# Supplementary Material 2: MGA 2-4 Analysis

## MGA 2 Analysis in English

Fleet chess game: Mathematical expressions of graph coordinate relations (Junior high School Geometry) (SDiME, 2022; <https://www.youtube.com/watch?v=9D9jHq6TWsw&list=PLylUUAvq6ZNV_J90yUuYC1XFXjpdtQKB-&index=5>)

### Phase A. Enter (video time from 0:23 - 1:30 minutes): 1 affective and 3 cognitive key moments

The teacher (Ms. Zhao) starts the lesson by explaining the rules of the game (Phase B: entertain) (AC3). After that, the teacher shows some examples to students and asks them some questions.

***T: Is this a way to arrange the aircraft carrier?***

***Ss: Yes.***

***T: Is this aircraft carrier arranged in a straight or horizontal way?***

***Ss: In a straightway.***

The teacher offered a wrong situation by intention, then she invited students to point out the mistakes. Ms. Zhao also asked students further questions for the complete explanation of the mistake (AC2). By doing so, Students could review the game rules again at the same time (AC3).

***T: Is this aircraft carrier arranged in a horizontal way? Are there any problems?***

***Ss: Yes.***

***T: What is the problem?***

***Ss: It lacks a grid.***

 After the situation judgment, Ms. Zhao asked some basic questions about the coordinate system, which also reviewed students’ previous knowledge about the lesson (AC1).

***T: How many dots does it have?***

***Ss: Five.***

***T: Only five. However, how many dots do we need to make an aircraft carrier?***

***Ss: Seven.***

***T: Seven, good job. As a result, you cannot miss any dot, is that clear?***

The teacher gave positive feedback on the right answer. This strengthened students’ self-confidence and aroused students' interest. (AA3). In phase A, Ms. Zhao replaced the professional terms of the coordinate system and introduced the game to students. Combining prior knowledge and the new topic of the lesson reduced students’ tension. Interactions between the teacher and students also enhance class participation.

### Phase B. Entertain (1:31 - 1:51): 2 affective key moments

Students were divided into pairs to play the game (1:31). In this simple feet chess game, every student had only one aircraft carrier, which was constructed by seven points vertically or horizontally on the coordinate system. After students decided the turn, they shouted out the coordinates to attack the other person. The person who got it right could continue attacking until destroying the other’s aircraft carrier totally.

 According to the introduction in Phase A, this was a competitive game, since the game result went to victory or defeat (BA1). Judging from students’ facial expressions, they stayed focused on this game and enjoyed the game time (BA2).

In Phase B, students got the chance to practice how to describe the position. During the process, students were considered to learn some mathematical knowledge from their experience of playing the game, which was a less stressful learning environment.

### Phase C. Enlighten (1:52 - 3:00): 1 affective and 2 cognitive key moments

In Phase C (enlighten), Ms. Zhao divided students into pairs and asked them to discuss winning strategies because there was a second-round later (CC1). In the beginning, Ms. Zhao gave some brief instructions to guide students. Through the discussion with others, students could indirectly learn some knowledge of the course (CC2). Ms. Zhao also made her rounds during group discussions. It can be inferred that the teacher offered timely help for students’ confusion in the process (CA3).

***T: You need to start discussing the winning strategy because it’s really quick for you to tell who wins just now. Remember, you all have your techniques. I’m saying that two people will be in one group later. You need to form an alliance and become a fleet. Then you should aim at other people’s fleet. I’m counting down for 1 minute.***

When Phase C ended, Ms. Zhao only reminded students to keep in mind the winning strategy and then started the round two challenging game immediately.

### Phase B. Entertain (3:01 - 3:54): 2 affective key moments

The rule of the second round was the same as the first one. Students who destroy their opponents’ aircraft carriers completely would be the winner. Only the number of the participants was different from the previous game. This time a group consisted of two students (3:01). They worked together to beat the other team. As a result, students showed a strong desire to win (BA1). Most of the students were more concentrated and excited in the second game (BA2).

Although there was not any teaching (interventions) in the video, students still learn how to make inferences based on the information they had. In the second Phase B, students experience a competitive and cooperative game, which would reduce the stress of mathematical learning and help students to learn in an unintentional way.

### Phase C. Enlighten (3:55 - 4:35): 3 affective and 2 cognitive key moments

Ms. Zhao invited students to share their secret of winning (CC1). Students explained their observations from the activities and their opinion toward their game results. Because there was not any limitation or standard answer for the question, students were allowed to say anything in class. It can be inferred that the learning interests of students were boosted based on their energetic facial expressions when they were answering the questions (CA1).

***T: Is there anyone willing to share their winning strategies? Please raise your hand and tell me.***

***S: We guess. Yes, we just guess.***

***T: So you think it was pure luck. Okay.***

Ms. Zhao repeated the utterance right away to clear and double-check the answer (CC3). Then Ms. Zhao invited another student (Andy) to answer the same question. (4:11)

***S: If you get two points right, it will form a line. And you can continue guessing on the same line.***

 Then Ms. Zhao asked a further question to the whole class to repeat the answer in a more organized way (CC3). At the same time, Ms. Zhao pointed out the key point and links to the topic of the course.

***T: How many points do we need to decide the direction of the aircraft carrier?***

***Ss: Two.***

***T: Two points, right? We can decide the direction with two points. OK.***

 Then Ms. Zhao invited another student (Eric) to share his strategies (CA2). Furthermore, Ms. Zhao asked more questions to clarify Eric’s answer.

***S: Start from the origin.***

***T: What do you mean by origin?***

***S: (0,0)***

***T: Okay, anywhere else?***

***S: Also start from the left or right corner.***

In Phase C, Ms. Zhao had more interaction with students by asking for winning strategies. During the process, the reconstruction of the answers from students can be treated as positive scaffolding feedback (CA4). Ms. Zhao leads students to think about the experience of Phase B through some key questions. By doing so, students connected their observation to some preliminary mathematical knowledge step by step. Moreover, student participation has increased since everyone had a chance to share their opinions and join the class activities. Also, most of them had a lot of fun in this session, which would reduce the tension of learning.

### Phase D. Enrich (4:36 - 7:45): 1 affective and 3 cognitive key moments

Ms. Zhao started the group discussion by inviting students to come up on stage and write down the features of the images of the coordinates. Then Ms. Zhao asked some questions to let students be aware of the features that were in common (DC4). After that, Ms. Zhao invited students to give a math-related name for the fleet coordinates (DC2).

***T: If you want to name it, please take a look at the aircraft carriers that you see right now. Is there any special about its coordinates? We can start from the special part of it, right? What is special about it?***

***S: They all have -2.***

***T: Which coordinate is -2?***

***S: y-coordinate.***

Then Ms. Zhao pointed out the unique part of the image and connected the observation result to mathematical knowledge (DC3). She used many tag questions to check whether students understand or not.

***T: y-coordinate is -2. So what you are seeing here is. The special thing about the horizontal line is it has the same y-coordinate, right?* (pointing at the coordinates that list on the blackboard) *As a result, since its y-coordinate is the same, it’s called -2. So this student actually called it a horizontal carrier. But I said. A horizontal carrier is not very mathematical, right? So we want a name that is more mathematical. He called it y=-2. Do you think you can agree with his naming? You think that’s not okay. Do you have a better one?***

***S: horizontal carrier!***

***T: What do you think? (Ask another boy in class.)***

***S: I think the horizontal carrier is good!***

When students’ answers were as precise as Ms. Zhao expected. Rather than pointing out the mistake directly, she asked further questions to lead students to understand the math concept (DC4).

***T: The horizontal carrier is good, but is this naming precise enough? If I go up, or I go down, we can also call it a horizontal carrier, right? So do we have a more precise way to name it? To represent this line.***

***S: Using numbers.***

***T: So if we have one carrier like the one on the screen, what mathematical name will you name it? To make it more precise.***

***S: y=-2***

***T: y=-2 can represent this aircraft, right?***

After the horizontal one, Ms. Zhao also named the vertical line in the same way. Then she used DesmosAPP to operate the graphic change to explore more features of the coordinates with students. Before Ms. Zhao extended the idea to a broader mathematical concept, she reviewed the concept they just learned from the discussion (DC3).

In Phase D, Ms. Zhao often asked students whether they agree with the statement or not. She allowed them to express their true opinion and never criticized the wrong answer. In this way, students were treated respectfully and encouraged to demonstrate in class. Without the concern of making mistakes, students were more confident and more willing to join the class discussion (DA4).

### Phase A. Enter (7:46 - 8:35): 1 affective and 3 cognitive key moments

Before starting the oblique fleet chess game, Ms. Zhao explained the rules, especially focusing on the difference between the previous and the later one (AC3). Then Ms. Zhao offered some examples for students to figure out whether these coordinates were correct or not.

***T: Please take a look at this example. Is this a correct example?***

***Ss: Yes.***

***T: Is this correct?***

***Ss: Yes.***

***T: That’s correct. What about this one?***

If students answer correctly, Ms. Zhao would give positive feedback (AA3). Then Ms. Zhao showed another example that is wrong and invited students to explain the mistake of image (AC2).

***S: It’s not correct.***

***T: Which part is not correct?***

***S: It didn’t pass the origin.***

***T: It didn’t pass the origin. And…(turn to ask another student)***

 ***S: It is not in a line.***

***T: It also skips a unit. And then (turn to ask another student)***

***S: It can’t be connected to form a line.***

***T: Okay, it looks like there’s a missing hole. Okay, so this is not a correct example.***

If the answers were correct, Ms. Zhao would repeat the answers. On the other hand, Ms. Zhao would ask more questions or invite other students' opinions if the answers were incomplete (AA4). This helped to build up students' confidence in the course and arouse their interest in the game.

### Phase B. Entertain (8:36 - 10:11, 10:12 - 10:44): 2 affective and 1 cognitive key moments

Students were asked to find the opponents that they hadn’t fought before. This time they also had to work in pairs, which meant two people were going to fight against another group of two. In order to win, they must shout out the coordinates of the other team’s aircraft carriers. There were two rounds of oblique fleet chess games. The first one was the practice only for oblique coordinates. And the second game combined all kinds of coordinates. Ms. Zhao had a brief explanation of the rule between the two rounds. Although there was no formal teaching in Phase B, Ms. Zhao still circled around the whole classroom to check the situation in each group (BC1).

Two students formed a group, and they discussed the strategies to beat the opponents in the process (BA1). As a result, students had more interaction with their classmates and learned mathematical knowledge by discussing the game. All the students looked enjoyable and devoted their minds to the game (BA2).

### Phase C. Enlighten (10:45 - 12:39): 3 affective and 2 cognitive key moments

In Phase C, Ms. Zhao led the discussion about the oblique fleet chess game. Starting by figuring out the winning strategies, students analyzed the coordinates by answering the teacher's questions (CC1).

***T: How many directions are there to place the aircraft carrier? If I only allow you to place obliquely.***

***Ss: Four.***

Ms. Zhao asked students a general question to start the discussion. Then, she elaborated on more details to students with the coordinates on the slides.

***T: Some of you place the line like this in the first place, right? Okay, no problem. And what about the other? The other direction, right? So in the beginning, you only place these two directions. And then you were killed instantly, right?***

Students nodded their heads, which means that they agreed to the experience Ms. Zhao had mentioned.

***T: You are right because you only need to guess left or right. But are there only two directions to place the line? (Students shake their heads) No, so can anyone tell me. Come up and tell me what other directions are there?***

Ms. Zhao gave several questions to motivate the students to think about the features of oblique lines on the coordinate. After students had a basic concept of the features, Ms. Zhao invited a student (Andy) to demonstrate on the blackboard (CA2).

***T: What can you do?***

***S: You can place it obliquely.***

***T: Obliquely, okay.***

***S: And you can place it obliquely at the other side.***

***T: Place it obliquely at the other side.***

***S: It can be placed obliquely everywhere.***

***T: So how many kinds are there?***

***S: Infinite.***

***T: Infinite kinds. That is a good idea. However, on our coordinate plane worksheet, is it possible? Our coordinate plane states we need to have 7 points. The middle needs to cross the origin. Does anyone else have other answers? How many can you really place? (Ms. Zhao stood close to a girl.)***

After the previous question session, students were able to notice the change of the coordinates now. Therefore, they told the teacher about what they know from their observation. However, there was an incomplete answer. Ms. Zhao still praised Andy’s answer, while she came up with more instructions (CA4). Later, Ms. Zhao got close to another student (Amy) and invited her to share (CA1).

***T: How many types do you think you can draw?***

***S: Six types, I guess.***

***T: Ok, six types.***

Then Ms. Zhao showed some examples and asked students to judge based on the rules.

***T: Do you think there are more?***

***S: I think there are more.***

***T: You think there are more. Okay, please tell me. What other directions do you think it’s okay? (Teacher walked to the student.)***

***S: It can pass the origin. And place it horizontally. Two units horizontally.***

***T: So compared with the one on the blackboard, is it more vertical or more oblique?***

***S: More oblique.***

 In Phase C, in order to lead mathematical thinking, Ms. Zhao kept asking students questions and inviting them to demonstrate on stage (CA2). For example, when students learned the feature of the oblique aircraft carrier by figuring out the winning strategies (CC1). By doing so, Students gradually understand the concept of the coordinates. Furthermore, Ms. Zhao always repeated answers and reconstructed the sentences to point out the key points (CA4). Then, Ms. Zhao would give positive responses to their answers (CC3). Students’ confidence was strengthened, and they learned in a less stressful way.

### Phase D. Enrich (12:40 - 15:07): 1 affective and 3 cognitive key moments

 At the beginning of Phase D, Ms. Zhao invited students to name the oblique aircraft carrier based on its features (DC2). As a result, a student came up and wrote down the coordinates of the line. After that, Ms. Zhao asked another student (Ben) to tell the features of the coordinates.

***T: Ok, please tell me. What’s special about its coordinates?***

***S: The numbers are the same. The numbers of XY are the same.***

***T: The numbers are the same. So if I need you to help me, giving a mathematical name, what kind of name will you give?***

***S: Oblique carrier.***

***T: Help me write it down.***

First, Ms. Zhao let Ben observe the coordinates to find the features in common. This question was an important guide for the teacher to give the student some hints to name the line. However, the student’s answer was not as mathematical as Ms. Zhao had expected, so she mentioned the previous question to remind the student of the feature (DC4).

 ***T: But you just said what is special about its coordinates?***

***S: Numbers of XY are the same.***

***T: So if I want to give it a more mathematical name. Actually, how can I name it? (turn to face the whole class)***

***Ss: x=y***

***T: Someone said x=y or y=x, and both are okay.***

As soon as Ms. Zhao repeated the question, not only Ben but also the whole class understood how to give a mathematical name. Ms. Zhao used questions to link the observation from the game to professional mathematical knowledge (DC3). Without introducing the concept directly, Ms. Zhao helped students to clarify and transfer their creation into a mathematical expression.

After the students said their answer, Ms. Zhao used visual aids to show the coordinates image. When Ms. Zhao operated the APP, she was checking the answers and added some details of the coordinates (DA4). Ms. Zhao went back to discuss the vertical line. She led the discussion with an easy definition of a vertical line, and then she asked another student to answer in a mathematical way. At the end of the class, Ms. Zhao used a question to draw the conclusion of today’s lesson and also review the most significant key point of the course.

***T: If I want to give this oblique straight line a mathematical name, what should I do? How can I name it?***

***S: Depending on the relationship between x and y.***

***T: That’s great.***

This class was conducted by cycling around four phases. In Phase A, Ms. Zhao aroused students’ interest in the game. Then students had cheerful memories in the next phase. During the game, students found some interesting facts which would be discussed in Phase C and Phase D. In the last two parts, Ms. Zhao never told the correct answer to students directly. Instead, she asked lots of questions to guide students to learn the mathematical concept. By doing so, students learn mathematics in a more comfortable way without having difficulty understanding the professional terms.

## MGA3 Analysis in English

Comparison of the Area, or the Perimeter?: The relationship between plan shape, area and perimeter (Primary school space and shape) (SDiME, 2022; https://www.youtube.com/watch?v=OQye2Rkkl5Y&list=PLylUUAvq6ZNXcOo-1eRh0xzKYdkaNacZf&index=17)

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### Phase A. Enter (video time from 0:31 - 4:10 minutes): 4 affective and 3 cognitive key moments

The teacher (Ms. Huang) began the class with the story of land enclosement (AA2). Ms. Hunag invited students to share their opinion if they were the characters in the story. These questions help students to scaffold prior knowledge of today’s lesson (AC1).

***T: Now, I have a question for you. I have a rope in my hand and if I want to enclose a piece of land with this rope. Do you agree that no matter what I do with this enclosure, they are going to have a piece of land as big as all the others from the enclosure?***

Students raised their hands as a response to the question, and they were required to tell the teacher the reasons for their choices (AA4).

***T: Nice… why do you agree they would be of the same size?***

***S: Because all the strings are of the same length.***

***T: Raise your hand if you think they would be different. (Then the teacher asked another student)***

***S: Because they are in different shapes… and so of different sizes.***

Later, Ms. Huang explained the rules of the game in the next session (AC3). She showed students some examples and invited them to share their thoughts (1:37). Then, Ms. Huang concluded the key points in the answers. In this way, students can clarify the rule and the prior knowledge of the lesson again.

***T: For this and this (pointing)... who think they are the same? (students raise their hand) Who could tell me why you think they are the same?***

 ***S: Because it is just being rotated.***

***T: Whether it is rotated or flipped, they belong to the same figures. Let’s not count scores for this one, shall we?***

Before the game started, Ms. Huang made rules clear by showing the game flow and demonstrating (AA1). During the demonstration, Ms. Huang often asked students “Do you get it?” or “Do you understand?” to make sure students understood every rule, or she would invite students to come to the stage and count the perimeter and area (3:02). Ms. Huang asked the whole class to count together and use the correct unit. In the end, the teacher gave compliments to the right answers (AA3).

Sometimes Ms. Huang made an intentional mistake and asked students whether these images were correct or not (AC2). If students fail to recognize the mistakes, the teacher would ask further questions like “Really?” (AA4). It was a less obvious way to give hints to students, but it did help to strengthen the confidence of students.

### Phase B. Entertain (4:11 - 4:46): 2 affective and 1 cognitive key moments

Students played this game in groups. To start the game, students tossed the dice to decide the number of the pieces they got. Then students would use those pieces to make the figure based on two rules. The first rule was that the minimum number of the overlapped side was one. Secondly, students had to make a circle to connect the starting point and the endpoint. After that, students record the area and perimeter of the figure on the worksheet. As long as students finished all these things, they could get points and switch turn to the next person in their groups.

In Phase B, students had to join the game and complete the worksheet on their own. There wasn’t enough facial expression or utterance from students, so it was difficult to identify how strong the winning desire would be. However, since it is a scoring game, it was inferred that there should be some tension between students (BA1). Most of the time, students stayed focused on individual works in the assigned groups (BA2).

Ms. Huang checked students’ situations in each group and gave students timely help (BC1). What students needed were some instructions to finish their worksheets. With teachers' hints, students were able to conduct the tasks smoothly. Furthermore, the teacher's compliment also benefited students’ performance in the next phase.

### Phase C. Enlighten (4:47 - 6:38): 2 affective and 3 cognitive key moments

At the beginning of Phase C, Ms. Huang invited a student to point out the perimeter and area for everyone (CA2). The student came up to the stage, and the distance between the teacher and the student was shortened (CA1). When the student told his answer, Ms. Huang repeated and rearranged the answer in a more logical way (CC3).

***S: The area of this one is here (pointing)...the whole part of this.***

***T: The inside of this whole part. And the perimeter?***

***S: Just go from 1 here. And then 2 here. Then 3 by counting the 3rd square. Do it in a circle.***

***T: Do it in a circle...doesn’t it?***

***S: Right.***

***T: Do it in a circle...and you’ll find the perimeter.***

After the first part, Ms. Huang divided students into groups and asked them to discuss the special things they had discovered from playing the game (CC2). All the students were concentrated on the discussion. What’s more, some students found the pattern from their worksheets. When the time was up, Ms. Huang invited students to share what they discussed.

***S(Alice): The more numbers we get by tossing the dice, the more figures we then create.***

***S(Andy): We have rolled the dice, and then we get six. The area of them is all 6. But we have many times with the same perimeter. But we have few times when it’s different.***

Then, Ms. Huang went on to a further question to point out the key concept as the conclusion of the first part of the discussion. Followed by the second discussion topic, Ms. Huang told students to find out the winning tips and share them with the whole class(CC1).

***T: How did you win the game? Does anyone want to share tips about how to win?***

***S (Chris): When you put the pieces the same way other people do, you can move one square piece to another place instead.***

***T: Move one of the square pieces to another place, and you can find another kind of figure... right?***

***S: Right.***

In Phase C, Ms. Huang used the group discussion to draw students’ attention to the meaningful observation of the game. This helps to connect to the mathematical knowledge in Phase D.

### Phase D. Enrich (6:39 - 7:12): 1 affective and 2 cognitive key moments

Ms. Huang led the mathematical thinking by asking several questions (DC4). In this way, the teacher linked the discussion result and mathematical knowledge (DC3).

***T: If I build a figure with the same number of pieces, could you tell me what their area would be?***

***S: The same.***

***T: What is their area?***

***S: All the same.***

***T: Incredible. For the figure with the same area… we discover differences in their shapes. Some young children also told me that for the figures with the same area ...we also discover differences in their perimeter.***

When students answered correctly, the teacher would give positive responses like awesome and incredible (DA4). In the end, Ms. Huang clarified the relationship between the number of the pieces and the area. Ms. Huang also restated the relationship between perimeter and area as the first step to educate students of this lesson.

### Phase A. Enter (7:13 - 8:07 minutes): 1 affective and 1 cognitive key moments

Ms. Huang explained the rules of the second game and told students what they needed to discuss after the game (AC3). Then, the teacher demonstrated the game on her own and taught students how to do the record (AA1).

### Phase B. Entertain (8:08 - 9:10): 2 affective and 1 cognitive key moments

In this round, Ms. Huang designed teaching aids with the topic of the game and numbers. Ms. Huang invited a student to draw two straws to decide the theme of this round and the number of the pieces. After that, students collaborated with their group mates to make a suitable figure of the topic. The group that created the longest perimeter in ten pieces would be the winner. All the students stayed focused on this game (BA2). However, the competition between groups still existed. In order to win, each group had to use the same pieces to create the greatest perimeter (BA1).

Although there was no image of the teacher in this phase, the video record Ms. Huang’s voice, which sounds like some advice for the groups (BC1). Because of that, some students would be able to come up with better figures and find out more knowledge about the area and perimeter.

### Phase C. Enlighten (9:11 - 12:11): 3 affective and 1 cognitive key moments

Ms. Huang invited every group to share their result on the blackboard (CA2). Moreover, the teacher asked the students to come up to the stage and count in front of the class (CA1).

***T: We are going to invite the group with 30 to do the counts for us. Show us how they find the perimeter. Count out loud up to 30… shall we?***

***(The student, Eric, is counting, and the whole class is counting out loud with him.)***

***T: Could you understand what he is doing through the count?***

***S (Ben): He counts the upper and lower side of the pieces by 2’s. Then… add another 2 for the left and right sides of the figure.***

In Phase C, Ms. Huang led the discussion by checking the area and the perimeter of these figures with students. When other students had other ideas about the perimeter, the teacher would give positive scaffolding feedback and ask them to explain it by themselves (CA4). Then Ms. Huang repeated what the student had said and clarified the concept again (CC3).

### Phase B. Entertain (12:12 - 12:44): 1 affective and 1 cognitive key moments

After the first round of the discussion, Ms. Huang gave another task to each group. This time, they could use eleven square pieces to make a figure which had the shortest perimeter. Students were expected to discuss with their group mates (BA2). The teacher visited some groups and asked them some questions about their figure (BC1). This helped students to understand the tips to win.

### Phase C. Enlighten (12:45 - 15:25): 2 affective and 2 cognitive key moments

Like the previous discussion, each group put the best figure on the blackboard (CA2). Ms. Huang asked students’ discoveries about the figures to start this session. Based on students’ answers, Ms. Huang went on to ask further questions about the winning tips of the game and certain examples in the game (CC1). Students described their intuitive thinking in everyday language, and the teacher reconstructed the sentences (CC3).

Sometimes students’ answers were too short, Ms. Huang would ask them to explain more or demonstrate on the blackboard (CA2). When students came up to the stage, the distance between teacher and students was shortened, and students could be more comfortable sharing their thoughts (CA1). At the end of this phase, Ms. Huang unified all the answers and came to two conclusions of the relationship between area and perimeter.

### Phase D. Enrich (15:26 - 17:06): 1 affective and 3 cognitive key moments

By asking several questions about the relationship of area and perimeter, Ms. Huang asked students to share their opinion based on their worksheets and their experience of the game (DC4). When students answered correctly, Ms. Huang would praise them immediately (DA4).

Then Ms. Huang gave three concepts that retold the observation from students in a mathematical way (DC3). She prepared an example that had many kinds of figures to explain those concepts, and invited students to find examples that match the concept (DC1). The student’s performance could be the teacher's assessment of this lesson.

## MGA4 Analysis in English

Carpet factory: Factorization (junior high school algebra) (SDiME, 2022; https://www.youtube.com/watch?v=M8NjewNbYhg&list=PLylUUAvq6ZNU96CHPHltLf1EjjyFtsgHf&index=5)

### Phase A. Enter (video time from 0:16 - 4:00 minutes): 1 affective and 1 cognitive key moments

Mr. Lin introduced the story of the largest carpet dealer in the world (AA2). He used the example, Beitou Library, which was close to students' daily lives, to explain the amount of the dealer’s profit. Then, Mr. Lin explained the teaching aids of the activity which is related to the topic (AC3). With the teacher’s instruction, students could practice before the formal activity.

After the practice, Mr. Lin introduced the rules of the first activity and the worksheet in which students had to record the result of the game. Mr. Lin kept explaining different assigned tasks for everyone in each group.

###  Phase B. Entertain (4:01 - 7:12): 1 affective and 1 cognitive key moments

In each ground of Phase B, Mr. Lin would tell students the number of the pieces, and then they needed to compose a rectangle or square by using the teaching aids. Besides that, they needed to record three things on the worksheet, including the method of composing the carpet, the area of the carpet and the length and width of the rectangles.

After receiving the tasks, students were fully concentrated on the activity (BA2). Some students observed the pattern in the game, which was the mathematical knowledge of the topic. And Mr. Lin gave instant feedback to them. Mr. Lin and the students had many interactions, and they built close relationships. However, students didn’t show the desire to win because there weren’t any scoring systems in the game. During the game, Mr. Lin circled the classroom and helped students to compose the figure (BC1). The teacher only gave some instructions and then students had to solve the problem on their own.

### Phase C. & D. Enlighten & Enrich (7:13 - 14:18): 3 affective and 1 cognitive key moments

Mr. Lin designed another worksheet with five questions to be answered based on the record of the game. Then, Mr. Lin checked each answer by inviting students to answer the question (CA2). He would stand close to students, and let them share their opinions (CA1). Mr. Lin used a sentence to conclude the answers or simply repeated the answer (CC3). Then he would ask further questions for more details.

***T: OK. It will be the same as column B, right? Why?***

***S (Edward): Both of them are rectangles. Adding up the two rectangles and (the sum) will be equal to column B, isn’t it?***

***T: OK...very well. Okay, Edward mentioned that both of them are rectangles, so their sum is equal to the original quantity of rectangles. So the answer to the first question is Column B.***

At the beginning of this session, Mr. Lin let different students answer the questions on the worksheet. He gave them positive responses to the answers and used scaffolding to build up knowledge by going through these questions (CA4). Students had to use the knowledge which they had learned from the previous question (9:30). If students had answered correctly, the teacher would give a verbal or non-verbal compliment (DA4). This strengthened the student’s confidence and encouraged them to answer. If students had given incomplete answers, Mr. Lin would ask further questions and guide them to the right ones (DC4).

***(After the student shared his answer to the second question)***

***T: So the answer to the second question is Column C because we have found that multiplying the length by width is exactly the total number of all the small squares. That is exactly the area composed of all small squares. Good... Let’s try the next question.***

Starting from the third question, the difficulty of questions was increased since there were more mathematical terms in the sentence (10:30). For those questions, Mr. Lin invited students to teach on the podium as teachers (DC1). The rest of the class also checked the answer on the blackboard. If there were mistakes, they would point them out immediately (14:08). This was an assessment of students' learning situations. After a few questions, Mr. Lin gave the conclusion which related to the mathematical concept (DC3).