textual nature is one of the characteristics of most of the representations identified in the textbooks, we believe it is an aspect that could be further investigated in the future.

A Dominant Discourse Supported by Mathematics Textbooks

In this section we address the second purpose of this study, which is to problematize the role of mathematics textbooks in the formation of a dominant discourse on "the mathematician". As can be noticed, the analysis of

Mexican textbooks produces results very similar to those reported in the research on representations of mathematicians. This is, our results show stereotyped representations of mathematicians, such as those that have been identified around the world in other mass media, and in different populations. Although these stereotyped images have been identified in different communities and contexts, we think that these are discursive regularities that constitute a dominant discourse about who and how mathematicians are.

The problem, as we see it, is that this discourse shapes and influences the way in which mathematicians are perceived, but also, it is a discourse that constitutes taken-for-granted truths about mathematicians and their activity, and positions the students with respect to mathematics. That is, it is a discourse that impacts the subjectivity of the student and indicates whether he or she is a suitable or adequate person to develop mathematics. For example, our results show that the textbooks analysed promote a discourse from which women are practically exiled, we can affirm that such discourse generates a certain "reality" that presents mathematics as a masculine enterprise. As noted by Dowling (1998), the point here is not only whether the textbook provides "appropriate role models" for girls, the problem rather is that this discourse contributes to create a perception about what it means to be a woman in the world of mathematics, and apparently is a world where women are definitely not its protagonists.

The Mexican students who are exposed to this discourse on the representations of mathematicians, experience a situation similar to that encountered by the female students. The discourse that these textbooks promote erases all trace of the uses and mathematical developments that took place in the region that today is known as Mexico and Latin America. We argue that such discourse legitimizes some social groups as capable of developing mathematics and represents them as outstanding in this human activity, but in which neither Mexicans nor their ancestors have participatory roles. Such discourse favours dividing practices in which certain social groups are classified as able to contribute to the development of mathematics, while others are not.

We call on mathematics textbook researchers, authors, and mathematics teachers, to conceptualize the textbook as a device that disseminates this type of discourse in the school society, particularly in the mathematics classroom. If we want all our students to feel included in the world of mathematics—regardless of their gender or socio-cultural background—, and perceive themselves as competent people to develop them, then we need to begin to question ourselves, the representations about the mathematicians that are included in the mathematics textbooks. The mathematics classroom could function as a suitable experimentation space to identify, question and discuss with the students, the stereotyped images about mathematicians that may be present in their textbooks. Other future research directions that could be pursued in future studies are mentioned in the next section of the article.

Some Directions for Future Research

The results of this research show that the bestselling mathematics textbooks for the lower secondary level in Mexico privilege representations of mathematicians that could perpetuate students' stereotypes, such as the idea that a mathematician is a white man. Are the representations of mathematicians included in lower secondary textbooks from other regions of the world similar to or different from those reported in this study? Do representations of ancient mathematicians prevail in other textbooks? Is there a lack of representations of female mathematicians? These and other questions could be answered by research studies similar to the one reported in this article, conducted in other regions of the world; comparative studies such as those made in other areas of textbook research (see for example Kim, 2012) would also help to expand our knowledge of representations of mathematicians in mathematics textbooks.

Primary school is another context in which it would be relevant to develop this type of research. Due to the large number of children who are educated using mathematics textbooks in this educational level, and because it is an important stage in the development of attitudes towards mathematics, it would be important to extend this kind of research to include the analysis of the representations of mathematicians included in the mathematics textbooks at primary level.

Finally, we believe it is important to devise mechanisms to make those people responsible for designing and writing mathematics textbooks familiar with the results reported in this article. It is relevant that publishers, designers, and authors of mathematics textbooks be aware of the key role that the representations of mathematicians contained in the textbooks, may have in the constitution of students' images and attitudes towards mathematics. Furthermore, it is important that they know that several of the books that they produce and the students consume may be filled with stereotypical representations that do not contribute to the formation of a rich, diverse, and updated image of the mathematician.

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APPENDIX

Mathematics Textbooks Selected for the Study

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Arriaga, R. A., & Benítez, C. M. M. (2015b). *Matemáticas por competencias 2*. Mexico: Pearson Educación.

Arriaga, R. A., & Benítez, C. M. M. (2015c). *Matemáticas por competencias 2*. Mexico: Pearson Educación.

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Block, S. D., Mendoza, V. D. B., García, P. S., & García, Z. J. C. (2015). *Matemáticas 3. Conect@ Estrategias*. Mexico: Ediciones SM.

De Icaza, P. A. (2015). *Matemáticas 3. Serie Todos Juntos*. Mexico: Santillana.

Escareño, F., & López, E. O. L. (2015a). *Matemáticas 1*. México: Trillas.

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Grade 10 Learners' Science Conceptual Development Using Computer Simulations

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ABSTRACT
This study explored Grade 10 learners' science conceptual development when conducting practical work using Computer Simulations (CS) and compared achievements with those from Traditional-Chalk-and-Talk (TCT). A pre- and post-quasi-experimental research design was used. 53 learners were assigned to the Experimental Group (EG) and 52 to the Control Group (CG). The EG was taught using CS, while CG used TCT. Interviews were used to identify learners' experiences after interventions. The overall results show that learners in EG enjoyed science and developed conceptual understanding better than those in the CG (T-test, $p < 0.05$), (ANCOVA, $p < 0.01$). The achievements of girls ($M=54.60$, $SD=10.93$) and boys ($M=54.39$, $SD=7.90$) in EG after intervention were not significantly different $t(51) = -0.08$, ($p < 0.05$). Despite the high learner-to-computer ratio environment, these results compare well with those of low learner-to-computer ratio and this is good news to developing countries where there are limited resources.

Keywords: computer simulations, learner-computer ratio, practical work, performance

INTRODUCTION

Over the last decade, Information Communication Technology (ICT) has made rich and dynamic visual representations using personal computers (Annetta, et al., 2009). These visual representations give users of educational Computer Simulations (CS) the flexibility to make changes and to see the effects (Annetta, et al., 2009). CS provide learners with rich educational experiences and instantaneous feedbacks on the results of virtual experiments (Kaheru & Kriel, 2012; Podolesky, Perkins & Adams, 2010; Sentongo, Kyakulaga & Kibirige, 2013). Instead of teachers explaining what happens to one variable when altering another, learners will be able to observe on the computer screen. As a result, CS give immediate feedback to learners and help them to understand science concepts. CS experiments can alleviate the teachers' burden of buying expensive apparatus for use in a specific practical work. CS can engage learners in inquiry-based learning and enhance science conceptual development (Abdullah & Shariff, 2008; Folaranmi, 2002). Also, CS may provide learners with access to questions and methods of inquiry which are well aligned with the ways scientists use experiments for exploration and discovery (Podolesky, et al., 2010). While the use of technology can assist to improve learners' engagement in practical work in a science class, it has not been clarified on how technology can improve learners' science conceptual development (Taylor & Parsons, 2011). Consequently, many schools in South Africa do not use CS when conducting practical work in order to enhance science conceptual development. CS are envisaged to compensate for the resources required to perform practical work. In fact, CS saves the school from buying expensive equipment for the laboratory. Learners can do the practical work on virtual system. To the best of our knowledge there are very few studies in South Africa on learners' use of CS in practical work to develop science concepts. Therefore, this study explored Grade 10 learners' conceptual development of waves using CS during practical work. This study highlights the importance of learners using CS to enhance science conceptual development in high schools with limited laboratory resources. The findings from this study add to the existing literature of teaching high school science practical work using CS.

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Contribution of this paper to the literature

- This study is one of the few studies to investigate Grade 10 learners' science conceptual development and learners' interests in the science subject in the Southern Hemisphere. The study extends the use of simulations in class to the literature because most of the studies were carried out in the Northern Hemisphere with low learner to computers ratio whereas this study is in the Southern Hemisphere where the learner to computer ratio is high.
- Doing practical work using Computer Simulation enhances learners' interests in the science subject. The interest of learners, particularly of the below average, plays a significant role in enhancing science development.
- The study shows that the use of Computer Simulation can be a stepping stone for schools with limited laboratory facilities in developing countries to do practical work as well as problem-based learning.

LITERATURE REVIEW

The percentage of computer use in the developing South Africa is very low when compared with the developed countries. The discrepancy between developed and developing countries has been termed as 'digital divide' (Fuchs & Horak, 2008). Information Communication Technology (ICT) is not widely distributed in schools in Limpopo Province, South Africa. For instance, by 2012, only 13.5% of the schools in Limpopo had computer facilities installed and 4.9% of those schools used computers for teaching and learning. The percentage of computer use for teaching and learning is very low in Limpopo when compared with Gauteng Province which has 88.5% of the schools with computers installed and has 45.4% of the computers used for teaching and learning. Furthermore, Limpopo ranks the lowest in terms of the number of households with access to Personal Computers (Tlabela et al., 2007). Also, the learner-computer ratio in secondary schools in South Africa is as high as 17:1 (Sossa, Rivilla & González, 2015). This ratio is very high when compared with the developed countries in the Northern Hemisphere where the ratio is 1:1. Notwithstanding the discrepancies in computer access, use and ratio in the country, it is not clear how schools in Limpopo use the few computers to teach science and the effect of CS in practical work. It was hypothesized that the use of CS to teach practical work would enhance teaching and learning; and provide comparative advantage for learners to cope with the ever-demanding 21st century knowledge acquisition science process skills.

Computer Simulations for Science Teaching and Learning

Computer Simulations (CS) are instructional settings where the teacher can engage learners to acquire knowledge. CS motivate students' interaction with the real-life issues. During CS learners observe and experience the experiment on the computer screen. Since CS are closest to the reality, learners are most likely to experience concepts and their meanings. The use of CS can be incorporated into the learning environment. Furthermore, Hertel and Millis (2002) reported that learning through real-world scenarios and problems have been supported by *using* CS. Schools nowadays invest in CS to enhance conceptual understanding. Simultaneously, business leaders affirm that CS is a dynamic approach to teaching science to develop the 21st-century skills (Partnership for 21st Century Bosco, 2009; Skills, 2010).

In contrast, research continues to claim that most teachers do not incorporate CS in their teaching and any other technology remains at the periphery in Africa. In South Africa, only 16% of the teachers used computers for teaching purposes (Draper et al., 2006). The reasons for not using CS in teaching science are many and range from lack of infrastructure (DoE, 2007) to overcrowded classrooms (Onwu 1998). The lack of CS use in science in the Southern Hemisphere suggests that there is a challenge. This challenge needs to be resolved by providing computer infrastructure and training teachers on the use of CS to increase learners' science conceptual development in the 21st Century.

To teach is to engage learners in learning; thus, teaching consists of involving learners in the active construction of knowledge (Jenson, Lewis & Smith, 2002). A teacher is not only equipped with knowledge of subject matter but the knowledge of how learners learn and how to transform them into active learners (Jenson, Lewis & Smith, 2002). When a teacher goes into a classroom, s/he prepares a lesson by setting out the objectives that s/he wants to achieve at the end of the lesson. It then becomes the responsibility of the teacher to ensure that what is intended for learners to learn has been learned. The question will be whether the teacher transmits the information to the learners or provides an opportunity for the learners to construct their knowledge. Furthermore, Ivowi et al. (1992) found that the lack of funds to buy equipment for practical work in most schools was a contributing factor to poor achievement.

For a hundred years now, educators have tried repeatedly to reform science education (Fetters at al., 2002), educators help learners engage in scientific inquiry to improve learner achievement. In many instances, learners gain more through inquiry learning than in a passive class. The gain is in time with the aim of teaching which is to transform learners from passive recipients of other people's knowledge into active constructors of own knowledge (Palmer, 1998). Also, learners using CS may develop positive attitudes towards science (Shymansky, Kyle & Alport, 1983).

Computer Stimulating and Practical Work

Inquiry without practical work cannot assist the learners to easily attain scientific experiences (Hofstein & Lunetta, 2004). The reason is that inquiry-based learning includes laboratory work. Teachers use laboratory tasks to learn by doing, design experiments and investigate issues in a classroom (Ivgen, 1997). Laboratory tasks are envisaged to "provide model lessons and experiences, build relevant theory and content knowledge" (Lit & Lotan, 2013). Practical work has a central role in the science curriculum to generate interest (Hodson, 1993; Hofstein, 2004; Hofstein & Lunetta, 2004; Lazarowitz & Tamir, 1994; Lunetta, 1998; Lunetta et al., 2007; Tobin 1990). In many schools where learners are only taught using the traditional approach, learners have challenges to make sense of science. Many South African schools do not have access to laboratories (Makgato & Mji, 2006). Laboratory work provides learners with an opportunity to experience science. Meaningful learning of scientific theories and their application methods means that learning should be done using laboratory investigations (Kibirige & Tsamago, 2013). Moreover, engaging in practical work should encourage the development of critical thinking skills (Ottander & Grelsson, 2006). Tobin (1990 p. 405) states that "Laboratory activities appeal as a way of allowing learners to learn with understanding and, at the same time, engage in the process of constructing knowledge by doing science". Knowledge construction depends on the level of conceptual understanding of science. A learner exposed to practical work understands better how the world operates than a learner without exposure to practical work.

CS as an Alternative to Laboratory Work

Research shows that learners construct their understanding of scientific ideas within the framework of their existing knowledge (Bransford, Brown & Cocking, 2000). It means that learners must actively engage with the content and must be able to learn from the engagement (Osborne, Simon & Collins, 2003). When learners start to engage with the content, they start to fall in what the constructivists call "*Self-discovery learning.*" Scientific discovery learning is a highly self-directed and constructivist form of learning (De Jong & Van Joolingen, 1998) and this is possible especially in Problem Based Learning (PBL) (Barrel, 2010). Polman (1999) conducted a case study of a teacher who created a collaborative learning community and provided his high school learners with opportunities to "learn by doing" science. The teacher used constructivist pedagogy by giving special attention to collaborative visualization. Constructivist pedagogy can be supplemented in a number of ways, one being through CS.

Interactive CS can meet the needs of a learner where s/he can explore and build on his/her existing knowledge (Annetta et al., 2009). Thompson, Simonson and Hargrave (1996) defined simulation as a representation or model of an event, object, or some phenomenon and Geban, Askar, and Özcan (1992) confirmed that computers could be used in science education as teaching devices. Traditionally, computers are used in biology as tools for investigating various topics, collecting data, searching literature, planning experiments, and analysing data. These functions are very common in many science and biology laboratories. However, simulations are important for formulating and, improving the conceptual models that scientists and science teachers use in their practice and teaching (Geban, Askar & Özcan, 1992). In science education, a computer simulation according to Akpan and Andre (1999) is the use of the computer to simulate dynamic systems of objects in a real or imagined world.

Literature suggests that the success of CS in science education depends on its incorporation into the curriculum and how the teacher uses it (Sahin, 2006). If teachers do not use computers, it can be hard to use it for teaching. Furthermore, the non-availability of computers complicates the matter. For instance, in Zambia, there is learner to computer ratio of 143:1, and in such a case it is a challenge to use CS effectively (Shami, Mgaya & Nkwe, 2014; Wanjala, 2016; Chaamwe, 2017). Other challenges to computer users have been reported among teachers for instance, low computers competencies among teachers in Kenya (Wanjala 2016); and Botswana (Shami, Mgaya and Nkwe, 2014). CS is a tool for classroom instruction and laboratory work. Researchers studying the use of simulations in the classroom have reported overall positive findings. Literature indicates that simulations can be effective in developing content knowledge and process skills, as well as in promoting more complex goals such as inquiry and conceptual change (Bell & Smetana, 2008). Since in the 90s, new technologies have made rich and dynamic visual representations possible on common personal computers (Annetta et al., 2009). This visual representation gives users of educational simulations the control and flexibility to make changes and see the effects in real time (Annetta et al., 2009). With these advances, simulations can provide learners with rich educational experiences as well as instantaneous feedback on the results of a virtual "experiment," (Podolesky, Perkins & Adams, 2010). The fact that CS permits learners to understand abstract scientific concepts is an indication that their conceptual understanding of scientific concepts improves. Studies (Budoff, Thormann & Gras, 1984; Cherryholmes, 1966; Cruickshank & Tefler, 1980; Greenblat & Duke, 1975) have suggested that one way to enhance these kinds of cognitive skills is through educational simulations. Simulations are thought to increase learner participation (Boocock & Schild, 1968; Farran, 1968; Stembler, 1975) and allow low- achieving students much-needed practice in applying what they have learned to new situations to improve their achievement (Cohen & Bradley, 1978).

Another advantage of CS in science education is that it is cheaper than laboratory experiments regarding time and cost (Simmons & Lunetta 1993). During experiments, learners need to wait for the results of the experiment, and they seem to waste their time. Also, for the laboratory experiments, equipment is expensive, and that is a problem for poor schools (Jegede, Okebukola & Ajewole, 1991). For instance, generally a computer-simulated experiment can take 35 minutes, but hands-on experiment can take between 120 and 180 minutes. However, CS provides learners with a quicker way of understanding of concepts. While students do experiments with the computer, they can receive immediate feedback (Jegede, Okebukola & Ajewole, 1991). Also, learners using CS have opportunities for reinforced practice without having a teacher to spend extra time in preparing supportive materials. The use of simulation in a classroom aimed at changing the negative attitude the learners may have towards studying science. Most schools use textbooks to teach science, but hands-on science curricula have become increasingly popular over the last two decades in most countries (Harlen, 2004). Hands-on science typically engages students in the classroom. Engaging students in inquiry using CS can provide a powerful learning experience where learners not only learn about science content but also gain research skills.

METHODOLOGY

The study used both qualitative and quantitative approaches (Creswell, 2014). The quantitative approach was used to generate numerical data (Neuman, 2011; Sibanda 2009) and for its objectivity (Cohen, Manion & Morrison, 2007). The qualitative approach was used to provide tenets, such as learners' experiences after the intervention, which could not be captured through numerical data (Merriam, 2009) and for methodological triangulation (Denzin, 1978). It provides holistic understandings of rich and contextual non-numerical data (Mason, 2002) and it engages participants conversations in a natural setting (Creswell, 2009). Therefore, the inclusion of qualitative study complemented the quantitative data by providing appropriate learners' description regarding CS.

Design of the Study

A quasi-experimental design (Gersten, Fuchs, Compton, Coyne, Greenwood, & Innocenti, 2005; Campbell & Stanley, 2015) and phenomenological research design (Denzin, & Lincoln, 2005; Smith, Flowers, & Larkin, 2009) were used in this study. The quasi-experimental provided empirical data on the use of CS in the science classroom, and the phenomenology provided learners with the opportunity to describe their experiences regarding the use of CS in science lessons.

Sample

A purposive sample (Bernard, 2002) using criterion (Palys, 2008) was adopted, and the criterion involved searching for schools where learners were taught Science without laboratory work. 53 learners from school A (23 girls; 30 boys) and 52 learners from school B (28 girls; 24 boys) were randomly assigned to Experimental Group (EG) taught using CS and Control Group (CG) taught using TCT approach without CS, respectively. Considering normality and the context, in our case the whole class, Kraemer & Thiemann (1987) suggests 30 as the minimum number and in our case 52 and 53 are way above the minimum number of 30. Therefore these numbers were considered to be sufficient for data analysis.

Instruments

The achievement test was administered to both the EG and the CG before and after the intervention. During the intervention, the EG was taught the topic "Waves" using CS from Physics Education Technology (PhET). The PhET Interactive Simulations incorporate research-based practices to enhance the learning of Science and Mathematics concepts. For validity, a Content Validity Index (CVI) was calculated as 0.91. For internal consistency, "measure of the degree of the similarity between items" (Pieterson & Maree, 2007), the test with 25 items was piloted to 10 learners who were not part of the study. From the pilot study, a Cronbach Alpha coefficient (Cronbach, 1951) was computed, and questions with Alpha less than 0.7 were removed from the test. The final test consisted of 20 items with four possible answers and equivalent to 20 marks in total. The overall Cronbach Alpha coefficient of 0.89 was obtained and was deemed suitable for the study since it was ≥ 0.7 (de Vos, 2010). Also, interview questions were structured by the researcher to identify learners' experiences after teaching using CS and TCT. For Face Validity of the interview answers to the questions were assessed by two experts in education and their recommendations were effected before the interviews were conducted. Interview questions were piloted with six learners to determine their suitability. These learners were from similar environments and were not part of the study.

Data Collection

The CG and the EG were given a pre-test to determine their knowledge before the study. The EG was taught using CS, and the CG was taught using the TCT (Roth & Roychoudhury, 2003). Both groups were taught for four weeks during the second quarter of the 2015 academic year and worked on three sets of tasks by the second author to avoid personality effects. After that, a post-test was administered to both groups (**Appendix 1**). This post-test was the same as the pre-test they had completed previously. The only difference was that questions in the post-test were re-arranged to minimise recognition. For EG a central computer was connected to the overhead projector to enable all learners in class to see from a big screen in front. Also, three computers were available for use, and 17 learners accessed one computer at a time. The teacher started by demonstrating the concept of waves on the main computer and learners used computers assigned to their group to continue with the experiments. They used those computers to observe what happens to the transverse wave when the frequency of the wave is either increased or decreased. They also checked the relationship between the amplitude and the wavelength of the transverse wave. Learners in the EG observed the nature of different types of waves on the screen of a computer and tried to predict what would happen to the wavelength of the wave when the frequency of the wave was increased or decreased. They also predicted what would happen to the wavelength of the wave when the amplitude of the wave was increased or decreased. In each case, learners were asked to write their hypotheses. Here are three sample tasks from "Waves" that were used and characterised CS activities: 1) Learners were to establish the relationship between the frequency and wavelength of a wave. They hypothesised what would happen to the wavelength of a wave when the frequency was reduced or when the frequency was increased? 2) They were to establish how an increase or a decrease in the frequency of wave affects the amplitude of a transverse wave, and 3) They were to determine how constructive and destructive interferences occur. After that, they were required to demonstrate the movement of a transverse wave using the computer and observe what happened when a wave oscillated. For the CG, these concepts were explained to learners without CS. To reduce any harm done by omitting the CG from using CS, and to comply with ethical issues, the whole CG class was taught Waves using CS after the study was completed.

There were 15 lessons in total, 3 per week and the other 3 hours were for pre- and post-testing times. One lesson of one hour was used for learners to view the video on the concepts and make notes during week one. For acclimatisation, in a week each learner spent one hour watching the accessed website for simulation on waves and two hours were used to do practical work on the computer by manipulating the system. The other three weeks followed the same pattern for the following topics: motion of a single pulse travelling along a spring or a heavy rope; ripple tank to demonstrate constructive and destructive interference of two pulses; Identify the wavelength, amplitude, crests, troughs, points in phase and points out of phase on a drawing of a transverse wave. Finally, to extend their knowledge of waves on how to make sound using a "vuvuzela"- a native instrument that was popularised at the Soccer World Cup in 2010 played at every stadium; and how to make a string / or wire telephone as a way to include their indigenous knowledge systems.

The qualitative data was collected after the intervention of four weeks. Four learners from CG and four learners from EG were interviewed in order to determine their experiences. The percentages gained from the quantitative may not tell the feelings of learners. In fact, we used the qualitative approach to get it from the horse's mouth (Maoko & Kibirige, 2014). The interview schedule consisted of three questions: 1) What was your experience with Physical Sciences lessons?; 2) How did the teaching approach assist you in developing interest in Physical Sciences?; and 3) How much time do you spend studying Physical Science after the lesson?. Interviews were conducted with eight learners (labelled 1-8); one to four from the CG and five to eight from the EG (2 Females and 2 Males per group) in order to determine their experiences. Each learner was interviewed for a maximum of 20 minutes and the interviews were audio-recorded.

Data Analysis

Descriptive statistics (mean and standard deviations) and inferential statistics (T-test, Analysis of covariance-ANCOVA and a Mann Whitney U-test) were utilised from SPSS version 22. The differences between the EG and the CG for the pre- and post-tests were analysed using a T-test ($p < 0.05$). ANCOVA was used with a pre-test as a covariate to determine the impact of the CS after four weeks of teaching. Also, a t-test was used to determine if there were significant differences in achievements between boys and girls after four weeks of teaching in the EG. In addition, responses from semi-structured interviews from the two groups (EG, CG) were analysed thematically to identify learners' experiences in science. The audio recorded data were transcribed verbatim and transcripts were analysed using open, axial and selective coding (de Vos, 2010). During open coding transcripts were read sentence by sentence to determine key ideas followed by axial coding where key ideas were re-arranged to form subthemes. Lastly, during selective coding sub-themes were compared to the purpose of the study in order to generate main themes (de Vos, 2010).

Ethical Considerations

The participants were informed that they were under no obligation to participate in the study and that they were free to withdraw from the study any time with no negative consequences. Since participants in the study were Grade 10 learners (minors), they were provided with a Consent Form for their parents to allow them to participate in the study. Those learners from CG were later, after the study, taught similar wave concepts using CS to minimise the perception that they were discriminated against during the teaching.

RESULTS

The overall results reveal that learners in EG out-performed the learners in the CG. Learners in the EG (taught using CS) indicated that they enjoyed, developed interest and understood waves with ease, unlike learners from the CG (taught without CS).

Quantitative

The results of the pre-test for the EG achievements (mean 47.00 ± 14.00 SD) and the results for the CG (mean 35.60 ± 9.60 SD) did not differ significantly (T-test: -0.35, $p > 0.05$). After teaching for four weeks, the EG achievements (mean 56.00 ± 19.50 SD) was again compared to that of the CG (mean 39.60 ± 13.35 SD) and there were significant differences between the two groups (T-test: 0.52, $p < 0.05$) (**Table 1**). An effect size of 0.84 and a Cohen *d* of 0.41 were calculated for the EG.

Table 1. T-test results of the EG and the CG before and after (*Significant at p < 0.05)

		N	M	SD	T-test	P
EG	Pre – test	53	47.0	14.0	-0.35	0.00*
	Post – test	53	56.00	19.50		
CG	Pre – test	52	35.6	9.60	0.52	0.81
	Post – test	52	39.60	13.35		

An independent-samples t-test was performed to compare girls' and boys' achievement in the EG after intervention. The results show that the achievements for girls (M=54.60, SD=10.93) and boys (M=54.39, SD=7.90 in EG after intervention were not significantly different t (51) =-0.08, (p < 005) (**Table 2**). Therefore, the results show that CS did not discriminate between girls and boys in the EG.

Table 2. T-test results for girls' and boys' achievements after intervention. (*Significant at p < 0.05)

		Levene's Test for Equality of Variances		T-test for Equality of Means					95% Confidence Interval of the Difference	
		F	Sig	T-test	df	Sig.(2-tailed)	Mean Difference	Std Error Difference	Lower	Upper
EG, (Boys & Girls)	Equivalent variance assumed	1.42	0.24	-0.08	51	0.94	0.21	2.60	5.43	5.02
	Equivalent variance not assumed			-0.08	43.25	0.94	-0.21	2.65	5.55	5.14

Using pooled data for boys and girls per group with pre-test as a covariate, ANCOVA test shows that there were significant differences between the achievement of the two groups ($p < 0.05$) (**Table 3**).

Table 3. ANCOVA summary results of EG and CG before and after (** Significant at p < 0.01)

Source	SS	Df	MS	F	*p*
Pre-test	24.58	24.57	4.23	0.06	0.01**
Post-test	36.41	36.41	6.27		
*Error	273.02	47	5.81		
Total	3845.00	49			

Qualitative

From semi-structured interviews **Appendix 2**, three themes were identified and these are: 1) Enjoyment; 2) Interest in Science; and 3) Ease to understand concepts. The themes from the CG are presented first, followed by those from EG.

Themes from the CG

Theme 1: Enjoyment

From question one, the researcher wanted to know learners' experiences during Physical Sciences lessons. In response to this question, all learners from the CG stated that they did not enjoy Science at all. Some even expressed that they think it was a big mistake for them to have chosen to study Physical Sciences as one of their subjects because they were unable to cope with the work. They indicated that, they thought they would perform practical work like other learners from their neighbouring schools, but they did not due to lack of resources. The teachers explained what the waves are and drew a few illustrations on the chalk board. On the extreme cases some of the teachers read directly from the textbook (Grade 11 Physical Sciences. Everything science by Siyavula) and often times learners did not understand what they wrote down. They tried to re-read after the lesson, but they still could not make sense of most of the concepts. Below are a few specific direct quotes from learners:

Learner 1: "I do not enjoy learning Physical Sciences because it is very hard to understand";

Learner 3: "I do not like Physical Sciences; I don't even know why I chose it because it is really difficult for me. No matter how much effort I put to the subject I still do not succeed in the test";

Learner 4:" I tend to fall asleep in a Physical Sciences lesson because there is very little that excites.

Learner 2: "I thought Physical Sciences was a practical subject, since we don't do any practical work. I do not enjoy studying Physical Sciences."

Theme 2: Interest in the subject

The second question sought to assess learners' interest in Physical Sciences as a subject, learners from the CG indicated that they found nothing interesting about Science. However, they agreed that everything in their lives revolved around Science but they still could not relate Science to their daily living. They felt Science was boring and did not provoke interest and this impacted on their achievements because they never passed the subject. Below are a few specific direct quotes from four learners:

Learner 1: "I was told that Science gives clarity to everything happening around us, but I have never seen anything to suggest it."

Learner 3: "I hate Physical Science, I have never passed the test and it does not seem like I will ever pass it."

Learner 4: "I have no desire to continue with Physical Sciences. I have had enough. I work hard and fail and because of that I hate the subject. Whenever I try to understand I become even more confused."

Learner 2: "I realised that Physical Sciences is not for people like me, maybe I am not gifted enough to study Science. I think I'm about to quit the subject."

Theme 3: Ease to understand concepts

On the last question that asked learners about how much time they spent studying Physical Science after the lesson, it appeared that they had different experiences. Some learners from the CG differed on the amount of time they spent on the subject. Some spent little time or they did not even bother to give time to the subject at all. Their reason was that they were struggling so it became pointless for them to spend time on something they did not understand. Some indicated that they spent a lot of time on the subject but that did not yield any results. They saw the study of Science as time consuming. Below are specific direct quotes from learners:

Learner 1: "Physical Sciences subject is time consuming. If I want to understand the content I have to spend a lot of time. I may not even pass it. I mean, it seems impossible to cover all the content in a given time."

Learner 3: "No matter how much time I spend on this subject, I just can't understand it."

Learner 4: "I gave myself enough time to understand Physical Science; I think I had lost a lot of it without gaining anything."

Learner 2: "I can't spend time on something I do not understand; I will waste time for other subject. I realised that Physical Science needs geniuses."

Themes from the EG

Theme 1: Enjoyment

All four learners from the EG had positive attitudes judging from the responses they provided to the questions. As the first question stipulated, the researcher wanted to know how learners enjoyed Physical Sciences lessons. In response to this question, all learners from the EG stated that they really enjoyed the Physical Sciences lessons. Below are some direct quotes from four learners:

Learner 5: "For the first time I have enjoyed the Physical Science class, I particularly loved the game-like environment. It was like a game. I never thought a computer could be used in this way to learn Science."

Learner 7: "If all subjects were taught the way we have been taught Science, then I would not have any reason to bunk the class."

Learner 8: "I can't' wait to get to my science class each day I go to school, I really enjoy doing science and observing experiments on the computer."

Learner 6: "I used to sleep in a science class but not anymore, I enjoy every moment."

Theme 2: Interest in the subject

On the second question that sought to assess the interest of learners on Waves as a topic, learners from the EG indicated that they found it highly interesting. They indicated that they could not comprehend the concept of waves, now they understood how waves occur and they were able to explain the types of waves. Below are some specific direct quotes from learners:

Learner 5: "I became more interested in Science more especially to the concept of Waves because of the CS. It made me realize what real science is all about and the observations helped me relate to what I read in the textbook."

Learner 7: "I find Waves to be more fascinating especially when we do experiments on the computer; I enjoy working with a computer too."

Learner 8: "I always want to know more about the world we live in and Science makes it easier for me to understand."

Learner 6: "I can't believe that Science can be so interesting as it was now even without practical work. This new way of learning Science does wonders."

Theme 3: Ease to understand concepts

On the last question that asked learners how much time they spent studying Physical Sciences after the lesson, some learners from the EG indicated that they spent much time because they loved the subject therefore they wanted to know more. Some indicated that they were no longer spending as much time as they did before they used CS because they now understood the Waves concept better than before they were exposed to CS. Below are specific direct quotes from four learners:

Learner 5: "I used to read many times without understanding, but after CS I could understand content after reading once. I now understand the concept of waves more clearly than ever before because of CS. I was happy to perform an experiment in my school despite lack of resources because of CS."

Learner 7: "I spend lesser time on my books these days because the observations that I got from CS help me relate to what I read in the text book about Waves."

Learner 8: "After the CS actually you do not need a lot of time to read the content because the content is right in your mind. That is to say, when I learn using CS I understand content much quicker and better than ever before."

Learner 6: "I spend more time because of the love I have for the subject, each time I study there is something that tells me to study more. I enjoy the time I spend studying Waves in Physical Sciences."

DISCUSSION

The study explored Grade 10 learners' conceptual development of Science concepts using Computer Simulations (CS). A pre-test was administered to both groups (EG and CG), and the results show that there was no significant difference in achievements of learners from both groups (T-test, $p > 0.05$), suggesting that learners both EG and CG had more or less similar understanding of waves concepts before instruction. However, in the post-test, learners in the EG performed better than learners in the CG and that the differences were significant (T-test, $p < 0.05$). An effect size of 0.84 was obtained, suggesting large positive effects for the EG and Cohen *d* of 0.41 obtained was greater than ($p > 0.35$), suggesting a large gain (Cohen 1988) for the EG. These results imply that CS improved learners' conceptual understanding of the science concepts studied.

CS improve understanding of Science concepts, and this ultimately improves the quality of Science Education. When learners who were taught using CS were interviewed, they claimed to spend less time to understand Science content. Also, in the case where the concept is not understood, at first, learners could replay the simulation to relearn, and this may improve autonomy in their studies. The autonomy observed in the EG in this study agrees with Jilani and Yasmin, (2016) and Tsai (2018). The increased understanding of the concepts in turn may increase interest in the subject. Thus, the interest formed during CS is vital to developing a positive attitude towards Science. These findings agree with Dalgety et al. (2003) and Covington (2000) regarding the importance of attitudes and motivation towards Science, respectively. Dwyer and Lopez (2001) also talked about learners being motivated when they experience realistic and authentic problems.

The EG learners' answers during the post-test imply that they had developed a clearer understanding of the Waves concept than their counterparts from the CG. The conceptual understanding exhibited by the EG agrees with Slotta (2002) and Tsai (2018) who indicated that learners might benefit from computer use which allows them to learn from each other and also to make them develop autonomous learning. The results are surprising because the high learner-computer ratio seems not to have affected learners' achievements. Also, the Cohen d of 0.41 suggests a large knowledge gain to reckon in favour of the EG. Hence, the use of CS can be a levelling ground for the 'have-nots' learners of the Southern Hemisphere with the 'haves' learners of the Northern Hemisphere (Halverson & Smitt, 2010; Stegmeir, 2015).

The development of positive attitudes has been reported in cognitive, behavioural and affective domains (Ajzen, 2005). In this study, semi-structured interviews were intended to uncover the affective domain by seeking learners' emotional feelings regarding Physical Science (Rajecki, 1990). Learners from EG developed and enjoyed learning science, but this was not the case with learners from the CG. The interview questions and probing questions sought to identify the learners' views and interests regarding the use of CS. Although it may seem like much time was spent on experiments in CS classes, learners reported that they enjoyed the teaching approach and it made them understand concepts much better than when the TCT approach was used. This is why some learners suggested that it took them a shorter time to read and understand the concepts. Learners from EG explained confidently the concepts of Waves and this suggests that CS enhanced their science conceptual development. While CS have been widely studied in the developed countries, this is not the case in Africa and in other developing countries. Our study is unique in that it combined both the quantitative and qualitative study regarding CS. Furthermore, the learners' views on the use of CS are very limited on the African continent. The learners' views in this study give credence to the fact that there are few studies on the learners' views on the use of CS in Africa south of the Sahara. The study results add to the scanty literature of the use of CS in the developing countries. The limitations of this study are that the participants in EG (53) and CG (52) were too few for generalisation and the content was limited to Waves only. Nevertheless, the findings of the study provide an insight on the potential for CS to arouse interest, as well as enhance conceptual development of Science concepts which are likely to improve the quality of education.

After four weeks of teaching, there was a significant difference in achievements of boys and girls in the EG ($p < 0.05$) when compared to those in the CG. These results are not surprising because CS as a strategy was applied to both boys and girls in EG. This suggests that CS was successful in narrowing the achievement gap between boys and girls because, as young girls, learners tend to be attracted to technology more than their older generation (Mavhunga, Kibirige, Chigonga & Ramaboka, 2016). This is in line with Baker's (2013) and Michael's (2013) findings regarding strategies focusing on learners' interests. In addition, learners benefit through engagement with concepts especially when they do practical work through "interactions, hands-on activities, and application in Science" (Hampden-Thompson & Bennett, 2013). During CS learners may interact with one another and share their conceptual understanding. Some of the concepts shared were constructive and destructive interference of two pulses; definitions and the meaning of wavelength, amplitude, crests, troughs, and sound waves in our everyday life. This is a remarkable outcome when one considers the high learner-computer ratio (Butcher 2003) which is 17:1 in high school and this ratio is better than 28:1 in the primary schools in the country (Sossa, Rivilla & González, 2015). In schools where there are limited or no laboratory resources, CS may improve conceptual understanding of practical work. This suggests that it is possible to use CS in place of a physical/manual laboratory experiments in the teaching of Science concepts (Choi & Gennaro, 1987). In the same vein, these results are good news to developing countries with limited resources to consider using computers to teach science regardless of the high learner to computer ratio. Also, our results are in agreement to Gerick, Eickelmann & Bos (2017) who contend that the higher the learner-computer ratio the lower the achievement. Currently, it is not clear as to what the results would be if the learner-computer ratio was lower than 17:1 used in this study and the effect of the delayed testing on learners' achievement and these need further studies.

Ideally, both simulations and hands-on practical work should be used to enhance conceptual understanding. CS may offer extra advantages in time so that when learners conduct manual laboratory work, they have a clear conceptual understanding and this may reduce the time learners spend on practical work experiments. This observation agrees with Kennepohl (2001) who contends that learners using CS have a slightly better conceptual knowledge of the practical work before manual practical work is done. Raimi's (2002) study in Pakistan found that laboratory work positively affected learners' achievements in Physical Sciences. Furthermore, Adesoji and Olatunbosun (2008) argued that learners tend to understand and recall what they see more than what they hear and this improves their achievements. The results indicate that EG achievements improved more than the CG. This improvement can be attributed to CS, which granted learners an opportunity to visualise the processes and conceptualise what they observed.

TCT used in the CG did not make learners enjoy Physical Sciences and their interest in the subject was low. This might have been due to the expository approach being too abstract in Science classes. It is no wonder learners spent a lot of time to understand Science content. Conversely, the EG spent less time to grasp Science concepts. This is because CS are potentially useful for stimulating learners' interest and providing a print on their minds better than TCT. Also, some laboratory experiments that may be expensive or dangerous to conduct may be observed on the screen (Slotta, 2002) especially if one considers the cost of apparatus for 17 learners as compared to a computer for the whole year! Thus, the use of CS contributes to conceptual understanding and may provide open-ended experiences for scientific inquiry and problem-solving skills. Such a teaching strategy is most likely to improve the quality of education (Livingston, 2017).

In conclusion, learners who were taught using CS were happy with the approach used. CS allowed the learners to gain conceptual development. The learners were "highly motivated", and it was not surprising that their achievements improved more than the CG and this suggests that CS enhanced learners' conceptual learning in Science.

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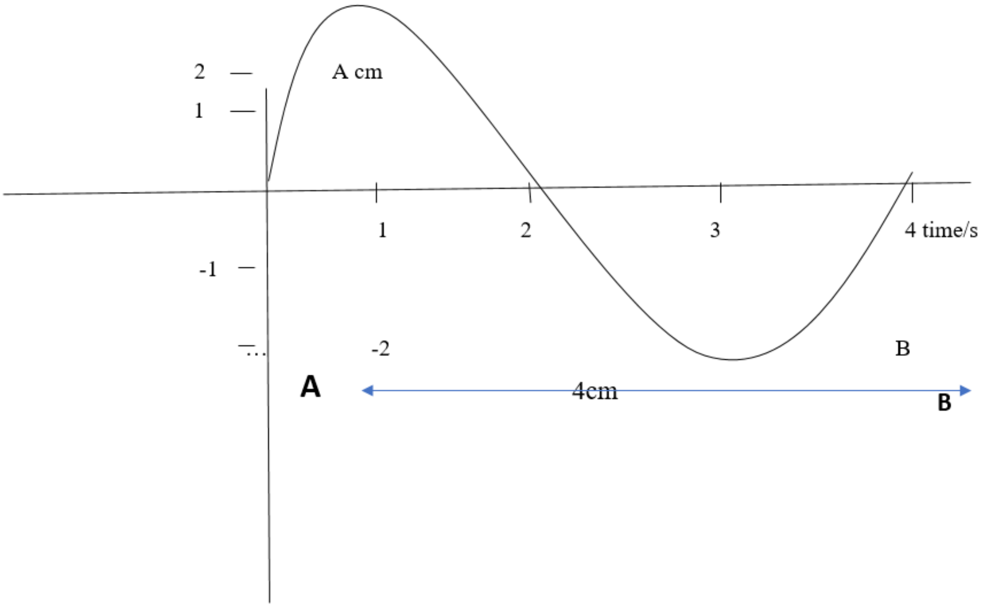
APPENDIX 1

Physical Science Grade 10 Achievements Test 1

MULTIPLE-CHOICE QUESTIONS

The diagram below is not drawn to scale; it illustrates the type of wave of a certain frequency. Refer to the wave to answer questions

Each question has only ONE correct answer. Write only the letter (A-D) next to the number.



1.1 What is the type of wave illustrated by the diagram above?

A. Microwave
B. Transverse wave
C. Longitudinal wave
D. Medium wave

1.2 What is the wavelength of this wave?

A. 3 cm
B. 2 cm
C. 4 cm
D. 12 cm

1.3 What is the amplitude of the wave?

A. 2 cm
B. 4 cm
C. 6 cm
D. 3 cm

1.4 How many wavelengths are illustrated in the diagram?

A. 4
B. 3
C. 2
D. 1

1.5 When the frequency of the wave increases, the amplitude…

A. Increases
B. Decreases
C. Stay the same
D. Decreases and then increases.

1.6 The increase in frequency will result in….

A. Increase in the number of wavelengths
B. A decrease in the number of wavelengths
C. Doubling wavelength
D. Tripling wavelength

1.7 Period of a wave refers to…

A. The number of complete vibrations per second
B. Time taken to form a wave
C. Time taken for a complete wavelength to pass a point
D. Frequency of a wave

1.8 When a crest of one wave coincides with a crest of another wave and it results in a bigger crest, it is called

A. destructive interference
B. cancellation
C. constructive interference
D. amplitude

1.9 T calculate the velocity one needs to take

A. frequency
B. wavelength
C. wavelength and divide it by frequency
D. wavelength times frequency

1.10 An increase in the frequency of wave means the pitch of sound…

A. Decreases
B. Remains the same
C. Increases
D. Fluctuates

1.11 The frequency of this wave is…

A. 0.0009Hz
B. 0.008Hz
C. 0.12Hz
D. frequency not given

1.12 What is the relationship between frequency and wavelength of a wave?

A. Frequency is directly proportional to the wavelength
B. Frequency and wavelength do not have a relationship.
C. Frequency is inversely proportional to wavelength
D. Frequency of a wave does not affect the wavelength

1.13 How long does it take for a complete wave to form?

A. 2s

B. 3s

C. 1.3s

D. no answer because no frequency given

1.14 The quality of sound is attributed to…

A. Amplitude of a wave

B. Speed of a wave

C. Waveform

D. Period

1.15 A dolphin emits an ultrasonic wave with frequency of 0.15 MHZ. The speed of the ultrasonic wave in water is $1500 m.s^{-1}$. What is the wavelength of this wave?

A. 0.1 mm

B. 10 cm

C. 100 m

D. 1 cm

1.16 Position **A** on the graph represents…

A. Trough

B. Wave

C. Point in phase

D. Crest

1.17 Position **B** on the graph represents…

A. Crest

B. Point in phase

C. Wave

D. Trough

1.18 The amplitude and the frequency of a sound wave are both increased. How are the loudness and the pitch of the sound affected?

	Loudness	Pitch
A.	Increased	raised
B.	Increased	unchanged
C.	Increased	lowered
D.	Decreased	raised

1.19 What points are considered points in phase?

A. Points that move perfectly in step with each other

B. Points those are close to one another

C. Points that are closely parked

D. Points on crest and trough

1.20 Low amplitude represents a…

A. loudness of sound
B. sound waves
C. No sound
D. Quality sound

2 x 20 = 40

______________________END____________END________________

APPENDIX 2

- Tell me anything you were told about Physical Science before you enrolled to study it.
- Do you enjoy learning PhysiRcal Science?
- What is your experience of studying Physical Science? Elaborate on the learning environment.
- How easy are Physical Science concepts for you to understand? Elaborated.
- What can you about the methods used to teach Physical Science?
- Is your study period for Physical Science the same as other subjects? Explain
- Do you have any other comment about Physical Science

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Students' Motivation to Engage with Science Learning Activities through the Lens of Self-Determination Theory: Results from a Single-Case School-Based Study

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ABSTRACT

Self-Determination Theory (SDT) is a sociocultural motivational theory that has been extensively applied within schools as a means of developing teachers' evidence-based practice, where the satisfaction of three basic psychological needs for relatedness, competence and autonomy has positive impacts upon students' motivation to engage with classroom-based learning activities. SDT has proved to be an effective theory for suggesting why selected key classroom-based behaviours and variables influence the students' engagement more than others. Whilst SDT emphasizes the centrality of autonomous motivation to students' engagement with scientific learning, it may be such engagement is more likely to be an outcome of students' perceived competence and teacher-student relationship quality. This potential outcome was explored through focus group interviews with 70 students, aged between 9 and 13. The responses consistently revealed key common variables that the students regarded the quality of the teacher-student relationship as being influential upon their perceptions of the teacher's effectiveness at enhancing students' perceived competence as opposed to satisfying any wish they had for their teacher to be autonomy-supportive. Such perceptions were consistently self-reported as being influential upon their engagement with, in this case, science-based learning activities.

Keywords: self-determination theory, student motivation, student engagement, perceived competence, teacher-student relationship quality, science education

INTRODUCTION

Teachers are the key factor in motivating students to engage with learning activities within their specific educational contexts (Ryan and Deci, 2009). A teacher whose behaviours reveal a positive attitude and enthusiasm for learning within a specific curricula subject is more likely to have students who develop positive affect and enthusiasm for learning and achievement within the subject (Fredricks et al., 2004; Tymms et al., 2008). Teachers' ability to engage students' interest and participation in their schooling in general (Christenson et al., 2012; Klem and Connell, 2004; Skinner and Belmont, 1993) is regarded as essential for a sustained academic achievement (Christenson et al., 2012; Fredricks et al., 2004; Marsh and Martin, 2011; Reeve, 2002, 2012).

Prior research suggests that "… at the classroom level, teacher support, positive teacher-student relationships … autonomy support and authentic and challenging tasks have been associated with student engagement" (van Uden et al., 2013, p. 44). Three essential factors consistently emerge as having a positive influence upon the optimal development of students' self-regulated academic motivation and achievement within classroom learning activities (Connell and Wellborn, 1991; Hattie, 2012). These are the quality of teacher-student interpersonal relationships, the extent to which learning activities are autonomously directed by students, and the nature and timing of competence-related feedback given by the teacher to the student. Within the research reported here, student engagement has been approached as an outcome of motivational informants, taking the form of a combination of observable behaviours and self-reported affect-driven perceptions (Christenson et al., 2012; Fredricks et al., 2004;

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Contribution of this paper to the literature

- The current research posits that student engagement is an outcome manifested in response to the motivation that students gain from the teacher satisfying the need for competence or autonomy, or both. A hierarchy amongst types of engagement has previously been proposed by Reschly and Christenson (2006, 2012) in that they argue that cognitive and emotional engagement precede and inform the quality and persistence of behavioural engagement.
- The evidence and interpretations within the current research are significant in that it reveals student-reported means of informing teachers' understanding of how they have and can have a direct impact upon their students' motivated engagement.
- This understanding may be used to inform practitioners' evidence-based practice. For example, these identified factors and the associated understanding of the interplay between them may be used in the design and implementation of interventions with the objective of teachers successfully enhancing their students' engagement with learning.

Klem and Connell, 2004; NRC, 2004). These perceptions and their informants are common to the three needs within Self-Determination Theory (SDT: Ryan and Deci, 2000a).

The research discussed herein utilized SDT as a theoretical lens for identifying and interpreting some of the key contextual variables that enhance students' self-determined engagement with learning activities. The research question - What do students regard as the key influences that have an impact upon their motivated engagement with learning activities? – led to a focus upon variables included teacher behaviours and methods that have a positive impact upon students' motivation to engage with learning, such as students' perceptions of the quality of the teacher-student relationship. SDT is a sociocultural motivational theory that has been effectively applied within schools as a means of developing teachers' evidence-based practice (Ryan and Deci, 2009), where the satisfaction of three basic psychological needs (BPNs: relatedness, competence and autonomy) has positive impacts upon students' motivation to engage with learning activities. Indeed, SDT has been empirically shown to be an effective theory for explaining why some key classroom-based behaviours and variables influence the students' engagement more than others (Reeve, 2002, 2012). Fredricks et al., (2004) note that the degree to which the three SDT needs mediate between teacher behaviour contextual factors and engagement had not been investigated by most studies seeking to understand engagement, and that least studied are the motivational relationships between perceived competence and students' persistent engagement with learning (p. 82). The research outlined here is one contribution to reducing such a paucity of studies.

LITERATURE REVIEW

Motivation has been defined as a cognitive and affective force that initiates, sustains and directs engagement behaviours, as an internalised process of formation drawn from the individual's experiences, perceptions and interpretations (Reeve, 2012). It consists of an inner psychological drive leading to action, i.e. engagement behaviours (Abrahams, 2011; Bandura, 1986). Engagement is defined as a motivation-driven mental construct predictive of and predicted by students' perceptions of positive interpersonal relationships (relatedness) at school in tandem with the cognitive and affective desire to initiate and sustain participation in a range of learning contexts and activities therein (Fredricks et al, 2004). Engagement is observable as manifestations of the motivated desire to be involved within learning activities. Engagement has been argued as being synonymous with self-regulated learning through motivation-informed and driven desires or needs, as common behaviours include persistence, attitude, concentration, the management of time, focus upon the main ideas and objectives, and the processing of information (de Bilde et al, 2011; Zimmerman and Schunk, 2008). Fredricks et al. (2004) discuss three other forms of engagement – affective, behavioural and cognitive: however, for simplicity, this study primarily focuses upon motivational leading to behavioural engagement: that is, prosocial behaviours exhibited through participation in school-based activities, and involvement in, for example, related extra-curricular activities and actively studying a subject area beyond the classroom out of personal interest (NRC, 2004). Parsons and Taylor (2011) cite three substantive reasons for researching and understanding the motivational processes and influences that impact upon student engagement: defining different types of engagement and their observable indicators;

> "… to help disengaged and disadvantaged students achieve and participate (or to reduce drop outs); to assist in classroom management (reduce classroom disruptions and discipline issues); and, finally, to engage students in learning about learning (to help them to become skilled life-long learners as opposed to well-behaved, attentive students)."
>
> (p. 9)

Student engagement is, therefore, of immense significance within classrooms as a measurable multidimensional construct in the form of a dynamic, malleable outcome of students' motivation for learning through affective, verbal and behavioural responses that are, reciprocally, predictive of students' motivational inclinations (Klem and Connell, 2004; NRC, 2004). Engagement has been posited as a significant predictor and indicator of students' motivation and well-being within formal learning environments (Baumeister and Vohs, 2007; Fredricks et al., 2004; Ryan and Deci, 2009). Therefore, students' engagement with learning in general is regarded as essential for the long-term commitment of students to their learning goals and prosocial approaches to academic success (Fredricks et al., 2004; Lawson and Lawson, 2013; Reeve, 2002, 2012). The converse of engagement is disengagement (also known as disaffection). Disengagement has been empirically asserted to be a cause of increased school drop-out rates, reduced attendance levels, and ultimately students not achieving their own self-perceived or their teacher-regarded potential (OECD, 2000). As engagement has a positive association with improved educational outcomes such as achievement levels, for teachers, "… the primary appeal of the engagement construct is that it is relevant for *all* students." (Christenson et al., 2012, p. vii) With regards to academic achievement and enjoyment of learning, "… considerable evidence now reveals that students who are intrinsically motivated and inherently interested or engaged in the learning process will more effectively master classroom assignments and achieve at higher levels" (Harter, 2012a, p. 273).

The three most frequently mentioned forms of engagement within classroom-based learning are affective engagement, cognitive engagement and behavioural engagement (Fredricks et al., 2004; Fredricks and Eccles, 2002; Parsons and Taylor, 2011). In addition, a further form of engagement – agentic engagement – has recently been proposed by Reeve and Tseng (2011). Affective, cognitive and behaviours engagement are asserted as combining to inform behaviours indicative of student engagement within classroom-based learning (NRC, 2004). However, Reeve (2012) proposes that all four subtypes of student engagement should be considered together when seeking to understand and enhance students' academic engagement. Within each, engagement behaviours are viewed as initiated by psychological responses and physical actions underpinned by motivational constructs (Connell and Wellborn, 1991; Deci and Ryan, 1985; Reschly and Christenson, 2012; Skinner and Pitzer, 2012).

Affective engagement "… encompasses positive and negative reactions to teachers, classmates, academics, and school and is presumed to create ties to an institution and influence willingness to do the work" (Fredricks et al., 2004, p. 60). Indicators of positive affective engagement during learning activities include enthusiasm, interest, enjoyment, satisfaction, pride, vitality and zest (Ryan and Deci, 2008; Skinner and Pitzer, 2012). Conversely, indicators of affective disaffection include boredom, disinterest, frustration, anger, sadness, worry, anxiety, shame and self-blame (Skinner and Pitzer, 2012, p. 25). Behavioural indicators of positive affective engagement include excitement, elation, happiness, hope, joy, pride and gratitude. Negative indicators include tension, anger, sadness, frustration, anxiety and shame (Pekrun and Linnenbrink-Garcia, 2012, pp. 261 – 262).

Cognitive engagement consists of inherently different internal psychological processes from those of affective engagement. Cognitive engagement acts as the mediating bridge between context and learning outcomes (Connell and Wellborn, 1991; Reschly and Christenson, 2012). Cognitive engagement has been defined as drawing "… on the idea of investment; it incorporates thoughtfulness and willingness to exert the effort necessary to comprehend complex ideas and master difficult skills" (Fredricks et al., 2004, p. 60). Indicators of positive cognitive engagement include observations that a student is purposeful, approaches learning activities with enthusiasm, strives to achieve a variety of learning goals, is a willing participant in learning activities, actively seeks challenges, and exhibits a thoroughness and desire to achieve the best possible learning outcomes (Skinner and Pitzer, 2012). Indicators of cognitive disengagement include a lack of self-direction, presenting themselves as helpless, unwilling or opposed to tackling learning challenges, avoiding or being apathetic during learning activities, and presenting themselves as incapable, incompetent or under undue pressure (Skinner and Pitzer, 2012).

Agentic engagement centres upon the active and volitional cognitive contributions that students make to learning activities (Reeve and Tseng, 2011). Agentic engagement is asserted as predictive of cognitive, affective and behavioural engagement, as well as an independent predictor of achievement within the classroom, particularly in terms of students feeling empowered to make constructive contributions to their learning activities (Reeve, 2013; Reeve and Tseng, 2011).

Motivation and engagement are usually dependent on an individual's self-perceptions of their actual achievements and perceived competence (Schunk and Pajares, 2005). These two perceptions are purported to act as motivational precursors of self-efficacy, which act, in turn, as predictors of sustained and effortful engagement within an activity (Bandura, 1997). Such engagement involves the expenditure and sustaining of effort which is optimally catalysed when the causes of competence are regarded as controllable. Zimmerman (1995) states that the evolution and sustaining of academic competencies is one of the most demanding motivational and cognitive challenges that developing children face (p. 202). Such perceptions of competence are constantly evolving and are usually informed by factors such as feedback from teachers, personal aspirations, intrinsically motivated goals, self-endorsed values, and a self-determined approach to activities through perceived autonomy-orientated causation.

All are informed by and internalised through context-specific experiences and self-perceptions (Reeve, 2012). These may act as the causality orientations within learning contexts, and, especially, a student's predictions regarding a teacher's verbal and non-verbal responses to the student's efforts and achievement. From perceived verbal and behavioural indicators of teacher warmth and expectation, each individual student will form their own worldview of a teacher based upon their experience of prior interactions. This appears to inform a student's perception of the strength of their attachment to each of their teachers, and is likely to influence future responses to the teacher and learning activities. The worldview formed is based upon criteria that experience has moulded as a means of interpreting a teacher's intentions, reliability and trustworthiness (Bretherton, 1987).

Self-Determination Theory (SDT) is a sociocultural motivational theory, involving the psychological and philosophical interplay of three basic psychological needs (BPNs) - *relatedness, autonomy* and *competence* (Ryan and Deci, 2000a). Relatedness is a basic psychological need, in that individuals have a "… psychological sense of being with others in secure communion or unity" (Ryan and Deci, 2002, p. 7). As a theory of human motivation and personality, SDT encompasses a continuum from proactive intrinsic motivation via passive extrinsic motivation to inactive amotivation. It has been empirically shown to be both predictive and indicative of an individual's sense of relatedness, perceived competence and behavioural regulation within classrooms as specific sociocultural environment (Reeve, 2002, 2012; Ryan and Deci, 2009). SDT differs from other sociocultural motivation theories in two distinct ways (Ryan and Deci, 2000). First, it emphasises the *quality* of the unseen motivational regulator as opposed to the *quantity* of the motivational regulator when considering the impact of different forms of motivation upon students' engagement with learning activities. A distinction is made between the different qualities of motivation, which range along a continuum from the most positive quality (self-determined motivation) to the most negative: a complete lack of motivation (amotivation) (Ryan and Deci, 2009, p.173). Second, it is the only sociocultural motivational theory that places the importance of autonomy as central, taking the form of an individual's self-regulated, volitional and sustained engagement during an activity.

Relatedness in the classroom involves the development of meaningful relationships with significant others, such as teachers and peers, through a sense of shared purpose and meaning (Painter, 2011). Competence is the psychological need to feel effective and confident within learning activities, so that students feel or perceive that they are capable of successfully performing within and completing a learning task (Ryan and Deci, 2002), and "… refers to the need to experience oneself as effective in one's interactions with the social and physical environments" (Skinner and Edge, 2002, p. 301). Perceived competence is a precursor that informs an individual's sense of self-efficacy in terms of perceived capability of achieving further competence within a specific domain or context, and self-agency, in the form of motivation to be autonomous and self-determined in working towards further competence. These perceptions, motivational drives and need for competence are at the heart of achievement motivation (Elliot and Dweck, 2007b). Therefore, perceived competence appears to be predictive of and predicted by self-efficacy and self-concept, with all three energizing self-regulated learning (Schunk and Zimmerman, 1989, 1998, 2008). That is, an individual's self-efficacy has been shown as predictive of their motivation to be autonomous, and for the development of self-determined, self-regulating learners who are able to make the most of opportunities to enhance their competence, engagement and social mediation within the classroom (Bandura, 1986, 1997; Connell and Wellborn, 1991; Reeve, 2012; Reeve et al., 2008; Ryan and Deci, 2009). An individual's need for and satisfaction of autonomy is linked to their cognitive and affective perceptions of their ability to achieve self-determined or externally-regulated goals (Bandura, 1997). Therefore, perceptions of competence act as initiators of persistence, autonomy and sustained engagement during learning activities (Roberts et al., 1981). Autonomy is the psychological need to feel agentic through being able to exercise some freedom of choice and to make contributions to learning activities (Ryan and Deci, 2000), and "…refers to the need to express one's authentic self and to experience the self as the source of action" (Skinner and Edge, 2002, p. 298). Striving to feel that one can direct and organize one's behaviour, that one can choose and is not controlled and that one can develop and realize goals and values that feel authentic and give a sense of direction and meaning (from Kaplan and Assor, 2012, p. 253).

Children's perceptions of the teacher-student relationship quality appear to be influential predictors of children's engagement with learning activities (Hughes et al, 2008). Several studies have reported a specific relationship between teachers' interpersonal behaviours and students' positive engagement and attitudes to their learning in science (Ainley and Ainley, 2011ab; den Brok et al. 2005, 2006b; Lee and Anderson 1993).

Within the reviewed literature to date, there has been a plethora of research relating to specific teacher influences upon student engagement within schooling and the classroom in general. There is a wealth of empirical support for positively correlating student engagement as a predictor of academic achievement and motivated involvement within school in general (Connell and Wellborn, 1994; Finn, 1989; Klem and Connell, 2004; Voelkl, 1995, 1996, 1997). By comparison, there has been a paucity of research regarding domain-specific or subject-specific engagement factors in science. Despite such a paucity, engagement-enhancing factors specific to children's positive perceptions of science have been investigated within a number of prior studies (Abrahams, 2009, 2011; Abrahams and Millar, 2008; Ainley and Ainley, 2011a, 2011b; Blumenfeld and Meece; 1988; Darby, 2005; Lee and Anderson, 1993; Lee and Brophy, 1996; Murphy et al., 2012). These have been reported, to varying degrees, that there are several common key elements central to an engaging science education, including teaching behaviours and approaches that promote autonomous learning and strong teacher-student interpersonal relationships. There was a further identified gap in the research in that vast majority of studies were only informed by data collected using questionnaires. However, the emergent common themes were rarely explored by researchers through discussions with students through focus group interviews. Finally, none of the prior research has involved the in-situ testing of SDT within a British school as means of identifying and understanding some of the key antecedents that inform students' engagement within science education. To date, the majority of research applying SDT within education has been situated in the USA. The current research has addressed the identified gaps.

METHOD INCLUDING ANALYSIS

Focus Group Interviews (FGIs) were used to harvest data which has enabled the researcher to gain an improved understanding of how knowledge, ideas, experiences, perceptions and expectations have been formed and what the sources of these are (Cronin, 2008). As Krueger (1998) asserts, the FGI, as with all qualitative research methods, should not be regarded as scientific research, in that the "… goal of qualitative research is to understand and communicate, not to control or replicate a study" (p. 64). Therefore, the focus group interviews (FGIs) were conducted as a means of gaining in-depth insights into the extent to which perceived competence and need for autonomy may or may not be affected by teacher-student relationship quality.

An FGI is "… a group interview or discussion" (Cronin, 2008, p. 227) which enables the discussion and exploration of respondents' views in depth. Within the current research, FGIs facilitated the exploration of students' perceptions and experience-informed interpretations of the engaging or disengaging nature of their learning environment. An advantage of utilising FGIs is that they provide a means of exploring students' responses in depth, alongside enabling respondents to determine how they respond to the exploration of their own perceptions and experiences. The dialogic interactions between the FGI participants enable the in-depth discussion of ideas, alongside elaboration through agreement or disagreement with the views of other respondents, and clarification of thinking. Whilst two people may agree and share the same perceptions and experiences, they may still differ in the words they use and the order in which responses are expressed. This includes being aware of the emphasis and intensity of responses, which also, invariably, differs between respondents.

The pilot study involved 10 students (none of whom took part in the formal FGI stage), with a range of open-ended questions being tested to ensure that these were non-ambiguous and easy to understand. Responses to the pilot questions were used to adapt the wording of the questions common to all FGIs and to ensure researcher neutrality, so that the questions could not be regarded as leading the respondents towards pre-determined views or responses (see **Appendix A**). In addition, a semi-structured approach was taken to enable unplanned questions to be asked if, for example, the researcher required clarification if he either did not understand responses or wished to explore them in greater depth.

Analysis of the Transcribed FGIs

The analysis of the FGIs was undertaken with a view to enabling external validity (generalisation) for science teachers wishing to apply the analysis outcomes within their own evidence-informed professional practice (Altricher et al., 1993; Kincheloe, 2001, 2005, 2012; Smyth, 1996; Thomas, 2002, 2004). Analysis of the FGIs was, therefore, focused through the research questions. Therefore, individual FGIs and the FGIs as a collective group were interrogated using Thematic Content Analysis protocols (TCA: Anderson, 1998, 2004). TCA enables the descriptive presentation of data collected using qualitative methods such as focus group interviews. The advantage of conducting a TCA is that the outcomes are descriptive and analytical (Cresswell, 2009; Fischer, 2006; Smith, 2008). However, TCAs are also a form of intuitive inquiry: a constructivist epistemology and ontology based upon the intersubjective, inferential interpretations of subjective data that has been focused by the interviewer's selection of questions and the order in which they are asked (Anderson, 1998, 2004). As TCA is a form of interpretive inquiry, the common points arising from the data analysis usually leads to far more questions than have been posed and answered (Aldridge et al., 1999, p. 50).

The transcribed FGIs were analysed in order to identify common themes across the year groups, to provide an overview of:

1. The self-reported affective, cognitive and self-attributional factors that students regard as being influential upon their engagement with learning;
2. The key motivating teacher characteristics and behaviours regarded as mediating influences upon the initiation and sustaining of engagement behaviours;
3. Specifically, the key teacher behaviours that;

a. Inform students' views of the quality of the teacher-student interpersonal relationship during and outside science lessons;
b. Inform students' self-attribute perceptions, such as self-competence for learning science and self-efficacy within science lessons and activities, and;
c. Encourage or inhibit students' participation and autonomy within learning activities within the classroom and written assignments.

The emergent common themes are summarised in **Appendix B**.

The Research Setting

The research took place within an independent preparatory school in Great Britain. The headteacher and three science teachers gave their consent for timetabled science lessons to be set aside to allow students to participate in the FGIs. The children were mainly of white British origin, with parents, under the most recent social or socioeconomic classification, being of the elite and established middle classes (as defined by Savage et al., 2013). Science is taught as a general subject up until the end of Year Six (Fifth Grade), and from the start of Year Seven (Sixth Grade) to the end of Year Eight (Seventh Grade) the teaching and learning of science is separated into the scientific areas of biology, chemistry and physics.

Selection of Participants for the Focus Group Interviews

The FGI participants were selected so that there were, wherever possible, equal numbers of male and female students within each group. The children were interviewed in groups as the school's child protection policy stipulated that children should not be interviewed on a one-to-one basis within an enclosed space. In addition, the children were selected based upon their varying responses to topics covered within the aforementioned informal discussions, rather than their confidence levels or ability to articulate perceptions. Therefore, most FGI groups had six members as this would ensure that all of the selected children being able to discuss their perceptions as far as their individual confidence and articulacy enabled them to do so. With some groups, however, given the small class sizes, being rigid about the size of a focus group would have excluded one or two students. In this case, all of the students within the age group were included. One class group contained only male students, and therefore all nine male students from that group were interviewed. In total, 47 students participated: Group 1 – 6 students aged 10-11 (3 male, 3 female); Group 2 – 9 students aged 10-11 (all male); Group 3 – 8 students aged 11-12 (5 male, 3 female); Group 4 – 6 students aged 11-12 (3 male, 3 female); Group 5 – 6 students aged 12-13 (3 male, 3 female); Group 6 – 6 students aged 12-13 (3 male, 3 female), and; Group 7 – 6 students aged 12-13 (3 male, 3 female). Each group participated in one FGI, with each FGI lasting approximately 45 minutes. Each FGI was recorded and transcribed.

RESULTS: Initial Analysis

The analysis of results was focused through the research question: What do students regard as the key influences that have an impact upon their motivated engagement with learning activities? The key emergent themes are summarised in **Table 1** and **Appendix B**. The perceived quality of the teacher-student relationship consistently emerged as the variable having the stronger impact upon students' motivation to engage with learning in science lessons. Students' perceived quality of the teacher-student relationship was influenced by their affective and cognitive responses and perceptions, mediated by their perceived competence. This, in turn, was influenced by the methods that individual science teachers had used to enhance students' feelings of self-competence during learning activities. Students self-reported their enhanced positive perceptions of their engagement with science. Across all cohorts and genders, such improvements were informed by positive changes to variables relating equally to relatedness, autonomy support and competence within science lessons. For example, when a group of 10 to 11 year-olds were asked about the role that the science teacher plays in motivating their engagement with learning activities, they affirmed that the teacher is the most important factor influencing and informing their enjoyment of science. This is due, according to the students, the teacher having the direct ability to particularly enhance the pace and depth of their perceived competence.

Table 1. Summary of the themes discussed by students during the Focus Group Interviews: perceptions of SDT-related phenomena

AGE GROUP		RELATEDNESS								AUTONOMY SUPPORT	COMPETENCE				ENGAGEMENT		
	TSIPRQ within lesson	R (TC)	PosTreat	NegTreat	SLikeT	SDislikeT	R (TS)	TExp	TSRExt	PAS	PCom	SelfEff	SelfConf	ComStrat	PosAffec / NegAffec	CogEng / CogDiseng	AcaEng / AcaDiseng
Group 1	N (1)			•		•		N - TNRC	N	N – I, L N – Tcont PCI		P		N - I	N	N	N
10 – 11																	
Group 2	P				•		P		P	P - I	P	P	P	P - L	P	P	P
Group 3	V				V	V			V	V – I	N	P	P	P	V	V	V
11 – 12																	
Group 4	V / Imp				P / Imp		P /Imp		P/ Imp	P / V	P/ Imp	P/ Imp	P	P/ Imp	P/ Imp	P/ Imp	P/ Imp
Group 5	N	N		•		•	N	N (A)	N	N – I	N		N	N	N	N	N
12 – 13																	
Group 6	V		•		V	V	V		V	N – I	P		P	P	V	V	V
Group 7	P								P	A / Imp	P / Imp		P/ Imp	P/ Imp	P	P	P

Key: P = Positive responses / self-reported perceptions (mainly); N = Negative responses / self-reported perceptions (mainly); V = varied responses / self-reported perceptions: a mixture of positive and negative responses: I = Investigations; L = Lessons; TCont = Teacher Controlling; PCI = Preferred Choice within Investigations; NI = Not interesting; TNRC = Teacher Not Recognise / Acknowledge Student Competence; A = Ambiguity / Ambiguous; Imp = Improvement since last year.
Notes; Where a blank space has been left, the theme was either not discussed or insufficient information was available to form a perception as to whether the response was, on the whole, positive, negative or varied.
ABBREVIATIONS: TSIPRQ – Teacher-Student Interpersonal Relationship Quality; R(TC) = Relatedness (Teacher Care); PosTreat (Positive Treatment); NegTreat (Negative Treatment); SLikeT = Students Like Teacher; SDislikeT (Student Dislikes Teacher); R(TS) = Relatedness (Teacher Support); TExp = Teacher Expectations;
TSRExt = Teacher-Student Relationship external to science lessons; PAS = Perceived Autonomy Support; PCom = Perceived Competence; SelfEff = Self Efficacy; SelfConf = Self Confidence; ComStrat = Strategies that have helped the students achieve success; PosAffec = Positive Affect; NegAffec = Negative Affect; CogEng = Cognitive Engagement; CogDiseng = Cognitive Disengagement; AcaEng = Academic Engagement; AcaDiseng = Academic Disengagement

The FGIs revealed that students' perceived competence was a vital precursor that influenced their motivated-informed desire to be autonomous within learning activities. That is, more positive perceptions of perceived competence were associated with a stronger desire to be autonomous within learning activities. Whilst there was a weaker association between relatedness and autonomy support, the very strong correlative association between competence and autonomy support suggests that the students' perceptions of autonomy support may be enhanced by the students' perceived competence. The influence of perceived competence as the central variable of engagement and the development of positive teacher-student relationship quality was also posited by Archambault et al. (2013). They report that whilst a positive teacher-student relationship was a reliable predictor of sustained engagement across age groups, it was perceived competence that was the central variable informing engagement. Indeed, they suggest that "…trying to change the relationship a teacher shares with each student will be more challenging than attempting to alter student engagement" (p. 6).

A common response was that the students base their views of the quality of the teacher-student relationship (relatedness) upon their perceptions of the teacher's effectiveness at enhancing students' self-perceptions of competence as opposed to satisfying any wish they had for their teacher to be autonomy supportive. However, students often reported that they felt more positively about the quality of the teacher-student relationship when they were given opportunities to demonstrate and develop their subject-specific competence through the autonomous design and conducting of their own investigations.

The Teacher-Student Relationship Quality (SDT: Relatedness)

The students regarded teachers' relational behaviours, affective reactions and the feedback they provide during and following learning activities as central to their motivated and sustained engagement with science. These motivational perceptions, in turn, informed students' engagement with science through, for example, intrinsic interest, enjoyment, and, where the teacher made it possible, the exploration of students' ideas and understanding through inquiry-based learning. The quality of the teacher-student relationship was confirmed as cumulative outcomes of interactions. It was clear that the relationship quality was informed by the consistency of teachers' interactions with the students. For example, one student (aged 11-12) stated that, "…he's sometimes really nice to me, but then he sometimes gets really angry at me, for not much at all, so ... I'm a bit confused really, and so I don't really know". Another student, within the sane group, stated that, "last year, he could sometimes be very, very nice to me, and he could sometimes be very, very… I really, really hated him, and it was like so hard to tell if he liked me or not, sometimes I thought that he didn't really like me that much at all, and sometimes I thought that he sort of liked me…". Some of the 11 to 12 year-old students reported an ambiguity regarding the extent to whether their engagement within science lessons was based upon whether their perceptions of the teacher-student relationship were positive or negative. However, other 11 to 12 year-olds reported that the teacher was central to their enjoyment of and engagement with science. This viewpoint was articulated by a number of groups: that where there was a perceived positive quality to the teacher-student relationship, there were also positive perceptions of competence and autonomy within investigations. Stronger perceptions of positive relationships were sustained where the teacher was receptive to students' confidence levels and obvious competence, thereby enabling students to perceive themselves as more competent during learning activities. The students enjoyed and appreciated lessons where they had opportunities to demonstrate their competence through, for example, the discussion of their ideas, exploring their understanding of scientific concepts, and demonstrating their learning within practical activities. From the students' perspectives, teacher-student relationships were improved through science teachers' feedback that was regarded as positive. Such feedback included encouragement regarding the quality of work and the extent to which understanding of concepts has been gained. Feedback should also enable the correcting of misunderstandings and reinforcing the mastery of knowledge. Specific factors that influenced the students' perceptions that the teacher-student relationship is a positive one included the teachers' ability to help students feel competent during science lessons.

All students, with the exception of one group, perceived a positive sense of competence and the motivation to be autonomous when a perceived positive teacher-student relationship was in place. The responses of the other group differed, in that whilst this group reported negative relationships with their current science teacher, these students still felt that they were learning more than they had with their previous science teacher. In this case, competence, despite the lack of a positive teacher-student relationship or enjoyment, was attributed more in terms of how the teacher taught the subject rather than the sense of relatedness that this particular group of students attributed to the teacher-student relationship.

Autonomous Motivation (SDT: Autonomy)

The students expressed their desire to be autonomous within learning activities, especially investigations. Further to changes of science teacher at the start of the academic year, many perceived a more positive teacher-student relationship quality and motivation to be autonomous with their previous science teacher, comparative to their current teacher. All confirmed that the teacher was the most important factor influencing and informing their enjoyment of science. The students preferred having the choice as to whether they wanted the teacher to either direct the investigations or to allow them more freedom by, for example, encouraging them to be entirely self-directing. However, despite this factor having a positive influence upon their engagement with science, none of the focus groups felt that there were regular opportunities for them to exercise open-ended autonomy within investigations. Autonomy through inquiry-based learning, such as investigations, was important to the students as means of enhancing their perceived competence. All felt that, within their present science lessons, there were fewer opportunities for them to be autonomous than they would have liked. This did not, however, diminish their sense of perceived competence within science. Fewer, or indeed no, investigations had potentially resulted in missed opportunities for the children to learn or master concepts as autonomous and independent learners, especially where the children had been used to being so with their previous science teacher.

The students reported that the opportunities for autonomous inquiry-based activities were reduced as they progressed through the older age ranges within the school. The reason for this, in their opinion, was because, with the older cohorts, the teachers placed a greater emphasis upon the importance of acquiring competence and competence-related confidence through the understanding and retention of scientific concepts. One of the older groups stated that they would prefer a science teacher that teaches in such a way that students' feelings of competence were enhanced, especially as they were preparing for external examinations later in the academic year. However, this was not seen as a negative, as the students' preferred means of ensuring competence, understanding and retention whilst, for example, preparing for examinations was through on-going interactions and discussions with the teacher and with each other. This included the revision of previously encountered concepts through discussion with the science teacher: for example, "… I think [the current science teacher] is much better at explaining things than [the previous science teacher], because, [previous science teacher] kept just going on about things, and he didn't really explain them very well, so, I think [current science teacher] is a bit better, like at explaining them, and helping you understand".

Perceived Competence (SDT: Competence)

Students' perceived competence was enhanced when they are afforded opportunities to work together and help each other during learning activities. In addition, students expressed a universal desire to move on to new scientific concepts as and when they felt that *they* understood them, rather than always having to wait upon teachers' decisions to do so: for example, "…you are not spending ages on one subject, like you're not spending like five lessons …you're only spending one lesson… because you have done it in much less time, and if you're just doing that every single time, in detail, it can get quite boring …". Students felt their perceived competence was further enhanced by teachers who have a positive questioning style and that gave feedback which promoted further understanding of and confidence with concepts. Other means of enhancing students' perceived competence included students being given more time to investigate concepts, to develop their understanding and to complete work proficiently. The students preferred more direct input from their science teacher: careful explanations and feedback were welcomed by students as the basis for improving upon their current competencies, as long as it was accompanied by guidance upon how to improve. One student, for example, suggested that there was the need for more focused feedback upon the content of the work rather the presentation of the work: "I don't think he gives out enough feedback on what we've done" and "I know that he marked my work wrong, but he didn't explain it why, so, I didn't really know what to do".

FURTHER ANALYSIS AND DISCUSSION

For a classroom to stimulate engagement, students should perceive that there is a relevance and value to learning activities; a positive emotional climate within which students perceive a warm caring interpersonal relationship with their teacher; that the teacher is attuned and responsive to the individual responses and needs of students; that the students are making academic progress and are capable of making further progress (both independently and through teachers' autonomy-supportive behaviours), and; that the students enjoy their time in the classroom with that teacher (Pianta et al., 2012, p. 373; Reeve, 2009, 2012; Reeve and Halusic, 2009). Primarily, to promote a positive teacher-student relationship through the enhancement of students' perceived competence, teachers should aim to enhance students' abilities by recognising and celebrating their competence within current learning activities. This appears to be a basis for positive self-efficacious decisions when faced with further similar learning activities. These contextual factors are likely to have a reciprocal impact, in turn, upon students' perceived competence (Hipkins, 2012; Hughes et al., 2011; Lam et al., 2012).

Students' responses across the seven FGIs confirmed that perceptions of the quality of the teacher-student relationship were directly influenced by students' affective and cognitive perceptions of the methods that individual science teachers had used to enhance the students' perceived competence. Students reported that their teachers have the direct ability to enhance the pace and depth of the students' perceived competence. The students also confirmed that the teacher is the most important factor influencing and informing their enjoyment of and engagement with science. This was based upon a teacher's perceived ability to enhance the pace and depth of the students' perceived competence. In addition, responses revealed that they base their views of the quality of the teacher-student relationship upon their perceptions of the teacher's effectiveness at enhancing students' perceived competence. This was regarded as more important than satisfying any wish they had for their teacher to be autonomy supportive, with students' perceived competence influencing their motivation for learning during lessons.

It appears that such motivational influencers, if effectively afforded by teachers, should result in students being encouraged and supported to become more independent, self-competent and self-agentic learners who have positive perceptions of their self-efficacy. This, in turn, will inform their sustained desire to be autonomous (Bandura, 1986ab, 1997; Dewey, 1902. 1938ab; Vygotsky, 1978). This posit was supported by the students' responses to questionnaires across the three data waves, which revealed that the strongest correlative association informing their engagement was between relatedness and competence. The weakest correlative relationship informing their engagement was between relatedness and autonomy support.

During all FGIs, relatedness (the quality of the teacher-student relationship) emerged as the most influential SDT construct in terms of its impact upon students' motivated engagement with science learning activities. Students revealed that they based their views of the quality of the teacher-student relationship upon their perceptions of the teacher's effectiveness at enhancing students' perceived competence. Perceived competence was revealed as predictive of students' autonomous motivation during learning activities. It was also affirmed that the teachers were central to students' enjoyment of and engagement with science. The quality of the teacher-student relationship appears to be inextricably linked to the extent to which a teacher's behaviours and afforded learning provision during lessons promote the students' perceived competence specific to science, based upon repeated, confirmatory interactions. Students' perceived competence was based primarily upon the performance feedback provided by the teacher. Teachers' affordance of autonomy-supportive learning activities that were regarded as enjoyable, interesting and enjoyable also informed and predicted students' engagement with learning activities (Jang et al., 2012; Vansteenkiste et al., 2005, 2012). In addition, teacher care and affective support was revealed as a potential predictive basis for enhancing students' feelings of belonging, academic enjoyment, self-efficacy (perceived competence) and engagement (Hardre et al., 2006; Pat El et al., 2012; Zhou et al., 2012).

Student responses, therefore, confirmed that, whilst the satisfaction of all three SDT basic psychological needs is important, relatedness (positive teacher-student relationships) and competence are the two most influential SDT constructs upon their motivation to engage with science. As stated above, students' perceived competence was regarded as a stronger basis for a positive teacher-student relationship than satisfying any wish that the students had for their teacher to be autonomy supportive through, for example, the affordance of inquiry-based learning activities. However, where such opportunities were afforded, the students did confirm that this reinforced and promoted more positive perceptions of the quality of the relationship with their science teacher. Where students had increased and / or sustained opportunities to exercise their own autonomy through inquiry-based learning, they self-reported more positive perceived competence and progress in science. It may be that students who are afforded the autonomy to demonstrate their competence through, for example, inquiry-based learning activities, whilst supported by positive feedback from the science teacher, are more likely to develop a strong teacher-student relationship and, reciprocally, are more likely to be engaged with science. In addition, the reciprocal feedback perceptions of relatedness and competence have been asserted by students as having a direct impact upon their engagement with learning (within the main study, and, for example, Harter, 2012a; Mahatmya et al., 2012). The basis of this reciprocal relationship may be that perceived competence is influenced by an intrinsic motivation orientation, which, in turn, is informed by a student's perceptions that they have frequent opportunity to be autonomous and be supported in this by the teacher (Guay et al., 2013).

Feedback as a form of support by the teacher enhances perceived competence, which, in turn, appears to inform the quality of and balance between intrinsic and extrinsic forms of motivation (Deci and Moller, 2005; Ryan and Deci, 2009; Vallerand and Reid, 1984). The quality of the balance between intrinsic and extrinsic motivation determines the extent of engagement, as "Intrinsic motivation flourishes under conditions supporting autonomy and competence and wanes when these needs are thwarted" (Ryan and Deci, 2009, p. 174). Indeed, intrinsic motivation and other motivational variables are intrapersonal psychological responses to the sociocultural conditions of the classroom, including the teacher-student relationship quality (Hughes et al., 2008) and the regular enhancement of perceived competence during learning activities (Ryan and Deci, 2009). The nature of the learning activity will influence students' perspective of whether the activity encourages autonomy (such as inquiry-based open-ended learning) or controlling (such as a test of factual knowledge and retention, where motivation is more extrinsic than intrinsic (Benware and Deci, 1984; Fortier et al., 1995; Grolnick and Ryan, 1987). Perceived competence can still have a strong influence upon students' motivated engagement through the internalisation of an activity's value, such as its potential to lead to further cognitive progress (Ryan et al., 1985).

Relatedness, through the quality of the teacher-student relationship, has been referred to as a "supplement" within the SDT model, with autonomy and competence being more often emphasised as the basis for self-determined engagement (Ryan and Deci, 2009, p. 178). However, the current research suggests that relatedness is the essential central catalyst informing the quality of students' engagement through the enhancement of perceived competence. Students across all cohorts self-reported that they temporally developed a stronger sense of competence and autonomy support. This suggests that students are temporally able to develop positive perceptions of their competence and self-efficacy across the full continuum of teacher motivating styles from autonomy supportive to controlling (Close and Solberg, 2008).

From the results, of the three SDT constructs, the one that is most *resilient* with regards to engagement appears to be competence, in the form of an individual's sustained need and desire to be competent. This resilience was affirmed, even when the quality of the teacher-student relationship is regarded as negative and there are limited opportunities for students' autonomy to be exercised. Whilst none of the cohorts reported a consistent positive relationship with their science teacher, a small number of individuals within each group did report a positive relationship with the science teacher: in some cases, this was very positive. These students reported similarly positive perceptions of their competence within science learning activities, of positive levels of the autonomy allowed and, where afforded, autonomy support. However, with the increasing age of the groups it was interesting to note that although the quality of the teacher-student relationship was regarded overall as negative by the students there were still steady increases in the students' perceived competence and motivation to be autonomous. This suggests that whilst the teacher-student relationship quality appears to be predictive of students' perceived competence and autonomous motivation with younger students, the relationship may be less influential with the increasing age of the students as they developmentally move from dependence upon the teacher to interdependence (Harter, 2012a; Mahatmya et al., 2012; Ryan, 1982; Skinner and Pitzer, 2012). Also, it may be that younger students perceive the quality of the teacher-student relationship as being more important, comparative to their older peers, as the motivational basis for feeling engaged and competent within learning activities. It has been suggested that younger students' perceptions of the teacher-student relationship quality are based upon a form of learned helplessness: manifested as dependency upon the teacher for guidance, and for making the student's competence-based progress, successes and achievements overtly evident (Harter, 2012ab; Hattie, 2009, 2012; Pat El et al., 2012). As students mature, they usually become less dependent on their teacher (Harter, 2012a). However, there will still be adaptive help-seeking alongside an increasingly greater psychological need to be more independent, as well as engaging in tasks and behaviours where they increasingly feel more competent by making progress as a result of their own self-motivated and self-determined autonomy (Harter, 2012a; Mahatmya et al., 2012).

All groups reported their need to feel competent and to become more competent, even when the teacher-student relationship quality was viewed as negative. In addition, the positive affect generated in response to perceived and actual achievement was instrumental in enhancing students' perceived competence and, in turn, an a more positive teacher-student relationship quality. For example, enhanced engagement was observed during learning activities when there were positive associations between students' perceived competence and intrinsically regulated motivation (Cox and Williams, 2008). These motivated perceptions and increased engagement resulted in the student feeling more self-efficacious and, therefore, motivated and enthused by the challenges within new learning activities. This appears to be due to perceived competence and self-efficacy combining to create an overall academic self-concept which influenced the beliefs that the student has about their academic capabilities, skills and strengths, and the experiences that have informed these (Bandura, 1977, 1986ab, 1997; Cleary and Zimmerman, 2012; Hughes et al., 2011; Marsh and Shavelson, 1985; Pajares, 1996; Urdan and Turner, 2007).

It was interesting, also, that the students' regarded their perception of competence as sustaining their motivation for learning within science even if they did not always get the chance to translate this into autonomous behaviours often associated with optimum engagement. This differs from the findings of other studies that have focused upon the importance of the teacher-student relationship as the motivational basis for effortful engagement (Archambault et al., 2013; Birch and Ladd, 1997; Hamre and Pianta, 2006; Hughes et al., 2008; Pianta and Steinberg, 1992; Pianta and Stuhlman, 2004; Pianta et al., 1995, 2002, 2003). A possible reason for the responses of the current participants differing from those reported by other studies may be that they have learnt through experience to not only be less reliant upon the quality of the teacher-student relationship as the basis for informing their perceived competence but also at an earlier age than one would normally expect (Lynch and Cicchetti, 1997). The students' perceptions revealed that, rather than looking to their teachers, they had become more reliant upon their peers at this stage. They appeared to be using interactions with peers as an influential means of informing their perceived competence within science lessons. Although this could not be confirmed, it may be that the excellent quality of the teacher-student relationships that the children had when they were younger had helped them to internalise benchmarks for judging their perceived competence and self-efficacy earlier than one would expect developmentally. Indeed, Ryan (2001) highlighted the important compensatory role of peer influence, especially when the teacher-student relationship is either negative or regarded as less important, reporting that there is a tendency amongst young adolescents to group together according to perceived homophily: shared attributes including "…the norms, values, and standards that concern academic motivation and achievement. This shared peer group context is likely to influence adolescent motivation and engagement in school" (p. 1136).

Similar findings across FGIs revealed factors that were predictive of and are predicted by a positive teacher-student relationship include a teacher who is;

1. receptive to students' perceived competence and self-confidence;
2. mindful of students' competence levels, allowing learning to progress at an appropriate pace;
3. adept at explaining scientific concepts and theories in such a way that all students may understand them;
4. providing opportunities for the students to discuss their ideas and explore their understanding of scientific concepts;
5. providing opportunities for the students to demonstrate their mastery, understanding and application of scientific concepts;
6. listening to students, acknowledging their ideas and questions; positive and encouraging in his feedback about a student's progress and competence, including the correction of misunderstandings;
7. perceived to be working hard to help students develop their competence and understanding of scientific concepts and processes;
8. treating all students fairly and equally, avoiding nepotism, and;
9. is adept at maintaining good relationships with students outside of science lessons.

Conversely, factors that are predictive of negative teacher-student relationships include a teacher who plans lessons in such a way that is very different from the way that the students prefer to learn; is perceived to make no effort to make learning enjoyable; who ignores the responses of students, particularly when they are attempting to demonstrate that they are already able to do something or have completed something prior within the current concept area, and; who does not allow sufficient time for the students to investigate concepts. Therefore, it is posited that the teacher-student relationship quality may be used as a reliable predictor of perceived competence, academic achievement, and educational outcomes such as sustained engagement with learning activities (Hattie, 2003). Furthermore, the current research suggests that the motivation to be autonomous is an outcome dependent upon the combined motivational impact of students' perceptions of the quality of the teacher-student relationship and their own perceived competence. The main study students self-reported that their perceptions of competence, especially where there was perceived negative relatedness, were attributed more to the means by which the teacher taught the subject and emphasised learning, rather than the sense of relatedness that this particular group of students attributed to the teacher-student relationship. As none of the focus groups felt that there were regular opportunities for them to design and lead open-ended, autonomous investigations, the perceived quality of the interpersonal relationship with the teacher had become increasingly dependent upon the extent to which the teacher directly enhanced the students' perceived competence during science lessons.

CONCLUSIONS

Whilst Ryan and Deci (2009) acknowledge that "… both the social-contextual and personal motivation variables central to SDT have been found to predict engagement, performance and well-being" (p. 181) and assert that relatedness, autonomy and competence have salient motivational influences upon an individual's self-determined motivation to engage with learning activities, they have not, within their writing, specified if one SDT construct is central to the positive psychosocial development of the other two when applied to students' motivated engagement in the classroom. The emergent multivariate hierarchical influences of each of the three SDT needs upon the other two and students' motivated engagement with learning is at the heart of the students' self-reported perceptions.

For a classroom to stimulate engagement, students should perceive that there is a positive emotional climate within which students perceive a warm caring interpersonal relationship with their teacher; that the teacher is attuned and responsive to the individual responses and needs of students; that the students are making academic progress and are capable of making further progress (both independently and through teachers' autonomy-supportive behaviours), and; that the students enjoy the time they spend in the classroom with that teacher (Pianta et al., 2012, p. 373; Reeve, 2009, 2012; Reeve and Halusic, 2009). Primarily, in order to promote a positive cumulative and reciprocal teacher-student relationship through the enhancement of students' perceived competence, teachers should aim to support and encourage students through the enhancement of students' abilities to internalise the standards necessary for recognising and celebrating their competence within current learning activities. Such standards appear to act as a basis for positive self-efficacious decisions when faced with further similar learning activities. These contextual factors are likely to have a reciprocal impact, in turn, upon students' perceived competence (Hipkins, 2012; Hughes et al., 2011; Lam et al., 2012).

In summary, the students regarded their teachers as being central to the enhancement of students' engagement and achievement within learning activities (see **Appendix B**). This was based upon the view that the students perceived that the teacher has the direct ability to particularly enhance the pace and depth of the students' perceived competence, mediated by teacher feedback. Students' willingness to listen to and act upon competence-based feedback is informed by the perceived quality of the teacher-student relationship (Hipkins, 2012). Receptiveness to teacher feedback reciprocally informs students' self-efficacy, and, as a result, impacts upon their engagement within learning activities (Cleary and Zimmerman, 2012). The current research reveals that engagement is predictive of students' motivation, which is reliant upon the individual teacher satisfying the need for competence or autonomy, or both.

From the FGIs, it appears that there is a potential hierarchy between SDT BPNs. However, the quality of the teacher-student relationship has been referred to as a 'supplement' within the SDT model, with autonomy and competence more often being emphasised as the basis for self-determined engagement (Ryan and Deci, 2009, p. 178). However, the evidence within the current research suggests that relatedness, in the form of positive teacher-student relationships, is the essential catalyst informing the quality of students' engagement through the enhancement of perceived competence. The results of the school-based research suggest that the teacher behaviours and methods supporting students' perceived competence and motivation to be autonomous are optimized when students perceive that they have a positive relationship with the teacher within the classroom. Where there is a perceived positive teacher-student relationship, different forms of motivation were enhanced. These include intrinsic motivation, extrinsic motivation to work towards goals that are regarded as having a personal value, competence motivation and autonomous motivation (Hughes et al., 2008; Ryan and Deci, 2009). The desire for autonomy also appears to have a motivating impact upon perceived competence and the resultant competence motivation and intrinsic motivation to engage with learning. Students' motivation to exercise their own autonomy originated with the students' affect-driven feelings of perceived competence, self-agency and self-determination. Autonomy was self-reported as the least influential of the three SDT basic psychological needs in terms of its impact upon students' motivation to engage with learning activities.

A limitation of the current research was the sample size, which was necessarily small due to convenience sampling as the research was undertaken with the teacher-researcher's students. However, the seven focus group interviews effectively harvested a range of student perceptions, including the experiences and inferences that had shaped these, regarding the contextual factors and teacher behaviours that encouraged their motivated engagement with learning. Repetition of the study by teachers would enable the extension of each questionnaire to explore each area in greater depth. In addition, allowing more time to conduct FGIs may lead to greater insights regarding factors, experiences and perceptions that inform and influence students' self-perceptions. In addition, the presence of the researcher undoubtedly has an influence upon the behaviour of the people being studied, a phenomenon known as 'observer effect' (Cohen et al., 2007). However, as with most research designs, the researcher will never be successful in their attempts to eliminate their influence upon the people and settings they are studying and seeking to understand (Hakim, 2000, p. 67).

The current research has been approached throughout with the objective of enhancing teacher-researchers' contextual understanding of students' motivation to engage with learning activities. The discussed findings highlight areas that teachers may wish to focus their energies upon as means of enhancing the quality of the teacher-student relationship and the students' perceived competence through a focus upon feedback. The findings may be applied by teachers within their own classrooms as a means of improving and developing both their evidence-informed professional practice and further in-school research. The implications of this research are discussed herein in terms of the significance of the findings and their applicability as epistemological contributions to the substantive field of SDT within educational research. The findings have practical implications for teachers in their own classrooms, as well as school leaders and others involved in the formulation of educational policy based upon research-led teaching.

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APPENDIX A

Examples of Questions Asked and Areas Explored during the Focus Group Interviews

- Describe how well you get on with your current Science teacher, outside the Science lesson.
- Do you have much contact with him outside of your Science lessons? Do you see him for other lessons, or for other activities?
- Do you think that liking your Science teacher outside of the Science lesson makes a difference to how you feel about Science lessons at all?
- Would you say that your perceptions of your teacher affect how you enjoy Science?
- If you had that choice, would you have your previous Science teacher back, or would you stick with the one you've got now?
- Did your perceptions change over time? If they did, what might have caused those changes in your perceptions?
- If you're not feeling happy within Science lessons, what have your science teachers done to help?
- What would your ideal Science teacher be like? What would you want from your ideal Science teacher?
- Does your Science teacher listen to you? Do they use your ideas, and do you feel that your Science teacher is listening to you?
- Do you feel able to say to your current Science teacher, "Can we have a lesson where we can show you that we have actually learned this?" Would you be able to do that?
- Are you given lots of opportunities to discuss your ideas and thinking, or are you expected to have the right answer?
- What do you like most about your Science lessons?
- Do you get opportunities to design, and do your own investigations? Does that make a difference to your enjoyment of Science lessons?
- What would be your ideal Science lesson?
- Is the teacher the most important factor in your enjoyment of Science?
- How does your Science teacher help you to feel more confident within Science?
- How does your current Science teacher use feedback in terms of how well you're doing?
- Does this help you to feel more competent, and that you are doing well within your Science?
- In what ways has your Science teacher helped you to understand what you're learning? How do they help you as individuals?
- What could your teacher do to make the relationship between you and him stronger within the Science lessons?
- Is it important to like your Science teacher, and your Science teacher to like you, to make a difference to your enjoyment and your progress within the lessons?
- Is there anything else that you want to say about your enjoyment or improvement of Science, anything like that, that I haven't asked you about, or that you haven't had the opportunity to discuss?
- Does your current Science teacher's feedback help you to feel more competent during science lessons?

APPENDIX B

Summary of Characteristics of Students' SDT-Grounded Engagement with Learning in Science

The influence of the interplay between the three SDT basic needs

1. There is an emerging important relationship, in terms of the influence that relatedness, in the form of teachers' support behaviours, has upon students' perceived competence, intrinsic motivation, and motivation for engagement and achievement.
2. Students' classroom-based perceptions of teacher support (relatedness) influenced student's perceived competence, with both being predictive of students' motivation to engage in learning: a student's individual affect and motivation to engage with learning activities are based upon his/her perceptions of the classroom environment.
3. Motivation for learning takes many forms, including intrinsic, self-determined, extrinsic and amotivation, and are functions of and dependent upon age, mediated by students' perceptions of the teacher and teacher-provided support;
4. Regardless of age, perceived competence and the perceived satisfaction of competence emerged as the strongest mediators between the teacher-student relationship (relatedness) and self-determined motivation and engagement in learning;
5. Perceived competence was based upon performance feedback provided by the teacher, which, in turn, informed students' need for autonomy;
6. Both relatedness and competence, but not autonomy, mediate the effect of feedback upon students' motivation: the extent to which a student regarded feedback as either positive or negative was dependent upon the perceived quality of the interpersonal relationship with the teacher and the extent to which the teacher's feedback informs a student's positive perceived competence;

Perceived competence and the influence upon self-efficacy

7. Common across the age range cohorts was the association between higher levels of perceived competence and higher self-efficacy, higher self-esteem, and higher competence need satisfaction: this was equally associated with higher levels of self-determined motivation and intrinsic motivation;
8. Key factors which were asserted as mediating between social contextual factors provided the teacher and students' sustained engagement were the provision of learning activities which promote students' positive perceptions of competence and self-efficacy
9. Competence support by the teacher was central to students' expectancy-related and self-efficacious beliefs;
10. Perceived competence has the potential to inform students' self-efficacy, and, in consequence, impact upon their engagement within learning activities;
11. Relatedness and competence have a mediational influence upon students' motivational perceptions and reactions predictive of engagement;
12. Prior academic self-concept significantly predicted academic achievement, which is potentially mediated by students' perceived competence.

Relatedness through the quality of the teacher-student interpersonal relationship

13. There was an emphasis upon the importance of the teacher's role in ensuring that all of these factors are sustained through their interpersonal and instructional styles;
14. The strength of the social relationship with the teacher was more influential upon students' motivation for and engagement with learning, comparative to the students' perceptions of autonomy and competence:
    a. All three SDT constructs were found to partially mediate self-determined engagement with learning through the quality of the teacher-student relationship.
    b. A teacher's interpersonal style and associated behaviours had a long-term impact upon students' SDT basic need satisfaction, adjustment to learning within a formal context, and an enhanced sense of the desire to engage in learning;
    c. Relatedness through teacher support was the basis of the teacher-student relationship quality;
    d. Social-contextual factors afforded by the teacher within the classroom enable students to satisfy their basic needs for relatedness, competence and autonomy;

   e. Affective support by teachers is a sound basis for enhancing feelings of belonging, academic enjoyment, academic optimism and self-efficacy (perceived competence) and engagement (academic effort).
   f. Significant associations were reported between perceived teacher affective support and students' motivational, affective and engagement behaviour outcomes.
   g. Teachers' positive affective support behaviours include caring for and interest in students, demonstrating respect and concern as appropriate, listening and responding to students' ideas, recognition of effort, and fair treatment: these are argued to be positive predictors of students' optimistic self-concept, academic effort, academic achievement, and the pursuit and practise of prosocial behaviours.
   h. Relatedness-enhancing behaviours were the basis of the development of higher expectations of students, and were associated together: that is, a teacher who felt a stronger affective / relational bond with a student had higher expectations of the student than, conversely, where the teacher felt a weaker affective / relational bond.
   i. The positive teacher-student interpersonal relationship develops through frequent interactions with teachers during collaborative projects, focus upon relationship enhancement, modelling enthusiasm for and confidence in students' ideas, providing academic and emotional support which result in regular opportunities to achieve success, and provide informative feedback in a positive manner, including what was done well, and what may be done next to achieve further competence and success.
15. There were positive associations between teacher support, enhanced feelings of relatedness towards the teacher, and students' feelings of self-determined motivation;

The need for autonomy and factors informing perceptions of autonomy supportive behaviours

16. Autonomy support and engagement may be enhanced over time, mediated by relatedness enacted as teacher support. Reciprocal effects were found between earlier perceptions of engagement and later perceptions of the motivating and engaging nature of the classroom.
17. Students who perceived a higher level of autonomous self-determination, as opposed to feeling controlled by their teacher, were more likely to feel that all of their SDT basic needs were being satisfied as a direct result of their teacher's behaviours;
18. There is a positive relationship between students' subject-specific science achievement, intrinsic motivation for and engagement with learning in science lessons, which are influenced by perceptions of autonomy support and perceived subject-specific competence in science;
19. Intrinsic motivation has a positive influence upon subject-specific self-concept, such as perceived competence and self-efficacy, and, in turn, achievement, and further enhanced perceptions of autonomous motivation, engagement and achievement over time;
20. The need to be autonomous may be a motivational outcome of the combination of teachers' relational-enhancing behaviours and the extent to which teachers' competence-based feedback enhances students' perceived competence.
21. Teachers' autonomy supportive behaviours have a positive mediating influence upon intrinsic motivation via the influence of perceived competence;
22. Students' perceptions of teachers' autonomy supportive behaviours was predicted by autonomous motivation, dependent upon whether goals are perceived by students as intrinsic or extrinsic
23. Relatedness and autonomy support can enhance students' enhanced positive perceptions of higher need satisfaction, self-determined motivation and engagement;

Teachers' behaviours supportive of enhancing students' positive engagement with learning

24. Teachers should afford and create an optimal learning context which enhances students' affective perceptions of well-being and the motivation to persistently engage in learning;
25. Teachers who deliberately increase the frequency of behaviours regarded as being central to the three SDT constructs can enhanced their students' quality of motivation and their subsequent wish to engage further in learning;
26. Teacher-student interactive dialogue supports students' sense of autonomy and competence:
   a. Teachers' performance feedback had an impact upon students' sense of relatedness and intrinsic motivation for engaging in learning activities;
   b. Key mediating variables that have an impact upon intrinsic motivation and subsequent engagement were the teachers' interpersonal style and instructional behaviours during lessons, and the influence of these upon students' motivational needs;

27. Perceptions of relatedness are enhanced by teachers' supportive dialogue that is meaningful to the student;
    a. allowing students to take leadership roles within the classroom;
    b. involving students in decision making;
    c. affording a motivational climate that emphasises the competence of students;
    d. encouraging students to develop their perceived competence as the basis for becoming more self-efficacious when approaching new learning activities, and;
    e. positive, autonomy-encouraging phrases, such as "You could" and "You might", as opposed to "You should" and "You must", when used by teachers will be regarded as more autonomy supportive, and therefore more motivating and predictive of engagement, than controlling.
28. Teacher support manifested as autonomy supportive behaviours were positively predictive of intrinsic motivation and self-determined motivation to engage with learning activities.

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The Language Used in the 8th Grade Mathematics Textbook

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ABSTRACT
This study aimed at identifying the form of the language, which is expressive of the mathematics and mathematics learner in the 8th grade mathematics textbook. The results showed that the mathematics textbook tends to exhibit an absolute, symbolic, mathematics specific image and a negative image of the mathematics learner as just who executes the orders (scribbler), rather than a thinker or engaged in authentic problem solving. Possibly, these two images may explain the difficulties and problems the students encounter while learning mathematics.

Keywords: language and mathematics, mathematical symbolism, systemic functional linguistics, mathematics textbook

INTRODUCTION

Over the few past decades, language became the focus of research in mathematics teaching. Such focus is one of the recent, relative change features in teaching, which main characteristics are represented in a new understanding of the student, and an increase in the complications of the learning contexts, such as those resulting from the cultural and linguistic diversity. Mathematics language was a concern of the mathematics teaching community; many efforts were spent to describe its characteristics, and the ways that may support or cause difficulties for mathematics learners. Development of the relationship between language and mathematics began by considering mathematics as a language exiting by itself, with its own symbols, figures, words and structures. Then, language was seen as a form of communication used in mathematics learning, and a vehicle to build meanings. (Eisenmann, 2007; Morgan, 1996, 2000, 2010; NCTM, 2000; O'Keeffe & Donoghue, 2015; O'Keeffe & Donoghue, 2011; Petocz, et al, 2006; Setati, 2002).

Mathematics textbooks are a form of communication used in mathematics learning-teaching. They are important for both the teacher and students as they are the main adopted source in mathematics learning-teaching. In Jordan, for example, the mathematics schoolbooks constitute the major source for teachers and students in the mathematics classes. However, what are the mathematics meanings and ideas, which we could build through our reading of these books? This study is made as an attempt to analyze and explore the functions the language may perform in mathematics learning-teaching. The study proposes some possible interpretations of both the mathematics image and mathematics learner image, which could help in unfolding the difficulties and problems the student may face during mathematics teaching (Morgan, 2010).

The theoretical framework of the study relies on Morgan's (1996) linguistic framework for mathematical texts analysis. Morgan developed a linguistic framework to analyze the mathematics texts depending on the Systematic Functional Linguistics formed by Halliday (1978, 1985). It depends on the idea that "any text can realize a number of functions, such as "Interpersonal and Ideational" functions. She based her work upon a substantial idea, i.e. the options. The language enjoys the ability to provide diversified options to use singular, plural, past, present or future tense verbs, first, second and third person pronouns, and words expressing doubt or certainty (O'Keeffe, 2011). Each of these usages has its different connotation from others, which requires us to take care when choosing one option rather than the others. This implicates that the explanation the language reader or listener provides can be affected by this option accordingly (Morgan, 1996). Based on the options idea, the language analysis was made represented by the texts. This is because the analysis is a description of the explanation that could be built from the option the author made versus other interpretations that could be built from other choices he left, which were also available to him (O'Keeffe & Donoghue, 2015).

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Contribution of this paper to the literature

- The study provides an analysis of the language used in school mathematics textbooks and suggests some possible explanations for both the mathematics image and the image of the mathematics learner. These explanations may help to identify the difficulties and problems students may face while learning mathematics.
- This study can be used to develop the writing of mathematics textbooks, to consider mathematics as a social act, it is not absolute, and constantly changing, which may be useful in redesigning the curriculum.
- The study of the functionality of the languages in mathematics learning, and the possible language difficulties facing learners require more attention.

Halliday (1978, 1985) sees that the mental function expresses the mathematics image and nature: Are mathematics suitable for every time and place? Or else, are they a human activity inseparable from reality (Dozzey, 1992; Ernest, 2004)? Morgan uses the "transitivity" system to identify the mathematics image through the options idea. However, transitivity is present in two aspects: types of processes and types of participants in these processes. Halliday (1985) defined six main types of these processes: physical, mental, relational, behavioral, emotional and verbal processes, the first three processes are the most commonly used.

Choosing a certain process rather than the others may make us build a different explanation of the mathematics image. To identify the physical process in the mathematics texts, we look for the verbs that require carrying out practical activities, such as "I use, I find". On the other hand, we can identify the mental process through the verbs that require mental efforts, while we can identify the relational process through the existence of the mathematics objects. For instance, linking the measurement of the straight angles by measuring two right angles (Morgan, 1996).

More important, when looking at the participants in the mathematical processes, is to define their roles in these processes: are they present through the active voice verbs, or disappear with the passive voice verbs and nominalization? In other words, transforming the mathematics verb into a noun. We can look into the types of the participants in the mathematics process with active voice verbs such as: "I explain that the measurement of > 1 is equivalent to > 3. The verb "explain" is in the active voice mode, i.e. the learner shares in learning, and this participation illustrates that mathematics are a human action. Meanwhile, the example "in the triangle ABC, its sides AB and AC were divided into four identical parts." The verb "were divided" is in the passive voice mode, which means disappearance of the human existence, as if the two sides of the triangle were divided into four parts without the action of an actor (grammar: subject). This may leave the impression that math is an independent world by itself.

However, the choice of using active voice does not necessarily mean that the image built for mathematics is a human action. To be so, it is inevitable that the actor the verb refers to is the learner, but, when the actor is the symbols of the mathematics terms, it indicates the absolute image of mathematics. For instance, two triangles are congruent if they have three similar, equal sides. Here the congruence is assigned to the actor (the two triangles), as if the two triangles are carrying out the congruence process by themselves. Furthermore, we can identify the mathematics image by nominalization through the example, "the total of the triangle angles is 180°."

The interpersonal function looks at the role of both the author and the learner in the mathematics text, and the mutual relations between them. Discovery of these relations can be with the first person pronouns (I, we). Their use indicates engagement and participation of the author in the mathematics action, and that the reader is interested in it. This use gives him a kind of responsibility in building the mathematics ideas, such as, "I put the symbol (more than or less than "< >) in the box", and, "we arrange the similar sides in the two equations under each other." In addition, the use of the second person pronoun (you) indicates a close relation between the book and the learner, causing the learner to engage in studying the mathematics. However, the author draws the attention of the reader with a "certain degree" of power.

Disappearance of the pronouns indicates an official relation between the author and the reader. In some cases, no relation seems between them. For example, "the line connecting between the midpoints of two sides of the triangle is parallel to the third side and half its length. Another example is the use of the imperative mode, such as, "consider, suppose, define", because this mode provides the learner a role to participate in building the mathematics ideas. In this context, Morgan (1996) distinguishes between two types of the imperative mode. The general, such as "let's assume", which looks at the learner as a (thinker), and the special, which looks at the learner as a (scribbler) in the mathematics process, such as "draw a column on the midpoint of a straight line."

The aforementioned interpretations of each of the ideational and interpersonal functions do not necessarily mean that this is the intention of the author. Rather, they are the meanings the reader builds after reading mathematics texts and his interaction with them. Furthermore, both functions are intertwined with each. For instance, looking into the mathematics text, when the actor (subject) is not the human, displays the mathematics image as an absolute one in the ideational function. On the other hand, the interpersonal function, with the same indicator, (disappearance of the actor) means that the relationship between the author and the reader is official.

Due to the importance of language and its role in mathematics learning and teaching, many researchers sought to analyze mathematics texts from many aspects and through different tools. Setati (2002) researched the relation between the language and mathematics teaching-learning. Morgan (1996) analyzed the mathematics texts to identify the ideational, interpersonal and textual relations, which the students can build through these texts. Some researchers utilized Morgan's framework (1996) to analyze the mathematics texts, such as Alshuwaikh (2012), Morgan (2001, 2005, and 2006) and Setati (2002). Still other researchers analyzed the mathematics texts to identify the learner's nature (Eisenmann & Wagner, 2007).

Haggarty and Pepin (2002) attempted to discover the most commonly used mathematics textbooks in three countries, England, France and Germany, to identify the image of mathematics in these countries. They found that the prevalent style of the mathematics image is that it is certain and unquestionable, not affected by the cultural and social contexts where this image takes place, and that the authors appeared as if they were the "knowledge owners". In his analysis of how the concept of "probability" is introduced in the Palestinian mathematics textbooks, Alshuwaikh (2012) found that the concept reveals the human activity in the 4th and 5th classes. Meanwhile, the passive voice verb begins to appear as of the 6th class, and found that the mathematics symbols and terms dominated the probability concept in both the 10th and 12th classes. Eisenmann (2007) analyzed the mathematics text and found a noticeable use of nominalization, and O'Keeffe and Donoghue (2011), in their analysis, found similar results about the use of nominalization in the books of the basic stage mathematics courses.

It seems that the absolute image of mathematics dominates expression about the mathematics ideas. In this concern, Morgan (2001) presented three sentences on a number of female teachers namely: "the rectangle has equal diagonals", "if you measure the lengths of the rectangle, you will find them equal, and "the measure of the rectangle diagonals lengths is always equal." Then she asked them a question about which of the three sentences is the most mathematical? Here, most of them indicated that it was the first sentence, although the three sentences talk about the same information, but their wordings were different. The selection of the first sentence is because it emphasizes the absolute nature of mathematics.

The absolute view of Mathematics also controls many of the students. Crawford, Gordon, Nicholas & Prosser (1994) found that 73% of 300 students in one of the Australian universities see that mathematics are abstract, not more than figures, symbols, forms and rules used for problem-solving. This result also appeared in the works of Petocz, et al (2006), O'Keeffe and Donoghue (2015). This absolute view held by numbers of students could be ascribed to many factors, such as the Mathematics image in the Mathematics textbooks; they study, in addition to the Mathematics image held by their teachers, which is apparent through the teaching strategies they use in teaching the Mathematics ideas (Dossey, 1992). Furthermore, the students consider that their task does not go beyond learning (by heart) the information they take during the Mathematics lessons, and then they "download" at the exam time (Crawford, et al, 1994; Moschkovich, 2007). Accordingly, the study attempted to answer the following questions:

- What is the shape of the language that expresses mathematics in the 8th grade mathematics textbook?
- What is the shape of the language that expresses the mathematics learner in the 8th grade mathematics textbook?

METHODOLOGICAL FRAMEWORK

The study followed the qualitative method, especially ways of analyzing the, texts, because this method can provide an intensive description about a given subject (Creswell, 2012). The lessons were examined and read in a preliminary reading to take a general idea of them, Then the analysis was taken by taking each lesson alone, and the colors were used for shading, based on the indicators of the analysis tool, which is known "color coding", After the shading process, each sentence is interpreted qualitatively, because some of the indicators may be part of a particular interpretation, but when the sentence is understood and interpreted within the context in which it was written, it appears to belong to another interpretation, especially since Morgan (1996) In its linguistic context, she pointed to the importance of knowing the context in which the mathematical text is being analyzed.

After completing the text analysis, deeply standing on the meaning it performs within the language option in which it was written, all words and sentences that have the same meaning are grouped together and placed in their own cursor.

Data Sources

Part Two of the eighth grade mathematics textbook, which included four units: linear equations with two variables (6 lessons); geometric constructions (4 lessons); triangles (4 lessons), and objects (7 lessons). This book constitutes the main source for both the students and teachers, as the Ministry of Education prepares the books and distributes over the schools. The analysis focused on the shape of the language, which expresses mathematics and that which expresses the mathematics learner.

Study Instruments

Analysis tool developed by Tang, Morgan, and Sfard (2012), which was designed to analyze the mathematics texts and their functions. The tool is divided into two sides, and each side is subdivided into a number of discourse properties which they lead to.

For answering the first question: "What is the shape of the language that expresses mathematics in the 8th grade mathematics textbook?" the researcher considered the following five properties:

- **Specialization**: to identify the extent of a specialized math language through three indicators: vocabulary used according to the mathematics definitions (vocabulary borrowed from the daily life and used in mathematics contexts); mathematical expressions (vocabularies borrowed from mathematics context), and, mathematical symbols.
- **Objectification:** to identify whether the text is talking about characteristics of objects or functions through three indicators: shift to nominalization, specialized names that encase the functions, and the relational, material and intellectual functions.
- **Alienation**: to identify the extent of mathematics alienation through two indicators: presence of two human actors (subjects) in the mathematics functions, and concealing the actor/s (through the active voice where the actor (subject) is a mathematics object, and through the passive voice and shift to nominalization).
- **Logical construct**: in which we can look into the types of the logical relations presented, and how far such construct is explicitly expressed using connectivity tools.
- **State of the mathematical knowledge**: to identify how far the text displays the possible decisions and choices during the mathematics activity; and whether mathematics are discovered or invented. This could be achieved through indicators of the alternate (stead) and conditional sentences, and the explicit decisions whether taken or have to be taken, as well as types of the mind/verbal functions.

For answering the second question, "What is the shape of the language that expresses the mathematics learner in the 8th grade mathematics textbook?" the researcher looked in the three following characteristics:

- **Effectiveness**: to find out the type of activity in which the learner will be engaged, through two indicators of the learner as scribbler and thinker.
- **Power**: to know where the authority is, whether choices are available, and who makes these choices, through the personal pronouns, and the indicators of the use and certainty.
- **Formality**: to identify the relation between the author and reader, and whether there is an apparent pedagogical relation between both through the text.

The Validity and Reliability

The validity of the study instrument was verified by presenting it to specialists in the mathematics curriculum and making necessary adjustments, Because the tool in its original form is written in English, and the researcher translated it into the Arabic language, and the researcher presented it to the reviewers to verify the integrity of the translation and clarity, and verify the suitability of the tool with the Arab culture, the reviewers decided on the validity of the tool and its applicability in the Arab countries. After that, the researcher and his colleague analyzed the language of the mathematics book independently, and explained every sentence specifically. This is because some indicators may apparently belong to a certain explanation, but when the sentence was interpreted and understood through the context it was written, it was found belonging to another explanation. In this concern, Morgan (1996) underlined the importance of knowing the context in which the mathematics text occurs.

Following the text analysis and deep understanding of the meaning it performs within the language choice in which it was written, the researcher collected all the words and sentences that perform the same meaning, and put them within their corresponding indicator. The analysis was continuous and subject to ongoing feedback, and the agreement rate between the two analyzers ranged between (92%-97%).

Table 1. Shape of the Language Expressing Mathematics in the 8th Grade Mathematics Textbook

Discourse Characteristics	Result of the Analysis
Specialization	Use of very highly specialized language, which appeared at the level of the vocabularies used according to the mathematics definitions, mathematical expressions and mathematical symbols.
Objectification	- Talk about mathematics objects using nominalization and specific names that encapsulate up the relational processes to a very wide extent. - Use of material processes carried out by the student at a very wide scale. - Use of mental processes much less than the material processes.
Alienation	- Use of the human "actors", only through first person pronoun "we" (very few times), but the second person pronoun "you" (very frequently). There was no use of the first person pronoun "I". - Concealing the "actor" through: A- Frequent use of the active voice (where the actor is a mathematics object). B- Frequent use of the passive voice. C- Too many shifts to nominalization.
Logical structure	Frequent use of "wa" followed by "fa" ***(two conjunctions in Arabic meaning and, but with different connotations)***; "la (No) that denotes negation; but; i.e.; or; and conditional "if".
Mathematical knowledge state	- Frequent use of "alternates" and "conditional sentences. - Presenting the mathematics facts "readymade". - The language used is decisive and assertive, particularly in generalizations. - Very much use of the mental/verbal functions.

RESULTS

Question 1: What is the Shape of the Language that Expresses Mathematics in the 8th Grade Mathematics Textbook?

This question was answered through the analysis of five properties: specialization, objectification, alienation, logical structure, and mathematics knowledge condition.

The results showed that mathematics in the 8th grade is characterized by being very highly specialized. It was embodied as a world having its own mathematics objects of logical structure. They also showed that the mathematics facts are certain, accepting no doubt; so that these mathematics facts are approached through the material processes, which make mathematics strange to the students. The passive voice and active voice were used to a very wide scale, particularly in generalizations; with the actor is a mathematics object. **Table 1** shows this.

Specialization characteristic

Generally, there is a wide degree of specialization, as the vocabularies used according to the Mathematics definitions, conventional idiomatical expressions and mathematics symbols, were widely used, **Table 2** presents examples of indicators.

Table 2. Indicators of specialization, Frequency and Examples

Indicators	Frequency	Examples
Vocabularies used as per the mathematics definitions	400	Ordered pair (120), net (21 times), flat (10); plane (12); ball (44); objectify (14); column (24); tangent (3); side (41); head, base, line, sides (41).
Mathematical expressions	300	Circle (2); triangle (2); right angled triangle (1); triangle (42); straight line (8); external angle of the triangle (17); square (6); midline point (3); half the circle (5); center of the circle (2); radius (1); trapezoidal (2); circular segment (2); right cone (26); tri-prism (34); prism quartet (4); linear equation (22); two-variable linear equation (22); equation solving (31); graphic representation of a two-variable linear equation (11); graphical solution of a system of two-variable linear equations (4); solution of a system of two-variable linear equations by compensation (5); solution of a system of two-variable linear equations by deletion (4); intersection of two straights, square of the hypotenuse length, two orthogonal lines.
Mathematical symbols	350	To express the names of lines (52), for example: two orthogonal lines AB, CD. Te express the equation system (25); example: A-B +3= 0 B+A+1=0 To express A coefficient and B coefficient and the absolute value (12); example: 2A + 4B= 9 To express names of the points (33), example: M center, A head. To express straight lines (33); example: S height; radius; L drawing. To express theories: Pythagoras theory. To express size measurement unit (28). To express area measurement unit (39). To express the mathematical laws (19). To express geometrical shapes: triangle (AMB), circle, trapezoidal, square. The angle **(>)** (45)

The results showed a very specialized image of mathematics in their three indicators. All the lessons of the mathematics textbook, for the 8th grade, usually begins with vocabularies used according to the mathematics definitions, which are words used in our daily life and in mathematics too. Sometimes, these vocabularies have different meanings from those in the mathematics context, which confuses the students and impedes their learning (Tang, Morgan, & Sfard, 2012). After that, the mathematics textbook use of special cognitive idiomatic expressions specially coined for the world of mathematics, which the students know in mathematics only, adding more difficulties to the students because they did not know them before (Morgan, 2010). Finally, the previous two indicators are reduced in the form of mathematical symbols. They are not mere written numbers and symbols, as every symbol enfolds a mathematics expression, whether from inside or outside mathematics. This symbol is tied with other mathematics symbols through mathematics relations (O'Keeffe & Donoghue, 2015; Schleppegrell, 2011).

This specialized image was not affected by the diversity of the topics it dealt with. Presenting mathematics in a specialized image is a reason for certain ambiguity as a result of using words borrowed from "inside" and "outside" mathematics. In addition, this specialized abstract image of mathematics does not match the students' cognitive development, especially basic stage students, who are still in their physical process stage, which Piaget talked about. Therefore, they are unable to think abstractly (AlShwaikh & Morgan, 2013), which is quite contrary to this abundant use of the mathematics symbols. Even more, certain studies, for instance AlShuwaikh (2012) stated that students in the advanced educational stage could not approach the abstraction level, which Piaget talked about. As for the vocabularies used according to the mathematics definitions, mathematics lessons of the 8th grade included may "borrowed" words and expressions from daily life, that are utilized in the mathematics context. For instance, net, angle, intersection, orthogonal, line, plane, surface, ball, size, area; in addition to the mathematical definitions related to these expressions: acute angle, right angle, obtuse angle and other expressions about the angle.

As for the conventional, idiomatic expressions, the 8th grade mathematics textbook contained a large number of idiomatic expressions (300/1050= 29%). For instance, hypotenuse length square in Pythagoras theory, which is stated in a sentence not easily understood by the reader, as every word contains a mathematics meaning that requires the student retrieve the related experiences (square concept, length concept, hypotenuse concept). Thereafter, the student should connect them with other expressions to understand the mathematics context of the entire sentence. The conventional, idiomatic expressions may enfold many concepts whether from inside or outside the mathematics contexts. Examples of the expressions used in the previous sentence are expressions used in our daily life, such as length, square, hypotenuse. The use of the conventional, idiomatic expressions increases the difficulties the students experience in mathematics learning, in addition to the vocabularies borrowed from our daily life, and used in the mathematics context. (Schleppegrell, 2011).

The results further showed that the mathematics book is "overloaded" with mathematics symbols, which are used for multiple purposes. The issue is not limited to reading the mathematics symbols, but to dive in each of

Table 3. Indicators of Objectification Characteristic, Frequency and Examples

Indicator	Frequency	Examples
Shift to nominalization	105	Drawn (feminine pronoun, 7 times); adjacent (4); total (12); drawn (masculine pronoun 14); located (9); given (5); subtracted (3); adjoining (4); surrounding (3); orthogonal, intersecting, shared (twice each); prism, parallel (feminine) (5); parallel (masculine) (5); cylinder, aside (3); composed, objectified, two sides (3); triangles congruence (1).
Specialized nouns encapsulate the functions	490	System (63 times); intersection (5); solution (101), general picture (5); two-variable linear equation, circumference (13 each); height (34); length (25); algebraic expression (2); area (89); size/volume (78); change (8); bisectors, two linear equations with two variables each (19); coefficient (39 times).
Relational functions	40	* Solution of two equations system is the intersection point of the two lines resulting from their graphical representations. * The two-variable linear equation AX+BY+C=0 is an algebraic rule, in which the values of one variable depend on the other. When we put Y in the equation in terms of X, we call Y subject of the rule, and the process of writing one variable in terms of the other is called changing the subject of the rule. * Size of the right cone is one third the size of the cylinder which shares the cone its base and height. * Relation between the length, height and radius is: L= 2H =2 radius^2. * Size of the pyramid is one third the size of the prism in which it has the same base and height. * The net is a plane object, which could be folded to make an object, it consists of base, face and heads. * The shortest distance between two parallel lines equals the length of the column connecting them. * Columns drawn from the centers of the triangle sides meet in one point. * Bisectors of the triangle angles meet in one point.
Materialism	423	Find (72 times); arrange (20); complete (3): draw (25); solve (62); connect (7); calculate (33); limit (14); write (20); project(v) (7); create (4); use (4); represent graphically (8); fold (1); express (1); check (9); bring (7); fill (2); cut (2); locate (1).
Intellectuality	45	Note (5 times); realize (9); conclude (5); estimate, clarify and discuss (1 each); compare (2); justify your answer (12); talk (1).

these symbols to probe the mathematics facts it holds, which, in turn, forms additional difficulty to the students. Some difficulties arise from their weakness in mathematics reading skills, others from their failure to read the mathematics symbols properly (Berger, 2013).

Objectification characteristic

Five indicators in the mathematics text were explored to identify the mathematics image through objectification; namely: nominalization, specialized nouns that encapsulate the functions, relational functions, physical, and intellectual. In general, the mathematics book objectified mathematics, is if they have their own objects, and used the materialistic functions more and more at the cost of the mental functions. **Table 3** presents examples of indicators.

The results showed that the mathematics textbook "works" on objectifying the mathematics as if they were self-existing work, with its own mathematics objects. Moreover, that such mathematics objects have the ability to carry out the mathematics functions alone without the help of the human. The results also showed that the analyzed units are consistent with the large use of "shift to nominalization" and specialized nouns that encapsulate the functions, which show mathematics as a number of relations in which the mathematics objects carry out the mathematics functions that form the relations on which the mathematics are founded.

As for the relational functions, the results indicated that they were very widely used, reflecting a mathematics image as a self-existing, independent from the human existence. For instance, the example, "Total Square of the two sides' lengths of the right angle triangle is equal to the square length of the hypotenuse." Here the relation between the total lengths of the sides through the "equality" term, without the reference of any human intervention, who found the lengths, squared them, and asserted the equality; a state that embodies mathematics as a universe independent from the human existence. In the matter of the materialistic and mental functions, the book units "agreed" on using the former more than the latter. In spite of the use of the material functions in the mathematics activities, yet, these functions soon disappear when approaching conclusion or generalization. All the practical efforts the students made are "blown out" and transferred into "words" that make the mathematics objects protagonists of the approached generalization. In this concern, the wide use of the relational processes is not confined to the Jordanian mathematics schoolbooks. We find this case in the study of Morgan and Tang (2012), who

Table 4. Indicators of Alienation, Frequency and Examples

Indicator	Frequency	Examples
Presence of human actors in the mathematics functions		
Subject (actor): I	0	None
Actor: We	82	We: follow (2 times); call (5); compensate (7); suppose (7); write (3); make (8); solve (11); find (15); represent (4); add (6); notice (2); e conclude (7); use (2).
Actor: you	366	You: find (72); arrange (20); complete (3); draw (25); solve (62); connect (7); calculate (33); locate (14); write (20)'; project (v) (7); create (4); use (4); represent graphically (8); fold (1); express (1); check (9); bring (7); fill (2); cut (2); show (1); notice (5); make sure (9); conclude (5); estimate-explain and discuss (1); compare (2); justify your response (12); prove (1); talk (1).
Concealing the actor (grammar: subject)		
Active voice (actor-subject is a mathematics object)	20	- If the two straight lines became congruent (the subject/actor: is the two lines AB, CD) - The system has (system is the actor) (3 times) - Perpendicular to (actor: the straight line AB). - Intersect with (actor: the two straight lines). - Congruent to (subject: one of two straight lines (twice). - Two straight lines result. - The two straight lines intersected in the point. - Their heads meet (heads is the actor/subject). - The figure shows. - The line divides. - Two sides became congruent.
Passive voice.	20	The ordered pair is called (twice); it is said: five doubles of the first subtracted from the second equals (10 times); to represent the equation, it is graphically represented; these two equations are called; it is concluded that; called the point; to be bisected (3 times); drawn (2), folded….. etc.
Shift to nominalization	105	Drawn (feminine) (7 times); adjacent (4); total (12); drawn (masculine) (14); located at (9); suggested (5); subtracted (3); adjoining (4); surrounding (3); perpendicular –intersected- parallel (5 times); joint cylinder (2); joint prism (1); triangles with two congruent sides (3); compound figure drawn aside (3).

analyzed the exam papers made in England between 1995 and 2011, and found that the rate of using these processes increased during that period. This is also found in the study of AlSharafa (2015), who analyzed the Palestinian mathematics books.

Alienation

To identify the image of mathematics through alienation property, the researchers explored two main indicators. First, the existence of two subjects (human actors) in the mathematics functions through the use of first person pronouns (I and we), and second person pronouns. Second, "hiding" the subject through using the active voice (the actor is a mathematics object), and the passive voice and shift to nominalization. In general, the book included revealing and concealing the actors at the same time, where the first person pronoun "we 18%" was rarely used, but the second person pronoun "you 82%" was widely used. Both pronouns produce the human subjects (actors) in the mathematics functions, while the first person pronoun "I" totally disappeared. On the other hand, there was too much (24%) use to conceal the actor with its three tenses: active voice (actor is mathematics object), passive voice tense and shift to nominalization. **Table 4** presents examples of indicators.

The results showed that the predominant feature is keeping mathematics "far" from the students, as the words used in this regard imply that there is a "far" distance between them. In conjunction with alienating mathematics to the eighth graders, indicators are used to reduce the distance between them and mathematics. However, that was not sufficient to eliminate the distance, especially the indicators of "actor's" presence increase when solving the book exercises (Golding, 2010).

Too many indicators of concealing the human doers (subjects of the verb) would take mathematics away from the students, and keep a distance between both. Through hiding the human role, the mathematics functions appear as if they were out of the human control (Morgan & Tang, 2010). This contributes in rooting the prevailing view of mathematics as an independent, autonomous world where there is no place for human action (AlSharafa, 2015; Morgan, 1996).

Logical structure characteristic

This trait increases through the use of different conjunctions, which were used in the book, such as "and", "or", conditional "if", "no", and other conjunctions. The results show the presence of two main traits of the mathematics

Table 5. Indicators of State of the Mathematics Knowledge Property, Frequency and Examples

Indicator	Frequency	Examples
Alternates	4	We can use the protractor to find out the measurement of any angle through the following steps.... The triangle could be called by naming its heads. We can generalize that every triangle has six outer angles. It could be said that: measurement of outer angle of the triangle equals the total of the two non adjacent interior angles.
Conditional Sentences	6	* If two lines intersect and one of the intersection angles was right angle, then the other three are also right angles, and the lines become orthogonal. * The line falls on the plane if all the points of the line fall on that plane. * Two straight lines, if you try to extend them as per their straightness from both sides, they will never meet. * If you extend the two straight lines, they will meet in one point. * If two lines intersect, they intersect in one point only. * If a line crossed two parallel lines, every two alternate angles are equal.
Types of the mental/material functions	480	**Mental:** Note (5 times); realize (9); conclude (5); estimate, clarify, discuss (1); compare (1); check (3); justify your answer (12); prove (1); talk (1). **Material**: Find (72 times); arrange (20); complete (3); draw (25); solve (62); connect (7); calculate (33); locate (14); write (20); project (v) (7); create (4); use (4); represent graphically (8); fold (1); express(1); make sure (9); bring (7); cut (2); locate (1).
Available Choices	4	* How many solutions are there for this problem? * Find (in two ways) measurement of the angle BCD. * Express (in your language) about what you see. * Justify your answer by providing examples.
Language Used	510	A definite, decisive language was used in several places, such as: * Write every two-variable linear equation in its general shape. * Rewrite the equation. * To represent the equation graphically, follow these steps. * Draw a column on a straight line. * Project a column on a straight line. * Move the triangles.

texts, use of conjunction tools, especially "and", and prominence of the logical relations among the mathematics objects. These two properties display the mathematics texts as logical structures, which components interconnect through logical relations, whether linguistically, through the use of conjunction (and in particular), or mathematically, through the use of specialized words that characterize the mathematics discourse. The mathematics logical structures idea supports the specialty degree of these texts in the schoolbooks, and represent mathematics as a universe having its own mathematics objects with their logical structure.

State of the mathematics knowledge property

There are five indicators of the mathematics state knowledge trait, namely: alternates (stead in grammar), conditional sentences, types of mental/verbal functions, available choices, and the used language. In general, the researchers found that the mathematics textbook widely used mental/verbal functions and critical language in presenting the mathematics facts. On the other hand, alternates and conditional sentences were less used. **Table 5** presents examples of indicators.

Through these results, mathematics appeared as a pool of readymade mathematical facts, presented through a definite, decisive language that does not allow doubt. The mathematics facts, in which the students are allowed the chance to conclude them, are made through following a number of definite steps. The use of the alternates and conditional sentences varied, as in Unit Two, where there was little use, while they were abundantly used in the other units.

Question 2: What is the Shape of the Language that Expresses the Mathematics Learner in the 8th Grade Mathematics Textbook?

This question was answered through analyzing three properties: doer (grammar; subject), control and formality. The results showed that the language used in the mathematics book for Grade 8 gave the learner a small role in the learning process, where the second person pronouns appeared in lieu of the first person ones, which highlight the role of the learner. However, in both types of pronouns, the role tends to simply implement the orders, but the basic

Table 6. Indicators of The actor characteristic, Frequency and Examples

Actor (Grammar subject)	Frequency	Examples
Second person pronouns dominated the mathematics text, but mostly were executing the orders implementer.	60 220	**Student as an order implementer (scribbler):** **First Person Pronouns:** (We): call (8 times); symbol (5); say (5); can (5); draw (twice)…. **Second Person Pronouns**: (You): find (7 times); arrange (20); complete (3); draw (25); solve (7); calculate (33); define (14); write (20); project "verb" (7); create (4); use (4); write (9); represent graphically (8); fold (1); express (1); make sure (9); bring (7); fill (2); cut (2); locate (1)….
	82 50	**Student as a thinker:** **First person pronouns:** (We): notice (twice); conclude, locate, calculate and find. **Second person pronouns**: Mental: (You): notice (5 times); ensure (9); conclude (5); estimate, clarify, discuss (1); compare (2); realize (3); justify your answer (12); prove (1); talk (1).

Table 7. Indicators of Control characteristic, Frequency and Examples

Indicator	Frequency	Examples
The basic power is the author's (book), due to dominance of 2nd person pronouns over the mathematics text as compared with the 1st person pronoun (I).		**Personal pronouns**: **(I):** None **(We):** 82 times: examples: Follow (2); call (5); compensate (7); suppose (7); write (3); make (8); solve (11); find (15); represent (4); add (6); note (2); put (3); conclude (7); use (2) **2nd Person pronoun (command): (more than 250 times). Examples:** Note (5 times); check (9); conclude (5); estimate- clarify, discuss (1); compare (2); assert (3); justify your answer (12); prove (1); talk (1); find out (72); arrange (20); complete (3); draw (25); solve (62); connect (7); calculate (33); define (14); write (20); project (v) (7); create (4); use (4); write (9); represent graphically (8); fold (1); express (1); check (9); bring (7); fill (2); cut (2); localize (1).
Connotations, using party and certainty: very widely used.	240	***** I name the number of pieces I can form from the four basic points. * If two lines intersected and one of the intersection points' angles was right angle, then all the other three angles are right angles and the two lines are perpendicular. * The two straight lines that never intersect are parallel. * In a triangle in which one of its angles is obtuse, what would be the measurement of the remaining two angles?

power was for the author (the book). Still, the pedagogical relation between the learner and the author appeared at low degree.

The actor characteristic

In general, it seemed that the second person pronouns dominated the mathematics text, but these pronouns tended to implement the order. **Table 6** illustrates the two indicators of the actor characteristic; **Table 6** presents examples of indicators.

Control characteristic

Generally, the basic power was clearly the author's (the book), due to the dominance of the second person pronouns over the mathematics text, as compared with the first person pronoun (I). The pronouns the book used tended to make students implement orders rather than to think. As for the connotations of the modality for the control and certainty, they were widely used. **Table 7** shows examples of the two indicators of the control property; **Table 7** presents examples of indicators.

Formality

In general, there is a relationship between the author and the learner, due to the use of the first person pronoun (we) to a medium extent (36%). The passive voice was "much 64%" used, especially when it comes to generalizations. Finally, it seems that the nature of the text is specialized because of the too frequent use of the specialized mathematics symbols and vocabulary. **Table 8** shows the indicators of the three characteristics of formality; **Table 8** presents examples of indicators.

Table 8. Indicators of Control characteristic, Frequency and Examples

Indicator	Frequency	Examples
- There is a relation between the author and the student through the use of the first person pronoun (we) to a medium extent	82	Use of the pronoun "We": (We): call (8 times); say (5); can (5), note (twice); it leads us to; shall know; put; discover; take out; described; conclude; define; use; find out … etc.
- Passive voice was used to a wide extent, especially in generalizations.	145	• Active voice (actor-subject is a mathematics object) (20) • Passive voice (20). • Shift to nominalization (105)

These results of the characteristic indicators of the language expressing the mathematics learner, represented by the eighth grade mathematics schoolbook, show that the learner was given a role; still, to a little extent. The learner's role begins decreasing so that the book becomes the basic role through the increase of second person pronouns. Nonetheless, the student still builds the mathematics thoughts through implementing the orders required by the book, to achieve the mathematics generalizations.

In the few times when the main role was given to the learner, they also tended to implement the orders. The student is assigned the task of thinking using the second person pronouns. However, what changes is the nature of the verbs used, which are more likely to think than to implement orders. The researcher noticed that they are, apparently, more likely tend to thinking, but they are only present in solved examples; thence they are used in exercises that simulate the solved examples through the lesson presentation. This eliminates the thinking process, and pushes the student to repeat the solution method already used in the solved example. This also appeared in the analysis of Alshwaikh (2015), when he indicated that the exercises come after presenting the theory and submitting its solved examples. Thus, the exercises appear as if asking the students to recur the way the examples were solved, with the same words, although the question uses the word "prove", which is among the indicators that introduce the students as thinkers.

As for the control, it was only the schoolbooks', which appeared using pronouns, so that the first person pronoun (I) disappears, and (We) is used at a low degree. As for the using party, and certainty connotations, which are the second indicator of the control characteristics, they were very widely used in the eighth grade mathematics book. This book is characterized by too many and long lessons, and the frequent use of the first person pronoun (We), particularly in the second unit, in a manner that decreases the formality between the author and learner. On the other hand, use of the passive voice is too much increasing, as the nature of the text is "very tightly specialized". The formal relation may create barriers between the students and mathematics learning, who may see mathematics as a very special and strange universe. As such, they fear taking it because it is a difficult, complicated subject, full of specialized mathematics vocabularies and symbols, which cause disturbance for them. This result was also found in (Dossey, 1992; Morgan, 1996).

The results of the three characteristics of the discourse show that they tend to display a negative image of the mathematics learner as a mere scribbler. Giving the learner the basic role in the learning process indicates the learner's engagement in the mathematics action (Morgan, 1996, 2006); and that students have an active role in building the mathematics ideas, which is in compliance of the modern theories (structural and social structure) (Oliver, 1989). But it seems that students are not sufficiently active in the learning process, due to the use of indicators that describe them as mere scribblers; they have not space for free thinking; not given different choices to follow the way they see fit to approach mathematics facts. All the book lessons have one planned, drawn way by the book designers, and students should only follow this way to reach the mathematics facts. Lack of the learners' presence as thinkers indicates that the traditional, behavioral view of the students still exists. In this way, learners are only vessels wherein information is placed and stored, which will be retrieved at the exam time, which implies that learners have no role in building their own knowledge (Moschkovich, 2010).

CONCLUSIONS

It is clear through the study that mathematics appears at first as the subject stems from the human existence, Then this presence begins to decrease gradually, The stereotype of mathematics is replaced by absolute and symbolic, And it is a self-world, based on independent groups of relationships between mathematical objects. As it can be seen that the mathematics learner appear as passive recipients and just executing orders, Based on the results of the study, attention should be given to the definition Teachers should identify the different views about the mathematics image and mathematics learner's image, which enables them make students perceive this different image, even if the schoolbooks were not developed.

Since the analysis results showed that the mathematics books tend to present students as mere scribblers, teachers could be qualified to produce students able to think. This also provides students chances to be mathematicians who build mathematics ideas through encouraging them to engage in written activities and tasks.

Such activities do not focus on making students display their comprehension of the mathematics content only, but also of the thinking mechanism, they performed to show how did they approach the solution.

Developing the mathematics schoolbooks in a manner that shows the other image of mathematics as a human activity, and students as engaged in the learning process actively, so they will be given space for thinking more than being mere "scribblers", and followers of predetermined steps.

Conducting research works that focus on exploring the relationship between the language and their uses choices with the students' believes. There are other forms of communication including images and gestures, which are also of a vital role that can not be ignored or minimized in mathematics learning-teaching. This calls the necessity to explore these two aspects and find out their impact on mathematics and mathematics learner's image.

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Verbal Interaction Types in Science Inquiry Activities by Group Size

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ABSTRACT

The extant research on linguistic interactions in scientific inquiry has focused solely on the quantitative analysis of the verbal interactions among the students and poses limitations to investigating the internal characteristics of such interactions. This paper presents connections among the members of a student group based on its interaction patterns and size to find out the ideal size of a student group in order to allow the student to have optimum participate in the group interactions. In this regard, this study sought to analyze the patterns of verbal interaction that occur during inquiry activities using social network analysis (SNA). 144 first-grade middle school students in science class in South Korea were participated and organized into small groups with three to six members each, and the language network of the 32 small groups that were thus formed was analyzed. The conclusions of this study are as follows: (1) the groups of students formed in school for learning activities should consist of only three members each to avoid alienation among the members, and (2) in student groups with four or more members each, there are many participation-type interaction structures that can be used if there is a group leader. The interactions among students within small learning groups can allow them to fully understand other students' opinions which may otherwise not be very clear, and to solve a problem after considering all the opinions students have expressed, and support the effective learning by positive interactions among the members.

Keywords: group size, social network analysis, verbal interaction types

INTRODUCTION

Interaction is a necessary activity for human beings to connect ideas and knowledge building. The view is also supported by Vygotsky's social constructivism that suggests knowledge formed and acquired through active interactions with members of society, such as one's colleagues or adults, rather than through individual cognitive activities (Vygotsky, 1978). Through active interaction, students form knowledge and construct new ways of understanding in the process of exchanging opinions and negotiating with others (Howe, Rodgers, & Tolmie, 1990; Palincsar, 1998). In addition, interaction not only helps reduce students' fear of expressing their opinions to induce active participation and helps develop their logical reasoning abilities but also contributes to knowledge formation and problem-solving (Lumpe & Staver, 1995). Therefore, it can be said that interaction plays an important role in learning.

In addition, the interaction among the students constituting small groups formed for learning activities has advantages in that it allows the students to understand the otherwise unclear opinions of other students fully and to solve a problem after considering all the opinions expressed (Alexopoulou & Driver, 1996; Johnson & Johnson, 1985; Palincsar, 1998). Therefore, to learn effectively, it is essential to encourage active and positive interactions among the members of small student groups formed for learning activities.

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Contribution of this paper to the literature

- Provides linguistic interactions in scientific inquiry through the quantitative analysis of students' verbal interaction.
- Analyzes the patterns of verbal interaction that occur during inquiry activities using social network analysis (SNA).
- The findings of this study indicate that the groups of students formed in school for learning activities should consist of only three members each to avoid alienation among the members.
- In student groups with four or more members each, there are many participation-type interaction structures that can be used if there is a group leader.
- The interactions among students within small learning groups can allow them to fully understand other students' opinions.

In particular, inquiry activities, which are mainly used as methods of scientific learning, are processes for solving problems through suggestions from the members of the small groups formed for such inquiry activities; through the processes of acceptance, criticism, revision, and consultation among them; and through the learner-centered activities that they carry out, in which interactions are important (Driver, Newton, & Osborne, 2000). Inquiry activities provide the members of the small student groups formed for such purposes with opportunities to actively and critically exchange various assertions and opinions among themselves; their importance to inquiry learning, which overcomes challenges through collaborative interactions within the small student groups, has been emphasized in the science curriculum (Ministry of Education, Science, and Technology, 2010). According to one report, the fewer the interactions among the members of small student groups formed for scientific inquiry activities, the more difficult it is for the students to improve their understanding of the subject matter being taken up (Nattiv, 1994). In other words, the quality of interaction among group members might be a strong predictor of students' learning outcomes in scientific inquiry activities (Cheng, Lam, & Chan, 2008). Therefore, it can be said that the formation of small student groups is an important factor that should be considered when planning effective scientific inquiry activities, as it can allow active interactions to occur in such activities.

As the formation of efficient small student groups is important, there has been vigorous debate on how students should be grouped in learning (Cheng, Lam, & Chan, 2008; Kyza, Constatinou, & Spanoudis, 2011). Above all, various methods have been proposed to determine the appropriate size of a student group or the number of people who should constitute the group (Jang & Kim, 2017). In this context, Brown (1993) said that the student groups formed for a scientific inquiry activity should consist of fewer than six members each, and Johnson and Johnson (1989) claimed that such groups should consist of two to six members each, but that a four-member group would be most effective. Slavin (1987) also showed that a two-, three-, and four-member students group would help improve the students' ability to conduct. The research results also showed that a three- or four-member student group would help improve the students' ability to conduct inquiries (Kim, 2009; Kim & Kim, 2004). Most studies on the appropriate size of a student group formed for a scientific inquiry activity, however, dealt mainly with the relationship between the size of the group and the members' ability to conduct inquiries, and there is a dearth of research on the interaction patterns among the group members. Therefore, it is important to analyze the verbal interaction patterns among the group members according to the size of the group because this may provide additional clarity about why the size of a student group formed for an inquiry activity can influence the students' ability to conduct inquiries.

The interactions among students are mostly carried out using language, which is the outcome of personal thinking (Ernest, 1994; Hennessy, 1993). Many studies have been conducted to analyze the verbal interactions among the members of small student groups carrying out scientific inquiry activities (Anderson et al., 2001). However, most of these studies were based on the linguistic analysis framework and were limited to a quantitative analysis of the verbal interactions among the students, thereby posing limitations to identifying the internal characteristics of the interactions among the members of the small student groups in the inquiry process (Kim et al., 2017).

Recently, social network analysis (SNA) has been used to overcome the limitations above (Hansen, Shneiderman, & Smith, 2009). SNA is a technology that quantitatively analyzes the interactions that occur among the members of a group when communication or information exchange takes place. It visualizes the interaction patterns of the group through a picture consisting of dots and lines (Derfel & Sonnaughton, 2009; Storberg-Walker & Gubbins, 2007).

It can be seen how each member of a small group is connected to the other members, or how all the members are related to one another. It helps in intuitively determining if any member plays a central role in the group or if there is an alienated member, and who the central or alienated member is (Kim et al., 2017). Kim and Kim (2015) analyzed the types of interactions occurring in inquiry activities through social network analysis. The study presented eight types of interactions shown by gifted students. Another study by Kim (2018) with general students presented 13 types of interactions including seven types of interactions with alienated students and six types of interactions with all students. The previous study analyzed only small groups of five students and found that some students could not join the interaction. Based on this result, they suggested the research questions on the sample size which allows all students to join the interaction (Kim & Kim, 2015). Therefore, this study sought to analyze the characteristics of the interaction patterns of small groups of general students by adjusting the size of such groups. This would help determine the kind of connections present among the members of a small student group based on its interaction patterns and size and would indicate the ideal size of a small student group in order to allow the student members to participate meaningfully in the group interactions.

METHODS

Participants

The participants of this study were 180 first-grade (male 52.1% and female 47.9%) students in middle schools located in Daegu City in South Korea. The school in this study have a relatively higher academic achievement than other schools, and the students are highly motivated to study.

They were split up into small groups with three to four members each (70 students in the first grade of D Middle School) and small groups with five to six members each (110 students in the first grade of K Middle School). Of the 40 small groups, 8 cases were excluded from the analysis because the students turned off the recorder in the middle of the class or had trouble hearing due to the excessively loud background noise, or because the recorder's sound quality was not good enough for analysis purposes. Thus, 32 groups (144 people) were selected as analysis subjects. The small groups were made to consist of three to six members each based on the studies of Brown (1993) and Johnson and Johnson (1989). As these studies suggested that two- to six-member groups are expected to have only one type of interaction in a straight line, such groups were excluded from this study.

Procedures

The class was conducted in the general circumstances related to science classes in South Korea, in which the teacher lets the students identify the learning content, presents the learning goals to the students, and creates a classroom atmosphere that is conducive to learning. After the teacher explains the basic concepts to be learned and the experimental procedures and methods to be employed, the students proceed with the inquiry activities. In this study, the process that was employed in the students' inquiry activities was recorded for analysis. The content of the inquiry activities is as follows. The three- to four-member groups measured the specific heat capacities of metals like copper, iron, and aluminum through an experiment, and the five- to six-member groups performed the experiment on the photosynthesis of hydrilla.

Data *Collection* and Analysis

Data collection

In class, either a voice recorder or a cell phone with a recording function was used for recording purposes. Either one was set up at the center of the experimental desk for each small group to record its experimental inquiry activity. Camcorders were installed in front of the classroom, at an angle that could capture images of two groups or more per camcorder, so that the speaker could be identified if he/she could not be distinguished by voice. Before each class, the teacher informed the students that the recorded content would be used only for research purposes and then asked for the students' consent and cooperation.

To create a database containing information about each material, the recorded media files were transferred from a recorder or camcorder to a computer and then transcribed using Microsoft Word. However, cases in which the students' voices were not recorded because the students covered the recorder, or in which the students' voices could not be recognized due to the confused babble or background noise, were excluded from the transcription.

To evaluate the quality of our data, we assessed the validity of task used to measure students' interaction with three content experts in science education. Reliability of the measures was assessed using inter-rater reliability where the observer (in-service teacher) and experts as reliable rater agree with the official rating of performance which based on the frequency of utterances and responses made in the verbal interactions that occurred. By using Cohen's kappa measures, the result was exceeded by 0.8 (Cohen's kappa > 0.8).

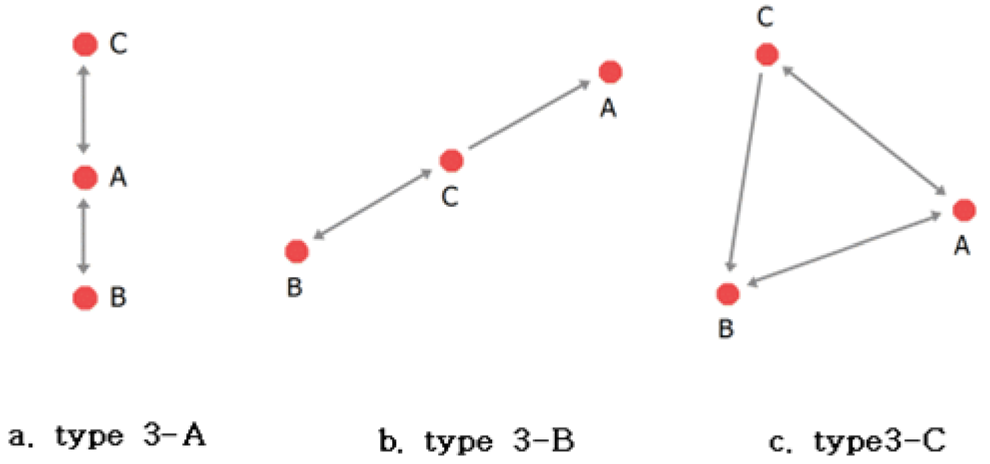

Figure 1. Interaction patterns that appeared in the three-member groups

Data analysis

The frequency (i.e., number of times of occurrence) of verbal interactions was measured based on the transcribed content. Only the verbal interactions among the members of each small group were recognized as interactions, and the soliloquies of individuals and the interactions with the teacher or with the members of another small group were excluded. The frequency of interaction was organized as the data of a matrix consisting of "person-to-person interaction."

To exhibit the structure of the interactions among the members of each small group, the data were converted to binary matrix data. The cut-off value that was used to convert the data to binary matrix data was based on the average interaction frequency in the previous research (Kim et al., 2017).

The converted binary matrix data were visualized using NetMiner 4, a software tool for SNA. NetMiner 4 identifies the utterance-response relationship of binary data and graphically depicts the structure of the interactions within small groups. In this study, the network data were visualized using the Spring-Eades algorithm, which can be utilized in NetMiner 4. The Spring-Eades algorithm has the advantage of being able to easily identify the interaction structure because it allows the nodes present in the matrix to be located close to one another and clarifies the relational structure of each node from a relative viewpoint (Choi & Choi, 2010).

RESULTS

Three-*Member* Groups

Figure 1 shows the interaction patterns that appeared in the three-member groups. The most common interaction pattern type in the three-member groups was that showing a linear structure rather than a geometric figure (**Figure 1a**, **b**). This type appeared in six of the eight three-member groups, accounting for 75% of the total. In the linear-structure interaction pattern type, one student actively interacts with the two other students in a meaningful way, but the two other students do not show frequency of meaningful interaction with each other and are thus connected on both sides, while one student is in the center.

For the linear structures, all the four groups showed type 3A, in which the student playing a central role has a two-way (utterance-response) interaction with the other students. That is, in most cases, one leader led the inquiry activities while maintaining proper two-way interaction with the two other students. A look at the interactions in the small groups that showed the type-3A linear structure will indicate that student A alternately interacted with students B and C, and led the inquiry activity (Case 1).

(Case 1)

A: 200 g temperature change... originally at 27 degrees.

C: Hey, what is the temperature?

A: OO, the temperature is zero degrees?

B: Huh?

A: Is it not zero?

A: 200 g…

B: Yeah, 0.2 Kg.

A: Hey, the temperature of this metal is 37 degrees? At the beginning….

C: 33 degrees. The first temperature of the metal is 33 degrees.

A: 3.3.

C: Is it 3.3?

A: Yes.

C: 3.3?

The two remaining groups out of the six groups that had a linear-structure interaction pattern type showed a structure in which a student playing a central role performs two-way (utterance-response) interaction with one student but shows one-way (utterance) interaction with the remaining student (**Figure 1b**). This structure appeared when the student forming a one-way interaction with the central student was passive or distracted in the course of the inquiry activity.

A look at the interactions in the small groups with a type-3B interaction structure will show that although student C mentioned an irrelevant issue, the remaining two students began to concentrate on the inquiry activity again when student C also gave responses related to the inquiry activity. This suggests that if two members of a three-member group concentrate on the inquiry activity, they are carrying out, the remaining member cannot continue uttering irrelevant things by him/herself alone and will thus be forced to participate again in the inquiry activity. Therefore, it can be confirmed that even if one student is not active in an inquiry activity, a three-member group is likely to form a participation-type interaction structure rather than an alienation-type one as another student forms a response relationship with the student leader.

Another type of interaction structure appeared in two of the eight three-member groups, showing a triangle structure, where all the members of the group interacted with one another (**Figure 1c**). This type of interaction structure can be seen when all the members of a three-member group actively participate in the inquiry activity they are undertaking. This type also shows that the three students form interaction relationships with similar frequencies regarding both the frequency and directionality of their statements and that two students respond to one student's remarks at the same time.

Type 3-4 and 3-B showed a similar structure; but two were differentiated based on whether the center student has the two way interaction with other two students. In type 3-B, there were students who were not active in inquiry activities. The type of verbal interaction structure that appeared in the three-member groups was the participation type, in which there were no alienated members. This result is in line with those of studies (Kim & Kim, 2004; Lee, 1995) that suggested that a small student group should be made to consist of only three members so that all the members could actively participate in the inquiry activity they were undertaking through a reduction of the number of alienated members.

It was also noticed that in five of the six groups that showed a leader-centered linear interaction structure, the student who acted as the leader was the first student to make an inquiry-related comment after the start of the inquiry activity. Although it was likely that the student at the center of the interaction made more comments than the other students and delivered the initial remarks, it was confirmed that taking the initiative in inquiry activities also positively affects the interactions in the group.

Four-Member Groups

Figure 2 shows the interaction patterns that appear in four-member groups. The most common type of interaction pattern among four-member groups shows a radial or similar form as one student takes the lead and the three other students interact with one another (**Figure 2a**, **b**). This type was found in four groups, accounting for 50% of the eight four-member groups. It was also confirmed that in this interaction pattern type, even if some students are passive in the inquiry activity, they have meaningful interactions with the student who acts as the leader, forming a participation-type interaction structure. Concerning the degree of centrality, the student positioned at the center has the highest value (3). Three of the four groups in this study that showed this interaction structure type had a complete radial structure (type 4A), with no interactions among the students except with the leader, and the remaining group exhibited a structure with a small triangle on one side (type 4B) as two of the three students (not the leader) interacted with each other. The interaction patterns of the small groups with the type-4B interaction structure also showed mostly similar aspects. Despite interactions between the two students who were not the leader, the leader was found to take the lead in the inquiry activity in the whole structure. This interaction structure type is different from type 4A as some of the students constantly interact in pairs to create a meaningful

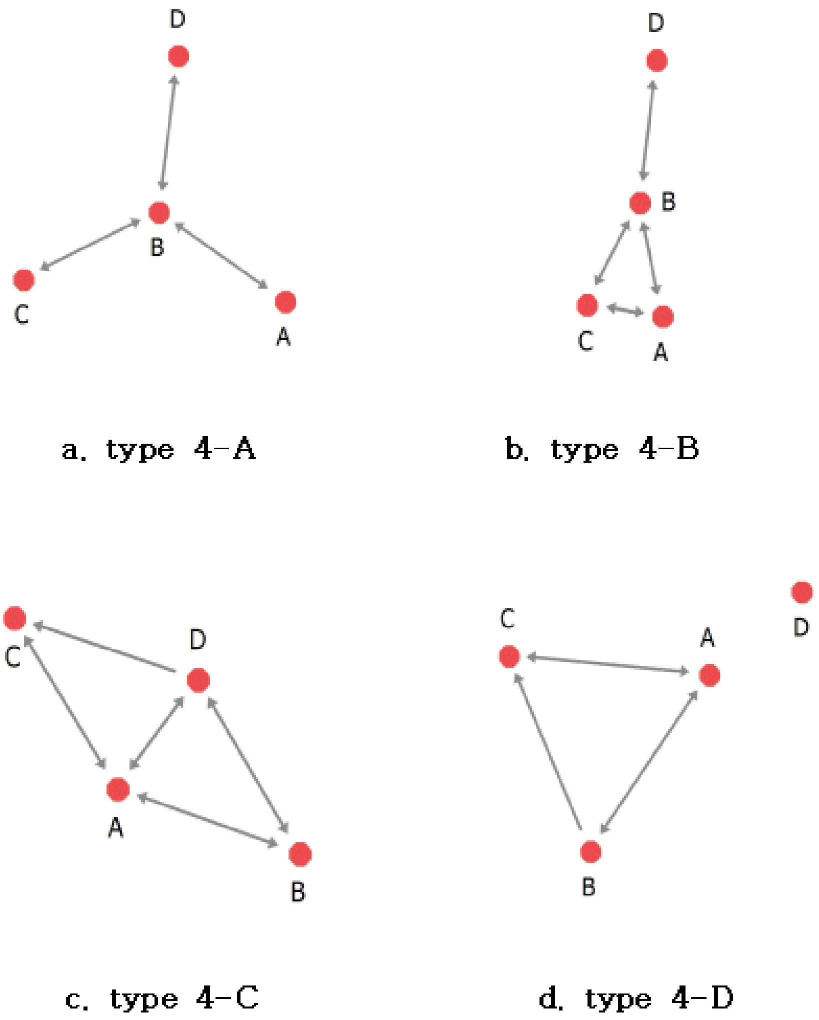

Figure 2. Interaction patterns that appear in four-member groups

interaction relationship. A look at the interaction example in Case 2 will reveal that student B, who played a central role in the inquiry activity, led the activity by helping the other members write their reports and informing them of matters requiring their attention, and the other students asked questions about the information on the inquiry activity that the leader gave them and about how to write a report. This can be said to be an example of a typical leader-centered interaction pattern.

(Case 2)

C: Is it 0.07?

A: Yeah, kilogram.

B: OO, it is 0.07 kilograms here, not there.

C: What?

B: Don't use a ball-point, I told you.

C: Why?

B: Can you ask the teacher why?

C: Why?

D: Hey, where should I write it?

B: Here, 0.07.

D: Mass. 0.07.

Two of the eight four-member groups in this study exhibited a diamond-shaped interaction structure as most of the students participated in the interaction in a balanced way. Although not all the students demonstrated interaction relationships, interaction relationships were formed between two students, except for one section (**Figure 2c**). Given that students D and A had the same degree of centrality (3), they led the inquiry activity, and the two other students actively followed; that is, two students acted as leaders.

Concerning the interaction patterns observed in the four-member groups, two groups exhibited a triangular-branched structure as one student was isolated (did not interact with anyone) and the three other students participated in the interaction on the whole. This type of interaction structure is formed when one student is alienated from the verbal interactions in the group due to his/her lack of linguistic participation in the inquiry activity compared with the three other students, and the rest of the students all form interaction relationships (**Figure 2d**).

In small groups of four students, there was a difference of types depending on whether there were students with centrality. The results showed that one student with centrality was Type 4-A and Type 4-B, two students with centrality was Type 4-C, and any student without centrality was Type 4-D. The difference between type 4-A and type 4-B was the presence of interaction between students. Types 4-B and 4-C were different depending on how many students with centrality interacted with each other.

In relation to the verbal interaction structures that appeared in the four-member groups, six of the eight groups (75%) showed a participation-type structure, whereas the remaining two groups (25%) showed an alienation-type structure. Unlike in the three-member groups, the alienation-type interaction structure began to appear in the four-member groups. In contrast to the findings of Kang, Kim, and Noh (2000), however, which showed that the rate of the cases in which all the four students participated in the inquiry activity was about 10%, a high proportion of the small groups (75%) exhibited a participation-type interaction structure, in which all the members participated in the inquiry activity. In this regard, further analysis is needed to investigate the factors that determine the participation level of the small-group members in their inquiry activity.

Five-Member Groups

There were six types of interaction structures in the five-member groups in this study, as shown in **Figure 3**. The first type of interaction was a radial-shaped structure in which one student was alienated and the four other students engaged in interactions around the leader (**Figure 3a**). This type of interaction structure appeared in one group and showed one student having minimal verbal interactions with the others and thus being alienated, and the remaining four students building interaction relationships with one another around the student playing a central role.

The interaction case of a small group with the type-5A interaction structure showed that student B understood the inquiry process and told the others what to do next and that the other students got involved in the inquiry activity by relying on student B for information. As a result, student B came to build the interaction structure as a leader, with the highest degree of centrality (3) in this case. The next interaction structure type is similar to type 5A but is a structure that forms a small triangle as the leader, and the other students participate in the interactions (**Figure 3b**). The conversations of the small groups that exhibited the type-5B interaction structure revealed that student A, positioned at the center of the interaction structure, coordinated and directed the overall progress of the interaction activity, as was the case for student B in the small groups that exhibited the type-5A interaction structure. Students B and E received confirmation from student A about what they should do in the inquiry process, and student C showed frequent interactions with student E while following the directions of student A. Student D. However, made comments unrelated to the inquiry or engaged in too much self-talk, which made it difficult for him/her to respond, thus failing to form an interaction relationship.

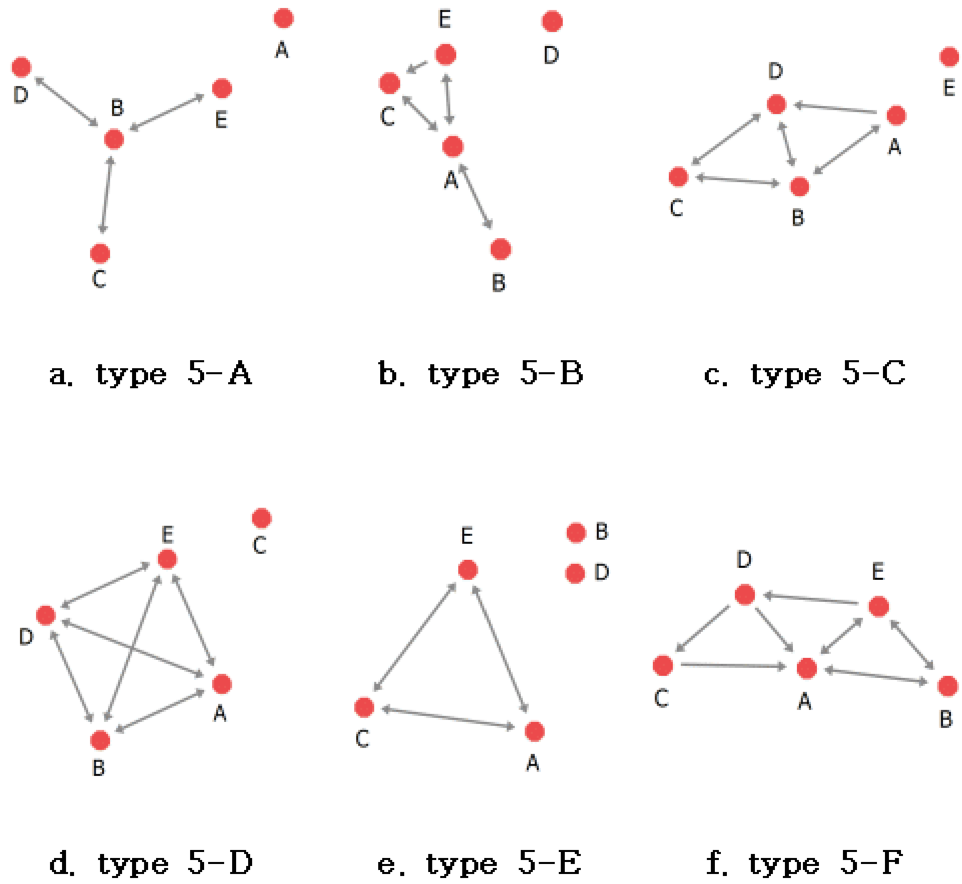

Figure 3. Interaction patterns that appear in five-member groups

In addition, there was a structure that showed overall interactions among the students. In this case, although one student was alienated, the remaining four students did not fully rely on the leader and interacted with one another in the course of their inquiry activity. This type of interaction occurs when one student fails to form an interaction relationship and is thus alienated, but the rest of the students interact indiscriminately with one another to form interaction relationships without a specific degree of centrality (**Figure 3c**, **d**).

This type of interaction structure can again be divided into type 5C, in which the interaction structure that has four students participate in the interaction forms a diamond shape, and type 2D, which shows a square shape. The difference between these two structures is that in the case of a square-shaped structure, four students have the same status as they all have a degree of centrality of 3, whereas in a diamond-shaped structure, two students have a degree of centrality of 3, and two have a degree of centrality of 2. A look at the structure of type 5C can confirm that almost all the students in the groups that exhibited such interaction structure interacted with one another, but no interaction occurred between students A and C.

The interaction example in Case 3 of a small group exhibiting the type-5D interaction structure showed an aspect that makes it slightly different from the type-5C interaction structure. The most noticeable finding was that the frequency of asking questions or of carefully guessing word meanings was much higher than that of issuing claims or commands. Although student B showed a tendency to sort out and answer the other students' questions, the other students also actively interacted with one another. As a result, all the students except student C came to achieve two-way (utterance-response) interaction.

(Case 3)

A: Well, why is there a thermometer?

B: Temperature, the temperature rises…

D: Does it come out at this time?

B: It comes out even at this time.

E: Alas! Turn it off after the light is turned on… You have to do it when the light starts flashing…

C: Hey, did OO put this thing after cutting it?

B: Yes.

D: It seems that the temperature is gradually rising.

B: The temperature is going up.

A: It seems easy when two people are there.

B: Yeah, it seems very easy.

D: Do you think the temperature goes up?

E: Well…. it's cold.

B: It's still cold.

A: Hey, put it at this point, not there.

B: Is it different at this point?

D: Different? I think it will be.

A: Maybe we should direct the light towards a different point?

D: Huh? Is it all right to put the thermometer here?

One of the eight five-member groups showed two students alienated and the remaining three students interacting to form a triangular interaction structure (**Figure 3e**). The type-5E interaction structure appeared in 9 of the 31 groups in the research conducted on five-member groups (Kim et al., 2017). Such research claimed that this type of interaction structure appears when no members encourage the alienated members to participate in the inquiry activity or lead them in a positive direction (Kim & Kim, 2015).

While there were small groups with two alienated members, some small groups showed a participation-type interaction structure (**Figure 3f**). The interaction of these small groups exhibited a trapezoidal structure connected with three triangles. This interaction structure type is a kind of leader-centered structure in which all the other students interact with one another around student A; students B and E perform two-way interaction, and students E and D, and students D and C perform the one-way interaction. In addition, the leader usually plays a central role and takes the initiative in the inquiry activity. In this leader-centered structure, the students usually engage in two-way interaction or spoken (utterance) interaction with the other students even in the case of one-way interaction. Student A is playing a central role in this small group, however, was observed to have response interactions with students C and D.

Except for alienated students, the types 5-A, B, C and E were the same as those found in small groups of four. Type 5-D appeared only in a five-person group. This type of interaction occurred when four students interacted with each other and students interacted in two directions, such as types 5-E, 4-D, and 3-C.

The verbal interaction structures that appeared in the five-member groups in this study were all alienation-type interaction structures except for one group; as such, there were more alienation-type interactions than participation-type interactions in the five-member groups. It can be said that this is the same result as in the research that was conducted using the same method (Kim et al., 2017), which showed that the proportion of small groups that exhibited participation-type interaction structures was less than 30%.

In addition, the research conducted on scientifically gifted students (Kim et al., 2017), found that the structure in which two of the five students in the group were alienated accounted for 29% of the total, but only one group showed an interaction structure with two alienated members in the present study conducted on general students. The students' participation in the inquiry activity was also higher than that in the previous studies, starting with the data on the groups with four or more members. In this regard, further research is needed to investigate the probable causes of these differences, such as the external factors (e.g., the level and personality of the students) or the methods of analyzing the research results.

Six-*Member* Groups

The interaction structures that appeared in the six-member groups in this study can be largely divided into four types, as shown in **Figure 4**. In four of the eight six-member groups, accounting for 50%, one student was alienated, and the five other students demonstrated the interaction structure with the shape of a webbed foot (**Figure 4a**). The interaction structure among the five students (except for the alienated student) took the shape of a rhombus, whereas the diamond shape was connected with the alienated student, as in the type-5C interaction structure, an interaction structure type that appears in five-member groups. Regarding the degree of centrality, student D seems to have played the most central role, interacting with the four other students, while student C interacted with three students and led the inquiry activity together with student D.

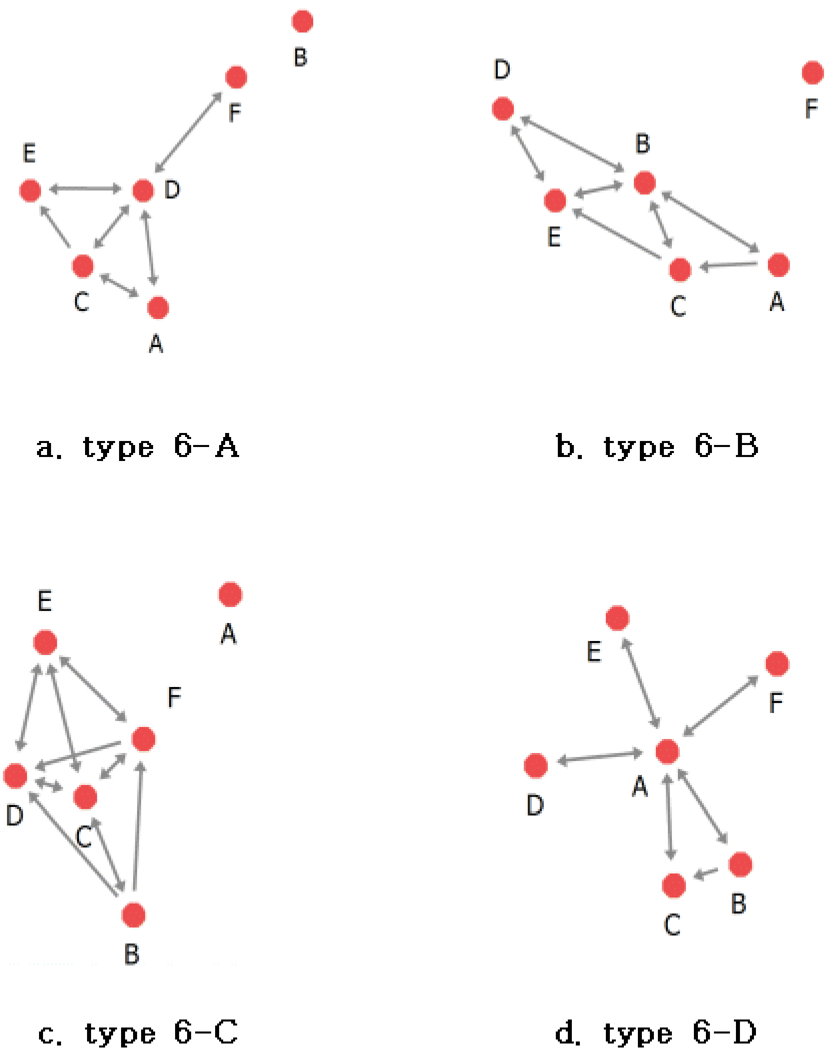

Figure 4. Interaction patterns that appear in six-member groups

Type 6B appeared in two small groups, and it showed a trapezoidal pattern of five students interacting with one another, with one alienated student (**Figure 4b**). There was a leader-centered structure in which student B, who had a degree of centrality of 4, played a central role, except for one alienated student, as in the trapezoidal pattern in the interaction structure type found in the five-member groups, and all the members except student F participated in the inquiry activity around student B.

A look at the interaction example in Case 4 of a small group that showed the same interaction pattern as type 6B will reveal that student F was alienated because he/she did not participate in the verbal interactions and instead concentrated on writing the group report, as the group's secretary. Student B looked confident and took the leadership role as a centrally positioned member of the group in terms of the interactions, and students C and E, who interacted with three students, also provided many comments and directed the other students to proceed with the inquiry activity as part of the efforts for direct involvement (Case 4).

(Case 4)

B: One

E: Put it down.

D: I'll put it down.

F: I'll be using it.

C: All right. I'll do it. Give it to me. Do not start yet.

B: Hey, no…. I'll do it, I….

C: No, I'll do it. No, stop it!

B: Towards the bottom. Completely downward

C: Huh? I need to turn the stem so that you can see it. I have to turn it so you can see....

B: Just coming up?

E: Just hold it from above. Continue.

B: Just press it down. Ask OO to continue to press it down. We can't help you with it.

D: Are you guys ready?

B: Okay. Ready. Go.

A: Bubbles are rising?

B: One, two, alas! We have to do it again.

A: Why?

B: Something went wrong. I'm getting confused….

D: Are you ready?

A: No.

C: Start!

B: No, I can't see….

A: Here it is.

B: How many?

The third type of interaction in six-member groups was observed in one group, and it has a structure that shows a flat shape with straight sides as one student was alienated, and there were interactions among all the other students, except for the interaction between students E and B (**Figure 4c**). In this structure, three students have the same degree of centrality (4), and two other students (except alienated student A) also show active interactions. In addition, it can be predicted that as student D shows one-way interaction as a response relationship with two students, he/she will form an interaction relationship by responding to the opinions of others rather than presenting his/her opinions first.

The interaction structure to be introduced last shows an extreme one-person-centered interaction structure. More specifically, it has a radial structure in which student A, playing a central role, interacts with all the students, but there are almost no interactions among the other students, except for student A (**Figure 6d**).

In the small group of six students, several central students showed up, indicating that the leader of the group was unclear. In the absence of a leader, it is reported that interaction and cooperation between members are limited (Lee & Chun, 2017) Therefore, if a leader was unclear in six small groups, it was difficult to direct members, provide information, or prioritize activities, etc. Therefore, it is considered inappropriate to comprise six members for the group's activities.

With respect to the interaction structures in the six-member groups, the alienation type appeared in seven groups, The ratio of participation to alienation-type interaction structures was the same as the ratio that appeared in the five-member groups. As one-on-one verbal interactions are most prevalent in small groups, the number of bystanders for a single verbal interaction increases as the group becomes bigger. In addition, although the same number of comments is made, the average number of interactions decreases as the small group becomes bigger, with an increasing number of members. For this reason, a lower interaction frequency, which is not significant in terms of the interaction structures, was recognized as an interaction relationship due to a low average interaction frequency or a low cut-off value.

In addition, the percentage of not being responded to was higher in the six-member groups than in the groups with fewer than five members. This is because as the group becomes bigger, the responsibility for answering the questions raised or responding to the opinions expressed is distributed. This result supports the results of the research conducted by Johnson and Johnson (1989), who pointed out the risk of free-riding and the difficulty of reaching a group consensus in small groups with more than five members.

DISCUSSION

So far, the verbal interaction structures that appeared in the small groups in this study according to the size of the group were analyzed. There were no alienated students in the three-member groups, and alienated students began to appear in the four-member groups. In the case of the four-member groups, there were alienated students in two of the eight groups; further, seven of the eight five- to six-member groups exhibited an alienation-type interaction structure. These results are consistent with those of the previous research that suggested that there would be free riders in groups with more than four members (Johnson & Johnson, 1989; Park, 2006).

The diversity of verbal interaction structures among middle school students leads to the expectation that various changes will occur during the process of inquiry activities. This in turn suggests that the interaction structures may vary depending on the strategies used and that teachers can encourage all the students constituting small groups to participate in the interactions within such groups. That is, the strategy of forming small groups will have to be used for general students who show a high percentage of participation in structured-learning-type interactions (Lee, Kang, & Huh, 2009; Kim, 2006). It is also expected that studies will be conducted on teacher feedback from inquiry activities and on changes in interaction structures after teacher intervention or feedback. The study will analyze the impact of teacher intervention on students' interactions in autonomous inquiry classroom.

As alienated students appear in groups with more than four members, both divisions of roles and assignment of responsibilities are required in such groups (Kim & Kim, 2004). This strategy has proven to be very effective in encouraging the students to actively participate in experiments and classes, and in improving their scientific inquiry abilities.

There was a significant difference in the percentage of participation- and alienation-type interactions according to the presence or absence of a leader in the four-member groups. Specifically, four groups exhibited participation-type interactions in the presence of a leader, but only two groups showed participation-type interactions in four group activities without a leader. Therefore, there is a need to designate a group leader when forming groups with more than four members each so that no student is alienated.

Therefore, it is expected that research will be added to ensure that the interaction type is stable according to the presence of the leader. It also suggests research on how changes in interaction types and students who are alienated participate in interactions when they are assigned roles.

On the other hand, a leader was positioned at the center of the verbal interaction structure through active utterances and responses to the current situation. This type of leader consistently checked the current situation, asked questions about the subsequent activities, and answered the questions raised by the other group members while exploring information. This student generally responded to the other students' remarks before the other group members did, resulting in poor verbal interactions among the other members.

Research has so far been limited to analyzing the types of interactions. In future studies, it is expected that studies will be conducted on the types of interactions by individual characteristics such as gender, the type of leader's communication style. It will also be necessary to add research into whether there is a difference in the ability to explore science depending on the type of interaction.

CONCLUSIONS AND IMPLICATIONS

Conclusions

This study was designed to analyze the interaction structures that occur in a small group according to the size of the group, by applying such interaction structures to scientific inquiry activities in general middle schools. The interaction structures were visualized based on the frequency of utterances and responses made in the verbal interactions that occurred in the course of the scientific inquiry activities. The relationships of the members of small groups were analyzed by identifying the characteristics of the interaction structures. Students' verbal interaction was found different depends on group size. The optimal number of members of a small group is three, to avoid alienated members. In the three-member groups in this study, the participation-type interaction structure appeared regardless of whether or not there was a leader actively leading the inquiry activity in the group and it minimize the number of people alienated from the group's inquiry activities. It also found that most of the student groups consisting of more than five members in this study were found to have alienated members. Furthermore, the result of study also found the ratio of proportion's participation- and alienation-type interaction structures also depend on presence or absence of a leader.

Implications

Three members are required for a group to avoid alienated members in a scientific inquiry activity. In this study, when a group consisted of three members, all three participated in the verbal interactions during the inquiry activity, regardless of the presence of a leader with centrality within the group. Therefore, the formation of three-member groups is expected to increase the level of participation in scientific inquiry activities.

In this study, when there were more than five students in a group, there were alienated members who did not participate in the inquiry activity, which suggests that fewer than four members are required for a group to conduct a scientific inquiry activity. Most of the small groups with more than five members in this study exhibited an alienation-type interaction structure as they could not attend to the members who did not participate in their group interactions despite a leader and interaction centrality. This is because, as the size of a group increases, the responsibility for answering the questions raised or responding to the opinions expressed is distributed, and thus, the likelihood that active interactions will occur is lowered.

The formation of four-member groups is suggested herein so that the members who can lead the inquiry activity in each group can be positioned to reduce the number of alienated students who will not participate in the inquiry activity. In this study, when a leader actively led the inquiry activity in a four-member group, he/she interacted with all three other members and thus helped create a participation-type interaction structure. In the absence of a leader, alienated members appeared in 50% of the small groups in this study. Therefore, it is expected that if an active and positive student who can play a leading role is positioned in each group, the number of alienated members in inquiry activities will be minimized.

ACKNOWLEDGEMENT

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Exploring Students' Understanding of Integration by Parts: A Combined Use of APOS and OSA

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ABSTRACT

Our goal in this paper is to study students' understanding of integration by parts based on two theories, APOS and OSA. We make an epistemic configuration (EC) of primary objects that a student activate for solving tasks in relation to the integration by parts, and then we design a genetic decomposition (GD) of mental constructions that he/she might need to learn the integration by parts. We then describe the EC and GD in terms of the levels of development of Schema (i.e., intra, inter and trans). Three tasks in a semi-structured interview were used to explore twenty three first-year students' understanding of integration by parts and classify their schemas. Results showed that students had difficulties in integration by parts, especially in using this technique to obtain a simpler integral than the one they started with. Using APOS and OSA gave us a clear insight about students' difficulties and helped us to better describe students' understanding of integration by parts.

Keywords: student's understanding, integration by parts, mental constructions, primary objects, schema

INTRODUCTION

The integral is a key tool in calculus for defining and calculating many important quantities, such as areas, volumes, lengths of curved paths, probabilities, averages, energy consumption, population predictions, forces on a dam, work, the weights of various objects and consumer surplus, among many others (Thomas, Weir, Hass & Giordano, 2010). As with the derivative, the definite integral also arises as a limit. By considering the rate of change of the area under a graph, Calculus proves that definite integrals are connected to anti-derivatives, a connection that gives one of the most important relationships in calculus. The Fundamental Theorem of Calculus (FTC) relates the integral to the derivative, and it greatly simplifies the solution of many problems. The FTC enables one to compute areas and integrals very easily without having to compute them as limits of sums. Because of the FTC, one can integrate a function if one knows an anti-derivative, that is, an indefinite integral (Anton, Bivens, & Davis, 2010).

Some research studies reported that integration is more challenging than differentiation for students (Kiat, 2005; Mahir, 2009; Orton, 1983; Thompson, 1994). These researchers explained that in finding the derivative of a function it is obvious which differentiation formula we should apply. But it may not be obvious which technique students should use to integrate a given function. Integration is not as straightforward as differentiation; there are no rules that absolutely guarantee obtaining an indefinite integral of a function (Pino-Fan, Font, Gordillo, Larios, & Breda, 2017).

Radmehr and Drake (2017) explored students' mathematical performance, metacognitive skills and metacognitive experiences in relation to the integral questions by interviewing students. Their findings showed that several students had difficulty solving questions related to the FTC and that students' metacognitive skills and experiences could be further developed. Pino-Fan et al. (2017) presented the results of a questionnaire designed to evaluate the understanding that civil engineering students have of integrals. The questionnaire was simultaneously

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Contribution of this paper to the literature

- In this article, to analyze students' understanding of integration by parts, we used APOS and OSA.
- Although many studies have been done in Calculus Education about students' understanding of integrals, very few focused on the teaching and learning of integration by parts.
- The combined use of APOS and OSA gave us a better insight to explore students' understanding of integration by parts, so the networking of these theories can help researchers to analyze students' understanding of other mathematics concepts.

administered to samples of Mexican and Colombian students. For the analysis of the answers, they used some theoretical and methodological notions provided by the OSA to analyze mathematical cognition and instruction. The results revealed the meanings of the anti-derivative that are more predominantly used by civil engineering students. Llinares, Boigues, and Estruch (2010) described the triad development of a Schema for the concept of the definite integral. Data for their study was gathered from earth science engineering. The results demonstrate students' difficulty in linking a succession of Riemann sums to the limit, which forms the basis for the meaning of the definite integral. Mateus (2016) presented an analysis of the structure and functioning of a sequence of math classes, with Colombian sophomore bachelor's degree in mathematics, where the method of integration by parts explained was presented. The model of analysis proposed by the focus Onto-semiotic of Cognition and Instruction Mathematics was used. The didactic analysis of Mateus led to the conclusion that the sequence analyzed classes can be considered as a mechanistic degeneration of the formal class. Since the development of the same are used partially formal characteristics mechanistic paradigms. Moreover, it was observed that the structure and operation of the analyzed classes ignores the complexity of integrated onto-semiotic, which is one of the reasons why certain learning difficulties occur in students.

Although many studies have been done in Calculus Education about students' understanding of integrals (Jones, 2013; Kiat, 2005; Kouropatov & Dreyfus, 2014; Mahir, 2009; Pino-Fan et al., 2017; Radmehr & Drake, 2017; Thompson, 1994), very few focused on the teaching and learning of integration by parts (Mateus, 2016). For this reason, and also the importance and necessarily of integration by parts in Calculus II, Differential Equations and Engineering Mathematics, where students need to use this technique for solving many questions in these subjects, this article focused on this technique of integration. Every differentiation rule has a corresponding integration rule. The rule that corresponds to the Product Rule for differentiation is called the rule for integration by parts (Stewart, 2010). The formula for integration by parts becomes: $\int udv = uv - \int vdu$. The aim in using integration by parts is to obtain a simpler integral ($\int udv$) than the one started with ($\int vdu$).

In the research studies of Mathematics Education there is an interest to find connections between theories to have a better analysis of students' understanding of mathematical concepts (Badillo, Azcárate, & Font, 2011; Haspekian, Bikner-Ahsbahs, & Artigue, 2013, Pino-Fan, Guzmán, Duval & Font, 2015). In recent research, relationships between the APOS (Action, Process, Object, & Schema) (Arnon et al., 2014) and the OSA (Onto-Semiotic Approach) (Font, Godino, & Gallardo, 2013; Godino, Batanero, & Font, 2007) have been explored in relation to the Calculus concepts (Badillo et al., 2011; Borji, Font, Alamolhodaei, Sánchez, 2018; Font, Trigueros, Badillo, & Rubio, 2016). It is possible to connect APOS and OSA for exploring students' understanding of mathematical concepts (Bikner-Ahsbahs & Prediger, 2014; Borji et al., 2018; Font et al., 2016) due to each of these theoretical approaches uses the term object. Thus, both theories consider the constructive nature of mathematics and take the institutional component into account. In both of them the mathematical activity of individuals plays a central role and both use notions involved in their description that show similarities (e.g. action, process or object). They also share a constructivist position in relation to the nature of mathematics. These similarities led Font et al., (2016) to conclude that there are no intrinsic contradictions between the two theories, and that possible connections between them could be expected through their comparison.

In this article, to analyze students' understanding of integration by parts, we used APOS and OSA. APOS theory describes mental constructions which one student might needed to learn a mathematical concept. Much research has used this theory to analyze students' mathematical understanding, especially Calculus notations (Arnon et al., 2014). In addition, OSA is a theory that analyzes mathematical practices by identifying primary objects that are activated during engaging in such practices (Godino, et al., 2007). Recent studies showed that OSA is a useful theory for exploring primary objects and help to have a better understanding of students' learning (Font & Contreras, 2008; Font, et al., 2013; Pino-Fan et al., 2017). Font et al. (2016) showed that APOS and OSA complement each other to conceptualize the notion of a mathematical object. Borji et al. (2018) applied the complementarities of APOS and OSA for the analysis of the university students' understanding on the graph of the function and its derivative. They explored the students' graphical understanding regarding the first derivative and characterized their schemas in terms of levels (intra, inter and trans) of development of the schema. Their results showed that most of the students had major problems in sketching graph f' when given the graph f. A similar methodology has been used in the present study.

To date, APOS and OSA theories have not been used together as a complementary combination for analyzing students' understanding of the integration by parts. In this research, we use the combination of these two theories to investigate how students understand the integration by parts. The research question that we are looking for an answer to in this article is: *What are students' main mental constructions and primary objects regarding integration by parts?*

THEORETICAL FRAMEWORK

In this section, the theoretical frameworks (i.e., APOS & OSA) used in this study and their relationships are described to frame the article.

APOS Theory

APOS is a theory that introduces Action, Process, Object and Schema as mental constructions that one learner might performs to make meanings of a certain cognitive request (Arnon et al., 2014). Internalization, encapsulation, coordination, reversion and thematization are the mental mechanisms that allow the above mental constructions to be made.

With action conception the student perceives the mathematics object as something external. When the student repeats an action and reflects on it, action conception can be interiorized to a process conception. The process conception is a transformation which is an internal construction. Having a process conception the student can explain the steps engaged in the transformation, coordinate them, and skip some and inverse the steps. When the student reflects on the process and needs to make transformations or operations on it, the process conception is encapsulated into an object conception. With an object conception the learner is aware of the concept as a whole, and he/she can make transformations on it. The student can interconnect the objects and processes when they have been constructed. For example more than one or two process can be coordinated in a one process. A schema is a collection of actions, process and objects that organized in in a structured way. Having a schema of the concept the student invokes it when facing related problems. In fact, the schema is a cognitive construction which formed by action, process, object and other schemas or even their interrelations (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997).

To describe the development of the schema of a concept in APOS, the triad (intra, inter and trans) of Piaget and García (1983) is used. As APOS-based research advanced, it was recognized that the schema structure was important and necessary in order to characterize certain learning situations. In APOS-based research, the triad advance of stages has been used to describe the development of students' schemas associated with specific mathematical topics and to better find how schemas are thematized to become mathematical cognitive Objects. Schema development (triad) has proven to be a useful way to understand this facet of cognitive construction and has led to a deep understanding of the construction of schemas (Trigueros, 2005). One student at the intra level concentrates on the repetition of actions and recognizes relationships between them in different elements of the schema. The inter level characterized by the constructions of relationship and transformation between the actions, processes and objects that make the schema. The trans level occurred when the student becomes aware of the relationships and transformations in the schema and gives them coherence (Clark et al., 1997). The analysis of the mathematical concept focused on the cognitive constructions that might be required for student learning is the starting point of the research based on APOS theory. This analysis is a hypothetical model which called the Genetic Decomposition (GD) of the mathematical concept. The GD describes a possible way in which a learner constructs a mathematical concept in terms of the mental constructions and mechanism of the APOS.

It should be noted that a GD is not unique, one mathematical concept can has more than one GD. A GD is as a useful cognitive model, as evidenced by the results of several empirical studies that show the effectiveness of the APOS as an efficient tool for design and analysis of instruction (Borji, Alamolhodaei, & Radmehr 2018; Weller, Arnon, & Dubinsky, 2011).

OSA Theory

OSA theory describes the processes by which mathematical objects emerge from mathematical practices which is complex and must be distinguished, at least at two levels. At the first level, primary objects including definitions, language, procedures, propositions, problems and arguments emerge (Font et al., 2013).

Font and Contreras (2008) in their research about the relation between particular and general in mathematics education show some of the theoretical notions proposed by the OSA theory on the emergence of primary objects from mathematical practices (Font et al., 2013). By particular and general in mathematics education, Font and

Contreras (2008) want to describe how students develop their understanding from the specific mathematical situations to the universal situations. For example the function $y = 2x + 1$ is a particular case of a more general class of functions (i.e., the family of functions $y = mx + n$). From the mathematical practices emerge the different types of primary objects (language, procedures, definitions, problems, propositions and arguments) organized in the Epistemic Configurations (EC), depending on whether a personal or institutional point of view is adopted. OSA used the metaphor "climb a ladder" to explain how the primary objects emerge. The step on which we rely to perform the practice is a configuration of primary objects already known, while the upper step that we access, as a result of the practice, is a new configuration of objects in which some of such objects were not known before. The new primary objects appear as a result of mathematical practice and become primary institutional objects by a process of institutionalization that are part of the teaching-learning process (Godino et al., 2007).

In the OSA theory the second level of emergency is considered. The mathematical object emerges as a global reference associated with different epistemic configurations that allow performing practices in different mathematical contexts. For example the derivative concept as a mathematical object has been interpreted as the slope of the tangent line, as a limit process or as a velocity, as well as an operator that transforms a function into another function, which leads to the understanding that the derivative represented in different ways, can be defined in several ways, etc. The result, according to the OSA theory, is that it considers the existence of a mathematical object which plays the role of global reference of all configurations (Godino et al., 2007).

In the OSA, the mathematical object that plays the role of global reference considered as unique for reasons of simplicity and, at the same time, as multiple metaphorically, since it can be said to explode in a multiplicity of primary mathematical objects categorized in different configurations. The perspective of the emergence of mathematical objects from the mathematical practices proposed by the OSA theory highlights the complexity of such mathematical objects and the necessary the articulation of the elements in which such complexity explodes. The OSA theory offers an explanation of the complexity in terms of epistemic configurations, and at the same time, how this plurality of configurations can look in a unitary way (Font et al., 2013).

Relation between APOS and OSA

The use of the notion of mathematical object in both theories, APOS and OSA, is the starting point for connection between two theories (Font et al., 2016). The research in mathematics education has had questions about the nature of mathematical objects, their construction process, their various types and their participation in mathematical activity. These two theories, APOS and OSA, are samples of a set of theories that use the term mathematical object as a relevant construct of their theoretical (Font et al., 2016).

In the passage from the action to the process and its subsequent encapsulation as an object, from the perspective offered by the OSA, many aspects intervene that inform its complexity. First, the student must understand that the actions performed can be performed according to a certain procedure (a rule that says how actions should be done). At this time, a certain level of reification already occurs, in the sense that the procedure can be treated as a unit (an object). Next, the student must consider a new object, the result of the process, and finally must understand the meaning of the definition that informs about the nature of the new object. In the APOS theory this transit is also considered complex, but unlike OSA, a procedure is not considered to be a cognitive object, but a process; the object in APOS would only be the result of the encapsulation of a process. On the new object actions can be exercised. The look that the OSA provides on the encapsulation allows one to appreciate that in this one a change of double nature takes place, on the one hand it passes from a process to an object (primary according to the OSA), as it indicates the APOS, but on the other hand, it changes the nature of the primary object.

In Font et al. (2016), relationships were found between the encapsulation mechanism in the APOS and the emergence of primary objects in the OSA, highlighting the complexity of the mechanism in which primary objects of a different nature must be considered. When considering the APOS thematization mechanism, a relation was found with the second level of emergence in the OSA, since the object resulting from thematization plays the role of global reference for a set of semiotic representations.

METHOD

This research is a multiple-case study in which 23 students from a university of Iran participated voluntarily. All of them had completed a course of Calculus I (single-variable) in the 2015-2016 academic year and had used Stewart Calculus, (2010), as their textbook.

In the first phase, tasks in a semi-structured interview were used to explore students' understanding of integration by parts. In the second phase, following the methodology of onto-semiotic analysis (Pino-Fan, Godino, & Font, 2018), primary objects of EC that were activated during these tasks were identified. The third phase included designing a GD based on APOS theory. This GD predicts the mental constructions that might be needed when students use the primary objects to solve such tasks. Additionally, two experts in OSA and APOS have evaluated the EC and GD designed in this research, and confirmed their validity. In the fourth phase, the GD and EC were used to characterize the development of the integration by parts schemas in terms of the triad (i.e., intra, inter and trans levels). In the fifth phase, students' interview were video recorded and transcribed. Finally, in the sixth phase, students' responses and their schemas were analyzed in terms of the triad of the integration by parts schema.

Tasks

Evaluate the integrals using integration by parts.

a) $\int x\, e^{x} dx$ b) $\int x \ln x\, dx$ c) $\int x\, tan^{2}x\, dx$

The tasks posed for this research were rather complex, because they include different types of functions (exponential, natural logarithm and trigonometric functions), and also required the students to use some prerequisite concepts and rules of differentiation, differentials, anti-derivatives and basic integration, so they might needed to perform primary objects of OSA theory and also mental constructions (actions, processes, objects and schemas) of APOS theory in order to successfully find the answer to the tasks using integration by parts. Therefore, the mental constructions, based on APOS, and the primary objects, based on OSA, that a learner might make to develop her/his understanding of the integration by parts are described below. This allowed the authors to determine different levels of the development of the schema (intra, inter and trans) of the integration by parts.

The reason that all of the tasks included x as one of the functions was to examine whether students have rational reason to choose terms for u and dv or not. The tasks were designed by the authors and were justified by two mathematics education professors and two mathematics professors, all of which had at least 10 years of experience in the teaching Calculus.

A look from OSA: Epistemic Configuration of Primary Objects

Based on the OSA theory, a priori analysis of the mathematical activities needed to solve the tasks in terms of practices and objects performed is made.

Practice

1) Read the task.
2) Evaluate the Integral.

Problems: The task proposed.

Languages

Verbal: Substitution, Differentiation, Differentials, Integration.

Symbolic: u, v, du, dv, $\int f(x)\, dx$

Concepts/definitions: Function, derivative, anti-derivative, Integral.

Procedures

Pr0: A proper choice of u and dv.

Pr1: Calculating new terms du and v with differentiation and integration, respectively.

Pr2: Applying the formula of the equation $\int udv = uv - \int vdu$.

Pr3: Solving a new integral, $\int vdu$, that is easier than the last one ($\int udv$).

Propositions

1. If $u = f(x)$, where f is a differentiable function, then the differential dx is an independent variable. The differential du defined in terms of dx by the equation: $du = f'(x)dx$.
2. If $f'(x)dx = dv$, then using integration: $v = \int f'(x)dx$
3. If new integral ($\int vdu$) is not solvable or more difficult than the first Integral ($\int udv$), it should be better to change and choose better terms for u and dv.
4. If one choose proper u and dv then the integral $\int vdu$ will be easier than the integral $\int udv$.

Arguments

Answer to the task (b) which is $\int x \ln x \; dx$: For u, ln x is proper because its derivative, which is $\frac{1}{x}$, is easier than itself ($\ln x$) and it helps to get an easier integral. The rest part, which is $x\,dx$ equal to dv. Using integration, v is $\frac{x^2}{2}$. Using the formula for integration by parts ($\int udv = uv - \int vdu$) one can get $\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}dx$ and it is equal to: $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$.

A Look from APOS: Genetic Decomposition

As a part of her or his derivative schema, the student

1. Has developed a process or object conception of differentiation rules.
2. Has developed a process or object conception of differential ($u = f(x)$ then $du = f'(x)dx$).

As a part of her or his Integral schema, the student

3. Has developed a process or object conception of integration's table (e.g. knowing integrals such as $\int x^n\,dx$, $\int \sin x\,dx$ and …) and substitution rule.
4. The student then coordinates previously constructed schemas to a new process and applies them to the formula of Integration by parts, $\int u\,dv = uv - \int v\,du$. The coordination consists in first choosing terms of the integrand for u and dv and then finding du and v using differential and Integration, respectively, and then substituting them into the formula.
5. Process conception of integration by parts, encapsulates to an object conception as a totality. Having an object conception of integration by parts, one student can correctly and with reasoning recognize and choose proper terms of the integrand for u and v for having an easier Integral for next step ($\int v\,du$).
6. The student establishes relations between these processes, objects with other schemas to construct and complete his/her schema of integration by parts, so that he/she will be able to recognize whether a given integral can be solved using integration by parts, and if so, how.

Based on the initial description of how the technique of integration by parts might be learned (GD), an attempt to interpret the data using the Action-Process-Object-Schema theoretical framework is made. To describe students' development of schema, a theory of schema development based on ideas of Piaget and Garcia (1983) is used. The Piagetian triad is suggested as a mechanism to describe the schema development of integration by parts. The triad of the intra, inter and trans levels of schema development provides the structure for interpreting the students' understanding of integration by parts and classifying their responses to the interview questions about integration by parts. We use mental constructions of the GD and primary objects of the EC to characterize the development of the students' conceptualization of integration by parts in terms of the triad.

Intra: A student at this level does not have any rational reasoning for choosing u and dv. He/she usually puts the first part of the integrand equal to u and puts the rest equal to dv. The student has problems in finding du with use of the differential and also has problems in finding v using integration.

Inter: A student at this level can have correct choices for u and dv but won't always choose correctly. In some cases, he/she at first chooses incorrect choices for u and dv and when he/she gets confused in solving part $\int vdu$ then he/she changes correctly his/her choices for u and dv. The student can find du and v using the differential and integration.

Trans: A student at this level pays attention to the next step ($\int vdu$) for choosing proper choices for u and dv. The student has a coherence schema of integration by parts so that he/she recognizes whether a given integral can be solved by integration by parts or not.

In APOS Theory two levels of analysis are proposed, one in terms of action, processes, objects and schemas which in this case should be based in the genetic decomposition presented, and other in terms of the schema development. When using the later type of analysis, the genetic decomposition should be described in terms of the characteristics of the levels of development of the schema. We used in our analysis the notion of schema development from APOS. The aim of APOS Theory is not to classify students or to place them in certain levels. The objective of the theory is to understand how a concept of a topic is constructed. To do so, students' constructions are compared in order to determine those constructions that seem indispensable in the learning of that concept. It is not correct to talk about the levels of development of the schema as levels of students while they are levels of development of the schema, not of the students; in APOS Theory students show constructions corresponding to a level of the schema and they are characterized by the types of relations students demonstrate among the components of the schema (Trigueros, 2005).

Table 1. Relation between APOS and OSA

APOS	OSA
Processes in APOS	*Procedures in OSA*
• Has developed a process conception of differentiation rules. • Has developed a process conception of differential ($u = f(x)$ then $du = f'(x)dx$). • Has developed a process conception of integration's table and substitution rule. • The student then coordinates previously constructed processes to a new process and applies them into the formula of Integration by parts, $\int u\,dv = uv - \int v\,du$. The coordination consists in first choosing terms of integrand for u and dv and then finding du and v using differential and Integration, respectively, and then substituting them in the formula.	Pr0: A proper choice of u and dv. Pr1: Calculating new terms du and v with differentiation and integration, respectively. Pr2: Applying the formula of the equation $\int udv = uv - \int vdu$. Pr3: Solving new integral, $\int vdu$, that is easier than the last one ($\int udv$).
Objects in APOS	*Propositions in OSA*
• Has developed an object conception of differentiation rules. • Has developed an object conception of differential ($u = f(x)$ then $du = f'(x)dx$). • Has developed an object conception of integration's table and substitution rule. • Process conception of integration by parts, encapsulates to an object conception as a totality. Having an object conception of integration by parts, one student can correctly and with reason recognize and choose proper terms of integrand for u and v for having an easier Integral for next step ($\int v\,du$).	P1: If $u = f(x)$, where f is a differentiable function, then the differential dx is an independent variable. The differential du defined in terms of dx by the equation: $du = f'(x)dx$. P2: If $f'(x)dx = dv$, then using integration: $v = \int f'(x)dx$ P3: If new integral ($\int vdu$) is not solvable or more difficult than the first integral ($\int udv$), it should be better to change and choose better terms for u and dv. P4: If one choose proper u and dv then the integral $\int vdu$ will be easier then the integral $\int udv$.
Schema in APOS	*EC in OSA*
All actions, process, objects and other schemas	All primary objects

Relation between EC, GD and Levels of Development of the Schema

In this section we present a networking of two theories, APOS and OSA, and show some commonalities and some links between these theories and show the complementary nature of their constructs for integration by parts. In networking these theories, we considered an analysis in terms of comparing and contrasting some of the principles of both theories. In what follows in **Tables 1** and **2**, we describe specific links obtained as a result of our reflection on EC, GD and levels of the development of schema of integration by parts.

Table 1 shows that each of the mental constructions in designed GD are complementary with the primary objects in designed EC. In **Table 1**, one can see the relation between processes in APOS and procedures in OSA, and also objects in APOS and propositions in OSA. **Table 2** shows that each of the levels (intra, inter and trans) of the development schema is related to certain parts of GD and to certain parts of EC. For example, from **Table 2** one can see which parts of primary objects in EC are related to the intra level, and also which parts of GD are related to this level. These relations are also presented for the two other levels; inter and trans. These tables help to identify deeper understanding between APOS and OSA, which is consistent with our goal in this research.

RESULTS

Students' responses to the mathematical tasks were analyzed and their schemas were categorized into the intra, inter and trans levels described in **Table 2**. There were eleven students who had constructed their schemas at the intra level, eight students at the inter level, and four students at the trans level. An example of students' responses from each level for Task b is described below. For each example, to relate OSA and APOS, the students' responses were also analyzed based on EC of the OSA.

Table 2. Relation between GD, EC and levels of schema of implicit differentiation

EC (OSA)	**GD (APOS)**	**Levels of development of the Schema (APOS)**
Pr1: Calculating new terms du and v with differentiation and integration, respectively, but no always. Pr2: Applying the formula of the equation $\int udv = uv - \int vdu$.	Has developed a process conception of differentiation rules. Has developed a process conception of differential ($u = f(x)$ then $du = f'(x)dx$). Has developed a process conception of Integration's table (e.g. knowing integrals such as $\int x^n\,dx$, $\int \sin x\,dx$ and ...) and substitution rule.	*Intra:* The student at this level don not have any rational reason for choosing u and dv. He/she usually puts the first part of integrand equal to u and the rest puts equal to dv. The student has problems in finding du with use of the differential and also has problems in finding v using integration.
P1: If $u = f(x)$, where f is a differentiable function, then the differential dx is an independent variable. The differential du defined in terms of dx by the equation: $du = f'(x)dx$. P2: If $f'(x)dx = dv$, then using integration: $v = \int f'(x)dx$ Pr3: Solving new integral, $\int vdu$, that is easier than the last one ($\int udv$). P3: If new integral ($\int vdu$) is not solvable or more difficult than the first integral ($\int udv$), it should be better to change and choose better terms for u and dv.	Has developed an object conception of differentiation rules. Has developed an object conception of differential ($u = f(x)$ then $du = f'(x)dx$). Has developed an object conception of integration's table. The student then coordinates previously constructions to a new process and applies them into the formula of integration by parts, $\int u\,dv = uv - \int v\,du$. The coordination consists in first choosing terms of integrand for u and dv and then finding du and v using differential and Integration, respectively, and then substituting them in the formula.	*Inter:* The student at this level can have correct choices for u and dv but no always. For some cases he/she at first chooses incorrect choices for u and dv and when he/she gets confused in solving the integral $\int vdu$ then he/she changes correctly his/her choices for u and dv. The student can find du and v using the differential and integration.
P4: If one choose proper u and dv then the integral $\int vdu$ will be easier then the integral $\int udv$.	Process conception of integration by parts, encapsulates to an object conception as a totality. Having an object conception of integration by parts, one student can correctly and with reason recognize and choose proper terms of integrand for u and v for having an easier Integral for next step ($\int v\,du$).	*Trans:* The student at this level have attention to the next step ($\int vdu$) for choosing proper choices for u and dv. The student has a coherence schema of Integration by parts so that he/she recognizes whether a given integral can be solved by integration by parts or not.

First group: Intra Level

Student A was one of the students who had constructed a schema at the intra level. Some parts of her answer to Task b are presented below (**Figure 1**).

"I choose u *equal to* x *and* dv *equal to* $\ln x$*,… Ok now I find the differential (she pointed to the* $x = u$*)… So I have one* dx *equal to one* du*… For finding* v *I have to integrate the second equation (she pointed to the* $\ln x dx = dv$*)… Oh, what is the integral* $\ln x$*?!... I think its integral is one over* x*… Now I use the formula for integration by parts … The integral* x *to the power minus one is equal to* x *to the power minus one plus one over minus one plus one …".*

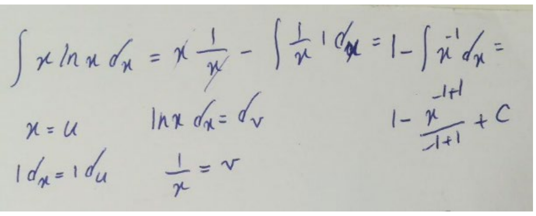

Figure 1. Student A's response (Intra level)

Student A did not have any rational reason for choosing u and dv. She put the first term of integrand (i.e., x) equal to u and put the rest (i.e., $\ln x\, dx$) equal to dv. Student A had difficulties in finding v using integration. Due to incorrect choices for u and dv the last answer was completely wrong.

In relation to EC of OSA, student A understood and did procedures Pr1 and Pr2. With respect to arguments in EC, she could not respond correctly to the tasks. It is noteworthy that student A could not do Pr0 and Propositions P1 and P2 correctly, consequently, she could not evaluate integrals correctly for the tasks.

Second Group: Inter Level

Student B was one of the students that had constructed a schema at the inter level (**Figure 2**). Some of his thinking about Task b is provided below.

"I put x equal to u and $\ln x\, dx$ equal to dv… from x equal to u I derive … so, dx is equal to du… from $\ln x\, dx$ equal to dv I integrate … I know that integral $\ln x$ using integration by parts is equal to $x \ln x$ minus x … (Then he used the formula for integration by parts and had difficulties with solving the next integral) … oh, integral $x \ln x$ minus x is more difficult than the first one… so, sorry my bad … I have to change my choices for u and dv… $\ln x$ equal to u, so one over x equal to du… It's good because one over x is much easier than $\ln x$… now I have to put $x\, dx$ equal to dv, so using integration I have v equal to x to the power two over two … Ok, I use the formula for integration by parts… "

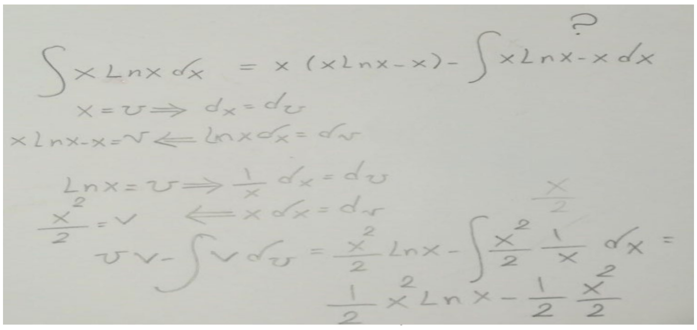

Figure 2. Student B's response (Inter level)

At first, student B chose incorrect choices for u (i.e., x) and dv (i.e., $\ln x\ dx$) and when he got confused in solving the integral $\int vdu$ (i.e., $\int (x \ln x - x)dx$) which was more difficult than the first one (i.e., $\int x \ln x\ dx$), then he correctly changed his choices for u (i.e., $\ln x$) and dv (i.e, $x\, dx$) and got an easier integral (i.e., $\int \frac{x^2}{2}\frac{1}{x}dx$) than the first one. The student could correctly find du and v using the differential and integration, respectively.

In relation to EC of OSA, student B understood and did procedures Pr0, P1, Pr2 and Pr3, and propositions P1, P2 and P3. With respect to arguments in EC, he had difficulties in procedure pr0 and proposition P4, so due to those he had problems in Task b.

Third group: Trans Level

Student C was one of the students that had constructed a schema at the trans level (**Figure 3**). Some of her thinking about Task b is provided below.

"For u I choose $\ln x$ because its derivative, which is one over x, is much easier than itself and it will help me to get an easier integral in the next step … So I put the rest part, which is $x\, dx$, equal to dv… My du is one over x dx and my v is equal to x to the power two over two … Now using the formula for integration by parts I can easily find the answer … "

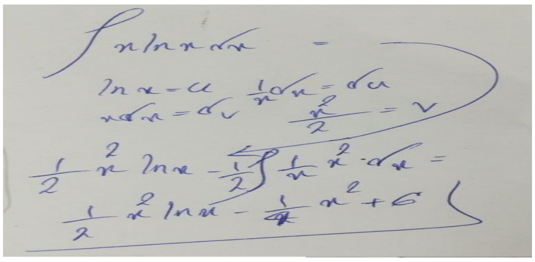

Figure 3. Student C's response (Trans level)

When Student C was thinking about proper choices for u and dv he had attention to the next step ($\int vdu$) in order to get an easier integral.

In relation to EC of the OSA, Student C correctly evaluated the tasks as he activated all the primary objects of the EC. He correctly identified the proper terms for u and dv, and correctly used differentials and integration to find du and v and plugged them into the formula for integration by parts and solved the integral $\int vdu$ correctly.

Table 3. Students' difficulties regarding integration by parts

Students' difficulty	Explanation
Students did not choose u and dv correctly.	By correctly, we mean that $\int vdu$ was not easier to solve compared to $\int udv$.
Students' difficulty regarding derivatives.	Students had difficulties in finding du using differentiation.
Students' difficulty regarding integrals.	Students had difficulties in finding v using integration.

Some of Students' Difficulties in Solving Integration by Parts Tasks

Since most students in our study have constructed their schemas at the intra and inter levels, we present some details about their problems of understanding, especially those problems that were common among them.

The students at the intra level had difficulties in integration to find v (48% of students). In the following we brought some parts of the explanations of one of students who showed this issue during his interview when he was solving Task a.

"… Ok, and $e^x dx$ equal to dv… for finding v I have to integrate … I don't know what is the answer of integral e^x… I can't continue …".

In regard to the framework of the study, these students cannot activate proposition P3 and have not developed an object conception of integration's table.

It seems that some students at the inter level had a procedural understanding of integration by parts. Hiebert and Lefevre (1986) described conceptual understanding as part of a network comprised of individual pieces of information and the relationships between these pieces of information. Hiebert and Lefevre also defined procedural understanding as including both a familiarity with the symbol representation system of mathematics and knowledge of rules for solving exercises in mathematics. They noted that, while conceptual understanding must be learned with meaning, procedural knowledge may or may not be learned meaningfully. In fact students with a procedural understanding of integration by parts did not know the reasoning of processes which they were doing. They followed the steps and rules without reasons and if they could not get the simpler integral then they came back to their solutions and changed their choices for u and dv and again followed all steps.

We now show as an example some parts of the explanations of one of the students at the inter level that showed this problem when he was solving Task a.

"I put u equal to e^x and dv equal to x…So, du equal to $e^x dx$ … and v equal to x to the power two over two… (Then the student used the formula for integration by parts and plugged u, v and du inside that but had difficulties in solving $\int vdu$)… Now I have to evaluate $\int \frac{x^2}{2} e^x dx$ … It's difficult for me… I try to change my u and my dv … I put my u equal to x and my dv equal to $e^x dx$… I hope I can solve with these choices… ".

In regard to the framework of the study, these students cannot activate proposition P4 and have not developed an object conception of integration by parts, actually they cannot correctly and with reasoning recognize and choose proper terms of integrand for u and dv for having an easier integral for the next step ($\int v\, du$). It should be noted that if a student did not explain his/her reason for choosing u and dv, we asked him/her to describe more about them. In **Table 3**, the students' difficulties, associated with implicit differentiation are reported.

DISCUSSION AND CONCLUSION

This research is a combined use of two theories, APOS and OSA (Font et al., 2016), as lenses to explore students' understanding of integration by parts. Results show that most of the students in our research have major difficulties in doing the practical work and developing the mental constructions needed to solve the tasks, particularly those mental constructions that have to be made to choose proper choices for u and dv so that the second integral will be easier than the integral they started with, which is why most of the students in our study have constructed their schemas at the intra or inter level of development of the integration by parts schema. We also observed that 56% students in our research, especially at the intra level, had some difficulties in prerequisite concepts like functions, differentiation, differentials and basic integration. This is line with findings of Mahir (2009) where the author concluded that several students could not solve conceptual questions regarding integrals because they had difficulties in prerequisite knowledge (e.g., derivatives) which is necessary for learning integrals.

The difference between students at the inter and trans level was that students at the inter level initially chose u and dv without precision, and then, if they had difficulties in solving the next integral (i.e., $\int vdu$), they returned and changed their choices for u and dv. But the students at the trans level from the beginning paid attention to the choices for u and dv in order to get a simpler integral. We can say that the students at the inter level had a procedural understanding while students at the trans level had a conceptual understanding of integration by parts (Borji et al., 2018). Similar to the finding of the present study, Mateus (2016) stated that students usually have difficulties to determine proper choices for u and dv.

Some mathematics concepts seem like they are only a series of symbolic procedures and formulas, and students might follow these procedures even without knowing their purpose, and can solve related questions correctly even without knowing why the methods for doing so work (Oaks, 1990). Integration by parts can be one example of these types of symbolic techniques in Calculus. Although symbolic and algebraic techniques and formulas can be taught with procedural teaching, behind each of these symbolic techniques and formulas there are reasons and causes (Oaks, 1990), so these techniques can be taught with conceptual teaching and can be learned conceptually with their reasoning. Although some students who might have a procedural understanding and who can follow some rules (without proper understanding of integration by parts) can somewhat successfully accomplish the tasks of integration by parts, as it has been said, the main difference between students who have made a coherent schema of integration by parts and such students are that the students at the trans level, know the reason for using any technique and concept and know, where, and when, and for what they have to use each of them. This was clearly visible during students' interviews.

Teachers, when teaching integration by parts, should emphasize that students choose u so that its derivative (i.e., du) is more convenient than itself, and choose dv so that they can find its integral ($\int dv$), which is v. In addition, the accuracy of students should begin with the point that the integral $\int vdu$ should be easily solvable, or at least that it should be easier than the first one ($\int udv$) they started with.

In recent studies of mathematics education, the use of networking and combining theories has grown to analyze students' understanding (Badillo et al., 2011; Haspekian et al., 2013; Pino-Fan et al., 2015). Two of the theories, that are appropriate for exploring students' understanding and their combination has been used in recent research, are APOS and OSA theories (Borji et al., 2018; Font et al, 2016). OSA theory helped us to analyze mathematical practices by identifying primary objects that are activated while engaging in integration by parts. Using APOS theory we could find mental constructions that one student might needs to learn and solve integration by parts tasks. The theory of development schema characterized students' schemas of integration by parts and showed their problems, misunderstandings and shortcomings. This theory, which is one part of APOS (Arnon et al., 2014) described mental constructions that the students at the intra and inter levels need to develop their schema for integration by parts and reach the trans level.

The future direction of this study is to analyze the way of teaching, both conceptual teaching and procedural teaching, that Calculus' lectures use to teach integration by parts and effects that each of these teaching ways has on students' understanding. As we said before, the combined use of APOS and OSA gave us a better insight to explore students' understanding of integration by parts, so the networking of these theories can help researchers to analyze students' understanding of other mathematics concepts. We also suggest that in future studies a larger sample or population be used to give consistency and stability to the conclusions of this work.

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Effects of Simulation-based Formative Assessments on Students' Conceptions in Physics

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ABSTRACT

Background: The paper presents effects of simulation-based formative assessments on students' conceptions in physics. In the study, two topics—motion in two dimensions and conservation of energy—were selected to explore students' conceptions in physics, and related assessment tasks incorporating computer simulations and formative assessment questions were developed.

Material and methods: The participant students were first-year college students with majors related to science or engineering. Analytic rubrics were developed to capture the students' normative and non-normative ideas revealed in their responses, and a holistic rubric was applied to categorize the responses into four response models.

Results: The results demonstrated that, overall, students predicted and explained the given scientific phenomena with more valid scientific ideas after experiencing a computer simulation. However, the results also indicated that students' non-normative ideas were still present even after experiencing computer simulations, especially when they were required to consider an abstract scientific concept such as energy dissipation.

Conclusions: The finding can be explained with knowledge-in-piece perspectives (diSessa, 1993), that students' naïve knowledge is fragmented, and thus they do not demonstrate a coherent understanding of abstract science concepts across different situations.

Keywords: computer simulation, misconception, conceptual understanding, formative assessment

INTRODUCTION

Online and blended learning have become increasingly important components in higher education. Many studies have investigated online education outcomes compared to those of traditional classes across a variety of subjects (e.g., Navarro & Schoemaker, 2000; Tallent-Runnels et al., 2006). Nguyen (2015) reviewed the literature, paying particular attention to meta-analyses studies on effects of online learning, and found robust evidence suggesting that online learning was as effective as traditional learning. Especially in science disciplines in higher education, although fully online college-level science courses are not yet commonly offered, the use of online homework has steadily increased over the past two decades (Bonham, Deardorff, & Beichner, 2003; Richards-Babb et al., 2011; Penn, Nedeff, & Gozdzik, 2000), which can be considered as an online learning component to a traditional class. In physics, Cheng et al. (2004) found that students who were assigned online homework showed similar performance results compared to those of students who completed paper-and-pencil homework. Bonham, Deardorff, and Beichner (2003) reported a similar result, that two homework methods—online and traditional paper homework—did not show significant differences on student learning outcomes. Even though previous studies have found that online homework is neither superior nor inferior to traditional homework, use of online homework is becoming a major component of college-level science courses (Bonham, Deardorff, & Beichner, 2003). The increasing popularity of online homework might be due to the fact that there are many online homework systems available; instructors

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Contribution of this paper to the literature

- This study contributes to the existing literature on using simulation-based formative assessments to promote students' conceptual learning in science.
- The results showed that simulation-based formative assessments had a positive effect on students' conceptual learning.
- The study, however, also argues that the improvement might be the case only in certain situations. In the study, students demonstrated better abilities to predict and explain the given scientific phenomena after running a simulation, whereas, when an abstract concept was required to apply to answer questions, their abilities regressed.

can easily collect students' homework and sometimes grade it automatically, especially in the case of multiple-choice questions, which gives the instructors more time for class preparation. College-level introductory science courses, especially, are often offered in large classrooms and taught in a traditional lecture-based format (Pascarella & Terenzini, 1991; Springer, Stanne, & Donovan, 1999). Instructors of these courses seek new methods that will keep students engaged in the class and in the learning process, but do not take much time in terms of collecting and grading homework. In these cases, an online homework system might be of interest to instructors as an alternative venue.

There is no doubt that formative assessment is an important component in education. The main role of formative assessment is diagnosing students' learning needs in order to improve their learning outcomes. There are an overwhelming number of studies showing the positive impact of formative assessment on student achievement in science. For example, Black and William (1998) reviewed more than 250 books and articles, and concluded that formative assessment has significant positive effects on student achievement.

Although the effects of formative assessment have been studied extensively, formative assessment as an online complement to the traditional face-to-face class has not been much illuminated. Gikandi, Morrow, and Davis (2011) asserted the importance of using online formative assessment in order to create learner-centered learning environments (Pachler et al., 2010); there was, however, a lack of studies on online formative assessment. Gikandi et al. (2011) reviewed published articles and reports regarding applications of online formative assessment in online and/or blended higher education contexts, and concluded that online formative assessment is beneficial to improve learners' engagement with valuable learning experiences. They also identified formative feedback and embedded authentic assessment activities within the teaching and learning processes as key factors for successful online formative assessment. To be specific, using online tools such as computer-based simulations or collaborative inquiry provides students opportunities to engage in activities and problem-solving relevant to their real-world situations (Crisp & Ward, 2008; Mackey, 2009). Thus, online formative assessments should be designed to encourage and promote student learning experiences through authentic tasks (Gikandi, Morrow, & Davis, 2011).

Computer simulations are programs presenting a representation of an authentic system, a phenomenon, or a process (de Jong & van Joolingen, 1998). Studies found that computer simulations promoted students' engagement in observing and exploring phenomena (Srisawasdi & Kroothkeaw, 2014), which offered students opportunities to promote conceptual change in science (Rutten, van Joolingen, & ven der Veen, 2012; Trundle & Bell, 2010).

In the current study, computer simulations and online formative assessment were incorporated as integral parts of pedagogy for enhancing students' conceptions in science through which they could perform their own investigations and develop conceptual understanding. Although simulation-based learning and formative assessment have been studied widely, there still is a lack in studies focusing on how online formative assessment incorporating computer simulations enhance college students' conceptions in physics. The research hypothesis is that computer simulation-based formative assessments will have a positive impact on students' conceptions in physics. The specific research question for this study is: "To what extent do simulation-based online formative assessments have an influence on students' conceptions in physics?"

Background and Review of Literature

Student knowledge in science

It has been generally agreed that students often have misconceptions in science that differ from expert conceptions (Bransford, Brown, & Cocking, 2000; Hammer, 1996; Treagust, 1988). An effective strategy to change their conceptions to scientific ones is to ask students to make predictions about various situations, then explain the reasons for their predictions. This elicits students' pre-existing understanding; and they can then be provided opportunities to build or challenge their initial understanding (Bransford, Brown, & Cocking, 2000).

Although student misconceptions have been extensively studied during past decades, there has been a debate over students' knowledge structure, as to whether their knowledge is theory-like or fragmented. Theory-like (framework theory) perspectives hypothesize that students' naïve knowledge structures are coherent, and students explain scientific phenomena consistently using the domain-specific structures in everyday life (Carey, 2009; Chi, 2005; Gelman, 1990; Ioannides & Vosniadou, 2002). On the other hand, knowledge-in-pieces perspectives assert that students' knowledge involves "phenomenological primitives" or "p-prims" (diSessa, 1993). This perspective posits that students' naïve knowledge is fragmented, in that their knowledge elements are not organized by overarching theories, but rather their naïve knowledge is a repertoire of multiple quasi-independent knowledge elements (diSessa, 2002). Thus, conceptual change is a process of restructuring and integrating knowledge elements by adding new elements to the existing ones to make appropriate connections among the knowledge elements (diSessa, 2002; Linn, Clack, & Slotta, 2003).

Ozdemir and Clark (2009) addressed the debate over students' knowledge structure using the same physics questions used by diSessa, Gillespie, and Esterly (2004) and Ioannides and Vosniadu (2002). They applied two coding schemes representing theory-like perspectives (Ioannides & Vosniadou, 2002) and knowledge-in-pieces perspectives (diSessa et al., 2004) to investigate if students demonstrated a coherent understanding of force meaning across questions, and found that their results supported knowledge-in-pieces knowledge structure over a framework theory.

Regarding student conceptions in the two scientific topics addressed in the study; projectile motion and conservation of energy, many empirical studies have been done to investigate students' misconceptions. For example, Hynd, Alvermann, and Qian (1997) summarized students' misconception in projectile motion that students believe that an object will move forward until its forward motion overpowers gravity, then begin to deviate downward once a force implanted in the object dissipates. They also do not believe that two different projectiles will reach the ground at the same time in a non-frictional situation. Many people believe that moving objects possess an internal force or impetus that keeps the object in motion; and the moving objects eventually slow down or stop as their impetus gradually dissipates (McClosky, 1983). Also, students believe that heavier objects fall faster; or that gravity finally acts on an object after its impetus is all dissipated (Gunstone & White, 1981). In regard to students' conception of energy conservation, Park and Liu (2019) found that the conservation of energy was significantly more difficult for students to understand than other energy aspects such as energy form or transfer across all science disciplines. Many students just recall the law of conservation of energy and fail to apply the law in solving science problems (Goldring & Osborne, 1994) or explaining living phenomena (Barak, Gorodetsky, Chipman, 1997). Tatar and Oktay (2007) reviewed studies on students' conceptions on energy conservation and summarized that students believe that energy is used up or lost; energy degradation is opposite to energy conservation.

In order to reveal students' misconceptions in science, various methods have been used including interviews, open-ended questionnaires, or ordinary multiple-choice tests. Although interviews are effective for investigating students' misconceptions in science, a large amount of time is required to train interviewers and interview students (Gurel, Eryılmaz, & McDermott, 2015). Treagust (1988) suggested two-tiered test items to overcome those difficulties and effectively diagnose students' misconceptions. A two-tiered item is composed of two related questions: the first tier question is a multiple choice question in which respondents should choose only one option out of multiple options from a list and the second tier question asks respondent to provide their reasoning for the answer given to the first question (Treagust, 1988). Peterson, Tregust and Garnet (1989) support that two-tiered items investigate students' understanding and diagnose their conceptions effectively.

Computer simulations in science teaching

Computer simulations can be used in science teaching to help students observe scientific phenomena that we could not accurately or easily do in real life. Through the meta-analysis of studies on using computer simulations over the last decade, Rutten, van Joolingen, and van der Veen (2012) found positive results on student performance, motivation, and attitude for the classrooms in which stimulations were used to replace or enhance traditional instructional practices. Smetana and Bell (2012) reviewed published research on the effectiveness of computer simulation, and found that science teaching that incorporated computer simulations produced, in general, better results than traditional teaching. In particular, they indicated that simulations were effective to enhance student content knowledge and to facilitate their conceptual change when the simulations were used to supplement traditional instruction.

Simply providing educational technologies or software without instructional strategies is not likely to result in positive effects on student learning, especially on conceptual change (Trundle & Bell, 2010). In order to promote student conceptual change using computer simulations, Tao and Gunstone (1999) suggested an instructional strategy that makes a prediction about scientific phenomena in order to confront students with discrepant events that contradict their preconceptions; explains the prediction; runs a simulation to test the prediction; and reconciles the conflict by reflecting on students' conceptions. In physics, Zacharia and Anderson (2003) used a similar strategy investigating the effectiveness of computer simulations on students' abilities to make predictions and to explain the results of physics experiments and concluded that the simulations facilitated students' conceptual change. There are many empirical studies supporting the use of computer simulations in science classrooms in order to effectively facilitate students' conceptual understanding of scientific ideas (e.g., Dori & Barak, 2001; Geban, Askar, & Ozkan, 1992; Huppert, Lomask, & Lazarowitz, 2002; Winn et al., 2006).

Although many studies have focused on the effectiveness of computer simulations employing a pre- and post-test design, fewer studies have been done using performance data collected while students were interacting with simulations (de Klerk, Veldkamp, & Eggen, 2015). In order to collect student performance data, it is necessary to embed formative assessment approaches into computer simulations used in science classrooms. Recently, some studies have been done focusing on performance data of simulation-based assessments. For example, Quellmalz et al. (2012) developed simulation-based science assessments for two middle school science topics, and collected data that included student responses to the assessments and classroom observations. Their findings showed that students performed better on simulation-based assessments than on conventional static assessments. Srisawasdi and Panjaburee (2014) investigated the effect of simulation-based inquiry integrated with formative assessment on students' conceptual learning of buoyancy-driven phenomena. They reported a positive effect of computer simulations embedded in assessments: students who had experienced the simulation-based formative assessments showed significantly higher performance scores than students who had not experienced the assessments. Park et al. (2017) developed simulation-based formative assessments to promote student understanding in high school chemistry, and found a significant effect of the frequency of using computer simulations as formative assessments on students' understanding of the nature of models (Treagust, Chittleborough, & Mamiala, 2002).

Instructors of physics were among the first in the various science disciplines to explore computer simulations in teaching and learning. As a result, there are currently many computer simulations available for various physics topics (Jimoyiannis & Komis, 2001). In this project, computer simulations were incorporated as an integral component in formative assessment to enhance student conceptions in physics, leading to the development of simulation-based formative assessments. The simulation-based formative assessments target high school physics classes and introductory-level college physics courses to assist instructors in assessing student conceptions as well as enhance students' deeper learning of core scientific ideas.

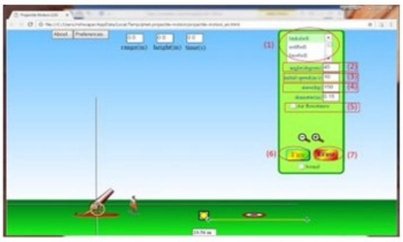
METHODS

Participants

An invitation email to participate in the study was sent to college students who were taking an introductory physics course at a research university in the United States. Initially, 75 students responded to the email. Among those students, 70 students participated in the first task, and 67 students participated in the second task (3 students who participated in the first task did not participate in the second task). The participants were first-year college students whose majors related to science or engineering (e.g., mechanical engineering, chemical engineering, physics, etc.). The physics course syllabus was obtained at the beginning of the semester, and each task was administered right after a relevant topic was taught. The participants were given five days to complete each task online.

Developing Simulation-Based Formative Assessments

The basic structure of the formative assessments consisted of a computer simulation and a series of two-tiered questions and constructed response (CR) questions. The questions were posed while students were running a computer simulation as the system directed. A two-tiered question consists of a simple multiple-choice question as its first tier and a justification question asking students to write their justification for their choice as its second tier (see **Figure 1**). Two-tiered questions have advantages over simple multiple-choice questions in that they provide information about students' reasoning or conceptions behind their selected responses (Gurel, Eryılmaz, & McDermott, 2015).



Note: (1)~(5) will be used while you are working on this task.
When you click the "Fire" (6) button, the cannon will shoot an object as you set the variables (angle, speed and mass). If you click the "Erase", the simulation will be reset.
Q. When air resistance is negligible, a cannon shoots a tank shell and a baseball. Before you run the simulation, speculate which one will go farther. The angle and initial speed are the same for both cases.
a. Tank shell b. Baseball c. Same

Q. Explain your choice without using a formula.

*Run the simulation (set the angle at 45, initial speed at 10 m/s).

Q. why do you think it happened?

Figure 1. Example questions in Task 1

The formative assessments were designed for online administration that allowed students to use the system at their convenience. A website was created to integrate the two components— computer simulations and formative assessment questions—so that students were able to answer the questions while they were running a simulation on the same web interface.

According to the course syllabus, an object's motion with the law of motion and mechanical energy with the concept of energy conservation were core ideas to teach in the first half of the semester. In the current study, two topics: (1) motion in two dimensions (projectile motion) and (2) conservation of energy were selected to align with the course syllabus. Computer simulations related to the two topics were selected from the Physics Education Technology (PhET) project, which develops research-based simulations (Perkins et al., 2006) and allows users to embed their simulations on their own websites. PhET simulations were designed to emphasize the connection between physics and everyday life and to facilitate the development of a robust conceptual understanding of physics (Perkins et al., 2006). The selected simulations were (1) Projectile Motion and (2) Energy Skate Park (retrieved on February 2017 from https://phet.colorado.edu). After selecting simulations, CR and two-tiered questions related to the simulations were developed. When developing questions, conceptual understanding in physics were targeted for assessment, thus students were not asked to calculate any values or to demonstrate their mathematical competence. The questions first asked students to predict what would happen in a given situation; then the assessment system asked students to run a simulation and posed questions asking them to explain the observed phenomena (**Figure 1**). In the process, the website system ensured that simulations were not enabled before students responded to the questions asking them to make a prediction. Each task included a series of questions asking students to predict and explain scientific phenomena. To be more specific, after students completed the first set of questions—making predictions and explaining results before and after running a simulation—they were asked to answer a new set of questions. The new set of questions used similar activities, but different simulation settings. For example, the first simulation in Task 1 was a cannon shooting a tank shell and a baseball. After students answered a series of questions related to the first simulation, they were asked to answer a second set of questions, which were associated with a simulation shooting a piano instead of a tank shell. After answering these questions, students interacted with more simulations, which were set in different situations such as different initial speeds and initial angles, and to responded questions in order to examine if they could transfer their conceptions to different situations. Task 1 for the topic (1) included 14 questions, and Task 2 for the topic (2) included 17 questions. Two physics professors (one a retired physics professor who had taught introductory physics courses over 20 years, and the other currently teaching an introductory physics course) were asked beforehand to review the content of the questions to ensure their scientific accuracy and relevance to college freshman. The experts conducted their reviews independently. They made specific suggestions on wording to improve a few questions' clarity, and revisions were made accordingly.

Table 1. Analytic Rubrics for Task 1 (a) and Task 2 (b)

a. Task 1 Scoring Rubric

Idea Types	Ideas	Idea Descriptions
Normative idea	Effect of initial factors	• Mass is not relevant to projectile motion • Initial velocity and/or angle affect(s) projectile motion
	Effect of external factors	• Gravity is the only force acting on an object in non-frictional situations • Object's acceleration is constant in non-frictional situations • Air resistance affects an object's projectile motion
Non-normative idea	Effect of initial factors	• Mass affects an object's projectile motion • Initial velocity or angle is not relevant to an object's projectile motion • Higher velocity or angle results in a greater acceleration or greater force acting on the object • Object size affects its motion in non-frictional situations
	Effect of external factors	• Different amounts of gravity force result in different acceleration rates • A heavier object will have greater air resistance
Off-task	Off topic, repetition	• Repeats the question • Off topic

b. Task 2 Scoring Rubric

Idea Types	Ideas	Idea Descriptions
Normative ideas	Effect of initial factors	• Mass is not relevant to its speed/peak height in non-frictional situations
	Effect of external factors	• Gravity is the only force acting on an object in non-frictional situations • Object's acceleration is constant in non-frictional situations • Friction affects an object's motion
	The concept of energy	• Mechanical energy is conserved in non-frictional situations (the law of energy conservation) • Mass is related to the amount of energy • Some energy is dissipated as thermal energy (heat)
Non-normative ideas	Effect of initial factors	• Mass affects an object's motion
	Effect of external factors	• Different amounts of gravity force result in different acceleration rates • A heavier object will have greater air resistance • Friction doesn't affect an object's motion
	The concept of energy	• The greater amount of energy results in greater speed, and vice versa • Mass is not related to the amount of energy • When mass changes, either PE or KE will change, not both • Heat is an addition to the total amount of energy (the amount of total energy increases when heat is created) • Gravitational potential energy is greater as closer to the ground • Mechanical energy is always conserved in frictional situations (No energy dissipation) • As mass increases, more energy is required to initiate its motion
Off-task	Off topic, repetition	• Repeats the question • Off topic

Scoring Rubric

The formative assessments included both two-tiered questions and CR questions; thus, rubrics for scoring students' written responses were necessary. In the study, analytic rubrics were developed to capture students' normative ideas (relevant scientific ideas) and non-normative ideas (misconceptions) revealed in their responses (**Table 1**). When developing analytic rubrics, student responses were first analyzed qualitatively using open-coding, as codes were developed to explore themes in student responses as they emerged. After a line-by-line analysis of each written response, response codes were categorized into *Ideas* as they shared commonalities, such as focusing on an object's initial factors (mass, shape, velocity, etc.), external factors (external force, air resistance, etc.), or the concept of energy (energy form, transformation, dissipation, and conservation). Those ideas were then classified into normative and non-normative ideas (*Idea types*). In the rubrics (**Table 1**), response codes were presented as *Idea descriptions*. Some students wrote off-topic, nonrelevant responses such as "I don't know," "I saw it in the simulation," or "I learned it from [the] last lecture." In those cases, student responses were classified into *Off-task*. The analytic rubric differs from the holistic rubric in that it breaks the holistic evaluation of students' explanations into normative scientific ideas or non-normative ideas assessed by questions. After developing scoring rubrics, each written response was quantified by tabulating the frequency of normative and non-normative ideas. During the process, two raters independently analyzed student responses and demonstrated high inter-rater

Table 2. Holistic Rubric

Model	Score	Description
Model 1	0	Off-task
Model 2	1	Non-normative ideas only
Model 3	2	Co-existence of normative and non-normative ideas
Model 4	3	Normative ideas only

reliability (kappa coefficients > 0.8). In cases of disagreement between raters, the discrepancies were discussed collectively until agreement was reached. Multiple-choice questions were scored dichotomously (i.e., "1" for correct answers and "0" for incorrect ones).

After analyzing students' responses to questions with the analytic rubric, a holistic rubric was applied to categorize students' responses into four different response models (**Table 2**). This method is similar to that of Moharreri, Ha, and Nehm (2014), who developed a web portal, EvoGrader, to grade students' written explanations in evolution. EvoGrader reports student holistic reasoning models: pure non-normative ideas (non-normative ideas only), mixed ideas (non-normative ideas and normative ideas), and pure scientific ideas (normative ideas only). In the current study, the holistic rubric includes the off-task response model (Model 1) along with the three response models (Moharreri et al., 2014).

Data Analysis

After scoring student responses, the scored data were subjected to analysis using the Rasch model. In particular, the partial credit Rasch model (Masters, 1982) was applied to incorporate differing numbers of response opportunities for different items (questions) in a test, including dichotomously scored items and partial credit scored items (Bond & Fox, 2007). As a result of applying the partial credit Rasch model, an item difficulty estimate for each item was produced in a logit scale. The larger the item difficulty estimate, the more difficult for students to achieve the maximum score on that item. Item difficulty estimates produced by the Rasch model are linear measures on the interval scale, allowing for direct comparisons between item difficulties. Thus, item difficulty estimates from the partial credit Rasch model were used to compare student performances on each question both before and after running a simulation. Winsteps 3.81 (Linacre, 2014) was used for the Rasch model analysis.

RESULTS

The reliability coefficients indicated by Cronbach's alpha was 0.70 for both tasks.

Partial Credit Rasch Modeling Analysis

The Rasch model provides fit statistics presenting how well each item fits within the underlying construct (Bond & Fox, 2007) as evidence of construct validity of measures. Item fit statistics include the mean square residual (MNSQ) and the standardized mean square residual (ZSTD). Rasch modeling analysis also produces INFIT statistics, giving more weight to better fitting responses, and OUTFIT statistics, giving no weight over all responses for MNSQ and ZSTD. Using a commonly used criterion for acceptable fit of items, MNSQ within the range of 0.7 to 1.3 and ZSTD within the range of -2.0 to +2.0, one item (Q1) in Task 1 was found with two statistics out of the acceptable ranges. Thus, Q1 was excluded from the analysis. Question 1 was a multiple-choice question asking, "Between a tank shell and a baseball, which one will go farther when they are shot under the same conditions? Even though it was removed, students' written responses to explain their predictions (Q2) were included in the analysis. After the deletion of the question, 13 questions were subjected to the Rasch modeling analysis to produce item difficulties. In case of Task 2, Q5 deviated from the item response function, but the other questions fit the Rasch model well. Q5 asked students if they observed how the skater's highest speed changed as her mass increased. All of the students chose the same option: that the skater's highest speed stayed the same. No one selected the other options, which were that the highest speed increased or decreased. If a question did not contribute to differentiate students' ability at all, the Winsteps software excluded the question in the analysis. Thus, Q5 was automatically dropped from the analysis.

Effects of Simulation-Based Formative Assessments

Each task was designed for the students to make a prediction about what would happen in a given situation, then explain the prediction. After explaining their prediction, they ran a simulation to test it, and wrote an explanation of the scientific phenomena. After they completed those questions, the students were asked to predict

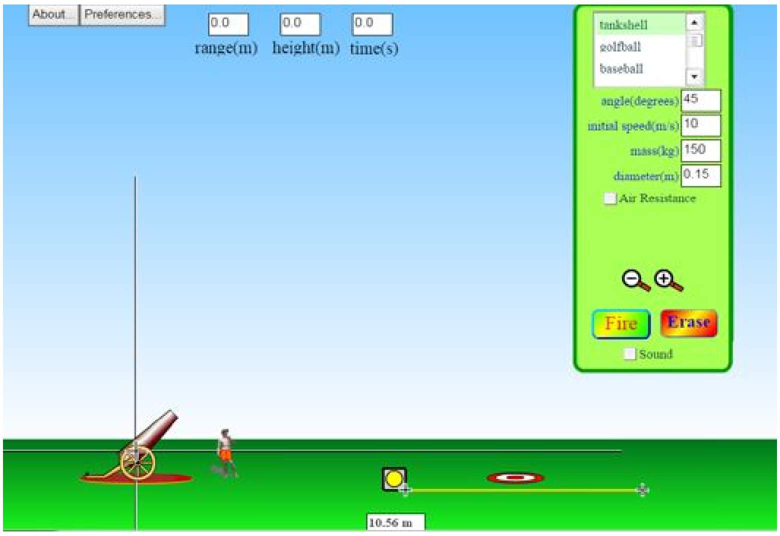


Figure 2. Projectile motion simulation. Source: https://phet.colorado.edu/en/simulation/projectile-motion

Table 3. Descriptions of Formative Assessment Questions in Task 1

Descriptions of questions and simulations	Question number	Question format	Before/After[1]	Item difficulty (logit)
(shooting a tank shell and a baseball without friction in the same condition of initial speed and angle) When a cannon shoots a tank shell and a baseball, which one goes farther?	2	SW[2]	Before	1.19
	3	SW	After	0.44
(shooting a piano and a baseball without friction in the same condition of initial speed and angle) When a cannon shoots a piano and a baseball, which one goes higher?	4	MC[3]	Before	-0.12
	5	SW	Before	0.06
	6	SW	After	0.05
(shooting two tank shells without friction and with the same initial angle) When a cannon shoots two tank shells in a row, which one goes farther? The first tank shell's initial speed is two times greater than the second one.	7	MC	Before	-1.65
	8	SW	Before	-0.11
	9	SW	After	-0.01
(shooting two tank shells without friction and with the same initial speed) When a cannon shoots two tank shells in a row, which one goes farther? The first tank shell's initial angle is 45 and the second one's initial angle is 10.	10	MC	Before	-0.25
	11	SW	Before	0.08
	12	SW	After	0.11
Describe what factors can be changed and how they can be changed to increase the horizontal range of the projectile object.	13	SW	After	-0.20
Elaborate on your explanations for Q12 using the idea of the object's velocity and acceleration in the vertical and horizontal directions	14	SW	After	0.42

Note. 1. Before/After; Before—a question was prompted before a running a simulation, After—a question was prompted after a running a simulation
2. SW: Short written response question
3. MC: Multiple-choice question

and explain a different situation to assess if they were able to transfer their conceptions to a new situation, and/or to generalize what they had learned, using ideas that they had experienced in the simulation.

Task 1. Task 1 included a simulation (Projectile Motion in PhET, **Figure 2**) providing a situation where a cannon shoots an object. The simulation allows users to choose objects (with different shapes and mass) to shoot, and to set an initial angle and speed.

Table 3 presents descriptions of questions and simulation settings, and shows item format and item difficulty estimates before and after experiencing a simulation. At the end of the task, students were asked to generalize what factors should be changed to increase the horizontal range of the projectile object (Q13); this was in order to examine if the students could integrate what they had learned through the activities. Q14 provided another opportunity for students to explain, using the vector concept in velocity and acceleration. This was to see if the students could apply the concept of vector in motion, which was important to understanding the projectile motion.

Three multiple-choice questions demonstrated lower item difficulties as compared to the short written response questions (e.g., second-tier questions or CR questions). The findings indicated that students were not always able to give an explanation using normative scientific ideas, even when they had selected the correct option in the

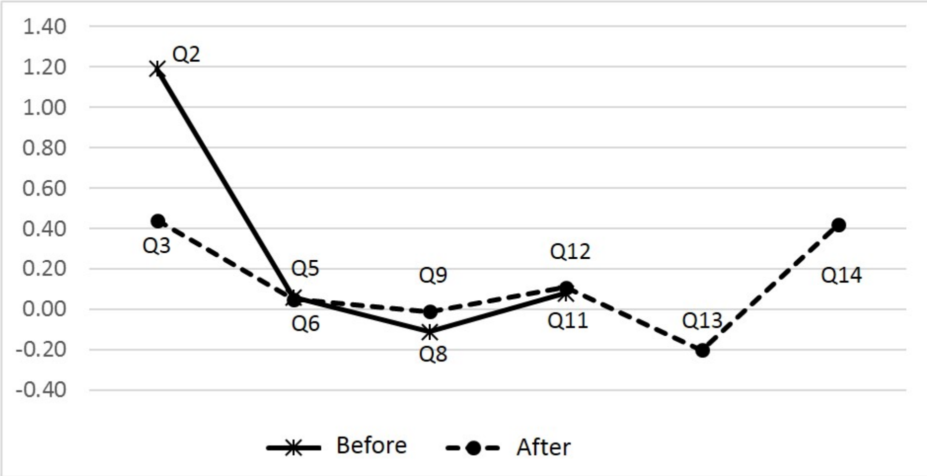


Figure 3. Task 1 item difficulties before and after running a simulation

multiple-choice question. For example, in the cases of Q10 and Q11, some students who chose the multiple-choice option that the first tank shell would go farther (Q10) wrote that gravity would more directly affect the tank shell with the angle of 10 degrees, or that, because its angle was greater, it would result in more area to cover. As such, in order to compare the students' responses before and after running the simulations, it was necessary to compare the students' responses to the same format of questions. Thus, item difficulties of the short written response questions were compared before and after a running simulation (**Figure 3**).

As seen in **Figure 3**, Q3's item difficulty was lower than Q2's, indicating that the students had less difficulty answering the question after running the simulation. This supported the effectiveness of computer simulations that helped students explain scientific phenomena using normative scientific ideas. Except for Q2 and Q3, the findings showed that item difficulties of questions posed after running a simulation—asking the students to explain observed scientific phenomena from the simulation (explanation questions)—were not always lower than those of questions students answered before a running the simulation—asking them to make a prediction about a given situation (prediction questions). Specifically, the explanation questions showed similar difficulty locations (Q6, Q9, and Q12) to their paired prediction questions (Q5, Q8, and Q11, respectively). When comparing Q2 and Q5 (prediction questions), although they were designed around a similar situation, Q5's item difficulty was less than Q2's item difficulty (the difference between the two item difficulties was 1.13 logits). The differences between the two questions were that the cannon would shoot a piano (Q5) instead of a tank shell (Q2); additionally, Q5 asked which object would go higher and Q2 asked which one would go farther. The result showed that after experiencing the first set of questions with a simulation, students had less difficulty making a correct prediction for the given situation. Students had seen in the simulation that two different objects (with different mass and shapes) would travel exactly the same, and revised or confirmed their initial thoughts in Q3 (explanation question), which helped them make a correct prediction in answer to the question. In the case of two objects with either different initial velocities or different initial angles, the item difficulties for both the prediction and explanation questions were located similarly. Their locations were also similar to those of the previous question set (Q5 and Q6), implying that students might demonstrate their conceptions consistently across slightly different situations asking about the same scientific ideas (factors affecting an object's projectile motion). Q13, which asked students to generalize their conceptions on projectile motion, showed the lowest difficulty location among all of the short written response questions, which might imply that, after running all of the simulations, their explanations were more scientifically valid. Q14's difficulty location explained that the students had more difficulty explaining projectile motions using velocity and acceleration ideas in two different directions, which required them to apply a new concept, vector.

Table 4 presents descriptive statistics for the short written response questions in Task 1 using the holistic rubric.

Table 4. Descriptive Statistics for Task 1 Short Written Response Questions

Question	Frequency (%)				
	Model 1 Score 0	Model 2 Score 1	Model 3 Score 2	Model 4 Score 3	Mean Score
Q2	0 (0)	24 (34.3)	0 (0)	46 (65.7)	2.31
Q3	6 (8.6)	5 (7.1)	1 (1.4)	58 (82.9)	2.59
Q5	2 (2.9)	11 (15.7)	0 (0)	57 (81.4)	2.60
Q6	3 (4.3)	2 (2.9)	0 (0)	65 (92.9)	2.81
Q8	2 (2.9)	2 (2.9)	1 (1.4)	65 (92.9)	2.84
Q9	2 (2.9)	5 (7.1)	4 (5.7)	59 (84.3)	2.71
Q11	2 (2.9)	10 (14.3)	3 (4.3)	55 (78.6)	2.59
Q12	2 (2.9)	13 (18.6)	1 (1.4)	54 (77.1)	2.53
Q13	1 (1.4)	2 (2.9)	15 (21.4)	52 (74.3)	2.69
Q14	5 (7.1)	7 (10.0)	3 (4.3)	55 (78.6)	2.54

In the case of Q3, the frequency of Model 4 (normative ideas only) responses increased and the frequency of Model 2 (non-normative ideas only) responses decreased, compared to Q2. Q5 and Q6 showed similar patterns: that the number of students' non-normative responses decreased, and their normative responses increased, after running a simulation. The result demonstrated that more students' responses included scientifically normative ideas instead of non-normative ones after experiencing a computer simulation. Q8 and Q11 asked students to make predictions about the projectile motions of the same object, but with different initial velocities or angles. In general, the students' predictions were scientifically correct (Model 4 responses) when the objects' initial velocities were different but other factors were the same (Q8), while the number of correct responses decreased in a situation that changed the initial angles (Q11). As **Table 4** presented, the number of the students' non-normative ideas increased in Q11, compared to Q8. Even after a simulation, the number of Model 2 responses did not decrease (Q12). Interestingly, in Q9, the number of responses in Models 2 and 3 (non-normative ideas only and mixed ideas, respectively) increased, compared to Q8 responses. Those results did not support the effectiveness of using simulations for student scientific learning. Examples of responses that changed from Model 4 to Models 2 or 3, between Q8 and Q9, and between Q11 and Q12 are presented below.

Student A's responses

Q8: x-displacement formula depends on the initial speed of the object. Tank shell's A initial speed was greater therefore will go higher. (Model 4)

Q9: The greater the speed the greater the acceleration and the object will go farther. (Model 2)

Student B's responses

Q8: The faster the velocity is while having the same angle will make the object go further in the x direction. (Model 4)

Q9: The slower object did not have enough speed to combat gravity the way an object at a faster speed would. (Model 3)

Student C's responses

Q11: They are fired at the same speed and 45 degrees is the optimal angle for an object to travel the most distance. The higher angle will travel higher and allow for more time in flight. (Model 4)

Q12: It (lower angle one) spent less time accelerating up. (Model 2)

These examples illustrate that the students used non-normative ideas to given an explanation after running a simulation. They were allowed to differentiate their responses from previous ones, which gave students an opportunity to express their non-normative ideas, implying that they already had the non-normative ideas before running the simulation. It is also possible that the simulation helped the students to uncover their non-normative ideas.

With the exceptions of those responses, the findings support the idea that computer simulations, coupled with a series of formative assessment questions, were effective for students to find their non-normative ideas and revise their answers to make them scientifically valid. After running a couple of similar simulations, the number of student response models did not show much difference between prediction questions and explanation questions.

Task 2. The Task 2 simulation (Energy Skate Park in PhET, **Figure 4**) was designed to demonstrate the concept of the conservation of mechanical energy using kinetic energy (KE) and gravitational potential energy (PE). The simulation allows users to change a skater's mass and track shapes and to include or remove friction.

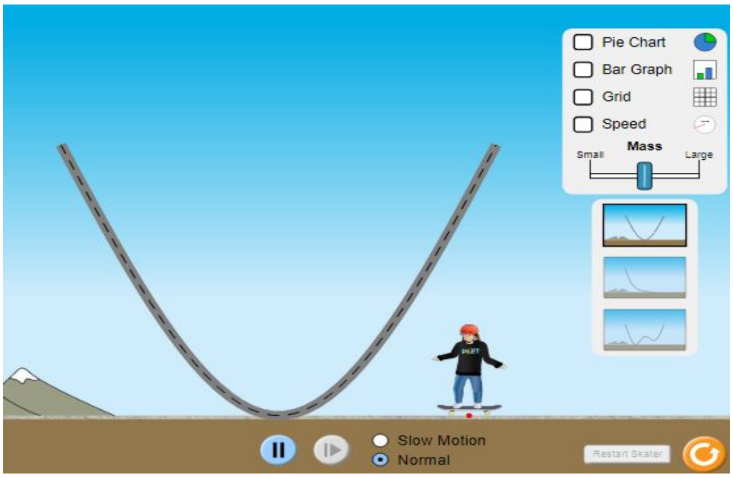


Figure 4. Energy skate park simulation. Source: https://phet.colorado.edu/en/simulation/energy-skate-park-basics

Table 5. Descriptions of Formative Assessment Questions in Task 2

Descriptions of questions and simulations	Question number	Question format	Before/After[1]	Item difficulty (logit)
(U-Shaped track without friction) What will happened to the skater's highest speed if you increase her mass?	1	MC[2]	Before	2.19
	2	SW[3]	Before	0.39
(U-Shaped track without friction) How will skater's total mechanical energy change when her mass increases?	3	MC	Before	-0.06
	4	SW	Before	0.63
(U-Shaped track without friction) How did the skater's total mechanical energy change according to the bar graph in the simulation?	6	MC	After	-0.06
(U-Shaped track without friction) Explain why her speed changed or didn't change.	7	SW	After	-0.15
(Ramp-shaped track without friction) How will the skater's speed be different when she arrives at the ground as her mass increases?	8	MC	Before	-0.44
	9	SW	Before	0.14
(U-Shaped track with friction) How will the skater's peak height change as she is skating along the track?	10	MC	Before	-2.18
	11	SW	Before	-0.88
(U-Shaped track with friction) How will the skater's total mechanical energy change as she is skating along the track?	12	MC	Before	0.65
	13	SW	Before	0.70
(U-Shaped track with friction) Explain the relationship between the amount of thermal energy changes, the skater's peak height, and total amount of energy.	14	SW	After	-0.46
(Ramp-shaped track with friction) Speculate the amount of kinetic energy of the skater when she arrived at the ground.	15	MC	Before	0.11
	16	SW	Before	0.11
(Ramp-shaped track with friction) Speculate about how the amount of kinetic energy of the skater will change as she travels along the ground after leaving the track.	17	SW	Before	-0.69

Note. 1. Before/After; Before—a question was prompted before a running a simulation, After—a question was prompted after a running a simulation
2. MC: Multiple-choice question
3. SW: Short written response question

Table 5 presents detailed descriptions of the questions and item difficulty estimates before and after experiencing a simulation. At the end of the task, students were asked to speculate how the amount of kinetic energy would change as the skater traveled along the ground after leaving the track. The question was intended to see if students would be able to transfer their energy conceptions to flat ground.

Task 2 questions included more prediction questions than explanation questions. As seen in Task 1, the explanation questions' difficulties were similar to the prediction questions' difficulties after students experienced simulations; however, student responses seemed to be affected when they were required to consider a new concept for the given situation. In that regard, Task 2 involved a greater variety of situations requiring students to consider new concepts than Task 1. Although students were not asked to explain observed phenomena every time, the website ensured that after answering prediction questions, students should run the simulation associated with the prediction question in order to move to the next set of questions. **Table 5** showed that Q7 and Q14 (explanation questions) were easier for the students than their paired prediction questions (Q2 and Q13, respectively).

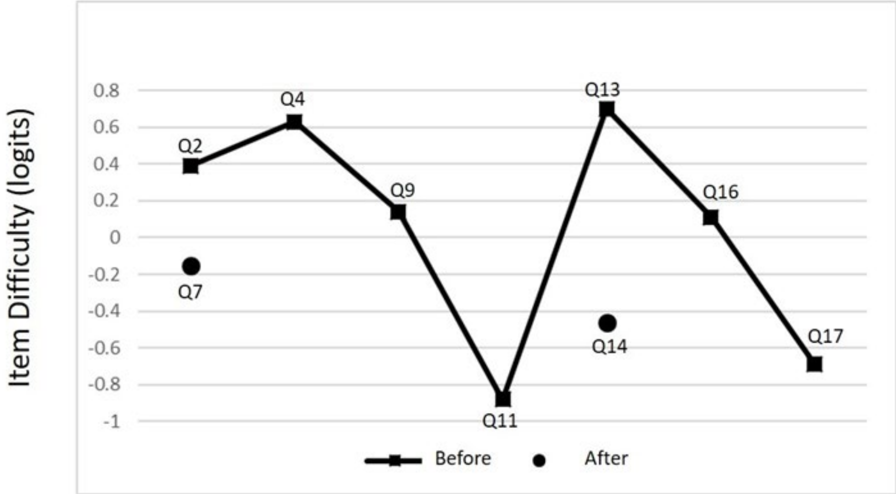


Figure 5. Task 2 item difficulties of questions before and after running a simulation

Table 6. Descriptive Statistics for Task 2 Short Written Response Questions

Question	Frequency (%)				Mean Score
	Model 1 Score 0	Model 2 Score 1	Model 3 Score 2	Model 4 Score 3	
Q2	2 (3.0)	24 (35.8)	27 (40.3)	14 (20.9)	1.79
Q4	0 (0.0)	18 (26.9)	11 (16.4)	38 (56.7)	2.30
Q7	3 (4.5)	13 (19.4)	6 (9.0)	45 (67.2)	2.39
Q9	0 (0.0)	11 (16.4)	8 (11.9)	48 (71.6)	2.55
Q11	1 (1.5)	3 (4.5)	3 (4.5)	60 (89.6)	2.82
Q13	0 (0.0)	20 (29.9)	9 (13.4)	38 (56.7)	2.27
Q14	2 (3.0)	2 (3.0)	14 (20.9)	49 (73.1)	2.64
Q16	3 (4.5)	23 (34.3)	8 (11.9)	33 (49.3)	2.06
Q17	1 (1.5)	9 (13.4)	5 (7.5)	52 (77.6)	2.61

The multiple-choice questions' item difficulties were lower than or similar to those of their paired short written response questions, with one exception. Q1 asked students to predict how a skater's highest speed changes when her mass increases; only 20 students out of 67 chose the correct option. This is a common misconception for students: that heavier objects fall faster than lighter objects. The findings showed that Q1 was the most difficult question in Task 2 for the participant students.

To investigate the student responses in more depth, the item difficulties and descriptive statistics for the short written response questions are presented in **Figure 5** and **Table 6**.

Students answered Questions 2 and 4 before running the first simulation. After running the first simulation, the explanation question (Q7) showed a lower item difficulty, which supported the idea that the simulation was effective for students in explaining the given scientific phenomena. When students were asked question similar to Q2 but in a different situation (Q9; skating on a ramp-shaped track rather than a U-shaped one), Q9's difficulty decreased. The situation in Q11 was close to what students had actually observed and experienced in real life, asking how a skater's peak height would change on a U-shaped track that had friction. The question demonstrated the lowest item difficulty in the task. However, when students were asked about total mechanical energy changes that they couldn't observe directly in real life, the item difficulty increased (Q13). This question assesses students' understanding of the energy dissipation concept, meaning that in order to answer the question, students should take into consideration that mechanical energy transforms into thermal energy. After they experienced a simulation, the students had less difficulty answering its paired explanation question (Q14), which again supported the idea that the simulation helped students better understand the concept of energy dissipation. Q16 and Q17 were situated on a ramp-shaped track with friction, asking how the skater's KE would change when she arrived at the ground and was traveling on the ground. The two questions' difficulties were lower compared to the previous questions (Q4 and Q13) asking about a similar concept, the skater's mechanical energy. In sum, the findings supported the positive effects of simulation-based formative assessment, not only on explaining given scientific phenomena, but also on students' predictions.

Table 6 presents descriptive statistics for the short written response questions in Task 2, using the holistic rubric (**Table 6**).

In the case of Q2, more than 75% of the students' responses were categorized as Model 2 or Model 3 responses, indicating that non-normative ideas were prevalent in the participant students. Once students experienced the first simulation, the frequency of Model 4 responses increased, while the frequency of Model 2 and Model 3 responses decreased, meaning that more students became able to explain the observed phenomena using normative ideas. Q13 also elicited many students' non-normative ideas (e.g., the total amount of mechanical energy will be always conserved). After experiencing the simulation, their explanations were improved by including normative ideas (Q14). Noticeably, Model 3 responses increased in Q14 responses compared to Q13 responses, implying that some of responses shifted from the only non-normative response model (Model 2) to the mixed model (Model 3) when the students explained the observed phenomena. Although the frequency of Model 2 responses decreased after running the simulation, as we have seen with Q14, their non-normative ideas emerged again in Q16. Q13 and Q14 were situated on a U-shaped track, while Q16 was in a ramp-shaped track, and they both were designed to examine if students could apply the energy dissipation concept along with the mechanical energy concept. Contrarily, Q17 asked students if they could provide the amount of KE's change on the ground with friction. In this case, more than half of the students' responses were categorized as Model 4 and only nine students' responses were found to be Model 2 responses. The findings indicated that the students had difficulty in applying the energy dissipation concept across different situations, especially when they were required to take gravitational PE into account; however, in a more common situation, such as skating on the ground (no need to consider gravitational PE), more students could explain the given phenomena using valid ideas such as the energy dissipation concept. Below are examples of a student's responses to Q13, Q14, Q16, and Q17, which changed from Model 2 to Model 4 after running a simulation but returned to Model 2 in a different situation (a ramp-shaped track), then demonstrated a Model 4 response in a more familiar situation (the ground).

Student D's responses

Q13: Stay the same because the mechanical energy will increase if the speed increases and the speed stayed the same. (Model 2)

Q14: The thermal energy is the energy lost to friction, which means the mechanical energy has decreased. (Model 4)

Q16: Same, because the energy in the potential gets transferred into kinetic. (Model 2)

Q17: As the skater leaves the track, it will turn into thermal energy which would decrease the kinetic energy. (Model 4)

Overall, the result supports the positive effects of simulations coupled with a series of formative assessment questions for helping students to predict and explain scientific phenomena. However, when students needed to apply an abstract scientific concept, such as energy dissipation, the result was not always promising.

DISCUSSION

As simulations-based assessments become ever more popular in science classes (de Klerk, Veldkamp, & Eggen, 2015), there is a growing need to study how students learn while interacting with the assessments. This study explored the effects of simulation-based formative assessments on students' conceptions in physics, especially focusing on the changes in student responses across different situations. Formative assessment questions were designed to first elicit students' prior knowledge and thinking by asking a prediction question, then to enable them to run a simulation, and then to provide them an opportunity to evaluate their previous answer by explaining the observed phenomena. Bransford, Brown, and Cocking (2000) and Tao and Gunstone (1999) emphasized the elicitation of students' pre-existing understanding and the provision of opportunities for challenging their initial understanding for concept change and conceptual learning in science. After giving students two assessment tasks, an exploratory study was conducted to gain insight into how the students' responses changed in quality while they were interacting with the tasks.

The results showed that, overall, students better explained the given scientific phenomena after they had experienced simulations. Also, after experiencing the first simulation, students were able to make a prediction for a similar situation using valid scientific ideas, which supports the effectiveness of using simulations coupled with formative assessment questions in learning science. This might be due to the possibility that students could add a new knowledge element or refine their existing ones through interacting with the simulations and answering questions. The findings indicated that the simulation-based formative assessment had a positive effect on enhancing students' conceptual development of scientific ideas, which is consistent with previous studies on the effectiveness of simulation-based formative assessment on student conceptual change (Quellmalz et al., 2012; Srisawasdi & Panjaburee, 2015).

However, the findings also indicated that students' non-normative ideas emerged even after experiencing simulations, especially when they were required to consider an abstract concept (e.g., energy dissipation). Previous studies have found that students have difficulty understanding energy dissipation (e.g., Black & Solomon, 1983) and the conservation of energy (e.g., Boyes & Stanisstreet, 1990; Driver & Warrington, 1985). Duit (2012) mentioned that students are able to understand energy conservation only if energy dissipation is also considered. Recently, research has suggested that energy dissipation and conservation should be thought of in conjunction (Park & Liu, 2016). The findings demonstrated that many students were able to predict and explain scientific phenomena with normative ideas related to energy conservation in non-frictional situations after they experienced the first simulation; however, their non-normative ideas emerged again in frictional situations, especially when they were required to consider both PE and KE. This might be due to the possibility that students predicted and explained scientific phenomena based on their rote memorization of the energy conservation law, e.g., the total amount of PE and KE is always conserved. This finding is consistent with the previous studies above that found that students lack a true understanding of energy conservation, which requires an understanding of energy dissipation.

The current study supports the claim that simulation-based formative assessments are effective to enhance students' learning in science (e.g., Quellmalz et al., 2012; Srisawasdi & Panjaburee, 2015). The study, however, also argues that the improvement might be the case only in certain situations. In the study, students demonstrated better abilities to predict and explain the given scientific phenomena after running a simulation, whereas, when an abstract concept was required to apply to answer questions, their abilities regressed. The finding can be explained with knowledge-in-piece perspectives (diSessa, 1993), that students' naïve knowledge is fragmented, and thus they do not demonstrate a coherent understanding of abstract science concepts across different situations. To predict and explain scientific phenomena coherently, a student must have integrated and organized the knowledge elements (Ozdemir & Clark, 2009).

Overall, although the students produced explanations with normative ideas in certain situations, the current study underlines that the use of computer simulations coupled with guiding questions did not always reduce invalid explanations and non-normative ideas, especially when considering abstract concepts across different situations. In a limited sense, simulation-based formative assessments had a positive effect on students' conceptual learning; they demonstrated this positive change by an increase in the number of responses that contained normative ideas close to the scientific explanations. But the findings also indicated that the students did not always show positive progress after they experienced simulations, especially when abstract concepts were introduced. Smetana and Bell (2012) suggest that high-quality support structures (e.g., feedback, training on how to use simulations, and scaffolding) embedded in simulations or provided by instructors, as well as multiple opportunities for students to reflect upon their conceptions, are critical aspects of the successful use of computer simulations. Given the current study's focus on students' individual performance data on assessment tasks, it is suggested that instructional strategies be aligned with assessment tasks, such as providing appropriate training on using simulations, instructors' feedback on student explanations and finding evidence, and more opportunities to reflect on their conceptions in the classroom. The results might be different if the assessment tasks were embedded in the course curriculum or were supported by instructor guidance (Rutten, Joolingen, & van der Veen, 2012). Trundle and Bell (2010) emphasize the connection between computer simulation and pedagogy, especially for promoting conceptual change. Further, they suggest that instructional activities using computer simulations should emphasize a climate of collaboration among students for successful learning. In that regard, the assessment tasks could be used as group tasks to enhance their discourse.

Study limitations and suggested future research are presented below. The study included a relatively small samples of students, a fact that might have compromised the generalization of the results. In an attempt to overcome this limitation, it is necessary to implement the study with a bigger number of students or employ a mixed-methods approach (Creswell, 2014) to provide a more comprehensive examination of the research question.

As students' competency in mathematical modelling plays an important role in learning physics (Redish, 2017), the results might be different if tasks would have been explicitly linked with mathematical modelling of simulated situations with students prompted to demonstrate their competency of mathematical modelling. Also, studies have found that computer simulations are more effective in group or whole-class settings to promote conceptual change (Trundle & Bell, 2010). Future research is required to examine the effectiveness of using the assessment tasks in group or whole-class settings. The study included two tasks to examine students' conceptions in physics; thus, generalizing the results to other topics requires additional research.

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Modelling Opportunities to Learn, Mathematical Belief and Knowledge for Teaching among Pre-service Teachers

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ABSTRACT

Issues about low level of mathematical knowledge for teaching among pre-service teachers has raised the question on the effectiveness of the mathematics teacher education program which has been planned and implemented by the Malaysian Institute of Teacher Education (MITE). This study was conducted to identify factors that affect mathematical knowledge for teaching (MKT) among pre-service teachers in Institute of Teacher Education (ITE). The influence of mathematical belief and opportunity to learn (OTL) have been tested to explain the factors affecting MKT. Using a structured questionnaire together with paper and pencil test adapted from the literature reviewed, data were collected from 105 pre-service teachers in MITE. Data were analysed using SmartPLS version 3.0. The result of the structural equation model indicated that OTL-Practicum (β=0.491, p<0.001) and OTL-Program (β=0.368, p<0.001) has a positive relationship with mathematical knowledge for teaching. Besides that, the result for the impact of OTL on mathematical belief, it showed that OTL-Practicum (β=0.208, p<0.001) and OTL-Program (β=0.243, p<0.001) has a positive relationship with constructivist belief, whereas OTL-Program was negatively related to traditional belief (β=-0.283, p<0.001). Overall, the model explained 53.9% of the variance in mathematical knowledge for teaching. Implications from these findings to the ITE were further elaborated.

Keywords: opportunities to learn, mathematical belief, mathematical knowledge for teaching, constructivist belief, traditional belief

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) (2000) states that one of the principles of mathematics education is to enhance mathematical knowledge for teaching to become an effective teacher. Issues related to the knowledge of mathematics teachers and their role in classroom practice have been a major issue in mathematics education from the past (Wasserman, 2018). In addition, according to Fitzallen (2015) the mastery of mathematical content knowledge also contributes to the success in the implementation of four disciplines namely Science, Technology, Engineering and Mathematics (STEM).

The framework of mathematics teacher knowledge can be divided into two parts, the content knowledge and content knowledge for teaching mathematics framework (Holmes, 2012). According to Holmes (2012), the content knowledge framework consists of Bloom's Taxonomy (Bloom, Englehard, Furst, Hill, & Krathwohl, 1956), Instrumental and Relational Understandings (Skemp, 1978); Procedural and Conceptual Understandings (Hiebert & Carpenter, 1992) and Cognitive Complexities (Porter, 2002). Content knowledge for teaching frameworks include Shulman's (1986), Type of Teachers Knowledge and Ball's (2000) Mathematical Knowledge for Teaching Framework.

The concept of pedagogical content knowledge (PCK) was introduced by Shulman (1986) as so-called "a missing paradigm" in the study of teaching and teacher education. Shulman has criticized the lack of attention given to the content of lessons related to teaching and evaluation of pre-service teachers as well as studies on the effectiveness

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Contribution of this paper to the literature

- This is one of the articles that examine the influence of opportunities to learn and mathematical belief on mathematical knowledge for teaching by using the structural equation modelling.
- The results indicate a significant effect of opportunities to learn on mathematical knowledge for teaching and constructivist belief.
- Opportunities to learn through teaching practice (OTL-Practicum) is the stronger factor affecting mathematical knowledge for teaching compare to opportunities to learn through a coherent teacher education program (OTL-Program).

of teachers and teaching. By introducing the PCK concept, Shulman intends to emphasize the content of lessons learned in teaching and teacher education and aims to address the differences between content and pedagogy (Depaepe, Verschaffel, & Kelchtermans, 2013).

The mathematical knowledge for teaching (MKT) is the concept of mathematical knowledge required by a teacher to teach effectively. It includes assessing student responses, responding to questions raised by students, preparing assignments and making lesson plans (D. L. Ball, Thames, & Phelps, 2008). According to Austin (2015), the concept of MKT developed by Ball et al. (2008), is a multi-dimensional construct consisting of subject matter knowledge (SMK) and pedagogical content knowledge (PCK).

Ernest (1989) has stated that, the differences between mathematics teachers are not only because of their knowledge, but also related to their beliefs. This is because it is possible for two different teachers to have similar knowledge, but they might be teaching the students with different approach. Teacher's beliefs have become a popular field in education-related studies because of their relationship to knowledge to teach (Thompson, 1992). According to Cross (2009) beliefs are conscious or unconscious opinions and views of the individual about himself, about the world or about his place in the world. According to Ernest (1989), there are generally three categories of beliefs associated with mathematics teachers, namely beliefs about the nature of mathematics, belief in teaching and learning, and beliefs related to principles of education. For this research, we are focusing on the belief in teaching and learning, which is also known as mathematical belief (MB) (Beswick, 2012).

In the Malaysian context, empirical studies related to the knowledge of mathematics teachers are less widely known. Among the studies conducted on the knowledge of mathematics teachers in Malaysia were by Zulhelmi Zulkpli, Mohini Mohamed, and Abdul Halim Abdullah (2017); Kwan Eu Leong, Chew Cheng Meng, and Suzieleez Syrene Abdul Rahim (2015); Yusminah Mohd Yusof, and Effandi Zakaria (2015); Harizon Suffian, and Shafia Abdul Rahman (2010); Tengku Zawawi Tengku Zainal, Ramlee Mustapha, and Abdul Razak Habib (2009) and Noraslina Hassan, and Zaleha Ismail (2008). The findings from the critical analysis that have been carried out on previous studies indicate that most previous studies are more focused on measuring the level of competence on teacher knowledge. Hence there is a need to conduct a study on MKT of mathematics teachers in Malaysia in the context of the factors that influence them.

An excellent mathematics teacher should master both domain of mathematical knowledge for teaching, either in terms of content knowledge or pedagogical content knowledge (Ball et. al., 2008). This is because the mastery of MKT is important indicator of the success of a teacher education program (Tatto, Rodriguez, & Lu, 2015). In addition, it also affects the mathematical achievement of a pupil (Goos, 2013). However, according to Kwan Eu Leong et al. (2015) the level of MKT among pre-service teachers in Malaysia is low. The low level of MKT among pre-service teachers will contribute to the implementation of less effective teaching and learning processes (Ball et al., 2008). Hence there is a need to conduct a study to identify the factors that affect MKT among pre-service teachers.

There are several studies that have been conducted to identify the factors affecting teacher's knowledge. Among them are studies by Konig et al. (2017); Blömeke, Jenßen, Grassmann, Dunekacke, and Wedekind (2016); Qian, and Youngs (2016); Tatto et al. (2015) and Rachel A. Ayieko (2014) who studied the influence of Opportunities to Learn (OTL) on teacher knowledge. The role of OTL on the mastery of mathematical knowledge among teachers have been studied in several aspects. Among them are the influence of OTL mathematical content (OTL-Content), OTL teaching practice (OTL-Practicum), OTL a coherent teacher education program (OTL-Program), OTL mathematics pedagogy and OTL general pedagogy. The findings showed that OTL affect the teacher's knowledge directly. In addition, the OTL factor had also been found to influence the belief of a teacher. Studies by Rachel A. Ayieko (2014), and Philippou, and Christou (2002) found that there was a positive relationship between OTL and teachers' belief.

Hence, this study implements a MKT model (Ball et al., 2008), mathematical belief model (Ernest, 1989) and the concept of OTL from Carroll (1963) to examine the role of OTL (OTL-Practicum and OTL-Program) in influencing mathematical belief (MB) and MKT among pre-service teacher in Malaysia, by using structural equation modelling (SEM).

Significance of the Study

Significance of this study is seen in terms of its contribution to theory and practice. The findings have contributed significantly to the body of knowledge by producing a comprehensive model to explain the factors affecting MKT among pre-service teachers. This model has combined both factors from the context of teachers' belief and OTL they have acquired during teacher education programs.

This study was also one of the studies on the factors affecting MKT among pre-service teachers in Institute of Teacher Education (ITE) by using SEM method. Therefore, the result of this study can be used by various stakeholders such as the Ministry of Education (MOE) Malaysia, especially the Malaysia Institute of Teacher Education (MITE) who is responsible for the training of future math teachers. The MITE can use the findings from this study as a guideline in developing a teacher education program capable of producing competent mathematics teachers. In addition, the findings of this study can also be used as references to other higher education institutions who are responsible for training potential math teachers to ensure that future teachers will master the MKT before they are placed in school.

Findings from this study can also be utilized by pre-service teachers who are studying in ITE and in any other higher education institutions to understand the factors that affect their MKT. Through that understanding, it will be able to create awareness for them to appreciate every opportunity they earned during the teacher education program.

Furthermore, the findings of this study can also be used as a reference to future researchers who study the factors affecting pre-service teachers' knowledge. The findings of this study are also expected not only relevant in the context of factors affecting pre-service teacher knowledge in mathematics, but it also includes teachers' knowledge in other disciplines. Hence this study is very significant to be carried out to contribute towards theory and practical.

THEORETICAL BACKGROUND

There are several models being used to study the role of mathematical belief and OTL in influencing CK, PCK and MKT by previous researchers. A number of studies on teacher's knowledge have been examined using the Shulman (1987) model, Fennema, and Franke (1992) model and Ball et al. (2008) model. However, the conceptual framework of the most influential teachers in the context of mathematics education is through the overlapping of some mathematical knowledge constructs for teaching (MKT) or content knowledge for the teaching of mathematics (CKTM) covering both content knowledge (CK) and pedagogical content knowledge (PCK) (Ball et al., 2008; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005).

MKT means the mathematical knowledge needed to carry out the work of teaching mathematics. In short, a mathematics teacher needs to know more, and different mathematics not less (Ball et. al., 2008). MKT covers three categories that relate to teachers' CK: (1) common content knowledge (CCK), (2) specialized content knowledge (SCK), and (3) horizon content knowledge (HCK). Another set of three categories within MKT concern teachers' PCK: (4) knowledge of content and students (KCS), (5) knowledge of content and teaching (KCT), and (6) knowledge of content and curriculum (KCC) (Ball et. al., 2008).

Teachers' beliefs have become a popular field in education-related studies because of their relationship to knowledge to teach (Thompson, 1992). Although the term "*belief*" is very popular among educational researchers, there is no definite definition (Pajares, 1992). For example, according to (Cross, 2009) "*beliefs are conscious or unconscious opinions and views of the individual about himself, about the world or about his place in the world. These opinions develop during the individual's joining in different social groups and they are considered as correct by the individual* ". Philipp et al. (2007) also defines belief as "*psychologically held understandings, premises, or propositions about the world that are thought to be true*". Meanwhile, Richardson (1996) defines belief as "*understandings, premises or propositions about the world that are felt to be true*".

Based on the views of most researchers, belief is a structure that is accepted as true and can influence behaviour (Kul & Celik, 2017). In addition, beliefs also influence the kind of knowledge teachers will use to teach in the classroom (Leinhardt & Greeno, 1986). According to Ernest (1989), there are generally three categories of beliefs associated with mathematics teachers, namely the beliefs about nature of mathematics, beliefs about teaching and learning, and beliefs related to educational principles.

In addition to the term beliefs related to teaching and learning, there are also researchers who use the term mathematical beliefs, it carries the same meaning (Beswick, 2012). There is agreement between previous researchers that teachers' beliefs regarding mathematics teaching and learning play an important role in determining the teaching objectives of teachers and directly affect their professionalism (Cross, 2009; Philipp, 2007).

Based on the critical analysis carried out, there are two main categories of teacher mathematical beliefs, namely (1) traditional beliefs or transmission views, it means the belief that teaching mathematics is a process of knowledge delivery by teachers and students accepting such passive knowledge, and (2) constructivist beliefs, it means the teacher's belief that mathematics teaching is the process of helping to build student knowledge (Lim & Chai, 2008).

The term Opportunity to Learn (OTL) was first used by Carroll (1963) to clarify the "*allowed time to study*", it was identified as a learning success factor during an educational program. The concept of OTL was introduced about half a century ago by the *First International Mathematics Survey* conducted by the *International Association for the Evaluation of Educational Achievement (IEA)* (Ting-Ying Wang, & Shu-Jyh Tang, 2013). The concept of OTL is usually used in evaluating the effectiveness of a teacher education program. The OTL serves as an indicator of the variation of the curriculum and the diversity of lessons learned by a pre-service teacher (Tatto et al., 2008).

RESEARCH MODEL

A study conducted by Swars, Hart, Smith, Smith, and Tolar (2007) found that mathematical beliefs factor has influenced teacher knowledge. The study was conducted among 103 elementary pre-service teachers. Additionally, recent studies conducted by Ren, and Smith (2017) also found that teachers' belief factor influenced the mastery of mathematical knowledge for teaching among teachers. The findings from the study on 396 early teachers found that there was a significant relationship between traditional beliefs and mathematical knowledge for teaching.

Furthermore, the research conducted by Meschede, Fiebranz, Möller, and Steffensky (2017) on teachers teaching elementary science has proven that the beliefs in teaching and learning had influenced the teacher's knowledge. The findings from their research had found that constructivist belief factor influenced the teachers' knowledge (β=0.52, p<0.001). While the traditional beliefs factor also had a significant relationship with the teacher's knowledge (β= -0.37, p<0.001). Therefore, this study will also examine the influence of mathematical beliefs on the MKT among pre-service teachers in ITE.

H1: Constructivist belief has a direct and significant influence on mathematical knowledge for teaching

H2: Traditional belief has a direct and significant influence on mathematical knowledge for teaching

Previous studies have shown that OTL variables influence the teacher's knowledge and teacher's belief. Among them are studies by Rachel A. Ayieko (2014) on the teachers from three country found that the opportunity to learn mathematics pedagogy has influence their belief and knowledge. Besides that, the study conducted by Konig et al. (2017) on prospective high school English teachers found that OTL content and OTL teaching practices have influenced their pedagogical content knowledge. Regression analysis showed that OTL was able to predict the positive score of the PCK test teacher (β= 0.28 (p<0.01) for OTL content, and (β= 0.29 (p<0.01) for OTL teaching practice (practicum). In addition, the study by Akkoç, and Yesildere (2010) also found that OTL teaching practice (practicum) influence teachers' PCK significantly.

The study by Kleickmann et al. (2013) and Tatto et al. (2012) on pre-service teachers also found the opportunity to follow the coherent teacher education program (OTL-Program) also influenced the mastery of CK and PCK of the teacher. This clearly shows that OTL is an important factor affecting the mastery of knowledge and academic achievement of future teachers. In addition, recent studies conducted by Livy and Downton (2018) also found that the OTL-Practicum factor has influenced the mastery of mathematical knowledge for teaching among pre-service teachers. The findings from the case study conducted among 20 second-year pre-service teacher found that the course experiences provided an opportunity to extend pre-service teachers' knowledge. Hence, this study will also examine the influence of OTL on the MKT among pre-service teachers at ITE.

H3: Opportunity to learn teaching practice (OTL-Practicum) has a direct and significant influence on mathematical knowledge for teaching.

H4: Opportunity to follow the coherent teacher education program (OTL-Program) has a direct and significant influence on mathematical knowledge for teaching.

H5: Opportunity to learn teaching practice (OTL-Practicum) has a direct and significant influence on constructivist belief.

H6: Opportunity to follow the coherent teacher education program (OTL-Program) has a direct and significant influence on constructivist belief.

H7: Opportunity to learn teaching practice (OTL-Practicum) has a direct and significant influence on traditional belief.

H8: Opportunity to follow the coherent teacher education program (OTL-Program) has a direct and significant influence on traditional belief.

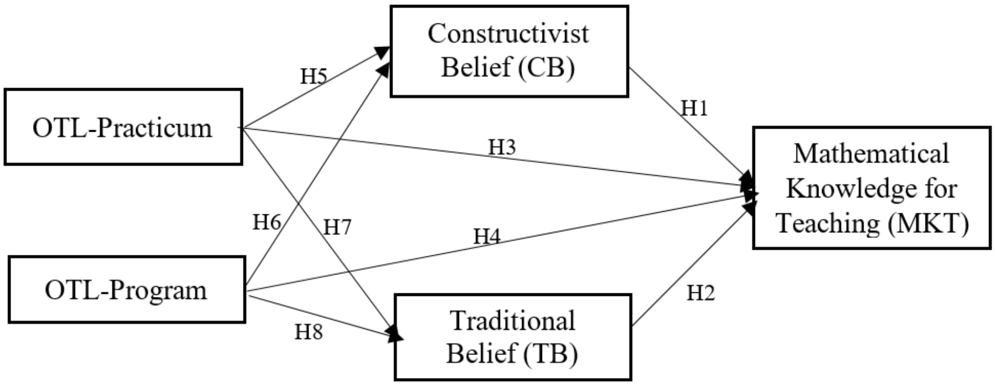

Figure 1. The research model

METHOD

Data Collection

Data was collected from 105 preservice teachers using a structured questionnaire and paper and pencil test. Both measures were adapted from previous research done by other researchers. We have used the clustered random sampling method to collect the data. This method was chosen because this study was conducted on populations involving large areas. The population of this study involves pre-service teachers who are currently pursuing Mathematics Education Program from semester 6 to 8 nationwide. Besides that, the clustered random sampling was used because the population of this study was widely scattered, and it is impractical to sample and select a representative sample of all the elements. Hence the sample of the study has been divided into several groups based on ITEs involved in the training of pre-service teacher in mathematics education. It was found that 13 out of 27 ITEs nationwide were involved in the training of pre-service teachers in mathematics education. Thus, the sample was divided into 13 groups, and then the sample selection from the cluster was randomly made to ensure that each sample had the same opportunity to be selected as a sample of the study (Acharya, Prakash, Saxena, & Nigam, 2013). The justification for the selection of all pre-service teachers of mathematics education in semesters 6 to 8 is that they have followed most courses offered and have undergone practicum training Phase 1. In addition, their selection as the study population is due to the issues studied have a direct connection with them.

Measures

The measure for OTL-Practicum and OTL-Program was adapted from Tatto, Senk, and Rowley (2008). These items are part of the items used in the TEDS-M 2008 study by the International Association for the Evaluation of Educational Achievement (IEA). It contains 8 items that measure OTL-Practicum and 6 items that measure OTL-Program. Permission to adopt the instrument has been applied and granted approval. They reported comparative fit index (CFI) for OTL-Practicum were 0.953, while for OTL-Program were 0.99 (Tatto, 2013). This illustrates that items are suitable for measuring the constructs. This is because the CFI value for the constructs exceeds 0.95 (Hu, & Bentler, 1999). It consisted of 14 Likert type items (OTL-Practicum = 8 items and OTL-Program = 6 items).

Mathematical belief measures were adapted from Effandi Zakaria et al. (2009). The instrument has been developed to measure mathematical beliefs among teachers in constructivist and traditional approaches. It contains 12 items, which is 8 items that measure constructivist beliefs and 4 items that measure traditional beliefs. Findings from Confirmatory Factor Analysis conducted by Mazlini Adnan, Mohd Faizal Nizam Lee Abdullah, and Che Nidzam Che Ahmad (2014) on the instrument indicates that the items are suitable for measuring mathematical beliefs. They reported the comparative fit index (CFI) value for the mathematical belief construct was 0.983.

Whereas MKT test was adapted from Hill, Schilling, and Ball (2004). It consisted of 32 multiple choice items. They reported the level of reliability of items that measure CK of primary school mathematics teacher for number and operation topics $\alpha = 0.784$, while for PCK was $\alpha = 0.888$ (Hill et al., 2004). This illustrates the level of reliability of both constructs is good. The validity of the items used in the MKT test has been determined by items analysis. Item analysis has been carried out to distinguish good items with poor items. Item analysis is intended to produce a high-quality test (Considine, Botti, & Thomas, 2005). Item analysis will be able to provide information regarding the response to each item whether they are able to answer or not that item. It also provides information on how each item works, whether the item is easy or difficult. In addition, an item analysis can discriminate between higher

Table 1. Descriptive statistics results of participants

	Frequency	Percent
Gender		
Male	36	34.3
Female	69	65.7
Ethnicity		
Malay	21	20
Chinese	49	46.7
Indian	30	28.6
Others	5	4.8
Cumulative Grade Point Average (CGPA)		
3.75 – 4.00	27	25.7
3.00 – 3.74	75	71.4
2.00 – 2.99	3	2.9
0 – 1.99	0	0

performance groups and lower performance groups (Si-Mui Sim, & Raja Isaiah Rasiah, 2006). ANATES 4.0.9 (Karno, & Wibisono, 2004) software was used to analyse the MKT test items.

Sample

The study sample was consisted of 105 pre-service teachers from ITE (65.7% were female). Majority of the preservice teachers involved in this study are Chinese (46.7%), whereas Malays about 20%, Indian 28.6% and others 4.8%. The CGPA obtained was quite high, which was almost 97% of them got the CGPA above 3.00. This showed that their academic achievements were good.

DATA ANALYSIS

For this study, the researcher has used the SmartPLS 3.0 software to analyse the data. SmartPLS 3.0 was used to analyse the data for this study because it was suitable to answer the research question. According to Hair, Ringle, and Sarstedt (2011) if the research goal is exploratory so we should use PLS-SEM. When analysing the data we have followed the analysis procedure as suggested by Hair, Hult, Ringle, and Sarstedt (2017). Firstly, we analyse the measurement model and then followed by analysing the structural model. This is to make sure the measures used in the study are reliable and valid to answer the research questions.

Measurement Model

When using multiple measures for an individual construct, the researcher should take into consideration the extent to which the measures demonstrate convergent validity (Hulland, 2002). Hair et al. (2017) has stated that a composite reliability (CR) of 0.70 or above and an average variance extracted (AVE) of more than 0.50 are considered acceptable. The result of Confirmatory Factor Analysis (CFA) stated in **Table 2** shows that all the composite reliability values are above 0.70 and the AVE is all above 0.50. Therefore, based on the CFA result obtained, we can conclude that convergent validity for this measurement model has been fulfilled.

Table 2. Result of CFA for measurement model

Construct	Item	Internal Reliability (Cronbach Alpha)	Convergent Validity		
			Factor Loading	Composite Reliability[a]	Average Variance Extracted[b]
OTL-Practicum	OTL_Prac1	0.840	0.687	0.880	0.552
	OTL_Prac2		0.798		
	OTL_Prac3		0.637		
	OTL_Prac4		0.709		
	OTL_Prac5		0.796		
	OTL_Prac6		0.814		
OTL-Program	OTL_Prog1	0.879	0.761	0.911	0.673
	OTL_Prog2		0.799		
	OTL_Prog3		0.877		
	OTL_Prog4		0.798		
	OTL_Prog5		0.862		
Constructivist Belief	CB1	0.777	0.714	0.846	0.523
	CB2		0.740		
	CB3		0.695		
	CB4		0.732		
	CB5		0.733		
Traditional Belief	TB1	0.704	0.694	0.805	0.584
	TB2		0.916		
	TB3		0.658		
Mathematical Knowledge for Teaching		N/A[a]		N/A[a]	N/A[a]

Note:
[a] Single item measures

Table 3. Discriminant validity

Constructs	(1)	(2)	(3)	(4)	(5)
(1) Constructivist Belief	**0.723**				
(2) MKT	0.293	**N/A[a]**			
(3) OTL-Practicum	0.304	0.651	**0.743**		
(4) OTL-Program	0.325	0.567	0.398	**0.821**	
(5) Traditional Belief	-0.502	-0.072	-0.007	-0.241	**0.764**

Note:
[a] Single item measures

Besides convergent validity, the researcher also need to take into consideration about discriminant validity in order to make sure the items used to measure a certain construct are different with other construct in the model. According to Fornell and Larcker (1981) discriminant validity can be established by calculating the square root of the AVE. Besides that, Hair et.al (2017) also stated that discriminant validity also can be establish by assessing the cross loading and heterotrait-monotrait ratio of correlations (HTMT) value. For this study we are only used square root of the AVE to assess the discriminant validity.

Structural Model and Hypothesis Testing

Structural model analysis are not only tests hypotheses but also estimates path coefficients of constructs by examining the relationship between the dependent and independent variables and the amount of variance which can be explained by the independent variables (R^2) as well as by the overall model. **Table 4** and **Figure 2** shows that H3, H4, H5, H6, and H8 were significant.

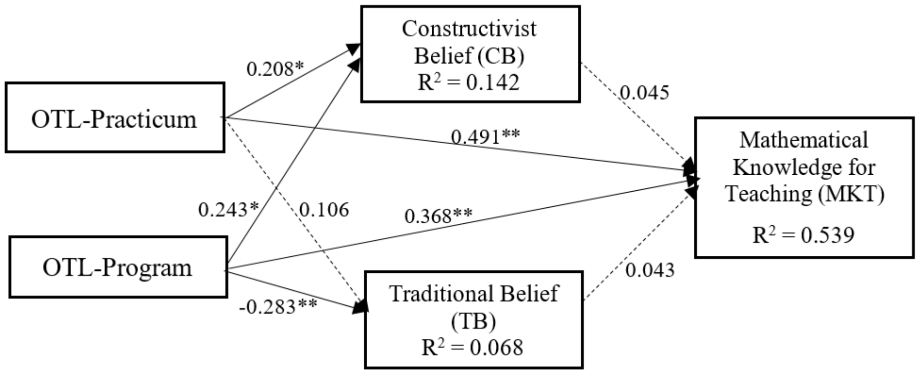

Figure 2. The research model

Table 4. Summary of hypothesis tests

Hypothesis	Standardized path coefficients (β)	t-value	Supported
H1. CB → MKT	0.045	0.647	No
H2. TB → MKT	0.043	0.549	No
H3. OTL-Practicum → MKT	0.491**	8.835	Yes
H4. OTL-Program → MKT	0.368**	6.381	Yes
H5. OTL-Practicum → CB	0.208*	2.164	Yes
H6. OTL-Program → CB	0.243*	2.557	Yes
H7. OTL-Practicum → TB	0.106	0.734	No
H8. OTL-Program → TB	-0.283**	2.754	Yes

Note: *p-value < 0.05; **p-value < 0.001

Table 5. Squared multiple correlations (R^2) of the proposed research model

Constructs	R^2
Mathematical Knowledge for Teaching	0.539
Constructivist Belief	0.142
Traditional Belief	0.068

The results demonstrated that, (1) OTL-Practicum had a positive effect on MKT (β=0.491, p<0.001); (2) OTL-Program had a positive effect on MKT (β=0.368, p<0.001); (3) OTL-Practicum had a positive effect on constructivist belief (β=0.208, p<0.05); (4) OTL-Program had a positive effect on constructivist belief (β=0.243, p<0.05) and (5) OTL-Program had a negative effect on traditional belief (β=-0.283, p<0.001). The results of the structural model analysis are illustrated in **Figure 2**. Overall, the model explained 53.9% of the variance in mathematical knowledge for teaching (**Table 5**).

RESULTS AND DISCUSSION

The purpose of this study was to test a model of factors affecting mathematical knowledge for teaching among preservice teachers in Malaysia. Due to that, this study examined the relationship between mathematical belief (in term of constructivist belief and traditional belief) and MKT, and the impact of opportunities to learn (OTL) on MKT and mathematical belief.

The results of the multiple regression analysis for this study are parallel with those found in a similar study by Rachel A. Ayieko (2014) where both dimensions of opportunity to learn were positively related to mathematical belief. Besides that, the findings are also consistent with Konig et al. (2017), and Akkoç, and Yesildere (2010) findings which stated that there is a significant relationship between preservice teachers' opportunity to learn through teaching practice (OTL-Practicum) and their pedagogical content knowledge. Furthermore, earlier findings by Toh Tin Lam, Berinderjeet Kaur, and Koay Phong Lee (2009) have established that OTL-Practicum affected Singapore preservice secondary mathematics teachers' content knowledge. In addition, a similar study by Tatto et al. (2015) and Kleickmann et al. (2013) also found that opportunity to follow the coherent teacher education program (OTL-Program) influenced the mastery of CK and PCK of the teacher. Surprisingly the result of this study was not consistent with finding from previous study done by Meschede et al. (2017). This is because it was found that MKT are not affected by both of the mathematical belief dimension (constructivist belief and traditional belief).

The role of MITE as an institution responsible for producing competent mathematics teachers is that it is desirable that MOE provide enough allocation to ensure that the widest possible learning opportunities can be enjoyed by pre-service teachers. Apart from providing opportunities to learn through a coherent teacher education programs, a pre-service teacher should also be given the opportunity to learn the pedagogical knowledge more effectively. This is because according to Blömeke et al. (2016) the OTL general pedagogical and mathematical pedagogy can influence the knowledge level of pre-service teachers. The delivery of pedagogical knowledge to pre-service teachers is directly related to the curriculum structure and quality of a teacher educators. Therefore, in order to ensure the delivery of more relevant pedagogical knowledge, it is recommended that the MITE regularly update the mathematics education curriculum while continuing to improve the mathematics teacher educator's competency in the delivery of pedagogical-related knowledge.

The model tested in this study shows that OTL-Practicum and OTL-Program can account for 14.2% of the variance in constructivist belief while about 53.9% of the variance in mathematical knowledge for teaching. These results suggest that the tested model are able to predict the mathematical belief and teachers' knowledge.

The contributions of this study towards Institute of Teachers Education (ITE) and implementers are there was a need for both of them to provide necessary opportunities to learn to the pre-service teachers in order to ensure they can increase their MKT. If preservice teachers' MKT is low because of lack of OTL, it will affect the teacher education program implementation. Enough and adequate opportunities to learn provided by the ITE seem to bring a greater teacher education program to the pre-service teachers.

LIMITATIONS AND FURTHER RESEARCH

This study empirically tested the effect of opportunities to learn and mathematical belief on mathematical knowledge for teaching. Although the finding of this study was really useful, there are certain limitations regarding this study. Therefore our findings need to be interpreted appropriately. The first consideration was that the sample size used in this study need to be take into account when generalizing the results of the study. This is because this study only involved a small sample size (N=105). Besides that, this study are only focused on testing the effect of opportunities to learn, mathematical belief and mathematical knowledge for teaching. There might be any other related variables that can affect MKT.

In the future, this study can be expand by (1) integrating the influence of mathematics teaching efficacy belief (MTEB) on the MKT among pre-service teachers, (2) including the effect of other OTL factors, such as OTL mathematics content, OTL general pedagogy and OTL mathematics pedagogy on the MKT among pre-service teachers and (3) expanding the model by adding other relevant variables found from latest literature. Besides that, future studies also can make a comparison of mathematical knowledge for teaching between pre-service and in-service teachers considering the fact that in-service teachers should have a better mathematical knowledge for teaching due to their teaching experiences.

CONCLUSION

According to the findings, we found that opportunity to learn teaching practice (OTL-Practicum) and opportunity to follow a coherent teacher education program (OTL-Program) are significant factors influencing mathematical knowledge for teaching among pre-service teachers in Malaysia. The findings of this study may enable the teacher education program provider to take into consideration on these variables that will influence mathematical knowledge for teaching. In addition, this study may provide an empirical justification for the ITE to develop a strategic plan that can improve the teacher education program by focusing on the pre-service teachers' opportunities to learn. In the future, there was a need to conduct further research to enhance this study. We believe this study are able to give a preparatory knowledge and comprehension on the role of opportunities to learn and mathematical belief in maximizing the mastery of mathematical knowledge for teaching.

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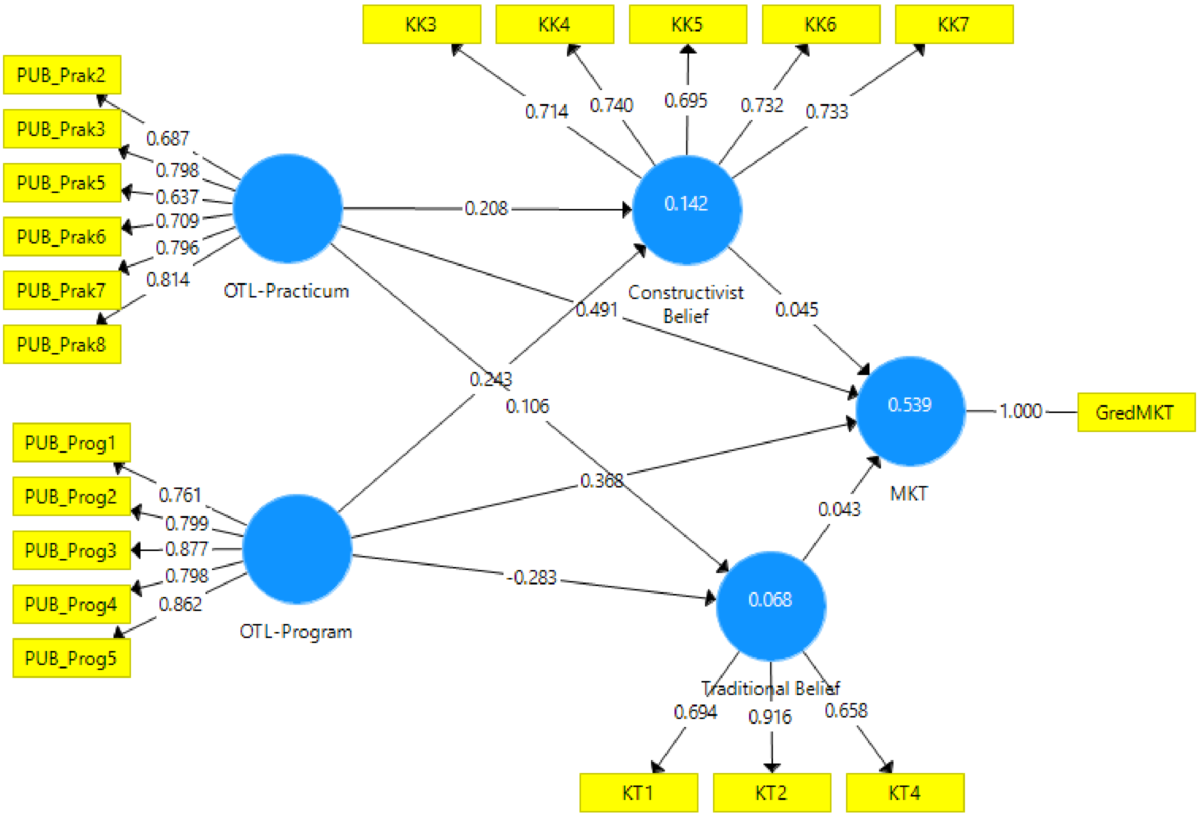
APPENDICES



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Age Heterogeneity of STEM Educators

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ABSTRACT

One of the ways to improve the quality of math and natural science education is to develop the pedagogical community of STEM education. On the one hand, according to the forecasts, an increase in the number of students and teachers is expected by an average of 20 % in both Russia and worldwide. On the other hand, there is definitely some specificity in the pedagogical community as compared with any other labor collective, since the pedagogical community develops alongside with the student one. In this perspective, the age heterogeneity of the teaching staff is of particular relevance. Consequently, the key point in the management of math and natural science education is the analysis of the age structure of STEM educators. The following subjects are taken to conduct a further analysis of STEM education: mathematics, handicraft, physics, biology, and chemistry. Accordingly, the purpose of the article is to analyze and forecast the heterogeneous development of the age structure of STEM teachers. The authors of the article justify the necessity for changes in the personnel policy on the basis of the assessment of the capacity of the teaching staff of STEM education in schools of the European part of Russia. The leading research approach is the method of the normal and natural distribution of age groups in the structure of the STEM education pedagogical community. As a result of a study conducted in 569 schools of the European part of Russia in 2016–2018, the authors of the article have found out the following: the average age of STEM school teachers in the European part of Russia is 6.8 years higher than the average age of teachers in Russia; there is a significant shortage of STEM teachers aged under 35; the average rate of the teaching load of a STEM school teacher in an academic subject is 0.72, but the load is distributed very unevenly, math teachers having the highest teaching load. The theoretical significance of the research lies in its contribution to the development of scientific ideas concerning the age heterogeneity of STEM school teachers. The research results can be used in building development trajectories of STEM education teaching staff by implementing a series of managerial and organizational measures to achieve the normal state of the age structure of teachers.

Keywords: STEM education, age heterogeneity of teachers, age structure of the pedagogical community, seniority of teachers, normal distribution of age groups

INTRODUCTION

The development of the regional pedagogical community largely determines the quality of education and the success of graduates of educational institutions at all levels of education (Dmitrieva et al., 2015; Filatov et al., 2018; Firsova et al., 2018; Novikova et al., 2018; Oborsky et al., 2018; Petrovskaya et al., 2016; Shcherbakov et al., 2017;

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Contribution of this paper to the literature

- The study defined the age structure of the STEM school teachers in the European part of Russia. The average age of STEM teachers (teachers of mathematics, handicraft, physics, biology and chemistry) is 51.8. It should be noted that the normal value of the average age is 45, and the average for the regions is 48.1 years. The group of STEM teachers under the age of 35 makes up only 11 % against 33 % characteristic of the normal distribution.
- It has been determined that the optimal age distribution of teachers should be close to the normal distribution. The coefficient of deviation from the normal age distribution of STEM teachers reaches up to 50 % which is significantly higher than the average coefficient for all teachers which does not exceed 32 %.
- The analysis of the teaching load of STEM teachers with regard to the subject has shown that the load of teachers of different subjects is uneven. Thus, the rate of the load of math teachers in the region averages out at 1.0; the average load rate of teachers of handicraft, biology and physics is 0.65; the one of teachers of chemistry is 0.56.

Zaitseva et al., 2017, 2018). The process of teacher training is multistage, but it should facilitate updating of professional teaching activities at each stage. It is important that it is provided with a transparent mechanism of interaction of all stakeholders (education authorities at various levels, teacher training institutions, general education institutions and other stakeholders) (NGPU, 2016). This mechanism should mainly rely on the existing age structure of teachers in each school subject (Faleeva et al., 2017; Lubnina et al., 2016; Rudenko et al., 2015). There is a quite limited number of applicants who want to get pedagogical qualifications, and neglecting the current situation concerning the age of teachers can lead to an inefficient allocation of state-financed places to teacher training institutions, and, consequently, to an increased risk of a shortage of subject teachers in the relatively short term. Math and natural science education is given special attention to when managing the school education. In this regard, it is important to analyze the age structure of STEM educators. Thus, we have taken the following academic subjects to conduct a further analysis of STEM education: mathematics, handicraft, physics, biology, and chemistry.

Additional risks of irrational allocation of state-financed places to higher education programs of STEM teacher training are associated with the lack of a comprehensive analysis. The analysis of the shift in the schoolchildren population with regard to the year of study and projecting the teaching load in STEM-subjects for several years in advance may become a normative forecast method. But if we take into consideration that, by 2020, the number of students in general education institutions only in one entity of the European part of Russia will reach up to 128.2 thousand people (according to the Center for Social Forecasting and Marketing, 2013 (Rosstat, 2016)) or 137.4 thousand people (according to the Institute of Sociology of the Russian Academy of Sciences, 2015 (TsSP and M, 2017)), then an increase in the number of students will be by 7 % as compared to 2016. It is necessary to check this growth against the projected number of STEM subject school teachers by 2020. Thus, the assessment of the age structure of the STEM education teaching staff of schools of the European part of Russia is a necessary condition to optimize the teacher training process and identify the development trajectories for the STEM schools teachers in the European part of Russia.

At the end of 2016, a similar study was conducted using the example of the Kirov region. The study results contain an objective generalized assessment of the structure of the teaching staff of general education organizations by age, education level, subject; the study also offers a projected estimate of the demand for teachers in the region until 2022 (Pugach & Utemov, 2016).

The analytical report based on the study results contained the data justifying the fact that it was necessary to take management decisions in the field of personnel policy in the sphere of education in the region (Kvon et al., 2017, 2018). Thus, for example, the most critical age groups of teachers are teachers of physics, chemistry, geography, biology, art and drawing, the Russian language and literature. On the other hand, there is a significant shortage of male teachers. Only 11 % of all school teachers are male. The research conducted by McGrath and Bergen (2017) shows that there is a decline in the share of male teachers worldwide, however, the development of STEM education shows a particularly strong dependence on male teachers (Kraker-Pauw et al., 2016). It should also be noted that one more reason for decreasing numbers of male teachers is that employers are more inclined to offer administrative positions to them rather than women (Wagner, Rieger & Voorvelt, 2016).

The results of the study (Pugach & Utemov, 2016), however, did not contain any conclusions concerning the STEM teachers, neither did they analyze the structure of the teaching load taking the subject into account.

In the summer of 2018, Minin University made a comparative analysis of the natural development of the age structure of the pedagogical community and gave a forecast of its development for the regions of the Russian Federation until 2050 in "Age structure of the pedagogical community: analysis and forecast of development: an analytical report" (Fedorov et al., 2018). As the analysis is of a generalized nature, we carried out an additional analysis in 2017–2018 the results of which are presented in the article.

LITERATURE REVIEW

The domestic and foreign studies devoted to the development of pedagogical communities with respect to age heterogeneity are rather sketchy and fragmented.

Kuo et al. (2011) carried out a prognostic analysis of the proportion of young and aged teachers at the beginning of each school year. Oerke and Bogner (2010) describe in their study the importance of taking into account the age characteristics of the teaching team and their influence on the education process. There are a few studies describing ideas concerning the structural constraints (age, disability, ethnicity and gender) which hinder teaching careers (Cau-Bareille, Teiger & Volkoff, 2019; Wilson et al., 2006). M. Sari (2012) writes in her article that female teachers take on their gender role of "being a mother" in their teaching practice which can influence teaching both positively and negatively. On the other hand, male primary teachers can have a greater educational effect on students (Bullough, 2015). At the same time, having a female teacher improves girls' academic performance in math (Xu & Li, 2018).

Taking into account the age characteristics of the teaching team is an important aspect in managing the teaching staff. Thus, Zhou et al. (2011) make a conclusion that middle-aged teachers are more motivated to develop professionally; as teachers become older, their motivation appears to be noticeably reduced. The phenomenon of the teaching staff ageing is also characteristic of higher education. Thus, the average age of teachers in French universities is over 49 years. Roschnik et al. (2004) argue that there is a need for young university teachers, increasing their number may give a boost to educational and research activities. Some studies on the age structure of the teaching team also describe the structure of working capacity in relation to the age of teachers. For instance, only 4 % of Finnish teachers aged over 45 have poor working capacity. However, the lowest level of working capacity among all teachers of this age is characteristic of teachers of vocational education institutions and teachers of special education (remedial) schools (Kinnunen, Parkatti & Rasku, 1994).

Seibt et al. (2015) conducted a research on mental abilities of two groups of female teachers (the first group is teachers aged under 45; the second group is teachers over 45 years old). The results showed slight differences in favor of younger teachers. It is stated that there are some minor differences in respect to coherence, susceptibility to stress and health complaints. This group of scientists concludes that some components of the mental ability of teachers suffer age-related changes. At the same time, continuous training allows maintenance or an improvement of mental ability at any age (Seibt et al., 2015). On the other hand, constant overload and stress characteristic of the teaching activity can gradually lead to declining health of the nervous system of female teachers with age (Burumbayeva et al., 2018). Age trends, although not clear cut, showed greater tension for older teachers (Powell & Ferraro, 1960).

Some research is devoted to how teachers respond emotionally to educational change at different ages (Hargreaves, 2018). According to Etherington (2011), the attitude of older teachers to change depends on how well established their careers are. Tumova (2012) makes a similar conclusion, stating that the effects of age and length of professional experience on teachers' attitudes to the curricular reform and its implementation are not as strong as they might be expected. Some research has shown that experienced teachers have noticeably lower levels of anxiety (Aslrasouli, Saadat & Vahid, 2014). It is worth mentioning that there are also critical reviews stating that the introduction of innovations into the educational process can lead to decreasing quality of education. In this regard, an adequate response to innovation could be useful (Pedro et al., 2018). Undoubtedly, it is necessary for teachers who have different seniority to implement changes in teaching from time to time (Dare, Ellis & Roehrig, 2018; Lynch, 2018). Some research proves that the practice of sport and physical activities and leisures can have a protective effect against burnout syndrome (Moueleu Ngalagou et al., 2018). Hildebrandt and Eom (2011) make a conclusion that teachers of different ages have different motivators for teacher professionalization and it should be taken into account.

Gampel and Ferreira (2017) analyzed adolescent evaluations of ageing teachers' voices. For example, teenagers showed a friendlier attitude to teachers whose voices were more expressive. Ahlander, Rydell and Lofqvist (2011), analyzing the influence of the age of math teachers on the quality of their performance, come to the conclusion that teachers who experience problems with voice also face problems in teaching. Ageing teachers suffering from the burnout syndrome often experience voice disorders (Mota et al., 2018).

Some studies analyze work ability and vitality of teachers of different age groups. It is found out that 24 % of teachers of the younger age group and 49 % of the elderly teachers show a poor/moderate work ability, which indicates an urgent need for measures for improving work ability (Freude et al., 2005). On the other hand, the length of experience of some teachers (special education programs, vocational training institutions, pre-school education, etc.) has a significant effect on the quality of their performance (Selzer King et al., 2018; Sheridan et al., 2018). The study conducted by Sasa, Borosb and Bonchisc (2011) reveals the dependence of the professional burnout of the teacher on the level of education where the teaching activity is performed.

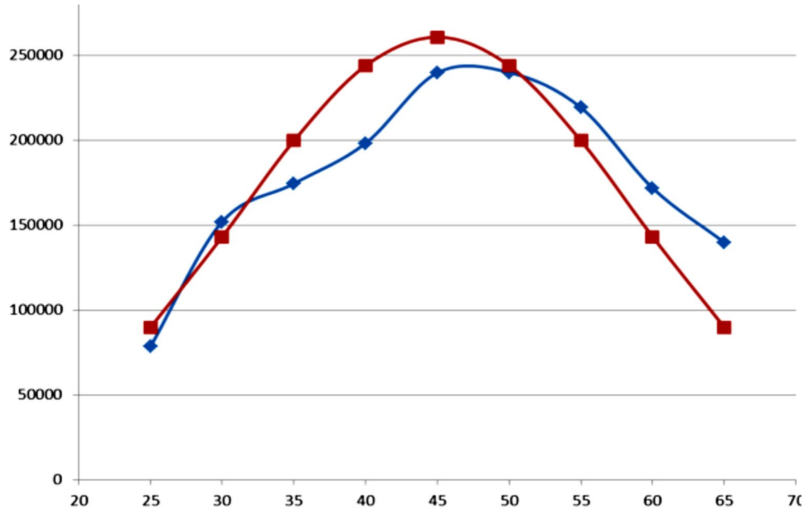

Figure 1. Age distribution of the RF teachers

Although knowledge is an integral part of the professional competence of teachers, it is not limited to knowledge only. Skills, relations and motivational incentives also contribute to mastering educational technologies. Blömeke and Delaney (2012) developed a model that identifies cognitive abilities and affective-motivational competencies as the two main components of the professional teacher knowledge. The first component contains professional knowledge, methodological knowledge (methods for teaching a particular subject) and general pedagogical knowledge. This component can be analyzed by assessing the students' opinions (Gargallo et al., 2018; Moe et al., 2019). The second component includes motivation, self-regulation and professional beliefs about teaching, and the content of the subject. It should be noted that the issue of the professional attitude to and methods for teaching a particular subject by various teachers is a separate area of research (Demonty, Vlassis & Fagnant, 2018; Lingo et al., 2018).

MATERIALS AND METHODS

Theoretical Framework

The study of the heterogeneity of the age structure of STEM educators is based on the method of the normal and natural distribution of age groups.

The normal distribution contributes to the stable functioning of the education system, retaining the balance and offering a possibility for an educational institution to both develop and preserve traditions at the same time. The natural distribution is based on the invariant nature of the development of the pedagogical community where each age group is represented by an equal number of members.

In the study, the optimal distribution of teachers by age in any pedagogical community should approach the normal distribution (Gauss distribution) having properties characteristic of the range from 25 to 65 with the spacing equal to 5. Thus, the distribution of the Russian Federation teachers by age is presented in **Figure 1**.

The red line is the normal distribution (Gauss distribution); the blue line is the natural distribution.

The analysis of the distribution of age groups in the structure of the pedagogical communities of the regions as of 2016–2017 has shown the deviation of the distribution of age groups in the direction of the middle and senior age groups.

By comparison, the age distribution of STEM teachers is presented in **Figure 2**.

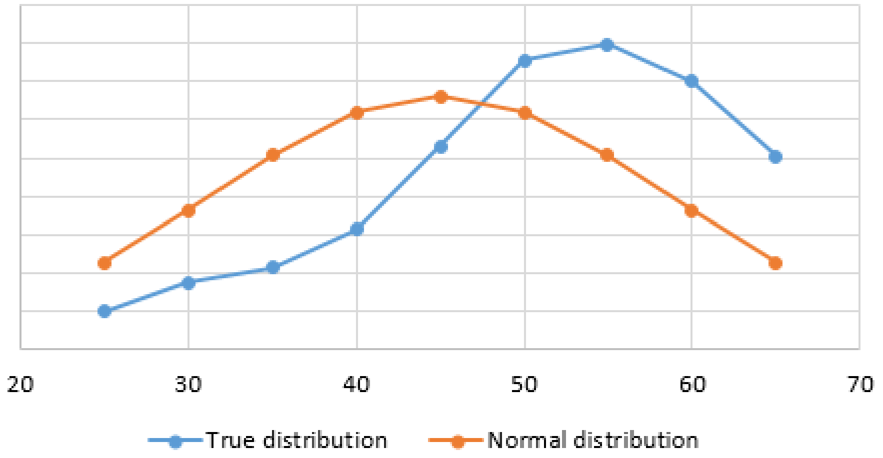

Figure 2. Age distribution of STEM teachers

It is evident that:

- the average age of STEM teachers of the European part of Russia (the peak of the natural distribution) is 6.8 years higher than the average age of the teachers in the Russian Federation;
- there is a significant shortage of teachers aged 30–45 in the region;
- the number of teachers in the region who are over 45 exceeds the norm dramatically.

Thus, the age structure of STEM teachers in the regions should be brought closer to the normal distribution.

Research Methods

To analyze and forecast the development of heterogeneity of the age structure of STEM school teachers in the European part of Russia, we employed the following methods: collecting empirical evidence by questioning the administrative personnel of the educational institutions; modeling the normal and natural distribution of age groups in the structure of the pedagogical community of STEM education; statistical processing of the experimental data.

Experimental Infrastructure

The research data to analyze and forecast the development of heterogeneity of the age structure of STEM teachers were collected, analyzed and generalized on the basis of 569 Russian schools (2016–2018):

– The research data were collected by means of questioning the administrative personnel of the educational institutions. The general education organizations of all educational districts of the regions were included in the study. The questionnaire consisted of 5 sections.

– The expert analytical assessment of the teaching load of the teaching staff of the educational institutions was conducted by means of the statistical processing of the experimental data. The results of the assessment were discussed with teachers of mathematics, handicraft, physics, biology and chemistry at seminars and round table discussions held within the framework of scientific and methodological workshops at the Vyatka State University (more than 200 participants annually).

– The results have been presented in the form of reports and presentations at scientific conferences and seminars of various levels, including international ones; they have also been published in collections of scientific articles and scientific journals.

Stages of the Research

The research consisted of four stages.

At the first stage, the state of the research problem in the theory and practice of teaching STEM subjects at school was investigated. The review of psychological, pedagogical and methodical literature on the problem of the research was carried out; conversations with teachers and the administrative personnel were held; the experience of teachers and the administrative personnel was examined and analyzed in order to predict the development of heterogeneity of the age structure of STEM teachers.

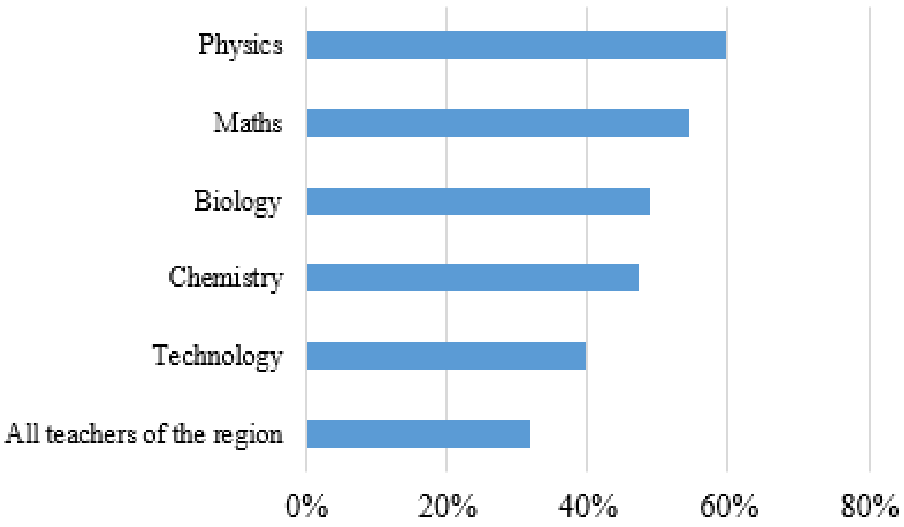

Figure 3. Coefficient of deviation from the ideal state for each group of STEM teachers in percentage

The methodological approach to how to analyze and forecast the development of heterogeneity of the age structure of STEM teachers was developed at the second stage of the research. The working laws were determined and the distribution of age groups in the structure of the pedagogical community was identified. The results of this stage were reported and discussed at conferences of various levels and seminars devoted to teaching school subjects.

The third stage of the research was carried out along with the second one. The administrative staff of 569 educational institutions were surveyed. They were asked to fill in a questionnaire consisting of 5 sections.

At the fourth stage, the expert analytical assessment of the capacity of the teaching staff of the educational institutions was carried out by means of the statistical processing of the experimental data. The research data on STEM teachers were generalized to form the grounds for justifying the need for management decisions in the personnel policy in the sphere of STEM education.

RESULTS

Differences in Distribution of STEM Teachers by Age with Regard to the Subject Taught

To identify the distribution of STEM teachers by age with respect to the subject taught, we have built the respective graphs. In order to compare the deviation, we calculated the coefficient of deviation from the ideal state for each group of teachers. The list of groups of teachers ranked by deviation is presented in **Figure 3.**

It is evident that there are significant differences in the age distribution of STEM teachers with respect to the subject taught. The deviation exceeds 50%, which is significantly higher than the average coefficient for all teachers, which is usually not higher than 32%.

Differences in the Distribution of STEM Teachers by the Structure of the Teaching Load with Regard to the Subject Taught

The collected data also make it possible to analyze the structure of the teaching load for the teachers in the region. Our calculations show that the average rate of the load for a school teacher in one subject is 0.70. But the rate of the load for teachers of mathematics is 1.0 on an average, the rate for teachers of handicraft, biology, physics is 0.65, for teachers of chemistry – 0.56.

The total share of teachers in the structure of the load is presented in **Figure 4**: it shows the percentage of teachers having the rate of the teaching load over 1 (> 1), less than 0.5 (<0.5) and in the range from 0.5 to 1.0 (<1).

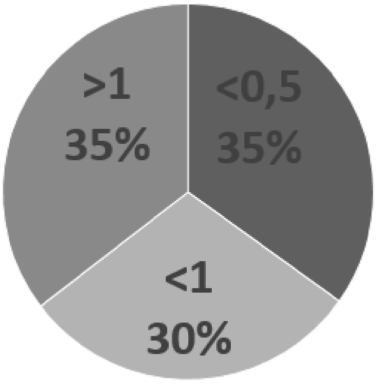

Figure 4. Structure of the teaching load for STEM teachers

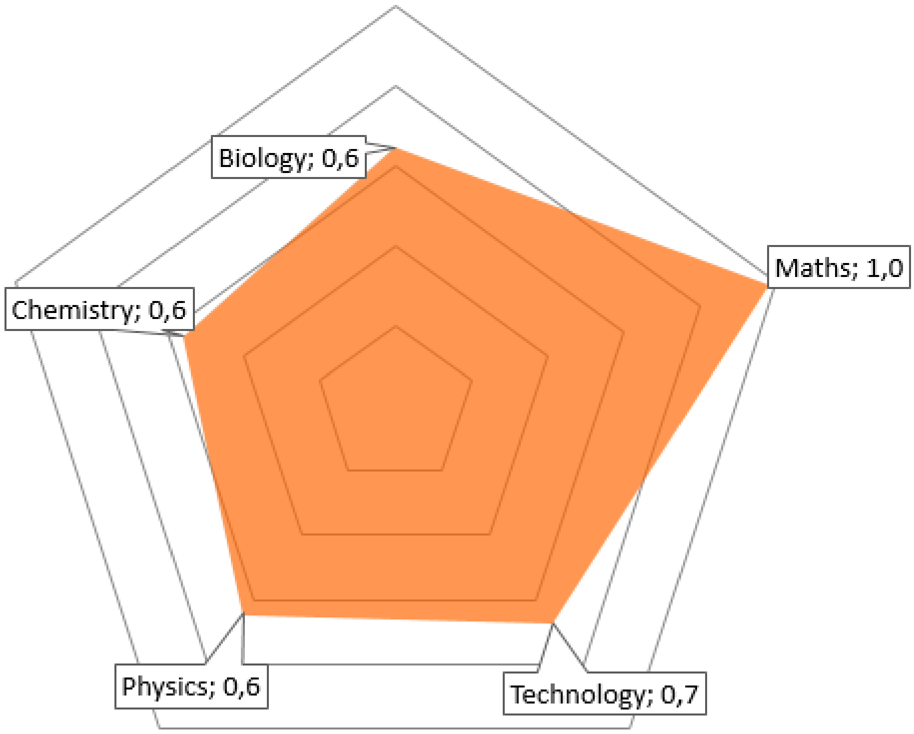

Figure 5. Teaching load for STEM teachers

The analysis of the structure of the teaching load with respect to the subject taught shows that the shares of STEM teachers having the rate of the load over 1.0, from 0.5 to 1.0 and less than 0.5 are approximately equal.

Another value that was calculated is the weighted average of the rate of the teaching load with respect to the subject taught (**Figure 5**).

It is seen that math teachers have the highest teaching load. Thus, the data obtained should be taken into consideration when combining the major subjects to train teachers at educational institutions.

Differences in the Average Age Distribution of STEM Teachers with Regard to the Subject Taught

The weighted average age of STEM teachers with regard to the subject taught is presented in **Figure 6**. The normal value of the average age is 45 years, and the average for the region is 48.1 years.

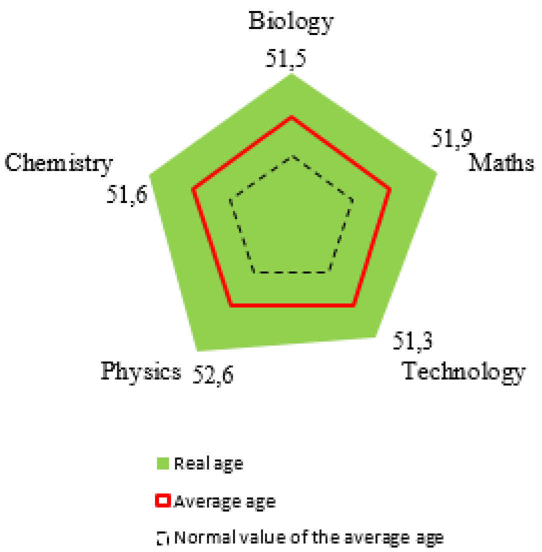

Figure 6. Average age of STEM teachers with regard to the subject taught

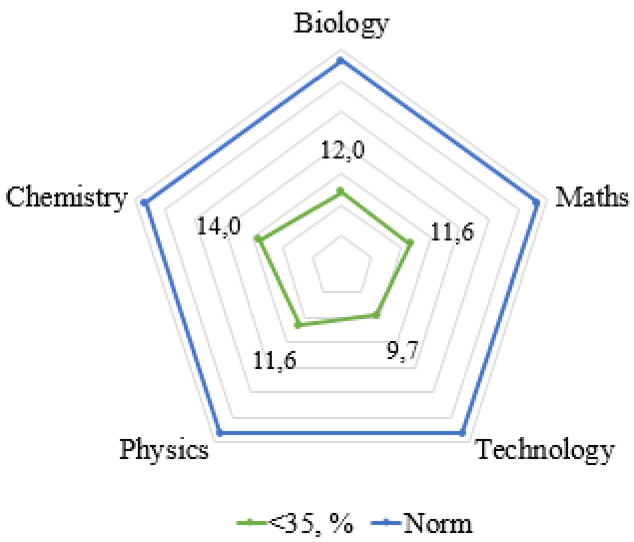

Figure 7. The share of STEM teachers under 35 years old with regard to the subject

The situation seems to be the most problematic with the teachers of physics.

Currently, the only quality characteristic of the age structure of the pedagogical community is the relative share of the number of teachers aged under 35. Thus, the indicator "the share of the number of teachers aged under 35" is established as a monitoring indicator for educational institutions by Order of the Ministry of Education and Science of the Russian Federation No. 955 of September 22, 2017 (the Ministry of Education and Science of Russia, 2017). The results of the analysis of these indicators are given in Figure 7.

It should be noted that the indicator of 33% of teachers under 35 years old in the total number of the teaching staff is considered to be normal in the natural distribution (a blue curve in the figure). Thus, it is evident that the most critical situation is in the sample of teachers of handicraft, physics, and mathematics.

Age Structure of STEM Teachers

The age structure of the pedagogical community in this study is presented as nine age groups integrated into three larger age groups:

– young teachers – under 25, 25–30, 30–35 years;

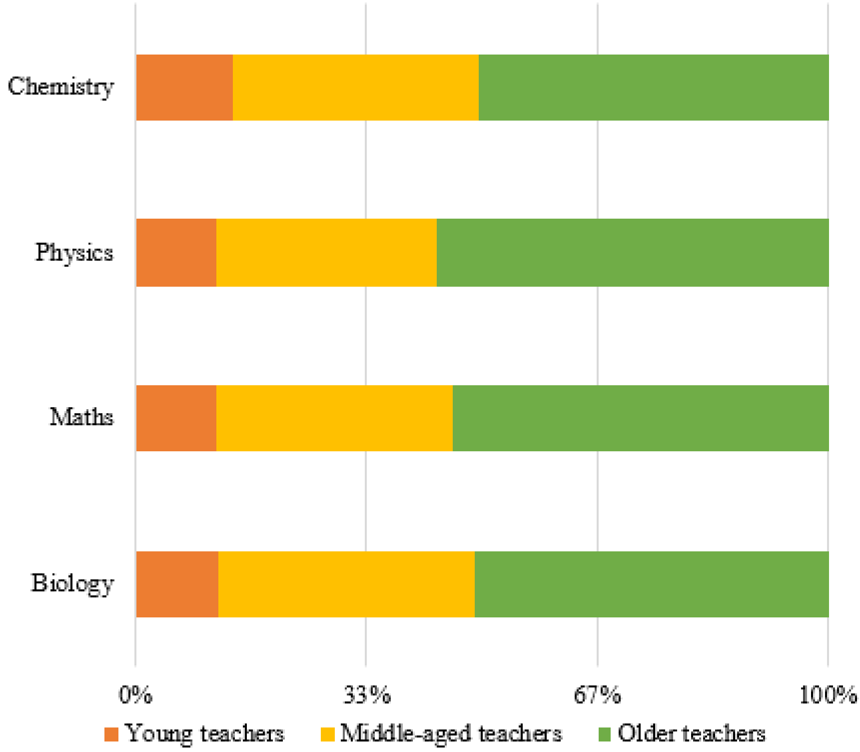

Figure 8. Age structure of STEM teachers by age groups

- middle-aged teachers – 35–40, 40–45,45–50 years;
- older teachers – 50–55, 55–60, 60 years and older.

The share of each age group of STEM teachers is shown in **Figure 8**. The analysis of age groups presented in **Figure 8** confirms the previously stated conclusion, namely: the groups of teachers of biology, mathematics, physics and handicraft demonstrate the most worrying situation in the age structure of the pedagogical community.

DISCUSSIONS

The results of the study conducted in 2016–2018 in 569 schools of the European part of Russia demonstrate the following: the average age of STEM teachers is 6.8 years higher than the average age of teachers in Russia; there is a significant shortage of STEM educators aged under 35; the average rate of the teaching load of a STEM school teacher in one subject is 0.72, but the load is distributed very unevenly. The teacher of mathematics has the highest teaching load. The results of the research make it possible to build the trajectory of development for the STEM-education teaching personnel by means of implementation of a set of managerial and organizational measures to achieve the normal condition of the age structure of teachers.

On the one hand, the method of the normal and natural distribution of age groups in the structure of the STEM pedagogical community allows us to analyze the heterogeneous development of the age structure of STEM teachers in the region. On the other hand, it may become a key point in the management of mathematical and natural science education by means of modeling the projected development of the age structure of STEM educators.

Another important aspect is identifying the factors negatively affecting the age structure of STEM teachers and minimizing their influence. This issue can be considered separately.

CONCLUSION

Having made the correlation analysis of the data on the teaching load and age structure, having made calculations to determine the degree of dependence, we have come to the conclusion that middle-aged teachers have the most of the teaching load while younger teachers most often have a smaller volume of the teaching load. With regard to older teachers, it has been revealed that the volume of their teaching load is minimal. This fact highlights the critical character of growth of the share of older STEM teachers in the pedagogical community of the regions.

The research results combined with the data of the Rosstat demographic forecast, for example, for the Kirov region for the period up to 2036, allow us to make a few more generalizations. (the Rosstat's demographic forecast for the Kirov region, 2018): reduced numbers of middle-aged teachers affect the volume of the teaching load most critically (teachers of the middle-aged group have on average a 30 % higher teaching load than the teachers of the younger age group); the number of middle-aged teachers is decreasing at a faster rate than the number of children in the region which may exacerbate the problem of teacher shortage.

To sum up the analysis and forecast of the development of heterogeneity of the age structure of STEM teachers, it should be stated that there is a need for building a trajectory of development for STEM teachers by means of implementation of a set of measures aimed at achieving the normal condition of the age structure of the pedagogical community in the region.

The main trajectories of the development of the pedagogical community with respect to age heterogeneity may include the activities along the following lines:

1) managerial:

- open contests held by the regional educational authorities giving the right to conduct advanced teacher training courses;
- implementing the Mobile Teacher project;
- creating educational clusters;
- a transition from employer-sponsored admission to targeted education;
- setting the region's order for the teacher training places financed by the federal budget;
- setting the region's order for personnel training financed by the regional budget;

2) organisational:

- creating educational consortia;
- creating the regional guidance programs and services;
- creating "pedagogical classes" in secondary general education schools;
- developing training and retraining programs for those who have a non-major education (up to one year);

3) content:

- creating an education network based on hub schools;
- integrating the schoolchildren of the region into the system of distance learning provided by a professional educator;
- updating the list and content of advanced training and retraining programs for subject teachers;
- creating pedagogical teams consisting of students learning different subjects to work at educational institutions;
- creating pedagogical e-communities.

The authorities of the RF entities and the Vyatka State University are working at the development and implementation of the above trajectories to develop the pedagogical community. Possible effects, risks, barriers are also investigated by the scientists.

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Addressing Alternative Conceptions about Transition Metals among Form Six Students using Information and Communication Technology based Instruction

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ABSTRACT

This study was conducted to reduce the incidence of alternative conceptions about transition metals among form six students. A quasi-experimental design was carried out involving 79 students from two intact classes that were randomly classified as the treatment (N=47) and the comparison (N=32) groups. Information and communication technology-based instruction (ICT) was used in this study with the treatment group while a traditional teaching method was used with the comparison group. For the quantitative study, nine two-tier items were administered in a conceptual test after instruction to the 79 form six students in both the groups to ascertain their understanding about transition metals. Five students from the treatment group were selected for interviews before and after the instruction to obtain further insights into their understanding. An independent samples t-test analysis was used to compare the total scores of the two groups in the transition metals conceptual test. The outcome revealed that there were statistically significant differences in the test mean scores between the comparison and treatment groups (M_{tre} = 8.47; SD_{tre} = 0.69; M_{comp} = 3.91; SD_{comp} = 0.96; t = 23.103, $p < 0.001$). The results from the analysis indicated that students from the treatment group showed significantly greater levels of achievement than the students from the comparison group. Furthermore, the percentage of alternative conceptions among students in the treatment group was lower than those of the comparison group students.

Keywords: alternative conceptions, form six students, ICT-based instruction, transition metals

INTRODUCTION

Almost all science topics including the study of transition metals involve the incidence of alternative conceptions among students. The study of transition metals is covered in inorganic chemistry in Form 6 (18 to 19 years old) in the Malaysian education system. This study focuses on alternative conceptions held by students about transition metals. ICT-based education is about incorporating technology across the curriculum by transferring knowledge, facts and information using ICT tools such as computers, lap tops, projectors, hand-held devices and more (Anderson, 2008). It was envisaged that the alternative conceptions about transition metals that are held by students could be reduced using ICT-based instruction.

Transition metals were first introduced to students in the Periodic Table topic in the first school term. Students would then learn about transition metals in detail in inorganic chemistry, in the second term. The transition metals topic in Form 6 covers physical properties and chemical properties of the first row of transition metals such as their nomenclature, the formation of different colours in transition metal compounds, the ability to form different oxidation states, formation and bonding of complex ions, transition metals as good catalysts and their uses.

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Contribution of this paper to the literature

- To this end literature on science education have documented about students developing alternative conceptions during the science lessons. However, very few studies reported on alternative conceptions about transition metals.
- The paper illustrates on using ICT based instruction in addressing alternative conceptions on transition metals. Many studies have employed different approaches in addressing alternative conceptions. This study specifically illustrates on using ICT in learning about transition metal.
- The ICT based instruction introduced in this study serves as a guide and exemplary for teachers to adapt the method in teaching about transition metal.

THEORETICAL BACKGROUND

The main objective of science education is to teach science concepts meaningfully to make students aware of how to use these concepts in their daily lives. Ausubel (1968) argued that meaningful learning would occur if new concepts are compatible with the old concepts. It is important to know what prior knowledge students bring to the classroom in order to help them build new concepts (Tsai, 2000).

Chemical knowledge seems to be difficult because it is learned at three levels which are "sub-microscopic," "macroscopic" and "symbolic" (Johnstone, 1991). The macroscopic level is something that is physical and visible, for example, students are able to see the colours of the transition metal compounds. Formation of variable oxidation states in terms of energies of the 3d and 4s orbitals are at the sub-microscopic level which refers to what is molecular and invisible. The symbolic level involves equations and chemical symbols such as formation of complex ions with different types of ligands. Alternative conceptions exist because of the difficulty in comprehending a topic. Thus, alternative conceptions will indirectly hinder the acquisition of the new knowledge. This is similar to the suggestion by Özmen (2004) who agrees that alternative conceptions will delay students' learning of subsequent concepts. In addition, if students' preconceptions do not match with scientifically acceptable concepts, they may lack the necessary basic ideas to build further knowledge that is needed to understand more advanced concepts (Mulford & Robinson, 2002).

Science education research has shown that children as well as adolescents have their own ideas and concepts about their surroundings and environments. For instance, Nieswandt (2001) stated that children develop their own ideas about nature and everyday life at a very early stage. According to Kao (2007) the sources of the students' alternative conceptions are diverse and are derived from their school teachers, experiments, textbooks, life experiences, and the use of anthropomorphism, analogies, and intuition. Alternative conceptions reflect situations in which students provide mistaken explanations to events about their daily experiences.

Several studies have been conducted to investigate the alternative conceptions that students hold in various science topics. These topics include:

(1) Chemical bonding (Coll & Treagust, 2003; Taber & Coll, 2002), heat, temperature and energetics of chemical reactions (Goedhart & Kaper, 2002);

(2) Conservation of matter, balancing of equations and stoichiometry (Agung & Schwartz, 2007); (3) acid strength in organic chemistry (McClary & Bretz, 2012);

(4) Chemical thermodynamics (Hadfield & Wieman, 2010; Sözbilir, Pinarbasi, & Canpolat, 2010);

(5) Solutions, solubility of salts and the particulate nature of salts in solutions (Smith & Nakhleh, 2011);

(6) Acid-base chemistry (Artdej *et al*., 2010; Drechsle and Schmidt, 2005; Furió-Más, Calatayud, & Bárcenas, 2007).

(7) Transition metals (Sreenivasulu & Subramaniam, 2014)

However, studies rarely have investigated high school students' understandings about transition metals.

The incidence of alternative conceptions can be reduced using ICT-based instruction to enable students to relate theory with practice easily. Hu, Gong, Lai & Leung (2018) supported that ICT benefits students learning science in wide range of topics. Teachers usually use technology-based instruments to reduce alternative conceptions about transition metals. Using ICT-based instruction will give students a better picture of the concepts compared to traditional methods of teaching. In fact, teaching and learning assisted with ICT has been increasing worldwide with its greater accessibility (Baytakar, 2000). Yushau, Mji and Wessels (2003) reported that visual representations on a computer screen are more beneficial to the students' understanding as compared to diagrams in books.

A study conducted by Sreenivasulu and Subramaniam (2014) revealed that transition metals is one of the topics from chemistry that is difficult to comprehend among the students. This is because of the abstract concepts that

Table 1. Major propositional content knowledge statements defining instruction on transition metals

Item No.	Propositional content knowledge statements
1.	A transition metal in zero oxidation state is able to attract ligands and form metal complexes, e.g., Ni in oxidation state zero forms nickel (0)tetracarbonyl, $Ni(CO)_4$.
2.	The tendency for transition metals to involve all 3d electrons in bonding will decrease once the d^5 configuration is exceeded because the electrons will pair up and will not be available for bonding.
3.	Cu^+ has unpaired d electrons which cannot take part in d-d transitions because the electronic configuration of Cu^+ is [Ar] $3d^{10}4s^0$, so it has completely filled 3d orbitals.
4.	Cu and K do not have the same ionisation energy for the loss of their 4s electrons because they do not have the same screening effect of their inner electrons.
5.	The reactivity of transition metals decreases from left to right across a period because the d orbitals are progressively filled up with electrons.
6	The ionisation energy of transition metals down a group in the Periodic Table is not similar to that of the alkali metals because the screening effect of the inner electrons of transition metals increases.
7.	Transition metal ions are the only ones that are coloured as ions of non-transition metals like sodium, calcium, sulfide, chloride, iodide and bromide ions are colourless.
8.	Transition metals can be good reducing agents because they have relatively low standard electrode potentials, e.g. E° of Sc^{3+}/Sc = -2.08.
9.	Transition metals are good catalysts because they have empty or partially filled d-orbitals that can be used to form temporary bonds with reactant molecules.

exist in chemistry especially about transition metals such as metallic bonding, d-d transitions of electrons which emit colours, formation of complex ions, ligands and so on. Deficiency in understanding the chemistry of transition metals results in students developing alternative conceptions about the topic. A study by Sreenivasulu and Subramaniam (2014) identified several alternative conceptions about transition metals. These included: transition metals with zero oxidation state cannot attract ligands to form complex ions, Cu^+ has unpaired d-electrons which can take part in d-d transitions, Cu and K have the same ionisation energy for the loss of their 4s electrons due to similar screening effects, etc. Furthermore, Johnstone (1971) found that students have difficulty in conceptualizing how the splitting of d-orbitals occurs in transition metals. He also reported that knowledge of d-orbitals is the basic knowledge that a student should acquire in order to learn about splitting of d-orbitals.

ICT-based Instruction

In recent years, the use of ICT has been widespread because the use of computers and other electronic forms of media in science teaching saves time and helps students better understand science concepts (Lai, Hwang & Tu, 2018). Most teachers and experts recognize the need for a teaching method using ICT facilities. Adam and Tatnall (2010) believe that ICT has positive impact on students' understanding of science concepts while Barak (2017) has suggested that ICT-based instruction plays a crucial role in current teaching and learning of science concepts. Computers are used as an additional tool in schools to achieve educational goals (Bayraktar, 2000). Education in the 21st century that involves the use of ICT in the classroom plays a vital role in enhancing the teaching and learning process. The role of ICT is to help students in particular, to learn and teachers to perform their teaching more effectively (Goktas & Yildrim, 2003). Integrating computers into the teaching and learning of inorganic chemistry can go a long way in solving the above problem, as it is an alternative approach that is available to teachers and students in the teaching and learning of chemistry (Anderson, 2002; Gyöngyösi, 2005).

The major propositional content knowledge statements covered in transition element instruction using ICT are shown in **Table 1**.

PURPOSE OF THE STUDY

This study was conducted with the purpose of reducing the incidence of alternative conceptions about transition metals among Form 6 students. The study was guided by the main research question (RQ): What is the effect of ICT-based instruction on reducing the incidence of alternative conceptions about the chemistry of transition metals among Form 6 students?

Name: ____

Class: ____________

Answer each question by choosing A for correct statement and B for incorrect statement. Provide a reason for each chosen statement.

No.	Question
1	A transition metal in zero oxidation state cannot attract ligands and hence cannot form complexes. A. True *B. False Expected reason: Transition metal with zero oxidation state can attract ligands because it has empty orbitals to be occupied by ligands. For example, nickel in oxidation state zero, forms $Ni(CO)_4$.
2.	The tendency for transition metals to involve all 3d electrons in bonding will not decrease once the d^5 configuration is exceeded. A. True *B. False Expected reason: Once the d^5 configuration is exceeded the electrons will pair up and will not be available for bonding.
3.	Cu^+ has unpaired d electrons which can take part in d-d transitions. A. True *B. False Expected reason: The electronic configuration of Cu^+ is $[Ar]3d^{10}4s^0$, so has completely filled 3d orbitals and cannot take part in d-d transitions.
4.	Cu and K are expected to have the same ionisation energy for the loss of their 4s electrons. A. True *B. False Expected reason: Cu and K do not have the same screening effect of their inner electrons.
5.	The reactivity of transition metals decreases from left to right across a period. *A. True B. False Expected reason: The d orbitals are progressively filled up with electrons.
6.	The ionisation energy of transition metals down a group in the Periodic Table is similar to that of the alkali metals. A. True *B. False Expected reason: The screening effect of the inner electrons of transition metals increases.
7.	Only transition metal ions are coloured. A. *True B. False Expected reason: Only transition metals undergoes d-d orbitals splitting to form coloured ions. Other ions of non-transition elements like sodium, calcium, sulfide, chloride, iodide and bromide ions are colourless.
8.	Transition metals can be good reducing agents. *A. True B. False Expected reason: They have relatively low standard electrode potentials, e.g. E^o of Sc^{3+}/Sc = -2.08.
9.	Transition metals are good catalysts. *A. True B. False Expected reason: They have empty or partially filled d-orbitals that can be used to form temporary bonds with reactant molecules.

(* correct answer)

Figure 1. Questions included in the conceptual test

METHODS

Sampling and Research Design

This study was aimed at reducing the alternative conceptions about transition metals among form six secondary school students. For this purpose, a quasi-experimental design, involving 79 form six (18 to 19 years old) science stream students was used in this study. The students were divided into two groups where one class of 32 students (referred to as the comparison group) was taught using a traditional method. The other group of 47 students from two classes (referred to as the treatment group) was taught using ICT-based instruction. Form Six is pre-university level studies. After completing Form Six students will be enrolled into degree courses of their choice. Chemistry is a requirement for students to enroll into science, technology, engineering and mathematics courses at tertiary level.

Research Instrument

Both quantitative and qualitative research methods were used in this study. A conceptual test was administered to both groups after the treatment. As the students were introduced to the chemistry of transition metals for the first time, it was not considered necessary to administer a pretest. In this test, true-false questions were administered to students to identify alternative conceptions that were held by them. This conceptual test that was adapted from a study by Sreenivasulu and Subramaniam (2014), consisted nine multiple-choice true-false questions. Each item was followed by an open-ended question that required students to give a reason for the answer chosen in the item. Items in the conceptual test are listed in **Figure 1** together with the expected reasons In addition, an interview session was conducted with the Form 6 students from the treatment group, before and after instruction, to further enhance the results of this study.

Figure 2. Shows a series of screen shots that the teacher could use to show that the colours are due to d electron transitions

Treatment

Nine periods a week were allocated for chemistry lessons and three periods were allocated for laboratory sessions together with the students' project work. Each period consisted of forty minutes of lesson time.

Traditional Teaching Method

The traditional teacher-centered method was used to teach transition metals to the comparison group. A white board, textbooks and additional notes were used in the classroom by the teacher. This instruction was more chalk-and-talk in nature where the students listened passively to the teacher and took down notes when necessary. This was followed by discussion of examples and a few exercises from the textbook. The teacher administered the conceptual test to identify the alternative conceptions at the end of the instruction.

ICT-based Instruction

ICT-based instruction was used with the treatment group. The teacher used the teaching courseware on a CD provided by the Ministry of Education. This courseware comprised of lessons in each topic as well as practical activities and problems that the students had to solve. During the lessons the students were shown the formation of complex ions, d-d transitions, etc. using the CD. This was followed by a group activity after each lesson. For example, after students had viewed the video of the formation of complex ions, they were required to form their own complex ion using a different central metal ion and ligands. A representative from each group then explained their complex ion to the class. The teacher then gave her opinions and comments on the students' presentation.

Figure 2 shows several screen shots from the CD that the teacher could use to show the colours of the ions of various transition metal ions.

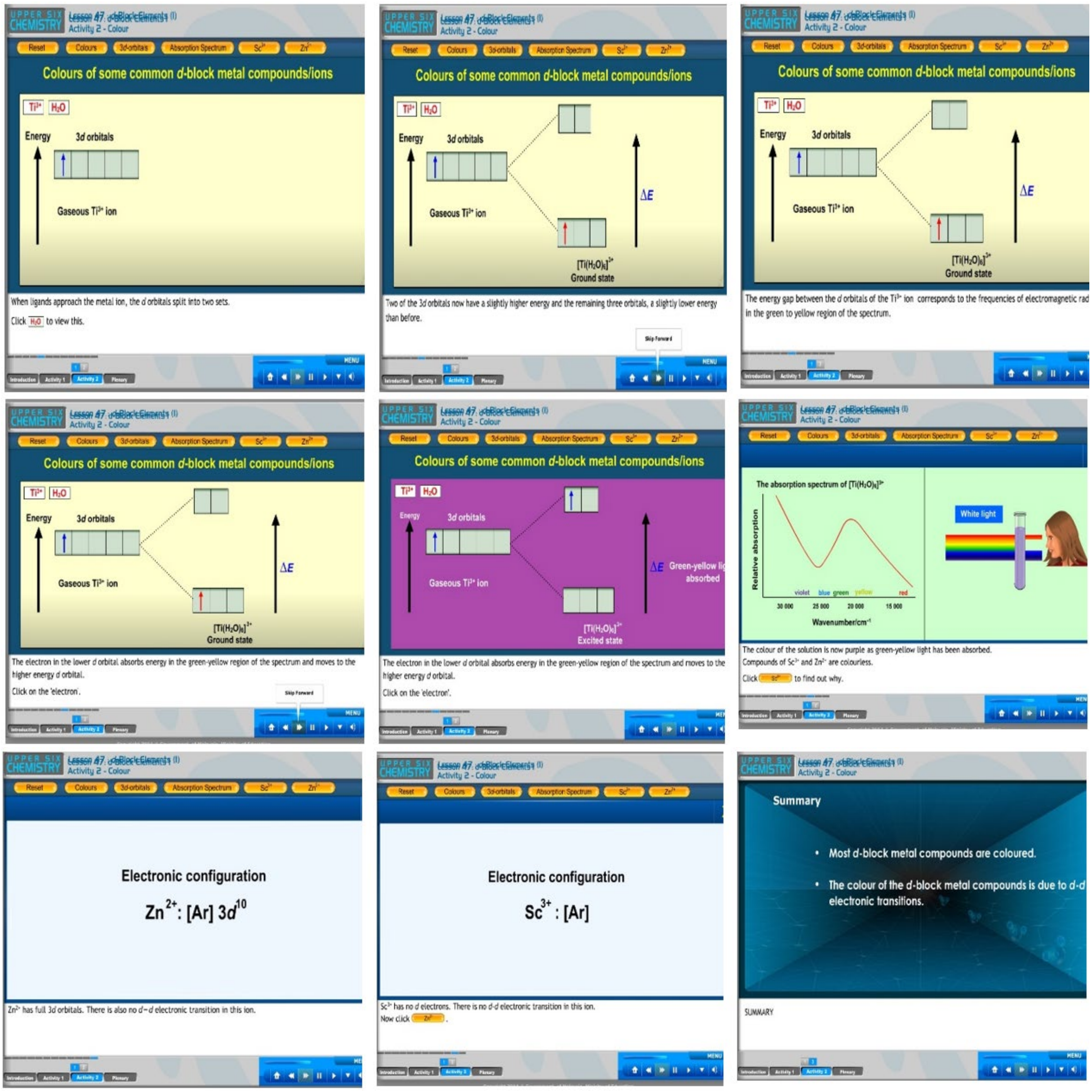

Figure 3. Colours of ions due to d electron transitions

To further enhance their knowledge about the formation of complex ions, the students carried out laboratory work. It helped students to have a clearer picture about the concepts and enhanced their understanding about the transition metals topic. Students were shown examples of complex ion solutions (e.g., Ni $(NH_3)_6{}^{2+}$ and $Cu(NH_3)_4{}^{2-}$) to emphasize the various colours of the solutions (see **Figure 4**).

Figure 4. Examples of coloured complex ion solutions

Table 2. Themes formed after the reviewing and refining process

No.	Initial theme	Theme
1.	The oxidation number of transition metals ions	Nature of transition metal ions to form complex ions
2.	Bonding in the complex ion	
3.	Comparison of ionization energy between copper and potassium	Comparison of ionization energy between alkali metals and transition metals
4.	Comparison of the trend in ionization energy between alkali metals and transition metals	
5.	Comparisons of color between transition metal ion and other ions	Colors in transition metal ions
6.	The explanation for some transition metal ions which are not colored	
7.	Reactivity trend of transition metals across the period	Chemical reactions of transition metals
8.	The strength of transition metals as reducing agents	

The teacher gave students exercises after each sub-topic to do as homework.

Data Collecting Method

An item was considered correct when only both parts of items were correctly answered (Treagust, 1988). No marks were awarded for any other combination of answers. Interviews were conducted with five students from the treatment group who were randomly selected by the first author, to ascertain the extent of their alternative conceptions before and after instruction. There was no time limit for the interview sessions.

Data Analysis

For the quantitative results, the posttest data of the comparison and treatment groups were analyzed using SPSS 21.0 (Statistical Package for the Social Sciences). For the qualitative results, five interview questions were used in interviews to ascertain the treatment group students' understandings about transition metals before and after instruction.

RESULTS

The results of quantitative and qualitative analysis of the data are reported below.

Comparison of the Total Scores in Transition Metals Posttest

The transition metals conceptual test was administered to the students from the comparison group and treatment group after the treatment had been carried out. Independent samples t-test analysis showed statistically significant differences in the test mean scores between the comparison and treatment groups (see **Table 2**). These results show that the students in the treatment group exhibited improved understanding about transition metals compared to the students from the comparison group.

Table 3. Means and standard deviations for the results of the posttest

Test	Treatment group (N = 47)		Comparison group (N = 32)		t	p
	Mean	SD	Mean	SD		
Post-test	8.47	0.69	3.91	0.96	23.103	< 0.001

Students' Alternative Conceptions about Transition Metals

The objective of this study was to reduce the alternative conceptions about transition metals among Form 6 students using ICT-based instruction. The two-tier questions in the conceptual test evaluated students' understanding in four areas which are formation of complex ions, variable oxidation states in transition metals, ionisation energy of transition metals, formation of colours in transition metal ions and uses of transition metals. The alternative conceptions that were identified are shown in **Table 3**.

As shown in **Table 3**, several alternative conceptions were identified about transition metals through the responses given by the students using the two-tier questions. There is a large difference in the percentage of the alternative conceptions between the comparison group and the treatment group students. The percentage of alternative conceptions held by the comparison group students ranged from 44% to 72%, while for the treatment group students the percentage of alternative conceptions ranged from 2% to 11%. This clearly shows that ICT-based instruction had resulted in a significantly lower incidence of alternative conceptions among the treatment group students compared to the traditional instruction with the comparison group. For qualitative data, the same five interview questions were used before and after instruction to determine the understanding about transition metals of the treatment group students. The questions that were used in the interviews involved the following areas:

(1) The formation of complex ions

(2) Variable oxidation states of the transition metals

(3) Ionisation energy of transition metals

(4) Formation of coloured compounds by transition metal ions

(5) Uses of transition metals

Formation of Complex Ions

Based on the quantitative analysis of the results of the conceptual test, 50% of the students from the comparison group and 2% of the students from the treatment group held the alternative conception *that a transition metal in zero oxidation state cannot attract ligands and hence cannot form complexes.* The alternative conception had decreased from the comparison group to the treatment group. Students understood better after demonstrating the formation of complex ions using ICT technology rather than traditional teaching. Students could observe that transition metals have empty orbitals that can be filled by the electrons from the ligands even without donating any of the electrons.

The interview responses below show students' understanding of transition metals having zero oxidation state before the instruction took place.

Student 1: *Transition metals have various oxidation states and it is one of their special properties.*

Student 2: *They should have more than one oxidation state like Cu has +1 and +2 oxidation states.*

Student 3: *Transition metals can donate more than one electron to form various oxidation states.*

Student 4: *All metals have oxidation states.*

Student 5: *Don't know………Maybe they can because if it is a solid then it will have zero oxidation state.*

Based on students' responses, it may be concluded that all the five students had some level of alternative conceptions about the formation of zero oxidation state by transition metals. Student 5 was unable to relate the oxidation number to a solid or ion. Students 1, 2, 3 and 4 basically knew that transition metals form various oxidation states.

After ICT-based instruction, the same students were interviewed using the same.

Student 1: *Transition metals such as* $Ni(CO)_4$ *can have zero oxidation state. Transition metals have various oxidation states including zero oxidation state.*

Student 2: *Transition metals can form complex ions with zero oxidation state.*

Student 3: $Ni(CO)_4$ *has zero oxidation state in its compound.*

Student 4: *Transition metals can have zero oxidation state. For example, nickel in* $Ni(CO)_4$ *has zero oxidation state.*

Student 5: *Zero oxidation state can occur in transition metal compounds.*

Based on the students' answers, it may be concluded that all the students had already overcome the alternative conception on variable oxidation state in transition metals. All the students agreed that transition metals can form zero oxidation state. Students 1, 3, and 4 could include the example of $Ni(CO)_4$ as having a transition metal with zero oxidation state.

Variable Oxidation States

There were three alternative conceptions that were identified about the variable oxidation states of transition metals. As many as 59% of the students from the comparison group and 6% of the students from the treatment group believed that *the tendency for transition metals to involve all 3d electrons in bonding will not decrease once the d^5 configuration is exceeded*. When the d^5 configuration is exceeded, the electrons will start to pair up and there will be less tendency to involve electrons from the d orbitals to form bonds. The CD provided by the Ministry of education helped students to understand that when the d orbital is half-filled, the electrons will start to pair up and will reduce the formation of bonds.

Interview responses below show that before the ICT-based instruction, the involvement of the electrons in bonding in transition metals decreases once the d^5 configuration is exceeded.

Student 1: *Metals form ionic bonds. Hmmm…I don't think so because transition metals need to donate higher number of electrons to form ionic bonds.*

Student 2: *No, because transition metals have many empty orbitals. They can fill the empty orbitals such as 4s, 4p and 4d orbitals to form bonds.*

Student 3: *Hmmm….Transition metals with higher number of electrons cannot form bonds with the ligands because higher energy is needed to donate all the electrons.*

Student 4: *To donate more electrons, more energy is needed.*

Student 5: *Hmm…not sure…but I think transition metals are reactive because they are catalysts for most reactions. So they can form more bonds.*

It may be concluded that students are not very sure about bond formation after the d^5 configuration is exceeded. Students 1, 3 and 4 held the alternative conception that transition metals can only form ionic bonds. Student 2 held the alternative conception that the electrons can be expanded to 4s, 4p and 4d orbitals. Student 5 held the alternative conception that the transition metals are reactive.

The interview responses below show the students' understanding after ICT-based instruction was carried out.

Student 1: *Yes, it decreases because after the d^5 configuration has exceeded, the electrons will start to pair up and will not be available for bonding.*

Student 2: *It decreases because electrons will pair up after the d^5 configuration has been exceeded.*

Student 3: *Once the electrons pair up in each orbital, they will not be available for bonding.*

Student 4: *Involvement of the electrons for bonding in transition metals decreases because the number of unpaired electrons also decreases after the d^5 configuration is exceeded.*

Student 5: *Involvement of the electrons for bonding in transition metals decreases because electrons will pair up in each d orbital.*

Based on the students' answers it may be concluded that most of them had overcome the alternative conception. All of them knew that the tendency for transition metals to involve all 3d electrons in bonding will decrease once the d^5 configuration is exceeded because the electrons will pair up and will not be available for bonding.

Reactivity of Transition Metals

Another alternative conception held by students was that *the reactivity of transition metals increases from left to right across a period.* Reactivity of the transition metals decreases as there is a decrease to involve the electrons from the d orbitals to form bonds. Fifty percent of students from the comparison group held this alternative conception but only 9% of the students held this alternative conception in the treatment group, showing a decrease in the alternative conception held by the students in the treatment group compared to comparison group.

Transition Metals as Reducing Agents

A total of 63% of students from the comparison group believed that *transition metals are not good reducing agents.* However, this alternative conception had decreased to 4% of the students in the treatment group.

Ionisation Energy of Transition Metals

Students held the alternative conception about the ionisation energy of transition metals compared to the ionisation energy of s-block metals with 53% of the students from the comparison group holding the alternative conception that *the ionisation energy of transition metals down a group in the Periodic Table is similar to that of the alkali metals*. Only 4% from the treatment group believed similarly.

Also, 63% of students from the comparison group held the alternative conception that *Cu and K are expected to have the same ionisation energy for the loss of their 4s electrons* while only 9% of the treatment group students held this alternative conception. This alternative conception arose due to the poor understanding of the students about the trend in the ionisation energy of transition metals and s-block metals.

Result from the interview before instruction about whether copper and potassium have the same ionization energy:

Student 1: *hmm…I don't think so.*

Student 2: *They have the same ionisation energy.*

Student 3: *No….*

Student 4: *Hmmm…not the same I think.*

Student 5: *Can be the same…*

Responses of students that copper and potassium have same or different ionization energy are given below:

Student 1: *Because they are different elements with different electronic configurations, different amount of energy is needed to remove the electron.*

Student 2: *Because potassium and copper are in the same row in the Periodic Table. So they can have the same ionisation energy.*

Student 3: *Potassium is a group 1 metal and copper is a transition metal. So the ionisation energy must be different.*

Student 4: *Both are metals and can donate electrons easily. Therefore, their ionisation energy can be the same with the same oxidation number.*

Student 5: *Because both are metals and both can form* Cu^{+} *and* K^{+}*, both involve the removal of one electron.*

Based on the students' answers it may be concluded that most of the students (Student 2, 3, and 4) held alternative conceptions. Student 2 held the alternative conception that if metals are in the same row they have the same ionisation energy. Student 4 held the alternative conception that metals can donate electrons, therefore, they have the same ionisation energy. Student 5 held the alternative conception that if the oxidation number is the same, the metals are most likely to have the same ionisation energy. Students understood the term ionisation energy but they could no explain the trend of the ionisation energy. Students 1 and 3 believed that they were different metals and so they have different ionisation energies.

Responses from interviews after ICT-based instruction are as follows:

Student 1: *The ionisation energy of potassium is lower than that of copper. The first electron is being removed from the 4s orbital of copper and potassium. The size of the copper atom is smaller than that of the potassium atom. The nuclear charge of copper is also higher than that of potassium. In conclusion, the 4s electron in copper is more strongly held by the nucleus and so is more difficult to be removed.*

Student 2: *The ionisation energy of potassium is lower than that of copper because the atomic size of potassium is larger than that of copper. Therefore, the valence electrons in copper are more strongly held by the nucleus compared to potassium. More energy is need to remove the valence electron in copper compared to potassium.*

Student 3: *: It doesn't have the same ionisation energy. Potassium has a larger size than copper. Potassium has lower nuclear charge than copper. Thus, the ionisation energy for potassium is lower than copper.*

Student 4: *Potassium with larger size and lower nuclear charge would be expected to have lower ionisation energy than copper. Less energy is needed to remove an electron from potassium compared to copper.*

Student 5: *Copper and potassium don't have the same ionisation energy. This is because the energy needed to remove an electron from copper is higher than potassium. Copper has a smaller atomic size than potassium and larger proton number than potassium. More energy is needed to remove an electron from copper compared to potassium because the outermost electron in copper has stronger attraction towards the nucleus compared to potassium. So the ionisation energy in copper is higher than potassium*

It may be concluded that all the students had overcome the alternative conceptions only up to a certain extent after instruction. Only students 1 and 5 had a complete idea by comparing the ionisation energy between a transition metal and an alkaline earth metal. These students compared the size, nuclear charge, strength of nucleus holding the outermost electron and energy needed to remove the electron. Students 2, 3 and 4 gave brief responses comparing the size and nuclear charge only to describe the ionisation energy.

Formation of Colours in Transition Metal Ions

ICT-based instruction plays an important role in reducing the alternative conception about the formation of coloured ions of transition metals. The teacher showed the experimental group students the video of d orbitals that can be split into two energy levels and the promotion of the electrons from a lower energy level to a higher energy level. Students also managed to see all the shapes of the d orbitals in the video presented as it is difficult to draw on the board. Thus, students were able to visualize the shapes of the d orbitals and the reason transition metal ions are coloured. As a result, only 11% of the students from the treatment group held the alternative conception that *Cu^+ has unpaired d electrons which can take part in d-d transitions.* In the traditional teaching method, students could not visualize d orbitals well because the teacher showed the shapes of the d orbitals from the textbook. This resulted in 53% of students from the comparison group holding an alternative conception. Another prevalent alternative conception among students was that *not only transition metal ions are coloured.* Students perceived that there are other coloured ions as well. Students from the treatment group understood that any metals that undergo d-d transition are coloured whereas students from the traditional teaching failed to understand this.

The teacher questioned the students to find out why the Cu^+ ion is colourless.

Student 1: *Hmm… don't know.*

Student 2: *No idea.*

Student 3: *Not sure.*

Student 4: *All transition metals ions are coloured except for Cu^+…hmm.*

Student 5: *Don't know.*

Based on the students' answers, it may be concluded that all the students have alternative conceptions about the formation of the colour of the Cu^+ ion. None of them gave a correct response. Students were not sure of the reason why the Cu^+ ion is colourless because they assumed that all transition metal ions are coloured.

After ICT-based instruction, students showed positive results in their interview responses.

Student 1: *Because the copper ion has fully filled d orbitals. Thus, Cu^+ cannot undergo d-d transition.*

Student 2: *Cu^+ cannot undergo d-d transition. There will be no electrons promoted to a higher energy level to absorb a certain amount of wavelength to reflect the colours. That's why it is colourless.*

Student 3: *Cu^+ has fully filled d orbitals and there will be no electrons promoted to a higher energy level. There is no d-d transition and hence Cu^+ is not coloured.*

Student 4: *d orbitals are fully filled. Any metals that undergo d-d transition are coloured. All transition metals ions are coloured except for Cu^+. So, Cu^+ cannot undergo d-d transition because electrons are not being promoted to higher energy level.*

Student 5: *d orbitals of Cu^+ are fully filled. None of the electrons from the Cu^+ get promoted to a higher energy level. So, it is colourless.*

It may be concluded that all the students held alternative conceptions before instruction on the formation of the colour. None of them gave a correct response. Students were not sure of the reason why the Cu^+ ion is colourless because they have a conception that all transition metal ions are coloured.

Uses of Transition Metals

One alternative conception that students held was that *transition metals are not good catalysts.* Forty-four percent of the students from the comparison group misunderstood that transition metals are not good catalysts whereas only 9% of the students from the treatment group managed to comprehend the correct concept. After the treatment it was found that students from the treatment group understood that transition metals have empty d orbitals that could be occupied by the electrons to form temporary bonds. Thus, they can be good catalysts compared to the other metals. The teacher in the ICT-based instruction helped to improve students' understanding by showing all empty orbitals that existed in transition metals with examples.

Table 4. The percentage of students' having alternative conception as determined in the conceptual test after instruction

Items nos.	Alternative conceptions	Comparison group (%)	Treatment group (%)
1	A transition metal in zero oxidation state cannot attract ligands and hence cannot form complexes.	50	2
2	The tendency for transition metals to involve all 3d electrons in bonding will not decrease once the d^5 configuration is exceeded.	59	6
3	Cu^+ has unpaired d electrons that can take part in d-d transitions.	53	11
4	Cu and K are expected to have the same ionisation energy for the loss of their 4s electrons.	63	9
5	The reactivity of transition metals increases from left to right across a period.	50	9
6	The ionisation energy of transition metals down a group in the Periodic Table is similar to that of the alkali metals.	53	4
7	Not only transition metal ions are coloured.	72	6
8	Transition metals are not good reducing agents.	63	4
9	Transition metals are not good catalysts.	44	9

Based on the post-instruction interviews, students' understanding about transition metals improved after the intervention. Students from the comparison group understood the transition metals concepts better than before. Students displayed alternative conceptions in the formation of complex ions, variable oxidation states of the transition metals, ionisation energy of transition metals, formation of the colours in transition metal ions and the uses of transition metals before instruction. However, after the intervention, the incidence of alternative conceptions had been reduced.

CONCLUSION

This study used a two-tier questionnaire about transition metals to ascertain the alternative conceptions that existed among form six students. The treatment group used ICT-based instruction to teach the students about transition metals whereas traditional teaching was used with the comparison group. In response to the research question of the study (What is the effect of ICT-based instruction on reducing the incidence of alternative conceptions about the chemistry of transition metals among Form 6 students?), this study showed that ICT-based education improved students understanding compared to the traditional teaching method. In addition, students were interviewed before and after the treatment to see if ICT-based instruction had improved their understanding about transition metals. It was found ICT-based instruction helped the students to reduce the incidence of alternative conceptions and improved their understanding about transition metals. Tsai and Chou (2002) in their research found that the achievement of the students increased with the use of computers in science education. Similarly, Kim *et.al*. (2013) described ICT as having an important role in improving teaching and learning. This study will contribute to the improvement of chemistry teaching especially about transition metals. Teachers' awareness of students' understanding before a lesson is very important to help reduce alternative conceptions, indirectly contributing to the improvement of teaching and achievement of better understanding in a particular topic.

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